AN EMPIRICAL ANALYSIS OF COMMODITY PRICING

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Abstract

Commodity pricing models generally explain the link between commodity prices and stock levels in terms of a stock-out constraint or a convenience yield. Analysis of these links is provided using monthly London Metals Exchange copper, lead and zinc prices obtained for the period November 1964 to December 2003. A Markov model, fitted to these data, supports the existence of two distinct pricing regimes while the impact of convenience yields is also identified with expansion of the model to include LME stocks.

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There are two models generally used to explain variation in commodity prices over time. The first focuses on the impact of stock-outs, modelled in Scheinkman and Schechtman (1983),[1] and the second is based on concept of convenience yields as discussed in Kaldor (1939).[2] While much of the research to date has focused on one or the other of these models, Ng and Ruge-Murcia (2000) and Routledge, Seppi, and Spatt (2000) combine the two models with some improvement in performance.

There is very little evidence of attempts to test the underlying characteristics of these models in a maximum likelihood setting. A major contribution of this paper is the application of Hamilton's (1989, 1990 and 1994) markov switching model to fit these models to monthly London Metal Exchange (LME) copper, lead and zinc prices collected over the period from November 1964 to December 2003. These commodities are important [3] in the world economy and while they can be stored for considerable periods of time, their prices can be quite volatile.

Evidence, provided in this paper, supports the existence of two regimes in the LME copper, lead and zinc prices. A stock variable is also included in the analysis to assess the impact of convenience yields and the stock variable coefficient estimates support the existence of convenience yield effects. Finally, the level of serial correlation in the

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[1] Also see Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) and Wright and Williams (1989) for further analysis of these models.

[2] Further discussion is provided in Brennan (1958), Stein (1961), Telser (1958) and Working (1949).

[3] Copper is well known for its use in electrical goods and in plumbing and zinc has a range of uses including in paints, in galvanizing other metals and in die-casting. Further, both copper and zinc are used in the manufacture of coins and metal currency. Lead is used in batteries, radiation shielding, cable covering, ammunition, plumbing, and in the manufacture of glass.

estimated model suggests that there still some way to go in fully explaining commodity prices, though convenience yields and stock-out effects are statistically important. While a review of the literature is provided in the next section, the data is described in the Section 3. Section 4 is devoted to analysis of the data and a summary is provided in section 5.

1. Literature Review

Keynes (1950) provides one of the early discussions of the behavior of commodity prices and the relationship between commodity price, production and stock levels. If we ignore hedging costs (Telser (1958)), then Keynes' argument is simply that when stocks are high the difference between futures prices and the underlying asset price (spot price) reflects the cost of storing or carrying the underlying asset but when stocks are low commodity prices tend to reflect the value of immediate consumption and the link to the value in storage is broken. For example, spot price could exceed the futures price when stocks are low.[4] Arguments relating to the impact of stock-outs on commodity prices are further clarified and extended in Scheinkman and Schechtman (1983) with additional modelling and testing evident in the work of Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) Routledge, Seppi, and Spatt (2000) and Wright and Williams (1982, 1984, 1989). One element that has gained some attention in this literature is the definition of stock-out. There can be considerable quantities of stock in manufacturing

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[4] For spot price to exceed futures price it is implicit that the market expects that there is sufficient time prior to futures contract maturity for production to adjust to the current underlying asset shortage.

that is stored on the factory floor as part of work in process. Considerable quantities may be found on conveyor belts, barges, lighters, and trucks or in specific storage areas and much of this stock is not available for immediate sale or purchase. Thus, while a zero stock constraint applies in the economic models, it may be difficult to determine when a stock-out has actually occurred in the market place.

Convenience yields provide another explanation for the changes in spot prices that occur when stocks are low. The convenience yield is said to arise from the benefit that producers obtain from physically holding stocks, a benefit not available to individuals holding a long futures or forward contract. The benefits are generally couched in terms of the value to the producer of "smoothing production, avoiding stock-outs and facilitating the scheduling of production and sales" (Pindyck (1993), p. 511) though there is no need to restrict the benefit in terms of final production as the stocks could be sold into the market rather than used in production if this were economically justified. Thus an alternative explanation for holding stocks is the existence of a sales timing option that accrues to the stockholder. The firm can always choose between selling existing commodity stocks or using the commodity in production of finished goods.

Convenience yield effects do not accrue to a futures contract holder and this proves useful in identifying the existence of convenience yields. It is generally observed that when stocks are low, commodity futures prices do not follow the spot price, or indeed earlier maturing futures contracts, as closely as the simple cost of carry relationship suggests. Thus the convenience value attached to holding stocks during periods of commodity shortage may explain variation in prices not explained by the storage cost model.[5]

While much of the early literature focuses on describing convenience yields and fitting non-linear functions to the data there is little evidence of economic modelling of convenience yields except perhaps for Weymar (1966). Nevertheless, option based models have been developed by Heinkel, How and Hughes (1990), Litzenberger and Rabinowitz (1995) and Milonas and Thomadakis, (1997a and 1997b) to explain the convenience yield effect in terms of a simple timing option. The producer (stock holder) holds a put option on the stored commodity that gives them the right to sell the commodity at a price at least equal to the marginal cost of production at some future time. The combination of this put option and the underlying stock holding creates a call option whose value is increasing in commodity price, much like the convenience yield discussed in the earlier literature.

If the impact of convenience yields is to be identified then the cost of carry pricing model is a useful starting point for analysis.[6] In its basic form this model captures the storage value noted by Keynes, with the futures price, F_{tT}, quoted at time t for a contract maturing at time T, expressed in terms of the underlying commodity price, P_t, quoted at time t, and the costs of storage which include, r, the continuously

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[5] Brennan (1958), Fama and French (1988), Kaldor (1939), Ng and Pirrong (1994), Pindyck (1993, 1994, 2002, 2003), Stein (1961), Telser (1958) and Working (1949).

[6] This is a forward contract pricing model. Although the use of a forward pricing model to value futures contracts can lead to errors in pricing arising from marking to market adjustments (Cox, Ingersoll and Ross, 1981) Pindyck (1994) shows that this error is economically small for LME metals. As a result the futures/forward difference is ignored in the following discussion and analysis.

compounding risk free rate of return for the period *t* to *T*, the physical costs of storage, *s*, continuously compounding for the period *t* to *T* and the exponential function term, *exp(.)*. The convenience yield, *cy*, is included for the period from time *t* to *T* in the model below though it should be noted that neither Keynes (1950) nor Scheinkman and Schechtman (1983) recognized convenience yields. The cost of carry model, adjusted for convenience yield, takes the form:

$$F_{tT} = P_t \exp\left(\left(r + s - cy\right) \times \left(T - t\right)\right) \qquad (1)$$

This can be rearranged to give the interest-adjusted basis (Fama and French (1988)), later used in analysis. Given natural logs, *ln(.)*, the interest adjusted basis is defined as:

$$IAB_{tT} = \ln\left(P_t / F_{tT}\right) + r = cy - s \qquad (2)$$

Drawing on the cost of carry model and extending it to deal with the impact of stock-outs, Scheinkman and Schechtman (1983) show that it is possible to model commodity prices in terms of two pricing regimes, value in consumption and value in storage, much like the process that Keynes described. The process driving the underlying commodity price is written as:

$$P_t = \max\left(E_t P_T \exp\left(-r - s\right), P_t\{x\}\right) \qquad (3)$$

where $P_t\{x\}$ is the commodity price in the market for immediate consumption given x units of commodity are available in the market. If we assume a risk neutral world then the futures price, F_{tT}, is equal to the expected spot price, E_tP_T and so it is possible to rewrite the relationship as:

$$P_t = \max\left(F_{tT} \exp(-r-s), P_t\{x\}\right) \tag{4}$$

Considerable measurement difficulties may arise when analysing the two pricing models based on stock-outs and/or convenience yields when using readily available aggregate price and aggregate stock data. Mathematically, stock-outs are simple to define though in reality this is a complex measurement question (Wright and Williams, 1989). Commodity stocks are often spread across the globe and though the LME is based in London the stock locations are not. For example it is possible in commodity markets for a stock-out to occur in one region with a consequent explosion of spot prices for delivery in that region with little or no effect elsewhere. The problem for the researcher lies with the tendency for recorded prices to reflect the average price and for recorded stocks to reflect total stocks regardless of location. For example, a simple average of prices taken across all markets for a commodity may suggest stock-out behavior even though there may be considerable stocks available in all but one region. This limitation should be noted in the following analysis though it should also be noted that the LME has an active warrant market designed to deal with stock location mismatches and the costs and time required for shipping would not preclude arbitrage where sufficient stocks exist on one area to meet shortages in other locations. The only restriction in this case is the time taken to move the stock from location with excess stocks to the stock-out location. Of course when stock is not available in another location, then it may take a considerable period to mine, process and ship the additional commodity to meet this demand.[7]

2. Data

There are a number of reasons for using LME data. First, both spot price and futures price are traded in the same market and quoted prices are obtained from the same trading session, the midday trading session. There is a considerable time series of prices and stock levels available through the Metal Bulletin and the LME web site. Few commodity markets provide the same access to matched spot and futures price data. Second, aggregate stock information is available and reported by the LME in the Metal Bulletin and on the LME web site. Third, the LME handles most of the world trade in the commodities that meet LME contract specifications. Clearly, the LME does not handle all trading in non-ferrous metals and its accredited warehouses do not store all stored non-ferrous metals in the world but the LME reported prices are sufficiently influential to be used as a reference rate for non-ferrous metal pricing throughout the world and a large proportion of the world trade in non-ferrous metals takes place through this market. Fourth, LME non-ferrous metal storage provides a measure of world stocks. I am not aware of an alternative measure of world stocks of copper, lead and zinc that is reported on a monthly basis for the period of this study.

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[7] Thus for commodities like copper, lead and zinc the longer term futures contracts, such as the three month contracts used here, are more likely to capture convenience yield effects than shorter term futures contracts.

Finally, LME quoted 3-month futures contracts are useful because it takes time to physically move commodities from mine to warehouse and then to consumer and so 3-month futures contracts are more likely to pick up convenience yield impacts.

Copper, lead and zinc are chosen for analysis because monthly observations of spot prices, 3-month futures prices and stock information are available for these commodities for the period of the study, November 1964 to December 2003. The copper and lead prices are denominated in Great Britain pounds (GBP) and the zinc price is denominated in USA dollars (USD). The spot price and three-month futures contract price are based on the official LME prices determined after the midday trading session each day.[8] Prices are obtained from the Metal Bulletin over the period, November 1964 to December 1988 and from the LME web site for the period from January 1989 to December 2003. All prices supplied on the LME web site are in USD and so, for consistency, the copper and lead prices are converted to GBP using the foreign exchange rates supplied with the LME web site based data. Although there is some variation among the copper and zinc contracts in terms of the spot asset definition (Sephton and Cochrane (1991)), the lead contract is essentially unchanged over the study period. The copper and zinc prices used in this study reflect an average price taken across the various categories of the metal for which prices are reported on the LME. This is not necessary for the lead contract where only one category of lead existed during the study period. Other important non-ferrous

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[8] LME prices are quoted as a representative range. The price used in analysis is the mid-point of this representative range of prices traded during the trading session.

metals such as tin, nickel and aluminium were excluded because prices were not available over the full period of the study.

Descriptive statistics are reported in Panel A of Table 1 for the commodity spot and futures prices as well as for stocks and the interest adjusted basis. Figures 1, 2 and 3 provide an indication of the range of values that both the spot and futures prices exhibit as well as the tight linkage that exists between the futures and the spot price. While copper spot prices vary from 341 GBP per tonne to 1946 GBP per tonne, lead varies from 79 GBP per tonne to 641 GBP per tonne and Zinc varies from 96 USD per tonne to 2050 USD per tonne. The futures prices show similar levels of variation. The average 3-month interest-adjusted basis (effective 12-month interest-adjusted basis) estimate is 0.021 (8.4% pa) for copper, 0.015 (6.0% pa) for lead and 0.011 (4.4% pa) for zinc. Stocks vary considerably over the period with a minimum of around 4700 tonne for copper, 2510 tonne for lead and 300 tonne for zinc and maximums of over 972,000 tonne for copper, 372,000 tonne for lead, and 1,234,000 tonne for zinc.

[Insert Table 1 and Figures 1, 2 and 3 about here]

Time series statistics are also provided in Panels B and C of Table 1 for levels, change in levels and squared levels with first order and 10th order correlation coefficients, AR (1) and AR (10), and chi-square test probabilities for serial correlation at lags 10 and 20. There is evidence of serial correlation in levels, change in levels and in squared levels suggesting that there is considerable serial correlation in the prices and the changes in prices along with time changing variance.

Three unit root test statistics are reported in Panel D of Table 1 and these include the Phillips-Perron (1988), the Augmented Dickey Fuller (1979, 1981) and the Kwiatowski, Phillips, Schmidt and Shin (1992) test. Results, given 10 lags, are reported in Table 1 though lag length has little impact on the Phillips-Perron and the Augmented Dickey Fuller results. While a unit root null underlies the Phillips-Perron and Augmented Dickey Fuller tests, a stationary null applies to the Kwiatowski, Phillips, Schmidt and Shin test. Spot and futures prices, stock levels and interest rate series appear to be non-stationary. For example, the prices, stock levels and interest rates all exhibit first order autocorrelation coefficients that are very close to one. Further, the null of a non-stationary process in the Phillips Perron and Augmented Dickey Fuller tests is rejected on only a couple of occasions. Rejection of the null for the Kwiatowski, Phillips, Schmidt and Shin tests occurs in all cases for these variables. The results for the interest-adjusted basis suggest that this is a stationary variable. The first order autocorrelation coefficients for the interest-adjusted basis are somewhat lower and the null of unit root is rejected for both the Phillips-Perron and the Augmented Dickey Fuller tests in all cases though the Kwiatowski, Phillips, Schmidt and Shin test is rejected for lead. Thus, while there is some contradiction for lead, it would seem reasonable to assume that the interest-adjusted basis is stationary for each of the currencies for the purposes of this study.

While Figures 1, 2 and 3 highlight the strong linear relationship between the spot price and futures prices for each of the commodities, copper, lead and zinc there is also increased variance in the relationship as price levels increase. Figures 4, 5 and 6 compare the level of stocks with the interest-adjusted basis and this shows that when stocks are high the interest-adjusted basis is close to zero and comparatively stable. When stocks are low the convenience yield is much more volatile and its magnitude tends to increase. There are a number of spikes in the price series consistent with non-ferrous metal shortages. Perhaps the most obvious are those occurring during the period of the Vietnam war (1960s-1973) with spikes appearing particularly in the periods from 1964 to 1970 and from 1972 to 1973.[9] There is also considerable volatility in prices from 1984 to 1990. This was a period of high non-ferrous metal consumption and was marked with uncertainties associated with Glasnost (USSR) and civil unrest in a number of countries in Africa, including South Africa (1983-1994). The dramatic build up of stocks in the first half of the 1990s seems to be driven by the break down of the USSR.

[Insert Figures 4, 5 and 6 about here]

Interest rates are obtained for USD (used for zinc) and for the GBP (used for copper and lead). The UK interest rate series consists of the minimum lending bank rate from November 1964 to December 1975, obtained from the Bank of England web site (www.bankofengland.co.uk), and the Euro Currency (London) Sterling 3 month middle rate obtained from Datastream from January 1976 to December 2003. The US interest

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[9] The Indonesian war (1965-1966) also occurred during this period.

rate series consists of the three month treasury bill secondary market rates obtained from the USA Federal Reserve Board (www.federalreserve.gov) for the period from November 1964 to December 1975 and the Euro Currency (London) USD 3 month middle rate obtained from Datastream is used for the remainder of the period through to December 2003. The rates are graphed in Figure 7.

[Insert Figure 7 about here]

3. Analysis

3.1. Interest-adjusted basis regime Switches

We ignore the impact of convenience yields in this section and, instead, focus on testing how well the stock-out model fits the data. If we assume that commodity price reflects either the cost of carry or the value in immediate consumption then it should be possible to model the price distribution as a Markov process with two states of the world, the storage value state and the consumption value state, with each state having a separate distribution. Given equation (4) we restate the pricing function in terms of the interest-adjusted basis (dividing through by the futures price, taking natural logs and adding the risk free rate to both sides). This model suggests a two state process for the interest-adjusted basis.[10]

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[10] We ignore the impact of convenience yields at this stage.

$$IAB_{tT} = \max\left(-s, \ln\left(\frac{P_t\{x\}}{F_{tT}} \right) + r \right) = \max\left(-s, IAB^{*}_{tT} \right) \qquad (5)$$

The term, IAB^{*}_{tT}, is based on the immediate consumption value of commodity. This suggests that in any period, prices are drawn from one of two possible distributions, either the storage-based distribution, state $S_t=1$, or the value-based distribution, state $S_t=2$. The means for the two distributions are $\mu(S_t=1) = -s$ which is the negative of the physical storage cost rate and applies in the state where commodity price reflects the value in storage and $\mu(S_t=2) = IAB_{tT}*$ which applies in the state where price reflects the value of immediate consumption. Given the definition of the price process we expect that $\mu(S_t=1) < \mu(S_t=2)$, Further, Fama and French (1988) and Ng and Pirrong (1994) observe that the variance in low stock periods is greater than the variance in high stock periods and so the variance is defined as $\sigma(S_t)$, with $\sigma(S_t=1)$ for state 1 and $\sigma(S_t=2)$ for state 2 with $\sigma(S_t=1) < \sigma(S_t=2)$.

Rather than rely on available stock data to identify the change dates in the distribution, we fit a Hamilton (1989, 1990, 1994) two state regime-switching model to the interest-adjusted basis data. This avoids the problem highlighted by Wright and Williams (1989) who argue that stock levels may not accurately reflect the existence of stock-outs for the purposes of testing the stock-out model. The aggregate nature of the price data ensures that only substantial stock-out effects are identified and so this technique should provide a fairly conservative picture of changes in commodity price distributions over time. Further, with application of Hamilton's method there is no decision made, a priori, about the price distribution coefficient values. The only structure imposed by the model is the requirement that there are two commodity pricing states, consistent with equation (5) above, along with fairly standard assumptions about the residuals. Hamilton's model is defined as follows:

$$IAB_{tT} = \mu(S_t) + \sum_{i=1}^{n} \phi_i \left(IAB_{t-iT} - \mu(S_{t-i})\right) + \sigma(S_t)\varepsilon_t \tag{6}$$

where $\varepsilon_t \sim i.i.d.N(0,\sigma^2)$ and two underlying states of the world exist with separate distributions. The model specifically allows for serial correlation in the interest adjusted basis values using a simple autoregressive structure. Hamilton defines a variable, s_t, as the outcome from a 2^n-state Markov chain with s_t independent of ε_t for all t and τ. Although there are two underlying states, $S_t=1$ and $S_t=2$, with n-lag terms in the model, it is possible to attain the current state in 2^n possible ways (Hamilton (1994)). For example if there were two underlying states and one lag in the model, it is possible to enter the current state from state 1 or from state 2 and so given that the current state is either 1 or 2 there are 4 possible combinations of the current state and the past state. To identify the current state and the relationship with past states we write each state as a vector of ones or twos with the first entry referring to the current state, the second entry referring to the previous period state, and so on.

$$s_t = \begin{cases} 1\, if\ (1,1,\ldots,1,1) \\ 2\, if\ (1,1,\ldots,1,2) \\ \quad\vdots \\ n\, if\ (2,2,\ldots 2,2) \end{cases} \tag{7}$$

Thus, the vector (1,1,…,1,1) identifies the event where prices are drawn from the state 1 distribution for the current state and all previous states that have an impact on the current realization of the interest-adjusted basis. Similarly the vector (1,1,…,1,2) is the event where the price was drawn from the state 2 distribution n lags ago and from the state 1 distribution since then.

The model is estimated separately for the three commodities, copper, lead and zinc and the coefficient estimates are reported in Panel A of Table 2. The probability of being in state one is graphed in Figures 8, 9 and 10. Due to the existence of serial correlation in the residuals it was necessary to include lagged values of the interest-adjusted basis with the final lag choice resulting from a search beginning with a maximum of four lags and dropping statistically insignificant lags as long as there is no residual serial correlation. This results in the inclusion of two lags for copper and three lags for both lead and zinc. As indicated in Panel B of Table 2, there is no evidence of serial correlation once lagged terms were included in the model.

[Insert Table 2 and Figures 8, 9 and 10 about here]

Hamilton's model appears to fit the data reasonably well. For example tests for equality of means and equality of the standard deviations across the two states are rejected for all three commodities (See Panel B of Table 2). Given the observed GARCH effects in the raw data (Table 1, Panel C) it is also important to note that the two state model seems to capture the time changing nature of the variance for both copper and lead, though there is still some residual GARCH effects for zinc.

Thus two states are identified in the data. The first state exhibits a statistically significantly lower mean and standard deviation when compared with the second state. This seems consistent with the existence of a value in storage state and an immediate consumption state as identified in the literature. While a relatively low standard deviation is expected in the value in storage state given past empirical analysis (Fama and French (1988)), equation (5) suggests that the mean value in this state should be equal to the negative of the storage cost rate. It is found that for each of the three commodities the estimated storage cost is not significantly different from zero (0.088% for copper, 0.063% for lead and –0.460% for zinc). Only zinc exhibits the expected negative sign.[11] Both the mean and the standard deviation are statistically significantly different from zero in the second state and both are considerably larger than the mean and standard deviation values reported for the first state. Given the characteristics exhibited by these states it seems reasonable to label state one, the value in storage state, and state two, the immediate consumption state.

Both states appear to be quite stable though the value in storage state is the more stable of the two states. This is true for each of the commodities, with the probability of remaining in state 1, the value in storage state, being 0.965 for copper, 0.922 for lead and 0.955 for zinc. The probability of remaining in state 2, the value in consumption state, is somewhat

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[11] The existence of a convenience yield may explain this result. In the value in storage state the interest adjusted basis will equal the convenience yield less the costs of storage.

less with 0.950 for copper, 0.832 for lead and 0.902 for zinc. Thus copper, lead and zinc prices spend fairly long periods of time in one or other of the two states with shifts from one state to the other occurring quite rapidly. This is consistent with the dramatic changes in prices reflected in Figures 4, 5 and 6.

While there is evidence of two states in pricing it is important to get a sense of the relationship between the pricing state and the level of inventories. Figures 8, 9 and 10 show the relationship between the value in the storage state (state 1) and periods of high levels of inventory. Almost invariably, when stocks are high the state identified using Hamilton's model is state 1. Once stocks levels drop there is generally a shift to state 2. The greater stability of state 1 price distribution is also apparent in Figures 8, 9 and 10 and this stability is consistent with fairly slow rates of consumption that is generally observed once stocks build up (Keynes (1950) and Bils and Kahn (2000)). It is important to note that the price distribution coefficients are measured independently of the recorded level of stock. It is also important to note that the absolute level of stocks is not the key driver apparent in Figures 8, 9 and 10. Stocks can be quite high, for example copper in 1997, and yet a sudden drop in stock levels can lead to a shift in price distribution. Perhaps this is expected, given the arguments of Wright and Williams (1989), because the link between price distribution and LME stock levels should reflect the increased dispersion of LME warehouses across the world after 1962.[12]

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[12] For example in 1962 the first overseas warehouses in Rotterdam were approved, in 1987 the first non-European warehouse location, Singapore, was approved and by the mid 1990s there were 43 locations covering the USA, Europe and the Far East. There are currently over 400 warehouses spread across the world.

3.2. Interest-adjusted basis regimes and the convenience yield

As indicated in the previous section there is support for two price regimes that are consistent with the stock-out models appearing in the literature. While we cannot explicitly identify stock-outs with our aggregate stock data, the two price distributions do reflect periods of low stocks and periods of high stocks as indicated in Figure 8, 9 and 10. In the simplest stock-out pricing models the level of stocks has no role to play in the pricing of commodities other than through the zero stock constraint. Convenience yield models provide a much more active role for stocks, with convenience yields being a decreasing non-linear function of the level of stocks.

Much of the convenience yield discussion is based on simple graphical analysis with little evidence of time series analysis except for Pindyck (1994) though inevitably convenience yield is modelled as some unspecified function of the level of stocks. The unit root tests discussed in the data section suggest that the stock variable is integrated of order one and so to regress the interest-adjusted basis on stock levels could lead to problems with statistical tests. The descriptive statistics, reported in Panel B of Table 1, suggest that the change in stocks follows a stationary autoregressive process and so convenience yield is modelled in terms of current and lagged change in the level of stocks.

$$cy_t = \alpha + \sum_{k=0}^{K} \beta_k \Delta \ln\left(Stk_{t-k}\right) \qquad (8)$$

Where $\Delta \ln\left(Stk_{t,k}\right)$ is the change in the natural log of the stock level. When the convenience yield effect is included in the cost of carry model, the interest-adjusted basis takes the form:

$$IAB_{tT} = \ln\left(P_t / F_{tT}\right) + r = -s + \alpha + \sum_{k=0}^{K} \beta_k \Delta \ln\left(Stk_{t-k}\right) = \mu + \sum_{k=0}^{K} \beta_k \Delta \ln\left(Stk_{t-k}\right) \tag{9}$$

The coefficient, μ, is the sum of the constant term in the convenience yield model (equation (8)), α, less the storage rate, s. It is now possible to rewrite equation (5) to give:

$$IAB_{tT} = \begin{cases} = \max\left(\mu + \sum_{k=0}^{K} \beta_{1,k} \Delta \ln\left(Stk_{t-k}\right), \ln\left(\dfrac{P\{x\}}{F_{tT}}\right) + r + \mu + \sum_{k=0}^{K} \beta_{2,k} \Delta \ln\left(Stk_{t-k}\right)\right) \\ = \max\left(\mu + \sum_{k=0}^{K} \beta_{1,k} \Delta \ln\left(Stk_{t-k}\right), IAB_{tT}^{\#} + \sum_{k=0}^{K} \beta_{2,k} \Delta \ln\left(Stk_{t-k}\right)\right) \end{cases} \tag{10}$$

Where $IAB_{tT}^{\#} = IAB_{tT}^{*} + \mu$ and the change in stock level coefficients are estimated separately for each state. Including the stock variables in equation (10) provides a test of the impact of convenience yields. The traditional convenience yield model is supported if there is a statistically significant relationship between stocks and interest-adjusted basis in both regimes. The simple model underlying the work of Chambers and Bailey (1996), Deaton and Laroque (1992, 1995, 1996) is favoured where stocks have no descriptive power at all over commodity prices in either of the two regimes. If stock effects are observed in the value in storage state but not in the value in consumption state then the Ng and Ruge-Murcia (2000) approach is supported. Stocks are assumed to be exogenous in this test and this seems resonable given the "sluggishness" stock movements noted by Bils and Kahn (2000) and given the time series based evidence of exogeneity (Heaney (1998)). We extend the Hamilton (1994) model to obtain:

$$IAB_{tT} = \mu(S_t) + \sum_{k=0}^{K} \beta_k(S_t)\Delta\ln(Stk_{t-k}) + \sum_{i=1}^{n} \phi_i(IAB_{t-iT} - \mu(S_{t-i})) + \sigma(S_t)\varepsilon_t \tag{11}$$

The coefficient, $\beta_k(S_t)$, measures the sensitivity of the interest-adjusted basis to the change in the level of stocks in state S_t in the current period ($k=0$) and prior periods ($k=1, 2, \ldots, K$). To identify the appropriate number of lags to be included for stocks, a search begins with a maximum of the current change in stocks plus 5 lags with statistically insignificant lags being dropped.

Commodity prices seem to be sensitive to the change in stocks regardless of whether the state reflects pricing under storage or under immediate consumption, contrary to the arguments of Wright and Williams (1989). The sensitivity to the stocks accords with the concept put forward in Brennan (1958), Kaldor (1939), Stein (1961), Telser (1958) and Working (1949) though stocks enter this model in the form of the current and lagged change in the level of stocks.

As is evident from Table 3 the stock coefficients are generally negative, with some exceptions for copper. This negative relationship aligns with the convenience yield argument that increases in stock lead to decreases in interest-adjusted basis. It is important to note the variation in the sensitivity of the interest-adjusted basis to the change in stock across the two states. The stock coefficients are generally smaller in the value in storage state (state one) than in the immediate consumption value state with some exceptions for copper. Thus a small change in stocks will have a larger impact on prices in the consumption state than in the value in storage state. This result is consistent with the non-linear model that has consistently appeared in the literature. There is little discussion in the literature about the impact of lagged stocks on commodity prices though coefficients for current and lagged values of the change in stock are important in this model (Table 3).

[Insert Table 3 about here]

It is important to note the impact of incorporating stocks on the identification of the two states. To this end, the Spearman rank correlation is estimated between the state one probability time series that is drawn from each of the two models reported in this paper. The correlation between the probability of being in state one for the model excluding stocks (equation (7)) and the probability of being in state one for the model including stocks (equation (12)) is 0.893 for copper, 0.922 for lead and 0.976 for zinc. These are statistically significant and positive. As might be expected, given the statistically significant stock parameters reported in Table 3, the correlation coefficients are not equal to one though they are close to one and this suggests some stability in the regime break points across the two models.

4. Conclusions

While some of the theoretical commodity price literature points to the possibility of two underlying states that determine commodity prices there is also a considerable literature supporting the existence of convenience yields. To some extent these two models of commodity price have been treated as alternatives though more recent modelling has recognized both the two state nature of commodity pricing and existence of convenience yields. This richer approach to modelling commodity prices appears to improve the explanatory power of the theoretical models.

There is little research evident in the literature addressing the issue of whether commodity prices actually move between two pricing states, a value state and a consumption state. Further, there is limited time series research concerning the existence of convenience yields under different market conditions. Statistical analysis reported in this paper support the existence of two pricing regimes for the commodities, copper, lead and zinc, and the existence of convenience yields that are a decreasing, non-linear function of stocks. Stocks are found to have explanatory power in both regimes and this suggests a more complex process in the consumption state than the simple white noise process often assumed in the stock-out constraint based literature.

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TABLE 1

DATA DESCRIPTION

The price is the mid-point of the reported representative price range reported in the Metals Bulletin for the period 1964 to December 1989. (N = 471). Copper and lead prices are in GBP and the zinc prices are in USD. For the remainder of the period prices are obtained from the LME web site with copper and lead prices converted to GBP using the LME FX rates to maintain consistency. COP3FWD is the 3 month copper forward price (GBP), COPSPOT is the spot price of copper (GBP), CYC is an estimate of the copper interest-adjusted basis, LEAD3FWD is the 3 month lead forward price (GBP), LEADSPOT is the spot price of lead (GBP), CYL is an estimate of the lead interest-adjusted basis, ZINC3FWD is the 3 month zinc forward price (USD), ZINCSPOT is the spot price of zinc (USD), CYZ is an estimate of the zinc interest-adjusted basis, UK interest rate consists of the minimum lending bank rate from November 1964 to December 1975 obtained from the Bank of England web site (www.bankofengland.co.uk) and the Euro Currency (London) Sterling 3 month middle rate obtained from Datastream, US interest rate consists of the three month treasury bill secondary market rates obtained from the USA Federal Reserve Board (www.federalreserve.gov) for the period November 1964 to December 1975 and the Euro Currency (London) USD 3 month middle rate obtained from Datastream is used for the remainder of the period, COPPER is the level of copper stocks in tonnes at all LME warehouses, LEAD is the level of lead stocks in tonnes at all LME warehouses, ZINC is the level of zinc stocks in tonnes at all LME warehouses. Levels refers to the spot price, forward price, interest-adjusted basis estimate or stocks (tonnes). Diff refers to the first differenced series. AR(n) is the nth order autoregression coefficient. PrQ(n) is the probability associated with the Ljung-Box Q-Statistic for given n lags. The cut off value the Phillips-Perron test and the Augmented Dickey-Fuller test is –3.41 and for the KPSS test it is 0.463. While the Phillips-Perron and Augmented Dickey Fuller tests have a null of unit root process the KPSS test has a null of stationary process. The KPSS test reported is the tau test value though there is little variation between the tau test and the mu test statistics. * significant at the 5% level of significance.

Panel A: Descriptive Statistics

	Mean	*Maximum*	*Minimum*	*Std. Dev.*	*Skewness*	*Kurtosis*
COPSPOT	975.620	1946.223	341.000	387.942	0.429	2.579
COP3FWD	975.615	1939.677	314.000	378.692	0.298	2.448
IABC	0.021	0.186	-0.019	0.032	1.717	7.494
Copper stocks	261738.9	971500.0	4700.0	236980.1	1.0	3.2
LEADSPOT	287.405	640.500	79.125	119.672	0.018	2.689
LEAD3FWD	289.266	611.500	79.500	117.931	-0.152	2.514
IABL	0.015	0.208	-0.031	0.032	1.711	8.575
Lead stocks	83743.5	371775.0	2510.0	80734.6	1.5	5.0
ZINCSPOT	638.724	2050.000	95.875	443.192	0.621	2.576
ZINC3FWD	640.168	1943.000	93.688	435.381	0.510	2.271
IABZ	0.011	0.180	-0.057	0.031	1.719	7.797
Zinc stocks	196908.1	1234150.0	300.0	280875.5	1.8	5.8
UK interest rate	9.030	20.875	3.406	3.485	0.577	2.545
US interest rate	6.736	19.938	1.063	3.34	1.321	5.357

Panel B: Serial correlation analysis on levels and change in levels

	Levels AR(1)	*Levels AR(10)*	*Levels PrQ(10)*	*Levels PrQ(20)*	*Diff AR(1)*	*Diff AR(10)*	*Diff PrQ(10)*	*Diff PrQ(20)*
COPSPOT	0.977	0.815	0.00	0.00	0.009	0.059	0.25	0.03
COP3FWD	0.982	0.832	0.00	0.00	0.037	0.040	0.34	0.02
IABC	0.848	0.426	0.00	0.00	-0.224	-0.035	0.00	0.00
Copper stocks	0.993	0.768	0.00	0.00	0.527	-0.064	0.00	0.00
LEADSPOT	0.981	0.790	0.00	0.00	0.032	0.039	0.90	0.00
LEAD3FWD	0.984	0.822	0.00	0.00	0.068	0.072	0.51	0.00
IABL	0.768	0.296	0.00	0.00	-0.225	-0.017	0.00	0.00
Lead stocks	0.995	0.811	0.00	0.00	0.438	0.041	0.00	0.00
ZINCSPOT	0.989	0.853	0.00	0.00	0.057	0.001	0.06	0.01
ZINC3FWD	0.992	0.873	0.00	0.00	0.159	-0.061	0.00	0.00
IAB	0.720	0.271	0.00	0.00	-0.366	0.016	0.00	0.00
Zinc stocks	0.998	0.901	0.00	0.00	0.666	0.247	0.00	0.00
UK interest rate	0.975	0.753	0.00	0.00	-0.044	-0.151	0.00	0.00
US interest rate	0.979	0.830	0.00	0.00	0.132	0.088	0.00	0.00

Panel C: Serial correlation analysis on squared levels – A test for ARCH effects

	Sq'd Levels AR(1)	*Sq'd Levels AR(10)*	*Sq'd Levels PrQ(10)*	*Sq'd Levels PrQ(20)*
COPSPOT	0.968	0.768	0.00	0.00
COP3FWD	0.974	0.783	0.00	0.00
IABC	0.619	0.169	0.00	0.00
Copper Stocks	0.991	0.652	0.00	0.00
LEADSPOT	0.965	0.635	0.00	0.00
LEAD3FWD	0.971	0.680	0.00	0.00
IABL	0.426	0.112	0.00	0.00
Lead Stocks	0.994	0.728	0.00	0.00
ZINCSPOT	0.974	0.722	0.00	0.00
ZINC3FWD	0.983	0.756	0.00	0.00
IABZ	0.375	0.097	0.00	0.00
Zinc Stocks	0.996	0.757	0.00	0.00
UK interest rate	0.960	0.676	0.00	0.00
US interest rate	0.957	0.774	0.00	0.00

Panel D: Unit root tests

	Levels PP(10)	*Levels ADF(10)*	*Levels KPSS(10)*	*Diff PP(10)*	*Diff ADF(10)*	*Diff KPSS(10)*
COPSPOT	-3.46*	-2.97	2.97*	-21.56*	-7.00*	0.03
COP3FWD	-3.35	-3.03	3.39*	-20.85*	-6.81*	0.03
IABC	-6.31*	-3.63*	0.40	-31.24*	-8.52*	0.03
Copper stocks	-2.96	-3.35	1.67*	-12.70*	-5.74*	0.03
LEADSPOT	-3.09	-3.46*	2.17*	-20.95*	-6.28*	0.03
LEAD3FWD	-2.90	-3.22	2.31*	-20.19*	-6.12*	0.04
IABL	-8.75*	-4.37*	0.69*	-32.95*	-8.09*	0.02
Lead stocks	-2.78	-3.61*	2.14*	-14.90*	-4.95*	0.04
ZINCSPOT	-3.19	-3.46*	3.22*	-20.62*	-6.72*	0.03
ZINC3FWD	-3.06	-3.19	3.35*	-18.74*	-6.65*	0.03
IAB	-9.77*	-4.25*	0.44	-39.13*	-8.92*	0.02
Zinc stocks	-2.02	-3.29	2.22*	-10.88*	-4.34*	0.10
UK interest rate	-2.70	-2.30	0.93*	-22.81*	-6.81*	0.11
US interest rate	-2.12	-2.06	0.78*	-18.93*	-6.05*	0.17

TABLE 2
HAMILTON TWO STATE REGIME SWITCHING MODEL FOR COPPER, LEAD AND ZINC

The coefficient estimates are obtained from the Hamilton two state switching regime model using maximum likelihood estimation over the interest-adjusted basis expressed as a percentage per month. P11 is the probability of being in state 1. P22 is the probability of being in state 2. The intercept term is the average interest-adjusted basis under the particular state, $\mu(S_t)$. It takes on a value of $\mu(S=1)$ in state 1 and a value of $\mu(S=2)$ in state 2. The terms, ϕ_1, ϕ_2, ϕ_3, are the lag coefficients. Lag choice is based on a general 4-lag model with exclusion of statistically insignificant lags subject to the requirement that there be no residual serial correlation. The residual term is the product of the state dependent standard deviation scale coefficient, $\sigma(S_t)$, which takes on values of $\sigma(S=1)$ in state one or $\sigma(S=2)$ in state 2, and a mean zero, unit variance residual term ε_t. The equation takes the form:

$$IAB_{tT} = \mu(S_t) + \sum_{i=1}^{n} \phi_i \left(IAB_{t-iT} - \mu(S_{t-i})\right) + \sigma(S_t)\varepsilon_t$$

PrQ(20) is the probability associated with the Ljung-Box Q-Statistic for 20 lags and PrQ(1) is the probability associated with the coefficient restriction . * significant at the 5% level of significance.

Panel A: Coefficient estimates

	Copper Coefficient	*Copper t-statistic*	*Lead Coefficient*	*Lead t-statistic*	*Zinc Coefficient*	*Zinc t-statistic*
μ (S=1)	0.088	0.56	0.063	0.17	-0.460	-1.77
μ (S=2)	2.228*	6.05	2.222*	4.19	1.310*	3.56
ϕ_1	0.592*	14.01	0.569*	12.03	0.705*	19.57
ϕ_2	0.227*	5.77	0.071	1.41	0.123*	3.57
ϕ_3	-		0.200*	6.17	0.057*	2.13
P11	0.965*	76.65	0.922*	43.35	0.955*	75.65
P22	0.950*	60.23	0.832*	17.78	0.902*	32.52
σ(S=1)	0.448*	16.17	0.821*	13.40	0.538*	20.55
σ(S=2)	2.424*	18.55	3.110*	13.46	3.489*	16.56

Panel B: Tests of restrictions and residual tests

	Copper	*Lead*	*Zinc*
Tests of coefficient restrictions			
μ (S=1) = μ (S=2), PrQ(1)	0.00*	0.00*	0.00*
σ(S=1) = σ(S=2), PrQ(1)	0.00*	0.00*	0.00*
Test for serial correlation			
Std. residual, PrQ(20)	0.52	0.26	0.45
Std. residual sqrd., PrQ(20)	0.86	0.93	0.00*

TABLE 3
HAMILTON TWO STATE REGIME SWITCHING MODEL FOR COPPER, LEAD AND ZINC INCLUDING THE IMPACT OF STOCKS

The coefficient estimates are obtained from the Hamilton two state switching regime model using maximum likelihood estimation over the interest-adjusted basis expressed as a percentage per month. P11 is the probability of being in state 1. P22 is the probability of being in state 2. The intercept term is the average interest-adjusted basis under the particular state, $\mu(S_t)$. It takes on a value of $\mu(S=1)$ in state 1 and a value of $\mu(S=2)$ in state 2. Similarly, the stock coefficient, $\beta_k(S_t)$, is estimated for both states with a value of $\beta_k(S=1)$ in state 1 and a value of $\beta_k(S=2)$ in state 2 with current (k=0) and lag terms, k=1, 2, … K. To identify the appropriate number of lags to be included for stocks, a search begins with a maximum of the current change in stocks plus 5 lags with statistically insignificant lags being dropped. The terms, ϕ_1, ϕ_2, ϕ_3 are the interest adjusted basis lag coefficients. Lag choice is based on a general 4-lag model with exclusion of statistically insignificant lags subject to the requirement that there be no residual serial correlation. The residual term is the product of the state dependent standard deviation scale coefficient, $\sigma(S_t)$, which takes on values of $\sigma(S=1)$ in state one or $\sigma(S=2)$ in state 2, and a mean zero, unit variance residual term ε_t. The equation takes the form:

$$IAB_{tT} = \mu(S_t) + \sum_{k=0}^{K}\beta_k(S_t)\Delta\ln(Stk_{t-k}) + \sum_{i=1}^{n}\phi_i\left(IAB_{t-iT} - \mu(S_{t-i})\right) + \sigma(S_t)\varepsilon_t$$

PrQ(20) is the probability associated with the Ljung-Box Q-Statistic for 20 lags and PrQ(1) is the probability associated with the coefficient restriction. * significant at the 5% level of significance.

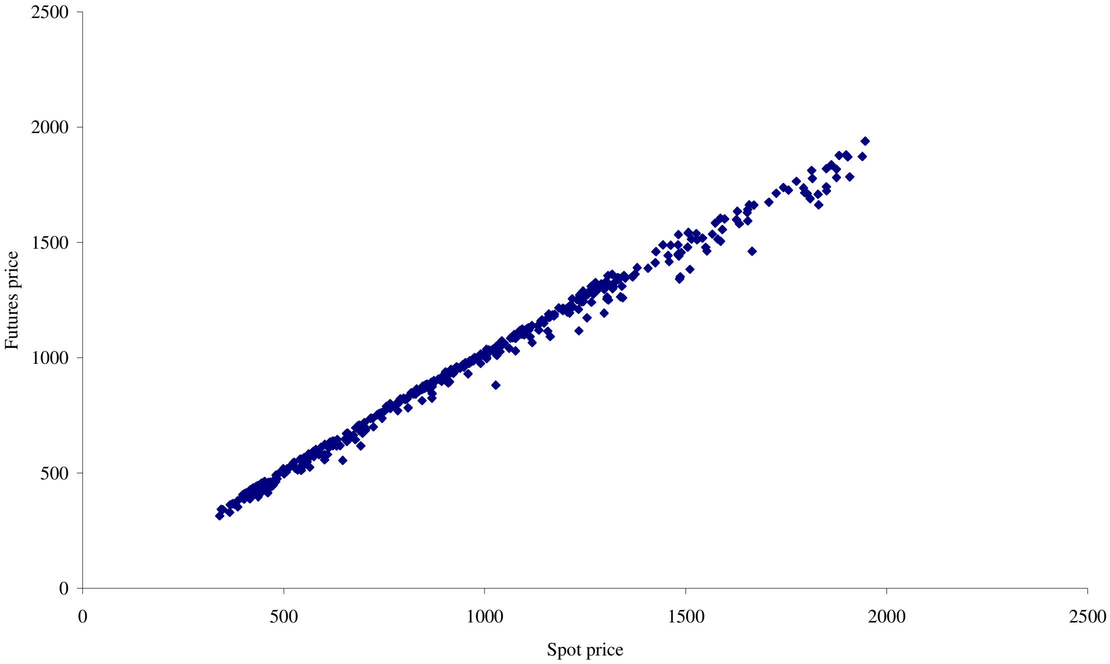
Panel A: Coefficient estimates

	Copper Coefficient	*Copper t-statistic*	*Lead Coefficient*	*Lead t-statistic*	*Zinc Coefficient*	*Zinc t-statistic*
μ (S=1)	0.727*	2.04	0.457	1.18	-0.155	-0.48
μ (S=2)	1.879*	4.60	2.657*	5.20	1.376*	3.24
φ 1	0.590*	12.95	0.513*	12.00	0.649*	14.58
φ 2	0.329*	7.30	0.143*	3.25	0.151*	4.02
φ 3	-		0.218*	6.53	0.101*	2.99
	-					
P11	0.926*	52.17	0.926*	48.63	0.954*	69.67
P22	0.889*	27.95	0.804*	15.93	0.904*	33.87
σ(S=1)	0.363*	13.65	0.814*	17.83	0.504*	21.90
σ(S=2)	2.178*	17.49	2.641*	15.30	3.315*	17.07
β_0(S=1)	-2.617*	-9.68	-1.655*	-3.68	-1.027*	-4.60
β_1(S=1)	0.066	0.25	-1.991*	-4.31	-0.658*	-3.16
β_2(S=1)	0.518*	2.20	-	-	-0.023	-0.10
β_3(S=1)	-0.118	-0.64	-	-	-	-
β_0(S=2)	-4.408*	-5.67	-8.480*	-6.78	-1.341*	-2.61
β_1(S=2)	-3.084*	-3.90	-3.741*	-2.98	-0.981*	-1.66
β_2(S=2)	-0.793	-1.05	-	-	-1.278*	-2.30
β_3(S=2)	0.695	0.87	-	-	-	-

Panel B: Tests of restrictions and residual tests

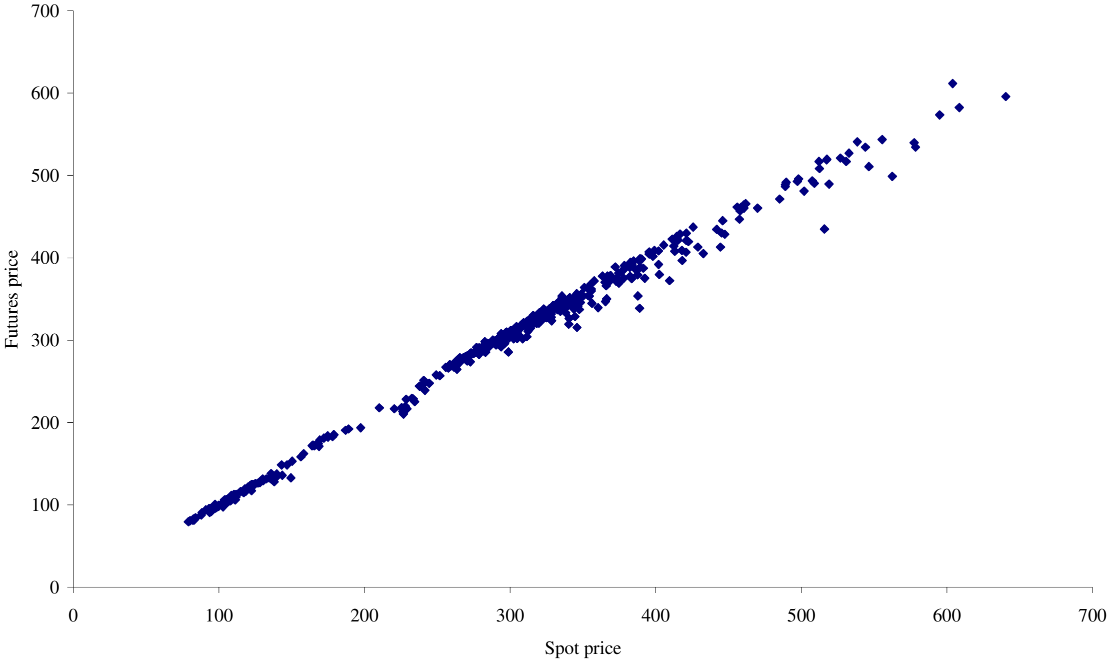
	Copper	*Lead*	*Zinc*
Tests of coefficient restrictions			
μ (S=1) = μ (S=2), PrQ(1)	0.00*	0.00*	0.00*
σ(S=1) = σ(S=2), PrQ(1)	0.00*	0.00*	0.00*
Test for serial correlation			
Std. residual, PrQ(20)	0.61	0.21	0.06
Std. residual sqrd., PrQ(20)	0.94	0.58	0.00*

Figure 1. The Relationship Between Copper Spot Price and Futures Price



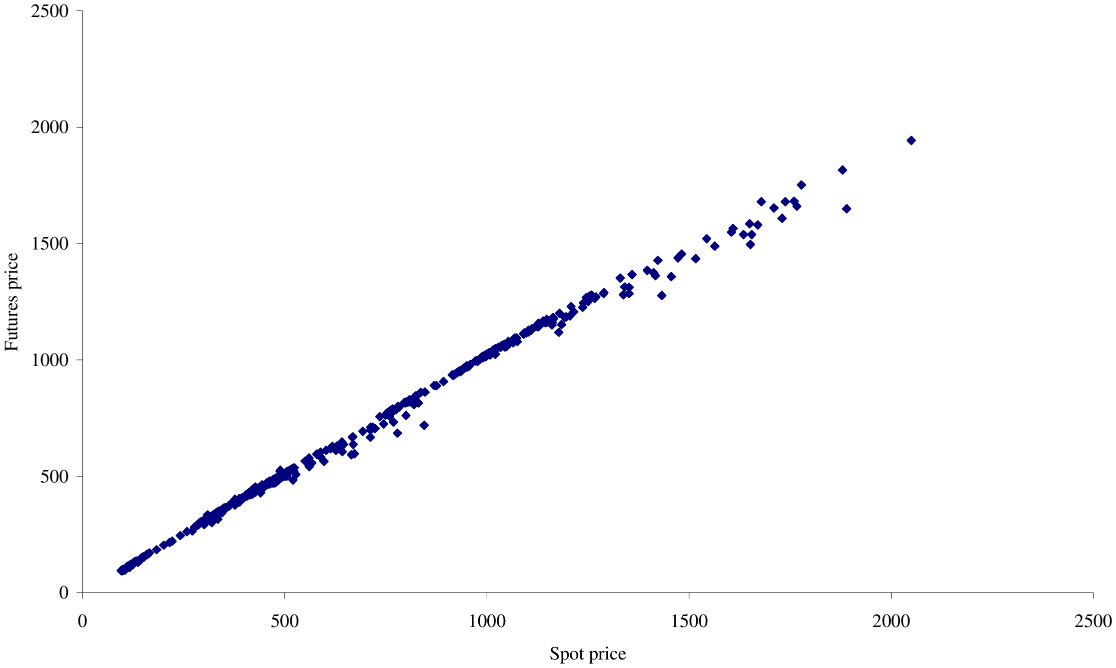
This figure consists of monthly GBP spot and futures price observations for copper for the period from November 1964 To December 2003.

Figure 2. The Relationship Between Lead Spot Price and Futures Price



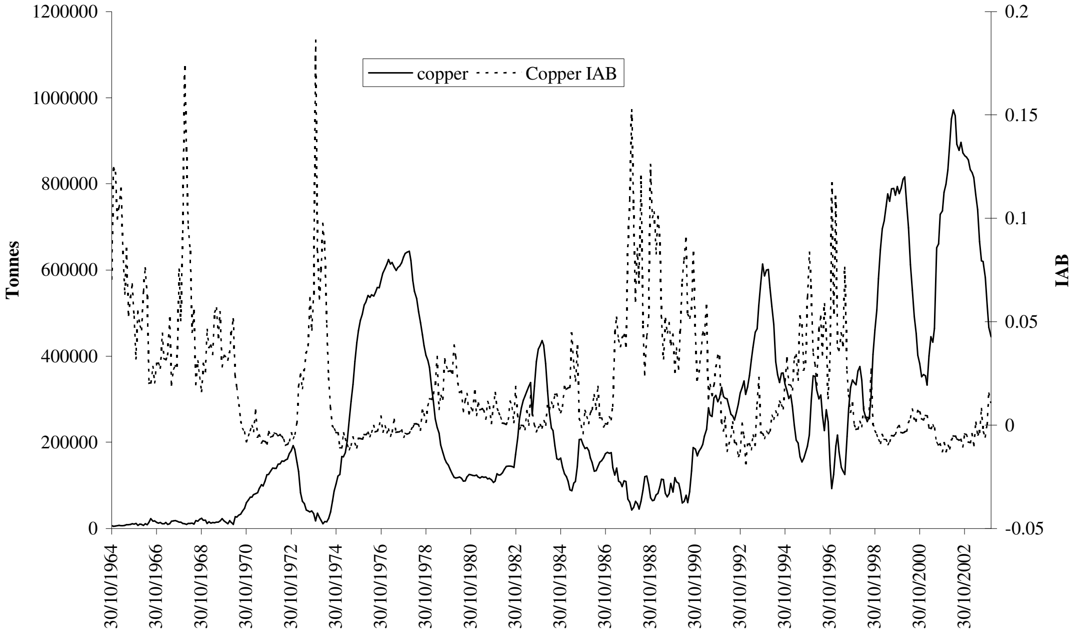
This figure consists of monthly GBP spot and futures price observations for lead for the period from November 1964 To December 2003.

Figure 3. The Relationship Between Zinc Spot Price And Futures Price



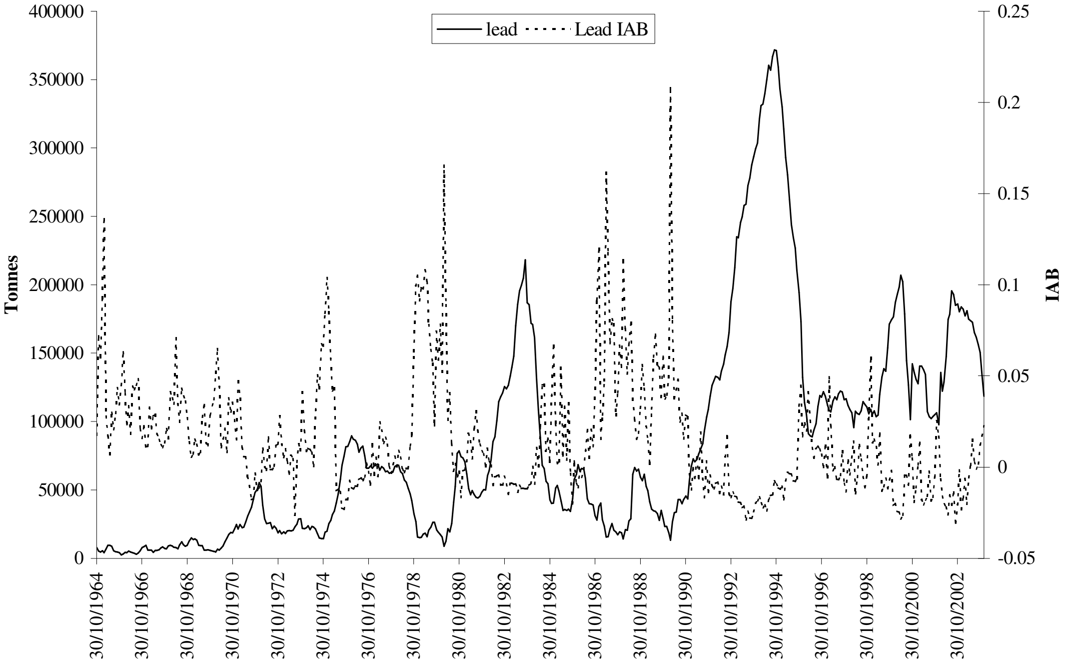
This figure consists of monthly USD spot and futures price observations for zinc for the period from November 1964 To December 2003.

Figure 4. Interest-Adjusted Basis (IAB) and Level of Stocks for Copper.



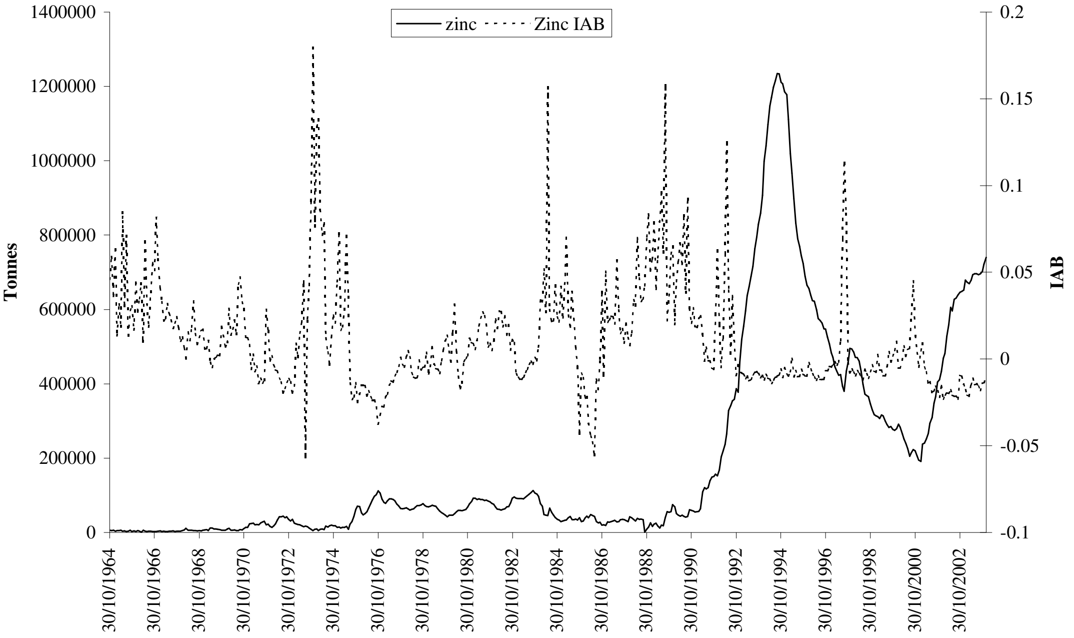
This figure plots the variation in the interest adjusted basis (dotted line), defined as $IAB_{tT} = \ln\left(P_t / F_{tT}\right) + r$, and the level of stocks in tonnes (unbroken line) over the study period for the commodity copper.

Figure 5. Interest-Adjusted Basis (IAB) and Level of Stocks for Lead.



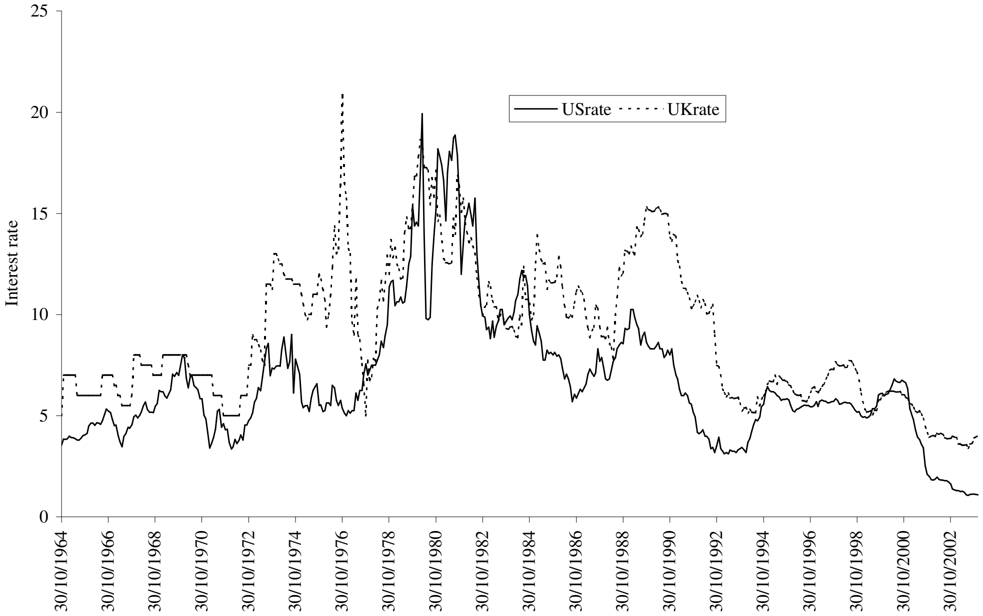
This figure plots the variation in the interest adjusted basis (dotted line), defined as $IAB_{tT} = \ln(P_t / F_{tT}) + r$, and the level of stocks in tonnes (unbroken line) over the study period for the commodity lead.

Figure 6. Interest-Adjusted Basis (IAB) and Level of Stocks for Zinc.



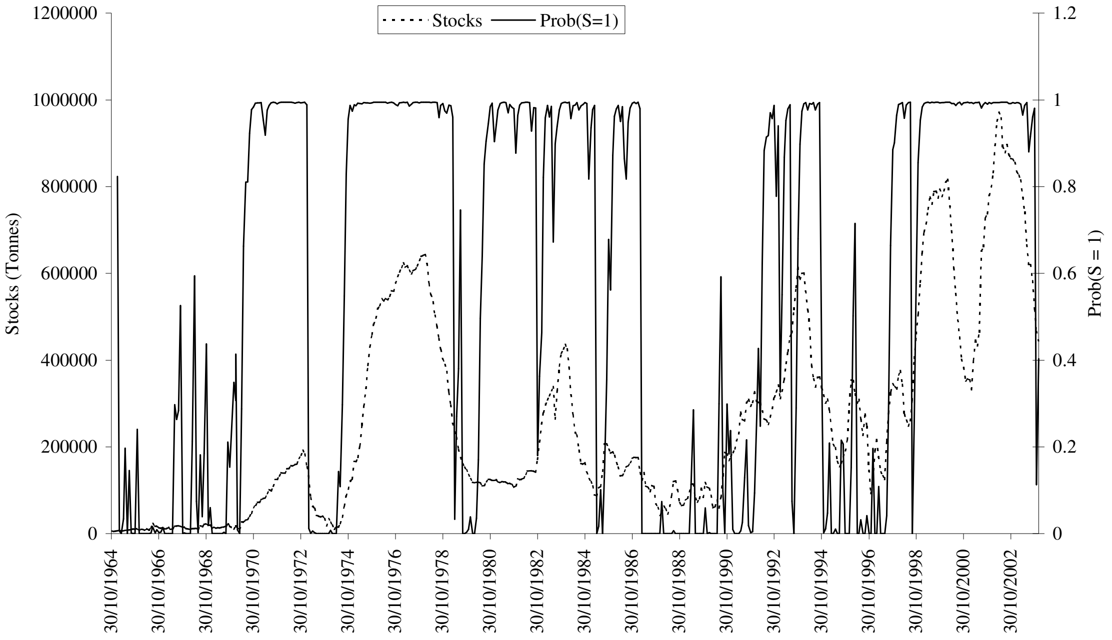
This figure plots the variation in the interest-adjusted basis (dotted line), defined as $IAB_{tT} = \ln(P_t / F_{tT}) + r$, and the level of LME stocks in tonnes (unbroken line) over the study period for the commodity Zinc.

Figure 7. Monthly Interest Rate Observations, November 1964 to December 2003.



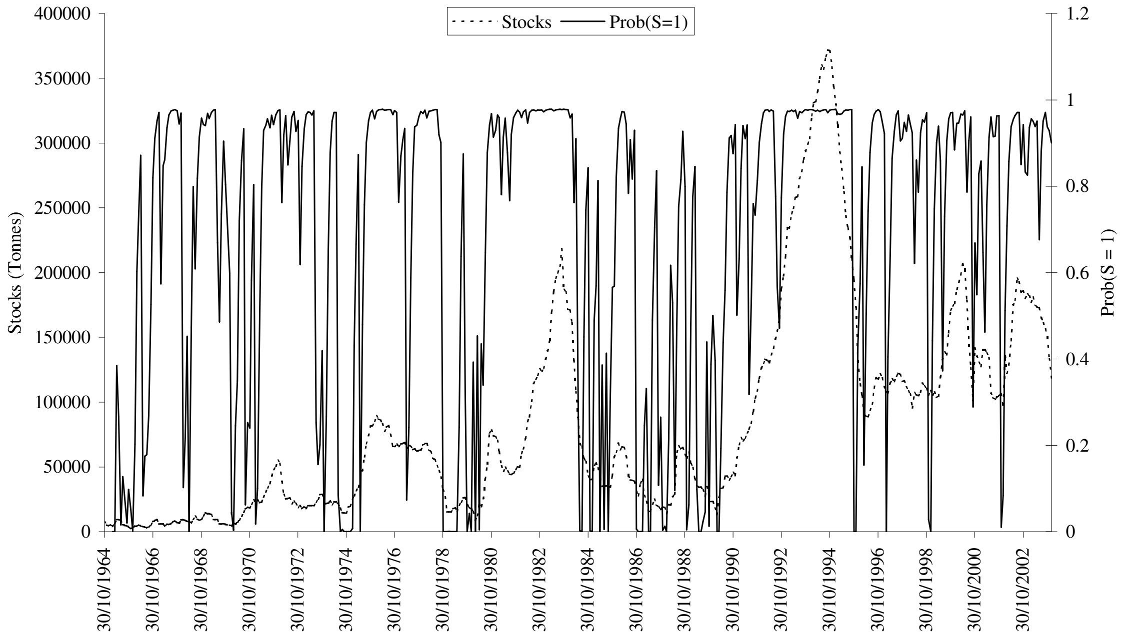
Both USD interest rates (unbroken line) and GBP interest rates (dotted line) are graphed in this figure.

Figure 8. Stock Levels and Probability of Being in State One for Copper.



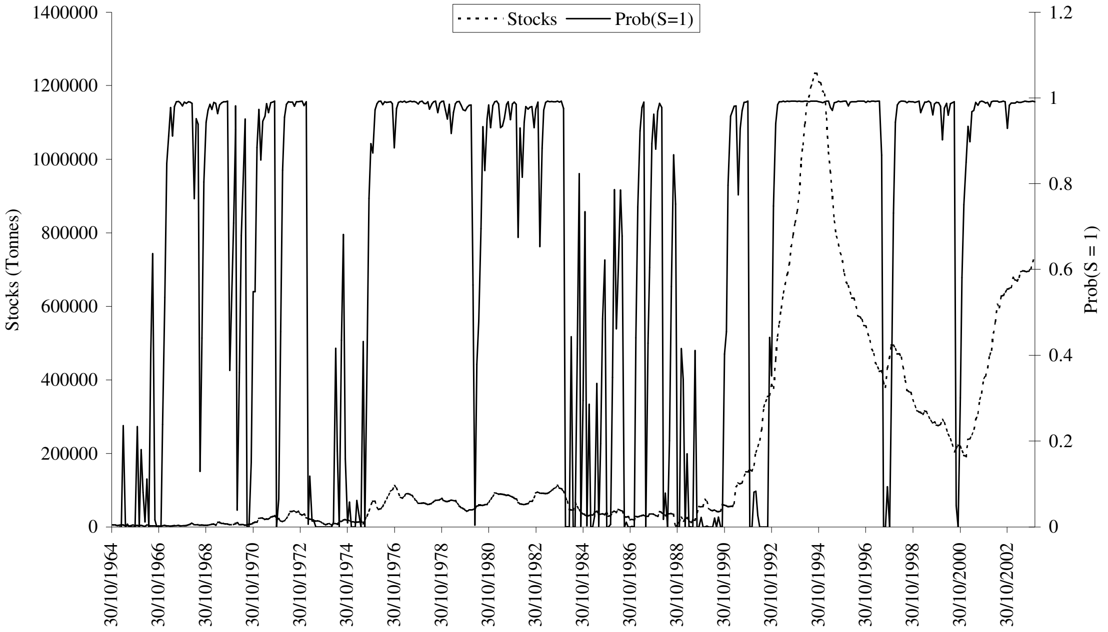
The level of LME stocks in tonnes (dotted line) for copper and the probability of being in the first of the state (unbroken line) are graphed in this figure.

Figure 9. Stock Levels and Probability of Being in State One for Lead.



The level of LME stocks in tonnes (dotted line) for lead and the probability of being in the first of the state (unbroken line) are graphed in this figure.

Figure 10. Stock Levels and Probability of Being in State One for Zinc.



The level of LME stocks in tonnes (dotted line) for zinc and the probability of being in the first of the state (unbroken line) are graphed in this figure.