

# How does a Shock Propagate? A Model of Contagion in the Interbank Market due to Financial Linkages<sup>†</sup>

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## Abstract

In a setting with multiple banks and differential information, we study how a shock propagates in the banking system due to strategic interactions between banks' managers and depositors. We construct a model in which a bank faces an exogenous shock and study the transmission of this shock in the interbank market due to financial linkages. We uniquely determine the unfolding of the financial crisis in equilibrium. Firstly, we show that an initial shock to a bank is transmitted to the banking system, thus increasing the financial fragility. The more interesting part of our analysis, however, is the role played by creditor banks in transmission of the shock. We show that, under certain circumstances, creditor banks increase the fragility of borrower banks by unwinding their claims to distance themselves from the line of contagion. Furthermore, our model predicts that even when creditor banks do not directly play an important role on the financial distress of their borrower banks, they amplify the sensitivity of the initial shock on their borrower banks. Our model shows that not only does the interbank market transmit shocks -- and acts as a channel for contagion -- but also that the endogenous liquidity in the interbank market can be reduced after the initial shock, thereby increasing the fragility of the whole system over and above the initial shock.

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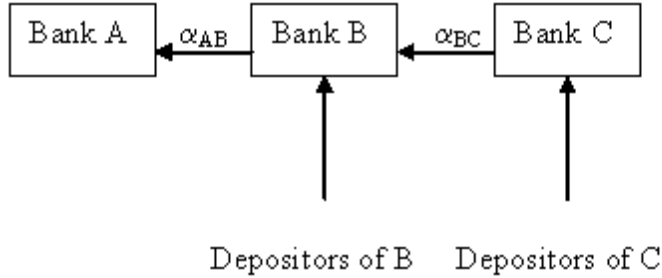
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# 1 Introduction

The importance of financial linkages among banks in the propagation of a crisis can hardly be disputed. Small shocks caused by the failure of a bank can easily snow ball into a crisis, the dimensions of which can be hard to imagine. There are several channels that could play an important role in the transmission of the initial shock. Focussing purely on rational explanations, the failure of a bank could provide adverse information about other banks with similar features. Thus, in a Bayesian framework, learning by rational agents could precipitate a financial crisis once the depositors have observed the failure of a bank (Chen, 1999). Another important channel that could play a role in the transmission of a crisis are the balance sheet connections among banks (Allen and Gale, 2000). For example, after an initial shock to one bank, banks connected to the failed bank would receive a negative shock. Furthermore, strategic interactions among banks in the interbank market could modify the impact of this initial shock in the system. In this paper, we focus primarily on how a shock is transmitted across the system due to balance sheet connections among banks. In particular, we analyze whether the interbank market amplifies or attenuates the fragility caused by the initial shock.

We have in mind the metaphor of a line of dominoes where the first piece goes down and starts hitting the other dominoes which are close to it –i.e., we think of banks as dominoes and, level of deposits in adjacent banks as a measure of proximity of the dominoes. In particular, we want to know what happens to the second domino in the line, after the first one has gone down. Thus, in the context of banks, the first question which we are interested in is the following: Does the probability of failure of a bank depend on the proximity to the bank that initially failed? The more interesting question that arises is that unlike a dominoes’ game, in which the pieces are static, when one considers a banking system, managers of banks can take actions after the initial shock to



**Figure 1**

move away from the line of contagion. To put it more simply, it is as if the pieces of the dominoes' game were able to adjust their positions. Furthermore, the actions of the managers of banks are influenced both by their own depositors and also by the depositors of other banks. Therefore, not only are our dominoes not static but they also have another layer down (the depositors of banks) that can completely alter the transmission of the initial shock. Hence, we study how banks behave in the interbank market –after an initial shock– taking into consideration the reaction of all the depositors. In consequence, to pursue our analysis on the transmission of an initial shock through the interbank market, we assume a model that consists of three banks: Banks A, B and C with their respective depositors (refer to figure 1):

Bank B has a deposit  $\alpha_{AB}$  in bank A, which fails exogenously at the beginning of the model. Bank C (the creditor) has a deposit  $\alpha_{BC}$  in bank B (the borrower). Both banks have their own small depositors, who receive private signals about the fundamentals of their respective banks. In addition, we assume that the manager of bank C has perfect information about the fundamentals of her own bank and bank B. After the initial shock (failure of bank A), depositors of bank B (the small depositors and the manager of bank C) decide whether to withdraw their deposits early. After

this game, a public signal is released about the level of early withdrawals in bank B. Intuitively, this public signal captures the extent of problems –queues of depositors– faced by bank B. Once depositors in bank C observe the public signal, they decide whether to withdraw their deposits early. Based on the previous structure, we analyze the following questions regarding the change in the ex-ante probability of failure of bank B: Firstly, how does the ex-ante probability of failure of bank B change with the level of deposit that bank B has with the failed bank A? We refer to this as the ‘first layer’ effect. Secondly, how does the level of deposit held by the creditor bank C in bank B affect the ex-ante probability of failure of bank B? We refer to this as the ‘second layer’ effect. And finally, how does an interaction of the first and the second layer affect the ex-ante probability of failure of bank B? In other words, we want to check whether there is contagion of the initial shock through the interbank market and, at the same time, whether the interbank market can reduce, or amplify, both the probability of financial distress of debtor banks and also the negative impact of the initial shock on the financial system.

In order to pursue our analysis, we assume differential information on the depositors’ set of information. Hence, we are able to determine the unique equilibrium of the game and the associated equilibrium probability of failure for a bank. The main results of our paper are the following: Firstly, we show that the probability of failure of bank B increases with its deposit in the bank that initially failed. Secondly, we highlight the dual nature of the interbank market, i.e. we show that bank C can either attenuate the crisis by provision of liquidity (to bank B) or can increase the liquidity risk of bank B. Finally, we show that in circumstances in which bank C does not directly play a significant role in the distress of its borrower B, bank C indirectly amplifies the negative impact of the initial shock in bank B.

The intuition driving the results is the following: after the initial shock, the depositors of bank B decide, based both on the fundamentals of their own bank and, on the exposure of their bank with

the failed bank, whether to withdraw their claims early. Higher the level of exposure of their bank with the failed bank, more likely it is that their bank faces higher level of early withdrawals (given that the liquidity of bank B has been reduced). Hence, under these circumstances, bank B might be solvent but illiquid, needing the support of its creditor bank C – it wants bank C not to liquidate the claim it holds in bank B. However, it is precisely under these circumstances that the potential liquidity that bank C can obtain from bank B is also reduced, thus increasing the fragility of the creditor bank C, which in turn increases the fragility of the borrower bank B. In consequence, the increase of coordination problems in the creditor bank (due to its depositors) increases the fragility of the borrower bank B. It is important to note that our result relies crucially on the fact that the large creditor is a bank i.e., the creditor has a fragile capital structure. Furthermore, we find the opposite results on the effects of the second layer if the creditor C was not fragile.

In sum, not only do we find that the interbank market transmits initial shocks, but also that the liquidity in the interbank market can be reduced after the initial shock, thereby increasing the fragility of the whole system even more. Though our model is very stylized, it helps us show that strategic interactions in the interbank market can amplify the initial shock. Our results also point to the fact that capital structure of agents in the interbank market can play a crucial role in the propagation of a crisis.

Though the results of our model highlight the importance of strategic interactions among banks in the interbank market (taking into account the actions of depositors) during the unfolding of a financial crisis, there is a dearth of empirical evidence regarding the behavior of banks during a crisis. However, one study to take note of was conducted during the great depression by the Federal Reserve (1938) that looked at deposit losses experienced by banks prior to closing. The main findings of the study were that large deposits decreased much more prior to bank suspensions than small deposits. More importantly for our analysis, the study also finds that interbank deposits

were extremely quick to leave banks experiencing trouble, declining at a rate of over three times that of demand deposits. This finding coupled with the finding that some of the suspended banks were financially sound, suggests that there might be other forces (as shown in our results) at work driving the behavior of banks in the interbank market.<sup>1</sup> The results of our model are also consistent with the findings of Iyer and Peydró-Alcalde (2004). Firstly, they show the significance of the first layer effect. Secondly, they show that the interaction of the first and the second layer matters, even though the second layer effect by itself is not an important factor in explaining depositor runs.

Our work is related to a number of others: Rochet and Vives (2004) also study runs on banks due to coordination problems among the participants in the interbank market, thereby showing how the interbank market may fail. Rochet and Tirole (1996) use monitoring as a means of triggering correlated crises: if one bank fails, it is implied that other banks have not been properly monitored, and a general collapse occurs. Kiyotaki and Moore (1997) analyze contagion through credit chains amongst lenders and entrepreneurs. Though their focus is to study balance sheet connections as a source of contagion, they do not concern themselves with depositors. Since our aim is to understand contagion via the interbank market taking into consideration depositors' reactions, the two papers which are closest to us are the ones by Allen and Gale (2000) and Dasgupta (2004).

Allen and Gale (2000) model contagion as an equilibrium phenomenon in a multiple bank setting. The main driving force behind their model is the liquidation by banks of their interbank holdings to meet excess demand for liquidity. They endogenize the interbank market claims and show that in equilibrium there can be contagion in the interbank market. Their model presents several equilibria and hence they do not have a specific prediction on how different level of deposits among banks can influence the expected probability of failure of a bank which is the main purpose of our paper. Dasgupta (2004), however, has a unique equilibrium in his model. He also studies

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<sup>1</sup>See also Wicker (2000) pages 2, 39, 82,130, 141, 143 and 144.

the spread of crises due to financial connections among banks. In his paper, financial contagion is modelled as an equilibrium phenomenon in a dynamic setting with incomplete information and multiple banks. The main result of the paper is that a contagious bank failure occurs with a positive probability and that reduces the incentive to use the interbank market. Furthermore, the direction of the contagion is from debtor to creditor banks. The emphasis of his paper is to endogenize interbank market claims when there is a positive probability of contagion ex-post, and to show that the main channel of contagion goes from borrower banks to creditor banks.

Our paper contributes to the existing literature on contagion due to financial linkages in the following way: Our study incorporates in a unified framework the three dominant forces which affect the unfolding of a financial crisis: the first layer effect (debtor banks' behavior), the second layer effect (creditor banks' behavior), and the effect caused by the interaction of the first and the second layers. In consequence, we obtain testable predictions on how these three forces affect the unfolding of a financial crisis. More importantly, we show that creditor banks (taking into account all the depositors' actions) can dramatically alter the propagation of the crisis. In a more general context, our paper highlights the role of the capital structure of agents in determining the fragility of the system

The rest of the paper is organized as follows. Next section presents the model. In section 3, we analyze the unique equilibrium of the model. Section 4 contains the main results of the paper and section 5 concludes.

## 2 Model

Consider a three period  $-t = 0, 1, 2-$  economy with three banks denoted by  $A$ ,  $B$  and  $C$ . We consider the following structure to be prevailing at time 0 (before any action in the model is taken):

<b>Bank B</b>			
Asset side		Liability side	
Real assets	$I$	Deposits from small depositors	$(1 - \alpha_{BC})$
Liquid Assets	$(m - \alpha_{AB})$	Deposits from bank C	$\alpha_{BC}$
Deposit in Bank A	$\alpha_{AB}$	Equityholders	$E_B$

<b>Bank C</b>			
Asset side		Liability side	
Real assets	$I$	Deposits from small depositors	$1$
Liquid Assets	$(m - \alpha_{BC})$	Equityholders	$E_C$
Deposit in Bank B	$\alpha_{BC}$		

**Figure 2**

Bank  $A$  is a failed bank. Bank  $B$  has  $\alpha_{AB}$  units of deposits in bank  $A$ . The liability side of bank  $B$  is composed by two classes of depositors:  $(1 - \alpha_{BC})$  units are held by infinitesimal depositors (henceforth small depositors), and the remaining  $\alpha_{BC}$  units are held by bank  $C$ , whose capital structure is composed by small depositors with mass equal to one. On the asset side, the two ongoing banks ( $B$  and  $C$ ) have  $m$  liquid assets and  $I$  illiquid assets. Their liquid assets (apart from their respective deposits in other banks) are composed by cash. We can respectively summarize at time 0 the balance sheet of both bank  $B$  and bank  $C$  as follows (refer to figure 2):

where:

- The  $I$  units of illiquid assets pay off a total random return  $R(\theta_i)$  at  $t = 2$ , where  $\theta_i$  represents the underlying fundamentals of bank  $i$ , where  $i = B, C$ . There is, however, a liquidation cost if assets are liquidated prematurely (i.e., at  $t = 1$ ). In this case, bank  $i$  can only obtain a maximum payoff (i.e., liquidating all its  $I$  units) of  $\lambda(\theta_i) \leq R(\theta_i)$ . We assume that both banks' fundamentals are distributed independently and uniformly between  $L$  and  $U$ .



- The cash amount held by each bank grows at the risk-free rate, which is set at zero.
- The depositors are risk neutral and do not have liquidity needs at the intermediate date -i.e., there are no depositors who necessarily need to consume at  $t = 1$ .
- All deposits are demandable debt and pay off the following:
  - Unless the bank fails at the intermediate date, a depositor obtains a payoff of 1 if she withdraws before maturity (we will refer to them as early depositors); and a return  $r(\theta_i)$  if she waits until maturity (we will refer to them as late depositors).
  - If the bank cannot satisfy the claims by its early depositors, the bank fails. The early depositors obtain the value of the liquidated bank, and the late depositors receive a payment of zero.<sup>2</sup>
- The equityholders are risk neutral. They are passive and they receive the residual payoffs at maturity.
- The decision over a bank's deposit in other banks is delegated to the manager of the bank, who maximizes her bank's equityholders' value.<sup>3</sup>

## 2.1 Information and Timing

At  $t = 0$  bank  $A$  fails. In consequence, there is a shock to the asset side of bank  $B$  since it has a deposit of  $\alpha_{AB}$  units in bank  $A$ . Afterwards, each depositor of bank  $B$  receives a private signal

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<sup>2</sup>We do not need depositors who may necessarily have to consume at the intermediate date because of two reasons. The first one is due to the fact that we assume demand deposits in order to study the resulting equilibrium. Hence, we want to study the resulting equilibrium given the demand deposits, thus we do not want to endogenize the demand deposit contract, thus we do not need liquidity preferences to endogenize the demand contract. The second one is because the initial liquidity shock -in our model- comes from the initial failure of bank  $A$ , thus we do not need stochastic liquidity preferences at the intermediate date to generate the initial shock in our model.

<sup>3</sup>Alternatively, the manager could maximize the value of the whole bank, i.e. the value of shareholders and depositors altogether.

about  $\theta_B$ , the underlying fundamentals of bank  $B$ . The signal received by a small depositor  $i$  is  $b_i = \theta_B + \varepsilon_i$ , where  $\varepsilon_i$  is uniformly distributed on  $[-e_B, +e_B]$  and is independent of the rest of errors in the model. We assume that the manager of bank  $C$  receives a noiseless private signal  $(b_M, c_M)$  about the underlying fundamentals of bank  $B$  and bank  $C$ .

After all depositors of bank  $B$  have received their private information, they play -i.e., they withdraw early from -or remain in- bank  $B$ . Once they have initially played, depositors of bank  $C$  receive a public signal about the early withdrawals by the small depositors of bank  $B$ . Intuitively, this public signal represents the fact that depositors of other banks can observe queues of depositors running to their bank or, they learn that through media releases. Apart from this public signal, each depositor of bank  $C$  receives a private signal about  $\theta_C$ , the underlying fundamentals of bank  $C$ . The signal received by depositor  $i$  is  $c_i = \theta_C + \epsilon_i$ , where  $\epsilon_i$  is uniformly distributed on  $[-e_C, +e_C]$  and is independent of the rest of errors in the model. Given the private signals about the underlying fundamentals of bank  $C$  and the public signal about the early withdrawals in bank  $B$ , the depositors of bank  $C$  withdraw early from -or remain in- bank  $C$ .

### 2.1.1 Public Signal

The public signal reflects the level of early withdrawals by small depositors in bank  $B$ .<sup>4</sup> The public signal -that we assume- captures the following: when there is a low level of withdrawals in bank  $B$ , the public signal  $s(w_B)$  will indicate that the level is low ( $L$ ) with probability one (where  $w_B$  represents the level of early withdrawals by small depositors of bank  $B$ ). We define low level ( $w_B \leq \underline{w}_B$ ) as the case in which even if bank  $C$  would withdraw completely its claim at the intermediate date, it would not imply the failure of bank  $B$ . In the same way, when there is a high level of withdrawals in bank  $B$ , the public signal  $s(w_B)$  will indicate that the the level

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<sup>4</sup>The intuition about this public signal is stated in the previous subsection.

is high ( $H$ ) with probability one. We define high level ( $w_B > \bar{w}_B$ ) as the case in which even if bank  $C$  would remain in bank  $B$ , this bank would fail due to the high level of early withdrawals by small depositors. However, we assume that there is an intermediate region of early withdrawals in which there is confusion. This specifically means that the public signal will either take a high value ( $H$ ) with probability one half or, a low value ( $L$ ) with probability one half. This intermediate region can have a measure that tends to zero and, it is defined by the level of early withdrawals ( $\underline{w}_B < w_B \leq \bar{w}_B$ ). In consequence, we assume the following public signal  $s(w_B)$  for the intermediate region:

$$s(\underline{w}_B < w_B \leq \bar{w}_B) = \begin{cases} H & wp\ 0.5 \\ L & wp\ 0.5 \end{cases} \quad (1)$$

### 2.1.2 Time Line

We can now summary the extensive form of the game as follows:

- At time  $t = 0$  :
  - Bank  $A$  fails.
- At time  $t = 1$  :
  - The small depositors of bank  $B$  receive private information about the underlying fundamentals of bank  $B$ . Manager of bank  $C$  receives perfect information about the underlying fundamentals of both bank  $B$  and bank  $C$ .
  - All depositors of bank  $B$  independently choose whether to withdraw early from bank  $B$  based on their information set. Early depositors are paid.
  - The public signal  $s(w_B)$  is released.

- Depositors of bank  $C$  receive private information about the underlying fundamentals of bank  $C$ , and independently decide, based on their information set, whether to withdraw early from bank  $C$ . Early depositors are paid.
- At time  $t = 2$  :
  - Late depositors of bank  $B$  and bank  $C$  are paid if their own banks have not failed at the intermediate period. After all depositors have been paid, the residual payoffs are shared by the equityholders.

We denote by  $\Gamma(s(w_B))$  the game defined by the previous extensive form.

## 2.2 Assumptions

- We assume that there exist a level of fundamentals  $\theta^U > 0$  such that if bank  $i$  liquidates -at the intermediate date- all its  $I$  units of illiquid assets, the value obtained is restricted to:

$$\lambda(\theta_i) = \begin{cases} 1 & \text{if } \theta_i \geq \theta^U \\ 0 & \text{if } \theta_i < \theta^U \end{cases} \quad (2)$$

This assumption implies the existence of an upper dominance region or "supersolvency" zone, which is needed for the uniqueness of the equilibrium.<sup>5</sup> It implies that when the fundamentals belong to this region, a depositor would remain in the bank independently of what she thinks other depositors would do.<sup>6</sup>

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<sup>5</sup>See Morris and Shin (2004) and Goldstein and Pauzner (2004) for complete details on the conditions needed for uniqueness of the equilibrium in global games.

<sup>6</sup>The upper dominance region or supersolvency zone is explained in detail when the equilibrium of the game is derived.

- If bank  $i$  does not fail at intermediate date, we assume that the payoff  $R(\theta_i)$  from the illiquid assets is  $1 + \theta_i$  and, the payoff  $r(\theta_i)$  for a late depositor is  $(1 + \gamma\theta_i)$ , where  $\gamma > 0$  and close to zero.<sup>7</sup>

## 2.3 Payoffs

### 2.3.1 Bank B

Let the fundamentals of bank  $B$  be on  $[L, \theta_B^U]$ .<sup>8</sup> We analyze the level of early withdrawals by small depositors of bank  $B$  for two separate regions; in one -as we will check- the action taken by the manager of bank  $C$  is irrelevant in determining whether bank  $B$  fails or survives. Whereas, in the other region, the action taken by the manager of bank  $C$  is crucial in determining the failure or not of bank  $B$ . Thus, depending on the specific region, the payoffs for the depositors and equityholders of bank  $B$  are the following:

- Action taken by the manager of bank  $C$  is irrelevant in determining failure of bank  $B$ :

Bank  $B$  survives at  $t = 1$  if the level  $w_B$  of early withdrawals by its small depositors does not exceed the cash minus the potential claim of bank  $C$  in bank  $B$  -i.e.,  $w_B \leq \underline{w}_B$ . Hence, if  $w_B \in [0, m - \alpha_{AB} - \alpha_{BC}]$ , the payoff for an early small depositor is 1 and the payoff for a late small depositor is  $(1 + \gamma\theta_B)$ . The payoff to bank  $C$  is  $w_{BC}$  at the intermediate date and  $(\alpha_{BC} - w_{BC})(1 + \gamma\theta_B)$  at maturity, where  $w_{BC}$  are the units withdrawn early by the manager of

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<sup>7</sup>All we need is that the return for the late depositors -given that the bank does not fail at the intermediate date- depends on the fundamentals. For instance, this could be due to a probability of failure on the risky investments, which -in turn- depends on the fundamentals of the bank (see Peydró-Alcalde, 2004). Alternatively, see Goldstein and Pauzner (2004) for an alternative justification on this assumption.

<sup>8</sup>It is easily shown that if the fundamentals are in the upper dominance region, i.e.  $\theta_B \in [\theta_B^U, U]$ , bank  $B$  does not fail at the intermediate date, and thus the payoffs for an early depositor and a late depositor are respectively 1 and  $(1 + \gamma\theta_B)$ . See more on this on the section about the equilibrium.

bank  $C$ .<sup>9</sup>

Bank  $B$  fails at  $t = 1$  if the level of early withdrawals by small depositors exceeds the cash held by bank  $B$  -i.e.,  $w_B > \bar{w}_B$ . The reason is twofold: firstly, it is due to the assumption that -if the fundamentals are not in the upper dominance region- the prematurely liquidated illiquid assets are worthless. Secondly, given that bank  $A$  has failed, the value that bank  $B$  can obtain from its deposit in bank  $A$  is zero. In consequence, for  $w_B > m - \alpha_{AB}$ , the payoff for an early small depositor is  $(m - \alpha_{AB}) / (w_B + w_{BC})$ . The payoff at  $t = 1$  for bank  $C$  is  $w_{BC} (m - \alpha_{AB}) / (w_B + w_{BC})$ . However, late depositors and equityholders obtain a payoff of zero.

- Action taken by the manager of bank  $C$  is crucial in determining the failure of bank  $B$ :

The last case is when bank  $B$  fails only if the manager of bank  $C$  withdraws early part of her bank's claims in bank  $B$ . This case corresponds to  $\underline{w}_B < w_B \leq \bar{w}_B$ . Hence, if the manager of bank  $C$  withdraws early  $w_{BC}$  from bank  $B$ , where  $w_{BC} > m - \alpha_{AB} - w_B > 0$ , bank  $B$  fails due to the early withdrawal by the manager of bank  $C$ . In this case, the payoff for an early small depositor is  $(m - \alpha_{AB}) / (w_B + w_{BC})$  and for bank  $C$  is  $w_{BC}(m - \alpha_{AB}) / (w_B + w_{BC})$ .<sup>10</sup> Otherwise, if the manager of bank  $C$  withdraws a smaller quantity  $w_{BC} \in [0, m - \alpha_{AB} - w_B)$ , there is no failure at bank  $B$ . In this case, the payoff for an early small depositor is 1 and for a late small depositor is  $(1 + \gamma\theta_B)$ .<sup>11</sup>

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<sup>9</sup>The shareholders always receive the following residual payoffs at maturity:  $\max[0, 1 + \theta_B + m - \alpha_{AB} - w_B - w_{BC} - (1 - w_B - w_{BC})(1 + \gamma\theta_B)]$ . In this particular case, they would receive:  $1 + \theta_B + m - \alpha_{AB} - w_B - w_{BC} - (1 - w_B - w_{BC})(1 + \gamma\theta_B)$ .

<sup>10</sup>The payoffs both for late depositors and shareholders are zero.

<sup>11</sup>Bank  $C$ 's payoff would be  $w_{BC}$  at the intermediate date and  $(\alpha_{BC} - w_{BC})(1 + \gamma\theta_B)$  at time  $t = 2$ . Instead, the payoff for shareholders would be  $1 + \theta_B + m - \alpha_{AB} - w_B - w_{BC} - (1 - w_B - w_{BC})(1 + \gamma\theta_B)$ .

### 2.3.2 Bank C

Let the fundamentals of bank  $C$  be on  $[L, \theta_C^U]$ .<sup>12</sup> The payoffs for its depositors and equityholders are the following ones:

Let  $q(t)$  be the payoff that bank  $C$  obtains as a depositor from bank  $B$  at time  $t$  (both at the intermediate date  $t = 1$  and at maturity  $t = 2$ ). There are two differences between the payoffs for bank  $C$  and for bank  $B$ : The first one is that bank  $C$  does not have a large depositor. The second one is that bank  $C$  obtains some payoff via its deposit in bank  $B$  -reflected in the random variable  $q(t)$ . Hence, the payoffs are as follows:

Bank  $C$  fails at the intermediate date if the level  $w_C$  of early withdrawals by its depositors exceeds the liquidity that bank  $C$  has at  $t = 1$ . Hence, for  $w_C > m - \alpha_{BC} + q(1)$ , an early depositor obtains  $(m - \alpha_{BC} + q(1))/w_C$  and a late depositor obtains zero payoff.<sup>13</sup> Otherwise, if  $w_C \leq m - \alpha_{BC} + q(1)$ , bank  $C$  does not fail. In consequence, an early depositor obtains a payoff of 1 and a late depositor obtains an payoff of  $(1 + \gamma\theta_C)$ .<sup>14</sup>

## 3 Equilibrium

### 3.1 Preliminaries

The decision on early withdrawal by a depositor depends on her expected value of remaining in the bank versus withdrawing early. Since we have assumed that a depositor does not have

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<sup>12</sup>Otherwise, if bank  $C$ 's fundamentals are in the upper dominance region, i.e.  $\theta_C \in [\theta_C^U, U]$ , bank  $C$  does not fail at the intermediate date. Thus the payoffs for an early depositor and a late depositor would be 1 and  $(1 + \gamma\theta_C)$  respectively. See more on this on the section about the equilibrium.

<sup>13</sup>The shareholders of bank  $C$  would also obtain a zero payoff.

<sup>14</sup>The shareholders would receive at maturity the residual payoffs:  $1 + \theta_C + q(1) + q(2) + m - \alpha_{BC} - w_C - (1 - w_C)(1 + \gamma\theta_C)$ .

liquidity needs at the intermediate date, her decision on early withdrawal depends only on both the fundamentals of her bank and on her beliefs about other depositors' actions. Intuitively, the belief of higher fundamentals will favor the decision of remaining in the bank, whereas the belief of higher early withdrawals by other depositors will favor the decision of running to the bank. Moreover, as we will see in more detail below, a depositor is not only interested on the decisions of other depositors from her own bank, but also, on the decisions by depositors of the other on-going bank, i.e. the one linked to hers through the interbank market.<sup>15</sup>

There are, however, situations in which a depositor takes her decision only based on the fundamentals of her own bank (i.e., her belief about other depositors' actions does not matter in deciding whether to withdraw early or not). On one hand, when the fundamentals of a bank are very strong -the bank is "supersolvent"- a depositor from this bank will remain in it, regardless of what she thinks about other depositors' actions. This is because the fundamentals are very strong; hence, the bank will survive independently of the level of early withdrawals that it will face. On the other hand, when the fundamentals of a bank are very weak -the bank is insolvent- a depositor will withdraw early, regardless of what she thinks about other depositors' actions. Obviously, since the bank is insolvent, the value, which the bank will generate at maturity, will not be attractive enough, precipitating the run from this depositor. In consequence, in those states of nature, a depositor would take her decision only based on the fundamentals of her own bank.

We now proceed to delimit these two zones or regions. In particular, if the fundamentals of bank  $i$  are such that  $\theta_i \in [\theta_i^U, U]$ , it will be optimal for a depositor of bank  $i$  not to withdraw early. This follows from the fact that even if all the other depositors of bank  $i$  would withdraw, the bank would not fail. Thus a depositor would obtain a higher payoff remaining in the bank

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<sup>15</sup>That is, depositors in bank  $i$  are interested on actions both by depositors of her own bank  $i$ , and also, by depositors of the other bank  $-i$ , where  $i \in \{B, C\}$ .



than withdrawing early ( $(1 + \gamma\theta_i)$  versus 1, where  $\theta_i > 0$ ). This follows from the assumption that the value of the liquidated assets, at the intermediate date, is higher as the value of the assets are higher -i.e.,  $\lambda(\theta_i) = 1$ , for a  $\theta_i \in [\theta_i^U, U]$ , which implies that the bank will not fail at  $t = 1$ .<sup>16</sup> The previous range of fundamentals is called upper dominance region or "supersolvency" zone. Instead, if the fundamentals of the bank  $i$  are such that  $\theta_i \in [L, 0]$ , it will be optimal for a depositor of bank  $i$  to withdraw early. This follows from the fact that even if all the other depositors of bank  $i$  would remain, she would obtain a higher payoff withdrawing early than remain in bank  $i$  ( $1$  versus  $(1 + \gamma\theta_i)$ , where  $\theta_i \leq 0$ ). The previous range of fundamentals is called lower dominance region or insolvency zone.<sup>17</sup>

We know that if the fundamentals of a bank  $i$  are in one of the two extreme zones, a depositor of bank  $i$  will have an easy strategy: withdraw early in the insolvent zone and remain in the bank in the "supersolvent" zones. However, when the fundamentals of bank  $i$  are not in those extremes zones, i.e.  $\theta_i$  is in the intermediate region  $[0, \theta_i^U]$ , bank  $i$  is solvent but illiquid: if all its depositors remain, bank  $i$  does not fail. However, if all of them withdraw early, bank  $i$  fails. As a consequence, this region of fundamentals presents a strategic problem for the depositors -i.e., there is a coordination problem among the depositors. Furthermore, the problem is even more complicated since the coordination problem is not only among the depositors of one bank, but also, there is a coordination problem among the depositors of the two on-going banks. For instance, if we would assume that the depositors have perfect information about the fundamentals -or, more generally, they do not have differential information-, depositors would coordinate themselves perfectly on their actions and, in consequence, the game would present four equilibria: both banks fail, both banks survive, and one bank survives and the other fails. Since the ex-ante probabilities of the different

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<sup>16</sup>See Rochet and Vives (2004), Allen and Gale (2000) and Visny and Sleifer (1992) for the justification of this assumption and a more detailed analysis of this process.

<sup>17</sup>The assumptions about these regions are explained in detail in the survey on global games by Morris and Shin (2004). See, also, Goldstein and Pauzner (2004) and Rochet and Vives (2004).

equilibria are not endogenously determined, it would not be possible to analyze the questions that we want to address in this paper: how does the probability of failure of the borrower bank  $B$  change depending on its claim with the failed bank  $A$  ( $\alpha_{AB}$ ), on its claim with the creditor bank  $C$  ( $\alpha_{BC}$ ) and, on the interaction of both claims? We can, however, answer to the previous question if we assume -as we do in this paper- that depositors have differential information on the fundamentals. In this case, we are able to link both banks' depositors' strategies and we can determine the unique equilibrium of the game -even when the precision of their signals tend to infinite. Hence, we are able to perform the comparative statics of the unique equilibrium and answer our initial questions on the unfolding of a financial crisis.

### 3.2 Strategies and Equilibrium

We focus on threshold strategies for the depositors. This is not restrictive since -as we will show- they use those ones in the unique equilibrium of the game. However, the manager of bank  $C$  apparently has a more complicated strategy as we can see in the following characterization of the strategies for each agent:

- Small depositors of bank  $B$ : A small depositor  $i$  of bank  $B$  withdraws at the intermediate time if and only if her private signal  $b_i$  is such that  $b_i \leq b^*$ .
- Small depositors of bank  $C$ : A small depositor  $i$  of bank  $C$  withdraws at the intermediate time if and only if her private signal  $c_i$  is such that  $c_i \leq c^*(s(w_B))$ .
- Manager of bank  $C$ : The manager of bank  $C$  withdraws  $w_{BC} \in [0, \alpha_{BC}]$  at the intermediate time depending on the fundamentals  $(\theta_B, \theta_C)$  and the public signal  $s(w_B)$ .<sup>18</sup>

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<sup>18</sup> A depositor of bank  $B$  with signal  $b^*$  is indifferent between withdrawing early or remaining in the bank. Therefore, she can withdraw early a part of her claim since she is indifferent at that point, and her action does not affect other's actions. Therefore, her action is not important for the equilibrium; hence, we can assume, without loss of generality,

We check in the appendix that as the errors of depositors' private signals  $e_i$ ,  $i \in \{B, C\}$ , tend to  $0^+$ , the strategy of the manager of bank  $C$  converges to a threshold one -i.e., she withdraws  $\alpha_{BC}$  at the intermediate date if and only if her private noiseless signal  $b_M$  about the fundamentals of bank  $B$  is such that  $b_M < b_M^*(c_M)$ , where  $c_M$  is her private noiseless signal about fundamentals of her own bank. Instead, she remains in bank  $B$  if  $b_M > b_M^*(c_M)$ . Hence, her strategy converges to the threshold function  $b_M^*(c_M)$  as  $(e_i)_{i \in \{B, C\}} \rightarrow 0^+$ .

Since we are interested in this paper on the comparative statics of the equilibrium with respect to the interbank market claims ( $\alpha_{AB}$  and  $\alpha_{BC}$ ). And, given that this analysis is more simple as the error of the private signals tend to zero, we prove -in the appendix- the existence and uniqueness of the equilibrium as the error is positive but very close to zero. We now state the result about existence and uniqueness and, then, explain intuitively how to obtain the equilibrium threshold strategies of the depositors and the equilibrium strategy of the manager.

**Proposition 1** *There exists a unique equilibrium in  $\Gamma(s(w_B))$  which is characterized by the following vector:  $(\bar{\theta}_B, \underline{\theta}_B, \theta_B^E(\theta_C, s(w_B)), b^*, w_{BC}(\theta_B, \theta_C, s(w_B)), c^*(H), c^*(L), \theta_C^E(s(w_B), \theta_B))$ .*

We assume that all agents use the equilibrium strategies defined by the previous proposition. Hence, a depositor  $i$  with a private signal  $b_i$  equal to  $b^*$  would be indifferent between withdrawing early and remaining in bank  $B$ . The payoffs that she expects depends on the fundamentals of both bank  $B$  and bank  $C$ , and on the public signal  $s(w_B)$  about the early withdrawals that will be released after they initially play. Let  $\pi_B(\theta_B, s(w_B), \theta_C)$  be the payoff of remaining in bank  $B$  versus withdrawing early. We know -from the subsection on payoffs- that the payoffs  $\pi_B(\theta_B, s(w_B), \theta_C)$  are affected by the fundamentals of bank  $C$  via the early withdrawal  $w_{BC}(\theta_B, \theta_C, s(w_B))$  by the

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that a depositor with a signal  $b^*$  withdraws early. However, the decision of the manager of bank  $C$  is very different. We will see that she can have a strict preference of withdrawing only part of the claim of bank  $C$  in bank  $B$  early. Therefore, her strategy is defined from the fundamentals to the whole range between 0 and  $\alpha_{BC}$ .

manager of bank  $C$ . Furthermore, if a complete early withdrawal by bank  $C$   $-w_{BC} = \alpha_{BC}$ - implies the failure of bank  $B$  -i.e., if bank  $B$ 's fundamentals are such that  $\theta_B \in [\underline{\theta}_B, \bar{\theta}_B]$ - the payoffs  $\pi_B(\theta_B, s(w_B), \theta_C)$  are crucially determined by the action of the manager of bank  $C$ . Therefore, the payoffs -of remaining in bank  $B$  versus withdrawing early- for a depositor of bank  $B$  are the following:

$$\pi_B(\theta_B, s(w_B), \theta_C) = \begin{cases} \gamma\theta_B & \text{if } \theta_B \in [\bar{\theta}_B, U] \\ \gamma\theta_B - [\gamma\theta_B + \frac{m-\alpha_{AB}}{w_B+w_{BC}}]\Lambda & \text{if } \theta_B \in [\underline{\theta}_B, \bar{\theta}_B] \\ -\frac{m-\alpha_{AB}}{w_B+w_{BC}} & \text{if } \theta_B \in [L, \underline{\theta}_B] \end{cases} \quad (3)$$

Where  $\Lambda$  denotes the indicator function, which takes the value one if there is an early withdrawal  $w_{BC}$  from the manager of bank  $C$  such that implies the failure of bank  $B$ . Therefore, since a depositor  $i$  of bank  $B$  with a private signal  $b^*$  has an expected value of withdrawing early which is equal to the expected value of remaining in bank  $B$ , thus, we obtain the following equation:

$$\int_L^U \sum_{s(w_B) \in \{L, H\}} \left[ f(s(w_B)/\theta_B) \int_L^U \pi_B(\theta_B, s(w_B), \theta_C) d\theta_C \right] dF(\theta_B/b^*) = 0 \quad (4)$$

where  $F(\theta_B/b^*)$  denotes the cumulative distribution function of the posterior belief about the fundamentals of bank  $B$  -given the threshold signal  $b^*$ . And,  $f(s(w_B)/\theta_B)$  denotes the density function of the public signal  $s(w_B)$  given the fundamentals  $\theta_B$ . We want to stress three points about the previous two equations: Firstly, since the manager of bank  $C$  has perfect information about the fundamentals of both banks, she will withdraw early if  $\theta_B \in [L, \bar{\theta}_B]$ . Otherwise, she would obtain a zero payoff, while, withdrawing early, she gets a positive payoff. Therefore, her strategy  $w_{BC}$  is equal to  $\alpha_{BC}$  when the fundamentals of bank  $B$  belong to  $[L, \bar{\theta}_B]$ . Secondly, we can notice that as the withdrawals in bank  $C$  increases, the indicator function  $\Lambda$  will be positive for

more states of nature. Thus increasing the incentive of early withdrawals by depositors of bank  $B$ . Thirdly, the indicator function  $\Lambda$  depends both on the behavior of the manager of bank  $C$  and, also, on the behavior of the depositors of bank  $C$  -i.e., depends on  $(\theta_B, s(w_B), \theta_C)$ .<sup>19</sup> Therefore, we need to analyze the game backwards in order to determine the strategies of both the depositors and the manager of bank  $C$ .

Since the manager of bank  $C$  has perfect information, her strategy  $w_{BC}$  is simple. She will withdraw early from bank  $B$  if the fundamentals of bank  $B$  are sufficiently low -i.e. if the level of early withdrawals by depositors of bank  $B$  imply the failure of their bank -i.e.,  $w_B > \underline{w}_B$  or, identically, if the fundamentals are such that  $\theta_B < \underline{\theta}_B$ . However, even if the fundamentals are better ( $\theta_B \geq \underline{\theta}_B$ ), the manager of bank  $C$  can withdraw early due to the particular realization of both her bank's fundamentals  $\theta_C$  and the public signal  $s(w_B)$ . As a consequence, we can see that the failure of bank  $B$  depends on the realization of the fundamentals of bank  $C$  and on the public signal. Therefore, the level of fundamentals  $\theta_B^F$  of bank  $B$  such that this bank fails can be between  $\underline{\theta}_B$  and  $\bar{\theta}_B$ , and depends on  $(\theta_C, s(w_B))$ .

Once the depositors of bank  $B$  have initially played, the depositors of bank  $C$  receive a public signal  $s(w_B)$ , which reflects the level of early withdrawals by small depositors of bank  $B$ . We assume a depositor  $i$  of bank  $C$  who has a private signal  $c_i$  equal to  $c^*(s(w_B))$ , where  $s(w_B) \in \{H, L\}$ . Once this depositor has observed the public signal  $s(w_B)$ , she updates her belief about her bank's potential liquidity at time  $t = 1$  -i.e., she uses the updated cumulative distribution function  $F(w_B/s(w_B))$  over the level of early withdrawals by the small depositors of bank  $B$ . The maximum liquidity  $q_M(1)$ , which her bank can obtain at time  $t = 1$  through its deposit with bank  $B$ , depends on the early withdrawals  $w_B$ , and is the following:

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<sup>19</sup> As it is shown in the appendix, the fundamentals of bank  $C$  -with the public signal  $s(\theta_B)$ - determine the number of early withdrawals. Therefore,  $(s(\theta_B), \theta_C)$  is a sufficient statistic of the behaviour of the depositors of bank  $C$ .

$$q_M(1) = \begin{cases} \alpha_{BC} & \text{if } w_B \in [0, \underline{w}_B] \\ \alpha_{BC} \frac{(m - \alpha_{AB})}{w_B + \alpha_{BC}} & \text{if } w_B \in [\underline{w}_B, 1 - \alpha_{BC}] \end{cases} \quad (5)$$

where  $\underline{w}_B$  is the level of early withdrawals in bank B such that if bank C withdraws completely from B, bank B fails. Therefore,  $\underline{w}_B$  is equal to  $(m - \alpha_{AB}) - \alpha_{BC}$ . The decision of early withdrawing by a depositor of bank C depends on her bank's fundamentals  $\theta_C$  and on the public signal  $s(w_B)$  that she observes. However, her payoffs are also affected by the particular realization of the fundamentals of bank B or, alternatively, the number of early withdrawals  $w_B$ . In particular, the creditor bank's fundamentals' level  $\theta_C^F$  -the level of fundamentals that bank C fails- depends on the value of the liquidated assets at the intermediate date, which -in turn- depends on  $q_M(1)$ . The payoffs for a depositor  $i$  of bank C -of remaining in her bank versus withdrawing early-are the following:

$$\pi_C(w_B, s(w_B), \theta_C) = \begin{cases} \gamma \theta_C & \text{if } \theta_C \in [\theta_C^F, U] \\ -\frac{m - \alpha_{BC} + q_M(1)}{w_C} & \text{if } \theta_C \in [L, \theta_C^F) \end{cases} \quad (6)$$

where  $\theta_C^F$  is implicitly defined by  $w_C(\theta_C^F) = m - \alpha_{BC} + q_M(1)$ , where  $q_M(1)$  is a function of  $w_B$  -i.e., the level of fundamentals of bank C such that the early withdrawals in bank C are equal to the value of the liquidated assets. Therefore, the depositor  $i$  of bank C with signal  $c^*(s(w_B))_{i \in \{L, H\}}$  has an expected value of withdrawing early from bank C which is equal to the expected value of remaining in her bank. Hence, we obtain the following equation:

$$\int_0^{1 - \alpha_{BC}} \int_L^U \pi_C(w_B, s(w_B), \theta_C) dF(\theta_C / c^*(s(w_B))) dF(w_B / s(w_B)) = 0 \quad (7)$$

From the previous equation, we obtain the thresholds for a depositor of bank  $C$  -i.e.,  $c^*(H)$  and  $c^*(L)$ . Solving for the equation (4) given the thresholds for a depositor of bank  $C$ , we obtain the threshold  $b^*$  for a depositor of bank  $B$ . As the errors of the private signals of depositors of bank  $B$  and bank  $C$  tend to zero, we obtain the following solution:

$$b^* = \frac{\ln(m - \alpha_{AB} + \alpha_{BC})^{-\frac{1}{\gamma}} + \frac{P(L)+P(H)}{2} \ln\left(1 + \frac{\alpha_{BC}}{m-\alpha_{AB}}\right)^{\frac{1}{\gamma}}}{1 - \frac{P(L)+P(H)}{2} \frac{\alpha_{BC}}{m-\alpha_{AB}}} \quad (8)$$

$$\text{where } P(i) = \frac{c^*(i)-L}{U-L}, i \in \{L, H\} \quad (9)$$

$$c^*(H) = \ln m^{-\frac{1}{\gamma}} \quad (10)$$

$$c^*(L) = \ln(m - \alpha_{BC}(1 - m + \alpha_{AB}))^{-\frac{1}{\gamma}} \quad (11)$$

**Remark 2** *It is easy to notice that if  $P(i)$  -i.e. the probability of liquidity needs from bank  $C$ , given the public signal  $i$ - increases, the threshold  $b^*$  increases, thereby increasing, the ex-ante probability of financial distress for bank  $B$ . However, the strategies of the depositors of one bank are not strategic complements of the strategies of the depositors of the other bank.<sup>20</sup> For instance, once bank  $C$  has failed, as more depositors of bank  $B$  run to their bank, the maximum liquidity  $q_M(1)$  -that the manager of bank  $C$  can obtain from bank  $B$ - decreases. Thus the incentive by a depositor of bank  $C$  to withdraw early (versus remaining in her bank) decreases. As a consequence, their strategies are not strategic complements. However, as the early withdrawals in bank  $C$  increases, the incentive for a depositor of bank  $B$  to withdraw early does not decrease, even if her bank has failed. To see this, notice that if the level of fundamentals in bank  $B$  is high ( $\theta_B > \bar{\theta}_B$ ), higher number of early withdrawals by depositors of bank  $C$  does not affect bank  $B$ 's small depositors' incentive to withdraw early (see the payoff function()). Instead, if the level of fundamentals in bank  $B$  are such*

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<sup>20</sup> *The strategies of the depositors of each bank are not either strategic complement among themselves. This is due to the fact that once a bank fails, more early withdrawals from depositors in this bank imply less incentives for the other depositors to withdraw early, as compared to remain in the bank.*

that  $\theta_B \in [\underline{\theta}_B, \bar{\theta}_B]$ , higher number of early withdrawals by depositors of bank  $C$  tend to increase the incentive of bank  $B$ 's small depositors to withdraw early (see the payoff function()). Finally, if the level of fundamentals is lower than  $\underline{\theta}_B$ , an increase on early withdrawals at bank  $C$  does not increase the incentive to withdraw early by small depositors of bank  $B$ . This is due to the assumption of perfect information by the manager of bank  $C$ , which implies that she completely unwinds her bank's position in bank  $B$  once this bank is going to fail.

## 4 Results

The purpose of our paper is to analyze how strategic interactions among different agents in the interbank market affect the unfolding of a financial crisis. In particular, we are interested in the strategic interactions of two kinds of agents: the manager of a creditor bank and both her own bank's depositors and the borrower bank's depositors. We want to know how the strategic interactions of these agents affect the possibility of contagion of the initial shock and, moreover, the role of the interbank market in attenuating/amplifying the initial shock.

In order to pursue our analysis, we want to know how the probability of failure of the borrower bank  $B$  changes: Firstly, with respect to the level of its claim  $\alpha_{AB}$  with the initial failed bank  $A$  (first layer effect). Secondly, with respect to the level of the claim  $\alpha_{BC}$  that the creditor bank  $C$  has in the borrower bank  $B$  (second layer effect). And, finally, with respect to the interaction of the level of the two claims  $\alpha_{AB}$  and  $\alpha_{BC}$  (interaction of the first and second layer). By answering these questions, we want to check if the interbank market serves as a mechanism for transmitting an initial shock. Furthermore, we want to understand under which circumstances the interbank market is good at providing liquidity and, secondly, whether this latter function is altered when there has been a negative shock in the system.



**Proposition 3** *If the level  $\alpha_{AB}$  of deposit of bank  $B$  in bank  $A$  increases, the threshold for a depositor of bank  $B$  strictly increases, i.e.  $\frac{\partial b^*}{\partial \alpha_{AB}} > 0$ .*

The intuition is simple. The higher the level of deposits that bank  $B$  has in the failed bank  $A$ , the higher the extent of negative shock it faces. This implies a higher level of deterioration of the assets of bank  $B$ . Thus increasing the aggressiveness of the depositors of bank  $B$  and, hence, increasing the threshold  $b^*$ . As a consequence, there is an increase in the financial distress of bank  $B$ , hence, the interbank market transmits shocks.

We have just seen that the interbank market can act as a channel for contagion of an initial shock. But, at the same time, scholars like Goodhart, among others, have said that nowadays the interbank market can always provide liquidity to solvent but illiquid banks. More formally, it would imply that interbank markets would reduce the liquidity problems that solvent banks could receive from negative shocks.<sup>21</sup> The following propositions tries to address the validity of this claim.

**Proposition 4** *There exists a unique, and finite, level of banks' fundamentals  $\underline{U}(\alpha_{AB}, \alpha_{BC}, m)$  such that  $\frac{\partial b^*}{\partial \alpha_{BC}} < 0$  if and only if  $U > \underline{U}$ .*

If the fundamentals of the banking system are strong, the expected intermediate liquidity needs for bank  $C$  are low, then, higher level of deposits held by bank  $C$  in bank  $B$  lowers the strategic uncertainty of the small depositors of bank  $B$ .<sup>22</sup> In consequence, the threshold of small depositors of bank  $B$  is reduced -in turn- reducing the probability of failure of bank  $B$ .

The following proposition, and the associated corollary, state the most important result of the

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<sup>21</sup>See Rochet and Vives (2004) for an analysis of breakdown in the interbank market with multiple creditor banks.

<sup>22</sup>We can notice that since the model is symmetric, the proposition refers to the level of fundamentals of all the banks. Furthermore, we check that there are incentives ex-ante both for the participation of the depositors in the demand contracts and also for the participation of the banks in the interbank market.

paper, which implies that the interbank market not only transmits initial shocks –contagion via interbank claims– but also it can amplify these shocks via a reduction of the liquidity available in the market.

**Proposition 5** *There exists a unique, and finite, level of banks' fundamentals  $\bar{U}(\alpha_{AB}, \alpha_{BC}, m)$  such that  $\frac{\partial^2 b^*}{\partial \alpha_{AB} \partial \alpha_{BC}} > 0$  if and only if  $U < \bar{U}$ , where  $\bar{U}$  is strictly higher than  $\underline{U}$ .*

The implication of this proposition is that the interbank market, depending on the fundamentals of the banking system, worsens the fragility of a bank which has an initial negative shock (in our model, this negative shock comes through the exposure, via the interbank market, to the failed bank  $A$ ). To put the previous claim into the context of the model: from the first proposition, we know that higher percentage of deposits of bank  $B$  in the failed bank  $A$  implies higher negative impact on bank  $B$ . Furthermore, proposition (5) states that this negative impact can even be amplified as the level of deposits that bank  $B$  has from bank  $C$  increases. In this case, this implies a higher probability of financial distress for the borrower bank  $B$ . In consequence, the interbank market can amplify the initial shock.<sup>23</sup> In sum, the previous effects imply that not only does the interbank transmit initial shocks from borrowers to creditors, but also, the initial shock can be amplified by creditor banks via a reduction of the expected liquidity available in the interbank market.

**Corollary 6** *There exists an  $\varepsilon > 0$  such that if  $\frac{\partial b^*}{\partial \alpha_{BC}} \in (-\varepsilon, +\varepsilon)$ , the threshold of a depositor of bank  $B$  satisfies  $\frac{\partial^2 b^*}{\partial \alpha_{AB} \partial \alpha_{BC}} > 0$ .*

The corollary is derived due to the result that  $\bar{U}$  is strictly higher than  $\underline{U}$ . This is because for certain region of fundamentals, higher level of fundamentals imply a reduction of the negative

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<sup>23</sup>This result is robust to the possibility of a limited bail-in by the creditor bank  $C$  (see more on this issue in Peydró-Alcalde, 2004).

impact that the creditor bank  $C$  can have on the borrower bank  $B$  –i.e., basically as the fundamentals are higher, the liquidity needs of the creditor bank  $C$  are lower, thus reducing the negative interaction (vicious circle) from the creditor bank’s depositors with those depositors in the borrower bank  $B$ .

The corollary implies that even under circumstances when higher level of deposits held by bank  $C$  in bank  $B$  by itself does not play an important role on the probability of failure of bank  $B$ , not only do we still find that the level of depositor runs are increasing in the extent of the initial shock, but also that the negative effect of initial shock is amplified by increasing the level of deposits held by bank  $C$  in bank  $B$ . Since the first layer always affects bank  $B$  negatively, the result states that even though an increase of deposits from the creditor bank would not significantly affect the incentive for the threshold depositor to withdraw early, it would amplify the negative threshold depositor’s reaction to a higher shock from the initial failed bank.<sup>24</sup> That is, the corollary states that, even if higher level of  $\alpha_{BC}$  does not significantly affect the threshold  $b^*$ , it affects the threshold depositor’s sensitivity to the first layer  $\partial b^*/\partial \alpha_{AB}$ . Furthermore, it amplifies the negative impact of the first layer  $\partial b^*/\partial \alpha_{AB}$ .<sup>25</sup>

Therefore, even though the level of the deposits held by the creditor bank does not -by itself- increase the financial fragility of its borrower, it amplifies the sensitivity of the negative shock that its borrower bank  $B$  received via its deposit with the failed bank. We now proceed to explain in more detail why the interaction of the first and second layer matters in the model when the second layer effect is not relevant. This is due to the following reasoning: If the level of deposit of bank  $C$  in bank  $B$  increases, this affects the the threshold depositor’s sensitivity to the first layer (i.e.,  $\alpha_{BC}$  affects  $\partial b^*/\partial \alpha_{AB}$ ) through three ways:

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<sup>24</sup>We sometimes abuse the notation by calling threshold depositor a depositor, who has a signal equal to threshold one.

<sup>25</sup>This results does not rely on the nature of the public signal or the number of creditor banks.

Firstly, it changes the borrower bank's small depositors' incentive of early withdrawal -i.e., it changes the threshold  $b^*$ . This first effect can be positive or negative depending on the level of the fundamentals (as it has been explained in the second proposition). However, since we are assuming that  $\partial b^*/\partial \alpha_{BC}$  is close to zero, we know that the first effect is not relevant.

Secondly, it changes the creditor bank's small depositors' incentive of early withdrawal -i.e., it changes the threshold  $c^*$ . Given the initial level of shock, higher the linkage of the bank  $C$  with bank  $B$ , higher the negative impact of the initial failed bank  $A$  on bank  $C$ . In consequence, this increases the aggressiveness of depositors of bank  $C$ , thus increasing the probability of distress of bank  $B$ .<sup>26</sup>

And, thirdly, it makes the borrower bank's small depositors' threshold ( $b^*$ ) and their reaction to the initial shock ( $\partial b^*/\partial \alpha_{AB}$ ) more sensitive to the behavior of the depositors of bank  $C$ . Thus, even if it was the case that an increase in  $\alpha_{BC}$  would not increase the aggressiveness of depositors of bank  $C$ , increasing the size of the claim of bank  $C$  in bank  $B$  makes depositors in bank  $B$  internalize more the expected liquidity needs of depositors in bank  $C$  at the intermediate date. Thus -caeteris paribus- increasing the financial distress of bank  $B$ .

## 5 Conclusions

In this paper, we analyze whether the interbank market serves as a mechanism to transmit shocks and if further interaction among agents in this market can reduce, or amplify, these initial shocks. The agents that we are interested in are managers of banks and depositors. Thus in order to pursue this analysis, we set up a simple model with only two on-going banks – one is a creditor bank and the other is a borrower bank, which receives an initial negative shock. These banks have

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<sup>26</sup>See remark (2) for how the behavior of depositors of bank  $C$  changes the behavior of depositors of bank  $B$ .

depositors, who can withdraw their deposits early based on their information set. Moreover, there is an active manager for the creditor bank, i.e. she takes her decisions based on internalizing all the depositors' actions.

We ask ourselves the following questions: How does the probability of failure of the borrower bank change with the level of its claim with the failed bank, and how does the creditor bank affect this probability? In order to answer these questions, we assume a model in which the agents have differential information, and we determine the unique equilibrium of the game. We find the following results: Firstly, the probability of failure of the borrower bank increases with its linkage with the failed bank (first layer effect). Secondly, even if higher fraction of deposits held by the creditor bank does not directly affect the probability of distress of the borrower bank, it amplifies the sensitivity of the borrower bank to the initial shock. These results are consistent with the findings of Iyer and Peydró-Alcalde (2004). Thus, we show that not only does the interbank market transmit initial shocks but it can also amplify the initial shocks by reducing the liquidity in the interbank market, and hence, increasing the financial fragility of the banking system.

### *Appendix*

In this appendix, firstly, we completely characterize the equilibrium of the game  $\Gamma(s(\theta_B))$  and show its existence and uniqueness. And, secondly, we provide the partials derivatives that support the main results of the paper.

Proof of Proposition 1.

We separate the proof on four steps. In the first step, assuming the existence of the threshold equilibrium, we characterize some important variables. In the second step, we find the threshold equilibrium both for a depositor of bank  $B$  and bank  $C$  -i.e., we find  $b^*$  and  $c^*(i)_{i \in \{L,H\}}$ . And,

then, we take the errors of the private signals by depositors as zero. In the third step, we show that indeed the threshold equilibrium –as the errors of the private signals converge to zero– converges to the equilibrium found in step two. Furthermore, we show that there cannot be another equilibrium. Finally, in the step four, we characterize the rest of the variables of the equilibrium -i.e.,  $(\bar{\theta}_B, \underline{\theta}_B, \theta_B^F(\theta_C, s(w_B)), w_{BC}(\theta_B, \theta_C, s(w_B)), \theta_C^F(s(w_B), \theta_B))$ .

First step: Firstly, we want to know at the threshold equilibrium for a depositor of bank  $i$ , the distribution of both the fundamentals of bank  $i$  and also the number of early withdrawals  $w_i$ . We assume that a depositor of bank  $C$  receives a signal  $c^*(i)$  and she observes that the public signal is  $s(w_B) = i$ ,  $i \in \{L, H\}$ . We called this depositor -abusing the notation- the threshold depositor. For her,  $\theta_C | c^*(i)$  is distributed uniformly between  $c^*(i) - e_C$  and  $c^*(i) + e_C$ . This is due to the assumption that  $\min\{(0 - L), (U - \theta^U)\} > 2e_C$ , which implies that  $L + e_C < c^*(i) < U - e_C$ . Furthermore, for the threshold depositor,  $w_C | c^*(i)$  is distributed uniformly between 0 and 1. The reason is the following:  $\forall w_C^+ \in [0, 1]$ , there exists a unique  $\theta_C^+ \in [c^*(i) - e_C, c^*(i) + e_C]$  such that  $w_C^+ = P[c_i \leq c^*(i) | \theta_C^+]$  which is equal to  $(0.5 + 0.5(c^*(i) - \theta_C^+)/e_C)$ . Hence,  $P[w_C \leq w_C^+ | c^*(i)] = \int_{\theta_C^+}^{c^*(i)+e_C} (\frac{1}{2e_C}) d\theta_C$  which is equal to  $0.5 + 0.5(c^*(i) - \theta_C^+)/e_C$  which is equal to  $w_C^+$ ,  $\forall w_C^+ \in [0, 1]$ . We can also notice the following relation between level of early withdrawals and fundamentals:

$$w_C(c^*(i)) = \begin{cases} 0 & \text{if } \theta_C \in [c^*(i) + e_C, U] \\ \frac{1}{2} \left(1 + \frac{c^*(i) - \theta_C}{e_C}\right) & \text{if } \theta_C \in [c^*(i) - e_C, c^*(i) + e_C] \\ 1 & \text{if } \theta_C \in [L, c^*(i) - e_C] \end{cases} \quad (9)$$

Applying the same reasoning for the threshold depositor at bank  $B$ , we find that  $\theta_B | b^*$  is distributed uniformly between  $b^* - e_B$  and  $b^* + e_B$ ; and,  $w_B | b^*$  is distributed uniformly between 0 and  $(1 - \alpha_{BC})$ . Furthermore, when the fundamentals are such that  $\theta_B \in [b^* - e_B, b^* + e_B]$ , the relation between early withdrawals by small depositors and fundamentals is defined by  $w_B =$

$$\frac{1-\alpha_{BC}}{2} \left(1 + \frac{b^* - \theta_B}{e_B}\right).$$

Secondly, we want to know -given the public signal  $s(w_B) = i$ ,  $i \in \{L, H\}$ - the distribution of the level of early withdrawals by the small depositors at bank  $B$  -i.e.,  $f(w_B/s(w_B))$ . Since we are not interested specific  $w_B$ , but on zones of  $w_B$ , we present the density function in the following way:

$$f(w_B/L) = \begin{cases} 0 & \text{if } w_B \in (\bar{w}_B, 1 - \alpha_{BC}] \\ \frac{\bar{\theta}_B - \underline{\theta}_B}{U - \underline{\theta}_B} & \text{if } w_B \in (\underline{w}_B, \bar{w}_B] \\ \frac{U - \bar{\theta}_B}{U - \underline{\theta}_B} & \text{if } w_B \in [0, \underline{w}_B] \end{cases} \quad (10)$$

$$f(w_B/H) = \begin{cases} \frac{\theta_B - L}{\bar{\theta}_B - L} & \text{if } w_B \in (\bar{w}_B, 1 - \alpha_{BC}] \\ \frac{\bar{\theta}_B - \underline{\theta}_B}{\bar{\theta}_B - L} & \text{if } w_B \in (\underline{w}_B, \bar{w}_B] \\ 0 & \text{if } w_B \in [0, \underline{w}_B] \end{cases} \quad (11)$$

Second step: We solve the game by backward induction. We substitute the above expressions in the equation (7) of the threshold depositor of bank  $C$  -given the public signal  $i$ - which implies the following equation:

$$\int_0^{m - \alpha_{BC}(1 - q_M(1, i))} \gamma [c^*(i) + (1 - 2w_C)e_C] dw_C = \int_{m - \alpha_{BC}(1 - q_M(1, i))}^1 \frac{m - \alpha_{BC}(1 - q_M(1, i))}{w_C} dw_C \quad (12)$$

If we take the errors of the private signals as zero, then, the threshold for a depositor of bank  $C$  is the following:

$$c^*(i) = \frac{\ln(m - \alpha_{BC}(1 - q_M(1, i)))}{\gamma} \quad (13)$$

$$\text{where } q_M(1, i) = \begin{cases} 1 & \text{if } i = L \\ (m - \alpha_{AB}) & \text{if } i = H \end{cases} \quad (14)$$

The behavior of the depositors of bank  $C$  -summarized in the threshold  $c^*(i)$ - determine the probability  $P(i)$  of liquidity needs by bank  $C$ , which is -in turn- internalized by the threshold depositor of bank  $B$ . Hence, substituting the equation (9) about  $P(i)$  in the equation (4) of the threshold depositor of bank  $B$ , we obtain the following expression:

$$\int_0^{m-\alpha_{AB}} \gamma \theta_B(w_B) dw_B = \int_{m-\alpha_{AB}}^1 \frac{m-\alpha_{AB}}{w_B+\alpha_{BC}} dw_B + \int_{m-\alpha_{AB}-\alpha_{BC}}^{m-\alpha_{AB}} (\gamma \theta_B(w_B) + \frac{m-\alpha_{AB}}{w_B+\alpha_{BC}}) P dw_B \quad (15)$$

Where  $P = \frac{P(L)+P(H)}{2}$ . Taking into consideration the function  $\theta_B(w_B)$  found in the first step of this proof, we obtain the threshold equilibrium represented by the equations 8, 9, 10 and 11 of page 16.

Third step: Firstly, we need to check that the threshold equilibrium of the game converges to the previous one -equation 8- as the error of the signals tends to zero. The only aspect, which we need to take care of, is that if the errors are not zero, the probability  $P(i)$  contains the threshold  $b^*$ . Hence, it implies that the equation for the threshold depositor of bank  $B$  is of second degree on  $b^*$ . However, as we will check now, one solution of the equation tends to infinite, and the other one converges to the threshold represented by equation 8. To see this, we can notice that we can write  $P(i)$  as  $(A + Bb^*e_B)/(C + Db^*e_B)$ , where  $A, B, C$  and  $D$  depend on the specific public signal  $i$  and do not depend on the error of the signal. Substituting  $P(i)_{i \in \{L, H\}}$  into the equation 18, we obtain the following equation of second degree on the threshold  $b^*$ :



$$A'e_B(b^*)^2 + (B' + C'e_B)b^* + (D' + E'e_B) = 0 \quad (16)$$

where  $A', B', C', D'$  and  $E'$  are functions of the parameters of the model, and do not contain the error of the private signal. We solve this equation as the error  $e_B$  of the signal tends to zero. One solution does not converge -tends to infinite- whereas the other solution tends to  $-D'/B'$  -i.e., tends to equation (8).

Secondly, we need to see that this threshold equilibrium is unique and there is no other equilibrium with different strategies. We can notice first that the public signal  $s(w_B)$ , which depositors of bank  $C$  receive, depends on the result of the actions from depositors of bank  $B$ , not on the fundamentals.<sup>27</sup> In consequence, once the depositors of bank  $C$  know the public signal  $i$  -as the error of the private signal tends to zero- they do not care about the strategies of the small depositors of bank  $B$  -since they know the money that their manager can extract from bank  $B$  and this is the only aspect that they internalize. Therefore, given the public signal, depositors of bank  $C$  do not take into account the strategies of depositors of bank  $B$ , and the game for the depositors of bank  $C$  is like the one that Goldstein and Pauzner (henceforth, G&P) analyzed. Therefore, the threshold is unique and there is not other equilibrium with different strategies for the depositors of bank  $C$ . The game of the depositors of bank  $B$  -however- is different from the one in G&P. In here, there is the manager of bank  $C$ , which has to take into consideration both the fundamentals in bank  $B$  and the fundamentals of bank  $C$  (plus the public signal). However, for a given level of fundamentals of bank  $C$ , the threshold depositor of bank  $B$  knows that if  $w_B > m - \alpha_{AB}$ , the manager of bank  $C$  will withdraw completely her bank's claim ( $w_{BC} = \alpha_{BC}$ ). Furthermore, if  $w_B < m - \alpha_{AB}$ , the threshold depositor of bank  $B$  knows that the number of withdrawals by the manager of bank

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<sup>27</sup>They depend on the fundamentals via the strategies.

$C$  (taking into consideration the action of her bank's depositors) will be increasing on  $w_B$ . This implies that there is not other threshold equilibrium. Hence, the threshold equilibrium is unique. Moreover, the game is almost identical to the one analyzed by G&P. In consequence, thanks to the fact that  $w_{BC}$  is increasing in  $w_B$ , we could use a very similar proof to the one by G&P and show that there cannot exist other strategies in an equilibrium.

Four step: We need to provide a characterization of the other variables of the unique equilibrium of the game. In concrete, the following variables:  $(\bar{\theta}_B, \underline{\theta}_B, \theta_B^F(\theta_C, s(w_B)), w_{BC}(\theta_B, \theta_C, s(w_B)), \theta_C^F(s(w_B), \theta_B))$ .

The manager of bank  $C$  has an strategy  $w_{BC}$  very simple: Since she has perfect information, she would withdraw from bank  $B$  only if either there is a liquidity problem in her own bank or, alternatively, if the fundamentals of bank  $B$  are low:<sup>28</sup>

On one hand, if the fundamentals of bank  $B$  are low,  $\theta_B \in [L, \underline{\theta}_B)$ , the manager of bank  $C$  knows that bank  $B$  is going to fail independently of her action. Thus she obtains a higher expected payoff withdrawing early. Hence, when  $\theta_B \in [L, \underline{\theta}_B)$ , her strategy is:

$$w_{BC} = \alpha_{BC} \tag{17}$$

On the other hand, if the fundamentals of bank  $B$  are high,  $\theta_B \in [\bar{\theta}_B, U]$ , the manager of bank  $C$  knows that bank  $B$  will not fail regardless of her action. Thus she obtains a higher expected payoff not withdrawing early. Therefore, she will withdraw only if there is liquidity problems on her bank. Hence, when  $\theta_B \in [\bar{\theta}_B, U]$  her strategy is:<sup>29</sup>

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<sup>28</sup>If the error  $e_C$  tends to zero, the strategy of the manager of bank  $C$  converges to:

$$w_{BC} = \begin{cases} \alpha_{BC} & \text{if } b_M < b_M^*(\theta_C) \\ 0 & \text{if } b_M > b_M^*(\theta_C) \end{cases}$$

<sup>29</sup>Notice that we can write these conditions with either bank  $i$ 's fundamentals, i.e.  $\theta_i$ , or with the number of bank's depositors, i.e.  $w_i$ , given that there is a one to one relation between  $\theta_i$  and  $w_i$ .

$$w_{BC} = \min(\max(w_C(\theta_C, s) - (m - \alpha_{BC}), 0), \alpha_{BC}) \quad (18)$$

Finally, when the fundamentals of bank  $B$  are intermediate,  $\theta_B \in [\underline{\theta}_B, \bar{\theta}_B)$ , the manager of bank  $C$  has an strategy which is almost identical to the previous case, except for one aspect. If she withdraws part of her bank's deposit on bank  $B$  to obtain liquidity for her own bank, she may cause the failure of bank  $B$ . This happens if the liquidity needed in her bank  $w_C(\theta_C, s) - (m - \alpha_{BC})$  is higher than the available cash in bank  $C$  :  $(m - \alpha_{AB}) - w_B$ . Hence, the optimal decision in this case would be  $w_{BC} = \alpha_{BC}$  -given that bank  $B$  will fail after she withdraws. In consequence, when  $\theta_B \in [\underline{\theta}_B, \bar{\theta}_B)$  her strategy is:

$$w_{BC} = \begin{cases} \alpha_{BC} & \text{if } w_C(\theta_C, s) - (m - \alpha_{BC}) > (m - \alpha_{AB}) - w_B \\ \max(0, w_C(\theta_C, s) - (m - \alpha_{BC})) & \text{otherwise} \end{cases} \quad (19)$$

The level of fundamentals of bank  $B$  such that this bank fails independently of the action taken by the manager of bank  $C$  is:

$$\underline{\theta}_B = b^* + e_B - \frac{2}{1 - \alpha_{BC}} e_B (m - \alpha_{AB}) \quad (20)$$

The level of fundamentals of bank  $B$  such that this bank does not fail independently of the action taken by the manager of bank  $C$  is:

$$\bar{\theta}_B = b^* + e_B - \frac{2}{1 - \alpha_{BC}} e_B (m - \alpha_{AB} - \alpha_{BC}) \quad (21)$$

The level  $\theta_B^F(\theta_C, s(w_B))$  of fundamentals of bank  $C$  such that this bank fails is given implicitly

by the following equation:

$$w_B(\theta_B^F) + w_{BC}(\theta_B^F, \theta_C, s(w_B)) = m - \alpha_{AB} \quad (22)$$

and it depends on the strategy by the manager of bank  $C$  and, in consequence, on both the fundamentals of bank  $C$  and on the public signal  $s(w_B)$ .

The level  $\theta_C^F(\theta_B, s(w_B))$  of fundamentals of bank  $C$  such that this bank fails is given implicitly by the following equation:

$$w_C(\theta_C^F(\theta_B, s(w_B))) = m - \alpha_{BC} + \alpha_{BC}q_M(1) \quad (23)$$

where  $q_M(1)$  is given by equation 5 and depends both on the fundamentals of bank  $B$  and on the public signal  $s(w_B)$ .

Proof of Proposition 2.

We need to differentiate the threshold  $-b^*$  for a small depositor of bank  $B$  with respect to bank  $B$ 's deposit in bank  $A$   $-\alpha_{AB}$ . Let  $P$  be the expected probability of the failure of bank  $C$  given that the fundamentals of bank  $B$  are such that  $\theta_B \in [\underline{\theta}_B, \bar{\theta}_B]$  - i.e.  $P = \left( \frac{c^*(L)+c^*(H)}{2} - L \right) / (U - L)$ , where  $b^*$ ,  $c^*(L)$ ,  $c^*(H)$  are defined in the equations 8 to 11.

Differentiating the small depositors' threshold, we obtain the following expression:

$$\frac{\partial b^*}{\partial \alpha_{AB}} = \frac{1 + \frac{\alpha_{BC}P}{m - \alpha_{AB}}}{m - \alpha_{AB} + \alpha_{BC}} + \frac{\frac{\partial P}{\partial \alpha_{AB}} \ln\left(1 + \frac{\alpha_{BC}}{m - \alpha_{AB}}\right) + \left(\frac{\partial P}{\partial \alpha_{AB}} + \frac{P}{m - \alpha_{AB}}\right) \frac{\alpha_{BC}}{m - \alpha_{AB}} \gamma b^*}{\gamma \left(1 - \frac{\alpha_{BC}P}{m - \alpha_{AB}}\right)} \quad (24)$$

Since all the terms in the expression above are positive, the proof is completed.

Proof of Proposition 3.

Differentiating bank  $B$ 's small depositors' threshold  $-b^*$  with respect to bank  $C$ 's deposit in bank  $B$ ,  $\alpha_{BC}$ , we obtain the following expression:

$$\frac{\partial b^*}{\partial \alpha_{BC}} = \frac{-\frac{1-P}{m-\alpha_{AB}+\alpha_{BC}} + \frac{\partial P}{\partial \alpha_{BC}} \ln(1 + \frac{\alpha_{BC}}{m-\alpha_{AB}}) + (\alpha_{BC} \frac{\partial P}{\partial \alpha_{BC}} + P) \frac{\gamma b^*}{m-\alpha_{AB}}}{\gamma(1 - \frac{\alpha_{BC}P}{m-\alpha_{AB}})} \quad (25)$$

Let  $\underline{U}$  be the level such that the above expression is zero. Solving the above equation for  $U$ , we obtain -implicitly- the value of  $\underline{U}$ :

$$(\alpha_{BC} \frac{\partial P}{\partial \alpha_{BC}} + P) \frac{\gamma b^*}{m - \alpha_{AB}} + \frac{\partial P}{\partial \alpha_{BC}} \ln(1 + \frac{\alpha_{BC}}{m - \alpha_{AB}}) = \frac{1 - P}{m - \alpha_{AB} + \alpha_{BC}} \quad (26)$$

It is easy to note that  $\underline{U}$  exists, is unique and finite. To see the existence, we notice that if  $U$  tends to infinite, the left part of the equation would tend to zero, whereas the right side of the equation would tend to  $(m - \alpha_{AB} + \alpha_{BC})^{-1}$ , which is strictly positive. Alternatively, if  $U$  -and  $\theta^U$ - tend to  $c^*(L)$  the left side is higher than the right side. To see this, notice that if  $\alpha_{BC}$  is equal to zero, the left side is  $(P \frac{\gamma b^*}{m-\alpha_{AB}})$  while the right side is  $(\frac{1-P}{m-\alpha_{AB}})$ . Thus the left side is lower only if  $P(\gamma b^* + 1) > 1$ , which implies that  $(c^*(L) + c^*(H) - L)(\gamma b^* + 1) > 0.5(U - L)$ . And this implies the following condition:  $1 < 2[(\frac{c^*(L)-L}{U-L}) + (\frac{c^*(L)-L}{U-L})](\gamma b^* + 1)$ . A sufficient condition it is that  $U$  tends to  $c^*(L)$ . Furthermore, if  $\alpha_{BC}$  increases the difference will increase even more, thus, the left side of the equation (29) is higher than the right side. As a consequence, there is a level of fundamentals  $\underline{U}$  that satisfies the equation (29). Finally, the expression (29) is decreasing on the upper bound  $U$  of fundamentals.<sup>30</sup> To see this, we just need to see that the threshold  $b^*$  is decreasing in  $U$  -see remark 2. In consequence,  $\underline{U}$  is unique. And, this concludes the proof.

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<sup>30</sup>Maintaining constant the rest of the parameters- in particular, the lower bound of fundamentals  $L$ .

Proof of Proposition 4.

We divide the proof in two steps. In the first one, we prove that the level  $\bar{U}$  exists, is unique and finite. In the second step, we show that  $\bar{U}$  is strictly greater than  $\underline{U}$ .

First step: we first need to obtain the cross partial derivative, which is the following:

$$\begin{aligned}
\frac{\partial^2 b^*}{\partial \alpha_{AB} \partial \alpha_{BC}} &= \left[ \left( \frac{\partial P}{\partial \alpha_{AB}} + \frac{P}{m - \alpha_{AB}} + \alpha_{BC} \left( \frac{\partial^2 P}{\partial \alpha_{AB} \partial \alpha_{BC}} + \frac{\frac{\partial P}{\partial \alpha_{BC}}}{m - \alpha_{AB}} \right) \right) \frac{\gamma}{m - \alpha_{AB}} b^* \right. \\
&\quad - \frac{1 - P}{(m - \alpha_{AB} + \alpha_{BC})^2} + \frac{\partial^2 P}{\partial \alpha_{AB} \partial \alpha_{BC}} \ln \left( 1 + \frac{\alpha_{BC}}{m - \alpha_{AB}} \right) \\
&\quad + \left( \alpha_{BC} \frac{\partial P}{\partial \alpha_{BC}} + P \right) \frac{\gamma}{m - \alpha_{AB}} \frac{\partial b^*}{\partial \alpha_{AB}} + \frac{\frac{\partial P}{\partial \alpha_{AB}} + \frac{\alpha_{BC}}{m - \alpha_{AB}} \frac{\partial P}{\partial \alpha_{BC}}}{m - \alpha_{AB} + \alpha_{BC}} + \\
&\quad \left. + \left( \frac{\partial P}{\partial \alpha_{AB}} + \frac{P}{m - \alpha_{AB}} \right) \frac{\gamma \alpha_{BC}}{m - \alpha_{AB}} \frac{\partial b^*}{\partial \alpha_{BC}} \right] / \gamma \left( 1 - \frac{\alpha_{BC} P}{m - \alpha_{AB}} \right)
\end{aligned} \tag{27}$$

We can notice that this expression is decreasing in  $U$ , and by the argument used on the previous proof, we know that there exist a unique and finite level of fundamentals  $\bar{U}$  such that the previous expression is zero when  $U = \bar{U}$ .

Second step: the last point is to check that  $\bar{U} > \underline{U}$ . In order to check it, we use the expression resulting of  $\frac{\partial b^*}{\partial \alpha_{BC}} = 0$  at  $U = \underline{U}$ . And, we substitute it on previous equation and show that  $\frac{\partial^2 b^*}{\partial \alpha_{AB} \partial \alpha_{BC}}$  is positive at  $U = \underline{U}$ . Substituting (29) in (30), we can write (30) as  $(A + B) / \gamma \left( 1 - \frac{\alpha_{BC} P}{m - \alpha_{AB}} \right)$ , where:

$$A = \left[ \frac{\gamma \left( 1 - \frac{\alpha_{BC} P}{m - \alpha_{AB}} \right)}{m - \alpha_{AB} + \alpha_{BC}} + \left( \frac{\partial P}{\partial \alpha_{AB}} + \frac{P}{m - \alpha_{AB}} \right) \frac{\gamma \alpha_{BC}}{m - \alpha_{AB}} \right] \frac{\partial b^*}{\partial \alpha_{BC}} \tag{28}$$

$$\begin{aligned}
B = & \left[ \frac{\partial P}{\partial \alpha_{AB}} + \alpha_{BC} \left( \frac{\partial^2 P}{\partial \alpha_{AB} \partial \alpha_{BC}} + \frac{\frac{\partial P}{\partial \alpha_{BC}} + P}{(m - \alpha_{AB} + \alpha_{BC})(m - \alpha_{AB})} \right) \right] \frac{\gamma}{m - \alpha_{AB}} b^* \\
& + (\alpha_{BC} \frac{\partial P}{\partial \alpha_{BC}} + P) \frac{\gamma}{m - \alpha_{AB}} \frac{\partial b^*}{\partial \alpha_{AB}} + \frac{\frac{\partial P}{\partial \alpha_{AB}} + \frac{\alpha_{BC}}{m - \alpha_{AB}} \frac{\partial P}{\partial \alpha_{BC}}}{m - \alpha_{AB} + \alpha_{BC}} + \\
& + \left( \frac{\partial^2 P}{\partial \alpha_{AB} \partial \alpha_{BC}} - \frac{\frac{\partial P}{\partial \alpha_{BC}}}{m - \alpha_{AB} + \alpha_{BC}} \right) \ln \left( 1 + \frac{\alpha_{BC}}{m - \alpha_{AB}} \right)
\end{aligned} \tag{29}$$

At  $U = \underline{U}$ ,  $\frac{\partial b^*}{\partial \alpha_{BC}}$  is equal to zero, thus,  $A = 0$ . Notice that all the elements in  $B$  are positive except for one. Hence,  $B$  is positive if  $\frac{\alpha_{BC}}{m - \alpha_{AB}} \frac{\partial P}{\partial \alpha_{BC}} - \frac{\frac{\partial P}{\partial \alpha_{BC}}}{m - \alpha_{AB} + \alpha_{BC}} \ln \left( 1 + \frac{\alpha_{BC}}{m - \alpha_{AB}} \right)$ . Since

$\alpha_{BC}/(m - \alpha_{AB}) \geq \ln \left( 1 + \frac{\alpha_{BC}}{m - \alpha_{AB}} \right)$ , for  $\alpha_{BC} \geq 0$ ,  $B$  is strictly positive. This concludes the proof.

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