Managerial Compensation Contracts and Overconfidence*

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Abstract

In this paper we analyze how overconfidence affects the principal–agent relationship when both the principal and the agent are assumed to be overconfident with respect to the quality of a common signal on the future state of nature. We study the impact of that psychological bias on both the compensation contract which the principal offers to the agent and the severity of the moral hazard problem. Most notably, our analysis indicates that a more pronounced overconfidence bias generally reduces the agency costs but enhances the incentive component of the compensation contract as well as the agent's effort. Therefore we conclude that overconfidence plays a crucial role in the design of incentive compatible compensation contracts. Furthermore, we find that from the principal's perspective overconfidence is advantageous only if favorable information about the future state of nature is available. If poor signals are available the overconfidence bias is detrimental to the principal.

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1 Introduction

Entrepreneurs and managers among others are reported to exhibit overconfidence in their judgements. Evidence concerning the former group can be found in Cooper, Dunkelberg and Woo (1988) whereas evidence as regards the members of the latter category is presented in Russo and Schoemaker (1992), and Malmendier and Tate (2002). Busenitz and Barney (1997) report that both entrepreneurs and managers are subject to this psychological bias. The basic relation between entrepreneurs and managers is that an entrepreneur employs a manager to perform a certain task. The entrepreneur — below referred to as the principal — writes a remuneration contract which guarantees the manager — henceforth called the agent — at least to some extent a compensation for the accomplished effort once the agent has accepted the labor contract. Since the agent's effort is assumed to be unobservable on the part of the principal and consequently cannot be contracted upon there is room for moral hazard. Facing the evidence of entrepreneurial and managerial overconfidence the purpose of this paper is to provide an analysis of how the overconfidence bias affects the compensation contract and the severity of the moral hazard problem.

This paper adds to the emerging body of literature in behavioral corporate finance by explicitly merging an empirically well–documented aspect of human behavior — namely overconfidence — with the principal–agent paradigm of corporate finance. There exists already a bunch of papers which are concerned with the analysis of behavioral aspects of corporate finance. Shefrin (2001*a*) stresses that incentive compatibility is a necessity for value maximization but that incentive effects alone cannot overcome the impact of behavioral obstacles that are internal to the firm such as for example overconfidence. Heaton (2002) explains a variety of corporate finance phenomena even in the absence of both asymmetric information and moral hazard by relying on managerial optimism as behavioral bias. Additionally managerial overconfidence is taken into account by Gervais, Heaton and Odean (2002) who find that both behavioral biases — overconfidence and optimism — can increase the value of the firm. Thus, the decisions of overconfident and optimistic managers align better with the interest of the shareholders than those of rational managers. Furthermore, they make the case for hiring an overconfident manager instead of realigning the decisions of a rational manager with the shareholders' interest by employing convex compensation schemes. In Goel and Thakor (2000) the competition among managers for leadership is identified as mechanism which fosters overconfidence among managers since a stronger overconfidence bias of a manager increases the probability to become leader. The persistence of entrepreneurial overconfidence is discussed in Bernardo and Welch (2001). An equilibrium proportion of overconfident individuals is derived in a group selection framework. It is shown that the overconfident behavior on the part of entrepreneurs provides a positive externality concerning information aggregation compared to an otherwise herding behavior. Thus the persistence of overconfidence is justified.

In contrast to the above approaches that are concerned with the effects of overconfidence that are internal to the firm there exists already a body of literature which continuously grows and discusses external effects of overconfidence. More precisely these papers deal with overconfidence from the financial markets' perspective. Kyle and Wang (1997) argue in a game theoretic setting that for a fund management an overconfident strategy may dominate a rational one. Daniel, Hirshleifer and Subrahmanyam (1998) demonstrate in a representative agent model that the variation of overconfidence over time according to biased self–attribution can explain security price patterns. Odean (1998) examines the impact of overconfidence in various market architectures and reports — besides other results — that overconfidence can improve or worsen the informational efficiency of the market depending on the distribution of information among market participants. The case for persistence of overconfident traders in financial markets is made by Hirshleifer and Luo (2001) in both a static and a dynamic analysis because overconfident traders successfully exploit mispricing stemming from noise trading. In a multiperiod model Gervais and Odean (2001) relate the dynamics of an individual's overconfidence to successes and failures from trading and find that overconfidence is not driven out of the market since successful and thus overconfident traders accumulate wealth. Daniel, Hirshleifer and Subrahmanyam (2001) explain the cross–sectional predictive power of fundamental/price ratios for expected security returns by introducing overconfidence with respect to the quality of signals on some risk factors that drive security returns. Hence they present a behaviorally–based explanation of some empirical findings that hardly can be accounted for under the hypothesis of full rationality and are therefore usually considered as anomalies. Shefrin (2001*b*) discusses how traders' errors affect the pricing kernel that underlies the pricing of all assets and suggests a behaviorally–based modification of the stochastic discount factor. In essence the modification of the pricing kernel introduces an additional degree of freedom that gives rise to an explanation of otherwise anomalous effects that are at odds with predictions based on full rationality.

Besides these more or less specialized treatments of behavioral biases — especially overconfidence — there are some papers that attempt to consolidate the different strands of the literature. Hirshleifer (2001) provides an up–to–date summary of how behavioral aspects besides overconfidence have already been taken into account by the asset pricing literature. Finally Daniel, Hirshleifer and Teoh (2002) beyond providing an overview of how investor biases affect asset prices suggest policies in order to promote the informational efficiency of the capital market, to avoid systematic mispricing of assets and consequently to prevent the misallocation of resources. An overview of the literature on overconfidence and a discussion that points out why overconfident behavior can be found in financial markets is provided in the second section of Odean (1998).

Our analysis of the impact of overconfidence on the compensation contract and the agency costs as measure for the severity of the moral hazard problem proceeds along the steps taken in Holmström's (1979) seminal approach. However, we modify the information structure by introducing a noisy signal on the environmental impact which in turn partly determines the monetary outcome that is to be shared ultimately. The signal is supposed to be available to the parties to the contract which both are assumed to be overconfident with respect to that signal's quality. In our model the overconfidence of the players is modelled as judging the quality of the signal to be higher than it really is. Hence, the overconfidence bias implies that in an inference process — that is by conditioning on the private signal — the players put more weight on the available information than would be rational.

The general idea behind the model can be described as follows. The common noisy signal can be interpreted as a forecast of the future state of nature which affects the final monetary outcome. In a broader sense one might think of the noisy signal as a forecast of the business cycle where good future states of the economy are associated with a higher signal indicating a favorable environmental effect on the final outcome and vice versa. Since the signal is noisy the forecast is not perfect. In this respect the signal noise represents the forecast error. Hence, the application of the model might be as follows. Think of a company whose shareholders hire a highly specialized manager to run the business on their behalf. The manager's expertise makes him believe to have an above average ability to forecast the future state of nature. Thus the agent is overconfident with respect to the signal's quality. Since it is unlikely that the shareholders hire a manager who differs with respect to the assessment of outcome relevant information the shareholders and the manager are supposed to exhibit the same degree of overconfidence.

The model which we analyze in a behaviorally–based framework by taking into account the players' overconfidence allows to address the question how the available signal and the psychological bias affect the principal–agent relationship. For the sake of tractability and the ability to provide closed–form solutions which in turn allow to derive comparative static results we restrict our analysis to the class of linear sharing rules. Each sharing rule determines how the monetary outcome or the final payoff from the agent's action and the environmental effect are shared between the principal and the agent finally.

We obtain the following results from the analysis of the second–best problem. The compensation contract depends on both the overconfidence bias and the available signal. Consequently, different combinations of (a.) the information about the future state of nature and (b.) the level of overconfidence with respect to the quality of that information imply different sharing rules. This accounts for the variety of compensation arrangements that can be observed for performing the same task. In detail, the fixed compensation component depends on both the overconfidence bias and the available signal. In contrast, the incentive component, the agent's effort and the agency costs solely depend on the overconfidence bias.

The comparative static results indicate that irrespective of the available information a more pronounced overconfidence bias mitigates the moral hazard problem by reducing the agency costs stemming from the unobservability of the agent's effort. Contrary to this finding, both the incentive component of the sharing rule and the agent's effort are generally increased by a stronger overconfidence bias. The impact of the overconfidence bias on the fixed compensation depends on the available signal. If favorable information is observed then a more pronounced overconfidence bias decreases the fixed remuneration. For poor signals the impact of a stronger overconfidence bias on the fixed compensation is reversed. Finally, the fixed compensation ceteris paribus is found to be decreasing in the available signal whereas the variable compensation as well as the agency costs are not affected by the common signal at all. These insights let us conclude that the overconfidence bias plays a crucial role in the design of incentive compatible compensation contracts.

In addition to the common analysis of the principal–agent relationship we shed light on the dependence of the principal's expected utility on the overconfidence bias. Thus we extend the usual discussion by analyzing the impact of the overconfidence bias from the shareholders' perspective. The comparative statics show that the principal's expected utility is increased by a more pronounced overconfidence bias if favorable information is available and vice versa. The most striking finding is that we identify ranges for the common signal where a stronger overconfidence bias generally is detrimental to or advantageous for the shareholders respectively. Since this observation is true irrespective of the actual level of overconfidence we can formulate general policy implications. In essence, we find that if good signals are observed then being more overconfident is advantageous for the shareholders whereas if bad signals are available being less overconfident is favorable for the shareholders. Put differently, overestimating the quality of good signals does not harm the shareholders whereas the shareholders suffer in terms of expected utility from the overestimation of the quality of bad signals.[1]

The remainder of the paper is organized as follows. Following this introduction section 2 delivers the basic structure underlying the principal–agent

[1] The meaning of "favorable" and "poor" information or "good" and "bad" signals is made concrete in propositions 5 and 8 as well as in corollary 5.

relationship to be analyzed in the next two sections. In section 3 the first–best and second–best compensation contracts as well as the agency costs are derived. Additionally, the principal's second–best expected utility is determined. The comparative static analysis with respect to the two major ingredients of the model — that are the signal and the coefficient of overconfidence — are provided in section 4. The conclusions are left to section 5 where further research avenues both empirical and theoretical are outlined. All proofs are collected in the appendix.

2 Setup of the Principal–Agent Relationship

The purpose of this section is to outline the basic structure that underlies the model which is analyzed in the next two sections. At first we describe how uncertainty affects the principal–agent relationship. Then we specify the structure of the compensation contract as well as both the principal's and the agent's preferences. Furthermore we characterize how the overconfidence bias enters the principal–agent problem studied later in this paper. At the end of this section the information structure of the model is described.

The principal–agent relationship is subject to uncertainty since the monetary outcome or payoff $\tilde{x}$ which is to be shared between the principal and the agent finally is affected by an ex ante unobservable state of nature $\tilde{\theta}$. We assume that the monetary outcome $\tilde{x}$ results as

$$\tilde{x} = e + \tilde{\theta}, \tag{1}$$

where e represents the agent's effort. The state of nature $\tilde{\theta}$ comprises the random environmental effect which the monetary outcome is subject to. The environmental effect $\tilde{\theta}$ is supposed to be normally distributed having law $\mathcal{N}(0, \sigma_\theta^2)$. Consequently, the final payoff $\tilde{x}$ is random and has a normal distribution too. To make things more concrete one might imagine that the environmental effect $\tilde{\theta}$ subsumes the part of the monetary outcome which results from economic conditions that are beyond the control of both the agent and the principal.

The sharing rule $r(\tilde{x})$ which the principal offers to the agent aims at aligning the agent's action with the principal's interest. The sharing rule is assumed to be a linear function of the monetary outcome $\tilde{x}$ that is

$$r(\tilde{x}) = \gamma + \delta \cdot \tilde{x}, \tag{2}$$

where γ and δ are some real numbers that determine the components of the compensation contract. The agent's fixed compensation amounts to γ whereas the variable compensation results as product of δ and the final monetary outcome $\tilde{x}$.

The principal and the agent are supposed to be expected utility maximizers. The principal is assumed to be risk neutral and his preferences are represented by the utility function V which is defined over final wealth. The agent however is assumed to be risk averse having the utility function U as preference representation over end of period wealth. Specifically the agent's preferences are represented by negative exponential utility which exhibits constant absolute risk aversion. The agent's coefficient of absolute risk aversion is denoted by $a > 0$.

The principal's end of period wealth depends on both the monetary outcome $\tilde{x}$ which is to be shared between the parties to the contract and the sharing rule $r(\tilde{x})$. The principal's end of period wealth amounts to $\tilde{x} - r(\tilde{x})$. Thus the principal claims the residual monetary outcome which remains after the agent is paid the contracted remuneration $r(\tilde{x})$.

The final wealth of the agent depends on both the sharing rule $r(\tilde{x})$ and the accomplished effort e. In particular we suppose that the agent's final wealth results as $r(\tilde{x}) - \frac{1}{2}e^2$. This definition implies that the effort e reduces the end of period wealth by decreasing the remuneration $r(\tilde{x})$. This decrease generates a kind of disutility what in turn captures the notion that the agent suffers from the effort. Since the effort costs change disproportionately and the remuneration only depends linearly on the effort — see the latter from plugging (1) into (2) — it is guaranteed by this implicit trade–off that there exists a finite optimal effort. Furthermore, the agent has a minimum or reservation level of wealth m that he requires at least from acting on the principal's behalf. Therefore the wealth level m produces the agent the so–called reservation utility $U(m)$. One might think of m as being the agent's final wealth from working elsewhere.

The principal cannot observe the agent's effort directly. This is the guiding principle underlying the design of optimal or incentive compatible compensation contracts and the source of moral hazard. Consequently the sharing rule which the principal offers to the agent is based on the monetary outcome $\tilde{x}$ that the principal observes ultimately instead of being based solely on the agent's effort e. This in turn implies that the agent's remuneration depends to some extent on the environmental effect $\tilde{\theta}$ too. Although the environmental effect cannot be controlled neither by the principal nor by the agent we assume that a noisy signal $\tilde{s}$ on the environmental effect is available initially. The noisy signal is given by

$$\tilde{s} = \tilde{\theta} + \tilde{\varepsilon}, \tag{3}$$

where $\tilde{\varepsilon} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ denotes the normally distributed signal noise which is uncorrelated with the environmental effect. The overconfidence bias is introduced with respect to the quality of the signal $\tilde{s}$. Being overconfident with coefficient κ means that the variance of the signal noise is taken to be $\kappa\sigma_\varepsilon^2$ where $0 < \kappa < 1$. Thus the overconfidence bias reduces the dispersion of the signal $\tilde{s}$ what corresponds to the notion that the signal is judged to be more accurate, more precise, or more informative than it really is. Note that $\kappa = 1$ corresponds to an unbiased assessment of the signal and that a stronger overconfidence bias stems from a lower coefficient of overconfidence. Since the signal gives an indication for the future state of nature it is informative for the final monetary outcome at least to some extent. Thus it is rational not to ignore the signal and to use the signal in contracting although it is less than perfect.

The model's information structure obeys to the above general description. In particular all features mentioned above are presumed to be common knowledge. Thus the information structure is symmetric and the players are subject to the overconfidence bias in the manner described previously.[2]

[2]The assumption that the overconfidence bias is common knowledge needs some further comment. One might raise the objection that once the players are aware of the overconfidence bias they correct their behavior accordingly. In response to that objection one can motivate this assumption alternatively. It is sufficient to claim that the principal and the agent agree on $\left(\sigma_\theta^2 + \sigma_{\varepsilon,\kappa}^2\right)^{-1}$ as the precision of the signal where $\sigma_{\varepsilon,\kappa}^2 < \sigma_\varepsilon^2$ that is the parties to the contract underestimate the true variance of the forecast error $\tilde{\varepsilon}$. This fits the

Before we turn to the analysis of the principal–agent relationship we have to mention some words about the special structure we imposed on the model. The special structure allows to derive closed–form solutions and comparative statics of the optimal compensation contracts in the presence of overconfidence. This benefit comes at the cost of (a.) specifying the players' utility functions as well as the agent's disutility, (b.) making specific distributional assumptions, and (c.) restricting to linear compensation schedules. We trade off the generality of the modelling approach and the ability to study the impact of overconfidence on the optimal compensation contracts.

3 Sharing Rules and Agency Costs

This section addresses the problem of the optimal compensation contract's design when a principal hires an agent to perform a certain task and both players are overconfident with respect to the quality of a common signal on the future state of nature. The setup of the model corresponds to that given in section 2. This section is devoted to the determination of both the optimal sharing rules as well as the according agency costs and the principal's second–best expected utility. The comparative static analysis is left to section 4.

The compensation contract we determine at first corresponds to the situation where the principal cannot control the agent's action due to the lack of perfect monitoring. This compensation contract is referred to as the second–best sharing rule. Thereafter we focus on the case of perfect monitoring where the principal chooses the effort level of the agent in addition to the compensation contract which then is referred to as the first–best sharing rule. Having the first–best and the second–best solution at hand allows to determine the agency costs immediately.

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empirical evidence that both entrepreneurs and managers are overconfident that is they overestimate the quality of information at hand. Then the comparative static analysis with respect to $\sigma^2_{\varepsilon,\kappa}$ yields qualitatively similar results. From the perspective of this alternative approach it is more or less a modelling convenience to employ the parameter κ in order to capture the degree of overconfidence. Put differently, the assumption of overconfidence boils down to an agreement on a biased assessment — especially a underestimation — of the variance of the forecast error $\tilde{\varepsilon}$. In this respect our modelling approach is equivalent to that taken for example in Daniel, Hirshleifer and Subrahmanyam (1998).

3.1 Second–Best Contract

The determination of the optimal second–best sharing rule $r_2(\tilde{x})$ boils down to the principal's choice of γ_2 and δ_2 which represent the second–best compensation contract's parameters. The timing of the players' actions and events in the second–best case is depicted in figure 1. At $t = 0$ both the principal and the agent observe the common signal $\tilde{s}$. The principal offers the sharing rule γ_2 and δ_2 to the agent at $t = 1$. At $t = 2$ the agent chooses the effort e_2. The resolution of uncertainty occurs at $t = 3$ and the principal observes the monetary outcome $\tilde{x}$.

The second–best compensation contract is the solution of the following constrained program

$$\max_{r_2(\tilde{x})} \quad \mathrm{E}_\kappa[V(\tilde{x} - r_2(\tilde{x}))|\tilde{s}] \tag{4}$$

$$\text{s.t.} \quad \mathrm{E}_\kappa[U(r_2(\tilde{x}), e_2)|\tilde{s}] \geq U(m) \tag{5}$$

$$e_2 \in \operatorname*{argmax}_{e} \mathrm{E}_\kappa[U(r_2(\tilde{x}), e)|\tilde{s}], \tag{6}$$

where the subscript κ of the expectation operator reminds of the fact that the players' expected utilities are derived subject to the overconfidence bias. The constraint (5) ensures that it is rational for the agent to enter the principal–agent relationship because the engagement produces at least the reservation level of wealth or the according reservation utility respectively. Note that this constraint is referred to equivalently as the individual rationality constraint, the participation constraint or the reservation utility constraint. The constraint (6) which guarantees that the agents chooses the effort which maximizes his expected utility is known as the incentive compatibility constraint. The second–best contract is given in proposition 1.

Proposition 1 *The second–best compensation contract $r_2(\tilde{x})$ that the risk neutral principal offers the risk averse agent is given by*

$$\gamma_2 = m - \frac{\mu_{\theta,\kappa}}{1 + a\sigma^2_{\theta,\kappa}} - \frac{1 - a\sigma^2_{\theta,\kappa}}{2\left(1 + a\sigma^2_{\theta,\kappa}\right)^2}$$

and

$$\delta_2 = \frac{1}{1 + a\sigma^2_{\theta,\kappa}} > 0,$$

where $\mu_{\theta,\kappa}$ and $\sigma^2_{\theta,\kappa}$ represent the expected environmental effect and the variance of the environmental effect conditional on the private signal $\tilde{s}$ subject to the overconfidence bias as given in lemma 1 respectively.

By inspection of proposition 1 one realizes that the second–best remuneration contract depends on the conditional moments of the environmental effect. Thus — besides the coefficient of risk aversion a and the reservation level of wealth m — both the common signal $\tilde{s}$ and the coefficient of overconfidence κ are determinants of the second–best sharing rule. Since $\delta_2 > 0$ the agent shares in the final monetary outcome. Thus there exists an incentive on the part of the agent to exert an effort which increases the final monetary outcome. The derivation of proposition 1 delivers the insight of corollary 1 immediately.

Corollary 1 *The agent's second–best effort amounts to $e_2 = \delta_2$ where δ_2 is given in proposition 1.*

Combining proposition 1 and corollary 1 allows to express the second–best fixed compensation as

$$\gamma_2 = m + \frac{1}{2}e_2^2 + \frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa} - \delta_2\left(e_2 + \mu_{\theta,\kappa}\right) \tag{7}$$

equivalently. Hence, the agent's second–best fixed compensation consists of the reservation level of wealth m, the compensation for his effort costs $\frac{1}{2}e_2^2$ and a risk premium $\frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa}$ which compensates for the remaining state uncertainty $\sigma^2_{\theta,\kappa}$. Finally, the conditional expected variable compensation $\delta_2\left(e_2 + \mu_{\theta,\kappa}\right)$ is subtracted from those three components. Using (7) yields

$$r_2(\tilde{x}) = m + \frac{1}{2}e_2^2 + \frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa} + \delta_2\left(\tilde{\theta} - \mu_{\theta,\kappa}\right) \tag{8}$$

for the agent's second–best compensation which is subject to state uncertainty although the signal $\tilde{s}$ is observed. Note that the agent's second–best remuneration depends on the conditional unexpected innovation of the future state of nature, $\tilde{\theta} - \mu_{\theta,\kappa}$. For positive (negative) surprises with respect to the conditional expected future state of nature $\mu_{\theta,\kappa}$ the agent's second–

best compensation becomes larger (smaller).[3] It is exactly that risk which is compensated by the risk premium mentioned previously.[4]

Note, even if the common signal is judged to be perfect — that is if $\kappa = 0$ what implies $\mu_{\theta,\kappa} = \tilde{s}$ at once — there remains state uncertainty in the agent's second–best compensation and thus in the principal's second–best final wealth. Hence, the overconfidence bias does not eliminate the fundamental state uncertainty which affects the principal–agent relationship. Solely, the players believe to have perfect information on the future state of nature in that case. Furthermore, in the limiting case where $\kappa = 0$ the remaining state uncertainty is judged to be zero that is $\sigma^2_{\theta,\kappa} = 0$. Thus, the agent does not receive any risk premium at all.[5] Moreover, the agent has the maximum exposure to the conditional unexpected innovation of the future state of nature since $\delta_2 = 1$ in that case.[6]

3.2 First–Best Contract

The optimal first–best compensation contract $r_1(\tilde{x})$ requires the determination of γ_1 and δ_1 on the part of the principal. In contrast to the second–best case where the agent chooses the optimal effort it is the principal who in the first–best case controls perfectly the agent's action and chooses the effort level of the agent additionally. Figure 2 shows the timing of the players' actions and events in the first–best case. At $t = 0$ both the principal and the agent observe the common signal $\tilde{s}$. The principal decides on the optimal compensation contract γ_1 and δ_1 as well as on the agent's effort e_1 at $t = 1$. At $t = 2$ the state uncertainty is resolved and the principal observes the monetary outcome $\tilde{x}$. Note that all decisions are taken by the principal. The agent does not take any action at all. Consequently, the principal is only subject to the state uncertainty since there is no room for moral hazard on the part of the agent. The absence of moral hazard ensures the first–best

[3] Recall that $\delta_2 > 0$. The principal's second–best final wealth is affected inversely.

[4] Note,

$$\sigma^2_{\theta,\kappa} = \mathrm{E}\left[\left(\tilde{\theta} - \mu_{\theta,\kappa}\right)^2\right].$$

[5] Cf. figure 7.

[6] Cf. figure 4.

result for the principal.

The first–best compensation contract is obtained as solution of the following constrained programm

$$\max_{r_1(\tilde{x}),e_1} \quad \mathrm{E}_\kappa[V(\tilde{x} - r_1(\tilde{x}))|\tilde{s}] \tag{9}$$

$$\text{s.t.} \quad \mathrm{E}_\kappa[U(r_1(\tilde{x}), e_1)|\tilde{s}] \geq U(m), \tag{10}$$

where the subscript κ indicates that the players are overconfident with respect to the quality of the common signal $\tilde{s}$. In the first–best case the principal's choice of the agent's effort has only to ensure that the agent accepts the offered sharing rule. Therefore the optimization is solely subject to the agent's participation constraint (10). The first–best compensation contract is given in proposition 2.

Proposition 2 *The first–best compensation contract $r_1(\tilde{x})$ that the risk neutral principal offers the risk averse agent is given by*

$$\gamma_1 = m + \frac{1}{2}$$

and

$$\delta_1 = 0.$$

Most notably, the first– best compensation contract is independent of the conditional moments of the future state of nature. Hence, it does not depend on neither the common signal nor the coefficient of overconfidence. Solely the agent's reservation level of wealth m enters the fixed component of the sharing rule. Since the principal optimally determines the agent's effort the principal does not provide the agent with any incentive at all. Consequently, the variable compensation amounts to zero. The proof of proposition 2 yields corollary 2.

Corollary 2 *The agent's first–best effort amounts to $e_1 = 1$.*

Having established both proposition 2 and corollary 2 the interpretation of these results is straightforward. The agent's first–best remuneration amounts to $r_1(\tilde{x}) = \gamma_1$. Thus the reservation level of wealth m is guaranteed since the effort costs $\frac{1}{2}e_1^2$ are compensated. Since the agent is not subject to any compensation uncertainty at all — recall that the variable compensation amounts to zero — the agent does not receive a risk premium. Consequently, in the first–best case the state uncertainty is carried solely by the principal.

3.3 Agency Costs

The major difference between the first–best program and the second–best program is that in the latter the agent chooses his effort in order to maximize the expected utility according to the incentive compatibility constraint whereas in the first–best case the principal determines the agent's effort. Since, ultimately, the principal and the agent share the monetary outcome $\tilde{x}$ it is obvious that in the second–best case the agent's optimization comes at the cost of a lower expected utility on the part of the principal. The amount by which the principal's expected utility in the second–best case is reduced compared to the first–best case is referred to as the agency costs. Thus agency costs are costs from hiring a selfish agent to perform a certain task when the principal cannot monitor the agent perfectly. Hence the agency costs quantify the severity of the moral hazard problem. The agency costs are given in proposition 3.

Proposition 3 *The agency costs that the risk neutral principal suffers are given by*

$$\frac{a\sigma_{\theta,\kappa}^2}{2\left(1+a\sigma_{\theta,\kappa}^2\right)},$$

where $\sigma_{\theta,\kappa}^2$ represents the variance of the environmental effect conditional on the private signal $\tilde{s}$ subject to the overconfidence bias as given in lemma 1.

Note that the agency costs are only affected by the coefficient of overconfidence κ through the remaining state uncertainty $\sigma_{\theta,\kappa}^2$. The actual common signal is irrelevant for the severity of the moral hazard problem. Proposition 3 delivers corollary 3 immediately.

Corollary 3 *The agency costs that the risk neutral principal suffers are positive.*

Note, corollary 3 restates the well–known result of principal–agent theory. Namely, the moral hazard problem strictly produces disutility for the principal that is the principal suffers from the agency relationship.

3.4 Principal's Second–Best Expected Utility

Although the severity of the moral hazard problem is judged by the magnitude of the agency costs it might be of interest how the principal's expected utility in the second–best case is affected by the common signal $\tilde{s}$ and the coefficient of overconfidence κ. Consequently this allows an interpretation of the impact of the overconfidence bias from the shareholders' perspective what in turn allows to derive some policy implications later on. Proposition 4 reports the shareholders' expected utility.

Proposition 4 *The principal's second–best expected utility amounts to*

$$-m + \mu_{\theta,\kappa} + \frac{1}{2\left(1 + a\sigma^2_{\theta,\kappa}\right)},$$

where $\mu_{\theta,\kappa}$ and $\sigma^2_{\theta,\kappa}$ represent the expected environmental effect and the variance of the environmental effect conditional on the private signal $\tilde{s}$ subject to the overconfidence bias as given in lemma 1 respectively.

First of all one realizes the dependency of the principal's second–best expected utility from both the common signal $\tilde{s}$ and the coefficient of overconfidence κ. Alternatively, the principal's second–best expected utility can be expressed as

$$e_2 + \mu_{\theta,\kappa} - \left(m + \frac{1}{2}e_2^2 + \frac{1}{2}ae_2^2\sigma^2_{\theta,\kappa}\right). \tag{11}$$

Due to the principal's risk neutrality the second–best expected utility (11) simply results as the conditional expected monetary outcome less the agent's conditional expected second–best compensation. The latter can be calculated as the expectation of the right hand side of equation (8) conditionally on the common signal $\tilde{s}$.

Obviously, the above propositions 1, 3, and 4 report a dependency of the second–best compensation contract, the agency costs, and the principal's second–best expected utility on the common signal $\tilde{s}$ as well as on the coefficient of overconfidence κ which captures the strength of the overconfidence bias with respect to the quality of the common signal. It is the sensitivity of these dependencies with respect to variations of the common signal and the overconfidence bias that we study in the comparative static analysis in section 4.

4 Comparative Static Analysis

After having derived the optimal sharing rules and the agency costs as well as the principal's second–best expected utility we now turn to studying the impact of changes of the coefficient of overconfidence κ and the common signal $\tilde{s}$ on these results ceteris paribus. The comparative static results allow us to formulate some implications concerning the relevance of the overconfidence bias and the common signal for the principal–agent relationship finally.

The first–best sharing rule was found to be independent of the common signal and the overconfidence bias. Therefore and since due to imperfect monitoring on the principal's part it is the second–best contract which can be implemented we carry out the comparative static analysis of the second–best compensation contract. Recall that the second–best sharing rule is represented by γ_2 and δ_2 as specified in proposition 1. The comparative statics of the second–best contract's fixed component γ_2 are summarized in proposition 5.

Proposition 5 *The fixed compensation γ_2 is*

- *increasing for $\tilde{s} > s_1$ and decreasing for $\tilde{s} < s_1$ in the coefficient of overconfidence κ and*
- *decreasing in the signal $\tilde{s}$.*

The signal s_1 is given in the proof.

Proposition 6 collects the comparative statics of the second–best contract's variable compensation δ_2.

Proposition 6 *The variable compensation δ_2 is*

- *decreasing in the coefficient of overconfidence κ and*
- *independent of the signal $\tilde{s}$.*

Corollary 1 in turn implies corollary 4 immediately.

Corollary 4 *The comparative static results of proposition 6 apply to the agents effort e_2 too.*

The sensitivities of the agency costs as given in proposition 3 with respect to the coefficient of overconfidence κ as well as with respect to the signal $\tilde{s}$ are pooled in proposition 7.

Proposition 7 *The agency costs that the risk neutral principal suffers are*

- *increasing in the coefficient of overconfidence κ and*
- *independent of the signal $\tilde{s}$.*

In order to interpret these comparative static results with respect to the coefficient of overconfidence properly we here emphasize again that it is a decreasing coefficient of overconfidence κ which actually comes along with a more pronounced overconfidence bias. This means that the lower is the coefficient of overconfidence κ the more precise is judged the common signal on the future state of nature by the parties to the contract.

The comparative statics of the second–best sharing rule's components indicate that a large menu of compensation schedules exists that differ with respect to the fixed and variable components. Thus the variety of sharing rules is spanned by the various combinations of the common signal $\tilde{s}$ and the parties' degree of overconfidence κ. Hence, our approach might serve to explain the multitude of labor contracts that are observed for performing the same task.

For example, if the parties to the contract ceteris paribus are subject to a more pronounced overconfidence bias then they generally agree on a higher variable compensation. This observation is depicted by the graph in the figure 4 which is based on the parameter values in table 1. In the case of a poor common signal $\tilde{s} < s_1$ they contract a higher fixed compensation too. If instead the parties have observed favorable information $\tilde{s} > s_1$ then the opposite is true with respect to the fixed compensation. In that case the more overconfident counterparties agree on a lower fixed compensation. These latter findings are illustrated in figure 3 drawing on the parameter values summarized in table 1.

The impact of the common signal $\tilde{s}$ on the second–best sharing rule's components ceteris paribus is quite simple. The variable compensation remains unaffected by a different signal whereas the contracted fixed compensation as shown in figure 3 is the lower the higher is the observed signal $\tilde{s}$.

The comparative static results as concerns the agency costs are clear cut. Generally, the stronger is the overconfidence bias of the parties to the contract the lower are the agency costs. Thus, judging the quality of the common signal better than it really is ceteris paribus reduces the severity of the moral hazard problem. The wedge between the principal's first–best expected utility and second–best expected utility becomes smaller by a more pronounced overconfidence bias. This comparative static result regarding the agency costs is confirmed by the graph in figure 5 for the chosen model parameters in table 1. The mechanics behind this striking result are as follows. Since a stronger overconfidence bias implies a higher variable compensation — and in case of poor information $\tilde{s} < s_1$ a higher fixed compensation additionally — the simultaneously increasing effort of the agent has to overcompensate these higher remuneration costs.

The effect of the common signal $\tilde{s}$ with respect to the agency costs remains to be discussed. Since the agency costs do not depend on the common signal observing a different signal ceteris paribus does not affect the agency costs at all. This independence is true generally.

Summarizing the comparative static analysis of the second–best sharing rule's components and the agency costs we record that (a.) a higher common signal generally decreases the fixed compensation but does not affect both the variable compensation and the agency costs and (b.) a more pronounced overconfidence bias increases the variable compensation and reduces the agency costs generally. The fixed compensation is reduced by a stronger overconfidence bias if favorable information is available and vice versa.

The above comparative static analysis of the sharing rule and the agency costs resembles the usual discussion of a principal–agent relationship. The following discussion extends that analysis by studying the impact of both the overconfidence bias and the common signal on the principal's second–best expected utility. This allows to assess the impact of the psychological bias and the common piece of information from the shareholders' perspective. The comparative static results as concerns the principal's second–best expected utility are collected in proposition 8.

Proposition 8 *The principal's second–best expected utility is*

- *increasing for $\tilde{s} < s_2$ and decreasing for $\tilde{s} > s_2$ in the coefficient of overconfidence κ and*
- *increasing in the signal $\tilde{s}$.*

The signal s_2 is given in the proof.

Before continuing with the interpretation of the comparative static results which are reported in proposition 8 again recall that a stronger overconfidence bias stems from a smaller coefficient of overconfidence κ. Thus, a more pronounced overconfidence bias ceteris paribus increases the principal's second–best expected utility if favorable information $\tilde{s} > s_2$ about the future state of nature is observed initially. If instead a poor common signal $\tilde{s} < s_2$ becomes available to the counterparties then the opposite result holds. In that case the principal's second–best expected utility is reduced by a stronger overconfidence bias. The interpretation of these results is straightforward. Putting more weight on good information increases the shareholders' expected utility whereas believing more strongly in a bad forecast has the opposite effect. These comparative static results are illustrated by the surface in figure 6 on the basis of the parameters in table 1.

Furthermore proposition 8 reports that ceteris paribus the better the common signal $\tilde{s}$ on the future state of nature is the higher is the principal's expected utility in the second–best case. Note that this comparative static result holds generally. Put differently, the shareholders' expected utility is the higher the better is the information on the future state of nature. Again figure 6 supports this observation.

Having established proposition 8 allows to derive general implications from the shareholders' perspective with respect to the handling of the common signal irrespective of the actual severity of the overconfidence bias. Corollary 5 states these implications formally.

Corollary 5 *Generally, the principal's second–best expected utility is increasing for $\tilde{s} < s_3$ and decreasing for $\tilde{s} > s_4$ in the coefficient of overconfidence κ where $s_3 < s_4$. The signals s_3 and s_4 are independent of the overconfidence bias and are given in the proof.*

In corollary 5 we have identified ranges of the common signal where it is advantageous or detrimental for the principal in terms of expected utility to exhibit a stronger overconfidence bias respectively. If favorable information $\tilde{s} > s_4$ on the future state of nature becomes available then being more overconfident with respect to the common signal's quality — and consequently hiring an agent which is subject to the same higher degree of overconfidence — is advantageous from the principal's perspective. However, in the case that poor information $\tilde{s} < s_3$ on the future state of nature is observed a more pronounced overconfidence bias affects the shareholders adversely by reducing their expected utility. In a few words, overestimating the quality of good common signals is beneficial whereas it is advisable to process bad common information as unbiased as possible. These findings of corollary 5 are supported by inspection of the endpoints of the curved line in the surface which is depicted in figure 6.

Summarizing, our previous findings indicate that depending on the common signal which is observed initially different information processing capabilities are desirable on the part of both the principal and the agent. Obviously, this finding has direct implications for the job market of managers. When the available information about the future state of nature shifts from good to bad it is favorable for the shareholders to recruit and employ a less overconfident management. In parallel, if the shareholders pursue this employment policy the management's compensation schedules also are affected. Then, the variable compensation is increased if a favorable common signal on the future state of nature is available whereas if poor information is observed a lower incentive component of the sharing rule is contracted.

5 Conclusion

These days a consensus emerges among financial economists that behavioral biases which reveal through systematic errors of economic agents affect securities prices. Behavioral finance models try to capture the empirically documented asset return anomalies that are at odds with predictions of models based on full rationality. These behaviorally based models build on the insights delivered by experimental economists. Predominantly the attention of financial economists was directed to the analysis of financial markets that is how companies are affected externally by the behavior of economic agents. But, by nature, companies are affected by behavior internally too. Consequently, this paper addressed the question how a well–documented behavioral bias — namely overconfidence — affects the principal–agent relationship which arises in companies with delegated management. Thus, this paper adds to a literature that makes the affirmative case for analyzing the internal effects of behavioral biases on companies and for establishing behavioral corporate finance as additional research field for financial economists.

We have presented a thorough analysis of a principal–agent relationship that deals with the design of an incentive compatible compensation contract in the presence of overconfidence and moral hazard. The model presumed a symmetric information structure except for the agent's unobservable effort. The psychological bias overconfidence on the part of the parties to the contract — the principal and the agent — is introduced with respect to the quality of a common signal on the future state of nature which in turn affects the monetary outcome that is to be shared finally.

The results we obtain from studying the impact of overconfidence on the principal–agent relationship are manifold. The most striking result is that a more pronounced overconfidence bias reduces the severity of the moral hazard problem, or mitigates the moral hazard problem. Put differently, the more overconfident are both the principal and the agent the lower are the agency costs. This means that the wedge between the first–best outcome and second–best outcome becomes smaller from the principal's perspective. Consequently, the overconfidence bias aligns the agent's action with the shareholders' interest and thus plays a positive role in the agency relationship.

Another clear cut finding is that the compensation contract's variable component increases the more overconfident are the counterparties. The agent's effort is affected identically by the overconfidence bias. A stronger overconfidence bias enhances the agent's effort. Thus the overconfidence bias provides the agent with an appropriate incentive. In these respects our findings compare favorably to those reported by Gervais, Heaton and Odean (2002) although we solely rely on overconfidence as psychological bias.

The discussion of the principal–agent problem from the principal's perspective extends the common analysis which the moral hazard problem undergoes usually. The comparative static results of the principal's second–best expected utility yields the insight that the principal is better off the more both parties to the contract overestimate the quality of a favorable common signal $\tilde{s} > s_4$. However if poor common information $\tilde{s} < s_3$ becomes available initially the principal profits from a less biased assessment of that information on the part of the counterparties. These results hold irrespective of the actual level of overconfidence. Consequently, these findings might account for the predisposition of shareholders to be more overconfident if good signals on the future state of nature are available and to judge bad signals on the future state of nature less biased that is more rationally. These conclusions have a direct impact on the quality of the employed management. If good signals on the future state of nature are available a more overconfident agent is desirable whereas in case of bad signals on the future state of nature a less biased agent is advantageous from the shareholders' perspective.

Tied together these insights on the impact of the overconfidence bias one might formulate the hypothesis that a company's management is replaced by a less overconfident one if the common information about the future state of nature shifts from good to bad. Although this employment strategy in turn aggravates the moral hazard problem by increasing the agency costs it is beneficial to the shareholders ultimately. In parallel, one should observe higher incentive components in compensation contracts if good signals on the future state of nature are available and vice versa.

The impact of the common signal on the principal–agent relationship ceteris paribus can be summarized quickly as follows. First, the agency costs, the contracted variable compensation as well as the agent's effort are not affected by the common signal at all. Second, a better common signal translates in both a lower fixed compensation of the agent and an increase of the principal's second–best expected utility and vice versa. Since — recall from equation (7) — the fixed compensation is affected negatively by the expected variable compensation which in turn depends positively on the common signal the former observation becomes obvious. The latter observation is straightforward from the perspective of the former result. The amount which the principal decides not to pay flatly to the agent adds to the principal's final wealth. Thus, the principal is better off in terms of expected utility. Consequently, the hypothesis that the contracted fixed compensation is low when good information about the future state of nature is available and vice versa can be explored.

In our approach we modelled explicitly an additional stage of information collection before the compensation contract is written. This allows to incorporate the overconfidence bias with respect to the quality of a common signal on the future state of nature. However, the way in which we incorporated the overconfidence bias is quite common and can be found for example in Kyle and Wang (1997), Daniel, Hirshleifer and Subrahmanyam (1998), and Daniel, Hirshleifer and Subrahmanyam (2001) among others. One might raise the objection that the results basically stem from a variation of both the principal's and the agent's beliefs. Well, this argument is only valid to some extent since we vary the beliefs consistently with an experimentally well–documented persistent behavioral bias, namely overconfidence. In essence, both conditional moments of the future state of nature are affected simultaneously by varying the degree of overconfidence. So, we rely our analysis on a well–founded behavioral bias that drives the counterparties' belief formation. Therefore, our approach is qualitatively and quantitatively different from an arbitrary belief variation.

The following further research avenues are indicated by this paper. Naturally, the hypotheses which we formulated previously can be tested empirically. Based on business cycle forecasts one might check the predisposition of shareholders and managers to be overconfident. Given access to real world data on managerial compensation contracts one could check the predictions of the model concerning the patterns of contracted compensation components on the basis of business cycle forecasts too.

On the theoretical level one might study relaxations of the models' setup with respect to the seemingly somewhat restrictive elements as there are the agent's quadratic effort cost function, the linearity of the sharing rule, the principal's risk neutrality and the distributional assumptions. But one should keep in mind that any relaxation comes at the cost of tractability and/or of solvability, and maybe does not add much to the economic intuition beyond that provided in this paper.

Another quite exciting exercise would be studying the impact of an asymmetric information structure where either the principal or the agent has access to the signal initially and is overconfident with respect to the signal's quality. In these settings the signal would constitute a source of asymmetric information. If the principal had access to the signal exclusively then the compensation contract offered to the agent provides some information about that observed signal. However, the principal should take into account that the agent in turn extracts that information from the sharing rule. If the signal were observed solely by the agent then one must think about a revelation mechanism that forces the agent to report his type — that is the signal — truthfully in order to contract on the signal. All these open issues are left for future research.

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A Proofs

Lemma 1 *The expectation* $\mathrm{E}_\kappa[\tilde{\theta}|\tilde{s}]$ *of the environmental effect* $\tilde{\theta}$ *conditionally on the signal* $\tilde{s}$ *and the according conditional variance* $\mathrm{Var}_\kappa[\tilde{\theta}|\tilde{s}]$ *subject to the overconfidence bias are*

$$\mu_{\theta,\kappa} \equiv \mathrm{E}_\kappa[\tilde{\theta}|\tilde{s}] = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \kappa\sigma_\varepsilon^2}\tilde{s}$$

and

$$\sigma_{\theta,\kappa}^2 \equiv \mathrm{Var}_\kappa[\tilde{\theta}|\tilde{s}] = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\sigma_\theta^2 + \kappa\sigma_\varepsilon^2}.$$

Proof. For a bivariate normally distributed random vector $(\tilde{x}, \tilde{y})' \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{pmatrix}$$

one knows

$$\mathrm{E}[\tilde{y}|\tilde{x}] = \mu_y + \frac{\sigma_{xy}}{\sigma_x^2} \cdot (\tilde{x} - \mu_x) \tag{12}$$

and

$$\mathrm{Var}[\tilde{y}|\tilde{x}] = \sigma_y^2 - \frac{\sigma_{xy}^2}{\sigma_x^2}. \tag{13}$$

Note that the environmental effect $\tilde{\theta}$ and the private signal $\tilde{s}$ are bivariate normally distributed. Straightforward application of (12) and (13) as well as taking into account the definition of the overconfidence bias yields the lemma. This completes the proof. □

A.1 Proof of Proposition 1

Proof. Using (1) and (2) the principal's expected utility results as

$$\mathrm{E}_\kappa[V(\tilde{x} - r_2(\tilde{x}))|\tilde{s}] = (1 - \delta_2)e_2 + (1 - \delta_2)\mu_{\theta,\kappa} - \gamma_2, \tag{14}$$

where $\mu_{\theta,\kappa}$ is given in lemma 1. Applying (1) and (2) yields the agent's final wealth to be $\gamma_2 + \delta_2 e_2 + \delta_2\tilde{\theta} - \frac{1}{2}e_2^2$ which has a normal distribution. The agent's constant absolute risk aversion preferences yield

$$\mathrm{E}_\kappa[U(r_2(\tilde{x}), e_2)] = U\left(\gamma_2 + \delta_2 e_2 + \delta_2\mu_{\theta,\kappa} - \frac{1}{2}e_2^2 - \frac{1}{2}a\delta_2^2\sigma_{\theta,\kappa}^2\right) \tag{15}$$

for the agent's expected utility where $\gamma_2+\delta_2 e_2+\delta_2\mu_{\theta,\kappa}-\frac{1}{2}e_2^2-\frac{1}{2}a\delta_2^2\sigma_{\theta,\kappa}^2$ denotes the agent's certainty equivalent wealth. These results allow an alternative formulation of the second–best program which becomes

$$\begin{aligned} \max_{\gamma_2,\delta_2} \quad & (1-\delta_2)e_2+(1-\delta_2)\mu_{\theta,\kappa}-\gamma_2 && (16) \\ \text{s.t.} \quad & \gamma_2+\delta_2 e_2+\delta_2\mu_{\theta,\kappa}-\frac{1}{2}e_2^2-\frac{1}{2}a\delta_2^2\sigma_{\theta,\kappa}^2 = m && (17) \\ & \delta_2-e_2 = 0. && (18) \end{aligned}$$

Thus the optimal compensation contract that the principal offers the agent leaves the agent exactly with the reservation level of wealth m. This is reflected in the individual rationality constraint (17). The incentive compatibility constraint (18) represents the first order condition of the agent's expected utility maximization with respect to the effort e_2. Note that (18) dictates

$$e_2=\delta_2 \tag{19}$$

for the agent's optimal effort. Plugging (19) into (17) yields

$$\gamma_2=m-\frac{\delta_2^2}{2}(1-a\sigma_{\theta,\kappa}^2)-\delta_2\mu_{\theta,\kappa} \tag{20}$$

for the optimal fixed compensation as function of δ_2. Plugging (20) into (16) allows to solve for the optimal δ_2 from the first order condition of (16) with respect to δ_2. The optimal δ_2 is given in the proposition. The optimal γ_2 as given in the proposition follows from (20) immediately. This completes the proof. □

A.2 Proof of Corollary 1

Proof. Equation (19) yields the corollary. □

A.3 Proof of Proposition 2

Proof. Along the same arguments as in the proof of proposition 1 the reformulation of the first–best program yields

$$\max_{\gamma_1,\delta_1,e_1} \quad (1-\delta_1)e_1 + (1-\delta_1)\mu_{\theta,\kappa} - \gamma_1 \tag{21}$$

$$\text{s.t.} \quad \gamma_1 + \delta_1 e_1 + \delta_1 \mu_{\theta,\kappa} - \frac{1}{2}e_1^2 - \frac{1}{2}a\delta_1^2\sigma_{\theta,\kappa}^2 = m. \tag{22}$$

From the individual rationality constraint (22) we solve for the optimal fixed compensation γ_1 as function of δ_1 and e_1 by rearranging terms. We obtain

$$\gamma_1 = m - \delta_1 e_1 - \delta_1 \mu_{\theta,\kappa} + \frac{1}{2}\left(e_1^2 + a\delta_1^2\sigma_{\theta,\kappa}^2\right). \tag{23}$$

After plugging (23) into (21) we solve for the optimal δ_1 and e_1 from the system of equations

$$\nabla\left(e_1 + \mu_{\theta,\kappa} - m - \frac{1}{2}\left(e_1^2 + a\delta_1^2\sigma_{\theta,\kappa}^2\right)\right) = \mathbf{0}, \tag{24}$$

where $\nabla(\cdot)$ represents the gradient of the principal's expected utility (21) with respect to δ_1 and e_1. The optimal effort results as

$$e_1 = 1 \tag{25}$$

and the optimal δ_1 is given in the proposition. The optimal fixed compensation γ_1 results from plugging the optimal δ_1 and the optimal e_1 into (23) and is given in the proposition too. This completes the proof. □

A.4 Proof of Corollary 2

Proof. Equation (25) yields the corollary. □

A.5 Proof of Proposition 3

Proof. Plugging γ_1 and δ_1 as given in proposition 2 as well as e_1 from (25) into (21) yields the principal's expected utility in the first–best case which amounts to

$$\frac{1}{2} - m + \mu_{\theta,\kappa}. \tag{26}$$

The principal's expected utility in the second–best case is stated in proposition 4. Subtracting the principal's second–best expected utility from the first–best counterpart and collecting terms yields the agency costs as given in the proposition. This completes the proof. □

A.6 Proof of Corollary 3

Proof. Straightforward inspection of the agency costs as given in proposition 3 yields that the agency costs are strictly positive for $0 < \kappa < 1$. This completes the proof. □

A.7 Proof of Proposition 4

Proof. The principal's expected utility in the second–best case results from applying γ_2 and δ_2 as stated in proposition 1 and e_2 from (19) to (16). The second–best expected utility is given in the proposition. This completes the proof. □

A.8 Proof of Proposition 5

Proof. Define

$$s_1 \equiv -\frac{a\sigma_\theta^2\left(3\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(3 - a\sigma_\theta^2\right)\right)}{2\left(1 + a\sigma_\theta^2\right)\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)}. \tag{27}$$

The inspection of the signs of the partial derivatives

$$\begin{aligned}\frac{\partial\gamma_2}{\partial\kappa} = &\frac{\sigma_\varepsilon^2\sigma_\theta^2}{2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^3}\\ &\cdot\Big(a\sigma_\theta^2\left(3\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(3 - a\sigma_\theta^2\right)\right) + 2\tilde{s}\left(1 + a\sigma_\theta^2\right)\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)\Big)\end{aligned} \tag{28}$$

and

$$\frac{\partial\gamma_2}{\partial\tilde{s}} = -\frac{\sigma_\theta^2}{\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)} \tag{29}$$

yields the proposition. This completes the proof. □

A.9 Proof of Proposition 6

Proof. Checking the signs of the partial derivatives

$$\frac{\partial \delta_2}{\partial \kappa} = -\frac{a\sigma_\varepsilon^2\sigma_\theta^4}{\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^2} \tag{30}$$

and

$$\frac{\partial \delta_2}{\partial \tilde{s}} = 0 \tag{31}$$

yields the proposition. This completes the proof. □

A.10 Proof of Corollary 4

Proof. The corollary in turn is a straightforward implication of corollary 1. This completes the proof. □

A.11 Proof of Proposition 7

Proof. The agency costs are given in proposition 3. The partial derivative of the agency costs with respect to the coefficient of overconfidence κ is

$$\frac{a\sigma_\varepsilon^2\sigma_\theta^4}{2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^2} \tag{32}$$

whereas the partial derivative of the agency costs with respect to the signal $\tilde{s}$ is zero. Checking the sign of the first partial derivative yields the proposition. This completes the proof. □

A.12 Proof of Proposition 8

Proof. Define

$$s_2 \equiv -\frac{a\sigma_\theta^2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\right)^2}{2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^2}. \tag{33}$$

The principal's second–best expected utility is given in proposition 4. The partial derivative of the principal's second–best expected utility with respect to the coefficient of overconfidence κ is

$$-\frac{\sigma_\varepsilon^2\sigma_\theta^2\left(a\sigma_\theta^2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\right)^2 + 2\tilde{s}\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^2\right)}{2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\right)^2\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^2} \tag{34}$$

and the partial derivative of the principal's second–best expected utility with respect to the signal $\tilde{s}$ is

$$\frac{\sigma_\theta^2}{\sigma_\theta^2 + \kappa\sigma_\varepsilon^2}. \tag{35}$$

Checking the signs of these partial derivatives yields the proposition. This completes the proof. □

A.13 Proof of Corollary 5

Proof. The signal s_2 is given in equation (33). Define $h(\kappa) \equiv s_2$. Thus, the function h determines a threshold signal $h(\kappa)$ for each coefficient of overconfidence $0 < \kappa < 1$ in the sense of proposition 8. Since

$$\frac{\partial h(\kappa)}{\partial \kappa} = \frac{a^2\sigma_\varepsilon^2\sigma_\theta^6\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\right)}{\left(\sigma_\theta^2 + \kappa\sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^3} \tag{36}$$

is positive generally we define the minimum threshold signal

$$s_3 \equiv \lim_{\kappa \to 0} h(\kappa) = -\frac{1}{2}a\sigma_\theta^2 \tag{37}$$

and the maximum threshold signal

$$s_4 \equiv \lim_{\kappa \to 1} h(\kappa) = -\frac{1}{2}a\sigma_\theta^2 \frac{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)^2}{\left(\sigma_\theta^2 + \sigma_\varepsilon^2\left(1 + a\sigma_\theta^2\right)\right)^2}. \tag{38}$$

Since $a\sigma_\theta^2 > 0$ we have $s_3 < s_4$. Equations (37) and (38) report the independence of s_3 and s_4 of the coefficient of overconfidence κ respectively. This completes the proof. □

B Tables

σ_θ^2	σ_ε^2	a	m
1	1	1	1

Table 1: The table collects the parameters which quantify the state uncertainty as well as the extent of signal noise and the agent's characteristics. These parameters are employed in the figures 3–6 which illustrate the comparative static results of propositions 5–8 as well as the corollaries 4 and 5 in section 4.

C Figures

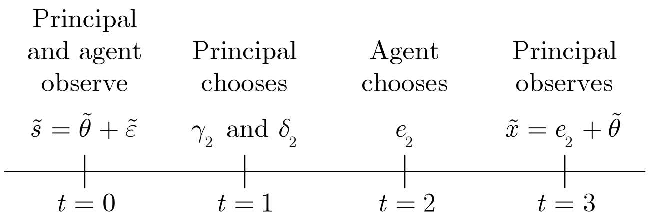

Figure 1: Timing of the players' actions and events in the second–best case

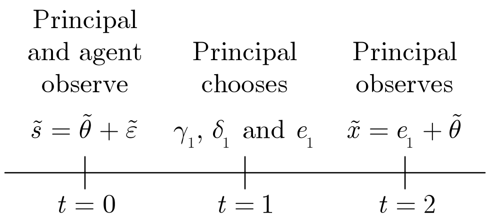

Figure 2: Timing of the players' actions and events in the first–best case

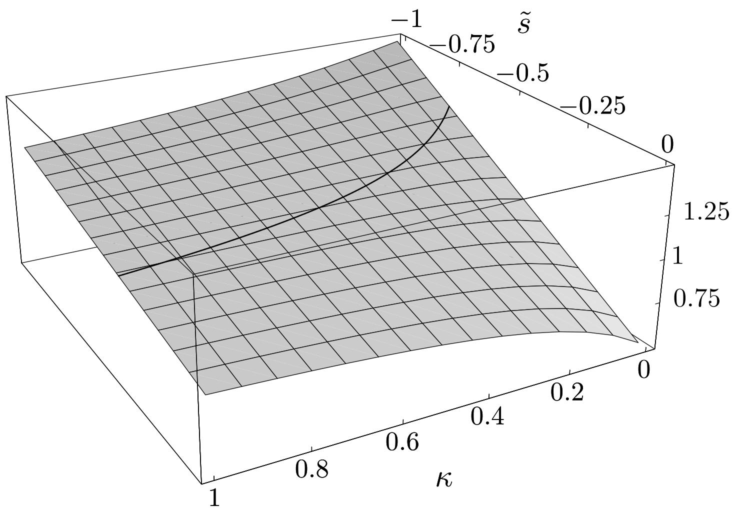

Figure 3: The surface depicts the second–best fixed compensation γ_2 for various combinations of the coefficient of overconfidence κ and the signal $\tilde{s}$. The shape of the surface illustrates that γ_2 always decreases in the signal $\tilde{s}$. The curved line on the surface collects the critical signals s_1 for each level of overconfidence as defined in proposition 5. For good signals — those in the front of the curved line — a stronger overconfidence bias reduces the fixed compensation and vice versa. The according model parameters are summarized in table 1.

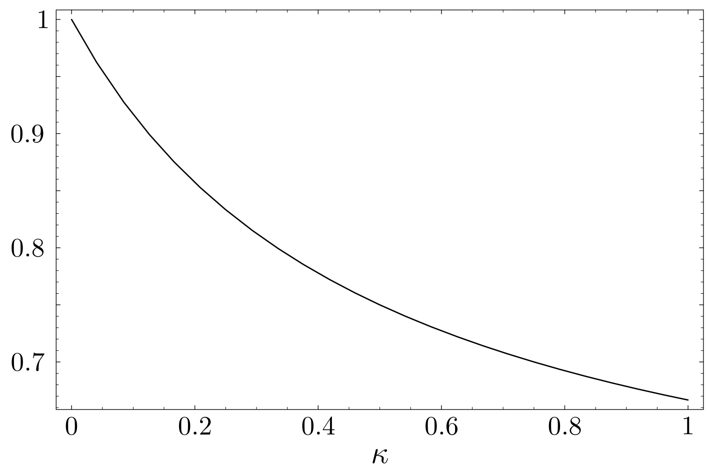

Figure 4: The graph illustrates the impact of the overconfidence bias on the second–best variable compensation component δ_2 according to proposition 5. Since $e_2 = \delta_2$ according to corollary 4 the graph also applies to the second–best effort e_2. The shape of the graph shows that the agent exerts a higher effort and increases his share in the final monetary outcome the more pronounced is the overconfidence bias. The according model parameters are summarized in table 1.

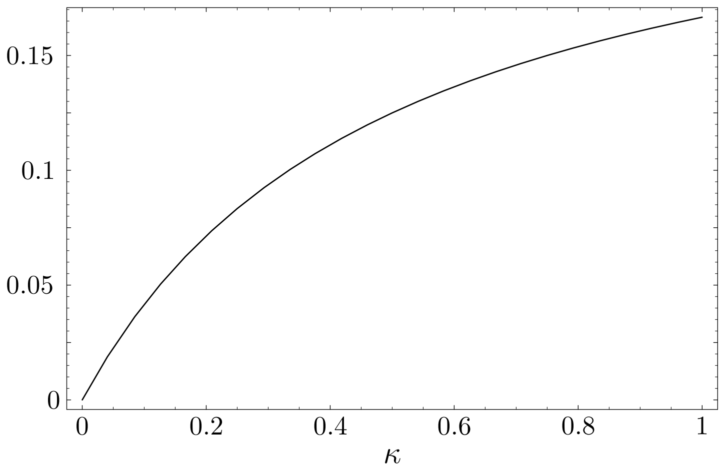

Figure 5: The graph illustrates the impact of the overconfidence bias on the agency costs according to proposition 7. The shape of the graph shows that the agency costs decrease the more pronounced is the overconfidence bias. Consequently, the wedge between the principal's first–best and second–best expected utility becomes smaller the more overconfident are the parties of the agency relationship. The according model parameters are summarized in table 1.

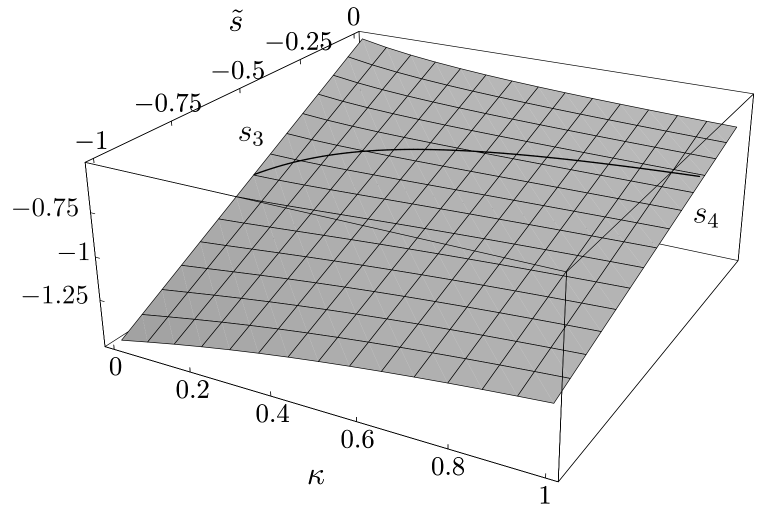

Figure 6: The surface depicts the principal's second–best expected utility for various combinations of the coefficient of overconfidence κ and the signal $\tilde{s}$. The shape of the surface illustrates that the principal's second–best expected utility generally increases in the signal $\tilde{s}$. The curved line on the surface collects the critical signals s_2 for each level of overconfidence as defined in proposition 8. For bad signals — those in front of the curved line — a stronger overconfidence bias decreases the principal's second–best expected utility and vice versa. The respective endpoints of the curved line depict the signals s_3 and s_4 as defined in corollary 5. The according model parameters are summarized in table 1.

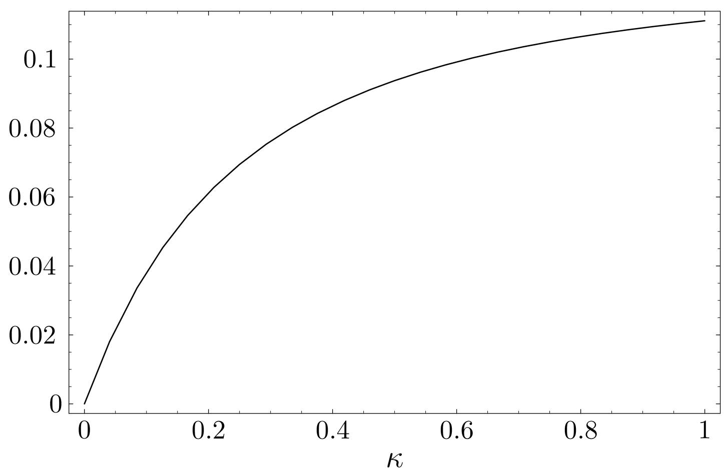

Figure 7: The graph illustrates the impact of the overconfidence bias on the risk premium $\frac{1}{2}ae_2^2\sigma_{\theta,\kappa}^2$ which is part of the second–best fixed remuneration and compensates for the remaining state uncertainty. The shape of the graph exhibits that the risk premium decreases the more pronounced is the overconfidence bias. The according model parameters are summarized in table 1.