Preferencing, Internalization and Dealer Inventory[1]

Laurence Lescourret[2]

Christian Y. Robert[3]

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[2]Assistant Professor, ESSEC Business School and CREST, avenue Bernard Hirsch, 95021 Cergy Pontoise, FRANCE. Tel: 33 (0)1 34 43 33 62, Fax: 33 (0)1 34 43 32 12. E-mail: lescourret@essec.fr

[3]Conservatoire National des Arts et Métiers and CREST, 15 bd G. Péri, 92245 Malakoff, FRANCE; Tel: 33 (0)1 41 17 36 05. Fax: 33 (0)1 41 17 76 66. Email: chrobert@ensae.fr

Abstract

This paper examines how preferencing practice affects the quote-setting behavior of dealers who differ in their inventory. In dealership market, retail trades are generally placed with brokers, who often direct them to a specific dealer regardless of his quotes. In return to this preferenced and captive order flow, this dealer has agreed in advance to match the inside spread. Depending on the market structure (centralized vs. fragmented market), this paper shows how preferencing alters dealers' incentives to narrow market spreads. In a centralized market, preferencing impedes price-competition between dealers. Typically, preferencing leads to wider market spreads and generates higher profits for dealers. In a fragmented market, the impact of preferencing is more ambiguous since it may cause preferred dealers to earn profits, but also to lose money. Actually, preferencing creates risks for the designated dealer in terms of inventory imbalance and price impact. However this market practice generally generates rents for dealers and surprisingly also for the unpreferred dealer, who competes less aggressively given his greater chance to post the best price at equilibrium.

Keywords : Dealership market, preferencing, inventory. **EFM Classification** codes: 360.

1 Introduction

Many securities[1] are traded in more than one markets: for instance, New York-and American Stock-Exchange-listed stocks are frequently traded on regional stock exchanges such as the Cincinnati Stock-Exchange. In the Nasdaq Stock Market, multiple dealers are in competition for the same security. On average, twelve dealers trade the same stock. In this competing environment, as the 1991 report of the NASD Board of Governors underline, "*order flow is a valuable commodity and the competition to attract retail order flow is intense*". In order to encourage brokers to send them aggregated retail orders, dealers use inducements of various kinds known as preferencing arrangements, allowing them to capture order flow. Under preferencing arrangements, brokers send their retail order flow to a *preferred* dealer who has guaranteed in advance to execute orders at the best price, even when that dealer is not quoting it. Orders are actually not exposed to the market. This price matching-like practice is widely used on equity markets (e.g. around 71% of total trades is preferenced in London[2], 30% in Germany, etc.) and it is suspected to sustain anticompetitive prices. For this reason, preferencing receives much attention from regulators.

Opponents argue that preferencing constitutes a captive order flow that impairs dealers' incentives to narrow market spreads on Nasdaq, leading to inferior executions for retail investors. However, in London, Hansh, Naik and Viswanathan (1998, 1999) find that preferenced order receive worse execution than public orders. They also find that the trading profits of preferred dealers are not significantly different from zero. Moreover the authors show that the best-quoting dealers in London still accommodate a significative greater share of public trades volume. Incentives to quote the best price still exist despite pervasive preferencing agreements. Klock and McCormick (2002) obtain similar results on Nasdaq where 75% of the total trades is preferenced. Consequently, it is still an open question whether preferencing impedes competition between dealers and whether it has some deleterious effects on the market performance.

To our knowledge, there exists no theoretical paper that explores systematically the link between preferencing, inventory costs and the quoting behavior of dealers. This paper constitutes a first attempt to model what impact preferencing has on the quote placement strategy of risk-averse dealers. More explicitely, we seek to answer the following questions:

[1] This paper focuses on preferencing in equity markets. But preferencing is also found in options markets.

[2] Figures originate from Hansh, Naik and Viswanathan (1998).

- How does preferencing alter dealers' incentives to compete for public (i.e. unpreferenced) orders?
- How does preferencing impact the formation of bid/ask spreads?

Dealers have an obligation to supply liquidity on their own inventory, regardless of their position which may be far away from the desired level. Each inventory imbalance represents a cost for a dealer, which is reflected in his spread as a compensation for the liquidity service. The effect of inventory on quotes is the main consideration of 'inventory' models (see Stoll (1978) or Ho and Stoll (1981, 1983)). The pure inventory 'paradigm' predicts that (i) dealers with extreme inventory position should post the best quotes ; (ii) an increase in the inventory after a buy trade leads to a decrease in the selling quote to attract trades in the opposite direction (the so called 'inventory' control effect). While numerous empirical studies have proved the relevance of the inventory control effect[3], the empirical significance of the link between inventories and dealers' quoting behavior is less obvious. Hansh *et al.* (1998) suggest that inventory models should reflect some additional market features such as preferencing to test more accurately the link between quotes and inventories. Our paper tries to fill this gap by proposing a new relation between quotes, inventories and preferencing.

To answer the previous questions, we consider two dealers with different inventory positions. We assume that the incoming order flow is partly pre-assignated to one of the dealers, regardless of his quotes. However, dealers still compete to accommodate the public part of the order flow. Preferenced trades clear at the best price in accordance with best execution standards. We model price-competition among preferred and unpreferred dealers in two settings: a centralized market and a fragmented one. In both settings we characterize how dealers alter their quote placement strategies and how the market spreads are affected by the existence of preferencing arrangements. Whether the market is transparent or not, we find that:

- Preferencing has an impact on the reservation price of the preferred dealer, which may impede dealers' incentives to narrow quoted spreads.
- Preferencing leads to wider market spreads in average.

[3]See Lyons (1995), Hansh et al. (1998).

The intuition for the alteration of the reservation price is a reminiscence of the dilemma faced by a monopolist between the cost of providing more liquidity and the profit to execute more shares. For instance, on the sell side, under a certain price, it is more profitable to execute only preferenced orders rather than the total order flow. Since preferencing changes the reservation price of the preferred dealer, it changes his possibility to narrow quoted spreads, which is fully anticipated by his opponent. As a result, it may finally soften price competition between both dealers

If the market is supposed to be transparent, we cast our analysis in the Ho and Stoll (1983)'s framework where dealers are supposed to observe each other inventory position. In this setup we show that under preferencing, it is not necessarily dealers with extreme inventory position that post the best quote. In other words, the link between inventories and quotes predicted by Ho and Stoll (1983) may sometimes be invalidated under preferencing. It may explain the lack of significance of the empirical findings by Hansh et al. (1998) mentionned above.

Then, we study the effects of preferencing on quotes in a market where dealers cannot observe each other's inventory position, as in a fragmented market such as the Nasdaq or the LSE. This alternative analysis is based on Biais (1993) model. Dealers do not know opponents inventory position neither the best price but observe which agent receives a preferenced demand and the scale of this demand. Despite the simplicity of the economic problem, the prices posted by dealers at equilibrium are quite complex. Actually the selling quotes correspond to those arising in a first price auction where bidders are asymmetric. The main problem of asymmetries is the lack of analytical solutions. First, this paper completely characterizes the Pareto-dominant equilibrium for dealers. Then, we adopt a numerical approach and we find that, for some levels of his initial inventory, the preferred dealer may incur *losses* in accommodating his captive preferenced orders. He faces indeed a risk in price execution whenever the market price he matches is below (resp. upper) his selling (resp. buying) reservation price.

This surprising result[4] could explain the zero profit of the preferred dealers on the LSE (see Hansh et al (1998)).

Our paper contributes to a growing literature on the effects of preferencing. Some early

[4]Losses are indeed quite counter-intuitive since (i) the preferred dealer is a monopolist on his captive demand, which generates rents, (ii) his quotations correspond to prices arising in a private-value first price auction where bidders do not post prices under their reservation price, in order to avoid losses.

papers (Chordia and Subrahmanyam, 1995 or Kandel and Marx, 1999) focus on the link between the development of preferencing and price discreteness. They argue that preferencing would disappear as price grids went finer. However, consistent with the prediction of Battalio and Holden (2001a), preferencing has not been eliminated despite decimalization (see the empirical findings of Chung, Chuwonganant and McCormick (2002)). Our results too do not depend on price discreteness.

Our paper analyzes order preferencing as a price matching-like practice. Such practices are suspected to reduce incentive to compete in price and to facilitate coordination between competitors (see Salop (1986)). This intuition is corroborated by several theoretical papers in market microstructure including Godek (1996), Dutta and Madhavan (1997), Kandel and Marx (1997) or Parlour and Rajan (2002). Bloomfield and O'Hara (1998) demonstrate that the negative effect of preferencing can also be found in laboratory financial markets. The framework of our paper differs from the standard assumptions of these models. We consider indeed the inventory position of competing dealers. We also suppose that dealers face two kinds of orders: preferenced orders already pre-assigned to a specific dealer and public orders for which all dealers compete. In this context, our model shows that the unpreferred dealer has less incentives to post aggressive quotes, but the reason comes from his greater chance at equilibrium to execute public orders. The unpreferred dealer anticipates indeed the less favorable position of the preferred dealer who faces an inventory imbalance caused by preferenced orders. Finally, we show that order preferencing enlarges market spreads and may increase dealers' rents as suspected.

Our paper complements also the model of Rhodes-Kropf (1999) who studies the impact on spreads of price improvement in a similar framework. Price improvement is a market practice which consists of filling the order inside the spreads. Rhodes-Kropft shows that dealers offer price improvement to mid-size and large trades because of the negociation power of these customers (generally institutional traders). Whereas his works deals with a market practice concerning institutional trading, we focus on preferencing which is dedicated to small orders from retail investors. Our conclusion is similar to his: preferencing too is a market practice that widens market spreads. However we are not able to conclude anything concerning the overall brokerage service since price-matching also allows retail traders to benefit from speed execution and price guarantee (almost no price disimprovement under such a practice).

This paper is organized as follows. Section 2 describes the institutional framework and the model. Section 3 shows which impact preferencing has on the link between quotes and inventories in a centralized market, whereas section 4 is dedicated to the analysis in a fragmented market. Section 5 explores some possible extensions and section 5 concludes. Proofs are in the Appendix.

2 Framework

2.1 Preferencing Practice and Institutional Concerns

Preferencing principally concerns retail orders and happens through three business arrangements: internalization, payment for order flow and payments in services (clearing, execution or research services, for instance).

Internalization - allowed in United States or in United Kingdom - is considered as self-preferencing. It leads to similar orders' execution: a firm (doing brokerage and market-making within a single entity) can 'internalize' its trades by executing them in-house against its own dealer inventory, provided that trades are executed at a price no worse than the consolidated best bid and offer (the NBBO[5]) in accordance with regulatory best execution standards. Internalization is cost-effective since it allows integrated firms to save costs related to transaction fees and clearing charges.

Concerning 'external' forms of preferencing, they vary according to the market. On the London Stock Exchange, cash payment to purchase order flow is not allowed. London preferencing agreements are 'soft-dollar' (i.e. noncash) arrangements, whereas, on Nasdaq, external preferencing principally happens through payment for order flow. Quantitatively preferencing represents 79% of the trading volume on Nasdaq [Chung *et al.* (2004)] and 71 % on the London Stock-Exchange [Hansh *et al.* (1998)]

Preferencing raises institutional and academics concerns. This practice indeed violates the principle of time priority which stipulates that orders have to be executed by the first dealer quoting the best price. Then, such arrangements forgo the opportunity of orders to interact and transact between the best bid and the best ask (to benefit from any price-improvement). As a

[5]The best market prices are also known as the National Best Bid or Offer: the so-called NBBO.

result, preferencing is argued to increase market spreads and to lead to higher execution costs for investors (see, for instance, Huang and Stoll (1996)[6]). Our model focuses on the impact of preferencing on the formation of bid/ask spread.

2.2 The Basic Setting

Consider the market for a risky asset, whose final cash flow is a normal random variable $\tilde{v}$ characterized by an expected value μ and a variance σ_v^2. There are two types of agents: (i) investors who demand liquidity $(|Q|)$[7] and (ii) dealers who supply liquidity by standing ready to execute incoming market orders $(\pm Q)$ at their bid or ask quote against their own inventory.

Dealers' reservation price and inventory cost

For ease of exposition, we focus on the *sell* side of the market and on the behavior of *two* strategic dealers who compete to post the lowest selling price (or *ask* price) so as to execute the incoming buy order $(+Q)$. Dealers, denoted by D_1 and D_2, are identically risk-averse but differ in their inventory position. In other words, the divergence in dealers' reservation prices is caused by the risk aversion of dealers facing each a more or less unbalanced position, as shown in a seminal paper of Stoll (1978). Adding inventory increases risks in moving the position away from the dealer's preferred level and alters his reservation price.

The reservation price to sell Q shares when a dealer holds an inventory position I_i is denoted by $a_r(I_i, Q)$, $i = 1, 2$. We use the result of Ho and Stoll (1983) to give a simple expression of $a_r(I_i, Q)$[8],

$$a_r(I_i, Q) = \mu + \frac{\rho \sigma_v^2}{2}(Q - 2I_i), i = 1, 2$$

where ρ is the coefficient of risk aversion of dealer D_i $(i = 1, 2)$, $+Q$ is the incoming buy order to accommodate and I_i is dealer D_i's initial inventory. It is common knowledge that I_i is a realization of the random variable $\tilde{I}_i$ uniformly distributed on $[I_d, I_u]$. We will, equivalently, consider that the reservation prices $a_r(I_i, Q)$ are random variable that is distributed according to a uniform distribution on $[a_r(I_u, Q), a_r(I_d, Q)]$.

[6]'We believe that preferencing has increased over time consistent with the increase in spreads, although we do not have direct evidence on this', Huang and Stoll (1996).

[7]'liquidity traders'

[8]This expression can be obtained in a mean-variance framework, as in Biais (1993).

The reservation price may also be interpreted as the average cost of the dealer to produce liquidity. Specifically, a dealer supplies liquidity against his own inventory, bearing risks that entail costs from which that dealer has to be compensated.

Preferenced vs. nonpreferenced Order flows and Best Prices

We make a distinction between two types of order flows: (i) the preferenced order $(+\kappa)$ which is pre-assigned to dealer D_2, and (ii) the public (i.e. unpreferenced) order $(+Q)$ which is not assigned to any dealer. While the preferenced order is routed exclusively to dealer D_2, the public order is attributed to the dealer who quotes the best price (dealer D_1 or D_2). Besides, we define the best offer by $\underline{a} = \min(a_1, a_2)$ where a_1 (resp. a_2) is the ask price posted by dealer D_1 (resp. D_2).

Obligation of execution by a preferred dealer

When the preferred dealer faces an unwanted inventory position, she might send her preferenced order flow to the best-quoting dealer to control her inventory risk. In practice, she must still pay her retail broker for receiving this order flow. Moreover, with fast-moving, narrower spreads due to decimalization, re-routing preferenced orders increases the risk of price-disimprovement and, then, the risk to lose the business relationship with the affiliated broker. Consequently, as the 2001 Nasdaq report underlines, preferred dealers "*rarely act in an agency capacity*". In this model, we do not model the business relationship between the discount broker and his preferred dealer, we simply assume that the potential costs to act as an agent are higher than the costs to act as principal. Consequently, the preferred dealer will not decline the order in re-routing it to the best-quoting dealer (D_1), but she will execute it instead.

The Best Offer

According to the usual standards of the Best Execution duty for retail order flow, the preferred dealer has to execute preferenced orders at the best available price (i.e. the lowest ask price in our model) even when she does not quote it.

The timing of the game and the payoffs of the dealers

At t=1, dealer D_i is endowed with an initial inventory position I_i. At $t = 2$ we suppose that an investor arrives and expresses his desire to buy Q shares. At the same time a broker sends a preferenced order flow $(\kappa > 0)$ to dealer D_2. Dealer D_1 knows that D_2 is committed to accommodate a preferenced order flow κ. At $t = 3$ dealers post simultaneously their ask quotes in order to execute the public order flow Q. The dealer with the lowest ask price executes Q. Besides D_2 executes κ at the lowest ask price whatever her own quote.

Dealers are supposed to have linear preferences over the surplus from trade. Doing so, we limit the impact of risk aversion to the determination of reservation prices[9]. Given that dealer D_1 does not execute any preferenced trade, his trading profit is given by:

$$\pi_1(a_1, a_2, I_1) = \begin{cases} (a_1 - a_r(I_1, Q)) \times Q & \text{if } a_1 < a_2 \\ 0 & \text{if } a_1 > a_2 \end{cases}$$

Dealer D_2's trading profit differs from dealer D_1 since D_2 executes for sure at least the preferenced order flow κ. Then, her trading profit is given by:

$$\pi_2(a_2, a_1, I_2) = \begin{cases} (a_2 - a_r(I_2, Q+\kappa)) \times (Q+\kappa) & \text{if } a_2 < a_1 \\ (\min(a_1, a_2) - a_r(I_2, \kappa)) \times \kappa & \text{if } a_2 > a_1 \end{cases}$$

When dealer D_2 posts the lowest ask price ($a_2 < a_1$), she accommodates the total order flow $(Q+\kappa)$ at that price, given that it is the best offer ($\underline{a} = a_2$). In the opposite case ($a_2 > a_1$), dealer D_2 executes only the preferenced trade κ at the best offer which is the quote posted by her opponent D_1. Because dealer D_2 does not execute the same volume whether she posts the best price or not, it is natural to consider two reservation prices, corresponding each to the quantity to supply: the reservation price to accommodate only the preferenced order flow is $a_r(I_2, \kappa)$ whereas $a_r(I_2, Q+\kappa)$ is the reservation price to execute the total order flow.

Note also that, because dealer D_2 is compelled to execute the preferenced order κ, she might face losses (as soon as $\underline{a} = a_1 < a_r(I_2, \kappa)$) which is consistent with the remark by Kandel and Marx (1999):

"*under preferenced arrangements, a dealer has less control over the trades she has to accommodate because she cannot withdraw from the market by adjusting quotes*".

Let us introduce a specific price termed as the *cutoff* price which leaves the preferred dealer indifferent between the trading profit earned from the execution of the total order flow $(Q+\kappa)$ and the one earned in executing only the preferenced order flow κ.

[9]Dealers' reservation prices depend on the risk-aversion coefficient, which would affect their quoting behavior in the auction models that are analyzed below. For simplicity, however, we ignore the effect of risk aversion on preferences by using the first order linear approximation proposed by Biais (1993) and used by Rhodes-Kropft (1999).

Definition 1 *Let $a_{r,2}^{\kappa}$ be the value of the posted price at which the preferred dealer is indifferent between trading κ shares or $(Q+\kappa)$ shares. That cutoff price $a_{r,2}^{\kappa}$ is defined as the solution of the following equation:*

$$\left(a_{r,2}^{\kappa} - a_r\left(I_2, \kappa\right)\right) \times \kappa = \left(a_{r,2}^{\kappa} - a_r\left(I_2, Q+\kappa\right)\right) \times (Q+\kappa)$$

Suppose that dealer D_2 posts a price below the cutoff price $a_{r,2}^{\kappa}$ and quotes the best price. Then she executes the total order flow. However, straightforward algebra shows that D_2 obtains a lower profit in doing so than in executing only her preferenced order flow at that price. Thus we can state the following result

Result 1 *The preferred dealer has no incentive to quote below the cutoff price.*

Since κ is a captive order flow, the preferred dealer faces the classic monopolist dilemma between cost and volume: accommodating only κ shares at a small cost $\left(a_r\left(I_2,\kappa\right)\right)$ or supplying more $(Q+\kappa)$ at a greater cost $a_r\left(I_2, Q+\kappa\right)$. Below the cutoff price $a_{r,2}^{\kappa}$, the cost to accommodate the total order flow is not offset by the increase in the revenue. Actually the cutoff price is the 'natural' reservation price of the preferred dealer. Consistently, we can re-write D_2's trading profit function:

$$\pi_2\left(a_2, a_1, I_2\right) = \begin{cases} \left(\underline{a} - a_{r,2}^{\kappa}\right) \times \kappa + \frac{\rho\sigma^2}{2} \times \kappa \times (Q+\kappa) & \text{si } a_2 > a_1 \\ \left(a_2 - a_{r,2}^{\kappa}\right) \times (Q+\kappa) + \frac{\rho\sigma^2}{2} \times \kappa \times (Q+\kappa) & \text{si } a_2 < a_1 \end{cases}.$$

with $a_{r,2}^{\kappa} = a_r\left(I_2, Q+\kappa\right) + \rho\sigma_v^2\kappa/2$.

Note that

$$a_{r,2}^{\kappa} > a_r\left(I_2, Q+\kappa\right) > a_r\left(I_2, \kappa\right), \forall\kappa.$$

The ranking is consistent with the monopolistic situation of the preferred dealer on the preferenced order. The cutoff price is strictly greater than the average costs to produce liquidity in both cases whether she supplies liquidity for the preferenced order flow κ or for the total order flow $(Q+\kappa)$.

A benchmark: the competitive case

We next introduce the 'competitive' case (or the No Preferencing case[10]) in which no orders cannot be preferenced. Consequently, the order flow κ is now executed by the best-quoting dealer. Then, the total quantity to accommodate is $(Q+\kappa)$, and dealers' trading profit is such that:

$$\pi_i^{NP}(a_i, a_{-i}, I_i) = \begin{cases} \left(a_i^{NP} - a_r(I_i, Q+\kappa)\right) \times (Q+\kappa) & \text{if } a_i^{NP} < a_{-i}^{NP} \\ 0 & \text{if } a_i^{NP} > a_{-i}^{NP} \end{cases} \quad i = 1, 2.$$

The best price (or the best offer) is $\underline{a}^{NP} = \min\left(a_1^{NP}, a_2^{NP}\right)$.

For the ease of the exposition of the results, we adopt the same notation as Biais (1993) and we note:

$$a_r(I_i, Q) \overset{Def}{=} a_{r,i}.$$

Note also that numerical results illustrated below are computed under the following values for parameters: $\rho = 1$, $\mu = 99.75\$$, $\sigma_v^2 = \frac{1}{10{,}000}$; $Q = 2,500$ shares; $I_d = 0$ and $I_u = 20,000$ shares. Hence, $a_{r,u} = 98\$$ and $a_{r,d} = 100\$$.

Let us discuss these basic assumptions.

2.3 Discussion

2.3.1 Preferencing and Dealers' competition

This section gives some intuitions on the positioning of prices under preferencing and regards to economic concerns on price matching-like practice.

(i) Risk aversion, inventory and preferencing

The positioning of dealers' quotes will depend on the relative ranking of their reservation prices since dealers do not quote under their reservation price. Because of risk aversion, dealers' reservation prices are increasing in the size of the transaction (for a given inventory). The reservation price is consequently higher in average for the preferred dealer who may execute $(Q+\kappa)$ shares and at least κ shares than the reservation price of the unpreferred dealer who trades at most Q shares. Thus, preferencing creates *asymmetric* average reservation prices between both dealers.

[10] We use the subscript NP to identify this 'competitive' case.

(ii) Preferencing as a price-matching practice: advantages and disadvantages

Preferencing is a price matching-like practice. Opponents argue that such practices facilitate cartel pricing by removing the incentive to undercut (see Salop (1986) for industrial organization or Dutta and Madhavan (1998) concerning dealer markets). In our setting, only a part of the total order flow is preferenced. Dealers have still incentives to narrow market spreads to attract the public part of the order flow[11]. Besides preferencing creates inventory risks for the preferred dealer. In case the best offer posted by the opponent is lower than one's own reservation price, matching the best price to execute the preferenced trade may cause the dealer to lose money. We refer to this risk as a risk in *price execution*.

However, preferencing does not free from the main concern of price-matching practice. The preferenced order flow is a captive demand[12]. There are some cases when it is more profitable for the preferred dealer to execute only κ preferenced shares at a smaller cost than a larger order flow $(Q+\kappa)$ at a greater cost. As a result, this monopolistic situation lowers her incentives to compete for the public order flow, which should lead to higher market prices.

Thus the questions raised are the following: may the competitive disadvantage of facing a risk in price-execution be more than offset by the effects of preferencing in favoring higher prices ? May dealers' incentives to execute the unpreferenced order flow be more than offset by the disincentives to face a preferenced demand ? The following paragraph exposes a way to study the impact of preferencing on the positioning of dealers' quotes.

2.3.2 How to capture the impact of preferencing on the bidding behavior of dealers?

In order to gain some intuitions on how preferencing affects the quote-setting behavior of dealers, we use two measures: (i) the probability to execute public orders and (ii) the relative surplus. The relative surplus relates to how close, on average, the ask price posted by dealer i is to his own reservation price. For instance let us denote θ_i a coefficient which measures the relative distance between the ask price posted by dealer D_i $(i=1,2)$ to his reservation price, i.e. $\theta_i\left(a_{r,i}\right)=\left(a_i\left(a_{r,i}\right)-a_{r,i}\right)/a_{r,i}$. The interpretation of this coefficient is straightforward: the

[11]This assumption is corroborated by the empirical findings of Hansh *et al.* (1998) that we mentionned in introduction.

[12]This feature clearly changes from usual economics model where the agreggate demand is still exposed to the whole market and to all competitors.

lower is the coefficient, the more competitive are dealers.

3 Inventory Paradigm and Preferenced Order Flow

In the following sections we analyze how preferencing interacts with dealers' quoting strategy in two different market settings: (i) the canonical one-period model of Ho and Stoll (1983) where dealers are assumed to perfectly observe each other's inventory ; (ii) a fragmented market where dealer do noy observe competitors' inventory position.

3.1 Preferencing and Equilibrium Quoting Strategy in a Transparent Market

We consider a fully transparent market (e.g. a centralized structure) where dealers are able to observe perfectly the inventory positions of their competitors. Without preferencing, Ho and Stoll (1983) show that the dealer with the most extreme inventory posts the best price in equilibrium and the (Nash) equilibrium strategy results in setting the best offer to the second best reservation price. Now, we analyze how preferencing affects this standard result.

In our setting dealer D_2 receives a preferenced order flow κ before trading. Regardless of the competitiveness of her quotes, she executes at least that order flow and her inventory I_2 necessarily shortens of a volume κ. In sum, the preferenced trade acts as an inventory shock arising at date 2 that changes her reservation price to reflect her effective position $(I_2 - \kappa)$: $a_r\left(\left(I_2-\kappa\right), Q\right) = a^{\kappa}_{r,2}$ (see Lemma 1). Since dealer D_1 is assumed to observe the magnitude of the preferenced trade, he anticipates correctly how dealer D_2 will modify her reservation price and her bidding strategy under preferencing agreement.

Theorem 1 *At equilibrium, when both dealers have a chance to post the best price ($a^{\kappa}_{r,u} \leq a_{r,d}$), then the dealer with the lowest reservation price ($\min\left(a_{r,1}, a^{\kappa}_{r,2}\right)$) posts a sell quote just below the second lowest reservation price. In other words, the Nash equilibrium consists of each dealer* *using the following pure strategy*[13]*:*

$$a_1^c = \begin{cases} a_{r,2}^{\kappa} - \varepsilon & \text{if } a_{r,1} < a_{r,2}^{\kappa} \\ a_{r,1} & \text{otherwise.} \end{cases},$$

$$a_2^c = \begin{cases} a_{r,1} - \varepsilon & \text{if } a_{r,2}^{\kappa} < a_{r,1} \\ a_{r,2}^{\kappa} & \text{otherwise.} \end{cases}.$$

where $\varepsilon > 0$ but ε is arbitrarily small.

At equilibrium, when the preferred dealer has no chance to post the best price ($a_{r,d} < a_{r,u}^{\kappa}$), then she quotes her reservation price, i.e. $a_2^c = a_{r,2}^{\kappa}$. Dealer D_1 quotes $a_1^c = a_{r,2}^{\kappa} - \varepsilon$.

Observe that if preferencing was not allowed, dealer D_1 would accomodate the total order flow $(Q + \kappa)$ if and only if his initial position is the longest[14] as in Ho and Stoll (1983). Under preferencing, dealer D_1 executes the public trade even if he is not initially the longest $(I_1 < I_2)$. Actually for a certain positioning of reservation prices $\left(a_{r,1} \in \left[a_r\left(I_2, Q + \kappa\right), a_{r,2}^{\kappa}\right]\right)$, dealer D_2 is not induced to undercut dealer D_1, letting him quoting the best price. Thus, we conclude that preferencing softens price competition between dealers by modifying their capacity to supply liquidity.

Preferenced order flow and dealers' quoting behavior

Preferencing alters the reservation price of the preferred dealer and thus her quoting behavior. Under preferencing, dealer D_2 is less likely to post the best price at equilibrium. Moreover as the volume of preferenced shares rises, she is induced to post quotes closer to her reservation price $a_{r,2}^{\kappa}$ than in the competitive case, i.e. she competes in average 'more' aggressively due to the impact of preferencing on her reservation price[15]. This result could be rather counter-intuitive compared with the arguments of Kandel and Marx (1997) or Dutta and Madhavan (1997) previously mentionned but it has to be moderated by the initial rising of her reservation price.

Preferencing alters also the bidding behavior of the *unpreferred* dealer: dealer D_1 posts higher selling prices. In other words, dealer D_1 quotes in average *less* aggressively which is

[13]We use the subscript c to identify dealers' quotes arising in a fully transparent market, in reference to Biais (1993)'s model that qualifies this transparent market structure as 'centralized'.

[14]$I_1^* = \max\left(I_1, I_2\right) \Leftrightarrow a_r\left(I_1^*, Q + \kappa\right) = \min\left(a_r\left(I_1, Q + \kappa\right), a_r\left(I_2, Q + \kappa\right)\right)$

[15]All the proofs are in the Appendix in the section dedicated to 'Bidding strategy characterization'.

associated with his greater chance to accommodate the unpreferenced order flow. In sum, preferencing is a disincentive to improve the quoted prices for the unpreferred dealer.

To sum up, in a centralized market, preferencing alters definitely the incentives of dealers to narrow quoted spreads because of the alteration of the reservation price of the preferred dealer. However the following questions are still open: what is the impact of such a practice on the expected market spreads, does preferencing necessarily lead to higher profits for dealer D_2 (remind that she faces a price-execution risk in matching the price of her opponent) ? Does it impair or not the expected profit of dealer D_1 who loses the opportunity to accommodate the preferenced trade compared with a 'competitive' situation ?

3.2 Market Performance and Preferencing

In order to analyze the impact of preferenced trade on the overall market performance, we use the competitive case, in which no preferencing is allowed, as a benchmark.

Best offer and preferenced order flow

In equilibrium, the Best Offer is: $\underline{a}^c = \max\left(a_{r,1}, a^{\kappa}_{r,2}\right)$. In order to measure the impact of preferencing agreement on execution costs, we turn to the analysis of the expected Best Offer.

Lemma 1 *The expected best offer denoted by $E\left(\underline{a}^c\right)$ worsens as preferencing is increasing ($\partial E\left(\underline{a}^c\right)/\partial\kappa > 0$). Moreover, the expected best offer is larger than the one which would prevail in the competitive case (No Preferencing allowed): $E\left(\underline{a}^c\right) > E\left(\underline{a}^{c,NP}\right)$.*

Increasing the scale of preferenced order flow increases the best ask price. In a symmetric way, it will decrease the best bid price. Hence, preferencing widens the expected bid-ask spreads. Thus, preferencing in a fully transparent market leads to an increase in transaction costs for investors. This supports the point of view of Huang and Stoll (1997) who argue that the larger execution costs on Nasdaq relative to NYSE are at least partially due to preferencing.

Dealers' expected Profit and Preferencing

To gain some intuitions, preferencing may be decomposed into three effects in this model: (i) the price effect, (ii) the chance effect and (iii) the volume effect. The price effect is obviously linked to the previous lemma: preferencing increases the expected trading profit since it enlarges expected bid/ask spreads. Then, under preferencing the ex ante probability to execute the public order flow increases for the unpreferred dealer and decreases for the preferred dealer, what we called the 'chance' effect. Finally, since the unpreferred dealer cannot compete on the captive order flow, he suffers from a loss in the total expected volume compared with the competitive case (the 'volume' effect).

Lemma 2 *(a) The preferred dealer' s expected profit is always larger under preferencing arrangements, , i.e.* $E\left(\Pi_2^c\right) > E\left(\Pi_2^{NP}\right)$.

(b) Depending on the value of the parameters, there exist cases in which the unpreferred dealer surprisingly expects higher profits when his opponent is preferenced: $E\left(\Pi_1^c\right) \geq E\left(\Pi_1^{NP}\right)$ *when (i)* $Q \geq \left(I_u - I_d\right)/3$ *and (ii) when* $Q < \left(I_u - I_d\right)/3$ *and* $\kappa \geq \underline{\kappa}\left(Q\right)$ *where* $\underline{\kappa}$ *is detailed in the Appendix.*

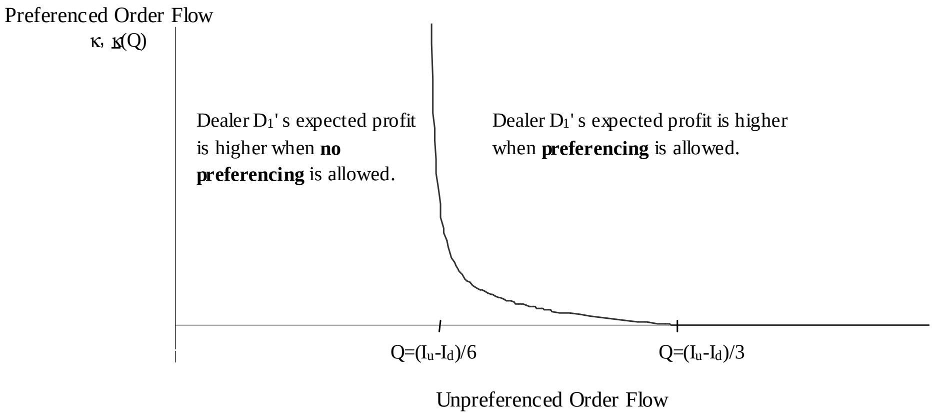


FIGURE 2: A comparison of dealer D_1's expected profit

Preferencing increases the expected profit of the preferred dealer even if she has less control on the price execution of preferenced trades. In this transparent two-dealer market, there is no price-execution risk since the best offer is equal to the second best reservation price and cannot be lower than the cutoff price of the preferred dealer[16]. Dealer D_2 takes fully advantage of the price-matching rule as a source of rents.

Surprisingly, the expected profit of the unpreferred dealer may also be larger in the preferencing case than in the competitive case (see Figure 2). Even if he is suffering from a truncated competition and a loss in trading volume, dealer D_1 may benefit from the increase in spreads (the price effect) and from a larger chance to execute the unpreferenced order flow (the chance effect). So, preferencing may create rents for all dealers.

[16]Analytically, the preferred dealer matches $\underline{a}^c = a_1^c = a_{r,2}^{\kappa} - \varepsilon > a_r\left(I_2, \kappa\right)$.Q.E.D.

These results show that preferencing can significantly affect (i) the market performance since it enlarges market spreads at investors' expense, (ii) dealer's profit that may be larger. These results provide a theoretical support to the experimental findings of Bloomfield and O'Hara (1998). Using laboratory financial markets, their research demonstrates that in a two-dealer market, increasing preferencing increases dramatically market spreads and enriches dealers at the expense of investors. However, they find also that these deleterious effects may be avoided when more than one dealer does not receive preferenced orders. We study whether this is the case in our framework in the next subsection. We first generalize the previous theorem to N dealers. Then we compute the best offer when one *unpreferred* dealer enters the two-dealer market ($N = 3$). Finally we turn to a study of the empirical implications of this model.

3.3 Extension

The previous setting at two dealers can easily be extended to N dealers.

Suppose that N dealers compete to execute a public (i.e. unpreferenced) order flow. Among the N dealers, M dealers have preferencing arrangements where $M \leq N$. It means that each of the M dealers receives a preferenced order flow large of κ_i shares where $\kappa_i \in [0, +\infty[, i = 1, ..., M$.

Following Lemma 1, each preferred dealer will not quote below one's cutoff price. The reservation price of a preferred dealer is given by $a^{\kappa}_{r,i} = \mu + \rho\sigma^2_v (Q - 2(I_i - \kappa_i))/2$, $i = 1, ..., M$. The remaining $(M - N)$ dealers who do not get any preferenced order flow are characterized by the Ho and Stoll (1983)'s reservation price: $a_{r,i} = \mu + \rho\sigma^2_v (Q - 2I_i)/2$, $i = M + 1, ..., N$. Observe that the reservation price of an unpreferred dealer is simply equal to the reservation price of a preferred dealer whose preferenced order flow is zero since $a_{r,i} = a^{\kappa}_{r,i}$ when $\kappa_i = 0$ for $i = M + 1, ..., N$. Consequently, to ease the exposition of the results, we denote by $a^{\kappa}_{r,i}$ the reservation price of any dealer D_i for $i = 1, ..., N$[17].

Corollary 1 *In a transparent market where a part of the total order flow is preferenced to $M \leq N$ dealers, the dealer with the lowest reservation price $\left(\min\limits_{i \in [1;N]} a^{\kappa}_{r,i}\right)$, denoted by D_T, posts the best price and executes the public part of the order flow. At equilibrium, the best-quoting*

[17] Implicitly, when dealer D_i is not preferred, his reservation price is equal to the Ho & Stoll's one, i.e. $a^{\kappa}_{r,i} = a_{r,i}$ for $i = M + 1, ..., N$.

dealer undercuts the second-lowest reservation price and the $(N-1)$ *other dealers quote their own reservation price, i.e.*

$$a_T = \min_{i\in[1;N]\backslash\{T\}} a^{\kappa}_{r,i} - \varepsilon$$
$$a_i = a^{\kappa}_{r,i}$$

for $i \in [1; N] \setminus \{T\}$.

The ranking of the *effective* inventory position $(I_i - \kappa_i)_{i\in N}$ of the dealers determines the ranking of dealers' reservation prices $\left(a^{\kappa}_{r,i}\right)_{i\in N}$ which yields the outcome of the quote-competition between dealers at date 3. Notice that the best-quoting dealer is the dealer with the following inventory position $(I_T - \kappa_T) = \max_{i\in N}(I_i - \kappa_i)$ which is not necessarily the dealer with the most extreme inventory at date 1.

Even if this Corollary is a straightforward generalization of Theorem 1, it allows us to examine how preferencing affects the market competitiveness when more than one dealer is unpreferred. Secondly, this theorem is useful to make a prediction about the relationship between inventories, quotes and preferenced order flow.

3.3.1 The Expected Best Offer in a Three-dealer Market

In this section, we assume that the number of dealers in the market is $N = 3$. In this setting, dealer D_2 receives a preferenced trade $(\kappa_2 > 0)$ whereas the two remaining dealers have no preferenced trades $(\kappa_1 = \kappa_3 = 0)$.

Lemma 3 *When the number of unpreferred dealers goes from one to two, expected bid/ask spreads narrow but remain wider than the competitive spreads:*

$$E\left(\underline{a}^c\right) > E\left(\underline{a}^{c,NP}\right) \text{ for N=3.}$$

In a three-dealer market, the additionnal dealer without preferenced order flow reinforces competition for the public order flow Q among unpreferred dealers. This competition effect decreases the best ask price on average. Symmetrically, it would increase the best bid price on average. Thus, expected market spreads narrow. Actually, the additional unpreferred dealer D_3 provides a competitive force that restores unpreferred dealers' incentives to narrow market spreads in order to attract the unpreferenced order flow. This result is consistent with the experimental finding of Bloomfield and O'Hara (1998) described above.

Figure 3 displays how the expected best offer is improved (lowered) when the number of unpreferred dealers goes from one to two.

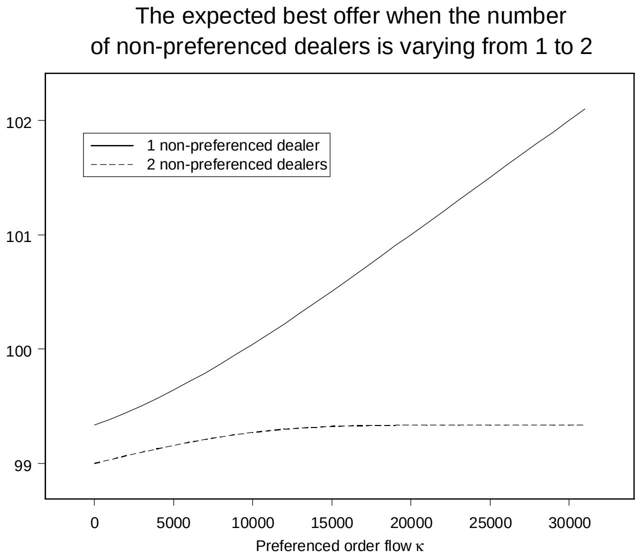


FIGURE 3: Expected best offers, κ varying.

3.3.2 Empirical Implications

Theorem 1 predicts that under preferencing, it is not necessarily the *longest* dealer who posts the best quote. This result invalidates partially the literal prediction of Ho and Stoll (1983)'s model. The aim of this paragraph is to propose a revised version of the link between quoted prices, inventories and preferenced order flow.

The link between inventories and best quotes

As we mention at the beginning of this section, Ho and Stoll (1983) show that the dealer with the most extreme inventory posts the best price and should consequently execute the *public* trades. In Ho and Stoll (1983), dealers' quotes can be expressed as a monotone function of their initial inventory positions. Hansh et al. (1998) deduce that there exists a simple relationship between the relative positionning of dealers' quotes and their relative inventory level. They express this link as follows

$$a_i - \underline{a}^c = \mathcal{F}\left(I_i - I_T\right) \qquad (E1)$$

where the position of the quote a_i posted by dealer D_i relative to the best market price ($\underline{a}^c$) quoted by the longest dealer D_T depends monotonically (though the decreasing function $\mathcal{F}$) on the difference between the level of his inventory I_i relative to that of the best-quoting dealer I_T .

Testing the previous equation on a dataset from the London Stock Exchange, Hansh et al. found that the dealers with extreme inventory position execute only 59% of the incoming public orders and not 100% as predicted by Ho and Stoll (1983). They argue that preferencing may cause this invalidation since Equation $E1$ does not take it into account.

Is there a link between preferenced order flows, quotes and inventories ?

Given Theorem 1, a testable link between inventories, best quotes and preferencing could be expressed as follows:

$$a_i - \underline{a}^c = \mathcal{F}\left(\left(I_i - \kappa_i\right) - \left(I_T - \kappa_T\right)\right) \tag{$E2$}$$

where κ_i and κ_T are respectively the preferenced trades executed by dealer D_i and by the best-quoting dealer D_T. Our model suggests that inventories should be shortened by the scale of preferenced trades in order to test a relation between the positioning of quotes, the level of dealers' inventories and the preferencing practice.

4 Preferencing in a Fragmented Market

In a fragmented market as the Nasdaq or the London Stock Exchange, dealers' bidding behavior will differ from the quoting behavior they would adopt in a centralized market since they cannot observe the inventory positions of their opponents. Actually, the preferred dealer only forms an expectation on the best price at which she could be constrained to execute the preferenced trade in case she does not post the best price. Does the preferred dealer take advantage of this lack of transparency ? What is the impact on the bidding behavior of her opponent ?

Dealer D_i does not know the reservation price of the opponent and forms an expectation on the positioning of quotes. Let the probability that the ask price posted by D_i is the lowest $\Pr\left(a_i < a_{-1}\right)$ where a_i is the price posted by dealer D_i and a_{-i} is that of the opponent D_{-i}. More explicetly, Before trading, the expected profit of dealer D_1 who posts a_1 is:

$$\Pi_1\left(a_1, a_{r,1}\right) = \Pr\left(a_1 < a_2\right) \times \left(a_1 - a_{r,1}\right) \times Q \tag{1}$$

Similarly, dealer D_2 sets her price a_2 given (i) her probability to sell (ii) the volume of preferenced trades. She expects the following profit:

$$\begin{aligned} \Pi_2\left(a_2, a_{r,2}^{\kappa}\right) &= \Pr\left(a_2 < a_1\right) \times \left(a_2 - a_{r,2}^{\kappa}\right) \times \left(Q + \kappa\right) + \Pr\left(a_2 > a_1\right) \times \left(E\left(a_1 \mid a_2 > a_1\right) - a_{r,2}^{\kappa}\right) \times \kappa \\ &+ \frac{\rho\sigma_v^2}{2}\kappa \times \left(Q + \kappa\right) \end{aligned} \tag{2}$$

We begin by characterizing the general case where the equilibrium bidding strategies are numerically investigated, then we turn to the determination of the analytical equilibrium solutions obtained when preferencing is large and when it is not allowed.

4.1 Preferencing and Equilibrium Quotes

We now turn to the detailed analysis of the Bayes-Nash equilibrium that consists of a pair of selling quote functions: $a_1 : [a_{r,u}, a_{r,d}] \rightarrow \mathbb{R}$, $a_2 : \left[a_{r,u}^{\kappa}, a_{r,d}^{\kappa}\right] \longrightarrow \mathbb{R}$. We assume that a_i are strictly increasing functions (see Lebrun (1999) for formal proofs). Then we can define the inverse bidding functions, which are more convenient to analyze. Consequently, we denote $v_1(y)$ and $v_2(y)$ the reservation prices drawn respectively by dealer D_1 and dealer D_2, that lead them to quote y. Note that $v_1 = (a_1)^{-1}$ and $v_2 = (a_2)^{-1}$.

Using the inverse functions, the dealers' profit expressions given by equations (1) and (2) write also:

$$(i) \quad v_1^{(-1)}\left(a_{r,1}\right) \in \arg\max_y \Pi_1\left(y, a_{r,1}\right)$$

with

$$\Pi_1\left(y, a_{r,1}\right) = \bar{F}_{\kappa}\left(v_2\left(y\right)\right) \times \left(y - a_{r,1}\right) \times Q \tag{3}$$

where $\bar{F}_{\kappa}$ is the survivor function: $\bar{F}_{\kappa} = 1 - F_{\kappa}$; and

$$(ii) \quad v_2^{(-1)}\left(a_{r,2}^{\kappa}\right) \in \arg\max_y \Pi_2\left(y, a_{r,2}^{\kappa}\right)$$

with

$$\begin{aligned} \Pi_2\left(y, a_{r,2}^{\kappa}\right) &= \bar{F}\left(v_1\left(y\right)\right) \times \left(y - a_{r,2}^{\kappa}\right) \times \left(Q + \kappa\right) + \left(1 - \bar{F}\left(v_1\left(y\right)\right)\right) \times \left(E\left(a_1 \mid y > a_1\right) - a_{r,2}^{\kappa}\right) \times \kappa \\ &+ \frac{\rho\sigma_v^2}{2}\kappa \times \left(Q + \kappa\right). \end{aligned} \tag{4}$$

where $\bar{F} = 1 - F$.

Technically, prices arising in this context correspond to those arising in a Dutch auction or, equivalently, in a first-price auction (FPA) (see also Biais (1993) or Rhodes-Kropf (2004)). In our set up (unknown reservation prices and preferencing), the equilibrium quotation strategies are quite complex because dealers expected profits from trade are different and because the supports of dealers reservation prices are not identical. Indeed, the distribution support for the reservation price of the preferred dealer is 'shifted' to the right compared with the distribution'support for dealer D_1's reservation price. That is, the reservation price of the unpreferred dealer D_1 is distributed uniformly on $[a_{r,u}, a_{r,d}]$ whereas the preferred dealer's cutoff price is distributed uniformly on $\left[a_{r,u}^{\kappa}, a_{r,d}^{\kappa}\right] = [a_{r,u} + \rho\sigma_v^2\kappa, a_{r,d} + \rho\sigma_v^2\kappa]$. Consequently, we distinguish F the uniform cumulative distribution function (c.d.f.) of dealer D_1' s reservation price on $[a_{r,u}, a_{r,d}]$ from F_κ the uniform c.d.f. of dealer D_2' s cutoff price on $\left[a_{r,u}^{\kappa}, a_{r,d}^{\kappa}\right]$. To sum up, preferencing creates a double asymmetry: (i) reservation prices are asymmetrically distributed ; (ii) expected profit functions are also asymmetric (see Equations (1) and (2)). It is well-known that these asymmetries preclude analytical solutions for FPA (see Lebrun (1999), Castillon (2000) and Maskin and Riley (2000)). Thus we use a numerical approach to derive equilibrium bidding strategies. In our setting, it is, however, possible to characterize analytical equilibrium strategies in two cases: (i) when the preferenced order flow is so large that the preferred dealer cannot post the best price at equilibrium ($\kappa \geq 2(I_u - I_d)$) and (ii) in the competitive benchmark where no preferencing is allowed (NP).

4.1.1 Case 1: the Equilibrium when Preferencing is Small $\kappa < 2(I_u - I_d)$

Dealers' bidding strategies have the same support $\left[a^{\inf}, a^{\sup}\right]$. Notice that, on this support, both dealers have a strictly positive probability to execute public orders.

The lower bound $a^{\inf}$ is the lowest possible ask price quoted by a dealer and it is defined such that $\bar{F}_\kappa\left(v_2\left(a^{\inf}\right)\right) \times \bar{F}\left(v_1\left(a^{\inf}\right)\right) = 1$. Intuitively, if dealer D_1 should post a lower price than dealer D_2 ($a_1^{\inf} < a_2^{\inf}$), then he could quote any price $a_1 \in \left[a_1^{\inf}, a_2^{\inf}\right[$ and be sure to post the best price. However, this strategy is strictly dominated by $\left(a_1 + a_2^{\inf}\right)/2$. Hence, it cannot be an equilibrium by elimination of iterated dominated strategy (the same holds in case when $a_1^{\inf} > a_2^{\inf}$). As a result, dealers' best reply must have the same lower bound $a^{\inf}$.

The upper bound $a^{\sup}$ is the largest possible ask price quoted by a dealer who has a strictly positive probability to execute the public order flow. This upper bound is defined such that $\bar{F}_\kappa\left(v_2\left(a^{\sup}\right)\right) \times \bar{F}\left(v_1\left(a^{\sup}\right)\right) = 0$. Using the same argument as before, we conclude that dealers must quote no more than the largest possible ask price to get a chance to execute public orders.

Theorem 2 *Assume that both dealers have a chance to post the best price ($\kappa < 2\left(I_u - I_d\right)$).*
(i) The equilibrium inverse bidding functions v_1 and v_2 are solutions to the following pair of differential equations:

$$\frac{-\bar{F}_\kappa'\left(v_2\left(y\right)\right)}{\bar{F}_\kappa\left(v_2\left(y\right)\right)} \times v_2'\left(y\right) = \frac{1}{y - v_1\left(y\right)} \tag{5}$$

$$\frac{-\bar{F}'\left(v_1\left(y\right)\right)}{\bar{F}\left(v_1\left(y\right)\right)} \times v_1'\left(y\right) = \frac{\left(1 + \kappa/Q\right)}{y - v_2\left(y\right)} \tag{6}$$

(ii) If $a_{r,d} \leq a^{\sup} \leq \frac{a_{r,d}+a^\kappa_{r,d}}{2}$, there exists an equilibrium.

Observe that when $a^\kappa_{r,2} > a^{\sup}$ dealer D_2 can never post the best price and she quotes her cutoff price: $a_2 = a^\kappa_{r,2}$. Note also that the equilibrium is not necessarily unique and that the lower bound $a^{\inf}$ is endogenously determined by the upper bound $a^{\sup}$

Among the multiplicity of equilibria, we use the Pareto-dominance criterion to select one of them. This criterion is defined as follows: the equilibrium denoted by the subscript $^{(2)}$ is Pareto-dominant under the initial conditions$^{(2)}$ if for each equilibrium $^{(1)}$ under other initial conditions$^{(1)}$, both following inequalities hold:

$$\begin{aligned} \Pi_1^{(1)}\left(a_1, a_{r,1}\right) &< \Pi_1^{(2)}\left(a_1, a_{r,1}\right) \text{ for each } a_{r,1} \in \left[a_{r,u}, a_{r,d}\right], \\ \Pi_2^{(1)}\left(a_2, a^\kappa_{r,2}\right) &< \Pi_2^{(2)}\left(a_2, a^\kappa_{r,2}\right) \text{ for each } a^\kappa_{r,2} \in \left[a^\kappa_{r,u}, a^\kappa_{r,d}\right]. \end{aligned}$$

Proposition 1 *The unique Pareto-Dominant equilibrium is obtained when the initial condition is such that $a^{\sup} = (a_{r,d} + a^\kappa_{r,d})/2$.*

(The proof of this Proposition has been omitted but is available upon request or in Lescourret and Robert (2002).)

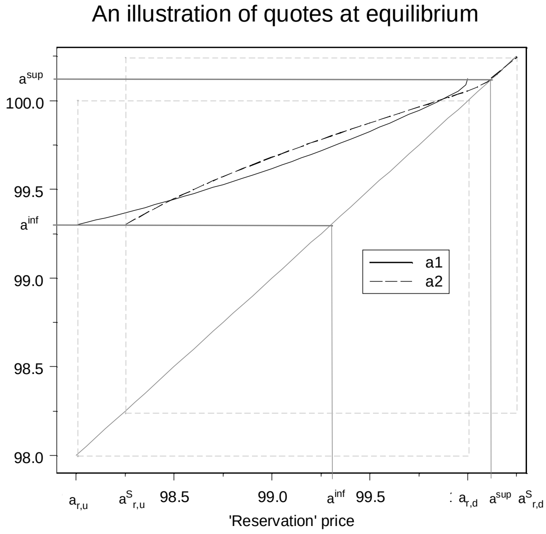


FIGURE 4: An illustration of quotes at equilibrium

It is worth stressing two facts about the equilibrium described by the system of the ordinary differential equations (4) and (5) and by the initial condition of Proposition 1. It is impossible to get an analytical solution to this asymmetric equilibrium (at least we have not been able to find one). Second, given that preferencing makes dealers asymmetric, they will in general have different bidding strategies. We will further analyze *numerical* solutions of the ODE system. However, there are two cases in which we can dispense from numerical solutions (i) when preferencing is so large that the preferred dealer cannot post the best price $(\kappa \geq 2\,(I_u - I_d))$ (ii) when no preferenced order flow is allowed as in a competitive situation.

4.1.2 Case 2: the Equilibrium when Preferencing is Large $(\kappa \geq 2\,(I_u - I_d))$

When $\kappa \geq 2\,(I_u - I_d)$, Theorem 2 does not apply. However, we can characterize the equilibrium in closed form.

Proposition 2 *Assume that dealer D_2 can never post the best price at equilibrium ($a^{\kappa}_{r,u} \geq a^{\sup}$ or $\kappa \geq 2\,(I_u - I_d)$). In this case, she quotes a selling price equal to her reservation price: $a_2 = a^{\kappa}_{r,2}$, and dealer D_1 posts $a_1 = a^{\kappa}_{r,u}$.*

In this case, the portion of the captive order flow is so large that it precludes any price competition between dealers. When the unpreferred dealer does not quote more than the lowest price posted by the preferred dealer, he is sure to post the best price and to execute public trades.

Link between the quote selling behavior of dealers equilibrium when preferencing is small (Case 1: $\kappa < 2\left(I_u - I_d\right)$) and when it is large (Case 2: $\kappa \geq 2\left(I_u - I_d\right)$)

This paragraph aims at giving some insights on the . As it may be proved, the initial condition on the upper bound $a^{\sup}$ determines the equilibrium (the lower bound $a^{\inf}$ is indeed endogenously determined by $a^{\sup}$). Given that the upper bound $a^{\sup}$ increases when preferencing increases, $a^{\inf}$ is also varying with preferencing as Figure 5 depicts. When $\kappa = 2\left(I_u - I_d\right)$ equilibria defined in Theorem 2 degenerate ($a^{\inf} = a^{\sup} = a^{\kappa}_{r,u}$) and are now characterized analytically by Proposition 2.

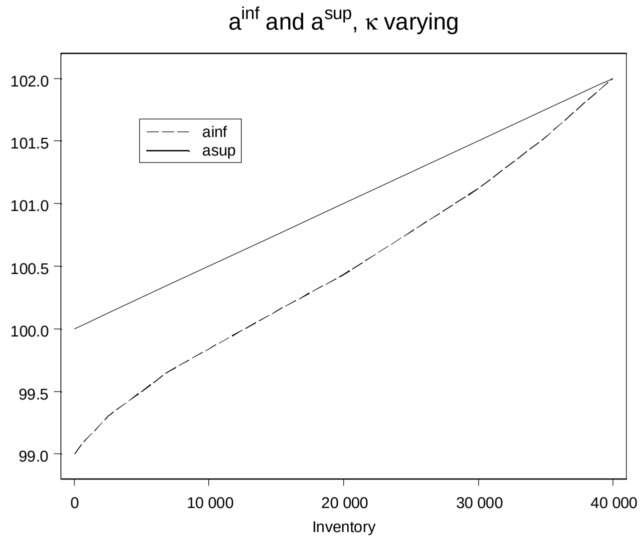


FIGURE 5: Evolution of the quotes' support $\left[a^{\inf}, a^{\sup}\right]$, κ varying.

4.1.3 The Competitive Case (Biais, 1993)

Now, we turn to the characterization of the competitive equilibrium (our benchmark) where the total order flow $(Q+\kappa)$ is public. In this case dealers draw their reservation price $a_r\left(I_i, Q+\kappa\right)$ from the same probability distribution F on a common support $\left[a_r\left(I_u, Q+\kappa\right), a_r\left(I_d, Q+\kappa\right)\right]$. Consequently, dealers are symmetric when there is no preferencing, that is $v_1 = v_2 = v$ and $a_1 = a_2 = a_{NP}$.

Then, the system of ODE described in Theorem 2 (equations (5) and (6)) simply writes:

$$\frac{-\bar{F}'(v(y))}{\bar{F}(v(y))} \times v'(y) = \frac{1}{y - v(y)}, \tag{7}$$

subject to the following boundary conditions:

$$a_{NP}^{\inf} = \frac{a_r(I_u, Q+\kappa) + a_r(I_d, Q+\kappa)}{2} \quad \text{and} \quad a_{NP}^{\sup} = a_r(I_d, Q+\kappa). \tag{8}$$

It is easy to verify that the symmetric equilibrium characterized by the ordinary differential equation (7) and by the initial condition (8) is unique. Furthermore, there exists an analytical solution, which is identical to the equilibrium described in Biais (1993, Corollary 1). Dealers post sell quotes which are equal to the sum of their reservation price and a mark-up: $a_{NP}(a_r(I_i, Q+\kappa)) = a_r(I_i, Q+\kappa) + \gamma(a_r(I_i, Q+\kappa))$, $i = 1, 2$. This quoting strategy shows that dealers post an ask price strictly above their reservation price. The mark-up $\gamma(a_r(I_i, Q+\kappa))$ allows them to make non zero profit.

In this symmetric case, the sell quotes and the mark-up are *linear* in the reservation price, as follows:

$$\begin{aligned} a_{NP}(a_r(I_i, Q+\kappa)) &= \frac{a_r(I_i, Q+\kappa) + a_r(I_d, Q+\kappa)}{2}, \\ \gamma(a_r(I_i, Q+\kappa)) &= \frac{a_r(I_d, Q+\kappa) - a_r(I_i, Q+\kappa)}{2} \geq 0. \end{aligned}$$

This mark up also writes:

$$\gamma(a_r(I_i, Q+\kappa)) = E\left[a_r(I_{-i}, Q+\kappa) - a_r(I_i, Q+\kappa) \mid a_r(I_{-i}, Q+\kappa) - a_r(I_i, Q+\kappa) > 0\right].$$

Actually, dealer D_i estimates how far upper his own reservation price the opponent's reservation price is on average and he submits a selling price equal to this amount. In this competitive case the dealer who executes the incoming order flow is the agent with the most extreme inventory. This result is consistent with the prediction of Ho and Stoll (1983)'s model: without preferencing, the dealer with the longest inventory position posts the best price at equilibrium.

4.2 The Impact of Preferencing on the Quotes Placement

In order to analyze how order preferencing alters the way to bid of dealers, we present first a numerical investigation on (i) the probability to post the best price and (ii) the relative surplus. Then, we explain qualitatively the numerical results obtained.

4.2.1 Preferencing and Bidding Strategy of the Unpreferred Dealer

This section details the quote-setting behavior of dealers in both market structures (centralized and fragmented).

As numerical results illustrate on Figure 6, the probability that the unpreferred dealer executes the unpreferenced order flow increases when the magnitude of the preferenced order flow rises (it grows from 1/2 to 1). Given that he has more chance to execute public orders as preferencing rises, dealer D_1 competes less aggressively and earns a larger surplus (see Figure 7).

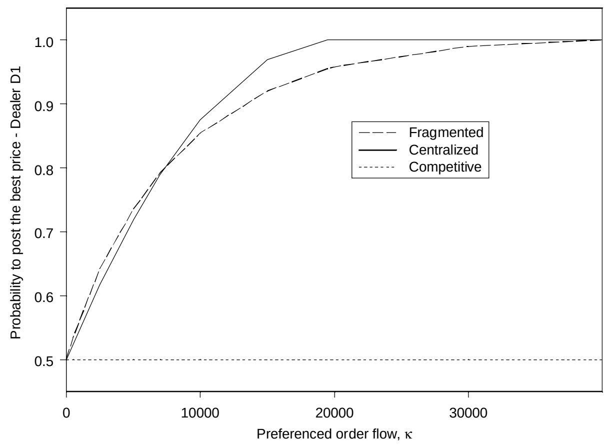


FIGURE 6: Probability that D_1 posts the best price

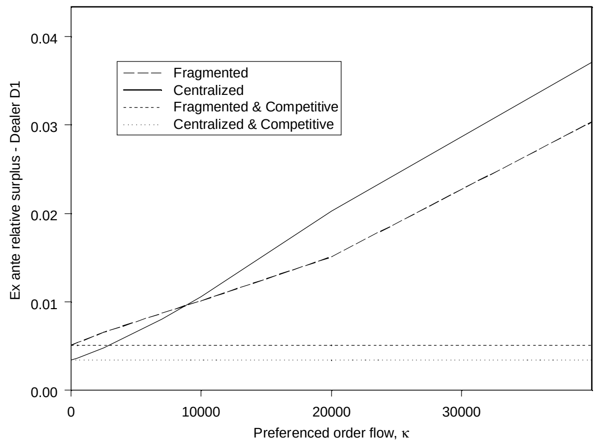


FIGURE 7: Relative surplus of dealer D_1

4.2.2 Preferencing and Bidding Strategy of the Preferred Dealer

Now, we turn to the analysis of the bidding behavior of the preferred dealer. When order preferencing becomes larger, dealer D_2 is less likely to draw a low reservation price and she has less and less chance to post the best price (it declines from 1/2 to zero when preferencing is large). In other words her quoting aggressiveness decreases since she is less in average on the best ask price (see Figure 8). Intuitively, because of her weaker probability to execute the unpreferenced trade, dealer D_2 is expected to post prices with a lower markup. Surprisingly, the relative surplus of dealer D_2 is *not monotonous* with the probability to post the best price. For instance, let us suppose that her inventory position is 15,000 shares, then her relative surplus is $\theta_2\left(a_{r,2}^{\kappa}\right) = 0.75$ when $\kappa = 0$, $\theta_2\left(a_{r,2}^{\kappa}\right) = 0.79$ when $\kappa = 500$ and $\theta_2\left(a_{r,2}^{\kappa}\right) = 0.83$ when

$\kappa = 2,500$. However, θ_2 decreases to 0.72 when $\kappa = 7,000$ (see an illustration on Figure 9).

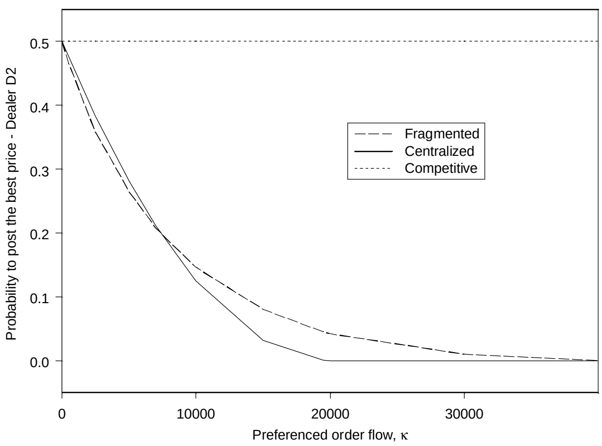


FIGURE 8

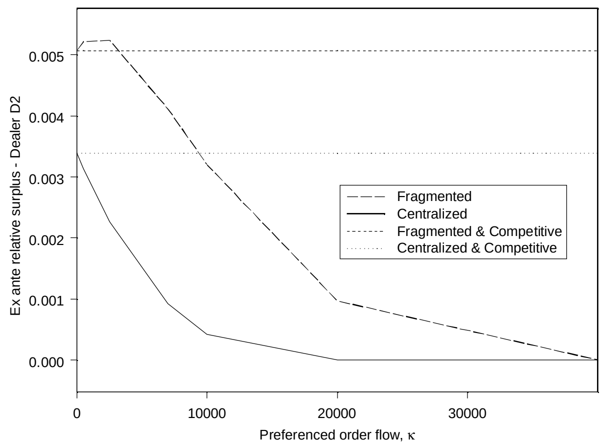


FIGURE 9

Actually, the preferenced order flow creates two types of asymmetry which generate opposite bidding behavior for the preferred dealer:

(i) on one side, it forces her to post lower ask price due to her lower probability to execute the public trade. With preferenced orders she is indeed less likely to draw a low reservation price;

(ii) on the other side, the private order flow creates a rent for the preferred dealer that destroys her incentive to compete in prices.

In other words preferencing changes (i) the supports of dealers reservation prices and (ii) the distribution of the probability function which changes the degree of price-competition between dealers. Unlike the previous works related to asymmetric auctions, this paper mixes two kinds of asymmetry which generate ambiguous bidding behavior for the preferred dealer. Specifically the combination of both asymmetries invalidates any condition related to the Conditional Stochastic Dominance. Then it is not easy to compare analytically dealers' bidding behavior as in Maskin and Riley (2000). It explains however the puzzling quoting behavior of dealer D_2, whose quoting-setting is not monotonous with the probability to post the best price. In conclusion, numerical examples indicate that even if the preferenced order flow has no clear impact on dealers D_2 's incentive to compete on the quoted prices, it deletes however her competitor's incentive to set narrower spreads (actually, we do not find numerical examples that invalidate this result).

4.3 Comparisons with a Centralized Market

Now, we analyze how the quote transparency alters market spreads and dealers profit regarding preferencing.

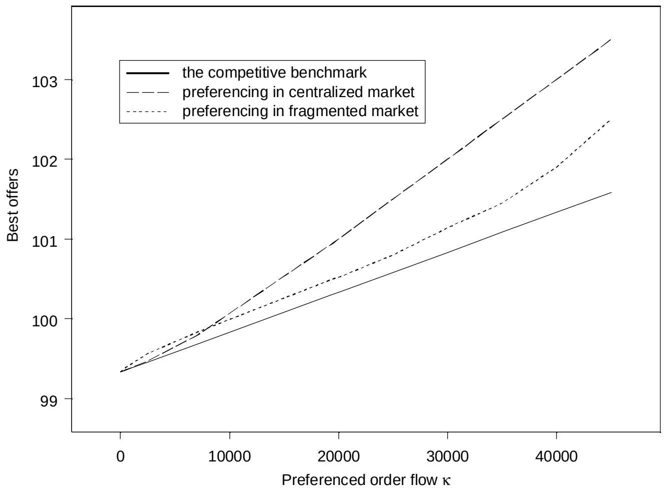
4.3.1 The Expected Best Offer

Result 2 *Under preferencing, the expected best offer in a centralized market differs from the best offer arising in a fragmented market. When preferencing is large ($\kappa \geq 2(I_u - I_d)$), a fragmented market offers better market prices than a centralized market, i.e.*

$$E(\underline{a}) < \frac{a^{\kappa}_{r,d} + a^{\kappa}_{r,u}}{2} = E(\underline{a}^{c})$$

where $E(\underline{a}) = E(a_1) = a^{\kappa}_{r,u}$.

When no preferencing is allowed (our competitive benchmark), the expected best offer in a centralized market and in a fragmented market are shown to be the same[18]. However preferencing creates asymmetries that invalidates this result (see an illustration in Figure 10). It is also a well-known that asymmetries prevent the 'revenue-equivalence theorem' to prevail in the theory of auctions.



[18]This result comes from the well-known 'revenue-equivalence theorem' obtained in the theory of auction as Biais (1993) explains in the Proposition 4 of his model.

FIGURE 10: Expected best offers with and without preferencing, quote transparency varying[19]

Remind that preferencing deteriorates the best offer in a centralized market. This result is also verified numerically in a fragmented market. Numerically, we find that wheareas the impact of small preferenced order flow is ambiguous, large preferenced order flow harms more centralized market than fragmented market, which is consistent with the experimental finding of Kluger and Wyatt (2002).

4.3.2 Preferencing, Market Structure and Dealers' expected profit

A. The unpreferred dealer's expected profit

Result 3 *In a two-dealer market where preferencing is large ($\kappa \geq 2(I_u - I_d)$), the expected profit of the unpreferred dealer is higher in a centralized structure than in a fragmented market,*

$$E(\Pi_1) = \left(\rho\sigma_v^2\kappa - \frac{(a_{r,d} - a_{r,u})}{2}\right) \times Q \leq \rho\sigma_v^2\kappa \times Q = E(\Pi_1^c).$$

When preferencing is small ($\kappa < 2(I_u - I_d)$), it is still numerically validated.

When preferencing is large, the expected profit of the unpreferred dealer is lower in a fragmented market than in a centralized market since market spreads are smaller. For example, when $\kappa \geq 2(I_u - I_d)$ (Case 2), the unpreferred dealer is sure to post the best price regardless of the market structure but behaves differently. In a transparent market, he quotes a best offer $\underline{a}^c$ that is such that $a_{r,u}^\kappa \leq \underline{a}^c \leq a_{r,d}^\kappa$, whereas in the fragmented market, dealer D_1 quotes $\underline{a} = a_{r,u}^\kappa$ (Proposition 2) that is always lower than his price $\underline{a}^c$ in the centralized market.

However, even if the expected profit of dealer D1 are lower in a fragmented market with preferencing, there exist some parameters values for which it is still higher than in a competitive market: $E(\Pi_1) \geq E(\Pi_1^{NP})$ for $\kappa \geq \underline{\kappa}^{\text{fr}}(Q)$ where it is numerically showed that $\underline{\kappa}^{\text{fr}}(Q) > \underline{\kappa}(Q)$.(Lemma 2 and Figure 2).

[19] Whether the 'competitive' market is centralized or fragmented, remind that the expected best offers are equal : $E(\underline{a}_{NP}) = E(\underline{a}_{NP}^c)$ (Revenue-equivalence theorem).

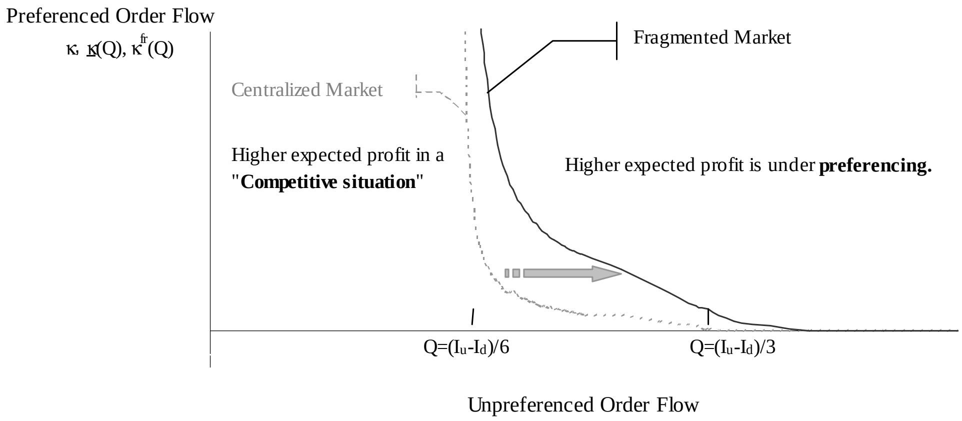


FIGURE 11: The unpreferred dealer's expected profit under preferencing vs. no preferencing allowed.

B. The preferred dealer

As discussed in Section 2, under preferencing agreements, the preferred dealer faces two risks. First, there is an inventory risk since the preferenced trade must be executed whatever her inventory position is. For that risk, she is compensated by an additional risk premium since her effective reservation price is higher under preferencing than under no preferencing (competitive benchmark): $a^{\kappa}_{r,2} > a_r\left(I_2, Q+\kappa\right)$. Second, there is also a risk in price execution. When she does not post the best price, dealer D_2 matches the best price which may be lower than her reservation price for clearing κ shares $\left(a_r\left(I_2,\kappa\right) < \underline{a} \in \left[a^{\inf}, a^{\sup}\right]\right)$.

Result 4 *In a fragmented market, the preferred dealer may incur losses in executing her captive order flow.*

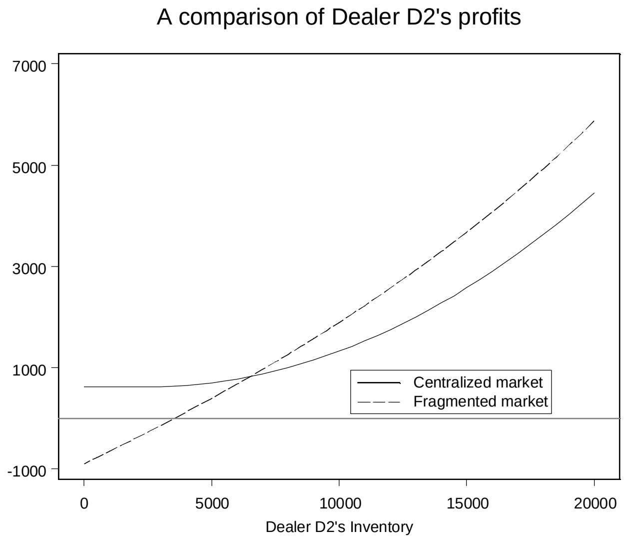


FIGURE 12: Dealer D_2 profit and loss, for I_2 varying

This result is consistent with the empirical evidence of Hansh, Naik and Viswanathan (1999) who find that preferred dealers on the LSE make zero profits over all trades. Losses could even be bigger if we now assume that the unpreferred dealer cannot observe whether a preferenced order flow is received or not by her opponent. Then, the best price to match is more competitive which makes the price execution risk rising for the preferred dealer (see Lescourret and Robert (2003)).

In which market structure is order preferencing the more profitable for the preferred dealer ?

Result 5 *In a two-dealer market, when preferencing is large ($\kappa \geq 2(I_u - I_d)$), the preferred dealer expects higher profits in a centralized market than in a fragmented market:*

$$E\left(\Pi_2^c\right) > E\left(\Pi_2\right) = \frac{\rho\sigma_v^2\left(\kappa + Q\right) - \left(a_{r,u} - a_{r,d}\right)}{2} \times \kappa > 0.$$

Observe that we numerically find that even if the preferred dealer may incur losses, she expects a higher profit when preferencing is allowed than when it is not allowed (competitive benchmark) as in a centralized market: $E\left(\Pi_2\right) > E\left(\Pi_2^{NP}\right)$. We also numerically find that there exist some cases where the expected profit of preferred dealer is higher in a fragmented market than in a centralized market even if she may face some losses.

Note that when preferencing is large ($\kappa \geq 2(I_u - I_d)$), even if the preferred dealer has no chance to accommodate the unpreferenced order flow due to a too large preferenced order, she secures however a positive expected profit due to the lack of competition that leads her opponent to post the highest quote.

5 Internalization

This section examines how internalization influences dealers' incentives to compete and finally how it impacts market spreads.

We consider that dealers compete for the public order flow Q, and that dealer D_2 may internalize order flow denoted κ. Under internalization, the dealer decides whether to match the best price or pass the order to the dealer with the best quote. Hence, dealer D_2' trading profit is such that:

$$\pi_2(I_2) = \begin{cases} \max\ (0, (\min(a_1, a_2, a_r(I_2, \kappa)) - a_r(I_2, \kappa)) \times \kappa) & \text{if } a_2 > a_1 \\ (a_2 - a_r(I_2, (Q + \kappa))) \times (Q + \kappa) & \text{if } a_2 < a_1 \end{cases},$$

whereas dealer D_1' expected profit is such that:

$$\pi_1(I_1) = \begin{cases} 0 & \text{if } a_1 > a_2 \\ (a_u - a_r(I_u, Q)) \times Q & \text{if } a_r(I_2, \kappa) < a_1 < a_2 \\ (a_u - a_r(I_u, (Q + \kappa))) \times (Q + \kappa) & \text{if } a_1 < a_r(I_2, \kappa) \end{cases}$$

We can easily verify that in a two-dealer market, dealer D_2 always internalizes order flow κ. However, in a three dealer market where only one dealer D_2 can internalize, dealer D_2 passes κ to the best quoting dealer with a strictly positive probability.

Work under progress

6 Conclusion

This paper investigates how preferencing alters the quoting behavior of two dealers with different inventory position. Dealers are supposed to undercut each other's quote to accommodate an incoming order flow. However, we assume that part of this order flow is already pre-assigned to one of the two dealers, regardless of his posted quotes. In accordance with best execution standards, that preferred dealer has guaranteed in advance to match the best price in executing the preferenced order flow. The best price to match results however from the price-competition with his opponent to attract the unpreferenced part of the order flow. In our framework, preferencing is analyzed as a price-matching practice which generates inventory risks for the preferred dealer. We find that these risk may entail some losses for that agent. However, consistent with institutional concerns on price-matching like practices, we show that preferencing generates negative effects on the market performance since it widens market spreads despite dealers' incentives to undercut to attract the unpreferenced order flow. Preferencing softens indeed price-competition among dealers. Moreover, under preferencing, the market mechanism fails to allocate efficiently the order flow: the longest dealer is not necessarily the dealer who posts the best price, which partially invalidates the literal prediction of Ho an Stoll (1983)' model.

Finally, we mention that to determine whether preferencing is good or not for markets is much more complex. Preferencing results from long-term relationships between brokers and dealers (or specialists) from whose investors may benefit, especially because of the guarantee to be executed at the best price. Indeed brokers could direct their orders to another place but incur the risk to be price-disimproved when the time of execution is taken into account. Preferencing yields to supra-competitive prices, which could also represent the remuneration of this execution guarantee. However, it remains that the unpreferenced order flow suffers then from the widening of market spreads without benefiting from any guarantee.

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7 Appendix

Let F be the uniform distribution function of the r.v. $a_{r,1}$, in the interval $[a_{r,u}, a_{r,d}]$ and let F_κ be the uniform distribution function of the r.v. $a^\kappa_{r,2}$, in the interval $\left[a^\kappa_{r,u}, a^\kappa_{r,d}\right]$.

7.1 Proof of Theorem 1

In Corollary 2, we show that dealer D_2 has no incentive to post a selling price below her cutoff price. It may be interesting to see why dealer D_2 modifies her reservation price. The natural reservation price would indeed be the reservation price that prevails in a competitive situation where the κ shares would not be executed by a preferred dealer but by the best-quoting dealer. This competitive reservation is defined in introduction by $a_r(I_2, Q+\kappa)$. To show that under preferencing, at equilibrium a preferred dealer raises one's reservation price from a competitive level to a preferenced level, we allow in the following proof dealer D_2 to quote as a function of her cutoff price or her competitive reservation price.

(i) Suppose that the ranking of reservation prices is such that $a_{r,1} > a^\kappa_{r,2} > a_r(I_2, Q+\kappa)$.

Then dealer D_2 posts the best price $a^c_2 = a_{r,1} - \varepsilon$ with probability 1. It is never optimal to quote lower than this price, given that the probability is still equal to 1, and the trading profit could only be lower.

Then, dealer D_1 quotes $a^c_1 = a_{r,1}$ since he cannot post the best price anyway.

(ii) Suppose that the ranking of reservation prices is such that $a^\kappa_{r,2} > a_{r,1} > a_r(I_2, Q+\kappa)$.

We suppose that dealer D_2 quotes $a^c_2 = a^\kappa_{r,2}$. Then, the best reply of dealer D_1 is to $a^c_1 = a^\kappa_{r,2} - \varepsilon$, which is the best price. If he quotes this price, it is indeed not optimal for dealer D_2 to undercut him, in posting $a^c_1 - \varepsilon$, till the competition yields to reach the reservation price of dealer D_1. Dealer D_2 would earn lower profit in this case than in not deviating from the quote equal to her cutoff price, since $(a_{r,1} - a_r(I_2, Q+\kappa)) \times (Q+\kappa) < \left(a^\kappa_{r,2} - a_r(I_2,\kappa)\right) \times \kappa$.

(iii) Suppose that the ranking of reservation prices is such that $a^\kappa_{r,2} > a_r(I_2, Q+\kappa) > a_{r,1}$.

Same than before. Dealer D_1 posts the best price $\underline{a}^c = a^c_1 = a^\kappa_{r,2} - \varepsilon$ and dealer D_2 quotes $a^c_2 = a^\kappa_{r,2}$. However since the competitive reservation price is bigger than the reservation price of her opponent, does dealer D_2 have any incentive to deviate from her strategy in undercutting her opponent ? Given that $a^c_1 = a^\kappa_{r,2} - \varepsilon > a_r(I_2, Q+\kappa)$, if dealer D_2 decides to undercut her opponent, she posts the best price equal to $\underline{a}^c = a_2^c = a_1^c - \varepsilon = a_{r,2}^{\kappa} - 2\varepsilon$. In this case she has to execute the total order flow at this price. However, the trading profit is lower in undercutting her opponent since, $\left(a_2^c - a_r\left(I_2, Q+\kappa\right)\right) \times \left(Q+\kappa\right) < \left(\underline{a}^c - a_r\left(I_2, \kappa\right)\right) \times \kappa$.

Consequently, at equilibrium it is not optimal for dealer D_2 to post a price below her cutoff price. This price plays the role of the reservation price of a prefrenced dealer. It combines indeed the value of two different order flows : Q unpreferenced shares and κ preferenced shares. It follows that at equilibrium the dealer executing the total order flow has the lowest reservation price : $\min\left(a_{r,1}, a_{r,2}^{\kappa}\right)$.

■

7.2 Bidding strategy characterization

STEP 1 : **Dealer D_1's probability to post the best price (ex ante)**

$$\begin{aligned}\Pr\left(D_1 \text{ posts the best price}\right) &= \int_{a_{r,u}}^{a_{r,u}^{\kappa}} 1 \times f\left(x\right) dx + \int_{a_{r,u}^{\kappa}}^{a_{r,d}} \frac{a_{r,d}^{\kappa} - x}{a_{r,d} - a_{r,u}} \times f\left(x\right) dx \\ &= \frac{1}{2} + \frac{\kappa}{\left(I_u - I_d\right)} - \frac{\kappa^2}{2\left(I_u - I_d\right)^2}\end{aligned}$$

STEP 1Bis : **Dealer D_1's ex ante aggressiveness**

$$\begin{aligned}\theta_1\left(a_{r,1}\right) &= \frac{\left(E\left(a_{r,2}^{\kappa}\right) - a_{r,1}\right)}{a_{r,1}} \mathbb{1}_{a_{r,1} < a_{r,u}^{\kappa}} + \frac{\left(E\left(a_{r,2}^{\kappa} \mid a_{r,1} < a_{r,2}^{\kappa}\right) - a_{r,1}\right)}{a_{r,1}} \times \Pr\left(a_{r,1} < a_{r,2}^{\kappa}\right) \mathbb{1}_{a_{r,1} \geq a_{r,u}^{\kappa}} \\ &= \left(\frac{a_{r,u}^{\kappa} + a_{r,d}^{\kappa}}{2 \times a_{r,1}} - 1\right) \mathbb{1}_{a_{r,1} < a_{r,u}^{\kappa}} + \frac{1}{2} \times \frac{\left(a_{r,d}^{\kappa} - a_{r,1}\right)^2}{\left(a_{r,d} - a_{r,u}\right) \times a_{r,1}} \mathbb{1}_{a_{r,1} \geq a_{r,u}^{\kappa}}\end{aligned}$$

and

$$E\left(\theta_1\right) = \frac{\frac{a_{r,u}^{\kappa} + a_{r,d}^{\kappa}}{2} \times \ln\left(\frac{a_{r,u}^{\kappa}}{a_{r,u}}\right) - \rho\sigma_v^2\kappa}{a_{r,d} - a_{r,u}} + \frac{\left(a_{r,d}^{\kappa}\right)^2 \times \ln\left(\frac{a_{r,d}}{a_{r,u}^{\kappa}}\right) + 2a_{r,d}^{\kappa}\left(a_{r,u}^{\kappa} - a_{r,d}\right) + \frac{1}{2}\left(a_{r,d}\right)^2 - \frac{1}{2}\left(a_{r,u}^{\kappa}\right)^2}{2\left(a_{r,d} - a_{r,u}\right)^2}$$

STEP 2 : **Dealer D_2's probability to post the best price (ex ante)**

$$\Pr\left(D_2 \text{ posts the best price}\right) = 1 - \Pr\left(D_1 \text{ posts the best price}\right) = \frac{1}{2} - \frac{\kappa}{\left(I_u - I_d\right)} + \frac{\kappa^2}{2\left(I_u - I_d\right)^2}$$

Step 2Bis : **Dealer D_2's ex ante aggressiveness**

$$\begin{aligned}
\theta_2\left(a^{\kappa}_{r,2}\right) &= \frac{E\left(a_{r,1} \mid a^{\kappa}_{r,2} < a_{r,1}\right) - a^{\kappa}_{r,2}}{a^{\kappa}_{r,2}} \Pr\left(a^{\kappa}_{r,2} < a_{r,1}\right) \mathbb{1}_{a_{r,d} > a^{\kappa}_{r,2}} \\
&= \frac{1}{2} \times \frac{\left(a_{r,d} - a^{\kappa}_{r,2}\right)^2}{\left(a_{r,d} - a_{r,u}\right) \times a^{\kappa}_{r,2}} \mathbb{1}_{a_{r,d} > a^{\kappa}_{r,2}}
\end{aligned}$$

and

$$E\left(\theta_2\right) = \frac{\left(a_{r,d}\right)^2 \times \ln\left(\frac{a_{r,d}}{a^{\kappa}_{r,u}}\right) + \frac{\left(a_{r,d}\right)^2}{2} - \frac{\left(a^{\kappa}_{r,u}\right)^2}{2} + 2a_{r,d}\left(a^{\kappa}_{r,u} - a_{r,d}\right)}{2\left(a_{r,d} - a_{r,u}\right)^2}$$

7.3 Proof of Lemma 1

Before proceeding to the computation of the expected best offer, it is worth noticing than when the preferenced order flow is large, then at equilibrium the preferred dealer is not able to post the best price anyway. Specifically, when $\kappa > \left(I_u - I_d\right)$, then $a^{\kappa}_{r,u} > a_{r,d}$.

Lemma 4 *When the preferenced order flow κ is so large that $\kappa > \left(I_u - I_d\right)$, then the preferred dealer can never post the best price at equilibrium. Dealer D_1 quotes $a^c_1 = a^{\kappa}_{r,2} - \varepsilon$, and then the expression of the expected Best Offer simply writes :*

$$E\left(\underline{a}^c\right) = \frac{a^{\kappa}_{r,d} + a^{\kappa}_{r,u}}{2}$$

Proof : When $\kappa > \left(I_u - I_d\right)$, at equilibrium, dealer D_1 posts the best price with probability 1 and quotes $a^c_1 = a^{\kappa}_{r,2} - \varepsilon$. Then, it is optimal for dealer D_2 to quote her cutoff price $a^c_2 = a^{\kappa}_{r,2}$. She would indeed earn lower profit in undercutting her opponent by $a^c_1 - \varepsilon$ since $\left(a^c_1 - \varepsilon - a_r\left(I_2, Q + \kappa\right)\right) \times \left(Q + \kappa\right) < \left(a^c_1 - a_r\left(I_2, \kappa\right)\right) \times \kappa$. Moreover, in this case, the expected best offer is simply equal to

$$E\left(\underline{a}^c\right) = E\left(a^{\kappa}_{r,2}\right) = \frac{a^{\kappa}_{r,d} + a^{\kappa}_{r,u}}{2}.$$

■

Now we have to consider the case where $\kappa \leq (I_u - I_d)$.

STEP 1 : **Determination of the expected Best Offer when** $\kappa \leq (I_u - I_d)$

By definition, the best offer writes : $\underline{a}^c = \min\left(a_1^c, a_2^c\right)$. In this two-dealer transparent market, the best offer is simply equal to $\max\left(a_{r,1}, a_{r,2}^{\kappa}\right)$.

Let us denote $F_M = FF_{\kappa}$, the c.d.f. of $\max\left(a_{r,1}, a_{r,2}^{\kappa}\right)$. Then

$$\begin{aligned} E\left(\underline{a}^c\right) &= E\left(\max\left(a_{r,1}, a_{r,2}^{\kappa}\right)\right) = \int_0^{+\infty} \bar{F}_M(x)\,dx = a_{r,u}^{\kappa} + \int_{a_{r,u}^{\kappa}}^{a_{r,d}^{\kappa}} \bar{F}_M(x)\,dx \qquad (9) \\ &= \frac{a_{r,d}^{\kappa} + a_{r,u}^{\kappa}}{2} + \frac{\left(a_{r,d} - a_{r,u}\right)}{6}\left(1 - \frac{\rho\sigma_v^2\kappa}{\left(a_{r,d} - a_{r,u}\right)}\right)^3 \mathbb{1}_{a_{r,u}^{\kappa} \leq a_{r,d}} \\ &\quad + \frac{a_{r,d}^{\kappa} + a_{r,u}^{\kappa}}{2}\mathbb{1}_{a_{r,u}^{\kappa} > a_{r,d}}. \end{aligned}$$

STEP 2 : **Determination of the expected Best Offer prevailing in the benchmark (the 'competitive' case).**

Remind that in a situation where No Preferencing is allowed, dealers are symmetric. In this case, the best offer is defined by $\underline{a}^{c,NP} = \max\left(a_{r,1}, a_{r,2}\right)$ and

$$E\left(\underline{a}^{c,NP}\right) = \frac{2a_{r,d} + a_{r,u}}{3}$$

STEP 3 : **Comparison of the expected best offers (competitive vs preferenced case)**

- When $\kappa > (I_u - I_d)$, it is easy to show that

$$E\left(\underline{a}^c\right) = \frac{a_{r,d}^{\kappa} + a_{r,u}^{\kappa}}{2} > E\left(\underline{a}^{c,NP}\right) = \frac{2a_{r,d} + a_{r,u}}{3}$$

- When $\kappa \leq (I_u - I_d)$, we denote $\psi(\kappa)$ the following expression : $\psi(\kappa) = E\left(\underline{a}^c\right) - E\left(\underline{a}_{NP}^c\right)$. After straightforward calculations,

$$\begin{aligned} \psi(\kappa) &= \frac{\rho\sigma_v^2}{2}\left(\frac{(I_u - I_d)}{3}\left(\left(1 - \frac{\kappa}{(I_u - I_d)}\right)^3 - 1\right) + \kappa\right) \\ \psi'(\kappa) &= \frac{\rho\sigma_v^2}{2}\left(1 - \left(1 - \frac{\kappa}{(I_u - I_d)}\right)^2\right) \\ \psi''(\kappa) &= \frac{\rho\sigma_v^2}{2(I_u - I_d)}\left(1 - \frac{\kappa}{(I_u - I_d)}\right) \end{aligned}$$

Since $\kappa \leq (I_u - I_d)$, then $\psi''(\kappa) > 0$, $\psi'(0) = 0$, $\psi'(I_u - I_d) = \rho\sigma_v^2/2$, then $\psi'(\kappa) > 0$ for each $\kappa \leq (I_u - I_d)$. Notice that $\psi(0) = 0$ and $\psi(I_u - I_d) = \frac{\rho\sigma_v^2(I_u - I_d)}{3} > 0$, then we can conclude that $\psi(\kappa) > 0$ for each $\kappa \leq (I_u - I_d)$. It follows that $E(\underline{a}^c) > E(\underline{a}_{NP}^c)$.

■

7.4 Proof of Lemma 2

STEP 1 : **The expected payoff of dealer D_1**

- When $\kappa > (I_u - I_d)$, then dealer D_1 posts the best price with probability 1, and his payoff is

$$\Pi_1^c(a_{r,1}) = \left(E\left(a_{r,2}^\kappa\right) - a_{r,1}\right) \times Q = \left(\frac{a_{r,d}^\kappa + a_{r,u}^\kappa}{2} - a_{r,1}\right) \times Q$$

Hence, at date 1 (ex ante), dealer D_1 expects the following profit :

$$E\left(\Pi_1^c\right) = \left[\int_{a_{r,u}}^{a_{r,d}} \left(\frac{a_{r,d}^\kappa + a_{r,u}^\kappa}{2} - x\right) f(x)\,dx\right] \times Q = \rho\sigma_v^2\kappa \times Q.$$

- When $\kappa \leq (I_u - I_d)$, then

$$\Pi_1^c(a_{r,1}) = \Pr\left(a_{r,2}^\kappa > a_{r,1}\right) \times \left[E\left(a_{r,2}^\kappa \mid a_{r,2}^\kappa > a_{r,1}\right) - a_{r,1}\right] \times Q$$

The uniform distribution $F_\kappa(.)$ of the r.v. $a_{r,2}^\kappa$ is in the interval $\left[a_{r,u}^\kappa, a_{r,d}^\kappa\right]$, then

  - If $a_{r,u}^\kappa \leq a_{r,1}$

$$\Pi_1^c(a_{r,1}) = \bar{F}_\kappa(a_{r,1}) \times \left(\frac{\int_{a_{r,1}}^{a_{r,d}^\kappa} x f_\kappa(x)\,dx}{\bar{F}_\kappa(a_{r,1})} - a_{r,1}\right) \times Q = \frac{1}{2} \times \frac{\left(a_{r,d}^\kappa - a_{r,1}\right)^2}{(a_{r,d} - a_{r,u})} \times Q$$

  - if $a_{r,u} \leq a_{r,1} < a_{r,u}^\kappa$

$$\Pi_1^c(a_{r,1}) = \left(\int_{a_{r,u}^\kappa}^{a_{r,d}^\kappa} x f_\kappa(x)\,dx - a_{r,1}\right) \times Q = \left(\frac{a_{r,d}^\kappa + a_{r,u}^\kappa}{2} - a_{r,1}\right) \times Q$$

and,

$$\begin{aligned} E\left(\Pi_1^c\right) &= \int_{a_{r,u}^\kappa}^{a_{r,d}} \frac{1}{2} \times \frac{\left(a_{r,d}^\kappa - x\right)^2}{(a_{r,d} - a_{r,u})} \times Q \times f(x)\,dx + \int_{a_{r,u}}^{a_{r,u}^\kappa} \left(\frac{a_{r,d}^\kappa + a_{r,u}^\kappa}{2} - x\right) \times Q \times f(x)\,c \\ &= \left(\frac{a_{r,d} - a_{r,u}}{6} - \frac{\left(\rho\sigma_v^2\kappa\right)^3}{6\left(a_{r,d} - a_{r,u}\right)^2} + \rho\sigma_v^2\frac{\left(a_{r,d} - a_{r,u} + \rho\sigma_v^2\kappa\right)}{2\left(a_{r,d} - a_{r,u}\right)} \times \kappa\right) \times Q \end{aligned}$$

STEP 2 : **The expected payoff of dealer D_2.**

- If $a_{r,d} < a^{\kappa}_{r,2}$, dealer D_1 posts the best price with probability 1 and he quotes $a^c_1 = a^{\kappa}_{r,2} - \varepsilon$. Since it is optimal that dealer D_2 quotes her cutoff price, her payoff is :

$$\Pi^c_2\left(a^{\kappa}_{r,2}\right) = \left(a^{\kappa}_{r,2} - a_r\left(I_2, \kappa\right)\right) \times \kappa = \frac{\rho\sigma^2_v}{2}\left(Q+\kappa\right) \times \kappa$$

- If $a^{\kappa}_{r,2} \leq a_{r,d}$, we get (see Theorem 1) :

$$\begin{aligned}
\Pi^c_2\left(a^{\kappa}_{r,2}\right) &= \Pr\left(a_{r,1} > a^{\kappa}_{r,2}\right) \times \left(E\left(a_{r,1} \mid a_{r,1} > a^{\kappa}_{r,2}\right) - a_r\left(I_2, Q+\kappa\right)\right) \times \left(\kappa + Q\right) \\
&\quad + \Pr\left(a_{r,1} < a^{\kappa}_{r,2}\right) \times \left(a^{\kappa}_{r,2} - a_r\left(I_2, \kappa\right)\right) \times \kappa \\
&= \bar{F}\left(a^{\kappa}_{r,2}\right) \times \left(E\left(a_{r,1} \mid a_{r,1} > a^{\kappa}_{r,2}\right) - a^{\kappa}_{r,2}\right) \times \left(Q+\kappa\right) + \frac{\rho\sigma^2_v}{2} \times \left(Q+\kappa\right) \times \kappa
\end{aligned}$$

This expression is quite natural since we argue in Lemma 1 that dealer D_2 will not post selling prices below her cutoff price. The latter expression rewrites,

$$\begin{aligned}
\Pi^c_2\left(a^{\kappa}_{r,2}\right) &= \bar{F}\left(a^{\kappa}_{r,2}\right) \times \left(\frac{\int_{a^{\kappa}_{r,2}}^{a_{r,d}} x f(x)\, dx}{\bar{F}\left(a^{\kappa}_{r,2}\right)} - a^{\kappa}_{r,2}\right) \times \left(Q+\kappa\right) + \frac{\rho\sigma^2_v}{2} \times \left(Q+\kappa\right) \times \kappa \\
&= \left(\frac{\left(a_{r,d} - a^{\kappa}_{r,2}\right)^2}{a_{r,d} - a_{r,u}} + \rho\sigma^2_v \times \kappa\right) \times \frac{\left(Q+\kappa\right)}{2}
\end{aligned}$$

Then, at date 1, when $\kappa < \left(I_u - I_d\right)$, dealer D_2 expects the following profit :

$$\begin{aligned}
E\left(\Pi^c_2\right) &= \int_{a^{\kappa}_{r,u}}^{a_{r,d}} \left(\frac{\left(a_{r,d} - x\right)^2}{a_{r,d} - a_{r,u}} + \rho\sigma^2_v \times \kappa\right) \times \frac{\left(Q+\kappa\right)}{2} f_{\kappa}(x)\, dx + \int_{a_{r,d}}^{a^{\kappa}_{r,d}} \frac{\rho\sigma^2_v}{2}\left(Q+\kappa\right) \times \kappa f_{\kappa}(x)\, dx \\
&= \frac{\left(Q+\kappa\right)}{2}\left[\frac{\left(a_{r,d} - a_{r,u} - \rho\sigma^2_v\kappa\right)^3}{3\left(a_{r,d} - a_{r,u}\right)^2} + \rho\sigma^2_v \times \kappa\right]
\end{aligned}$$

Finally,

$$\begin{aligned}
E\left(\Pi^c_2\right) &= \frac{\rho\sigma^2_v \times \left(Q+\kappa\right)}{2}\left(\frac{\left(I_u - I_d - \kappa\right)^3}{3\left(I_u - I_d\right)^2} + \kappa\right) \mathbb{1}_{\kappa \leq \left(I_u - I_d\right)} \\
&\quad + \frac{\rho\sigma^2_v}{2}\left(Q+\kappa\right) \times \kappa \mathbb{1}_{\kappa > \left(I_u - I_d\right)}.
\end{aligned}$$

STEP 3 : **Comparison with the competitive case**

If the preferenced order flow was directed to the first dealer who quotes the best price then dealers'expected profits would be :

$$E\left(\Pi^{NP}_1\right) = E\left(\Pi^{NP}_2\right) = \left(\frac{a_{r,d} - a_{r,u}}{6}\right) \times \left(Q+\kappa\right)$$

then,

STEP 3.1 : **Comparison of dealer D_2's expected profit**

$$E\left(\Pi_2^c\right) - E\left(\Pi_2^{NP}\right) = \left(\frac{3\left(a_{r,d} - a_{r,u}\right) - \rho\sigma_v^2 \times \kappa}{3\left(a_{r,d} - a_{r,u}\right)^2}\right) \times \frac{(Q+\kappa)}{2}\left(\rho\sigma_v^2 \times \kappa\right)^2 > 0$$

STEP 3.2 : **Comparison of dealer D_1's expected profit**

- When $\kappa \leq (I_u - I_d)$, after straightforward manipulation, we get

$$E\left(\Pi_1^c\right) - E\left(\Pi_1^{NP}\right) = \rho\sigma_v^2\kappa Q \times \left(-\frac{\left(\rho\sigma_v^2\kappa\right)^2}{6\left(a_{r,d} - a_{r,u}\right)^2} + \frac{\rho\sigma_v^2\kappa}{2\left(a_{r,d} - a_{r,u}\right)} + \frac{1}{2} - \frac{a_{r,d} - a_{r,u}}{6\rho\sigma_v^2 Q}\right)$$

- When $\kappa > (I_u - I_d)$, then

$$E\left(\Pi_1^c\right) - E\left(\Pi_1^{NP}\right) = \rho\sigma_v^2\kappa Q\left(1 - \left(\frac{a_{r,d} - a_{r,u}}{6\rho\sigma_v^2 Q}\right)\left(1 + \frac{1}{\frac{\kappa}{Q}}\right)\right)$$

Let us now define the following function :

$$\begin{aligned} g(\kappa, Q) &= \frac{1}{6}\left(-\frac{\kappa^2}{(I_u - I_d)^2} + 3\frac{\kappa}{(I_u - I_d)} + 3 - \frac{(I_u - I_d)}{Q}\right)\mathbb{1}_{\kappa \leq (I_u - I_d)} \\ &+ \left(1 - \frac{(I_u - I_d)}{6Q}\left(1 + \frac{1}{\frac{\kappa}{Q}}\right)\right)\mathbb{1}_{\kappa > (I_u - I_d)} \end{aligned}$$

$$g(0, Q) = \frac{1}{6}\left(3 - \frac{(I_u - I_d)}{Q}\right), \quad g((I_u - I_d), Q) = \frac{1}{6}\left(5 - \frac{(I_u - I_d)}{Q}\right), \quad \lim_{\kappa \to \infty} g(\kappa, Q) = 1 - \frac{(I_u - I_d}{6Q}$$

$$\frac{\partial g(\kappa, Q)}{\partial \kappa} = \frac{1}{6(I_u - I_d)}\left(-\frac{2\kappa}{(I_u - I_d)} + 3\right)\mathbb{1}_{\kappa \leq (I_u - I_d)} + \frac{(I_u - I_d)}{6\kappa^2}\mathbb{1}_{\kappa > (I_u - I_d)}$$

g is an increasing function, the sign of this function depends on the initial condition. Then,

- $Q \geq \frac{(I_u - I_d)}{3}$, $E\left(\Pi_1^c\right) \geq E\left(\Pi_1^{NP}\right)$
- $Q < \frac{(I_u - I_d)}{3}$ and
  - $\frac{(I_u - I_d)}{5} \leq Q < \frac{(I_u - I_d)}{3}$, $E\left(\Pi_1^c\right) \leq E\left(\Pi_1^{NP}\right)$, if $\kappa \leq \kappa^*(Q)$, $E\left(\Pi_1^c\right) > E\left(\Pi_1^{NP}\right)$, otherwise

- $\frac{(I_u-I_d)}{6} < Q < \frac{(I_u-I_d)}{5}$, $E\left(\Pi_1^c\right) \leq E\left(\Pi_1^{NP}\right)$, if $\kappa \leq \kappa^{**}(Q)$, $E\left(\Pi_1^c\right) > E\left(\Pi_1^{NP}\right)$, otherwise
- $Q < \frac{(I_u-I_d)}{6}$, $E\left(\Pi_1^c\right) < E\left(\Pi_1^{NP}\right)$

where

$$\begin{aligned}
\kappa^*(Q) &= \frac{(I_u-I_d)}{2}\left(3-\sqrt{21-4\frac{(I_u-I_d)}{Q}}\right) \\
\kappa^{**}(Q) &= \frac{(I_u-I_d)\,Q}{(6Q-(I_u-I_d))}
\end{aligned}$$

Now, we define $\underline{\kappa}$ such that

$$\underline{\kappa}(Q) = \kappa^*(Q)\,\mathbb{1}_{\frac{(I_u-I_d)}{5}\leq Q<\frac{(I_u-I_d)}{3}} + \kappa^{**}(Q)\,\mathbb{1}_{\frac{(I_u-I_d)}{6}\leq Q<\frac{(I_u-I_d)}{5}}$$

■

7.5 Proof of Lemma 3

In a three-dealer market, we suppose that only dealer D_2 is preferred. The reservation prices of dealers are respectively $a_{r,1}$, $a_{r,2}^{\kappa}$ and $a_{r,3}$. Then, the exepcted best offer writes :

$$\begin{aligned}
E\left(\underline{a}^c\right) &= E\left(a_{r,3}\mathbb{1}_{a_{r,1}<a_{r,3}<a_{r,2}^{\kappa}}\right)+E\left(a_{r,2}^{\kappa}\mathbb{1}_{a_{r,1}<a_{r,2}^{\kappa}<a_{r,3}}\right)+E\left(a_{r,1}\mathbb{1}_{a_{r,2}^{\kappa}<a_{r,1}<a_{r,3}}\right) \\
&\quad +E\left(a_{r,3}\mathbb{1}_{a_{r,2}^{\kappa}<a_{r,3}<a_{r,1}}\right)+E\left(a_{r,2}^{\kappa}\mathbb{1}_{a_{r,3}<a_{r,2}^{\kappa}<a_{r,1}}\right)+E\left(a_{r,1}\mathbb{1}_{a_{r,3}<a_{r,1}<a_{r,2}^{\kappa}}\right) \\
&= 2\left(E\left(a_{r,3}\mathbb{1}_{a_{r,1}<a_{r,3}<a_{r,2}^{\kappa}}\right)+E\left(a_{r,2}^{\kappa}\mathbb{1}_{a_{r,1}<a_{r,2}^{\kappa}<a_{r,3}}\right)+E\left(a_{r,1}\mathbb{1}_{a_{r,2}^{\kappa}<a_{r,1}<a_{r,3}}\right)\right)
\end{aligned}$$

Let us denote $x = a_{r,1}$, $y = a_{r,2}^{\kappa}$ and $z = a_{r,3}$

$$E\left(a_{r,3}\mathbb{1}_{a_{r,1}<a_{r,3}<a_{r,2}^{\kappa}}\right) = \int_{a_{r,u}^{\kappa}}^{a_{r,d}} \frac{z}{(a_{r,d}-a_{r,u})}\left(\frac{z-a_{r,u}}{a_{r,d}-a_{r,u}}\right)\left(\frac{a_{r,d}^{\kappa}-z}{a_{r,d}-a_{r,u}}\right)dz + \int_{a_{r,u}}^{a_{r,u}^{\kappa}} \frac{z}{(a_{r,d}-a_{r,u})}\left(\frac{z-\ldots}{a_{r,d}-\ldots}\right.$$

$$E\left(a_{r,2}^{\kappa}\mathbb{1}_{a_{r,1}<a_{r,2}^{\kappa}<a_{r,3}}\right) = \int_{a_{r,u}^{\kappa}}^{a_{r,d}} \frac{y}{(a_{r,d}-a_{r,u})}\left(\frac{y-a_{r,u}}{a_{r,d}-a_{r,u}}\right)\left(\frac{a_{r,d}-y}{a_{r,d}-a_{r,u}}\right)dy$$

$$E\left(a_{r,1}\mathbb{1}_{a_{r,2}^{\kappa}<a_{r,1}<a_{r,3}}\right) = \int_{a_{r,u}^{\kappa}}^{a_{r,d}} \frac{x}{(a_{r,d}-a_{r,u})}\left(\frac{x-a_{r,u}^{\kappa}}{a_{r,d}-a_{r,u}}\right)\left(\frac{a_{r,d}-x}{a_{r,d}-a_{r,u}}\right)dx$$

After straightforward manipulations, we get

$$
\begin{aligned}
E\left(a_{r,3}\mathbb{1}_{a_{r,1}<a_{r,3}<a^{\kappa}_{r,2}}\right) &= \frac{1}{\left(a_{r,d}-a_{r,u}\right)^{3}}\left(\begin{array}{c} -\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{4}}{4}+\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{3}}{3}\left(a^{\kappa}_{r,d}-2a^{\kappa}_{r,u}-\rho\sigma_{v}^{2}\kappa\right)\\ +\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{2}}{2}\left(\left(\rho\sigma_{v}^{2}\kappa+a^{\kappa}_{r,u}\right)\left(a^{\kappa}_{r,d}-a^{\kappa}_{r,u}\right)-\rho\sigma_{v}^{2}\kappa a^{\kappa}_{r,u}\right)\\ +\rho\sigma_{v}^{2}\kappa a^{\kappa}_{r,u}\left(a^{\kappa}_{r,d}-a^{\kappa}_{r,u}\right)\left(a_{r,d}-a^{\kappa}_{r,u}\right)\end{array}\right)\\
&\quad+\frac{\left(\rho\sigma_{v}^{2}\kappa\right)^{2}\left(3a_{r,u}+2\rho\sigma_{v}^{2}\kappa\right)}{6\left(a_{r,d}-a_{r,u}\right)^{2}}\\
E\left(a^{\kappa}_{r,2}\mathbb{1}_{a_{r,1}<a^{\kappa}_{r,2}<a_{r,3}}\right) &= \frac{1}{\left(a_{r,d}-a_{r,u}\right)^{3}}\left(\begin{array}{c} -\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{4}}{4}+\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{3}}{3}\left(a_{r,d}-2a^{\kappa}_{r,u}-\rho\sigma_{v}^{2}\kappa\right)\\ +\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{2}}{2}\left(\left(a^{\kappa}_{r,u}+\rho\sigma_{v}^{2}\kappa\right)\left(a_{r,d}-a^{\kappa}_{r,u}\right)+a^{\kappa}_{r,u}\rho\sigma_{v}^{2}\kappa\right)\end{array}\right)
\end{aligned}
$$

and

$$
E\left(a_{r,1}\mathbb{1}_{a^{\kappa}_{r,2}<a_{r,1}<a_{r,3}}\right)=\frac{1}{\left(a_{r,d}-a_{r,u}\right)^{3}}\left(\begin{array}{c} -\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{4}}{4}+\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{3}}{3}\left(a_{r,d}-2a^{\kappa}_{r,u}\right)\\ +\frac{\left(a_{r,d}-a^{\kappa}_{r,u}\right)^{3}}{2}a^{\kappa}_{r,u}\end{array}\right).
$$

Finally,

$$
\begin{aligned}
E\left(\underline{a}^{c}\right) &= \frac{\left(a_{r,d}-a_{r,u}\right)}{2}\left(1-\frac{\rho\sigma_{v}^{2}\kappa}{\left(a_{r,d}-a_{r,u}\right)}\right)^{4}-\frac{\rho\sigma_{v}^{2}\kappa}{3}\left(1-\frac{\rho\sigma_{v}^{2}\kappa}{\left(a_{r,d}-a_{r,u}\right)}\right)^{3}\\
&\quad+\left(2\rho\sigma_{v}^{2}\kappa+a_{r,u}\right)\left(1-\frac{\rho\sigma_{v}^{2}\kappa}{\left(a_{r,d}-a_{r,u}\right)}\right)^{2}+\frac{2\rho\sigma_{v}^{2}\kappa a^{\kappa}_{r,u}}{\left(a_{r,d}-a_{r,u}\right)}\left(1-\frac{\rho\sigma_{v}^{2}\kappa}{\left(a_{r,d}-a_{r,u}\right)}\right)\\
&\quad+\frac{\left(\rho\sigma_{v}^{2}\kappa\right)^{2}\left(3a_{r,u}+2\rho\sigma_{v}^{2}\kappa\right)}{3\left(a_{r,d}-a_{r,u}\right)^{2}}
\end{aligned}
$$

■

7.6 Proof of Theorem 2

STEP 1 : **Determination of the ordinary differential equations system**

Given the best reply of dealer D_2, dealer D_1 chooses y so as to maximize his profit,

$$\Pi_1\left(y,a_{r,1}\right)=\bar{F}_{\kappa}\left(v_2\left(y\right)\right)\times\left(y-a_{r,1}\right)\times Q.$$

Then the first order condition (FOC) yields

$$\frac{\partial\Pi_1\left(y,a_{r,1}\right)}{\partial y}=0 \text{ or } \bar{F}_{\kappa}\left(v_2\left(y\right)\right)+v_2'\left(y\right)\times\bar{F}_{\kappa}'\left(v_2\left(y\right)\right)\times\left(y-a_{r,1}\right)=0.$$

At equilibrium, if a_1 is the optimal strategy $\left(a_1\left(a_{r,1}\right)=y\right)$, then $v_1\left(y\right)$ must verify the FOC such that for each y :

$$\bar{F}_{\kappa}\left(v_2\left(y\right)\right)+v_2'\left(y\right)\times\bar{F}_{\kappa}'\left(v_2\left(y\right)\right)\times\left(y-v_1\left(y\right)\right)=0. \tag{10}$$

Now, given that dealer D_1 quotes $a_1 = (v_1)^{-1}$, then dealer D_2 chooses y so as to maximize her profit Π_2, where

$$\begin{aligned}
\Pi_2\left(y, a^{\kappa}_{r,2}\right) &= \bar{F}\left(v_1\left(y\right)\right) \times \left(y - a^{\kappa}_{r,2}\right) \times \left(Q + \kappa\right) \\
&\quad + \left(1 - \bar{F}\left(v_1\left(y\right)\right)\right) \times \left(E\left(a_1\left(a_{r,1}\right) \mid y > a_1\left(a_{r,1}\right)\right) - a^{\kappa}_{r,2}\right) \times \kappa \\
&\quad + \frac{\rho\sigma_v^2}{2}\kappa \times \left(Q + \kappa\right).
\end{aligned}$$

Then the first order condition yields :

$$\frac{\partial \Pi_2\left(y, a^{\kappa}_{r,2}\right)}{\partial y} = 0 \text{ or } \bar{F}\left(v_1\left(y\right)\right)\left(1 + \kappa/Q\right) + v_1'\left(y\right) \times \bar{F}'\left(v_1\left(y\right)\right) \times \left(y - a^{\kappa}_{r,2}\right) = 0$$

Now, at equilibrium, if a_2 is the optimal strategy, then $v_2\left(y\right)$ must verify the first order condition of dealer D_2 such that for each y :

$$\bar{F}\left(v_1\left(y\right)\right)\left(1 + \kappa/Q\right) + v_1'\left(y\right) \times \bar{F}'\left(v_1\left(y\right)\right) \times \left(y - v_2\left(y\right)\right) = 0 \tag{11}$$

At last, the equations (10) and (11) give the following system :

$$\begin{aligned}
\frac{-\bar{F}_{\kappa}'\left(v_2\left(y\right)\right)}{\bar{F}_{\kappa}\left(v_2\left(y\right)\right)} \times v_2'\left(y\right) &= \frac{1}{y - v_1\left(y\right)}, \\
\frac{-\bar{F}'\left(v_1\left(y\right)\right)}{\bar{F}\left(v_1\left(y\right)\right)} \times v_1'\left(y\right) &= \frac{\left(1 + \kappa/Q\right)}{y - v_2\left(y\right)}.
\end{aligned}$$

STEP 2 : **Existence of an equilibrium**

Given that $\bar{F}\left(x\right) = \frac{a_{r,d} - x}{a_{r,d} - a_{r,u}}$ and $\bar{F}_{\kappa}\left(x\right) = \frac{a^{\kappa}_{r,d} - x}{a_{r,d} - a_{r,u}}$, the system writes also :

$$v_1'\left(y\right) = \frac{\left(a_{r,d} - v_1\left(y\right)\right) \times \left(1 + \kappa/Q\right)}{y - v_2\left(y\right)}, \tag{12}$$

$$v_2'\left(y\right) = \frac{a^{\kappa}_{r,d} - v_2\left(y\right)}{y - v_1\left(y\right)}. \tag{13}$$

Following Theorem 3 of Griesmer et al. (1967), since $\frac{a_{r,d} + a^{\kappa}_{r,d}}{2} > a^{\kappa}_{r,u}$, we can prove that there exists a multiplicity of [20] equilibria parameterized by $a^{\sup}$. In such an equilibrium :

(i) $\max\left(a_{r,d}, a^{\kappa}_{r,u}\right) \leq a^{\sup} \leq \frac{a_{r,d} + a^{\kappa}_{r,d}}{2}$,

(ii) $v_2\left(a^{\sup}\right) = a^{\sup}, v_1\left(a^{\sup}\right) = a_{r,d}$

(iii) $a^{\inf}$ is such that $v_1\left(a^{\inf}\right) = a_{r,u}$ and $v_2\left(a^{\inf}\right) = a^{\kappa}_{r,u}$.

■

[20] Both dealers have a positive probability to accommodate the unpreferenced order flow $+Q$.

7.7 Proof of Proposition 1

(The proof of the Theorem has been ommitted but is available upon request or in Lescourret and Robert (2002).

7.8 Proof of Proposition 2

The captive order flow κ is such that $a_{r,u}^{\kappa} > \left(a_{r,d} + a_{r,d}^{\kappa}\right)/2$ i.e. $\kappa \geq 2\left(I_u - I_d\right)$. Then,

$$a_{r,u} \leq a_{r,d} \leq \frac{a_{r,d} + a_{r,d}^{\kappa}}{2} \leq a_{r,u}^{\kappa} \leq a_{r,d}^{\kappa}$$

Now, we suppose that the preferred dealer D_2 quotes an ask price equal to her reservation price : $a_2\left(a_{r,2}^{\kappa}\right) = a_{r,2}^{\kappa}$ (we will prove ultimately that this reply is the best one). When $a_1 \geq a_{r,u}^{\kappa}$, dealer D_1 chooses a selling quote that maximizes his profit,

$$\begin{aligned}
\Pi_1\left(a_{r,1}\right) &= \Pr\left(a_1 < a_2\right) \times \left(a_1 - a_{r,1}\right) \times Q \\
&= \Pr\left(a_1 < a_{r,2}^{\kappa}\right) \times \left(a_1 - a_{r,1}\right) \times Q \\
&= \bar{F}_{\kappa}\left(a_1\right) \times \left(a_1 - a_{r,1}\right) \times Q
\end{aligned}$$

The first order condition yields to

$$\begin{aligned}
\bar{F}_{\kappa}\left(a_1\right) - f_{\kappa}\left(a_1\right) \times \left(a_1 - a_{r,1}\right) &= 0 \\
\left(a_{r,d}^{\kappa} - a_1\right) - \left(a_1 - a_{r,1}\right) &= 0
\end{aligned}$$

Then, we deduce that

$$a_1 = \frac{a_{r,d}^{\kappa} + a_{r,1}}{2}$$

a_1 is increasing in $a_{r,1} \leq a_{r,d}$. Setting $a_1 = \frac{a_{r,d}^{\kappa} + a_{r,d}}{2} = a^{\sup} \leq a_{r,u}^{\kappa}$ gives dealer D_1 an equal probability to post the best price. However dealer D_1 maximizes his profit when he quotes $a_1 = a_{r,u}^{\kappa}$. Given the dealer D_1' s best reply, dealer D_2 has no chance to execute the unpreferenced order flow and quotes indeed $a_2\left(a_{r,2}^{\kappa}\right) = a_{r,2}^{\kappa}$ (since dealer D_2 never quotes a price under her cutoff price).

■

7.9 Proofs related to the characterization of the way to quote

STEP 1 : **The benchmark case**

1. **Aggressiveness**

$$\theta_i\left(a_{r,i}\right) = \frac{a_{NP}\left(a_{r,i}\right) - a_{r,i}}{a_{ri}} = \frac{a_{r,d}}{2a_{r,i}} - \frac{1}{2}$$

$$E\left(\theta_i\right) = \int_{a_{r,u}}^{a_{r,d}} \left(\frac{a_{r,d}}{2x} - \frac{1}{2}\right) f\left(x\right) dx = \frac{a_{r,d}}{2\left(a_{r,d} - a_{r,u}\right)} \ln\left(\frac{a_{r,d}}{a_{r,u}}\right) - \frac{1}{2}$$

2. **Probability to post the best price**

Given that $\Pr\left(D_i \text{ posts the best price} \mid a_{r,i}\right) = \bar{F}\left(v\left(a_i\right)\right) = \frac{a_{r,d} - v(a_i)}{a_{r,d} - a_{r,i}}$. At equilibrium, we must have $v\left(a_i\right) = a_{r,i}$, then

$$\Pr\left(D_i \text{ posts the best price}\right) = \int_{a_{r,u}}^{a_{r,d}} \frac{a_{r,d} - x}{a_{r,d} - a_{r,u}} f\left(x\right) dx = \frac{1}{2}$$

STEP 2 : **The preferencing case when $\kappa > 2\left(I_u - I_d\right)$**

1. **Aggressiveness**

$$\theta\left(a_{r,1}\right) = \frac{a_{r,u}^{\kappa} - a_{r,1}}{a_{r,1}}$$

Hence,

$$E\left(\theta_1\right) = \int_{a_{r,u}}^{a_{r,d}} \left(\frac{a_{r,u}^{\kappa}}{x} - 1\right) f\left(x\right) dx = \frac{a_{r,u}^{\kappa} \ln \frac{a_{r,d}}{a_{r,u}}}{a_{r,d} - a_{r,u}} - 1.$$

Note also that $E\left(\theta_2\right) = 0$.

2. **Dealers' expected profits**

$$E\left(\Pi_2\right) = \left(\int_{a_{r,u}^{\kappa}}^{a_{r,d}^{\kappa}} \left(a_{r,u}^{\kappa} + \frac{\rho\sigma_v^2}{2}\kappa + \frac{\rho\sigma_v^2}{2}Q - x\right) f_{\kappa}\left(x\right) dx\right) \times \kappa.$$

It is straightforward to show that $E\left(\Pi_2\right) = \left(a_{r,u} - a_{r,d} + \rho\sigma_v^2\left(\kappa + Q\right)\right)/2 \times \kappa$. Concerning the unpreferred dealer, he expects the following profit :

$$E\left(\Pi_1\right) = \left[\int_{a_{r,u}}^{a_{r,d}} \left(a_{r,u}^{\kappa} - x\right) f\left(x\right) dx\right] \times Q = \frac{2\rho\sigma_v^2\kappa - \left(a_{r,d} - a_{r,u}\right)}{2} \times Q.$$

■

7.10 Proofs included in the section 'Internalization'