Size and Value Anomalies under Regime Shifts*

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Abstract

This paper finds strong evidence of time-variations in the joint distribution of returns on a stock market portfolio and portfolios tracking size- and value effects. Mean returns, volatilities and correlations between these equity portfolios are found to be driven by underlying regimes that introduce short-run market timing opportunities for investors. The magnitude of the premia on the size and value portfolios and their hedging properties are found to vary significantly across regimes. Regimes are also found to have a large impact on the optimal asset allocation - especially under rebalancing - and on investors' welfare.

1. Introduction

Substantial empirical evidence links variations in the cross-section of stock returns to firm characteristics such as market capitalization (e.g., Banz (1981), Keim (1983) Reinganum (1981), Fama and French (1992)) and book-to-market values (e.g., Fama and French (1992, 1993), Davis, Fama, and French (2000)). Cross-sectional variations associated with these characteristics are non-trivial by conventional measures. Fama and French (1992) report that, over a recent sample, a portfolio comprising small firms paid a return of 0.74 percent per annum in excess of the return on a portfolio composed of large firms. Similarly, firms with a high book-to-market ratio outperformed firms with a low ratio by 1.35 percent per annum. In neither case could such differences be explained by variations in CAPM betas.

Far less is known about the extent to which the joint distribution of returns on these equity portfolios varies over time. This is clearly an important question. For a multiperiod investor the economic value of investments in size and value portfolios is determined not only by their mean returns but also by their volatilities and correlations with the market portfolio and by the extent to which these vary over time. To address this question, we propose in this paper a new model for the joint distribution of returns on the market portfolio and the size (SMB) and book-to-market (HML) portfolios proposed by Fama and French (1993). We find evidence of four economic regimes that capture important time-variations in mean returns, volatilities and return correlations. Two states capture outliers associated with periods of high volatility and thus accommodate skews and fat tails in stock returns. The other two states are

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associated with shifts in the distribution of size and value returns. Regimes continue to be important even if our model is extended to include the dividend yield as an additional state variable.

To quantify the economic significance of regimes in returns on US equity portfolios we consider their importance from the perspective of a small investor's optimal asset allocation. Optimal allocation to size and value portfolios has received some attention in the existing literature. Brennan and Xia (2001) solve the portfolio allocation problem of a long-term Bayesian investor assuming an asset menu similar to ours. They study optimal stock holdings obtained under different priors over the size and value effects. Their calculations suggest a substantial economic value of investments in the Fama French portfolios, on the order of 5% per annum, although the certainty equivalent value depends on the investor's coefficient of risk aversion, prior beliefs and the extent of pricing errors in the underlying asset pricing model. Pástor (2000) considers the single-period portfolio problem of a mean-variance investor. His calculations suggest that the HML portfolio should be in much greater demand than the SMB portfolio and that even investors with strong doubts about value effects should take substantial positions in the HML portfolio.

Here we focus instead on the presence of predictability linked to regimes underlying the joint distribution of returns on the market, SMB and HML portfolios. The economic value of investment strategies in the anomaly portfolios is of course related to the average size and value premium but further depends on how much these vary across economic states. As pointed out by Brennan and Xia (2001, p. 906), an important issue for a long-horizon investor is whether size and value effects, if genuine, can be expected to persist in the future. By allowing these effects to vary across regimes we can address this important question. Indeed we find strong evidence that optimal asset holdings vary significantly across regimes and across short and long investment horizons as investors anticipate a shift out of the current state.

We study several aspects of the portfolio allocation problem, such as the importance of the rebalancing frequency and the investment horizon. At long horizons we find that the size and value portfolios have moderate weights in a buy-and-hold investor's optimal allocation. This finding differs from previous estimates of a more substantial role for the SMB and HML portfolios in the optimal long-run asset allocation and is a reflection of the fat-tailed return distribution captured by the two high-volatility states. At short horizons, we find a more significant role for these portfolios linked to the market timing opportunities implied by the four-state model. By allowing for adjustments to portfolio weights following changes in the underlying state probabilities, rebalancing enhances the weights on the size and value portfolios in the optimal asset allocation.

We also study the hedging demand induced by regime switching and compare it to the hedging demand under predictability from the dividend yield or under learning about the drift of the asset price process. Consider the hedging demand for the market portfolio. Since shocks to the dividend yield are negatively correlated with shocks to asset prices, the market portfolio provides a hedge against shocks to future investment opportunities and the hedging demand for this portfolio is positive under predictability from the dividend yield. In contrast, when investors learn about the mean return − as assumed by Brennan and Xia (2001) − shocks to the investment opportunity set and shocks to returns are positively correlated so the hedging demand for the market portfolio will be negative. Under regime switching we see both positive and negative hedging demand depending on which state the market starts from. The hedging demand is negative when the investor starts from regimes favorable to the market portfolio−and mean-reversion to less favorable investment opportunities is anticipated−but it is positive when starting from bad states.

Consistent with recent findings by Barberis (2000) and Xia (2001), we find that parameter estimation uncertainty has a large effect on optimal asset holdings. Nevertheless, regime shifts continue to have a significant effect on the optimal asset allocation even after accounting for parameter uncertainty. Furthermore, welfare cost calculations confirm the economic significance of regimes in the distribution of returns on the market and Fama French portfolios even in the presence of uncertainty about parameter values. Our estimates of the costs from ignoring regimes are substantial irrespective of whether we include the dividend yield as an additional predictor variable.

Our paper is part of a growing literature that explores the asset allocation and utility cost implications of return predictability from the perspective of a multi-period, small investor who maximizes expected utility. In an exercise involving a single risky stock market portfolio Kandel and Stambaugh (1996) find that predictability can be small statistically yet still have a large effect on the optimal asset allocation. Barberis (2000) extends their result to long horizons. Campbell and Viceira (1999) derive closed-form expressions using log-linear approximations for a consumption and portfolio choice problem with continuous rebalancing and infinite horizon. Outside the framework of allocations to pure equity portfolios, Campbell and Viceira (2001, 2002) and Campbell Chan and Viceira (2003) have studied strategic asset allocation and documented large effects of predictability on asset holdings and welfare costs. Detemple, Garcia and Rindisbacher (2003) approach a wide class of portfolio choice problems in continuous time, including strategic asset allocation. Building on the evidence that both interest rates and the market price of risk(s) follow non-linear processes, they investigate the asset allocation implications of non-linear predictability. They show that findings obtained in linear models may be overturned in the presence of non-linearities. Finally, to our knowledge the only other paper to study the effect on asset allocation due to regime switching is Ang and Bekaert (2002), but their focus is on international asset allocation and the home country bias.

The outline of the paper is as follows. Section 2 presents our multivariate regime switching model for the joint distribution of returns on the market, size and book-to-market portfolios and its extensions to include additional predictor variables. Section 3 presents empirical results while Section 4 sets up the optimal asset allocation problem and Section 5 reports empirical asset allocation results. Section 6 provides utility cost calculations and also considers the impact of parameter estimation uncertainty. Section 7 concludes.

2. Models for Regimes in the Joint Return Process

A vast literature in finance has reported evidence of predictability in stock market returns, mostly in the context of linear, constant-coefficient models, c.f. Campbell and Shiller (1988), Fama and French (1988, 1989), Ferson and Harvey (1991), Goetzmann and Jorion (1993) and Lettau and Ludvigsson (2001). More recently, some papers have found evidence of regimes in the distribution of returns on individual stock portfolios or pairs of these (e.g., Ang and Bekaert (2002), Perez-Quiros and Timmermann (2000), Turner, Startz and Nelson (1989), Whitelaw (2001)). Following this literature we model the joint distribution of a vector of n stock returns, $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{nt})'$ as a multivariate regime switching process driven by a common discrete state variable, S_t, that takes integer values between 1 and k :

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} \mathbf{A}_{j,s_t} \mathbf{r}_{t-j} + \boldsymbol{\varepsilon}_t. \tag{1}$$

Here $\boldsymbol{\mu}_{s_t} = (\mu_{1s_t}, ..., \mu_{ns_t})'$ is a vector of mean returns in state s_t, $\mathbf{A}_{j,s_t}$ is an $n \times n$ matrix of autoregressive coefficients at lag j in state s_t and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{nt})' \sim N(0, \boldsymbol{\Sigma}_{s_t})$ is the vector of return innovations that are assumed to be joint normally distributed with zero mean and state-specific covariance matrix $\boldsymbol{\Sigma}_{s_t}$. Innovations to returns are thus drawn from a Gaussian mixture distribution that is known to provide a flexible approximation to a wide class of distributions, c.f. Timmermann (2000).[1]

Moves between states are assumed to be governed by the $k \times k$ transition probability matrix, $\mathbf{P}$, with generic element p_{ji} defined as

$$\Pr(s_t = i | s_{t-1} = j) = p_{ji}, \quad i, j = 1, .., k. \tag{2}$$

Each regime is hence the realization of a first-order Markov chain. Our estimates allow S_t to be unobserved and treat it as a latent variable.

The model (1) - (2) nests several popular models from the literature as special cases. In the case of a single state, $k = 1$, we obtain a linear vector autoregression (VAR) with predictable mean returns provided that there is at least one lag for which $\mathbf{A}_j \neq 0$. Absent significant autoregressive terms, the discrete-time equivalent of the Gaussian model adopted by Brennan and Xia (2001) is obtained. Allowing for regime shifts, the model is also consistent with observations of instability in US equity portfolio returns, c.f. Pastor (2000) and Davis et. al. (2000).

Even in the absence of autoregressive terms, (1) - (2) imply time-varying investment opportunities. For example, the conditional mean of asset returns is an average of the vector of mean returns, μ_{s_t}, weighted by the filtered state probabilities $(\Pr(s_t = 1|F_t), .., \Pr(s_t = k|F_t))'$, conditional on information available at time t, $\mathcal{F}_t$. Since these state probabilities vary over time, the expected return will also change. Similar comments apply to higher order moments of the return distribution.

Our model can be extended to incorporate an $l \times 1$ vector of predictor variables, $\mathbf{z}_{t-1}$, comprising variables such as the dividend yield or term and default premia that have been used in recent studies on predictability of stock returns (e.g. Aït-Sahalia and Brandt (2001) and Campbell, Chan and Viceira (2003)). Define the $(l + n) \times 1$ vector of state variables $\mathbf{y}_t = (\mathbf{r}_t' \ \mathbf{z}_t')'$. Then (1) is readily extended to

$$\mathbf{y}_t = \begin{pmatrix} \boldsymbol{\mu}_{s_t} \\ \boldsymbol{\mu}_{zs_t} \end{pmatrix} + \sum_{j=1}^{p} \mathbf{A}^*_{j,s_t} \mathbf{y}_{t-j} + \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_{zt} \end{pmatrix}, \tag{3}$$

where $\boldsymbol{\mu}_{zs_t} = (\mu_{z_1 s_t}, ..., \mu_{zl s_t})'$ is the intercept vector for $\mathbf{z}_t$ in state s_t, $\{\mathbf{A}^*_{j,s_t}\}_{j=1}^{p}$ are now $(n+l) \times (n+l)$ matrices of autoregressive coefficients in state s_t and $(\boldsymbol{\varepsilon}_t' \ \boldsymbol{\varepsilon}_{zt}')' \sim N(0, \boldsymbol{\Sigma}^*_{s_t})$, where $\boldsymbol{\Sigma}^*_{s_t}$ is an $(n+l \times n+l)$ covariance matrix. This model allows for predictability in returns through the lagged values of $\mathbf{z}_t$. It embeds a variety of single-state VAR models that have been considered in recent studies including

[1] Recent papers have emphasized the importance of adopting flexible modeling strategies (capable of capturing time-varying correlations, skewness and kurtosis) applied to the joint distribution of asset returns for optimal portfolio choices, see e.g. Manganelli (2004) and Patton (2004).

Barberis (2000), Campbell and Viceira (1999, 2001) and Kandel and Stambaugh (1996). This model is complicated by the joint presence of linear and non-linear predictability patterns, the latter arising due to time-variations in the filtered state probabilities.

3. Regimes in market, size and book-to-market returns

This section investigates the presence of regimes in the joint distribution of returns on the market, SMB and HML portfolios.

3.1. *The Data*

We study monthly, continuously compunded returns on US stock portfolios over the sample 1927:12 - 2001:12, a total of 889 observations. The basis for our analysis is the returns on six equity portfolios formed on the intersection of two size portfolios and three book-to-market portfolios. All portfolios are value-weighted with weights that are revised at the end of June every year and held constant for the following twelve months.[2] We also use data on the value-weighted stock index and the dividend yield.

To simplify the asset allocation problem, we follow Fama and French (1993) and consider two portfolios tracking size and book-to-market ratio effects. The first portfolio (SMB) is long in small firms and short in big firms, controlling for the book-to-market ratio:

$$r_t^{SMB} = \frac{1}{3}(\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) - \frac{1}{3}(\text{Big Value} + \text{Big Neutral} + \text{Big Growth}).$$

The second portfolio (HML) is long in firms with a high book-to-market ratio and short in firms with a low book-to-market ratio, controlling for size:

$$r_t^{HML} = \frac{1}{2}(\text{Small Value} + \text{Big Value}) - \frac{1}{2}(\text{Small Growth} + \text{Big Growth}).$$

Both SMB and HML are zero-investment portfolios. It is therefore appropriate to consider their simple returns as opposed to returns in excess of a T-bill rate. Conversely, we follow common practice and consider returns on the market portfolio in excess of the T-bill rate.

Table 1 reports summary statistics for the two spread portfolios and the market index. The mean excess return on the market portfolio is 8% per annum. The volatility of this portfolio is 19% per annum and it also has a thick-tailed, largely symmetric distribution. The HML portfolio earns a mean return of 5% per annum and, at 13% per annum, is less volatile than the market portfolio but with strongly skewed returns. The SMB portfolio earns a mean return of 3% per annum and has lower volatility and more right-skew than the HML portfolio. Correlations between returns on the three equity portfolios vary between 0.08 and 0.33.[3]

[2] The portfolios for July of year t to June of year $t+1$ include all NYSE, AMEX and NASDAQ stocks with market equity data available for December of year $t-1$ and June of year t, and book equity data for year $t-1$. The book-to-market ratio for June of year t is the book equity for the last fiscal year ending in $t-1$ divided by the market equity in December of year $t-1$. Further details on data construction are available from Ken French's web site at Dartmouth.

[3] These properties are similar to those reported by Davis et al. (2000) for a comparable sample 1929-1997. The correlation between HML and SMB is slightly higher with our data.

3.2. *Regimes in the individual portfolio returns*

Before undertaking the analysis of the joint distribution of returns on the three stock portfolios, it is informative to consider the presence of regimes in returns on the individual portfolios, r_t^{MKT}, r_t^{SMB} and r_t^{HML}. For each portfolio we first tested the null of a single state against the alternative of multiple states and found that the single state model was soundly rejected at the 1% significance level.[4] A two-state model was found to be appropriate for the market portfolio while three-state models were selected for the HML and SMB portfolios.

Figure 1 plots the smoothed state probabilities obtained from the models fitted to the individual portfolio returns. A high degree of coherence between the state probabilities indicates the presence of common factors and hence suggests that we only need a small number of states to model the joint return distribution. Indeed, the figure suggests similarities between the state variable underlying the SMB and HML returns.[5] A very different picture emerges for the market portfolio, however, whose state probabilities are nearly uncorrelated with those identified for the SMB and HML portfolios. Economic factors responsible for shifts in returns on the overall market thus appear to differ from those driving shifts in returns on the SMB and HML portfolios.

3.3. *Regimes in the joint return process*

No previous work seems to have attempted to identify regimes in the joint process of returns on the market, size and value portfolios $(r_t^{MKT}\ r_t^{SMB}\ r_t^{HML})'$. Economic theory offers little guidance on how to select the number of regimes and lags. To address these issues and to make sure that there is robust evidence of regimes in the first place we conduct a thorough specification analysis.

More specifically, we consider a range of values for the number of regimes consistent with our analysis of returns on the individual portfolios ($k = 1$, 2, 3, 4, and 6). This covers very parsimonious as well as heavily parameterized models. We use a consistent model selection criterion, namely the Schwarz information criterion (SIC), to select the basic design for the regime switching model such as the number of states and lags. We then use likelihood ratio tests to impose mean return and covariance restrictions and see whether a more parsimonious model is consistent with the data.

Table 2 reports the outcome of this analysis. For each model we show the value of the SIC in addition to the outcome of a test for a single state against the presence of multiple regimes. The single state model is universally strongly rejected. Looking across all models, the preferred specification has four states but no autoregressive terms. The absence of autoregressive terms is perhaps unsurprising given the lack of serial correlation in the individual return series. That four states are required to capture the dynamics of the joint returns on the market and Fama French portfolios is consistent with our finding of three (largely common) states for the HML and SMB portfolios and two (uncorrelated) states for the market portfolio. This suggests that the appropriate number of regimes for the joint process of stock returns will likely exceed three and possibly be as high as $2 \times 3 = 6$ regimes.

[4]Tests are performed using the statistic proposed by Davies (1977). This accounts for the fact that under the null of a single state ($k = 1$) some of the regime switching parameters are not identified.

[5]Correlation estimates for the smoothed state probabilities of matched pairs of regimes for the HML and SMB returns are all positive and large (0.33, 0.47 and 0.52).

To assist in the economic interpretation of the four-state model, Table 3 presents parameter estimates while Figure 2 plots the associated state probabilities.[6] Regime 1 is a moderately persistent bear state whose average duration is seven months. In this state the mean excess return on the market is significantly negative at -14% per annum. During bear markets, the size and value anomalies are largely absent from the data and mean returns on the SMB and HML portfolios are not significantly different from zero. Volatility is high and return correlations closely track their unconditional counterparts listed in panel A. Figure 2 shows that this regime captures major crashes and periods with sustained declines in stock prices such as the 1929 crash, the Great Depression, the two oil shocks in the 1970s and the recent bear market of 2000-2001.

Regime 2 is a highly persistent, low-volatility bull state with an average duration of 23 months that captures long periods with growing stock prices during the 1940s and 1950s. Mean returns in this state are significantly positive for the market and HML portfolios (15% in excess of the riskless rate and 5% per annum, respectively) but close to zero for the SMB portfolio. Hence the value effect is strong in this state while the size effect is much smaller. Returns on the HML portfolio are positively correlated with returns on the SMB and market portfolios while SMB returns are uncorrelated with the market.

Regime 3 is another highly persistent, low-volatility state where all equity portfolios earn positive mean returns (8%, 2%, and 3%, respectively). This state captures most of the bull markets since the mid-sixties. A clear difference between regimes 2 and 3 is found in their correlation structure. In the second state the SMB portfolio provides a hedge with respect to the performance of the market portfolio. In the third state the HML portfolio plays a similar role and its returns are also strongly negatively correlated with SMB returns.

Finally, regime 4 is a highly volatile, transient state that captures stock prices during parts of the Great Depression and 1999. Mean returns in this state are high (18, 10, and 12 percent per month) but not absurdly so since the average duration of this state is less than two months and volatilities in this state are also very high, i.e. 47, 53, and 48% per annum. Despite its short duration, regime 4 is clearly important for size and value effects to emerge in the data.

The steady state probabilities implied by the estimates, $\hat{P}$, are 21%, 25%, 53% and 1%, respectively. Furthermore, transition probabilities follow a very particular pattern in our model: The market either remains in the fourth, high return state (with a probability of one-third) or exits to the bear/crash state (with a two-thirds probability) so that states 1 and 4 jointly identify periods with clustering of high volatility.

These findings shed new light on some empirical regularities discussed in the literature. For instance, Davis et al. (2000) note that the size premium has declined after the mid-1980s, and Pástor (2000) finds that it fluctuates significantly over time.[7] The first observation matches our finding that most of the 1980s and 1990s was spent in the third regime where the size premium is not significantly positive. The second observation is consistent with our finding that SMB returns and volatilities vary substantially across states. Similarly, the evidence reported by Pástor (2000) that the value premium is more stable over time is consistent with our finding that the value premium exceeds the size premium and is positive in all four states.

[6] Parameter estimates were computed by maximimum likelihood methods.

[7] Early evidence on the instability of the size premium is discussed in Banz (1981) and Brown et al. (1983).

In sum, our findings confirm the presence of strong size and value effects but also show that they are subject to significant time-variation related to the presence of economic regimes. Mean returns on the SMB portfolio are positive and significant only in the short-lived fourth regime, while mean returns on the HML portfolio are positive and significant in three out of four regimes, including the highly persistent second and third states. Such variations are difficult to capture by a single-state model that largely misses the hedging properties that the SMB portfolio has in regime 2 and the HML portfolio has in regime 3 in relation to the market portfolio.

3.4. *Testing restrictions and ARCH effects*

Our very long data set on three relatively weakly correlated return series means that most parameters in Table 3 are reasonably precisely estimated. Even so, the number of parameters of the four-state model is quite large and it is worth investigating whether a more parsimonious specification can be obtained. In view of the imprecise mean return estimates often found for equity portfolios, we follow Ang and Bekaert (2002, pp. 1147-1149) and first test a model where mean returns are restricted to be identical across regimes:

$$\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{s_t}). \tag{4}$$

We can formally test the restrictions on the mean return parameters through a likelihood-ratio test:

$$LR = 2(5422.52 - 5408.40) = 28.24.$$

The implied p-value of 0.0009 strongly rejects the state-independence of mean returns.

Next, we test whether the regime switching model can be simplified by imposing covariance restrictions. Returns in regimes 1 and 4 are highly volatile so it is natural to test the hypothesis that $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_4$ which implies six parameter restrictions:

$$LR = 2(5422.52 - 5397.39) = 50.26.$$

This yields a p-value very near zero. Once again the restrictions are resoundingly rejected so we maintain the general four-state model from Table 3.

Finally, we test whether the preferred four-state model is misspecified or needs to be extended to incorporate ARCH-effects. To address this question, we estimated a bivariate Markov switching ARCH model similar to that considered by Hamilton and Lin (1996):[8]

$$\begin{aligned} \mathbf{r}_t &= \boldsymbol{\mu}_{S_t} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim N(0, \boldsymbol{\Sigma}_{S_t}) \\ \boldsymbol{\Sigma}_{S_t} &= \mathbf{K}_{S_t} + \boldsymbol{\Delta}_{S_t} \boldsymbol{\varepsilon}_t' \boldsymbol{\varepsilon}_t \boldsymbol{\Delta}_{S_t}'. \end{aligned} \tag{5}$$

Here $\mathbf{K}_{S_t}$ is restricted to be symmetric and positive definite and $\boldsymbol{\Delta}_{S_t}$ captures regime-dependent effects of past shocks on current volatility. To formally test for ARCH effects, we imposed the restriction $\boldsymbol{\Delta}_{S_t} = \boldsymbol{\Delta}$, $S_t = 1, 2, 3, 4$ and obtained the likelihood ratio test

$$LR = 2\left[5447.23 - 5422.52\right] = 49.42.$$

[8]It remains clearly possible that other, non-nested multivariate regime switching GARCH models might improve the fit, see e.g. Haas, Mittnik, and Paolella (2004). Notice though that these more general conditional heteroskedastic extensions would make more difficult to interpret portfolio choices in regime-specific terms.

The associated p-value is 0.301 so the null hypothesis of no ARCH effects fails to be rejected. We therefore maintain the simpler four-state model without ARCH effects. The absence of ARCH effects in our model can be explained by the fact that, at the monthly frequency, regime switching can capture volatility clustering through time-variations in the probabilities of (persistent) states with very different levels of volatility, c.f. Timmermann (2000).

3.5. *Predictor Variables: The Dividend Yield*

Many studies suggest that stock returns are predicted by regressors such as term and default spreads or the dividend yield, c.f., Campbell and Shiller (1988), Fama and French (1988, 1989), Ferson and Harvey (1991), Goetzmann and Jorion (1993). Most of the literature on optimal asset allocation has focused on predictability from the dividend yield, c.f. Barberis (2000) and Kandel and Stambaugh (1996). Standard linear predictors fail to explain much of the variation in the monthly returns of size- and book-to-market sorted equity portfolios. However, the dividend yield is the predictor variable that generates strongest variations in hedging demands. The possibility that the dividend yield might predict returns on the SMB and HML portfolios has not been considered in the context of regime switching models.

To investigate the effect on our model of adding predictor variables such as the dividend yield, again we used a battery of tests to determine the best model specification for $(r_t^{MKT}\ r_t^{SMB}\ r_t^{HML}\ dy_t)$, where dy_t is the dividend yield in period t. Reflecting the strong persistence in the yield, the SIC strongly suggests a VAR(1) model irrespective of the number of states, k. Even with a first order autoregressive term included, a four-state model continues to be selected.

The economic interpretation of the four regimes is aided by studying the smoothed state probabilities presented in Figure 3 and the parameter estimates reported in Panel B of Table 4. For comparison Panel A reports estimates for the single-state benchmark model. The basic interpretation of the regimes remains unchanged from the simpler model reported in Table 3. Regime 1 is a transient state with an average duration less than two months that mostly picks up bear markets such as the Great Depression, the two oil shocks in the 1970s and the more recent period 2000-2001. The main difference to the bear state in the simpler model in Table 3 is that this state now has a shorter expected duration, records large negative mean returns also for the SMB portfolio and a larger mean return on the HML portfolio.

Regimes 2 and 3 continue to be persistent, low volatility states with average durations exceeding ten months. Taken together, these states capture most bull markets between the 1940s and 1990s. State 2 has a low dividend yield (on average 2.1%) while state 3 has a high yield (on average 4.6%). While state 2 tracks periods with small size and value anomalies, state 3 captures periods where the size anomaly is strong. Three of four of the coefficients of the lagged dividend yield on the SMB and HML returns are significant in these two states.

Finally, regime 4 remains an outlier state with large positive mean returns on the market and SMB portfolio although it now has large negative returns on the HML portfolio. In this state the mean excess return on the market is 24% per annum while growth stocks outperform value stocks to the tune of 12% per annum and small firms outperform large firms by 30% per annum. Volatility is also high, ranging from 22% to 36% per annum for the three portfolios.

Equity return correlations continue to vary significantly across states. The correlation between the market and SMB portfolio varies from 0.09 to 0.28, while the correlation between the market and HML portfolio varies from -0.30 to 0.60. This again suggests important time-variations in the hedging properties of the Fama French portfolios.

4. **The Asset Allocation Problem**

So far we have documented the presence of regimes in the process underlying returns on the market portfolio and portfolios tracking size and value effects. We next explore the asset allocation implications of such regimes. Since it is clear that regime shifts generate predictability in future investment opportunities, we expect to find interesting horizon effects and hedging demands. Under the CAPM, investors should not hold the size or value portfolios. To see if this continues to be valid here, we consider the asset allocation problem of an investor with power utility over terminal wealth, W_{t+T}, coefficient of relative risk aversion, $\gamma > 1$, and time horizon, T:

$$u(W_{t+T}) = \frac{W_{t+T}^{1-\gamma}}{1-\gamma}. \tag{6}$$

The investor is assumed to maximize expected utility by choosing at time t a portfolio allocation to the market, SMB and HML portfolios, $\boldsymbol{\omega}_t \equiv (\omega_t^{MKT}\ \omega_t^{SMB}\ \omega_t^{HML})'$, while $1-\omega_t'\iota_3$ is invested in riskless, one-month T-bills. For simplicity we assume the investor has unit initial wealth and ignores intermediate consumption. Portfolio weights are adjusted every $\varphi = \frac{T}{B}$ months at B equally spaced points t, $t+\frac{T}{B}$, $t+2\frac{T}{B}$, ..., $t+(B-1)\frac{T}{B}$. When $B=1$, $\varphi = T$, so the investor simply implements a buy-and-hold strategy.

Let $\boldsymbol{\omega}_b$ $(b = 0, 1, ..., B-1)$ be the weights on the stock portfolios at these rebalancing times. Then $1-\boldsymbol{\omega}_b'\boldsymbol{\iota}_3$ is the weight on T-bills at time $t+b\frac{T}{B}$ and the investor's optimization problem becomes[9]

$$\begin{gathered}
\max_{\{\boldsymbol{\omega}_j\}_{j=0}^{B-1}} \quad E_t\left[\frac{W_B^{1-\gamma}}{1-\gamma}\right] \\
s.t.\ W_{b+1} = W_b\left\{(1-\boldsymbol{\omega}_b'\boldsymbol{\iota}_3)\exp\left(\varphi r^f\right) + \boldsymbol{\omega}_b'\exp\left(\mathbf{R}_{b+1} + \varphi r^f\mathbf{e}_1\right)\right\} \\
\mathbf{R}_{b+1} = \mathbf{r}_{t_b+1} + \mathbf{r}_{t_b+2} + ... + \mathbf{r}_{t_{b+1}}.
\end{gathered} \tag{7}$$

Here $E_t[.]$ denotes the conditional expectation given the information set at time t, $\mathcal{F}_t$. The term $\mathbf{R}_{b+1} + \varphi r^f\mathbf{e}_1$ $(\mathbf{e}_1 \equiv [1\ 0\ 0]')$ arises since we specified our model for the vector of excess returns on the market portfolio and returns on the zero-investment SMB and HML portfolios, both continuously compounded.[10] The wealth process in (7) reflects the fact that our return data are continuously compounded. Incorporating investors' use of predictor variables, $\mathbf{z}_b$, at the decision points, b, the derived utility of wealth is

$$J(W_b, \mathbf{y}_b, \boldsymbol{\theta}_b, \boldsymbol{\pi}_b, t_b) \equiv \max_{\{\boldsymbol{\omega}_j\}_{j=b}^{B-1}} E_{t_b}\left[\frac{W_B^{1-\gamma}}{1-\gamma}\right]. \tag{8}$$

[9] As is common in the empirical literature on optimal asset allocation, we assume that the risk-free rate is constant over time and also do not address market equilibrium issues so our investor is small relative to the total market.

[10] Both SMB and HML require short-selling stocks and thus depositing funds in margin accounts. If a proportion ω is invested in one of these portfolios, a percentage ω of the agent's wealth must be borrowed at the riskless rate to satisfy the deposit requirement. This is equivalent to investing a proportion $-\omega$ in T-bills, c.f. Pástor (1999, p. 201).

Here $\mathbf{y}_b \equiv (\mathbf{r}_b\ \mathbf{z}_b)'$, $\boldsymbol{\theta}_b = \left(\left\{\boldsymbol{\mu}_{i,b}, \{\mathbf{A}_{j,i,b}\}_{j=1}^{p}, \boldsymbol{\Sigma}_{i,b}\right\}_{i=1}^{k}, \mathbf{P}_b\right)$ collects the parameters of the regime switching model, and $\boldsymbol{\pi}_b$ is the state probabilities at point b. Investors face a large set of state variables, most obviously the regime probabilities, $\boldsymbol{\pi}_b$, and the vector of returns and predictor variables, $\mathbf{y}_b$. The parameter vector $\boldsymbol{\theta}_b$ could also be treated as a separate state variable that gets updated at each point in time, t. However, solving the associated problem implies using a very large set of state variables (as many as 140 in some of our applications) and is infeasible. We therefore solve a simplified version of the asset allocation program in which the model's parameters are fixed at their estimated values $\boldsymbol{\theta}_b = \hat{\boldsymbol{\theta}}$ for all $b = 0, 1, ..., B-1$.[11] Moreover, as in Ang and Bekaert (2002), we assume that states are observable at future rebalancing points so $\boldsymbol{\pi}_b = \mathbf{e}_i$ $(i = 1, ..., k)$. Assuming power utility, the expression for derived utility of wealth simplifies to

$$J(W_b, \mathbf{y}_b, S_b) = \frac{W_b^{1-\gamma}}{1-\gamma} Q(\mathbf{y}_b, S_b), \qquad \gamma > 1, \tag{9}$$

where S_b simply records the future state. Since

$$\begin{aligned} \frac{W_b^{1-\gamma}}{1-\gamma} &= \frac{W_{b-1}^{1-\gamma}}{1-\gamma} \cdot \left[(1-\boldsymbol{\omega}_{b-1}'\boldsymbol{\iota}_3)\exp\left(\varphi r^f\right) + \boldsymbol{\omega}_{b-1}'\exp\left(\mathbf{R}_b + \varphi r^f \mathbf{e}_1\right)\right]^{1-\gamma} \\ &\propto \left[(1-\boldsymbol{\omega}_{b-1}'\boldsymbol{\iota}_3)\exp\left(\varphi r^f\right) + \boldsymbol{\omega}_{b-1}'\exp\left(\mathbf{R}_b + \varphi r^f \mathbf{e}_1\right)\right]^{1-\gamma}, \end{aligned}$$

the first order conditions are

$$E_{b-1}\left\{(R_{b-1:b}^p(\hat{\boldsymbol{\omega}}_{b-1}))^{-\gamma}\left[\exp\left(\mathbf{R}_b + \varphi r^f \mathbf{e}_1\right) - \exp\left(\varphi r^f\right)\boldsymbol{\iota}_3\right] Q(\mathbf{y}_b, S_b)\right\} = \mathbf{0}, \tag{10}$$

where $R_{b-1:b}^p(\boldsymbol{\omega}_{b-1}) \equiv (1-\boldsymbol{\omega}_{b-1}'\boldsymbol{\iota}_3)\exp(\varphi r^f) + \boldsymbol{\omega}_{b-1}'\exp(\mathbf{R}_b + \varphi r^f \mathbf{e}_1)$ is the overall portfolio return over the interval $[t+(b-1)\varphi,\, t+b\varphi]$, $\hat{\boldsymbol{\omega}}_{b-1}$ is the vector of optimal portfolio weights, and $E_{b-1}[\cdot] = E[\cdot|\mathcal{F}_{b-1}]$. Since the information set $\mathcal{F}_{b-1}$ includes knowledge of the state, $S_{b-1} = i$, (10) can be re-written as:

$$\sum_{m=1}^{k} \Pr\{S_b{=}m|S_{b-1}{=}i\} E_{b-1}\left\{(R_{b-1:b}^p(\hat{\boldsymbol{\omega}}_{b-1}(i)))^{-\gamma}[\exp(\mathbf{R}_b + \varphi r^f \mathbf{e}_1) - \exp(\varphi r^f \boldsymbol{\iota}_3)] Q(\mathbf{y}_b, S_b{=}m)\right\} = \mathbf{0}, \tag{11}$$

where $\Pr\{S_b = m|S_{b-1} = i\} = \mathbf{e}_i'\mathbf{P}\mathbf{e}_m$ is the appropriate element of the transition matrix $\mathbf{P}$. (11) yields a system of three nonlinear equations in three unknowns that can easily be solved numerically. Once $\hat{\boldsymbol{\omega}}_{b-1}(i)$ has been found it follows that

$$Q(\mathbf{y}_{b-1}, S_{b-1} = i) = E_{b-1}\left\{\left[(1-\hat{\boldsymbol{\omega}}_{b-1}'(i)\boldsymbol{\iota}_3)\exp\left(\varphi r^f\right) + \hat{\boldsymbol{\omega}}_{b-1}'(i)\exp\left(\mathbf{R}_b + \varphi r^f \mathbf{e}_1\right)\right]^{1-\gamma} Q(\mathbf{y}_b, S_b)\right\}.$$

This expectation is a function of $\mathbf{y}_{b-1}$ which enters the conditioning information set used to compute the expectation $E_{b-1}[\cdot]$. At this point the problem can be solved recursively, starting at time T (where $Q(\mathbf{y}_B, S_B) = 1$ for all $(\mathbf{y}_B, S_B)'$) and moving backwards until time t, where, given $S_t = i$, $\hat{\boldsymbol{\omega}}_0(i) = \hat{\boldsymbol{\omega}}_t(i)$ provides the vector of optimal portfolio weights.

[11] Barberis (2000) considers a simple example with future updating limited to two parameter estimates.

4.1. *Quadrature Solution Methods*

A variety of solution methods have been applied in the literature on portfolio allocation under time-varying investment opportunities. Barberis (2000) employs simulation methods and studies a pure allocation problem without interim consumption. Ang and Bekaert (2002) solve for the optimal asset allocation using quadrature methods. Campbell and Viceira (1999, 2001) derive approximate analytical solutions for an infinitely lived investor when interim consumption is allowed and rebalancing is continuous. Campbell et al. (2003) extend this approach to a multivariate set-up and show that a mixture of approximations and numerical methods can deliver powerful results. Finally, some papers have derived closed-form solutions by working in continuous-time, e.g. Brennan et al. (1997) and Kim and Omberg (1996) for the case without interim consumption.

The approach most closely related to ours is proposed by Ang and Bekaert (2002) who study Markov switching for pairs of international stock market returns. Ang and Bekaert employ quadrature methods to approximate expected utility at the investor's decision points. We adopt their computational strategy, generalized to deal with asset allocation problems when $\varphi > 1$, so rebalancing occurs less frequently than new data is observed.

To illustrate our approach, consider first the case without predictor variables beyond asset returns (so $m = 0$) and rebalancing every period. We can then rewrite (11) as

$$\sum_{m=1}^{k} \left(\mathbf{e}_i'\mathbf{P}\mathbf{e}_m\right) E_{b-1}\left\{g(\mathbf{R}_b + \varphi r^f \mathbf{e}_1; \hat{\boldsymbol{\omega}}_{b-1}, m)\right\} = \mathbf{0}, \tag{12}$$

where $g(\mathbf{R}_b; \hat{\boldsymbol{\omega}}_{b-1}, m) \equiv (R^p_{b-1:b}(\hat{\boldsymbol{\omega}}_{b-1}(i)))^{-\gamma}[\exp\left(\mathbf{R}_b + r^f\mathbf{e}_1\right) - \exp\left(\varphi r^f\right)\boldsymbol{\iota}_3]Q(\mathbf{R}_b, S_b = m)$. Following Tauchen and Hussey (1991), an $N-$point quadrature rule for integration of a function g against a regime-specific density f is a set of N regime-dependent points $\mathbf{R}_b^j(i)$ and nonnegative quadrature weights $\delta^j(i)$ $j = 1, ..., N_i$, such that the conditional first order conditions of the problem are approximated by

$$\sum_{m=1}^{k} \left(\mathbf{e}_i'\mathbf{P}\mathbf{e}_m\right) E_{b-1}\left\{g(\mathbf{R}_b + r^f\mathbf{e}_1; \hat{\boldsymbol{\omega}}_{b-1}, m)\right\} \simeq \sum_{m=1}^{k} \left(\mathbf{e}_i'\mathbf{P}\mathbf{e}_m\right)\left[\sum_{j=1}^{N_m} h(\mathbf{R}_b^j(i); \hat{\boldsymbol{\omega}}_{b-1}, m)\delta^j(i)\right] = \mathbf{0}. \tag{13}$$

Notice that both the approximation grid points and the quadrature weights are regime-specific and depend only on the state-dependent density ("importance") function Δ. Different rules can be used to select Δ and consequently the weights. In our case we adopt an N^3-point multiplicative Gauss-Hermite rule for multivariate integrals, thus choosing Δ to be multivariate Gaussian. This is equivalent to using a discrete Markov chain approximation with transition matrix $\mathbf{P}$ from the regime switching model. When $\varphi > 1$, multiperiod portfolio returns are simulated sequentially from the approximating Markov chain using the transition probabilities in $\mathbf{P}$. The optimal portfolio choice can then be found using a nonlinear equation solver.

A crucial issue in our application is the possibility that wealth can become negative ("bankruptcy"), given that short-sale positions are admitted.[12] As pointed out by Kandel and Stambaugh (1996) and Barberis (2000), bankruptcy makes the asset allocation problem (7) ill-behaved and implies that no

[12]When short-sales are admitted, numerical quadrature makes it easy to control the truncation points for the joint distribution of asset returns in order to impose the no-bankruptcy constraint.

interior solution exists.[13] In our algorithm however, the no-bankruptcy constraint is easily imposed: given the grid points $\{\mathbf{R}_b^j(i)\}_{j=1}^N$ $(m = 1, ..., k)$ employed by the quadrature method, we check that, at all nodes on the grid and for $\boldsymbol{\omega}_{b-1}$ constrained to lie in the set Λ,

$$(1 - \boldsymbol{\omega}_{b-1}'\boldsymbol{\iota}_3)\exp\left(r^f\right) + \boldsymbol{\omega}_{b-1}'\exp(\mathbf{R}_b^j(i) + r^f\mathbf{e}_1) > 0, \tag{14}$$

Hence wealth must be positive at all points on the approximating grid provided that $\boldsymbol{\omega}_{b-1} \in \Lambda$. In our application we set Λ to the wide interval, $[-5, 5]$. This set includes plausible portfolio weights previously reported in asset allocation exercises focusing on value and size anomalies (e.g. Brennan and Xia (2001) and Pástor (2000)). The grid defining the approximating discrete Markov chain is shrunk when violations of the no-bankruptcy constraint appear. Hence the portfolio choice problem is solved by appropriately truncating the tails of the joint distribution obtained in Section 3.

We undertook a range of numerical experiments to assess the accuracy of the approximations provided by the multiplicative Gauss-Hermite quadrature rule. Setting $N = 8$, the unconditional and conditional (regime-dependent) moments for monthly portfolio returns under the approximating discrete Markov chain are close to those implied by the data and the four-state regime switching model estimated in Section 3.3. Furthermore, we checked that increasing N to 12 or higher values did not change our conclusions.

Introducing rebalancing and including the dividend yield as a predictor variable we find, similar to Ang and Bekaert (2002), that particular care should be used when constructing the discretization grid over which the approximating Markov chain is defined. Rebalancing complicates the calculations since $\mathbf{y}$ now contains additional state variables. Once again, the investor's first order conditions can be approximated by

$$\sum_{m=1}^{k}\left(\mathbf{e}_i'\mathbf{\Pi}_{i\to m}\mathbf{e}_g\right)\left[\sum_{g'=1}^{G} g(\mathbf{R}_b^{g'}; \hat{\boldsymbol{\omega}}_{b-1}(g,i), m)\right] = \mathbf{0}, \tag{15}$$

where $g(\mathbf{R}_b^{g'}; \hat{\boldsymbol{\omega}}_{b-1}(g,i), m) \equiv (R_{b-1:b}^{p,g'}(\hat{\boldsymbol{\omega}}_{b-1}(g,i)))^{-\gamma}[\exp(\mathbf{R}_b^{g'} + r^f\mathbf{e}_1) - \exp(\varphi r^f\boldsymbol{\iota}_3)]Q(\mathbf{y}_b^{g'}, m)$ and $\mathbf{\Pi}_{i\to m}$ gives the probability of moving from state i to state m on the quadrature grid. The expression requires that both g and g' be consistent with values of $\mathbf{y}$ that are admissible with regimes i and m, respectively. The optimal portfolio choice conditional on an initial state g for the predictors and state $i = 1, ..., k$ can then be found using a nonlinear equation solver. At this point the problem can be solved recursively, starting at time T and going backwards until time t. To define the $G-$point grid and the transition matrices $\mathbf{\Pi}_{i\to m}$ employed by (15), we discretize the space on a grid of $k \times G_1 \times \cdots \times G_{n+l}$ points, so each variable in the 4×1 vector $\mathbf{y}$ is replaced by G_v discrete values. Following Ang and Bekaert (2002), we calibrate an approximating discrete Markov chain to the full sample and to each of the regimes and then combine these using the transition probabilities. Hence, we choose regime-dependent points for each of the four variables on a (variable-specific) grid and a transition matrix $\mathbf{\Pi}_{i\to m}$ such that the unconditional distribution of $\mathbf{y}$ is correctly approximated. At this stage, the support of the discretization grid is truncated to reflect no-bankruptcy constraints.

[13] This occurs when in (10) $R_{b-1:b}^p(\boldsymbol{\omega}_{b-1}) \leq 0$, so the marginal utility of wealth $\left[R_{b-1:b}^p(\boldsymbol{\omega}_{b-1})\right]^{-\gamma}$ is either not defined (if $R_{b-1:b}^p(\boldsymbol{\omega}_{b-1}) = 0$) or becomes negative.

Provided the approximating grid is sufficiently dense (especially with respect to the persistent dividend yield), both conditional and unconditional means and covariance matrices are close to those implied by the four-state model estimated in Section 3.4. Furthermore, when a multivariate regime switching model is fitted to data simulated off the discrete Markov chain, we find that the majority of parameters fall well within one standard deviation of the estimates reported in Table 4, while parameters rarely switch signs. In particular, in our application we use $G_1 = G_2 = G_3 = 5$ grid points for the portfolio returns and $G_4 = 20$ points for the grid used to approximate the dividend yield. Further details are provided in an appendix that is available from the authors.

5. **Empirical Asset Allocation Results**

5.1. *Buy-and-Hold Investor*

We first consider the asset allocation strategy of a buy-and-hold investor who only solves the asset allocation problem once, namely at time t. Consistent with choices in the literature the coefficient of relative risk aversion is set at $\gamma = 5$. The levels of the risky asset holdings clearly depend on γ. However, following Ang and Bekaert (2002) and choosing a different value such as $\gamma = 10$ revealed robustness of our qualitative results on the allocations to the market, SMB and HML portfolios. We consider horizons that vary between 1 and 120 months, c.f. Aït-Sahalia and Brandt (2001). Figure 4 plots the optimal portfolio weights as a function of the investment horizon starting from each of the four states.

Asset allocations vary significantly across regimes in the four-state model, particularly at short horizons where market timing effects are strong. Regime 1 is dominated by the negative average return on the market portfolio and by the relatively high mean returns on the SMB and HML portfolios. Starting from this state, the allocation to the market portfolio is therefore small at short investment horizons though it rises in T. While the weights on the SMB and HML portfolios initially rise, they decline as a function of the horizon, T, for $T \geq 6$ months.

Turning to regime 2, due to its high expected return, the market portfolio features prominently in the optimal asset allocation with a weight above 100% at short horizons. Large, short positions are taken in the SMB and HML portfolios to finance the long position in the market. Regime 3 produces similar portfolio choices although the allocation to the market portfolio is far smaller than in regime 2, reflecting its lower mean return. An investor should also hold a long position in the HML portfolio in this state even at the shortest horizons. This is explained by the hedge that the HML provides with respect to the market portfolio. Finally, in the short-lived fourth regime the equity portfolios offer high mean returns and are generally held in long positions at short or medium horizons. The long equity holdings are financed by substantial borrowings in T-bills.[14]

At the longest horizon almost 60% is held in the market, 15% in the HML portfolio, -20% in the SMB portfolio and 45% in T-bills. These long-run asset allocation results are broadly consistent with those reported by Pástor (2000) for a single-period exercise under a tight prior tilted towards the CAPM. Our finding that the allocation to the HML portfolio is positive in three of four states and only negative in

[14]Consistent with findings reported by Ang and Bekaert (2002), the portfolio weights tend to already converge to their long-run levels at horizons of 2-3 years.

the fourth state for very short horizons is also consistent with Pastor's results.

It is also interesting to compare our results to those reported by Brennan and Xia (2001) in a model without predictor variables but with learning about mean returns on the equity portfolios. For a short horizon ($T = 1$) with a zero prior on the CAPM, Brennan and Xia obtain an allocation heavily tilted towards the HML (176%) and market portfolios (71%) and with slightly less weight on the SMB portfolio (41%). These weights do not match our weights in any of the four regimes and differences grow larger at the long horizon, $T = 120$. While Brennan and Xia report a weight on the HML portfolio that exceeds unity (132%) and a large positive weight on the size portfolio (37%), our results show a more reduced role for both the SMB and HML portfolios at long horizons. Our long-run allocations are, however, quite similar to those in Brennan and Xia based on a 50-50 mixed prior over the CAPM and the empirical distribution of asset returns which gives rise to weights on the HML, SMB and market portfolios of 14%, -3% and 35%, respectively. Hence, similar long-run allocations with reduced weights on the SMB and HML portfolios can be achieved either by putting a large prior on the CAPM or by adopting a model such as ours that accounts for fat tails - and thus higher risk - in the returns on the size and value portfolios.

5.2. *Predictability from the Dividend Yield*

Figure 5 shows the optimal asset allocation for a buy-and-hold investor when predictability from the dividend yield is incorporated in the regime switching model. Qualitatively, the results are quite similar to those shown in Figure 4. For example, the optimal allocation to the market portfolio is increasing when starting from the bear state (state 1) and decreasing from the other states. The slope of the investment demand for the SMB and HML portfolios also varies significantly across states. At short horizons the optimal allocations to the size and value portfolios are again highly sensitive to the current state probability, but quickly converge to their long-run levels as T grows. Comparing Figures 4 and 5, holdings in the SMB and HML portfolios become more extreme once the yield is included as a predictor variable.

The most notable difference with respect to the earlier results from Figure 4 is the large, positive holdings in the HML portfolio and the negative holdings in the market and SMB portfolios in the bear state (regime 1) at the shortest horizons. The reason for this change is the large negative mean returns on the market and SMB portfolios and the large positive mean return on the HML portfolio in this state. When combined with the fact that the bear state is highly transient in the extended model, this explains why the equity positions now become more extreme at the very shortest horizons and why these positions quickly revert to the steady-state weights as the horizon is expanded and a regime shift is anticipated.

To compare asset allocations under a broader set of models and to isolate the effect of regime switching, Figure 6 shows the optimal portfolio weights as a function of the investment horizon under three alternative specifications, namely regime switching without the dividend yield (MS), regime switching with the dividend yield included (MS-VAR(1)) and a VAR(1) model similar to that previously considered by Barberis (2000):

$$\mathbf{y}_t = \begin{pmatrix} \boldsymbol{\mu} \\ \boldsymbol{\mu}_{dy} \end{pmatrix} + \mathbf{A}^* \mathbf{y}_{t-1} + \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \varepsilon_{dyt} \end{pmatrix}, \quad (\boldsymbol{\varepsilon}_t' \ \varepsilon_{dyt})' \sim N(0, \boldsymbol{\Sigma}^*). \tag{16}$$

Estimates of this model can be found in panel A of Table 4.[15] As is common in the literature, averaging across states for the regime switching models is undertaken using the unconditional (steady-state) distribution.

Consistent with findings reported by Barberis (2000) under the VAR(1) model the allocation to stocks rises as a function of the investment horizon. For this model we also find that the allocations to the HML and SMB portfolios grow as a function of the investment horizon.

The large positive demand for the HML portfolio and T-bills and the large negative demand for the market and SMB portfolios at short horizons under the MS-VAR(1) model is explained by the large negative mean returns of the market and SMB portfolios in the short-lived bear state (state 1) which−due to the high marginal utility in this state−dominates results for this model. Increasing the investment horizon from one to six months leads to an increased demand for the market and SMB portfolios and a lower demand for the HML portfolio under the four state MS-VAR(1) model.

Clearly, the portfolio weights under the single state model (16) are quite different from those obtained under the four-state model irrespective of whether this includes the dividend yield. Most notably, the four regimes introduce short-run market timing effects while the single-state model is driven by slower, long-run movements in the dividend yield. Asset demand curves are therefore steeper at horizons shorter than six months under the four-state model.

5.3. *Rebalancing and Hedging Demands*

So far we have studied the optimal asset allocation for a buy-and-hold investor. Investors may, however, have access to rebalancing opportunities. Table 5 shows the effects of rebalancing every 1, 3, 6, 12 or 24 months on optimal holdings in the three stock portfolios. If frequent rebalancing is possible, the investor's horizon matters far less than under the buy-and-hold scenario. Effectively, only the period between the current time (t) and the next rebalancing point ($t + \varphi$) induces curvature in the investment demand which is flat when $T > \varphi$.[16] The investor also responds more aggressively to the current state. The reason is simple: an investor who can rebalance frequently will utilize information about the current state by taking large, short positions when the return distribution indicates poor prospective returns and large, long positions in a state with more attractive returns. If the perceived state probabilities change next period, the investor can simply adjust the portfolio weights. Such adjustment opportunities are not available to the buy-and-hold investor who must consider the probability of future states during the entire holding period.

The rebalancing frequency can clearly have a large effect on asset holdings, most notably when the rebalancing frequency is varied from $\varphi = 3$ to $\varphi = 1$ in state four. The fourth state only has a 'stayer' probability of one-third and exits to the 'bear' state with a two-thirds probability. Under monthly rebalancing, an investor will significantly increase holdings in the market portfolio (compared to scenarios with higher values of φ) largely by lowering investments in the HML portfolio which has returns with a

[15]We estimate an unrestricted VAR that allows lagged returns to forecast future returns and the dividend yield, c.f. Campbell, Chan and Viceira (2003). However, since lagged returns have weak predictive power our conclusions are robust to imposing restrictions ruling out such effects as Barberis (2000) does.

[16]For $\varphi \geq T$ the optimal portfolio weights are identical to the buy-and-hold values and thus omitted from Table 5.

lower mean that are strongly correlated with market returns in the fourth state. Conversely, moving to $\varphi = 3$ and starting from the fourth state, a switch to the bear state will almost certainly occur prior to the next rebalancing point. Since the bear state has low mean returns on the market and SMB portfolios but high mean returns on the HML portfolio, the weights on the former assets are reduced while the weight on the SMB portfolio increases substantially compared to the case with $\varphi = 1$.

We continue to observe large variations across states in the portfolio weights under rebalancing. Starting from the first (bear) state, as rebalancing happens more frequently (φ declines) the allocation to the market portfolio declines and becomes negative. Conversely, in states two and three the demand for the market portfolio rises as φ is lowered while the non-monotonicities found for state four are explained by the high probability of going from state four to the low return bear state (state 1). State two (four) is associated with very large negative (positive) holdings in the SMB portfolio. The SMB weight increases with the rebalancing frequency in regimes one and four while the opposite happens in regimes two and three. Less variation across states is generally observed in the holdings of the HML portfolio.

The introduction of rebalancing opportunities allows us to measure the optimal hedging demand defined as the difference $\hat{\omega}_t^i(T) - \hat{\omega}_t^i(1)$ ($i =$ MKT, SMB, HML) for $T \geq 2$ and $\varphi = 1$ month, i.e. when rebalancing occurs at the same frequency as the data is observed (see Ingersoll (1987, p. 245)). Results are reported in separate rows in Table 5. Hedging demands for the market and SMB portfolios are substantially larger than hedging demands for the HML portfolio. The sign of the hedging demand for the market portfolio has an intuitive interpretation. In the bear state, future regime switching will improve investment opportunities so the hedging demand is positive and quite large (21 percent); similarly, hedging demands remain positive in regime three. Conversely, shifts away from the high mean return states (two and four) imply a worsening of the investment opportunities, so hedging demands for the market portfolio are negative when starting from these states.

To compare hedging demands under multiple regimes with those derived under a VAR benchmark, Table 5 also reports buy-and-hold allocations and hedging demands under linear predictability. For simplicity, calculations are performed when all the variables in y (i.e. portfolio returns and the dividend yield) are set at their sample means. For the market portfolio, hedging demand is positive but moderate (14%) in the VAR(1) case. The positive value is consistent with findings in papers such as Barberis (2000) and Campbell and Viceira (1999). The reason for the positive hedging demand is the negative covariance between shocks to the dividend yield and stock market returns which leads investors with a long horizon to hold more in stocks. Interestingly, the hedging demand for the market portfolio under the VAR(1) model is relatively small compared to the positive hedging demand in regime 1 (21%) and the large negative demand in regime 4 (-29%).

In the case of the SMB and HML portfolios, it is interesting to note the contrast between the rather sizeable (47% and -39%, respectively) hedging demand under the VAR(1) model and the more modest ones under regime switching. Though small, systematic patterns remain in these hedging demands which, as in the case of the market portfolio, are positive in state 1 and negative in state 4. Brennan and Xia (2001) find a somewhat larger negative hedging demand for all three portfolios in a model without predictability but with learning about the unknown drift of the stock price process underlying the Fama-

French portfolios. This finding is easy to understand since a positive shock to the unknown mean return is always positively correlated with future investment opportunities, causing the hedging demand to be negative. Our finding that regime switching only induces relatively small hedging demands for the size and value portfolios is, however, consistent with Ang and Bekaert (2002)'s finding that for many of their international equity portfolios the null hypothesis of a zero hedging demand cannot be rejected under regime switching.

6. **Economic Importance of Regimes**

So far we have shown that regimes have a large effect on the optimal asset allocation. It does not necessarily follow that ignoring regimes leads to welfare costs sufficiently large to give investors strong incentives to use the more complicated model that we propose. To address this issue, we next investigate the effect of parameter estimation uncertainty on the optimal portfolio weights and then undertake utility cost calculations to quantify the economic significance of regimes.

6.1. *Parameter uncertainty*

A concern often expressed in the literature on optimal asset allocation under predictability (see Brandt (1999) and Ang and Bekaert (2002)) is that the relatively large standard errors surrounding many parameter estimates tend to result in imprecisely determined portfolio weights. Aït-Sahalia and Brandt (2001) refer to this as the "Achille's heel" of models of conditional asset allocation. Although the portfolio weights reported so far are determined by solving a complicated dynamic programming problem, these weights condition on the parameter estimates, $\widehat{\boldsymbol{\theta}}$, and are therefore themselves random variables. We quantify the effect of estimation uncertainty by forming confidence intervals for the optimal portfolio weights as follows. From asymptotic analysis (e.g., Krolzig (1997))

$$\sqrt{T}\left(\widehat{\boldsymbol{\theta}}-\boldsymbol{\theta}_0\right) \overset{A}{\sim} N(\mathbf{0}, V_\theta), \tag{17}$$

where $\boldsymbol{\theta}_0$ denotes the true but unknown vector of parameters. The optimal portfolio weights $\hat{\boldsymbol{\omega}}_t(T)$ maximizing expected utility over a $T-$period horizon are implicitly defined by the first-order condition (10). This can be re-written as a generic function $\Xi_t(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\omega}}_t(T)) = \mathbf{0}$ that depends on $\hat{\boldsymbol{\theta}}$ since the true value of $\boldsymbol{\theta}$ is replaced by its estimator, $\hat{\boldsymbol{\theta}}$. As in Ang and Bekaert (2002, p. 1145), it can be shown that when

$$\det\left[\left.\frac{\partial \boldsymbol{\Xi}_t}{\partial \boldsymbol{\omega}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0, \boldsymbol{\omega}_t(T)=\hat{\boldsymbol{\omega}}_{0,t}(T)}\right] \neq 0, \tag{18}$$

the implicit function theorem guarantees the existence of some function that maps the true parameters $\boldsymbol{\theta}_0$ into a vector of portfolio weights, $\hat{\boldsymbol{\omega}}_{0,t}(T) = \xi(\boldsymbol{\theta}_0)$, such that

$$D \equiv \left.\frac{\partial \xi}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = -\left(\frac{\partial \boldsymbol{\Xi}_t}{\partial \boldsymbol{\omega}}\right)^{-1} \left.\frac{\partial \boldsymbol{\Xi}_t}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0, \boldsymbol{\omega}_t(T)=\hat{\boldsymbol{\omega}}_{0,t}(T)}. \tag{19}$$

Hence the Jacobian matrix of the partial derivatives of portfolio weights with respect to the predictive density parameters is well-defined. The delta method then gives the (asymptotic) distribution of $\hat{\boldsymbol{\omega}}_t(T)$

as

$$\sqrt{T}\left(\hat{\boldsymbol{\omega}}_t(T) - \hat{\boldsymbol{\omega}}_{0,t}(T)\right) \overset{A}{\sim} N(\mathbf{0}, DV_\theta D').$$

Standard deviations of the portfolio weights $\hat{\boldsymbol{\omega}}_t(T)$ are thus obtained from the diagonal elements of $DV_\theta D'$.[17]

Table 6 presents 95% confidence intervals for the portfolio weights under the regime switching model calculated this way. Once again we consider scenarios starting from each of the four states and study investment horizons of 1, 6, 60 and 120 months. For comparison, we also report the confidence interval that applies to portfolio weights obtained under the assumption of an IID process for returns.

Unsurprisingly in view of the different complexity of the two types of models, confidence intervals for IID weights tend to be more narrow than those produced by the four-state model. Confidence intervals for the regime switching allocations are particularly wide at short horizons. The degree of uncertainty about $\hat{\omega}_t(T)$ varies significantly across states, however, with the fourth regime associated with the greatest uncertainty. This reflects the short duration of this state and the fact that a small change in the transition probabilities changes the likelihood of a transition to the very different low-return bear state (state 1). At the longest horizons the confidence intervals for the portfolio weights derived under regime switching are more narrow and more similar to those characterizing the IID weights.

Wide confidence intervals at short horizons are unsurprising: Aït-Sahalia and Brandt (2001) also report large standard errors for portfolio weights, especially when investment in cash is allowed as in our paper. Furthermore, as pointed out by Campbell, Chan and Viceira (2003) the parameters governing the dynamics of asset returns can have very large effects on the optimal asset holdings so that any uncertainty about their values tends to have a large effect on portfolio weights.

Despite this uncertainty, ignoring regimes would clearly lead to a suboptimal portfolio allocation: Most of the four-state intervals for the weights on the market and SMB portfolios do not overlap with the confidence intervals obtained from the IID model. Ignoring regimes would lead an investor to invest too little in (short) the market portfolio and too much in the size portfolio.

6.2. *Utility cost calculations*

Disregarding regimes or predictability from the dividend yield is equivalent to constraining investors to choose optimal portfolio weights, $\hat{\boldsymbol{\omega}}_t^{IID}$, under the assumption that asset returns are drawn from a single-state model. To quantify the costs of this constraint, we compute the increase in initial wealth η_t^{IID} – or compensatory variation – an investor requires to derive the same level of expected utility from the IID and unconstrained asset allocation problems:

$$(1+\eta_t^{IID})^{1-\gamma}\left\{\sum_{b=0}^{B} E_t\left[(W_b)^{1-\gamma}\right]\right\} = Q(\mathbf{y}_b, S_b),$$

[17] In practice D depends on the unknown weights $\hat{\boldsymbol{\omega}}_{0,t}(T)$ but it can be replaced by a numerical estimate obtained by perturbing each of the parameters in $\hat{\boldsymbol{\theta}}$ by a small quantity $\epsilon\hat{\boldsymbol{\theta}}\mathbf{e}_i$ where, e.g., $\epsilon = 0.0001$. Portfolio weights are then computed from $(1+\epsilon)\hat{\boldsymbol{\theta}}\mathbf{e}_i$ and $(1-\epsilon)\hat{\boldsymbol{\theta}}\mathbf{e}_i$, solving for $\hat{\boldsymbol{\omega}}_t^+(T)$ such that $\boldsymbol{\Xi}_t((1+\epsilon)\hat{\boldsymbol{\theta}}\mathbf{e}_i, \hat{\boldsymbol{\omega}}_\mathbf{t}^+(\mathbf{T})) = \mathbf{0}$ and $\hat{\boldsymbol{\omega}}_t^-(T)$ such that $\boldsymbol{\Xi}_t((1-\epsilon)\hat{\boldsymbol{\theta}}\mathbf{e}_i, \hat{\boldsymbol{\omega}}_\mathbf{t}^-(\mathbf{T})) = \mathbf{0}$. The $i-th$ column of D is then approximated by $\left(\hat{\boldsymbol{\omega}}_t^+(T) - \hat{\boldsymbol{\omega}}_t^-(T)\right)/2\epsilon\hat{\boldsymbol{\theta}}\mathbf{e}_i$.

where $Q(\mathbf{y}_b, S_b)$ is the scaled value function under regime switching defined in equation (9). Solving for η_t^{IID}, we have

$$\tilde{\eta}_t^{IID} = \left\{ \frac{Q(\mathbf{y}_b, S_b)}{\sum_{b=0}^{B} E_t\left[(W_b)^{1-\gamma}\right]} \right\}^{\frac{1}{1-\gamma}} - 1. \tag{20}$$

The compensatory variation - plotted in Figure 7 as an annualized percentage rate - ranges from about two percent at the one-month horizon to about six percent at the ten-year horizon. Figure 7 also shows 95% asymptotic confidence intervals obtained by the delta method. Although the confidence bands are quite wide there is no question that regimes in the return process for the market, size and value portfolios are economically important. The lower band suggests a minimum of about 150 basis points at most horizons. The upper band suggests higher compensatory returns.

Our estimates of utility costs of ignoring regimes are higher than those reported by Ang and Bekaert (2002) for a study of international equity portfolios. This is easy to explain due to our finding of larger and more significant mean return effects and the coincidence of the low mean return state with the high volatility state (state 1). Reducing the allocation to equity portfolios during this state will be highly beneficial to the investor, particularly if a risk-free asset is present as we assume here. Furthermore, although relatively high, our estimate of the annualized welfare loss is well within the range of values reported in the literature. For instance, Brennan and Xia (2001) report a certainty equivalence value of investing in the HML and SMB portfolio that exceeds 8% per annum even in the presence of parameter estimation uncertainty. Our estimates suggest that the utility costs arising from ignoring time-variations in the joint distribution of returns on these portfolios due to regime switching is roughly of a similar magnitude.

We also considered utility costs in the presence of predictability both from regime switching and from the dividend yield. Calculations based on the MS-VAR(1) model suggested that utility costs are higher at the shortest horizons compared to our estimates under the simple regime switching model. This is in part a reflection of the large variations in the optimal weights that we observed at the short horizons for this model in Figure 5. Utility costs under the two models are, however, very similar at the longest investment horizon of 10 years.

7. Conclusion

This paper documented the presence of four regimes in the joint distribution of equity returns on market, size and value portfolios. A single-state model appears to be misspecified as means, correlations and volatilities of returns on these portfolios vary significantly across states. This finding is perhaps not so surprising given the very different episodes and market conditions—such as the Great Depression, World War II and the oil shocks of the 1970s—that occurred during a sample as long as ours (1927-2001). It is difficult to imagine that the same single-state model is able to capture episodes of such diversity.

We quantified the economic value of investing in the three equity portfolios under regime switching by considering the optimal asset allocations of an investor with power utility. Economically large variations were found in the optimal portfolio weights as a function of the economic state and the investment horizon. Rebalancing opportunities make the investor respond more aggressively to the current state probabilities since portfolio weights can be adjusted rapidly should the state probabilities change. This option is not open to a buy-and-hold investor. Overall, our estimates suggest that it is important to account for regimes when analyzing investments in returns on the market, size and value portfolios. Furthermore, regimes and the dividend yield appear to identify quite different predictable components in stock returns.

There are several ways in which our framework could be extended. First, our paper relies on a parametric model that links state variables to asset returns. This has the advantage of tractability, but also the disadvantage that optimal portfolio holdings inevitably reflect the underlying model assumptions. We have argued that our model provides a flexible representation of the dynamics and distribution of asset returns, but another approach would be to use semiparametric methods as advocated by Aït-Sahalia and Brandt (2001) and Brandt (1999).

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Table 1

Summary Statistics for the Stock Portfolios

This table reports summary statistics for monthly returns on the value-weighted market portfolio (in excess of the 1-month T-bill rate) and Fama and French (1993) SMB (small minus big) and HML (high book-to-market minus low book-to-market) portfolios over the sample period 1927:12 – 2001:12.

	Mean	**Median**	**Minimum**	**Maximum**	**Standard deviation**	**Skew**	**Kurtosis**
A. Portfolio Returns							
SMB	0.0022	0.0005	-0.1626	0.2138	0.0341	2.1982	24.3045
HML	0.0040	0.0022	-0.1323	0.1367	0.0365	1.8777	18.2384
Market	0.0066	0.0099	-0.2901	0.3817	0.0552	0.2248	10.7615

B. Correlation Matrix

	SMB	HML	Market portfolio
SMB	1		
HML	0.3274	1	
Market	0.2122	0.0854	1

Table 2
Selection of Multivariate Regime Switching Model

This table reports the fit (log-likelihood), Schwarz information criterion (SIC) and a likelihood ratio test of a single state model against multivariate Markov switching models of the form:

$$\mathrm{r}_t = \mu_{s_t} + \sum_{j=1}^{p} A_{js_t} \mathrm{r}_{t-j} + \varepsilon_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state S_t, A_{js_t} is the matrix of autoregressive coefficients at lag $p \geq j \geq 1$ in state s_t and $\varepsilon_t \sim N(\mathbf{0}, \Sigma_{s_t})$. The unobserved state variable S_t is governed by a first-order Markov chain that assumes *k* distinct values. The three monthly return series are Fama and French's (1993) SMB and HML portfolios and the excess return on a broad market portfolio. The sample period is 1927:12 – 2001:12. Acronyms are as follows: MS stands for Markov Switching, I for the presence of regime-dependent intercepts, A for regime-dependent autoregressive terms, H for regime-dependent covariance matrices (heteroskedasticity). The first number in parenthesis (*k*) is the number of regimes, the second (*p*) is the VAR order.

Model (k,p)	Number of parameters	Log-likelihood	LR test for linearity	SIC
	Base model: MSIA(1,0)			
MSIA(1,0)	9	4806.03	NA	-10.7435
MSIA(1,1)	18	4852.31	NA	-10.7910
MSIA(1,2)	27	4851.62	NA	-10.7328
	Base model: MSIA(2,0)			
MSIA(2,0)	14	4848.03	83.9930 (0.000)	-10.7998
MSIAH(2,0)	20	5288.48	964.8905 (0.000)	-11.7448
MSIAH(2,1)	38	5340.67	976.707 (0.000)	-11.7380
MSIAH(2,2)	56	5345.74	988.2458 (0.000)	-11.6250
	Base model: MSIA(3,0)			
MSIA(3,0)	21	5048.75	485.4399 (0.000)	-11.1979
MSIAH(3,0)	33	5366.00	1119.9421 (0.000)	-11.8199
MSIAH(3,1)	60	5410.71	1116.8031 (0.000)	-11.7276
	Base model: MSIA(4,0)			
MSIA(4,0)	30	5100.29	588.5121 (0.000)	-11.2451
MSIAH(4,0)	48	5422.52	1232.9824 (0.000)	**-11.8325**
MSIAH(4,1)	84	5474.00	1243.3862 (0.000)	-11.6866
	Base model: MSIA(6,0)			
MSIA(6,0)	54	5188.70	765.3445 (0.000)	-11.2607
MSIAH(6,0)	84	5500.30	1388.5308 (0.000)	-11.7325
MSIAH(6,1)	138	5587.14	1469.6596 (0.000)	-11.5286

Table 3

Parameter Estimates of Regime Switching Model for Market, SMB and HML Returns

This table reports parameter estimates for the multivariate regime switching model:

$$r_t = \mu_{s_t} + \varepsilon_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state S_t and $\varepsilon_t \sim N(\mathbf{0}, \Sigma_{s_t})$ is the vector of unpredictable return innovations. The unobserved state variable S_t is governed by a first-order Markov chain that can assume one of four values. The return series are net returns on Fama and French's (1993) SMB and HML portfolios and excess returns on the value-weighted market portfolio. The sample period is 1927:12 – 2001:12. Panel A represents the single-state benchmark, while panel B refers to the four-state model. Values reported on the diagonals of the correlation matrices are annualized volatilities. All other estimates are monthly. Standard errors are shown in parentheses for mean coefficients and transition probabilities.

	Panel A – Single State Model		
	Market	SMB	HML
1. Mean excess return	0.0063 (0.0019)	0.0022 (0.0011)	0.0040 (0.0012)
2. Correlations/Volatilities			
Market Portfolio	0.1921***		
SMB Portfolio	0.3318**	0.1179***	
HML Portfolio	0.2130*	0.0848	0.1262***

	Panel B – Four State Model		
	Market	SMB	HML
1. Mean excess return			
Regime 1	-0.0120 (0.0060)	0.0015 (0.0032)	0.0017 (0.0040)
Regime 2	0.0124 (0.0021)	-0.0002 (0.0011)	0.0042 (0.0012)
Regime 3	0.0068 (0.0018)	0.0016 (0.0012)	0.0022 (0.0011)
Regime 4	0.1769 (0.0229)	0.0971 (0.0470)	0.1246 (0.0300)
2. Correlations/Volatilities			
Regime 1:			
Market Portfolio	0.2843***		
SMB Portfolio	0.3404***	0.1469***	
HML Portfolio	0.1508**	0.1032	0.1830***
Regime 2:			
Market Portfolio	0.1115***		
SMB Portfolio	-0.0227	0.0541***	
HML Portfolio	0.4392***	0.2649***	0.0672***
Regime 3:			
Market Portfolio	0.1378***		
SMB Portfolio	0.3310***	0.0911***	
HML Portfolio	-0.3546***	-0.2273***	0.0800***
Regime 4:			
Market Portfolio	0.4671***		
SMB Portfolio	0.1040	0.5263***	
HML Portfolio	0.7718***	-0.0729	0.4818***

3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1	0.8518 (0.0166)	0.0347 (0.0120)	0.0765 (0.0139)	0.0371
Regime 2	0.0271 (0.0119)	0.9572 (0.0406)	0.0208 (0.0118)	0.0013
Regime 3	0.0330 (0.0104)	0.0066 (0.0049)	0.9604 (0.0049)	0.0000
Regime 4	0.6436 (0.1663)	0.0000 (0.0259)	0.0000 (0.0317)	0.3534

* significance at 10% level, ** significance at 5%, *** significance at 1%.

Table 4

Estimates of Regime Switching Model for Stock Returns and the Dividend Yield

This table shows parameter estimates for the regime switching model

$$y_t = \mu_{s_t} + A_{s_t} y_{t-1} + \varepsilon_t$$

where $\mathbf{y}_t$ is a 4×1 vector collecting the market, SMB and HML portfolio returns in the first three positions and the dividend yield in the fourth. $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{js_t} is the matrix of autoregressive coefficients in state s_t and $\varepsilon_t \sim N(\mathbf{0}, \Sigma_{s_t})$. The unobservable state S_t is governed by a first-order Markov chain that can assume one of four distinct values. The sample period is 1927:12 – 2001:12. Panel A refers to the single-state case while panel B covers the four-state model. Values reported on the diagonals of the correlation matrices are annualized volatilities. All other estimates are monthly. Standard errors are shown in parentheses for mean coefficients and transition probabilities.

	Market	SMB	HML	Dividend Yield
Panel A – VAR(1) (single state) Model				
1. Intercept term	-0.0065 (0.0052)	-0.0056 (0.0031)	-0.0046 (0.0034)	0.0009(0.0003)
2. VAR(1) Matrix				
Market Portfolio	0.1074 (0.0361)	-0.0273 (0.0573)	0.1163(0.0516)	0.2935(0.1211)
SMB Portfolio	0.1618 (0.0215)	-0.0219 (0.0341)	0.0736 (0.0308)	0.1647 (0.0721)
HML Portfolio	0.0269 (0.0234)	-0.0988 (0.0372)	0.1837 (0.0335)	0.1996 (0.0786)
Dividend Yield	-0.0066 (0.0021)	0.0051 (0.0033)	-0.0112(0.0029)	0.9775 (0.0069)
3. Correlations/Volatilities				
Market Portfolio	0.1900***			
SMB Portfolio	0.3072***	0.1132**		
HML Portfolio	0.1941**	0.0561	0.1233**	
Dividend Yield	-0.8834***	-0.2055***	-0.3256**	0.0108***
Panel B – Four State Model				
	Market	SMB	HML	Dividend Yield
1. Intercept term				
Regime 1	-0.0475 (0.0021)	-0.0510 (0.0036)	0.0741 (0.0039)	-0.0003 (0.0001)
Regime 2	0.0008 (0.0006)	-0.0111 (0.0011)	0.0007 (0.0010)	0.0006 (1.9e-05)
Regime 3	-0.0072 (0.0009)	0.0107 (0.0012)	-0.0053 (0.0013)	0.0014 (0.0001)
Regime 4	0.0334 (0.0040)	0.0529 (0.0064)	-0.0654 (0.0063)	0.0017 (0.0003)
2. VAR(1) Matrix				
Regime 1				
Market Portfolio	0.1620(0.0263)	0.1307 (0.0380)	-0.0013 (0.0130)	-0.3997 (0.0505)
SMB Portfolio	0.0403 (0.0386)	-0.1406 (0.0634)	0.0838 (0.0486)	0.7428 (0.0834)
HML Portfolio	-0.1276 (0.0506)	0.0028 (0.0183)	-0.0103 (0.0267)	-1.5608 (0.934)
Dividend Yield	-0.0102 (0.0010)	-0.0016 (0.0015)	-0.0057 (0.0015)	1.0851 (0.0020)
Regime 2				
Market Portfolio	-0.0314(0.0129)	-0.0388(0.0257)	-0.0880 (0.0265)	0.3761 (0.0176)
SMB Portfolio	0.1809 (0.0274)	0.0875(0.0419)	-0.0246 (0.0494)	0.3624(0.0340)
HML Portfolio	0.0722(0.0229)	0.0019(0.5185)	0.1700 (0.0425)	-0.0163 (0.0232)
Dividend Yield	0.0012(0.0005)	0.0023(0.0007)	0.0039(0.0008)	0.9708 (0.0006)
Regime 3				
Market Portfolio	0.0174(0.0195)	-0.2286 (0.0379)	0.0396(0.0336)	0.3789 (0.0171)
SMB Portfolio	0.0935 (0.0284)	0.1470(0.0491)	0.0824(0.0436)	-0.1868 (0.0229)
HML Portfolio	0.0081 (0.0201)	-0.1656(0.0575)	0.2892 (0.0452)	0.1814 (0.0261)
Dividend Yield	-0.0028 (0.0010)	0.0113 (0.0019)	-0.0010 (0.0015)	0.9692 (0.0009)

- Significance at 10% level, ** significance at 5%, *** significance at 1%.

Table 4 (continued)

	Panel B (continued)			
	Market	SMB	HML	Dividend Yield
2. VAR(1) Matrix (cont'd)				
Regime 4				
Market Portfolio	-0.0700 (0.0360)	-0.0876 (0.0665)	0.6966 (0.0576)	-0.0653 (0.1197)
SMB Portfolio	0.1300(0.0619)	-0.1537 (0.1167)	0.2831 (0.0901)	-0.5294 (0.1027)
HML Portfolio	0.1701 (0.0641)	-0.2456 (0.1126)	0.1883 (0.0912)	1.3480 (0.1030)
Dividend Yield	-0.0009 (0.0030)	0.0176 (0.0053)	-0.0421(0.0045)	0.9451 (0.0051)
3. Correlations/Volatilities				
Regime 1				
Market Portfolio	0.1710***			
SMB Portfolio	0.2469***	0.1090***		
HML Portfolio	-0.0569	0.1533	0.1149***	
Dividend Yield	-0.9309***	-0.3438***	-0.0046	0.0067***
Regime 2				
Market Portfolio	0.1222***			
SMB Portfolio	0.2447***	0.0840***		
HML Portfolio	-0.2950***	-0.2393***	0.0775***	
Dividend Yield	-0.9408***	-0.2566***	0.2407***	0.0039***
Regime 3				
Market Portfolio	0.1196***			
SMB Portfolio	0.0919***	0.0641***		
HML Portfolio	0.3596***	0.2531***	0.0778***	
Dividend Yield	-0.9234***	-0.1573***	-0.3911***	0.0063***
Regime 4				
Market Portfolio	0.3617***			
SMB Portfolio	0.2762***	0.2174***		
HML Portfolio	0.5997***	-0.3057***	0.2546***	
Dividend Yield	-0.9212***	-0.1393***	-0.5147***	0.0265***
3. Transition probabilities	Regime 1	Regime 2	Regime 3	Regime 4
Regime 1	0.3882 (0.1868)	0.2952(0.1117)	0.0537 (0.0733)	0.2629
Regime 2	0.0638 (0.0541)	0.9282 (0.0670)	1.79e-10 (0.0440)	0.0080
Regime 3	0.0103(0.0727)	0.0135 (0.0106)	0.9118 (0.0696)	0.0644
Regime 4	0.2357(0.1939)	0.0376 (0.0935)	0.2020 (0.1063)	0.5247

* significance at 10% level, ** significance at 5%, *** significance at 1%.

Table 5

Optimal Portfolio Weights under Rebalancing

This table reports optimal weights on the market (Panel A), size (Panel B) and value (Panel C) portfolios as a function of the rebalancing frequency φ for an investor with a coefficient of relative risk aversion of 5. Returns are assumed to be generated by the regime switching model of Table 3. Allocations marked as 'NA' have φ ≥ T and imply portfolio weights identical to the buy-and-hold case. For comparison, portfolio weights under a Gaussian VAR(1) model (where the dividend yield and portfolio returns are set at their unconditional sample mean) are also shown.

Panel A: Market Portfolio

Rebalancing Frequency φ	Investment Horizon T (months)					
	T=1	**T=6**	**T=12**	**T=24**	**T=60**	**T=120**
	Gaussian VAR(1) (Linear Predictability)					
φ = T (buy-and-hold)	0.56	0.58	0.60	0.62	0.82	1.13
Hedging demand	NA	0.14	0.14	0.14	0.14	0.14
	Regime 1					
φ = T (buy-and-hold)	-0.01	0.28	0.36	0.47	0.55	0.59
φ = 24 months	NA	NA	NA	NA	0.53	0.53
φ = 12 months	NA	NA	NA	0.42	0.42	0.42
φ = 6 months	NA	NA	0.32	0.32	0.32	0.32
φ = 3 months	NA	-0.10	-0.13	-0.13	-0.13	-0.13
φ = 1 month	NA	-0.18	-0.18	-0.18	-0.18	-0.18
Hedging demand	NA	0.21	0.21	0.21	0.21	0.21
	Regime 2					
φ = T (buy-and-hold)	2.17	1.42	1.17	0.98	0.79	0.69
φ = 24 months	NA	NA	NA	NA	0.87	0.87
φ = 12 months	NA	NA	NA	1.05	1.05	1.05
φ = 6 months	NA	NA	1.33	1.30	1.30	1.30
φ = 3 months	NA	1.65	1.61	1.56	1.55	1.55
φ = 1 month	NA	2.12	2.12	2.12	2.12	2.12
Hedging demand	NA	-0.05	-0.05	-0.05	-0.05	-0.05
	Regime 3					
φ = T (buy-and-hold)	1.17	0.82	0.74	0.67	0.66	0.65
φ = 24 months	NA	NA	NA	NA	0.69	0.69
φ = 12 months	NA	NA	NA	0.71	0.71	0.71
φ = 6 months	NA	NA	0.78	0.78	0.78	0.78
φ = 3 months	NA	0.91	0.91	0.91	0.91	0.91
φ = 1 month	NA	1.29	1.29	1.29	1.29	1.29
Hedging demand	NA	0.12	0.12	0.12	0.12	0.12
	Regime 4					
φ = T (buy-and-hold)	2.04	0.33	0.40	0.50	0.56	0.62
φ = 24 months	NA	NA	NA	NA	0.54	0.54
φ = 12 months	NA	NA	NA	0.44	0.44	0.44
φ = 6 months	NA	NA	0.39	0.39	0.39	0.39
φ = 3 months	NA	0.84	0.84	0.84	0.84	0.84
φ = 1 month	NA	1.75	1.75	1.75	1.75	1.75
Hedging demand	NA	-0.29	-0.29	-0.29	-0.29	-0.29

Panel B – SMB (size) Portfolio

Rebalancing Frequency φ	Investment Horizon T (months)					
	T=1	T=6	T=12	T=24	T=60	T=120
	Gaussian VAR(1) (Linear Predictability)					
φ = T (buy-and-hold)	-0.57	-0.61	-0.55	-0.49	-0.37	-0.28
Hedging demand	NA	0.47	0.47	0.47	0.47	0.47
	Regime 1					
φ = T (buy-and-hold)	-0.10	0.03	-0.03	-0.11	-0.20	-0.22
φ = 24 months	NA	NA	NA	NA	-0.17	-0.17
φ = 12 months	NA	NA	NA	-0.09	-0.09	-0.09
φ = 6 months	NA	NA	-0.01	-0.01	-0.01	-0.01
φ = 3 months	NA	0.04	0.04	0.04	0.04	0.04
φ = 1 month	NA	-0.01	-0.01	-0.01	-0.01	-0.01
Hedging demand	NA	0.09	0.09	0.09	0.09	0.09
	Regime 2					
φ = T (buy-and-hold)	-2.17	-0.96	-0.68	-0.50	-0.30	-0.28
φ = 24 months	NA	NA	NA	NA	-0.41	-0.41
φ = 12 months	NA	NA	NA	-0.48	-0.48	-0.48
φ = 6 months	NA	NA	-0.91	-0.91	-0.91	-0.91
φ = 3 months	NA	-1.51	-1.47	-1.41	-1.38	-1.38
φ = 1 month	NA	-2.16	-2.16	-2.16	-2.16	-2.16
Hedging demand	NA	0.01	0.01	0.01	0.01	0.01
	Regime 3					
φ = T (buy-and-hold)	-0.89	-0.54	-0.43	-0.32	-0.28	-0.27
φ = 24 months	NA	NA	NA	NA	-0.30	-0.30
φ = 12 months	NA	NA	NA	-0.35	-0.35	-0.35
φ = 6 months	NA	NA	-0.49	-0.49	-0.49	-0.49
φ = 3 months	NA	-0.71	-0.71	-0.71	-0.71	-0.71
φ = 1 month	NA	-0.96	-0.96	-0.96	-0.96	-0.96
Hedging demand	NA	-0.07	-0.07	-0.07	-0.07	-0.07
	Regime 4					
φ = T (buy-and-hold)	0.72	0.31	0.14	0.00	-0.13	-0.17
φ = 24 months	NA	NA	NA	NA	-0.04	-0.04
φ = 12 months	NA	NA	NA	0.10	0.10	0.10
φ = 6 months	NA	NA	0.26	0.26	0.26	0.26
φ = 3 months	NA	0.45	0.45	0.45	0.45	0.45
φ = 1 month	NA	0.67	0.67	0.67	0.67	0.67
Hedging demand	NA	-0.05	-0.05	-0.05	-0.05	-0.05

Panel C – HML (Book-to-market) Portfolio

Rebalancing Frequency φ	Investment Horizon T (months)					
	T=1	T=6	T=12	T=24	T=60	T=120
			Gaussian VAR(1) (Linear Predictability)			
φ = T (buy-and-hold)	-0.01	-0.01	0.03	0.12	0.26	0.39
Hedging demand	NA	-0.39	-0.39	-0.39	-0.39	-0.39
			Regime 1			
φ = T (buy-and-hold)	0.02	0.22	0.20	0.16	0.12	0.12
φ = 24 months	NA	NA	NA	NA	0.14	0.14
φ = 12 months	NA	NA	NA	0.19	0.19	0.19
φ = 6 months	NA	NA	0.23	0.23	0.23	0.23
φ = 3 months	NA	0.24	0.24	0.24	0.24	0.24
φ = 1 month	NA	0.03	0.03	0.03	0.03	0.03
Hedging demand	NA	0.01	0.01	0.01	0.01	0.01
			Regime 2			
φ = T (buy-and-hold)	-0.42	-0.15	0.00	0.09	0.12	0.13
φ = 24 months	NA	NA	NA	NA	0.10	0.10
φ = 12 months	NA	NA	NA	-0.05	-0.05	-0.05
φ = 6 months	NA	NA	-0.07	-0.06	-0.06	-0.06
φ = 3 months	NA	-0.18	-0.15	-0.11	-0.11	-0.11
φ = 1 month	NA	-0.44	-0.45	-0.45	-0.45	-0.45
Hedging demand	NA	-0.02	-0.02	-0.02	-0.02	-0.02
			Regime 3			
φ = T (buy-and-hold)	0.23	0.10	0.09	0.10	0.13	0.15
φ = 24 months	NA	NA	NA	NA	0.13	0.13
φ = 12 months	NA	NA	NA	0.11	0.11	0.11
φ = 6 months	NA	NA	0.11	0.09	0.09	0.09
φ = 3 months	NA	0.00	0.00	0.00	0.00	0.00
φ = 1 month	NA	0.25	0.25	0.25	0.25	0.25
Hedging demand	NA	0.02	0.02	0.02	0.02	0.02
			Regime 4			
φ = T (buy-and-hold)	-0.10	0.40	0.32	0.24	0.18	0.14
φ = 24 months	NA	NA	NA	NA	0.24	0.24
φ = 12 months	NA	NA	NA	0.29	0.29	0.29
φ = 6 months	NA	NA	0.41	0.41	0.41	0.41
φ = 3 months	NA	0.50	0.51	0.52	0.52	0.52
φ = 1 month	NA	0.01	0.01	0.01	0.01	0.01
Hedging demand	NA	-0.09	-0.09	-0.09	-0.09	-0.09

Table 6

Effects of Parameter Estimation Uncertainty

This table reports 95% (asymptotic) confidence intervals for a buy-and-hold investor's optimal portfolio weights at different investment horizons, T, assuming a constant relative risk aversion coefficient of 5. Intervals are calculated by the delta method. Under regime switching, portfolio returns are assumed to be generated by the model

$$r_t = \mu_{s_t} + \varepsilon_t$$

where $\boldsymbol{\mu}_{s_t}$ are the intercepts in state s_t and $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \Sigma_{s_t})$ is the vector of return innovations.

		Investment Horizon T				Investment Horizon T			
		T=1	T=6	T=60	T=120	T=1	T=6	T=60	T=120
		A: Allocation to the Market Portfolio				B: Allocation to the SMB (Size) Portfolio			
I.I.D.	Mean + 2 SD	-0.30	-0.30	-0.30	-0.30	0.95	0.95	0.95	0.95
	Mean	-0.49	-0.49	-0.49	-0.49	0.81	0.81	0.81	0.81
	Mean - 2 SD	-0.68	-0.68	-0.68	-0.68	0.67	0.67	0.67	0.67
Regime 1	Mean + 2 SD	0.43	0.58	0.79	0.83	0.70	0.51	0.22	-0.02
	Mean	-0.01	0.28	0.55	0.59	-0.10	0.03	-0.20	-0.22
	Mean - 2 SD	-0.45	-0.02	0.31	0.35	-0.90	-0.45	-0.62	-0.42
Regime 2	Mean + 2 SD	2.41	2.02	1.15	0.97	-3.17	0.28	0.18	0.16
	Mean	2.17	1.42	0.79	0.69	-2.17	-0.96	-0.30	-0.28
	Mean - 2 SD	1.97	0.82	0.43	0.41	-1.17	-2.20	-0.78	-0.72
Regime 3	Mean + 2 SD	1.79	1.10	0.90	0.81	-0.07	0.08	0.16	0.15
	Mean	1.17	0.82	0.66	0.65	-0.89	-0.54	-0.28	-0.27
	Mean - 2 SD	0.55	0.54	0.42	0.39	-1.71	-1.16	-0.72	-0.69
Regime 4	Mean + 2 SD	3.40	0.69	0.80	0.88	2.04	0.81	0.25	0.21
	Mean	2.04	0.33	0.56	0.62	0.72	0.31	-0.13	-0.17
	Mean - 2 SD	0.68	-0.03	0.32	0.56	0.60	-0.19	-0.51	-0.21
		C: Allocation to the HML (Book-to-Market) Portfolio				D: Allocation to T-bills			
I.I.D.	Mean + 2 SD	0.14	0.14	0.14	0.14	0.80	0.80	0.80	0.80
	Mean	-0.00	-0.00	-0.00	-0.00	0.68	0.68	0.68	0.68
	Mean - 2 SD	-0.14	-0.14	-0.14	-0.14	0.56	0.56	0.56	0.56
Regime 1	Mean + 2 SD	0.60	0.60	0.56	0.56	1.66	1.09	1.10	1.11
	Mean	0.02	0.22	0.12	0.12	1.08	0.47	0.52	0.53
	Mean - 2 SD	-0.56	-0.26	-0.32	-0.32	0.50	-0.15	-0.06	-0.05
Regime 2	Mean + 2 SD	1.08	0.67	0.58	0.59	2.72	1.29	0.99	0.97
	Mean	-0.42	-0.15	0.12	0.13	1.42	0.69	0.39	0.39
	Mean - 2 SD	-1.92	-0.97	-0.34	-0.33	0.12	0.09	-0.21	-0.19
Regime 3	Mean + 2 SD	1.41	0.66	0.57	0.59	2.00	1.51	1.08	1.07
	Mean	0.23	0.10	0.13	0.15	0.48	0.63	0.48	0.47
	Mean - 2 SD	-0.95	-0.46	-0.31	-0.29	-1.04	-0.29	-0.12	-0.13
Regime 4	Mean + 2 SD	1.44	0.90	0.62	0.58	0.12	0.64	0.93	0.96
	Mean	-0.10	0.40	0.18	0.14	-1.66	-0.04	0.39	0.40
	Mean - 2 SD	-1.64	-0.10	-0.26	-0.30	-3.44	-0.72	-0.15	-0.16

Figure 1

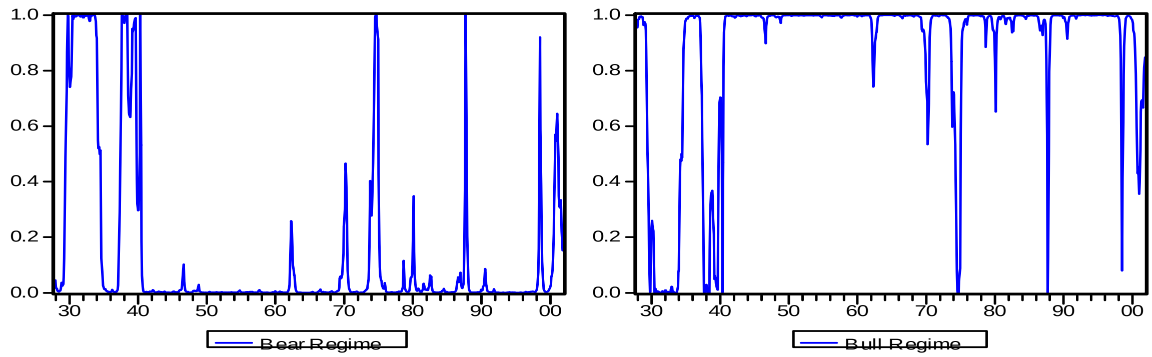
Smoothed State Probabilities for Regime Switching Models fitted to Individual Stock Portfolios

The graphs plot smoothed state probability estimates for the Markov switching model

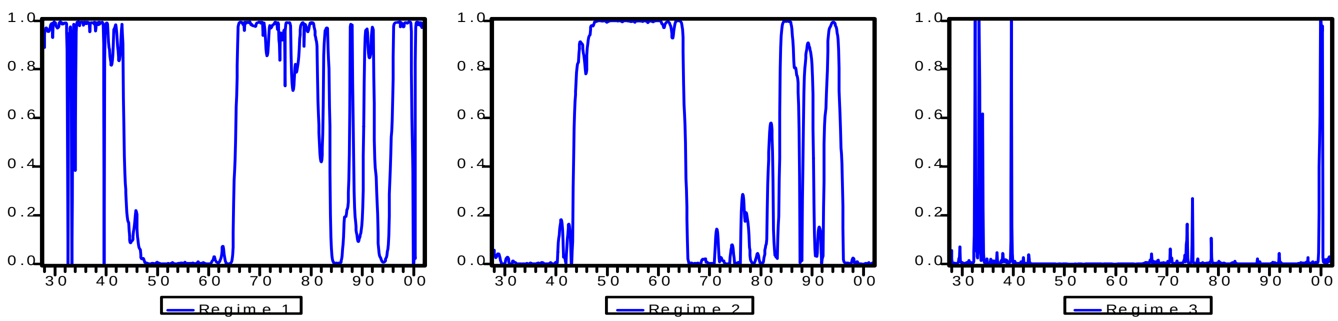
$$r_t = \mu_{s_t} + \sum_{j=1}^{p} a_{j,s_t} r_{t-j} + \sigma_{s_t} \varepsilon_t$$

where μ_{s_t} is the intercept vector in state S_t, a_{j,s_t} is the j-th order autoregressive coefficients in state S_t and $\varepsilon_t \sim N(0, \sigma^2_{s_t})$. The unobserved state variable S_t is governed by a first-order Markov chain that can assume *k* distinct values. The sample period is 1927:12 – 2001:12.

Market Portfolio – 2-state Model



SMB Portfolio – 3-state Model



HML Portfolio – 3-state Model

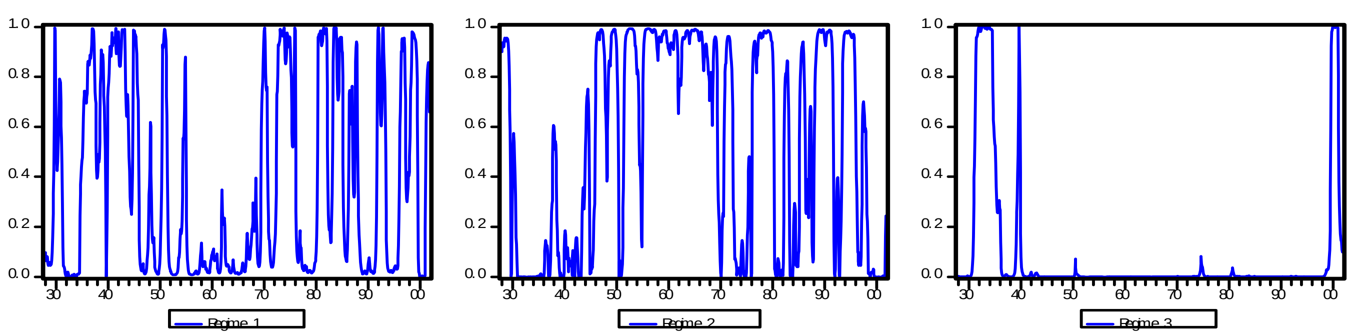


Figure 2

Smoothed State Probabilities: Four-State Model for SMB, HML and Market Portfolio Returns

The graphs plot the smoothed probabilities of regimes 1-4 for the multivariate Markov switching model comprising monthly return series on SMB and HML portfolios and excess returns on the value-weighted market portfolio. The sample period is 1927:12 – 2001:12. Parameter estimates underlying these plots are reported in Table 3.

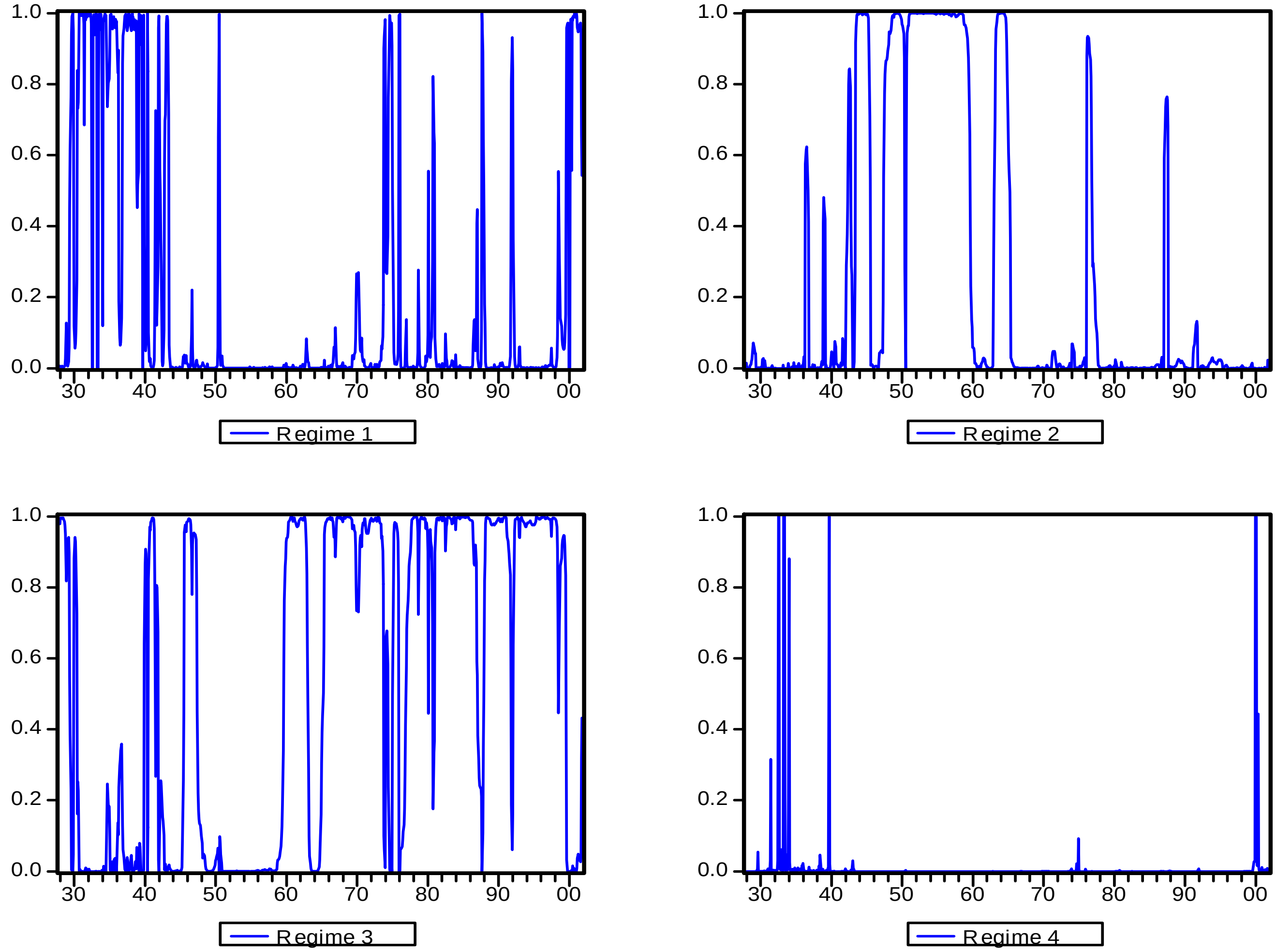


Figure 3

Smoothed State Probabilities: Four-State Model for Equity Returns and the Dividend Yield

The graphs plot the smoothed probabilities of regimes 1-4 for the multivariate Markov switching model comprising monthly return series on the SMB and HML portfolios, the value-weighted market portfolio and the dividend yield. The sample period is 1927:12 – 2001:12. Parameter estimates underlying these plots are reported in Table 4.

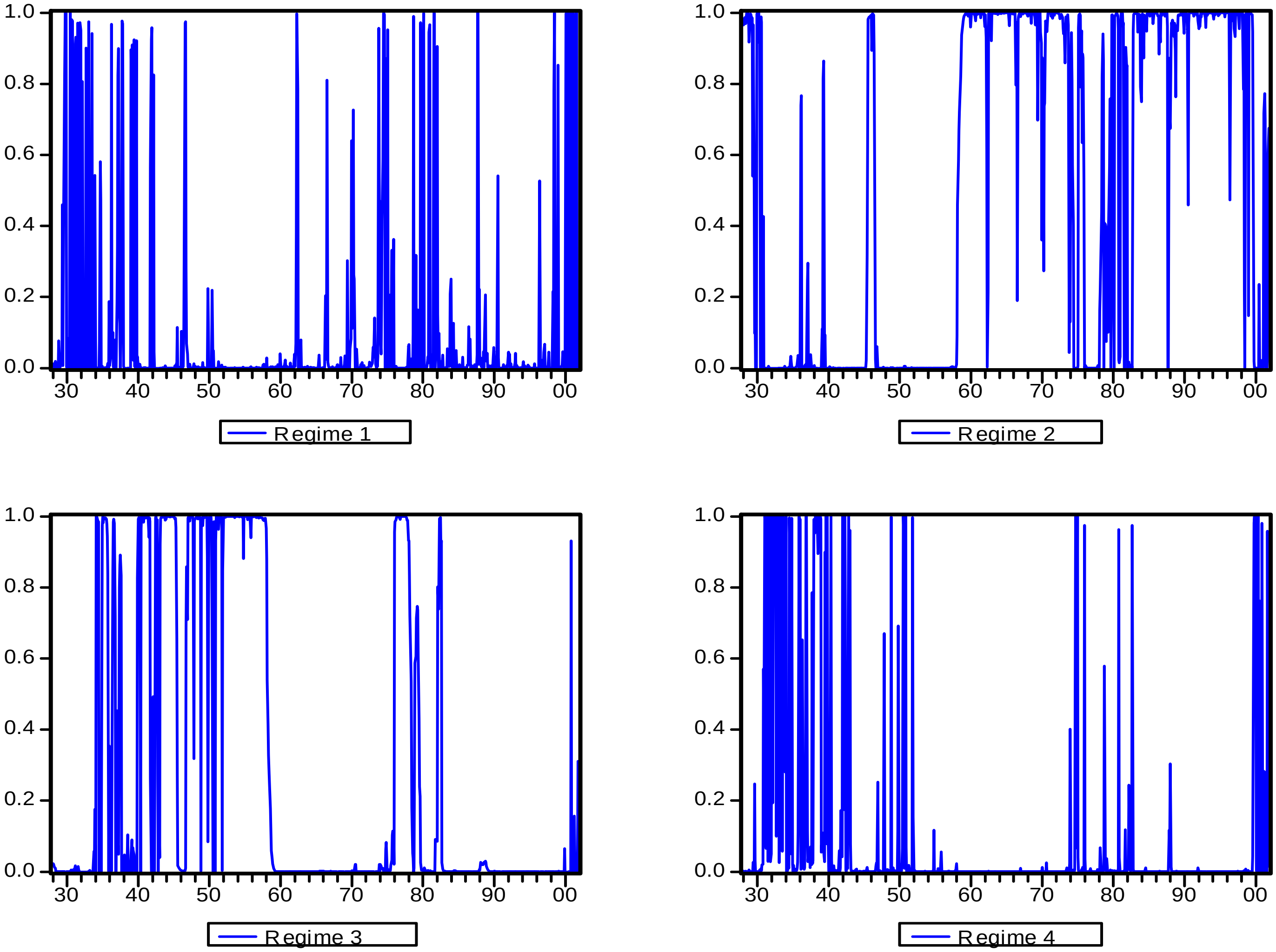


Figure 4

Optimal Asset Allocation as a Function of the Investment Horizon

The graphs show the optimal allocation to equity portfolios (market, SMB and HML) and risk-free T-bills under a four-state regime switching model as a function of the investment horizon for an investor with constant coefficient of relative risk aversion $\gamma = 5$.

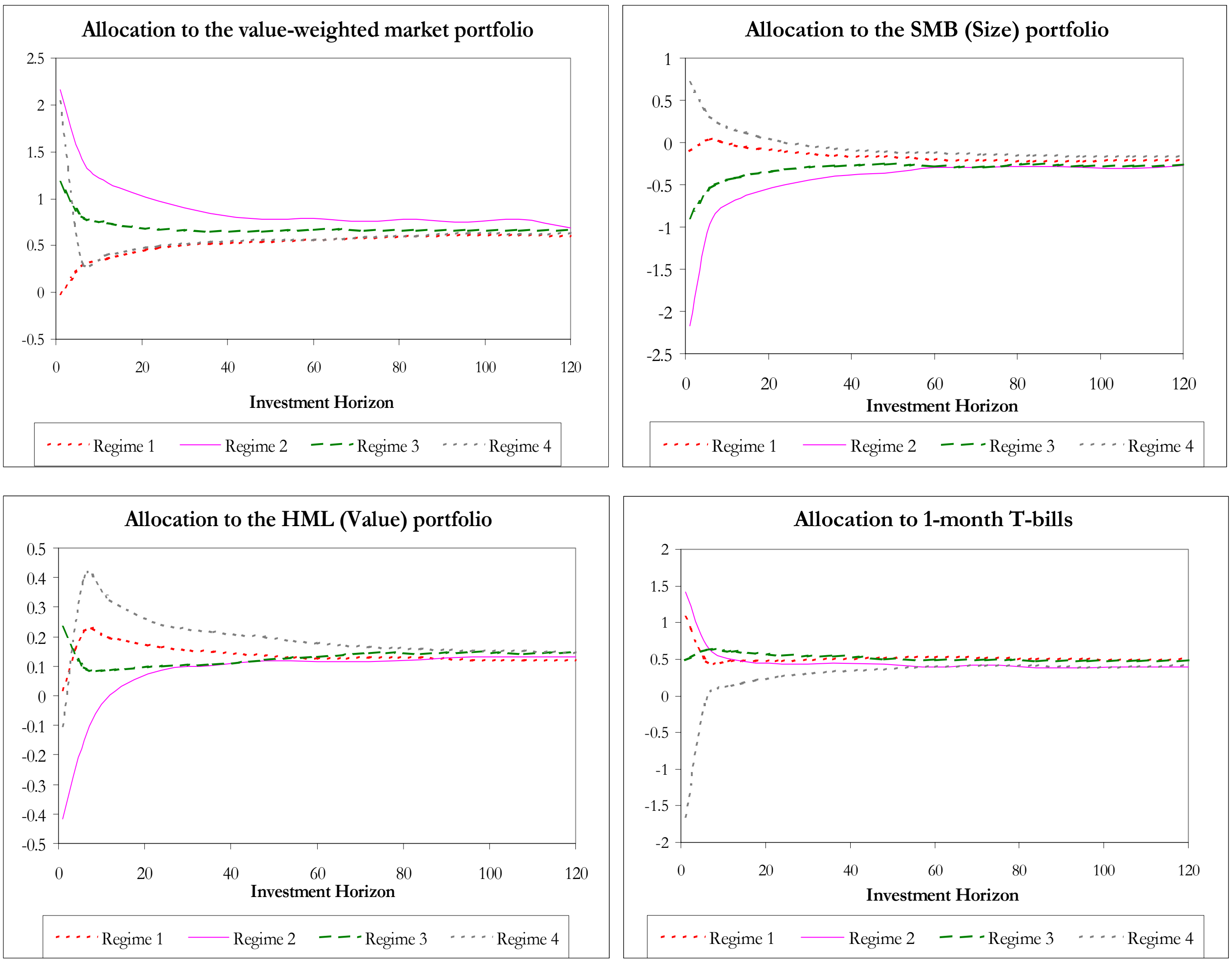


Figure 5

Optimal Asset Allocation under Predictability from the Dividend Yield

The graphs show the optimal allocation to equity portfolios (market, SMB and HML) and risk-free T-bills under a four-state regime switching model in which the dividend yield predicts portfolio returns as a function of the investment horizon for an investor with constant coefficient of relative risk aversion $\gamma = 5$.

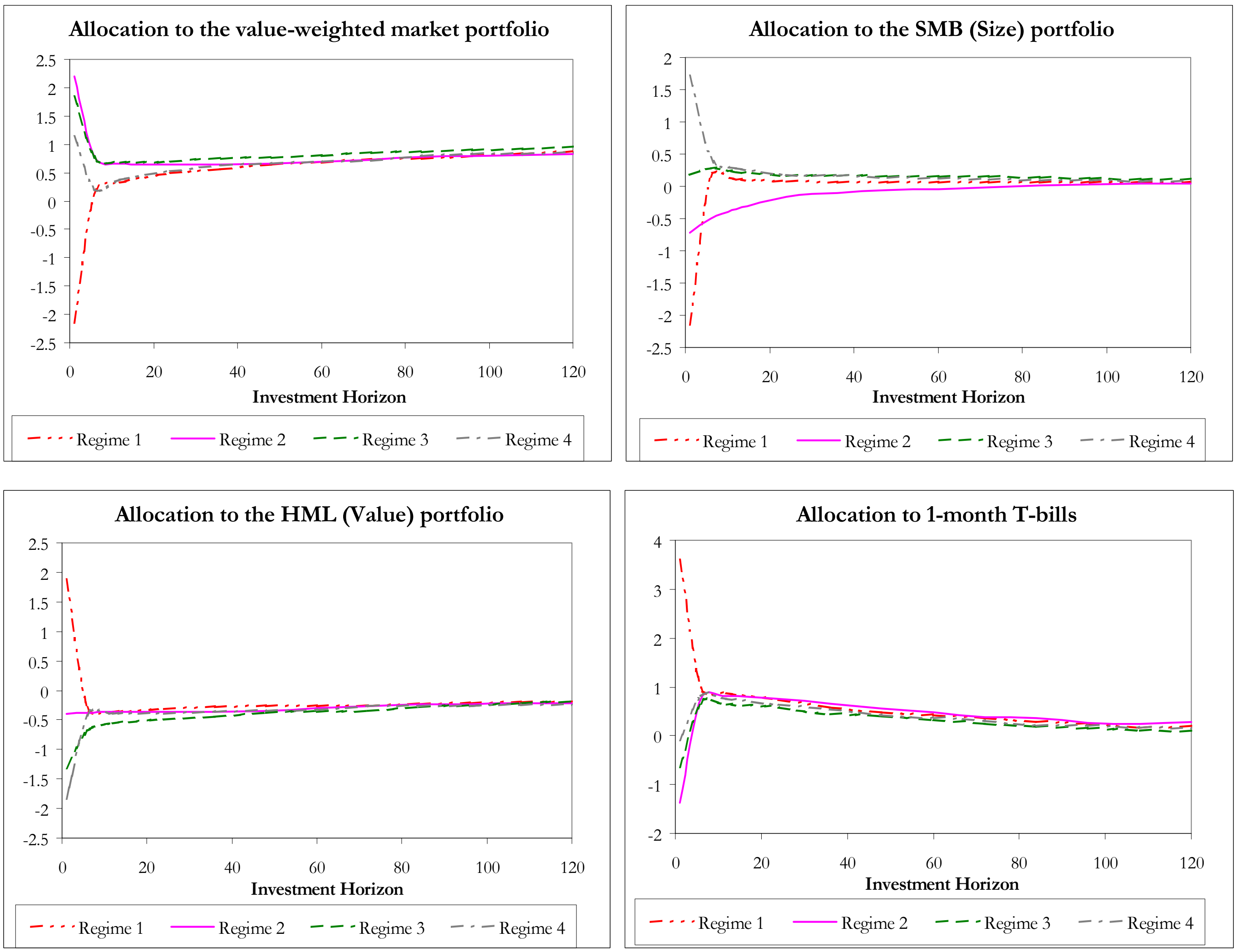


Figure 6

Comparison of Optimal Asset Allocation Across Models

The graphs show the optimal allocation to equity portfolios (market, SMB and HML) and risk-free T-bills as a function of the investment horizon for an investor with constant coefficient of relative risk aversion $\gamma = 5$. The VAR(1) model assumes predictability from the dividend yield. The MS model assumes the presence of four states while the MS-VAR(1) model allows for four regimes and predictability from the dividend yield.

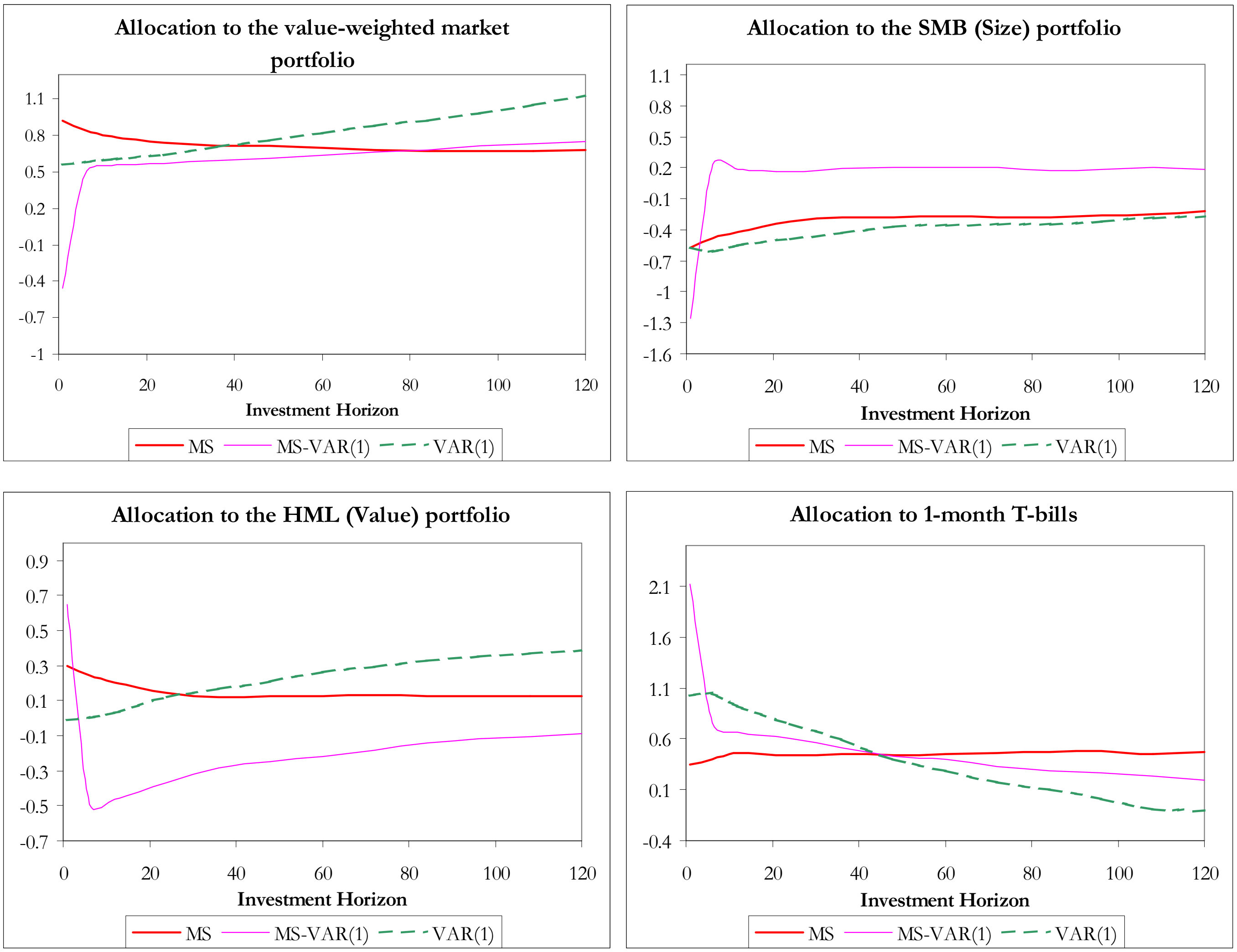


Figure 7

Welfare Costs of Ignoring Regimes

This graph shows the compensation required for a buy-and-hold investor with power utility (γ = 5) to be willing to ignore regimes in asset returns surrounded by 95% confidence bands computed using the delta method.