Volatility Threshold Dynamic Conditional Correlations: Implications for International Portfolio Diversification

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First version

Abstract

In this paper we extend the Dynamic Conditional Correlation multivariate GARCH specification to investigate how the correlations between the single pairs of the assets in our system behave in volatile markets. Our approach allows to identify asset pairs the correlations of which are less sensitive to extreme volatility values associated with bear markets. In this context the paper also discusses the possible portfolio diversification benefits.

JEL classification: C50, C51, F37, G11, G15

Keywords: Dynamic conditional correlations, Volatility threshold, International diversification benefits.

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1 Introduction

The seminal studies by Grubel (1968), Levy and Sarnat (1970), Lessard (1973) and Solnik (1974) laid the ground for considerable academic research advocating the benefits of international diversification on the basis of the low correlation between national stock markets. However, the recent empirical evidence indicates that over the last decade the correlations of the major security markets have increased significantly, by this strongly reducing the benefits of diversification among these markets. On the other hand a range of new markets has emerged, expanding the opportunity set of investors, and thereby offering new sources for the diversification of portfolio risk.

A strand of literature on international portfolio diversification has investigated the question whether the benefits of diversification are present when they are needed most, i.e. in times of extreme market volatility, often associated with bear markets.[1] Evidence from capital market history suggests that poor market performance was associated with an increase in international correlations. Goetzmann, Li and Rouwenhorst (2002) cite the chairman of the Alliance Trust Company, reflecting on the Crash of 1929:

"Trust companies...have reckoned that by a wide spreading of their investment risk, a stable revenue position could be maintained, as it was not to be expected that all the world would go wrong at the same time. But the unexpected has happened, and every part of the civilized world is in trouble..."

A range of studies has interpreted covariance asymmetry within the framework of a particular generalized autoregressive conditional heteroscedasticity model, where the asymmetry is defined to be an increase in conditional covariance or correlation resulting from past negative shocks to return processes. Specifically, Cho and Engle (2000), Bekaert and Wu (2000),

[1] See e.g. Lin, Engle and Ito (1994), Erb, Harvey and Viskanta (1994), Longin and Solnik (1995), Karolyi and Stulz (1996), Solnik, Bourcrelle, and Le Fur (1996), De Santis and Gerard (1997), Ramchmand and Susmel (1998), Ang and Bekaert (1999), Das and Uppal (1999), Longin and Solnik (2001), and Ang and Chen (2002).

Kroner and Ng (1998), and Conrad et al. (1991) examine the covariance asymmetry of domestic stock portfolios, and Capiello, Engle and Sheppard (2004) investigate the correlation asymmetry of international equity and bond returns, using multivariate asymmetric GARCH models.

In this paper we extend the multivariate GARCH Dynamic Conditional Correlation (DCC) model of Engle (2002) and its generalization by Capiello, Engle and Sheppard (2004) to investigate the relationship between the correlation and the volatilities of the underlying assets. The hypothesis the extended model tests for is whether high volatility values (exceeding some prespecified threshold) of the assets, implied by the model, are associated with an increase in their correlation values. The resulting specification could be interpreted as asymmetric in the level of volatility. The identification of asset pairs the correlations of which do not increase in volatile markets associated with bear markets, under *ceteris paribus* conditions, could be useful for leveraging the benefits of portfolio diversification.

To demonstrate practical relevance of our model we employ a sample of national stock indices from markets heterogeneous in the level of their development and integration into international securities markets. While there is a considerable body of research investigating the Asian and Latin American emerging stock markets, the transition markets of Central Europe have seen much less attention so far. Our sample includes the stock indices from the three largest transition stock markets of Central Europe: Hungary, Poland and the Czech Republic. We conduct two separate analysis, international and regional European. In the international part we consider the mixed sample of the transition indices with the U.S. and European composite indices, while for the regional part we analyse a sample of the transition indices with the major European market indices.

We start from investigation of the volatility and correlation dynamics of the considered markets over the last decade. Some interesting patterns in their reaction to global events appear. The response of the transition markets to these events, as expected, is not always similar to that of the developed markets. The empirical results of the application of the extended DCC model to our sample delivers strong evidence that over the considered time period the turbulent markets were associated with increases in the correlations of the developed markets. For the cross-correlations of the transition markets with the rest of the markets we do not observe a similar pattern. This potentially makes them attractive targets for the portfolio diversification of international investors.

The paper is organized as follows. Section 2 starts from the description of the data employed in this study. Section 3 presents the base multivariate GARCH models and analyses the empirical results. Section 4 proposes an extension of the base models considered in section 3 and presents the corresponding empirical results. Section 5 summarises our findings.

2 Data description

The empirical part of this paper concentrates on the investigation of the time-varying correlation dynamics of international stock markets over the last decade. To make our analysis richer, our sample includes markets heterogeneous in the level of their development, both mature and emerging stock markets. This allows us to test hypothesis for different market environments. The emerging markets chosen for this study are the three largest transition stock markets of Central Europe: Hungary, Poland and the Czech Republic. In the regional European part of the analysis, the following six stock market indices are considered: German DAX30, French CAC40, British FTSE100, Hungarian BUX30, Polish WIG20 and Czech PX50. For the international part, we consider S&P500 and the European blue chip stock composite STOXX50 from the developed world, as well as the same transition markets indices employed for the regional analysis. The choice of the specific transition indices among other indices available for these markets is primarily based on the fact that those are the national blue chip indices available for the longest time period. These indices are published by the national stock exchanges of the considered countries. All indices are observed at weekly frequency and are US dollar-denominated. The employed sample covers the period from the last week of April 1994 to the first week of June 2004, constituting the total of 528 return observations. The use of weekly data is preferred because daily data could suffer from frictions in the markets especially in the case of transition markets. Additionally, for the international analysis, the use of daily data would induce noise due to time-zone differences in the countries analysed. All data is obtained from Datastream. Table 1 presents some descriptive statistics for the indices.

Insert Table 1 here

All series show the typical non-normality of financial time series. Excess kurtosis and negative skewness are especially pronounced in the case of the Hungarian BUX. All series with exception of STOXX50 display negative skewness. The Ljung-Box statistics suggest autocorrelation in the return levels of S&P500 and BUX only. The squared returns, on the other hand, are highly autocorrelated, which can be taken as evidence of ARCH effects in the considered series.

Insert Table 2 here

Table 2 shows unconditional correlations of the returns. The correlations of DAX30, CAC40 and FTSE100 with STOXX50 are, non-surprisingly, very high, partially explained by the fact that a part of the component stocks of the German, French and British indices are also the components of STOXX50. These are followed by the correlation between DAX30 with CAC40 (0.80), FTSE100 with CAC40 (0.73), and STOXX50 with S&P500 (0.70). The correlations of the considered individual developed European market indices with the U.S. market are somewhat lower (around 0.64). The regional European impact on the transition stock markets is clearly stronger than that from the U.S., as the correlations of these markets with S&P500 is significantly lower than with the European indices. Finally, the correlations between the transition markets range from 0.41 for the Polish and Czech indices to 0.53 for the Hungarian and Polish indices. It is also interesting to note that the correlations between the transition indices are higher than the correlation of these indices with the rest of the markets considered.

3 Dynamic Conditional Correlations

3.1 The models

Multivariate modeling of the second moments of asset returns plays an important role in many different areas of financial management, like the assessment of Value-at-Risk and other risk measures estimates, portfolio allocation and asset pricing. It is now widely accepted that with the changes in market conditions the volatilities and correlations of assets change over time as well. The last two decades have produced a range of studies modeling the time-varying behavior of correlations and covariances between financial assets.

The problems associated with the estimation of the multivariate GARCH models, related to the tradeoff between their generality and the number of parameters to be estimated as well as the considerable restrictions on the parameters necessary for positive definiteness of the covariance matrix, are well known.[2] Bollerslev (1990) introduced a new class of multivariate GARCH models, the so-called Conditional Correlation models. The specification of the conditional covariance matrix for this class of models is implemented in a hierarchical way. First, volatility for each individual series is estimated using a univariate GARCH specification, then, based on the resulting standardized residuals, one models the conditional correlation matrix. The Constant Conditional Correlation (CCC) model by Bollerslev (1990) ensures the feasibility of the model estimation also in large dimensions and positive definiteness of the covariance matrix simply requiring each univariate conditional variance to be positive and the constant matrix of conditional correlations to be positive definite. Due to its computational simplicity, the CCC model is widely used in empirical applications. A range of studies (like e.g. Bera and Kim (1996), Tsui and Yu (1999) and Tse (2000)) find, however, that the assumption of constant conditional correlation can be too restrictive.

[2] For the recent review of the existing multivariate GARCH models see e.g. Bauwens et al. (2003).

Engle (2002) proposed a generalisation of the CCC model of Bollerslev, the Dynamic Conditional Correlation (DCC) model.[3] The new specification preserves the ease of the estimation of the Bollerslev's model, but allows time variation of the conditional correlation matrix. In Engle's model one fits to each asset return an appropriate univariate GARCH model (the models can differ from asset to asset) and then standardizes the returns by the estimated GARCH conditional standard deviations. The standardized return vector is then used to model the correlation dynamics. The model estimation is performed through $n+1$ numerical optimizations, each involving only a few parameters, regardless of the size of n (number of assets in the system).

Consider an n-variate conditionally normal return process r_t with mean zero[4] and covariance matrix H_t:

$$r_t \mid F_{t-1} \sim N(0, H_t) \tag{1}$$

$$H_t = D_t R_t D_t \tag{2}$$

$$D_t = diag\left\{\sqrt{h_{ii,t}}\right\} \tag{3}$$

$$\varepsilon_t = D_t^{-1} r_t \tag{4}$$

where $h_{ii,t}$ s could e.g. be thought of as univariate GARCH models, ε_{it} s are standardized residuals with mean zero and variance one, and $R_t = \left\{\rho_{ij,t}\right\}$ is the time-varying conditional correlation matrix of returns. R_t corresponds to the conditional covariance matrix of the standardized residuals[5], i.e. $\rho_{ij,t} = E_{t-1}(\varepsilon_{it}, \varepsilon_{jt})$.

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[3] For an alternative generalization of the CCC model see Tse and Tsui (2002). Additionally, Pelletier (2004) proposed a regime switching model for dynamic correlations, which can be seen as a midpoint between the CCC model of Bollerslev (1990) and the DCC model of Engle (2002), where the correlations change every period.

[4] r_{it} s can be either mean zero or the residuals of a filtered time series. In the empirical part of this paper the data were not filtered other than simple demeaning.

[5] $\rho_{ij,t} = \dfrac{E_{t-1}(r_{it} r_{jt})}{\sqrt{E_{t-1}(r_{it}^2) E_{t-1}\left(r_{jt}^2\right)}} = \dfrac{\sqrt{h_{ii,t}}\sqrt{h_{jj,t}} E_{t-1}(\varepsilon_{it}\varepsilon_{jt})}{\sqrt{h_{ii,t}}\sqrt{h_{jj,t}}\sqrt{E_{t-1}(\varepsilon_{it}^2) E_{t-1}\left(\varepsilon_{jt}^2\right)}} = E_{t-1}(\varepsilon_{it}\varepsilon_{jt})$.

The DCC model of Engle (2002) specifies the dynamics of the correlation matrix as follows:

$$R_t = (diag(Q_t))^{-\frac{1}{2}} Q_t (diag(Q_t))^{-\frac{1}{2}} \tag{5}$$

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha(\varepsilon_t \varepsilon_t^{'}) + \beta Q_{t-1} \tag{6}$$

where $\bar{Q}$ is the unconditional correlation matrix of ε_t. As a result, the typical element of R_t is of the form $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}$. This normalization ensures that all correlation estimates fall in the [-1;1] interval. The model is estimated subject to the unconditional correlation targeting constraint by which the long run correlation matrix is the sample correlation matrix. The specification is mean reverting as long as $\alpha + \beta < 1$. Note, in case α and β are zero, one obtains the CCC model by Bollerslev (1990).[6]

A drawback of this specification is that all the elements of the conditional correlation matrix are restricted to have the same dynamics. Cappiello, Engle and Sheppard (2004) propose the following generalization of the model[7], which allows the individual series specific news impact parameters:

$$Q_t = (\bar{Q} - A\bar{Q}A' - B\bar{Q}B') + A(\varepsilon_{t-1}\varepsilon_{t-1}^{'})A' + BQ_{t-1}B' \tag{7}$$

where A and B are $n \times n$ diagonal matrices. As a result, the dynamics of the individual elements of the covariance matrix Q_t is specified as follows:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} + \alpha_i\alpha_j\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_i\beta_j q_{ij,t-1} \tag{8}$$

Sufficient condition for the covariance matrix to be positive definite is that $(\bar{Q} - A\bar{Q}A' - B\bar{Q}B')$ in (7) is positive definite. Although this generalized model undoubtly adds flexibility to Engle's specification, the number of parameters to be estimated increases

[6] Testing for dynamic versus constant correlation for the data that have time-varying volatilities has proven in the literature to be a difficult problem. Some examples of such kind of tests are in Bera (1996) and Tse(1998).

[7] The full Asymmetric Generalized DCC in Cappiello, Engle and Sheppard (2004) includes additionally a variable matrix accounting for the asymmetric impact of the past negative shocks on the correlation processes.

considerably. Alternative attempts to generalize the standard scalar DCC by Engle (2002) can be found e.g. in Frances and Hafner (2003) and Billio, Caporin and Gobbo (2003).

A nice feature of conditional correlation multivariate GARCH models is that they allow for two-stage estimation. Specifically, the likelihood function of the DCC models outlined above can be written as a sum of a volatility part and a correlation part. Let the parameters of the volatility part be denoted ϕ, and the additional parameters of the correlation part ψ. The estimates of volatility parameters can be found by replacing R_t in (2) by an identity matrix of size n. The resulting first stage log-likelihood function gives the sum of the log-likelihoods of individual volatility equations of n series in the system:

$$
\begin{aligned}
L_v(\phi) &= -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + \log(|I_n|) + 2\log(|D_t|) + r_t^{'} D_t^{-1} I_n D_t^{-1} r_t \right] \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + 2\log(|D_t|) + r_t^{'} D_t^{-2} r_t \right] \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + \sum_{i=1}^{n}\left(\log(h_{it}) + \frac{r_{it}^2}{h_{it}} \right) \right] \\
&= -\frac{1}{2}\sum_{i=1}^{n}\left[T\log(2\pi) + \sum_{t=1}^{T}\left(\log(h_{it}) + \frac{r_{it}^2}{h_{it}} \right) \right] \qquad (9)
\end{aligned}
$$

The second stage log-likelihood is:

$$
\begin{aligned}
L_c(\psi \mid \hat{\phi}) &= -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + r_t^{'} D_t^{-1} R_t^{-1} D_t^{-1} r_t \right] \\
&= -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + \varepsilon_t^{'} R_t^{-1} \varepsilon_t \right] \qquad (10)
\end{aligned}
$$

Given the estimates of the volatility parameters, $\hat{\phi}$, the relevant part of L_c, that will influence the selection of the correlation parameters, ψ, is:

$$
L_c^{*}(\psi \mid \hat{\phi}) = -\frac{1}{2}\sum_{t=1}^{T}\left[\log(|R_t|) + \varepsilon_t^{'} R_t^{-1} \varepsilon_t \right] \qquad (11)
$$

Engle and Sheppard (2001), based on the results in Newey and McFadden (1994), demonstrate that the two-step estimation approach provides consistent, although not efficient, estimates of the parameters of the model. We use this two-step estimation procedure in the empirical part of the paper.

3.2 Univariate volatility model choice and estimates

We start from fitting the univariate volatility models for each of the eight series considered in this study.[8] The alternative specifications employed are the following GARCH models:

Model name	**Specification**
GARCH	$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 h_{t-1}$
NARCH	$h_t = \alpha_0 + \alpha_1 \lvert r_{t-1} \rvert^{\alpha_2} + \beta_1 h_{t-1}$
EGARCH	$\ln(h_t) = \alpha_0 + \alpha_1 \left(\left\lvert \frac{r_{t-1}}{\sqrt{h_{t-1}}} \right\rvert - \sqrt{2/\pi} \right) + \alpha_2 \left(\frac{r_{t-1}}{\sqrt{h_{t-1}}} \right) + \beta_1 \ln(h_{t-1})$
GJR	$h_t = \alpha_0 + \alpha_1 r_{t-1}^2 + \alpha_2 S_{t-1}^- r_{t-1}^2 + \beta_1 h_{t-1}$ with $S_{t-1}^- = \begin{cases} 1 & \text{if} \quad r_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases}$
AGARCH	$h_t = \alpha_0 + \alpha_1 \left(r_{t-1} + \alpha_2 \right)^2 + \beta_1 h_{t-1}$
NGARCH	$h_t = \alpha_0 + \alpha_1 \left(r_{t-1} + \alpha_2 \sqrt{h_{t-1}} \right)^2 + \beta_1 h_{t-1}$
VGARCH	$h_t = \alpha_0 + \alpha_1 \left(r_{t-1} / \sqrt{h_{t-1}} + \alpha_2 \right)^2 + \beta_1 h_{t-1}$

While GARCH and NARCH are symmetric[9], the rest of the models allow for an asymmetric impact of the positive and negative news on the volatility process. The asymmetry is achieved either by allowing the slopes of negative and positive sides of the news impact curve, with a

[8] A similar approach is also employed in Cappiello, Engle and Sheppard (2004).

[9] Note, as compared to GARCH, NARCH would imply a reduced response of volatility to news if $a_2 < 2$.

minimum at $r_{t-1} = 0$, to differ (EGARCH and GJR), or the minimum of the news impact curve to be located at $r_{t-1} \neq 0$ (AGARCH, NGARCH and VGARCH).[10]

The results of the estimation of the univariate volatility models outlined above indicate evidence of an asymmetric impact of news on volatility for all series in our study. For each of the series we select a univariate volatility specification based on the Schwarz Information Criterion. The selected models and the corresponding parameter estimates are presented in Table 3. As we see, while three of employed indices prefer a model (NGARCH), implying recentering of the news impact curve at a positive r_{t-1}, for the rest we select models (EGARCH and GJR), capturing the asymmetry by allowing a steeper slope of the negative side of the news impact curve compared to its positive side.

Insert Table 3 here

Table 4 presents the cross-correlations of the fitted volatility series, while figure 1 shows the development of the volatilities over the considered sample period. As expected, the correlations of the transition market volatilities with the developed market series are much lower than those between the developed markets. From figure 1 it is clear that the volatilities of the major markets comove, and react to significant international events in a similar manner.[11] It is interesting to note that while the reaction of the transition markets to the Russian default in August-September 1998 was very strong, other major international events like September 11 or the new economy bubble burst did not have such a strong impact on these markets.

Insert Table 4 and Figure 1 here

[10] For further details on the employed models and their news impact curves see Engle and Ng (1993).

[11] For the formal analysis of the cross country volatility comovements, particularly focusing on the periods of high volatility, see e.g. Edwards and Susmel (2001).

3.3 Conditional correlation estimates

On the basis of the individual standardized residual series, obtained as a result of the estimation of the univariate volatility models, the dynamics of the conditional correlation matrix is parameterised as a scalar DCC in (6) and as a generalized DCC (GDCC) in (7) above. We conduct two types of analysis, international and regional European. In the international part we model the correlation dynamics of three transition market indices with the S&P500 and STOXX50, while in the regional part with three European indices, DAX30, CAC40 and FTSE100. The estimation results are presented in table 5 for the international and in table 6 for the regional analysis.

Insert Tables 5 and 6 here

The last rows of tables 5 and 6 provide likelihood ratio tests between the standard DCC and its generalized version.[12] The test statistic for the international analysis cannot reject the null hypothesis of the scalar DCC. In the regional case the scalar DCC is rejected in favour of the GDCC. The plots of the resulting conditional correlation series are presented in Figure 2.[13] The first important feature we observe is that correlations of all developed market indices have increased since mid-nineties. It is especially pronounced for the correlation between French and German indices, which is obviously influenced by the fixing of the exchange rates in 1999. We observe a sharp drop of the correlations of FTSE100 with DAX30 and CAC40 in the first part of 2000. This drop is present for some other market pairs as well. Another regularility is that Asian-Russian crisis around 1998 has lead to a swing in the international correlations for almost all country pairs, including transition markets. Some other interesting thing to note is that the last part of our sample is characterized by a steady increase in the

[12] The t-statistics of the parameter functions for the GDCC model are calculated using the delta method.

[13] Given that for the regional European results the rejection rate of the scalar DCC against GDCC is not very high, and the shapes of the charts of conditional correlations for these two specifications differ only marginally, Figure 2 presents regional correlations implied by the scalar DCC (similar to the international case).

correlations of transition markets with the rest of the indices. This may be due to an anticipation of the accession of these countries to the European Union in May 2004.

Insert Figure 2 here

3.4 Volatility versus correlation: Some empirical regularities

Figure 3 presents the scatter plots of the conditional correlation series against the volatility of the underlying markets. The interesting regularity we note is that for the correlations between developed markets extreme volatility values are associated with high correlation values as well, while for the transition markets this pattern is not as pronounced (both for the correlations between transition markets and their correlations with developed markets). The high correlation values associated with the extreme volatility in the underlying markets would make international diversification benefits disappear, at least partially, in times when they are required most. For the investors who diversify internationally, it would be beneficial to identify markets the correlations of which are less sensitive to extreme values of the volatilities in these markets. This, *ceteris paribus*, could provide some protection in turbulent market times.

Insert Figure 3 here

4 Volatility Threshold Dynamic Conditional Correlations

4.1 The models

The varying relationship between high volatility and correlation values of the different asset pairs in the portfolio, if present but ignored, could have serious consequences for portfolio hedging effectiveness. The empirical regularities identified by observing the scatter plots suggest an extension of the DCC model considered in the previous section. Given the historical data for the assets under interest up to time period $t-1$, within the two-step estimation framework, the investor produces a volatility estimate for time period t for each of the series in the system. Because the parameters of the volatility models are determined exclusively in the first-step, the fitted volatility series could be considered as given for the second correlation step of estimation. The extension of the DCC model we propose tests the hypothesis whether high volatility values (exceeding a specific threshold) of the underlying assets are associated with an increase in their correlation values. An investor rearranging his portfolio would be grateful to identify assets for which this association does not hold, as, other things being equal, one could consider those assets as potentially attractive targets for portfolio diversification.

Let V_t be a dummy variables matrix with elements defined as:

$$v_{ij,t} = \begin{cases} 1 \quad if\ h_{i,t} > fh_i(k)\ or\ h_{j,t} > fh_j(k) \\ 0 \qquad\qquad otherwise \end{cases} \tag{12}$$

where $fh_i(k)$ is the k-th fractile of the volatility series h_i.

One could now extend the DCC and GDCC models in (6) and (7) in the following way:

$$Q_t = (1-\alpha-\beta)\bar{Q} - \gamma\bar{V} + \alpha(\varepsilon_{t-1}\varepsilon_{t-1}^{'}) + \beta Q_{t-1} + \gamma V_t \tag{13}$$

$$Q_t = (\bar{Q} - A\bar{Q}A' - B\bar{Q}B' - \Gamma\bar{V}\Gamma') + A(\varepsilon_{t-1}\varepsilon_{t-1}^{'})A' + BQ_{t-1}B' + \Gamma V_t \Gamma' \tag{14}$$

where $\bar{V} = E[V_t]$, and A, B and Γ are $n \times n$ diagonal matrices.

For the GDCC specification the dynamics of the individual elements of the covariance matrix Q_t would then be specified as:

$$q_{ij,t} = (1-\alpha_i\alpha_j - \beta_i\beta_j)\bar{q}_{ij} - \gamma_i\gamma_j\bar{v}_{ij} + \alpha_i\alpha_j\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta_i\beta_j q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t} \tag{15}$$

Sufficient condition for the covariance matrix, Q_t, to be positive definite is that $(\bar{Q} - A\bar{Q}A' - B\bar{Q}B' - \Gamma\bar{V}\Gamma')$ in (14) is positive definite.

In case the aim of the empirical analysis is to identify heterogeneity in the response of the markets to the volatility values exceeding some thresholds, it is more suitable to consider a version of the model where the diagonal elements of matrix Γ are allowed to vary. On the other hand, the restrictions on the GARCH dynamics of the conditional correlations (the scalar version) in some cases could be well justifiable, leading to a more parsimonious specification and/or making the model estimation feasible also in large dimensions.[14] The version of the model in (14), which restricts the GARCH dynamics but allows different volatility impacts on the correlations of different asset pairs, could be specified by restricting the diagonal elements of the parameter matrix A and B, for each of the matrices, to be identical. The expression in (15) then becomes:

$$q_{ij,t} = (1-\alpha^2-\beta^2)\overline{q}_{ij} - \gamma_i\gamma_j\overline{v}_{ij} + \alpha^2\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta^2 q_{ij,t-1} + \gamma_i\gamma_j v_{ij,t} \tag{16}$$

In the rest we refer to the specification in (15) as the *Volatility Threshold GDCC* (VT-GDCC), and to the specification in (16) as the *Volatility Threshold DCC* (VT-DCC).

As emphasized in the introduction to this paper, a range of studies have identified that the correlations between assets increase for downside moves, especially for extreme downside moves, rather than for upside moves. Below we propose a modification of the model in (14) which would consider the case of "extreme" volatility associated with bear markets.[15] In the framework of the DCC model this could e.g. be defined as the case when the fitted volatility for the period t exceeds the pre-specified threshold and at the same time the observed return at time $t-1$ is negative (which is equivalent to the corresponding standardized residual being negative). To integrate this feature in our specification, one could redefine the dummy variables matrix, V_t, as follows:

$$v_{ij,t} = \begin{cases} 1 & \left(if\ h_{i,t} > fh_i(k)\ and\ \varepsilon_{i,t-1} < 0\right)\ or\ \left(h_{j,t} > fh_j(k)\ and\ \varepsilon_{j,t-1} < 0\right) \\ 0 & otherwise \end{cases} \tag{17}$$

[14] As is shown in Engle and Sheppard (2001), the scalar DCC model leads to sub-optimal portfolio selection in case of many assets (like 20 or 30) as it assumes the same type GARCH dynamics for all the asset-specific conditional correlations. This assumption becomes, however, increasingly more likely to be satisfied in case of small number of assets.

[15] In this context, see Capiello, Engle and Sheppard (2004), who provide an extension of the GDCC model in (7), the Asymmetric Generalized DCC, to account for the asymmetric impact of the sign of the past innovations on the current correlation values.

In the rest we refer to the specifications in (15) and (16), with the elements of the matrix V_t defined as in (17), the *Volatility Threshold Asymmetric GDCC* (VT-AGDCC) and *Volatility Threshold Asymmetric DCC* (VT-ADCC), respectively.

All the models described in this section could be modified in such a way that the correlation values are conditioned on the observed past return series only (but not on the fitted volatility values). The idea similar to the specification with the matrix V_t defined in (12) would be to condition the correlation values on the past squared returns exceeding a pre-specified threshold. To test the hypothesis similar to the specification with the matrix V_t defined in (17) one would condition the correlation values on the large (exceeding some threshold) past negative returns.

4.2. Estimation results

Tables 7 - 10 present the results of the estimation of the Volatility Threshold DCC models specified above. The models are estimated for different predefined volatility threshold levels: 50 percent, 75 percent, 90 percent and 95 percent fractiles. As we were interested in the analysis of the heterogeneous impact of volatilities on correlations of different asset pairs in our sample, we did not consider the scalar model in (13), and estimated two versions of the model in (14), specified in (15) and (16), respectively. The last rows in the tables report the likelihood ratio statistics, testing the restrictions of the specification in (16) against the unrestricted model in (15). For most of the cases the restricted specification is preferred to the unrestricted one (perhaps with exception of VT-ADCC for the regional analysis in table 10, where, however, the rejection rate is not very high). Therefore, for the sake of parsimony, we report parameter estimates for the specification in (16) only.

Insert Tables 7 and 8 here

The results in tables 7 and 8, which are based on the model with the elements of the matrix V_t defined in (12), deliver strong evidence that the correlations of the developed market indices are significantly affected by the volatility in one of the markets or both exceeding a predefined threshold (the exception is the 50 percent threshold for the pair S&P500 and STOXX50). Different to that, the high volatility values seem not to affect the correlations of the transition markets with their developed counterparts on the one side, and the transition markets among each other on the other side. The exception is the correlation of the Polish WIG20 with S&P500 for the volatility threshold of 95 percent for the international analysis, and the correlation of the Czech PX50 with the European developed market indices for the volatility threshold of 75 percent for the regional analysis.

Insert Tables 9 and 10 here

Tables 9 and 10 are based on the specification with the matrix V_t defined as in (17). The general tendency of the estimates for the specifications in tables 7 and 8, on the one side, and tables 9 and 10, on the other, to be similar most probably indicates that the high volatility values are predominantly associated with negative returns.

The results in this section reflect the general picture illustrated by the scatter plots in figure 3, and indicate that transition markets, under *ceteris paribus* conditions, could potentially provide some protection for international investors in turbulent market periods.

5 Conclusions

In this paper we investigate the volatility and correlation dynamics of national stock indices from markets heterogeneous in the level of their development. We extend the multivariate GARCH Dynamic Conditional Correlation of Engle (2002) to analyse the relationship between the correlations on the one side and the volatility of the underlying assets exceeding a predefined threshold on the other side. The empirical results indicate that the correlations of the developed markets are significantly affected by high volatility levels (associated with bear markets), while high volatility seems not to have a direct impact on the correlations of the transition blue chip indices with the rest of the markets. This feature could be potentially relevant for the international portfolio diversification considerations.

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Table 1. Summary Statistics

	SP500	STOXX50	DAX30	CAC40	FT100	BUX30	WIG20	PX50
Mean	0.0017	0.0014	0.0012	0.0012	0.0011	0.0023	-0.0005	1.93e-05
Max	0.0749	0.1302	0.1261	0.1046	0.1091	0.1470	0.1621	0.1285
Min	-0.1233	-0.0894	-0.1203	-0.1024	-0.0879	-0.3324	-0.2180	-0.1308
Standard Dev.	0.0236	0.0249	0.0322	0.0282	0.0224	0.0432	0.0516	0.0332
Skewness	-0.5171	0.1001	-0.2182	-0.0579	-0.1780	-1.0083	-0.2862	-0.2156
Kurtosis	5.6542	5.4404	4.7200	3.9712	4.5893	11.1765	4.4393	4.1465
JB	178.5171	131.9019	69.2796	21.0469	58.3539	1560.2840	52.7840	33.0084
LB(6)	16.400	8.247	8.686	7.8417	4.368	19.983	9.374	13.338
LBS(6)	34.770	52.729	104.980	58.758	43.186	22.213	64.759	62.396

Notes: JB is Jarque-Bera test statistic, distributed χ_2^2. LB(6) and LBS(6) are Ljung-Box test statistics with 6 lags for return levels and return squares, respectively, distributed χ_6^2. The upper 1 and 5 percentile points of the χ_2^2 distribution are 9.21 and 5.99, respectively. The upper 1 and 5 percentile points of the χ_6^2 distribution are 16.81 and 12.59, respectively.

Table 2. Unconditional cross-correlations

	STOXX50	DAX30	CAC40	FTSE100	BUX30	WIG20	PX50
SP500	0.7041	0.6374	0.6462	0.6359	0.3125	0.3133	0.1421
STOXX50		0.8767	0.8895	0.8747	0.4308	0.3676	0.3075
DAX30			0.8009	0.6986	0.4360	0.3954	0.3143
CAC40				0.7343	0.3779	0.3654	0.2994
FTSE100					0.3701	0.3043	0.2385
BUX						0.5253	0.4908
WIG20							0.4091

Table 3. Univariate volatility model choice

Index	Model	Parameter estimates			
		α_0	α_1	β_1	α_2
SP500	EGARCH	-0.3731 (-2.4975)	0.1590 (3.9988)	0.9516 (49.1006)	-0.9692 (-2.3413)
STOXX50	EGARCH	-0.2765 (-2.3893)	0.2248 (4.2825)	0.9622 (60.7172)	-0.4204 (-3.9370)
DAX30	EGARCH	-0.4921 (-2.6399)	0.2841 (4.3492)	0.9301 (35.0480)	-0.4804 (-3.3761)
CAC40	GJR	8.86e-04 (1.7334)	0.0037 (0.1087)	0.8053 (10.4896)	0.1500 (2.4389)
FTSE100	NGARCH	6.51e-04 (3.3940)	0.0756 (5.4103)	0.7033 (96.8463)	-1.1036 (-5.2405)
BUX30	NGARCH	0.0004 (2.4748)	0.2312 (2.0873)	0.4765 (2.6514)	-0.6002 (-2.4854)
WIG20	NGARCH	9.26e-04 (2.5461)	0.0902 (2.9765)	0.8545 (24.0095)	-0.4540 (-2.1263)
PX50	EGARCH	-0.6899 (-1.8099)	0.2873 (3.2353)	0.9004 (16.4156)	-0.2513 (-1.8665)

Notes: This table gives the quasi-maximum likelihood estimates of the selected univariate volatility models. t-statistics are given in parentheses.

Table 4. GARCH volatility correlations

	STOXX50	DAX30	CAC40	FTSE100	BUX30	WIG20	PX50
SP500	0.7084	0.7535	0.7413	0.8088	0.2365	0.1079	0.2053
STOXX50		0.8426	0.7450	0.6799	0.1576	0.1521	0.2295
DAX30			0.8606	0.7302	0.2068	0.1667	0.2374
CAC40				0.7672	0.1452	0.1101	0.1837
FTSE100					0.2753	0.1890	0.3046
BUX30						0.4459	0.5391
WIG20							0.6788

Figure 1. GARCH volatility

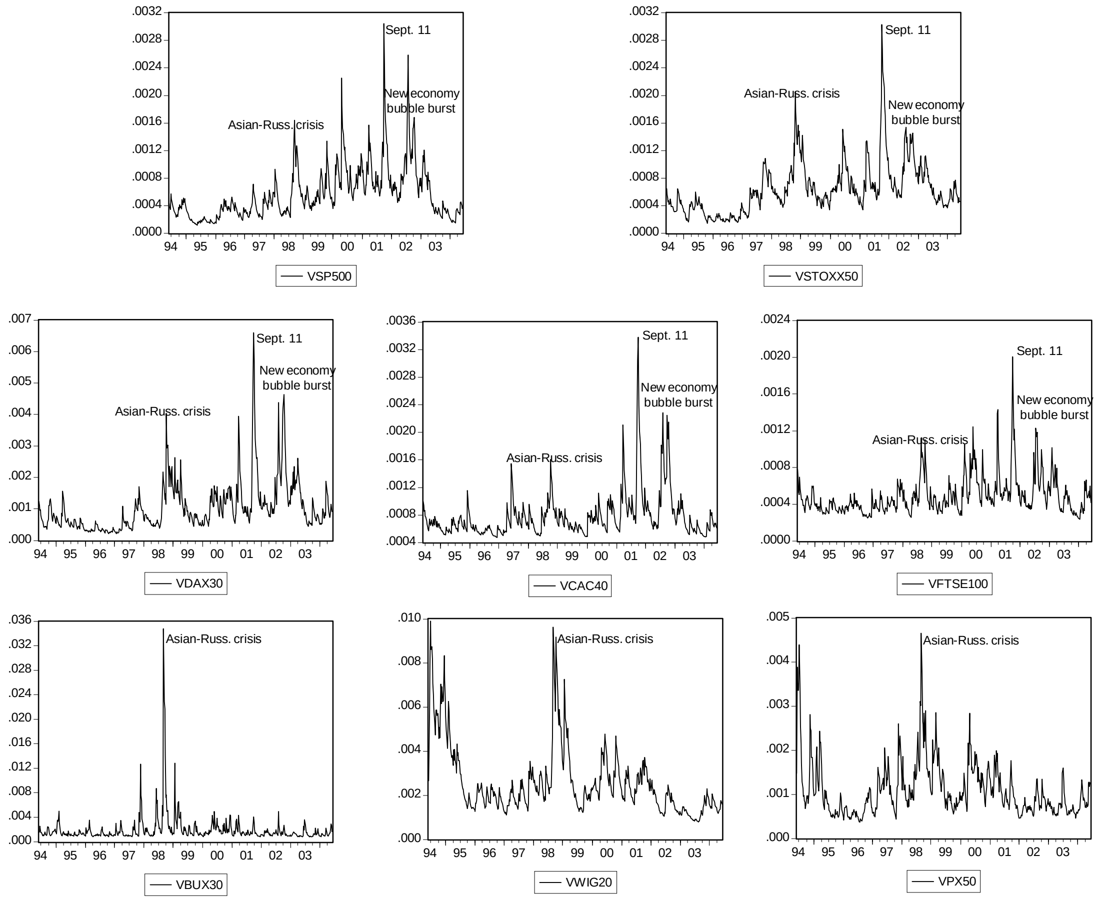


Table 5. DCC conditional correlation estimates: international analysis

DCC			
α	0.0269 (3.1183)	β	0.8702 (15.9396)
GDCC			
$\alpha_{SP}\alpha_{STOXX}$	0.0581 (3.2819)	$\beta_{SP}\beta_{STOXX}$	0.8772 (18.3055)
$\alpha_{SP}\alpha_{BUX}$	0.0335 (1.9021)	$\beta_{SP}\beta_{BUX}$	0.5614 (1.6476)
$\alpha_{SP}\alpha_{WIG}$	0.0141 (1.9569)	$\beta_{SP}\beta_{WIG}$	0.9630 (44.9918)
$\alpha_{SP}\alpha_{PX}$	0.0254 (1.8205)	$\beta_{SP}\beta_{PX}$	0.8531 (6.4270)
$\alpha_{STOXX}\alpha_{BUX}$	0.0524 (1.9408)	$\beta_{STOXX}\beta_{BUX}$	0.5249 (1.6471)
$\alpha_{STOXX}\alpha_{WIG}$	0.0220 (1.9256)	$\beta_{STOXX}\beta_{WIG}$	0.5249 (19.4970)
$\alpha_{STOXX}\alpha_{PX}$	0.0398 (1.8675)	$\beta_{STOXX}\beta_{PX}$	0.5249 (6.2290)
$\alpha_{BUX}\alpha_{WIG}$	0.0127 (1.3994)	$\beta_{BUX}\beta_{WIG}$	0.5761 (1.6480)
$\alpha_{BUX}\alpha_{PX}$	0.0229 (1.3770)	$\beta_{BUX}\beta_{PX}$	0.5761 (1.6038)
$\alpha_{WIG}\alpha_{PX}$	0.0096 (1.4379)	$\beta_{WIG}\beta_{PX}$	0.8755 (6.1848)
L_{DCC}	-3317.4947		
L_{GDCC}	-3313.3326		
LR	8.3242		

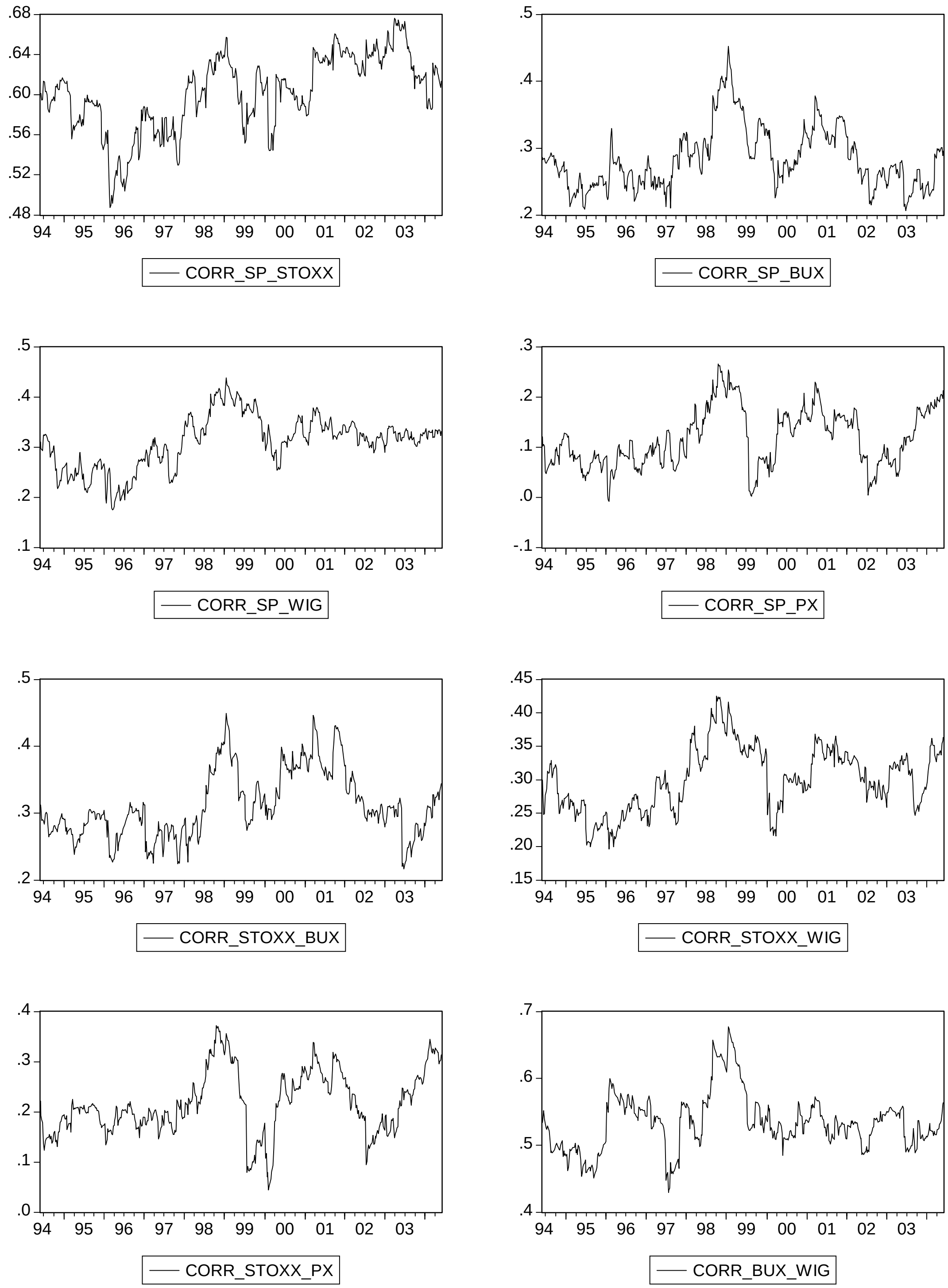
Notes: This table gives the quasi-maximum likelihood estimates of DCC model in (6) and GDCC model in (7) for the international part of analysis. t-statistics are given in parentheses. t-statistics of the parameter functions for GDCC model are calculated using the delta method. LR is likelihood ratio test statistic of GDCC against DCC specification, distributed χ_8^2. The upper 1 and 5 percentile points of the χ_8^2 distribution are 20.09 and 15.51, respectively.

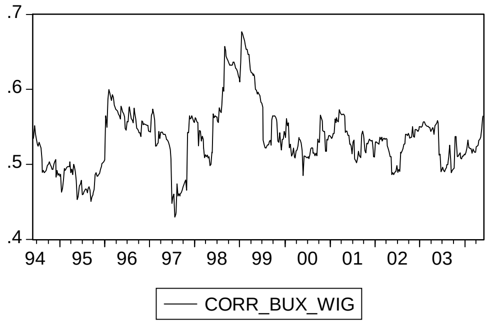
Table 6. DCC conditional correlation estimates: regional analysis

DCC			
α	0.0129 (7.7099)	β	0.9723 (216.9331)
GDCC			
$\alpha_{DAX}\alpha_{CAC}$	0.0302 (3.6231)	$\beta_{DAX}\beta_{CAC}$	0.9599 (21.1992)
$\alpha_{DAX}\alpha_{FTSE}$	0.0169 (2.8775)	$\beta_{DAX}\beta_{FTSE}$	0.9724 (21.8428)
$\alpha_{DAX}\alpha_{BUX}$	0.0079 (1.6855)	$\beta_{DAX}\beta_{BUX}$	0.9734 (22.7236)
$\alpha_{DAX}\alpha_{WIG}$	0.0112 (1.4986)	$\beta_{DAX}\beta_{WIG}$	0.9487 (16.7792)
$\alpha_{DAX}\alpha_{PX}$	0.0168 (1.8745)	$\beta_{DAX}\beta_{PX}$	0.9317 (14.8427)
$\alpha_{CAC}\alpha_{FTSE}$	0.0244 (3.3854)	$\beta_{CAC}\beta_{FTSE}$	0.9675 (23.2672)
$\alpha_{CAC}\alpha_{BUX}$	0.0114 (1.7486)	$\beta_{CAC}\beta_{BUX}$	0.9685 (24.5071)
$\alpha_{CAC}\alpha_{WIG}$	0.0162 (1.6210)	$\beta_{CAC}\beta_{WIG}$	0.9440 (18.4610)
$\alpha_{CAC}\alpha_{PX}$	0.0243 (1.9606)	$\beta_{CAC}\beta_{PX}$	0.9270 (15.3251)
$\alpha_{FTSE}\alpha_{BUX}$	0.0064 (1.8903)	$\beta_{FTSE}\beta_{BUX}$	0.9811 (28.7756)
$\alpha_{FTSE}\alpha_{WIG}$	0.0090 (1.4443)	$\beta_{FTSE}\beta_{WIG}$	0.9563 (17.1280)
$\alpha_{FTSE}\alpha_{PX}$	0.0136 (1.9331)	$\beta_{FTSE}\beta_{PX}$	0.9391 (15.8112)
$\alpha_{BUX}\alpha_{WIG}$	0.0042 (1.1135)	$\beta_{BUX}\beta_{WIG}$	0.9572 (15.8165)
$\alpha_{BUX}\alpha_{PX}$	0.0064 (1.2306)	$\beta_{BUX}\beta_{PX}$	0.9400 (13.1607)
$\alpha_{WIG}\alpha_{PX}$	0.0090 (1.1599)	$\beta_{WIG}\beta_{PX}$	0.9162 (11.5623)
L_{DCC}	-3759.3417		
L_{GDCC}	-3744.3135		
LR	30.0563		

Notes: This table gives the quasi-maximum likelihood estimates of DCC model in (6) and GDCC model in (7) for the regional part of analysis. t-statistics are given in parentheses. t-statistics of the parameter functions for GDCC model are calculated using the delta method. LR is likelihood ratio test statistic of GDCC against DCC specification, distributed χ^2_{10}. The upper 1 and 5 percentile points of the χ^2_{10} distribution are 23.21 and 18.31, respectively.

Figure 2. Conditional correlations

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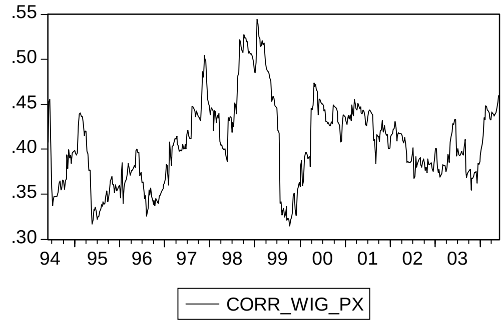
94 95 96 97 98 99 00 01 02 03
CORR_BUX_WIG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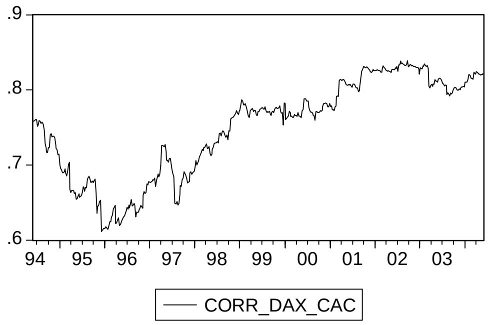
CORR_WIG_PX


.9
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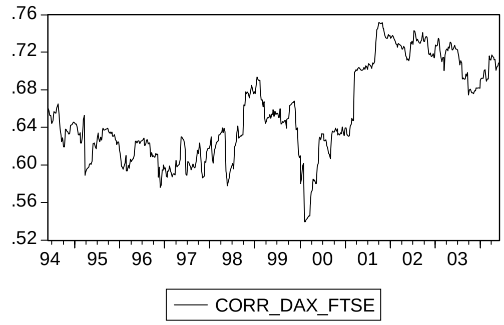
CORR_DAX_CAC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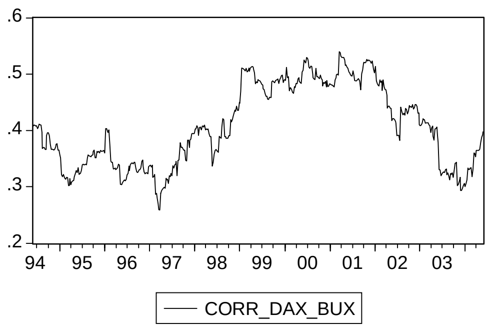
94 95 96 97 98 99 00 01 02 03
CORR_DAX_FTSE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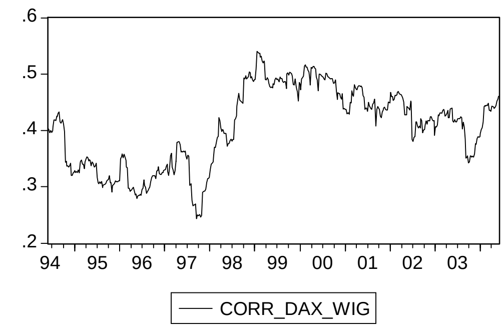
CORR_DAX_BUX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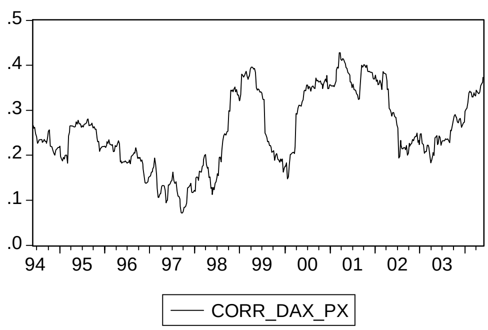
CORR_DAX_WIG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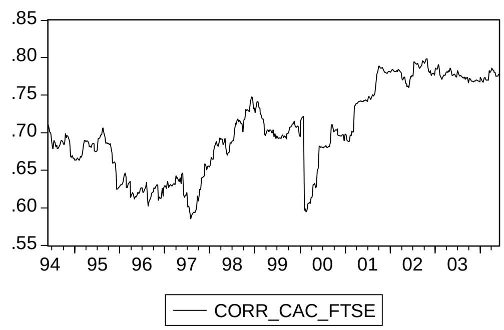
CORR_DAX_PX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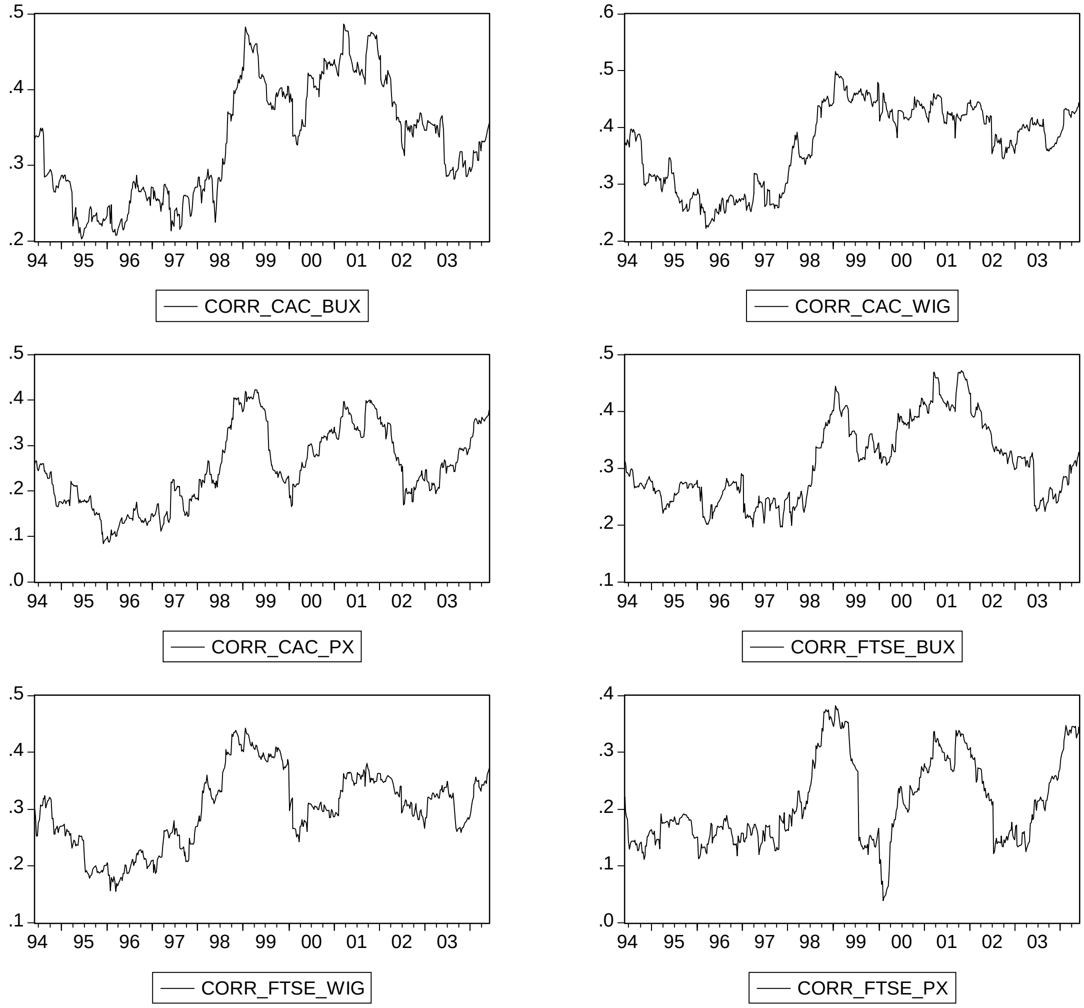
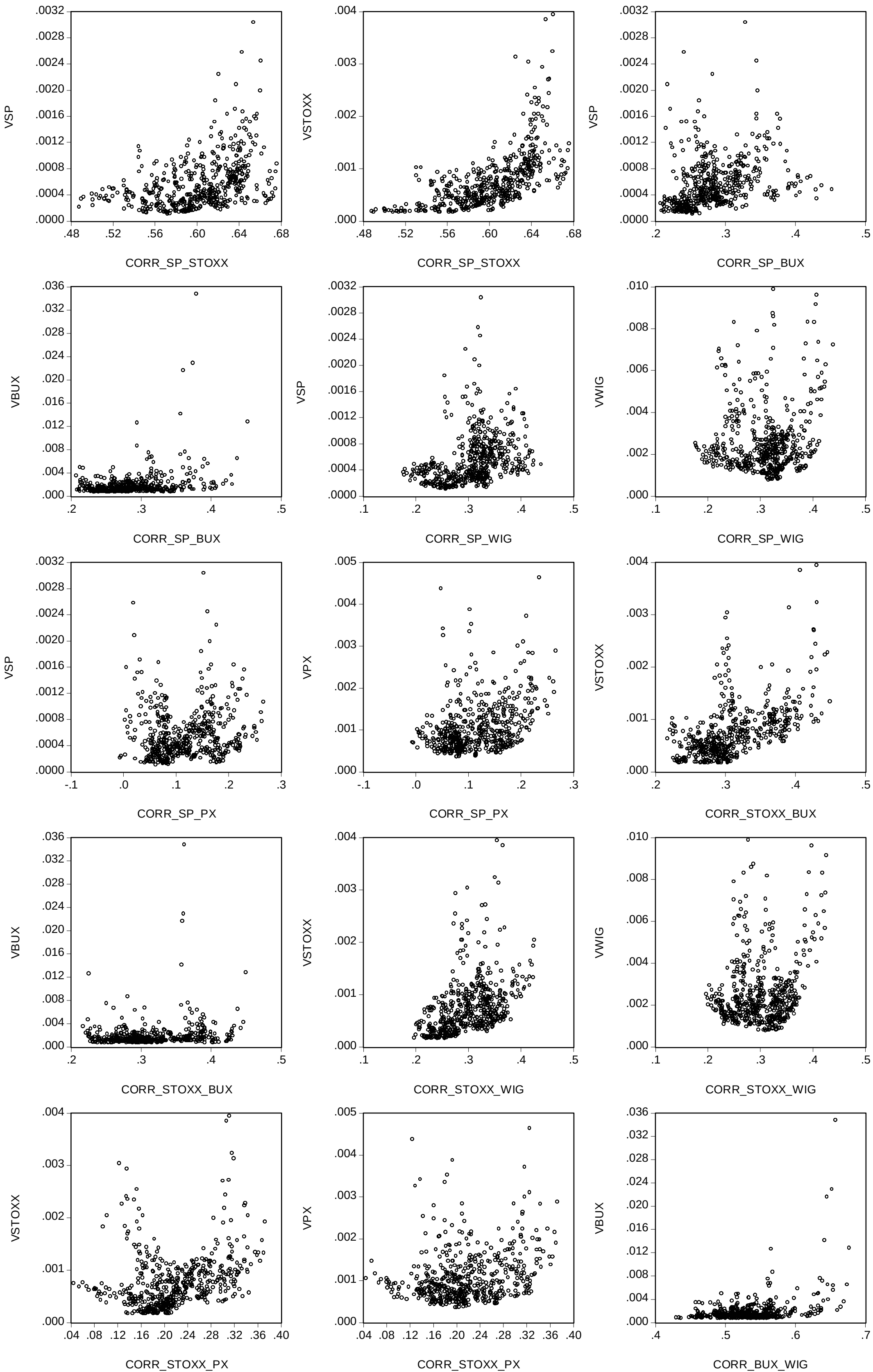
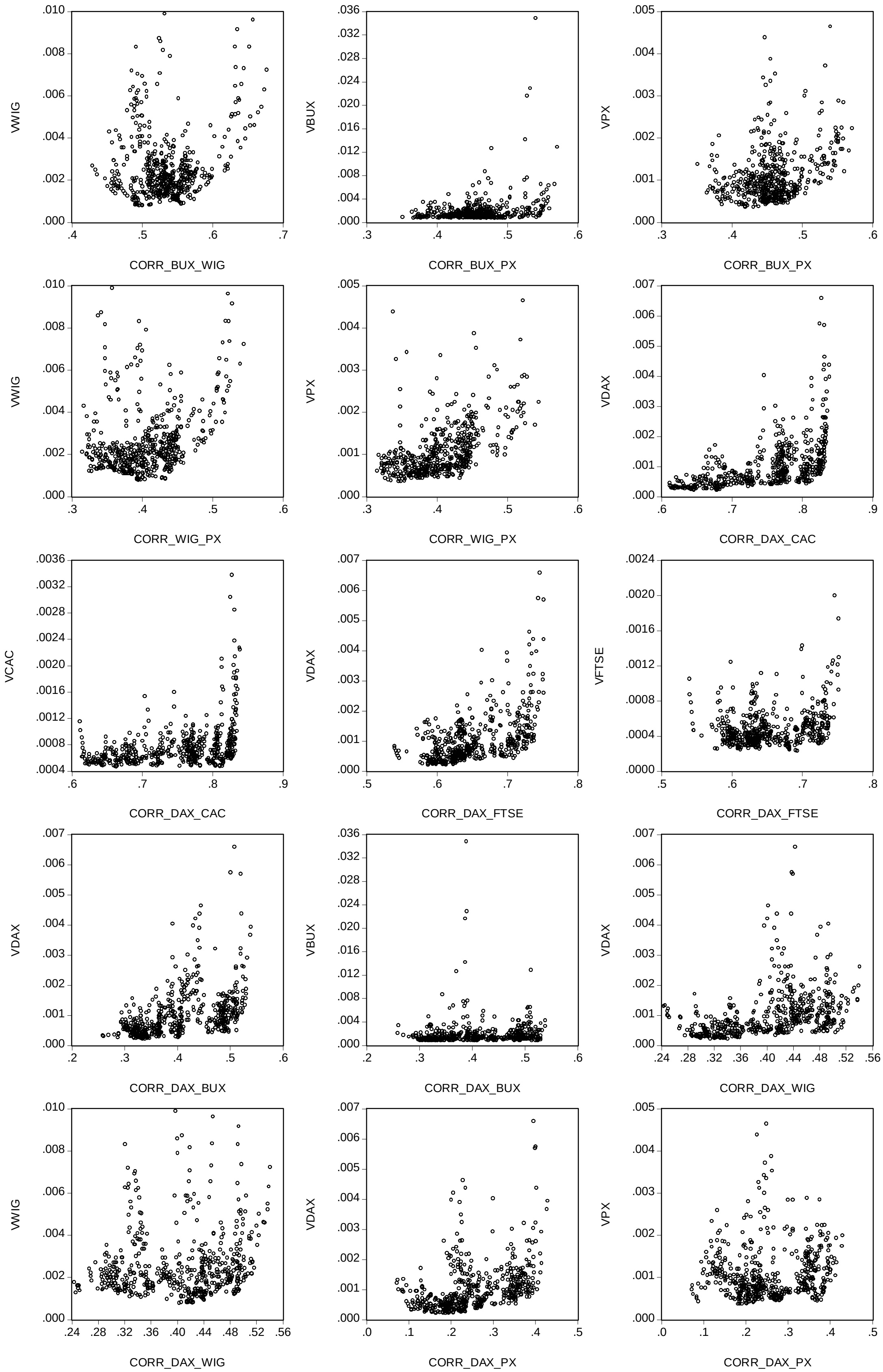
CORR_CAC_FTSE



CORR_CAC_BUX
CORR_CAC_WIG
CORR_CAC_PX
CORR_FTSE_BUX
CORR_FTSE_WIG
CORR_FTSE_PX
94 95 96 97 98 99 00 01 02 03

Figure 3. Volatility versus correlations





VWIG
CORR_BUX_WIG
VBUX
CORR_BUX_PX
VPX
CORR_BUX_PX
VWIG
CORR_WIG_PX
VPX
CORR_WIG_PX
VDAX
CORR_DAX_CAC
VCAC
CORR_DAX_CAC
VDAX
CORR_DAX_FTSE
VFTSE
CORR_DAX_FTSE
VDAX
CORR_DAX_BUX
VBUX
CORR_DAX_BUX
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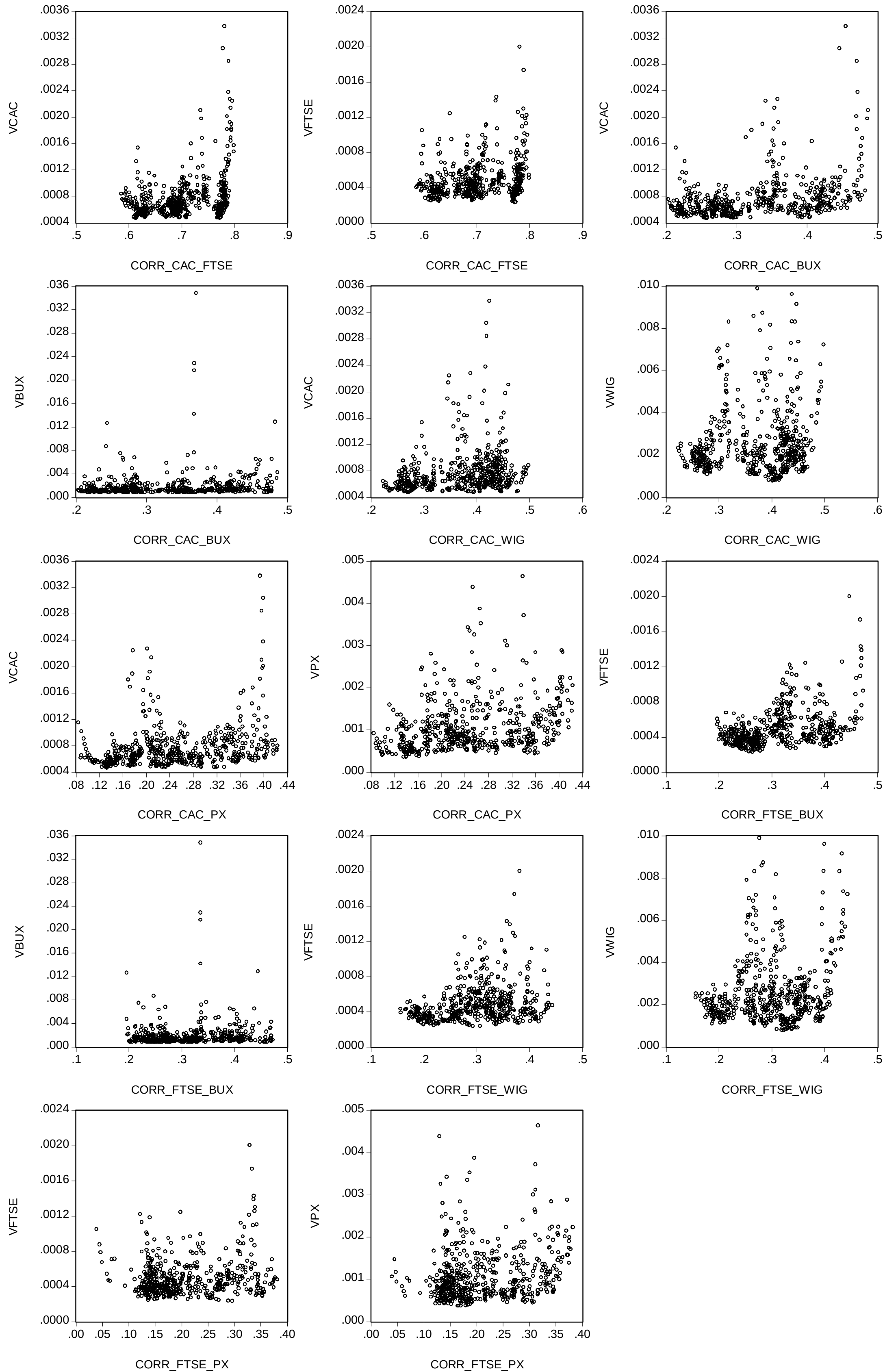
CORR_DAX_PX
VPX
CORR_DAX_PX


VCAC
CORR_CAC_FTSE
VFTSE
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VBUX
CORR_CAC_BUX
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CORR_CAC_WIG
VWIG
CORR_CAC_WIG
VCAC
CORR_CAC_PX
VPX
CORR_CAC_PX
VFTSE
CORR_FTSE_BUX
VBUX
CORR_FTSE_BUX
VFTSE
CORR_FTSE_WIG
VWIG
CORR_FTSE_WIG
VFTSE
CORR_FTSE_PX
VPX
CORR_FTSE_PX

Table 7. Volatility Threshold DCC: international analysis

	50%	75%	90%	95%
α^2	0.0250 (5.9788)	0.0214 (5.8023)	0.0172 (5.6149)	0.0145 (5.4494)
β^2	0.8554 (27.1674)	0.8945 (41.9447)	0.9270 (69.4995)	0.9338 (88.5245)
$\gamma_{SP}\gamma_{STOXX}$	0.0022 (0.8597)	0.0249 (1.9621)	0.0238 (2.4799)	0.0592 (3.6293)
$\gamma_{SP}\gamma_{BUX}$	0.0037 (1.1350)	-0.0054 (-0.5213)	-0.0087 (-0.7652)	-0.0083 (-0.5421)
$\gamma_{SP}\gamma_{WIG}$	0.0187 (1.4032)	0.0062 (0.6814)	0.0159 (1.4902)	0.0411 (2.0263)
$\gamma_{SP}\gamma_{PX}$	0.0036 (1.1075)	-0.0073 (-0.5924)	-0.0189 (-1.3258)	-0.0313 (-1.5231)
$\gamma_{STOXX}\gamma_{BUX}$	0.0027 (1.0123)	-0.0037 (-0.5031)	-0.0058 (-0.7753)	-0.0061 (-0.5499)
$\gamma_{STOXX}\gamma_{WIG}$	0.0138 (1.2213)	0.0042 (0.6711)	0.0106 (1.3134)	0.0299 (1.8090)
$\gamma_{STOXX}\gamma_{PX}$	0.0027 (1.0495)	-0.0050 (-0.6323)	-0.0126 (-1.4008)	-0.0228 (-1.6185)
$\gamma_{BUX}\gamma_{WIG}$	0.0231 (1.3677)	-0.0009 (-0.4989)	-0.0038 (-0.8369)	-0.0042 (-0.5873)
$\gamma_{BUX}\gamma_{PX}$	0.0045 (1.1102)	0.0011 (0.3598)	0.0046 (0.6579)	0.0032 (0.4942)
$\gamma_{WIG}\gamma_{PX}$	0.0226 (1.4453)	-0.0012 (-0.5155)	-0.0084 (-1.3152)	-0.0158 (-1.5154)
L_{VT-DCC}	-3314.163	-3314.053	-3314.244	-3310.933
$L_{VT-GDCC}$	-3297.788	-3305.210	-3309.883	-3309.066
LR	32.750	17.686	8.722	3.734

Notes: This table gives the quasi-maximum likelihood estimates of VT-DCC model in (14) with the restrictions on the GARCH dynamics of the conditional correlations, and the matrix V_t defined as in (12). t-statistics are given in parentheses. t-statistics are calculated using the delta method. LR is likelihood ratio test statistic of VT-GDCC against VT-DCC specification, distributed χ_8^2. The upper 1 and 5 percentile points of the χ_8^2 distribution are 20.09 and 15.51, respectively.

Table 8. Volatility Threshold DCC: regional analysis

	50%	75%	90%	95%
α^2	0.0058 (5.8697)	0.0043 (5.0309)	0.0053 (6.2938)	0.0090 (6.8836)
β^2	0.9836 (440.8916)	0.9867 (593.0920)	0.9855 (436.0248)	0.9731 (260.9315)
$\gamma_{DAX}\gamma_{CAC}$	0.0125 (3.4165)	0.0129 (3.8353)	0.0181 (3.9205)	0.0421 (2.7911)
$\gamma_{DAX}\gamma_{FTSE}$	0.0094 (3.3095)	0.0104 (3.9613)	0.0150 (3.7319)	0.0287 (2.8360)
$\gamma_{DAX}\gamma_{BUX}$	0.0035 (1.4627)	0.0044 (1.6508)	0.0061 (1.3424)	0.0072 (0.8045)
$\gamma_{DAX}\gamma_{WIG}$	0.0009 (0.4572)	-0.0003 (-0.1596v	-0.0011 (-0.3168)	0.0073 (0.7782)
$\gamma_{DAX}\gamma_{PX}$	0.0041 (1.6058)	0.0052 (2.1164)	0.0072 (1.4531)	0.0077 (0.7396)
$\gamma_{CAC}\gamma_{FTSE}$	0.0112 (2.8114)	0.0111 (3.3691)	0.0128 (3.1801)	0.0293 (2.3748)
$\gamma_{CAC}\gamma_{BUX}$	0.0042 (1.5312)	0.0047 1.5957)	0.0052 (1.3000)	0.0073 (0.7853)
$\gamma_{CAC}\gamma_{WIG}$	0.0011 (0.4779)	-0.0003 (-0.1587)	-0.0009 (-0.3106)	0.0074 (0.7721)
$\gamma_{CAC}\gamma_{PX}$	0.0049 (1.6639)	0.0055 (2.0895)	0.0061 (1.4263)	0.0078 (0.7183)
$\gamma_{FTSE}\gamma_{BUX}$	0.0032 (1.5514)	0.0037 (1.6979)	0.0043 (1.3386)	0.0050 (0.8155)
$\gamma_{FTSE}\gamma_{WIG}$	0.0008 (0.4649)	-0.0003 (-0.1595)	-0.0007 (-0.3128)	0.0050 (0.7713)
$\gamma_{FTSE}\gamma_{PX}$	0.0037 (1.5253)	0.0044 (1.9764)	0.0051 (1.4296)	0.0053 (0.7444)
$\gamma_{BUX}\gamma_{WIG}$	0.0003 (0.4154)	-0.0001 (-0.1633)	-0.0003 (-0.3233)	0.0013 (0.5198)
$\gamma_{BUX}\gamma_{PX}$	0.0014 (1.0223)	0.0019 (1.2052)	0.0021 (0.9425)	0.0013 (0.5361)
$\gamma_{WIG}\gamma_{PX}$	0.0004 (0.4312)	-0.0001 (-0.1608)	-0.0004 (-0.3167)	0.0014 (0.5048)
L_{VT-DCC}	-3751.576	-3750.113	-3751.727	-3753.884
$L_{VT-GDCC}$	-3738.453	-3739.610	-3742.214	-3742.657
LR	26.246	21.006	19.026	22.454

Notes: This table gives the quasi-maximum likelihood estimates of VT-DCC model in (14) with the restrictions on the GARCH dynamics of the conditional correlations, and the matrix V_t defined as in (12). t-statistics are given in parentheses. t-statistics are calculated using the delta method. LR is likelihood ratio test statistic of VT-GDCC against VT-DCC specification, distributed χ^2_{10}. The upper 1 and 5 percentile points of the χ^2_{10} distribution are 23.21 and 18.31, respectively.

Table 9. Volatility Threshold Asymmetric DCC: international analysis

	50%	75%	90%	95%
α^2	0.0241 (5.8548)	0.0267 (6.1420)	0.0103 (4.8527)	0.0078 (4.5522)
β^2	0.8594 (29.6925)	0.8446 (26.1097)	0.9569 (119.7003)	0.9658 (167.0667)
$\gamma_{SP}\gamma_{STOXX}$	0.0088 (1.3886)	0.0037 (0.9844)	0.0510 (2.6939)	0.0826 (3.1205)
$\gamma_{SP}\gamma_{BUX}$	0.0077 (1.5626)	0.0055 (1.3539)	-0.0077 (-0.6377)	-0.0050 (-0.3465)
$\gamma_{SP}\gamma_{WIG}$	0.0456 (2.0795)	0.0359 (1.6569)	0.0167 (1.5216)	0.0375 (1.7796)
$\gamma_{SP}\gamma_{PX}$	0.0082 (1.5206)	0.0060 (1.2907)	-0.0241 (-1.5173)	-0.0521 (-2.2039)
$\gamma_{STOXX}\gamma_{BUX}$	0.0066 (1.4384)	0.0044 (1.1497)	-0.0059 (-0.6271)	-0.0035 (-0.3451)
$\gamma_{STOXX}\gamma_{WIG}$	0.0392 (1.8807)	0.0289 (1.3374)	0.0129 (1.4787)	0.0265 (1.7875)
$\gamma_{STOXX}\gamma_{PX}$	0.0070 (1.4712)	0.0048 (1.1538)	-0.0186 (-1.5629)	-0.0369 (-2.3375)
$\gamma_{BUX}\gamma_{WIG}$	0.0346 (1.6168)	0.0434 (1.6443)	-0.0019 (-0.6576)	-0.0016 (-0.3588)
$\gamma_{BUX}\gamma_{PX}$	0.0062 (1.3200)	0.0072 (1.3036)	0.0028 (0.5731)	0.0022 (0.3302)
$\gamma_{WIG}\gamma_{PX}$	0.0366 (1.6854)	0.0473 (1.6164)	-0.0061 (-1.3294)	-0.0167 (-1.7372)
L_{VT-DCC}	-3311.5123	-3312.9189	-3312.3696	-3310.3283
$L_{VT-GDCC}$	-3304.8729	-3302.3611	-3308.7538	-3309.3240
LR	13.2788	21.1156	7.2316	2.0086

Notes: This table gives the quasi-maximum likelihood estimates of VT-ADCC model in (14) with the restrictions on the GARCH dynamics of the conditional correlations, and the matrix V_t defined as in (17). t-statistics are given in parentheses. t-statistics are calculated using the delta method. LR is likelihood ratio test statistic of VT-GDCC against VT-DCC specification, distributed χ_8^2. The upper 1 and 5 percentile points of the χ_8^2 distribution are 20.09 and 15.51, respectively.

Table 10. Volatility Threshold Asymmetric DCC: regional analysis

	50%	**75%**	**90%**	**95%**
α^2	0.0055 (5.7776)	0.0049 (5.5377)	0.0057 (6.5093)	0.0081 (7.5531)
β^2	0.9835 (429.3885)	0.9849 (535.1780)	0.9846 (425.7374)	0.9799 (315.1847)
$\gamma_{DAX}\gamma_{CAC}$	0.0167 (3.3683)	0.0165 (3.5847)	0.0182 (3.1927)	0.0222 (1.9090)
$\gamma_{DAX}\gamma_{FTSE}$	0.0130 (3.0191)	0.0156 (3.3344)	0.0197 (3.3256)	0.0237 (2.3488)
$\gamma_{DAX}\gamma_{BUX}$	0.0045 (1.3315)	0.0066 (1.7667)	0.0085 (1.4954)	0.0015 (0.3402)
$\gamma_{DAX}\gamma_{WIG}$	0.0013 (0.4379)	6.78e-04 (0.2271)	-0.0037 (-0.8916)	0.0042 (0.6645)
$\gamma_{DAX}\gamma_{PX}$	0.0051 (1.3800)	0.0051 (1.4802)	0.0048 (0.8612)	-0.0108 (-1.2343)
$\gamma_{CAC}\gamma_{FTSE}$	0.0175 (2.6473)	0.0203 (3.1061)	0.0243 (3.3160)	0.0438 (2.9305)
$\gamma_{CAC}\gamma_{BUX}$	0.0060 (1.3879)	0.0086 (1.7340)	0.0104 (1.4146)	0.0028 (0.3394)
$\gamma_{CAC}\gamma_{WIG}$	0.0017 (0.4484)	0.0001 (0.2274)	-0.0046 (-0.8791)	0.0077 (0.6694)
$\gamma_{CAC}\gamma_{PX}$	0.0069 (1.4334)	0.0067 (1.4924)	0.0059 (0.8604)	-0.0200 (-1.2677)
$\gamma_{FTSE}\gamma_{BUX}$	0.0047 (1.3537)	0.0081 (1.8237)	0.0113 (1.5288)	0.0030 (0.3434)
$\gamma_{FTSE}\gamma_{WIG}$	0.0013 (0.4371)	0.0008 (0.2257)	-0.0050 (-0.8921)	0.0082 (0.6693)
$\gamma_{FTSE}\gamma_{PX}$	0.0053 (1.3214)	0.0063 (1.4180)	0.0064 (0.8648)	-0.0213 (-1.2078)
$\gamma_{BUX}\gamma_{WIG}$	0.0004 (0.3894)	0.0004 (0.2164)	-0.0021 (-0.9260)	0.0005 (0.2930)
$\gamma_{BUX}\gamma_{PX}$	0.0018 (0.8878)	0.0027 (1.0562)	0.0027 (0.7131)	-0.0014 (-0.3494)
$\gamma_{WIG}\gamma_{PX}$	0.0005 (0.4018)	0.0003 (0.2212)	-0.0012 (-0.6653)	-0.0037 (-0.6809)
L_{VT-DCC}	-3752.4975	-3751.5902	-3752.7170	-3754.8699
$L_{VT-GDCC}$	-3739.0166	-3736.6451	-3740.8362	-3741.8587
LR	26.9618	29.8902	23.7616	26.0224

Notes: This table gives the quasi-maximum likelihood estimates of VT-ADCC model in (14) with the restrictions on the GARCH dynamics of the conditional correlations, and the matrix V_t defined as in (17). t-statistics are given in parentheses. t-statistics are calculated using the delta method. LR is likelihood ratio test statistic of VT-GDCC against VT-DCC specification, distributed χ^2_{10}. The upper 1 and 5 percentile points of the χ^2_{10} distribution are 23.21 and 18.31, respectively.