A Dynamic Model of Order Execution and the Intraday Cost of Limit Orders

Tai Ma* and I-Chun Tsai**

Abstract

We develop a dynamic model of limit order in an order-driven market, where traders differ in their share valuations. Taking into consideration traders learning process and allowing the conditional probability of limit order execution to vary, we can analyze the dynamics of order execution. Our results have interesting empirical implications that are closely related to existing literature on order sequences and order execution, and yield further insight into the dynamic process of order execution. Furthermore, the paper complements the literature on the transaction costs of limit orders: we show that the intraday pattern of the cost of limit order submitted by uninformed traders is U-shaped.

Keywords: order-driven market, probability of execution, order sequences, intraday patterns, execution cost

---

(Please do not quote without permission)

*Corresponding author: Professor Tai Ma, email: matai@finance.nsysu.edu.tw, address: Department of Finance, National Sun Yat-sen University, 804 Kaohsiung, Taiwan R.O.C., tel: +886-7-525000ext4810, fax: +886-7-5251523

** Ph.D Candidate I-Chun Tsai, email: trista12@mail.stut.edu.tw, address: Department of Finance, National Sun Yat-sen University, 804 Kaohsiung, Taiwan R.O.C., tel: +886-7-525000ext4830, fax: +886-7-5251523.

1. Introduction

The successful development of electronic limit order trading platforms in almost all major stock markets around the world has drawn increasing attention to the academic research on the order driven market. Among the growing body of theoretical literature on order driven markets, there are models that describe the price formation process of limit orders (e.g., Glosten(1989,1994), Foucault(1999), Parlour(1998), Sandas(2001), and Seppi(1997)), models that explore the probability of limit order execution(e.g., Angel(1994)), and trader's choice models of limit vs. market orders (e.g., Holden and Chakravarty, 1995). In this paper we address the following questions: (1) What is the dynamic behavior of order execution when uninformed traders can learn? (2) What is the intraday pattern of the execution cost of limit orders?

Traders in the order-driven markets face a dilemma in choosing the type of order to submit. A market order is executed with certainty at the quoted price. With a limit order, a trader has the possibility to improve the price of execution, but she runs the risk of non-execution and faces the adverse selection risk if the order is executed. The cost of non-execution and the adverse selection cost of execution have been well discussed in the literature; however, little has been said about the intraday behavior of the costs[1]. By analyzing the variation of the conditional probabilities of limit order execution, we are able to explore the intraday behavior of the non-execution cost and adverse selection risk of limit orders, thereby explain the intraday pattern of the liquidity and price volatility in an order-driven market.

---

[1] Although there are many empirical studies focus on the trading cost of limit order, none has discussed the intraday pattern. Some focuse on the costs and determinants of order aggressiveness (e.g., Keim and Madhavan (1997), and Giffiths, Smith, Turnbull, White (2000)), others on the comparison of trading costs for different stocks (e.g., Barclay, Christies, Harris, Kandel and Schultz (1997), Bessembinder and Kaufman (1997), Jones and Lipaon (1997)), and others on the survival analysis of limit order execution times and it's determinants (e.g., Lo, MacKinlay, Zhang (2002)).

Glosten (1994) derives the equilibrium price schedule in an open limit order book. He shows that limit order traders profit from liquidity driven traders but lose from information driven price changes. Handa and Schwartz (1996) analyze the rationale for limit order trading. They posit that those traders who have minimal non-execution costs have an incentive to submit limit orders, while those who have high non-execution costs prefer to submit market orders, though they do not explicitly model the trader's decision. Earlier theories of limit orders trading such as the studies mentioned above are mostly static models.

More recently, Parlour (1998) and Foucault (1999) develop dynamic models of limit order trading. In Parlour (1998) the endogenous probability of limit order execution depends on the state of the book as well as the agent's belief regarding further order arrivals. Foucault (1999) provides a game theoretic model of price formation and order placement decisions. Foucault, Kadan, Kandel (2001) develop a dynamic model of an order driven market populated by discretionary liquidity traders. They find that equilibrium pattern is determined by the degree of impatience of the patient traders, their proportion in the population, and the tick size.

However, while information asymmetry plays an important role in the real world, none of these dynamic models account for private information in their analysis. Hence, their predictions may not always be consistent with empirical findings. For example, Parlour (1998) predicts that the conditional probability of a limit buy order followed by the same type order should be less than the conditional probability of a limit sell order followed by a limit buy order. But Ranaldo (2003) finds the opposite empirical evidence on the Swiss Stock Exchange, and he points out the reason for the opposite finding may be that information asymmetry was ruled out from Parlour's model.

Handa, Schwartz, Tiwari (2003) highlight the issue of information asymmetry by extending the ideas of Foucault (1999) in a more general model in which the traders not only differ in share valuation but also in information availability. They show that the size of the spread in an order driven market is a function of adverse selection and the differences in valuation among investors. But their model mainly focuses on the determinants of bid-ask spread, and does not deal with the issue of order sequences and the dynamic behavior of order execution[2]. Although Foucault (1999) and Handa, Schwartz, Tiwari (2003) improve the previous models with respect to information asymmetry, the probability of order arrival or order execution is non-dynamic in their models, which contradicts to what many empirical papers have found.

Empirical evidences tell us that the probability of order arrival is not random, for example, orders are often followed by similar orders, which are referred to as the diagonal effect in Biais, Hillion and Spatt (1995) study on limit orders in Paris Bourse. Later, Al-Suhaibani and Kryznowski (2001) also find similar result in Saudi stock market. However, there is much less theoretical analysis on this issue. What is the dynamic behavior of order execution? Is there any intraday pattern in the execution cost of limit orders? Surprisingly, these questions have not been adequately addressed[3]. The objective of this study is to develop a dynamic order execution model. Our model is an extension of Foucault (1999) and Handa, Schwartz, Tiwari (2003), in

---

[2] Handa, Schwartz and Tiwari (2003) describe the unconditional probabilities of the arrival of uninformed traders and the limit order execution, but not the conditional dynamics of order execution.

[3] The foregoing researches examined the traders' choice between limit and market orders, they did not discuss the intraday pattern. And in the last few years, several empirical studies have devoted to the issue of the determinants of trader's order choice and the probabilities and times of limit order execution, for examples: using data from the Paris Bourse, Biais, Hillion and Spatt (1995) find evidence that traders submit more market orders when the order book is relatively full and more limit orders when the order book is relatively empty. Harris and Hasbrouck (1996) analyze the profitability of alternative order placement strategies in different market conditions. Hollifield, Miller and Sandas (2001) analyze order placement strategies in a limit order market, using data on the order flow from the Stockholm Stock Exchange. Lo, MacKinlay, Zhang (2002) develop econometric models of limit order execution times using survival analysis, and estimate them with actual limit order data. Bae, Jang, Park (2003) examine a trader's order choice between market and limit orders using a sample of orders submitted through NYSE SuperDot. Harris (1998) develops a dynamic model, in which separate solutions are obtained for quoted and order-driven markets.

addition to take into consideration the information asymmetry as Handa, et al., we further model the dynamics of the conditional probability of order execution and analyze the dynamics of the cost of limit orders in the order driven market. In short, this paper differs from previous studies in two aspects; first, unlike previous studies where the probability of limit order execution is invariant to time, the conditional probability of limit order execution in our model is free to vary; second, we explore the intraday patterns of the transaction cost of limit orders. While the price formation process and the intraday patterns of the spread and trading activities in the specialist and dealer markets have been extensively studied, this paper complements the literature on the intraday transaction costs in the order driven market.

As in Handa, Schwartz, Tiwari (2003), traders differ in their share valuation and the advent of information, in static equilibrium, the determinants of price and spread are obtained. The static equilibrium results conform to other studies on the determinants of bid ask spreads. Our results imply that the bid ask spread in the order driven market increases as the volatility of the asset increases, and it decreases as the number of uninformed traders increases. The result is similar to the positive relation found between bid ask spread and adverse selection in many studies on quote driven markets.[4] In addition, the result suggests that the relation between the difference in share valuation and the bid ask spread is positive when there is no serious order imbalance, however, if there is order imbalance, the greater the difference in share valuation among agents, the smaller is the bid ask spread.

The results of the dynamic analysis show that: First, the probability of order execution is influenced by the structure of traders, the expected value of the risky asset, and the expected aggressiveness of the other traders. Our result supports the

---

[4] For example, Copeland and Galai(1983), Glosten and Milgrom(1985), and Easley and O'Hara(1987)

empirical findings of Hollifield, Miller, Sandas (2001)[5], and Hollifield, Miller, Sandas and Slive (2002)[6]. Second, contrary to Parlour(1998), we show that following the execution of a buy(sell) order, the conditional probability of a buy(sell) order execution increases. Finally, the result suggests that the intraday pattern of the cost of limit order submitted by uninformed traders is U-shaped.

The findings and implications of the model are closely related to other empirical and theoretical studies, for example, the implication about order sequences helps to explain the empirical findings of Hamao and Hasbrouck (1995), Biais, Hillion, and Spatt (1995) and Ranaldo (2003)[7]; the implication about bid-ask spread and adverse selection is similar to the theories of bid-ask spread in the quote-driven market[8], and the determinants of the probabilities of execution are closely related to Foucault (1999), Foucault, Kadan, Kandel (2001), and Handa, Schwartz, Tiwari (2003).

The rest of the paper is organized as follows. Section 2 presents a model of a pure order driven market and discusses the static equilibrium and the determinants of price and bid-ask spread, followed by the dynamic analysis. Section 3 discusses the implications of the model. Section 4 concludes the paper.

---

5 Their findings imply that variation in the composition of the order flow can be explained by variation in the relative profitability of alternative order choices and the movements in the common value of the asset.

6 Hollifield, Miller, Sandas and Slive (2002) find that a trader's optimal order submission changes with market conditions.

7 Hamao and Hasbrouck (1995) find order persistence and suggest that the order continuation may depend on information motives. Biais et al. (1995) explain in more details, they report that the most likely incoming order type would be the same order type that just arrived, this phenomenon may be a result of order splitting, trading imitation, and the same response to the information. Ranaldo (2003) finds the sequence of a trade and a subsequent order in the same direction on the Swiss Stock Exchange.

8 This informational source of the spread has been first suggested by Bagehot (1971) and formally analyzed by Copeland and Galai (1983). Glosten and Milgrom (1985) use a formal model to show how the spread arises from adverse selection. Easley and O'Hara (1987), focusing on the learning process of market makers in dealer markets, find that the bid ask spread is positive related with the adverse selection risk.

2. Theoretical Model

2.1 Model assumptions

Assumption 1: Asset Valuation

There is a single risky asset in the order driven market, the true value of the asset is a random variable, $\tilde{v}$. There are N traders who buy or sell this asset during the trading periods. Traders differ in their information about the asset value. The uninformed traders can observe the public information of asset value, and their trades are driven by liquidity demand or influenced by noise. The other traders are the informed traders, who possess private information about the asset value. Their trades are information-driven. Assume uninformed traders believe the value of the asset is uniformly distributed as $\tilde{v} \sim U(V_L, V_H)$, and the expected asset value is $E_u[\tilde{v}] = \dfrac{V_L + V_H}{2} = V_u$. The informed believe the value of the asset is distributed as $\tilde{v} \sim U(V_l, V_h)$, and the expected asset value is $E_i[\tilde{v}] = \dfrac{V_l + V_h}{2} = V_i$. Due to the superiority of the private information, the precision of evaluation is higher for the informed, that is, $(V_h - V_l) < (V_H - V_L)$. The volatility of the asset value based on public information, $\dfrac{(V_H - V_L)^2}{12}$, is greater than that based on private information, $\dfrac{(V_h - V_l)^2}{12}$. The true value of the asset becomes public by the end of the trading period.

Assumption 2: Trading periods

The time horizon is one normal trading day. The trading day is divided into discrete time intervals denoted by t, $t=1,2,3,\ldots,\tilde{T}$. We assume that the payoff time $\tilde{T}$ is random. At time t, the probability that the trader's expected trading process stops and the payoff of the asset is realized is (1-ρ_t) >0, where ρ_t is the probability that the trading process continues. Furthermore, (1- ρ_t) is an increasing function of *t*; that is, $\frac{\partial(1-\rho_t)}{\partial t}$ >0.

Assumption 3: Order placement strategies

Traders arrive sequentially to trade one share of the asset. Upon arrival, an uninformed trader can choose to place a limit order or to place a market order. A limit order is held until the next trader arrives, at which point it is either executed or expired.

Assumption 4: Trading behavior

The behavior of the two types of traders is described in more detail as follows:

1.Uninformed trader: Suppose there are *U* uninformed traders out of the *N* traders. In addition to public information, these traders may be influenced by noise, so they have different reservation prices of the asset. Assume that there are U_b uninformed traders who think the asset value is equal to $V_{U_b} = \int_{V_u}^{V_H} \tilde{v} f(\tilde{v}) d\tilde{v} = V_u + \varepsilon$, $\varepsilon > 0$ and they are buyers, while there are U_s uninformed traders who think the asset value is equal to $V_{U_s} = \int_{V_L}^{V_u} \tilde{v} f(\tilde{v}) d\tilde{v} = V_u - \varepsilon$, and they are sellers. If the noise ε only influences the range of the volatility of asset value, but not the distribution of the value, then $\varepsilon = \frac{1}{4}(V_H - V_L)$.

2.Informed traders: Suppose there are *I* informed traders out of *N* traders. They profit by their superior information. When $E_i[\tilde{v}] = V_i > E_u[\tilde{v}] = V_u$, they will choose to buy the asset; when $V_i < V_u$, they sell the asset. If $V_i = V_u$, they will not enter the market. Since private information is short-lived when there is competition among informed traders, we assume informed trader submit only market orders.

Assume traders are risk-neutral expected utility maximizers. For a specified price *P*, the expected utility of a buy order is $E(U)=\lambda(V_i-P), i=I, U_b$, the expected utility of a sell order is $E(U)=\lambda(P-V_i), i=I, U_s$, where λ is the probability of execution of the order, *I* and *U* denote informed traders and uninformed traders, respectively.

2.2 Equilibrium of bid-ask prices and spread

Once a trader submits an order, the probability of limit order execution depends on the information they owned. There are three possible relations between public and private information, i.e., $V_i > V_u, V_i = V_u, V_i < V_u$, probability of each is 1/3. Since a limit order can only be executed when a market order arrives, trader's placement strategies are interrelated. Assume the expected probability of a buy limit order placed by other uninformed traders is π_1, and the expected probability of a sell limit order placed by other uninformed traders is π_2. π_1 and π_2 can describe the aggressiveness of the orders. For example, if there are more aggressive traders in the market, π_1 or π_2 would be small.

It is straightforward to see that:

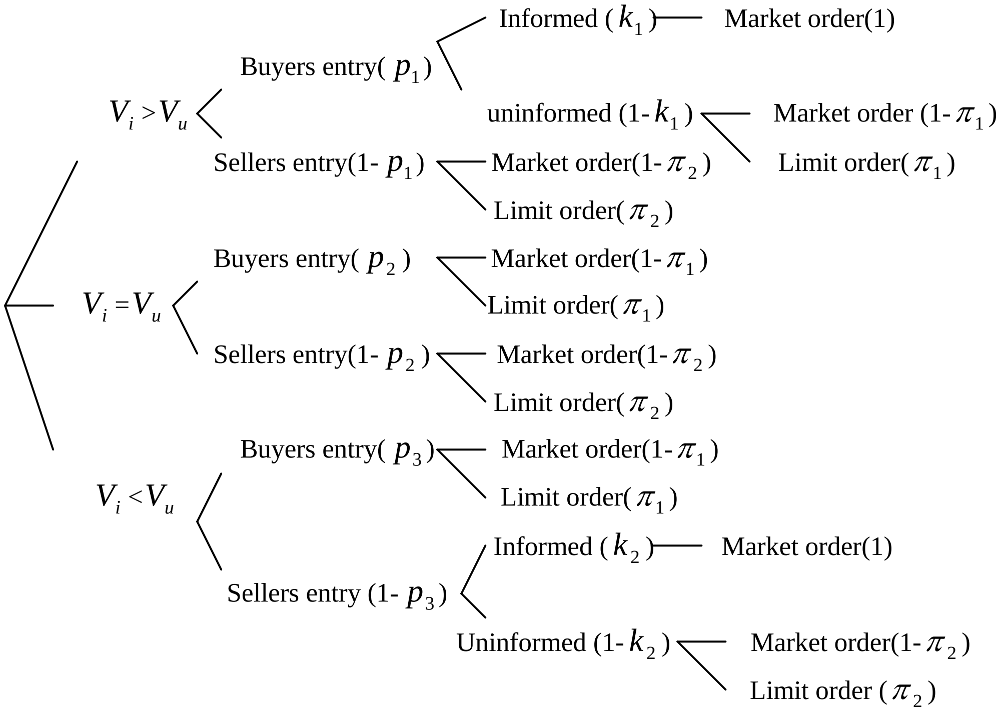
1.If $V_i > V_u$, there are $I+U_b$ buyers and U_s sellers, the probability of buyer arrival is $p_1=\frac{I+U_b}{I+U}$, so the probability of sellers entering the market is ($1-p_1$). The proportion of informed buyers to total buyers is $k_1=\frac{I}{I+U_b}$, and there are no informed sellers in this case.

2.If $V_i = V_u$, there are U_b buyers and U_s sellers, the probability of buyer arrival is $p_2=\frac{U_b}{U}$, and the probability of sellers entering the market is ($1-p_2$). Informed traders do not trade in this case.

3.If $V_i < V_u$, there are U_b buyers and $I+U_s$ sellers, the probability of buyers entering the market is $p_3 = \frac{U_b}{I+U}$, and the probability of sellers entering the market is (1- p_3). The proportion of informed sellers to total sellers is $k_2 = \frac{I}{I+U_s}$, and there are no informed buyers in this case.

Figure 1 summarizes the order paths faced by an uninformed trader.

Figure 1. The order path faced by uninformed traders.



This tree describe the possible paths faced by uninformed traders. V_i indicates the expected asset value of informed traders, and V_u indicates the expected asset value of uninformed traders. p_1, p_2, p_3 are the probabilities of buyers enter the market. k_1, k_2 are the probabilities of informed trading. π_1 and π_2 are the expected probability of a limit buy and limit sell orders placed by other uninformed traders.

Upon arrival, an uninformed trader can choose to submit a limit or a market order. Based on the order path in Figure 1, the expected utility of the uninformed trader is analyzed as follows.

First, consider an uninformed buyer who arrives at time *t* and places a limit order. Let B_t be the bid price. The order will be executed if (1) the trading interval does not stop before the arrival of the next trader, (2) the next trader is a seller, and (3) the next trader submit a market order. The expected utility of this buyer is:

$$E(U) = \frac{\rho_t}{3}\{(1-\pi_2)[3-(p_1+p_2+p_3)+p_3k_2]\}\{(V_u+\varepsilon)-B_t\} \tag{1}$$

Let A_{t-1} be the ask price of an uninformed seller arrives at time *t-1*. The expected utility of an uninformed buyer who arrives at time *t* and submits a market buy order:

$$E(U) = (V_u+\varepsilon)-A_{t-1} \tag{2}$$

For a buyer to be indifferent between a market order and a limit order, B_t must satisfy the following equality:

$$(V_u+\varepsilon)-A_{t-1} = \rho_t\,\frac{1}{3}\{(1-\pi_2)[3-(p_1+p_2+p_3)+p_3k_2]\}\{(V_u+\varepsilon)-B_t\} \tag{3}$$

Similarly, let A_t be the ask price of a limit sell. The expected utility of this seller is:

$$E(U) = \frac{\rho_t}{3}\{(1-\pi_1)[p_1(1-k_1)+p_2+p_3]+p_1k_1\}\{A_t-(V_u-\varepsilon)\} \tag{4}$$

Let B_{t-1} be the bid price of a buyer arriving at time *t-1*. Hence, the expected utility of a seller who arrives at time *t* and submits a market sell order is:

$$E(U) = B_{t-1}-(V_u-\varepsilon) \tag{5}$$

For a seller to be indifferent between a market and a limit order, A_t satisfies:

$$B_{t-1} - (V_u - \varepsilon) = \rho_t \frac{1}{3}\{(1-\pi_1)[p_1(1-k_1) + p_2 + p_3] + p_1 k_1\}\{A_t - (V_u - \varepsilon)\} \tag{6}$$

Stationary solution can be derived from Equations (5) and (6), and it does not depend on time[9]. The equilibrium is characterized by the optimal bid and ask prices { A^*, B^* } as established in Proposition 1.

Proposition 1.

Given the values for $I, U_b, U_s, V_H, V_L, \pi_1$ and π_2, the equilibrium bid and ask prices can be expressed as

$$B^* = V_u + \frac{2y\varepsilon - (1+xy)\varepsilon}{(1-xy)} \tag{7}$$

$$A^* = V_u + \frac{(1+xy)\varepsilon - 2x\varepsilon}{(1-xy)} \tag{8}$$

Bid ask spread: $s = A^* - B^* = 2\varepsilon\{\frac{(1-x)(1-y)}{(1-xy)}\} \geq 0$ (9)

Where $p_1 = \frac{I+U_b}{I+U}$, $p_2 = \frac{U_b}{U}$, $p_3 = \frac{U_b}{I+U}$, $k_1 = \frac{I}{I+U_b}$, $k_2 = \frac{I}{I+U_s}$

$$x = \rho \frac{1}{3}\{(1-\pi_2)[3-(p_1+p_2+p_3)+p_3 k_2]\}, \quad 0 \leq x \leq 1 \tag{10}$$

$$y = \rho \frac{1}{3}\{(1-\pi_1)[p_1(1-k_1)+p_2+p_3]+p_1 k_1\}, \quad 0 \leq y \leq 1 \text{且} 0 \leq x+y \leq 1 \tag{11}$$

Proof. See the Appendix 1.

x is the average probability of execution of a limit buy order, and y is the average probability of execution of a limit sell order. In equilibrium, the relationships between the limit price and the unconditional expected value of asset are as follows:

[9] The structure of this model is similar to that in Foucault (1999) where the order placement strategy is endogenous, hence there is no point in time from which we can start solving recursively for the equilibrium.

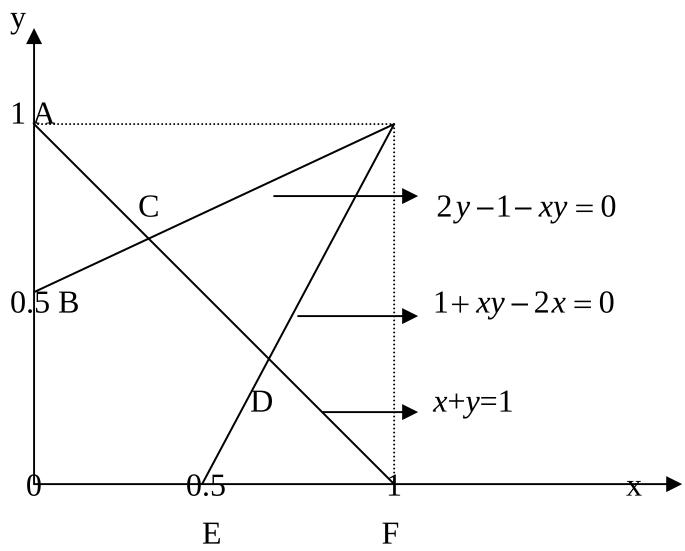
1. When $2y-1-xy\geq 0$, then $A^* \geq B^* \geq V_u$. (12a)

2. When $1+xy-2x\geq 0$ and $2y-1-xy\leq 0$, then $A^* \geq V_u \geq B^*$. (12b)

3. When $1+xy-2x\leq 0$, then $V_u \geq A^* \geq B^*$. (12c)

Equations 12a, 12b and 12c delineate how the level of limit prices is affected by the execution probabilities *x* and *y*. The three conditions of *x* and *y* are illustrated in Figure 2.

Figure 2. The space of *x* and *y*



The horizontal axis *x* is the average probability of execution of a limit buy order, and the vertical axis y is the average probability of execution of a limit sell order

Figure 2 shows that, when the values of *x* and *y* are in the area of △ABC (*y* is far greater than *x*), then $A^* \geq B^* \geq V_u$; when the values of *x* and *y* are in the area of □BCDE (*x* is close to *y*), then $A^* \geq V_u \geq B^*$; when the values of *x* and *y* are in the area of $\triangle$DEF (*x* is far greater than *y*), then $V_u \geq A^* \geq B^*$.

From Equations (10) and (11), we know *x* is the average probability of execution of a limit buy order, and y is the average probability of execution of a limit sell order. The intuition of the relationship between the value of *x*, *y* and the limit prices is straightforward: When *x* is high, indicating the numbers of sellers is larger or sellers are more aggressive, hence the bid and ask prices will be lower. When *y* is high, indicating the number of buyers is larger or buyers are more aggressive, hence the bid and ask prices will be higher. Although the intuition of this relationship is straightforward, the conditions in (12) are helpful in the following discussion for the determinants of equilibrium bid-ask prices.

2.3 The determinants of the equilibrium bid-ask prices

We analyze the determinants of equilibrium bid-ask prices in three cases.

Case 1. $1 + xy - 2x \geq 0$, $2y - 1 - xy \leq 0$ ($A^* \geq V_u \geq B^*$)

Proposition 2

When $1 + xy - 2x \geq 0$ and $2y - 1 - xy \leq 0$, it can be shown that $\frac{\partial A^*}{\partial \varepsilon} \geq 0$, $\frac{\partial A^*}{\partial x} \leq 0$, $\frac{\partial A^*}{\partial y} \geq 0$, $\frac{\partial B^*}{\partial \varepsilon} \leq 0$, $\frac{\partial B^*}{\partial x} \leq 0$, $\frac{\partial B^*}{\partial y} \geq 0$.

Proof. See the Appendix 2.

The size of ε indicates the noise in public information. When the volatility of asset value ε rises, the risk of adverse selection born by uninformed traders increases. The uninformed require larger premium in this case, consequently, ask price increases and bid price decreases.

When the average probability of execution for a limit buy order is large, the non-execution risk of the buyers is lower than that of the sellers. Consequently, the bid price decreases as the buyers require higher premium, while the ask price decreases as the sellers seek to lower the risk of execution.

When the average probability of execution for a limit sell order is large, the non-execution risk of buyers are higher then that of the sellers, Consequently, the ask price increases as the sellers require higher premium, while the bid price increases as the buyers seek to reduce the risk of execution.

Case 2. $2y-1-xy \geq 0$ $(A^{*} \geq B^{*} \geq V_{u})$

Proposition 3

If $2y-1-xy \geq 0$, then $\frac{\partial A^{*}}{\partial \varepsilon} \geq 0$, $\frac{\partial A^{*}}{\partial x} \leq 0$, $\frac{\partial A^{*}}{\partial y} \geq 0$ 及 $\frac{\partial B^{*}}{\partial \varepsilon} \geq 0$, $\frac{\partial B^{*}}{\partial x} \leq 0$, $\frac{\partial B^{*}}{\partial y} \geq 0$。

Proof. See the Appendix 3.

In this case, the probability of execution of limit sell order y is far greater than x[10], and the non-execution risk is very high for the uninformed traders with high valuations. As ε increases, buyer's valuation rises, to reduce the risk of non-execution they need to increase the bid price. Except for the relation between ε and B^{*}, all relationships hold as in Proposition 2.

Case 3. $1+xy-2x \leq 0$ $(V_{u} \geq A^{*} \geq B^{*})$

Proposition 4

[10] See the figure 2.

If $1+xy-2x\le 0$, then $\frac{\partial A^*}{\partial \varepsilon}\le 0$, $\frac{\partial A^*}{\partial x}\le 0$, $\frac{\partial A^*}{\partial y}\ge 0$, and $\frac{\partial B^*}{\partial \varepsilon}\le 0$, $\frac{\partial B^*}{\partial x}\le 0$, $\frac{\partial B^*}{\partial y}\ge 0$。

Proof. See the Appendix 4.

In this case, *x*, the probability of execution of limit buy order, is far greater than *y*[11], and the non-execution risk is very high for the uninformed traders with low valuations. So when ε rises, the seller's valuation of asset falls, to reduce the risk of non-execution they need to lower the ask price. Except for the relationship between ε and A^*, all relationships hold as in the proposition 2.

The following comparative statics show the relations between the bid-ask spread and its determinants.

Proposition 5.

The relationships between the bid-ask spread and ε, x, y, are

$$\frac{\partial s}{\partial \varepsilon}\ge 0,\quad \frac{\partial s}{\partial x}\le 0,\quad \frac{\partial s}{\partial y}\le 0.$$

Proof. See the Appendix 5.

As the volatility of asset value rises, ε rises, the risk of adverse selection perceived by uninformed traders also increases. Traders require higher premium, consequently, ask price increases and bid price decreases and the bid-ask spread widens.

When the number of uninformed traders increases, both *x* and *y* increase, the risk of adverse selection perceived by uninformed traders decreases, traders require less premium, consequently, ask price decreases and bid price increases, causing the

[11] See Figure 2.

bid-ask spread to narrow.

2.4 Dynamic Analysis

In the pervious static equilibrium analysis, we find that, consistent with other studies[12], the levels of *x* and *y* affect the equilibrium price and bid-ask spread. But in the real world, the probability of order execution is not constant. In this section, we proceed to the dynamic analysis.

In the above analysis, we assume there are three equally possible conditions between public and private information: $V_i > V_u, V_i = V_u, V_i < V_u$. However, as trading progresses, the expectation of the uninformed traders will adjust according to the order flow. For example, if only sell limit orders are executed, the uninformed traders would not perceive equal probability for the three situations. We will relax the equal probability assumption in the following analysis.

If the last executed limit order is a sell order, the conditional probabilities of the three market situations are:

$$\delta_{1t} \left| I_{t-1} = \Pr ob_t (V_i > V_u \right| EL_{t-1} = SO) = \frac{\delta_{1t-1}[p_1 k_1 + p_1(1-k_1)(1-\pi_1)]}{y_{t-1}} \tag{13}$$

$$\delta_{2t} \left| I_{t-1} = \Pr ob_t (V_i = V_u \right| EL_{t-1} = SO) = \frac{\delta_{2t-1}[p_2(1-\pi_1)]}{y_{t-1}} \tag{14}$$

$$\delta_{3t} \left| I_{t-1} = \Pr ob_t (V_i < V_u \right| EL_{t-1} = SO) = \frac{\delta_{3t-1}[p_3(1-\pi_1)]}{y_{t-1}} \tag{15}$$

If the last executed limit order is a buy order, then the conditional probabilities of the three market situations are:

$$\lambda_{1t} \left| I_{t-1} = \Pr ob_t (V_i > V_u \right| EL_{t-1} = BO) = \frac{\lambda_{1t-1}[(1-p_1)(1-\pi_2)]}{x_{t-1}} \tag{16}$$

[12] Foucault (1999) and Handa, Schwartz and Tiwari (2003) also find the probability of order execution affect the equilibrium price and bid-ask spread. .

$$\lambda_{2t}\left|I_{t-1} = \Pr ob_t(V_i = V_u\right|EL_{t-1} = BO) = \frac{\lambda_{2t-1}[(1-p_2)(1-\pi_2)]}{x_{t-1}} \quad (17)$$

$$\lambda_{3t}\left|I_{t-1} = \Pr ob_t(V_i < V_u\right|EL_{t-1} = BO) = \frac{\lambda_{3t-1}[k_2(1-p_3)+(1-k_2)(1-p_3)(1-\pi_2)]}{x_{t-1}} \quad (18)$$

Where I_{t-1} denotes the information set at time *t-1, Prob* denotes probability, EL_{t-1} is the executed limit order at time *t-1, SO* is sell order, *BO* is buy order.

Given that the last executed limit order is a sell order, the conditional probability of execution of a limit sell order is:

$$E(y_t|EL_{t-1} = SO) = \rho_t\{\delta_{1t}|I_{t-1} \times [p_1k_1 + p_1(1-k_1)(1-\pi_1)]$$

$$+\delta_{2t}|I_{t-1} \times p_2(1-\pi_1) + \delta_{3t}|I_{t-1} \times p_3(1-\pi_1)\} \quad (19)$$

Given that the last executed limit order is a buy order, the conditional probability of execution of a limit sell order is:

$$E(y_t|EL_{t-1} = BO) = \rho_t\{\lambda_{1t}|I_{t-1} \times [p_1k_1 + p_1(1-k_1)(1-\pi_1)]$$

$$+\lambda_{2t}|I_{t-1} \times p_2(1-\pi_1) + \lambda_{3t}|I_{t-1} \times p_3(1-\pi_1)\} \quad (20)$$

Given that the last executed limit order is a sell order, the conditional probability of execution of a limit buy order is:

$$E(x_t|EL_{t-1} = SO) = \rho_t\{\delta_{1t}|I_{t-1} \times [(1-p_1)(1-\pi_2)] + \delta_{2t}|I_{t-1} \times [(1-p_2)(1-\pi_2)]$$

$$+\delta_{3t}|I_{t-1} \times [k_2(1-p_3) + (1-k_2)(1-p_3)(1-\pi_2)]\} \quad (21)$$

Given that the last executed limit order is a buy order, the conditional probability of execution of a limit buy order is:

$$E(x_t \left| EL_{t-1} = BO \right.)= \rho_t \{ \lambda_{1t} \left| I_{t-1} \right. \times [(1-p_1)(1-\pi_2)] + \lambda_{2t} \left| I_{t-1} \right. \times [(1-p_2)(1-\pi_2)]$$

$$+ \lambda_{3t} \left| I_{t-1} \right. \times [k_2(1-p_3) + (1-k_2)(1-p_3)(1-\pi_2)] \} \quad (22)$$

From equations (13) to (22), we find that the average probability of execution is influenced by the probability of the stop of trading process (1-ρ_t), the structure of traders (I, U_b, U_s), the expected value of asset ($\delta_{it}, \lambda_{it}$), and the aggressiveness of traders (π_1, π_2). Hence, the optimal limit price is also influenced by these factors.

2.5 Order Sequences

Previous studies have found a conditional order flow pattern, for instance, after the arrival of a limit buy order at the best bid price, the incoming order is most likely to be the same order type, this is called the diagonal effects. This result is first documented in Biais, Hillion and Spatt (1995) in Paris Bourse, followed by Al-Suhaibani and Kryznowski (2000) on the Saudi Stock Market. Biais, Hillion and Spatt (1995) offered three explanations to the diagonal effect: strategic order splitting, trade imitation, or similar reaction to information event.

The finding on order sequences of this model is consistent with the documented diagonal effect. From equations (19) to (22), we can see that the conditional probability of order execution followed by the same type order is higher than the conditional probability of order execution followed by the different type order. If the last executed order is a sell order, then the conditional expected probability of $V_i > V_u$ increases, as a result, the probability of execution of limit sell order increases. If the last executed order is a buy order, then the conditional expected probability of $V_i > V_u$ decreases, as a result, the probability of execution of limit buy order increases. The following equations define the “diagonal effect”:

$$E(y_t \mid EL_{t-1} = SO) > E(y_t \mid EL_{t-1} = BO) \tag{23}$$

$$E(x_t \mid EL_{t-1} = SO) < E(x_t \mid EL_{t-1} = BO) \tag{24}$$

2.6 The cost of limit order trading

The cost of limit order trading has two components. One is the cost of non-execution risk, the other is the cost of adverse selection risk. Conditional on the private information, the non-execution cost of a limit buy order of the uninformed trader is:

$$1\text{-}\{\rho_t \times \theta_{1t} \mid I_{t-1} \times [(1-p_1)(1-\pi_2)]\}(V_i - B_t) \tag{25}$$

The cost of adverse selection risk of a limit buy order is:

$$\rho_t\, \{\theta_{3t} \mid I_{t-1} \times [k_2(1-p_3) + (1-k_2)(1-p_3)(1-\pi_2)]\}\, (V_i - B_t) \tag{26}$$

Conditional on the private information, the non-execution cost of a limit sell order of the uninformed trader is:

$$1\text{-}\{\rho_t \times 1\text{-}\theta_{3t} \mid I_{t-1} \times p_3(1-\pi_1)\}\,(A_t - V_i) \tag{27}$$

The cost of adverse selection risk of a limit sell order is:

$$\rho_t\, \{\theta_{1t} \mid I_{t-1} \times [p_1 k_1 + p_1(1-k_1)(1-\pi_1)]\}\,(A_t - V_i) \tag{28}$$

θ are the conditional probabilities of the three market situations. If the last executed limit order is a buy order, $\theta_{it} \mid I_{t-1} = \lambda_{it} \mid I_{t-1}$, if the last executed limit order is a sell order, $\theta_{it} \mid I_{t-1} = \delta_{it} \mid I_{t-1}$, i=1,3.

From equations (25) and (27), we find that the cost of non-execution risk is an increasing function of *t*, because (1-ρ_t) is an increasing function of *t*. On the other hand, from equations (26) and (28), we find that the cost of adverse selection risk is a decreasing function of *t*. Since as trading proceeds the information is disclosed, the expectation and the limit price of the uninformed traders will adjust to the order flow accordingly to reduce the risk of adverse selection. Then limit price of uninformed will be more efficient, i.e., $(V_i - B_t(\theta_{3t}|I_{t-1})) \leq (V_i - B_\tau(\theta_{3\tau}|I_{\tau-1}))$ and $(A_t(\theta_{1t}|I_{t-1})$-$V_i) \leq (A_\tau(\theta_{1\tau}|I_{\tau-1})$-$V_i)$ where τ <t. Therefore, the intraday pattern of the cost of limit order submitted by uninformed traders is U-shaped.

3. Model Implications and relations to the literature

Implication 1: The equilibrium limit buy and sell prices of liquidity traders are equal to the unconditional expected value of the asset.

If the uninformed traders are not different in terms of share valuation, i.e., the noise of the expected asset value (ε) is zero, then from equations (7) and (8), the equilibrium limit buy and sell prices are equal to V_u. Hence, if trades are driven by liquidity demand then the optimal limit price is equal to the unconditional expected value of the asset.

Implication 2: The bid-ask spread is an increasing function of the volatility of the unconditional expected value of the asset.

Implication 2 is consistent with theoretical models of the bid-ask spread in dealer markets (see, for example, Copeland and Galai, 1983; Easley and O’Hara, 1987) and in order-driven markets (see, for example, Handa et al., 2003). They all find that the bid-ask spread is increasing in information asymmetry and in the degree of asset value uncertainty.

Implication 3: The bid-ask spread is a decreasing function of the number of uninformed traders.

By proposition 5, when the number of uninformed traders increases, both *x* and *y* increase, the risk of adverse selection perceived by uninformed traders decreases, the traders will require less premium, consequently, ask price decreases and bid price increases, causing the bid-ask spread to narrow.

Glosten and Milgrom (1985) suggest that the bid-ask spread in dealer markets contains an informational component, while the market maker loses to informed traders on average, but recoups these losses on noise trades. The market maker must trade off the reduction in losses to the informed from a wider spread against the opportunity cost in terms of profits from trading with uniformed traders with reservation prices inside the spread. The situation faced by uninformed limit order traders is similar to the market maker, therefore when the number of uninformed traders increases, the risk of adverse selection decreases, then the bid-ask spread of limit prices is narrow.

Implication 4: When there is no order imbalance in the market, greater asset value noise leads to lower bid price and higher ask price. If there are far more sellers than buyers, then increasing asset noise will lead to lower ask price; and if there are far more buyers than sellers, the greater noise will lead to higher bid price

Handa et al. (2003) show that the spread in an order driven market is highest when the buy and sell orders are balanced, and the spread is minimized when there is large order imbalance. In their model, only the risk of adverse selection is considered. In our model, the cost of limit order includes adverse selection as well as non-execution risk. We show that the structure of traders not only influences the size of spread, but also influences the relation between noise and limit prices.

Implication 5: The probabilities of limit order execution are influenced by the probability of the trading process stops, the structure of traders, the expected value of asset, and the expected aggressiveness of the other traders.

From equations (13) to (22), we find that the average probability of execution is influenced by the probability of the trading process stops (1-ρ_t), the structure of traders (I, U_b, U_s), the expected value of asset ($\delta_{it}, \lambda_{it}$), and the aggressiveness of traders (π_1, π_2). Hence, the optimal limit price is also influenced by the above factors.

The determinants of the probabilities of execution in this paper are closely related to Foucault (1999), Foucault, Kadan, Kandel (2001), and Handa, Schwartz, Tiwari (2003); supports the empirical findings of Hollifield, Miller, Sandas (2001), and Hollifield, Miller, Sandas and Slive (2002).

Implication 6: The conditional probability of limit order execution followed by the same type order should be higher than the conditional probability of limit order execution followed by different type order.

Previous studies have found a conditional order flow pattern, for instance, after the arrival of a limit buy order at the best bid price, the incoming order is most likely to be the same order type, this is called the diagonal effects (see, Biais, Hillion and Spatt, 1995; and Kryznowski, 2000). The finding on conditional “execution” order flow pattern of this model is consistent with the previous studies. Furthermore, the implication supports the empirical findings of Ranaldo (2003) that is contrary to Parlour (1998).

Implication 7: The intraday pattern of the cost of limit order submitted by uninformed traders is U-shaped.

Admati and Pfleiderer (1988) develop a theory to explain the concentration of trading at the open and the close of a day. They propose that discretionary liquidity trading and informed trading will concentrate at the open and the close, due to higher liquidity trading in these periods. Following their thought, suppose that the discretionary liquidity traders choose trading time to minimize the trading cost, then the commonly observed U-shaped pattern of trading activities[13] should suggest an inverse U-shaped pattern of trading cost, contrary to our finding. But studies have also found that the variance of price and the variance of returns follow a U-shaped pattern[14]. Glosten (1994) argues that limit order traders profit from liquidity driven traders but lose from information driven price changes. Therefore, if the large price changes in the beginning and the end of the trading day are attributed to informed trading, then the trading cost of the uninformed limit order traders will be large in the beginning and the end of the trading day, as predicted by our model.

Foster and Viswanathan (1990) develop an adverse selection model and examine the interday variations in volume, variance and adverse selection costs. They find that on Monday the trading costs and the variance of price changes are highest. Our finding on adverse selection costs is similar in that the adverse selection costs of the uninformed limit order traders is large in the beginning of the trading periods. In addition to intraday patterns of volume and volatility, many empirical studies have documented the intraday U-shaped behavior of the bid-ask spread[15]. Our model helps to explain this phenomenon by exploring changes in the total trading costs of the

---

[13] The U-shaped pattern of the average volume of shares traded has been documented in a number of studies, for example: Jain and Joh (1986).
[14] For example, see Wood, McInish and Ord (1985).
[15] Declerck (2000) used the data of the Paris Bourse to show the relative spread intraday pattern is U-shaped. Ranaldo (2003) find that the spread in the Swiss Stock Exchange shows a intraday U-shaped pattern. McInish and Wood (1992), Brock and Kleidon (1992), Lee, Mucklow, and Ready (1993), and Chan, Chung, and Johnson (1995) show that the spread of NYSE stocks is widest at the open, drops during the first hour of trading, and increases slightly before the market close.

uninformed traders during a trading day.

4. Conclusion

We develop an information asymmetric model, in which the conditional probability of order execution and the cost of limit orders are dynamic.

There are several interesting implications of this model that are closely related to existing empirical and theoretical studies, for example, the implication about order sequences is consistent with the empirical evidences in Hamao and Hasbrouck (1995), Biais, Hillion, and Spatt (1995) and Ranaldo (2003); the implication about bid-ask spread in the order driven market is similar to the theories of the bid-ask spread in the dealer market. Furthermore, our result complements the literature on the trading costs of limit order: We show that the intraday pattern of the cost of limit order submitted by uninformed traders is U-shaped. Our findings may shed more light on the dynamics of order execution and the intraday pattern of market performances in the order driven market.

Appendix 1

Proof of proposition 1.

Given our framework, consider the optimal order placement decision of uninformed buyers. The price that they are indifferent between a buy market order or a buy limit order will satisfies:

$$(V_u + \varepsilon) - A^* = \rho \, \frac{1}{3}\{(1-\pi_2)[3-(p_1+p_2+p_3)+p_3k_2]\}\,\{(V_u+\varepsilon)-B^*\} \qquad \text{(A1)}$$

The uninformed sells face exactly the same type of problem, so we can write:

$$B^* - (V_u - \varepsilon) = \rho\,\frac{1}{3}\{(1-\pi_1)[p_1(1-k_1)+p_2+p_3]+p_1k_1\}\,\{A^* - (V_u-\varepsilon)\} \tag{A2}$$

Let x= $\rho\,\frac{1}{3}\{(1-\pi_2)[3-(p_1+p_2+p_3)+p_3k_2]\}$ (A3)

y= $\rho\,\frac{1}{3}\{(1-\pi_1)[p_1(1-k_1)+p_2+p_3]+p_1k_1\}$ (A4)

The Eqs. (A1) and (A2) become:

$$(V_u+\varepsilon) - A^* = x\{(V_u+\varepsilon) - B^*\} \tag{A5}$$

$$B^* - (V_u-\varepsilon) = y\{A^* - (V_u-\varepsilon)\} \tag{A6}$$

Solving Eqs. (A5) and (A6), we obtain:

$$A^* = V_u + \frac{(1+xy)\varepsilon - 2x\varepsilon}{(1-xy)} \tag{A7}$$

$$B^* = V_u + \frac{2y\varepsilon - (1+xy)\varepsilon}{(1-xy)} \tag{A8}$$

$$A^* - B^* = 2\varepsilon\{\frac{(1-x)(1-y)}{(1-xy)}\} \tag{A9}$$

And x= $\rho\,\frac{1}{3}\{(1-\pi_2)[3-(p_1+p_2+p_3)+p_3k_2]\}$ (A10)

Because x is the average execution probability of a limit buy order so $0 \le x \le 1$。

y= $\rho\,\frac{1}{3}\{(1-\pi_1)[p_1(1-k_1)+p_2+p_3]+p_1k_1\}$ (A11)

Because y is the average execution probability of a limit sell order so $0 \le y \le 1$, and $xy \le 1$。

And adding Eqs. (A10) and (A11), we obtain:

$$x + y \le \frac{1}{3}[3-(p_1+p_2+p_3)+p_3k_2] + \frac{1}{3}[p_1(1-k_1)+p_2+p_3]+p_1k_1 = 1 \tag{A12}$$

Appendix 2

Proof of proposition 2.

Using Eqs. (A7)(A8), we obtain:

$$\frac{\partial A^*}{\partial \varepsilon}=\frac{(1+xy-2x)}{(1-xy)}\geq 0 \tag{A13}$$

$$\frac{\partial A^*}{\partial x}=\frac{(1-xy)(y\varepsilon-2\varepsilon)+[(1+xy)\varepsilon-2x\varepsilon](-y)}{(1-xy)^2}\leq 0 \tag{A14}$$

$$\frac{\partial A^*}{\partial y}=\frac{(1-xy)(x\varepsilon)+[(1+xy)\varepsilon-2x\varepsilon](-x)}{(1-xy)^2}\geq 0 \tag{A15}$$

$$\frac{\partial B^*}{\partial \varepsilon}=\frac{2y-(1+xy)}{(1-xy)}\leq 0 \tag{A16}$$

$$\frac{\partial B^*}{\partial x}=\frac{(1-xy)(-y\varepsilon)+[(2y\varepsilon-(1+xy)\varepsilon](-y)}{(1-xy)^2}\leq 0 \tag{A17}$$

$$\frac{\partial B^*}{\partial y}=\frac{(1-xy)(2\varepsilon-x\varepsilon)+[(2y\varepsilon-(1+xy)\varepsilon](-x)}{(1-xy)^2}\geq 0 \tag{A18}$$

Appendix 3

Proof of proposition 3.

By the condition: $2y-1-xy\geq 0$ and equations from (A13) to (A18), we obtain:

$$\frac{\partial A^*}{\partial \varepsilon}\geq 0,\quad \frac{\partial A^*}{\partial x}\leq 0,\quad \frac{\partial A^*}{\partial y}\geq 0 \text{ 及 } \frac{\partial B^*}{\partial \varepsilon}\geq 0,\quad \frac{\partial B^*}{\partial x}\leq 0,\quad \frac{\partial B^*}{\partial y}\geq 0\text{。}$$

Appendix 4

Proof of proposition 4.

By the condition: $1+xy-2x\leq 0$ and equations from (A13) to (A18), we obtain:

$$\frac{\partial A^*}{\partial \varepsilon}\leq 0,\quad \frac{\partial A^*}{\partial x}\leq 0,\quad \frac{\partial A^*}{\partial y}\geq 0 \text{ 及 } \frac{\partial B^*}{\partial \varepsilon}\geq 0,\quad \frac{\partial B^*}{\partial x}\leq 0,\quad \frac{\partial B^*}{\partial y}\geq 0\text{。}$$

Appendix 5

Proof of proposition 5.

$$\frac{\partial s}{\partial \varepsilon} = 2\{\frac{(1-x)(1-y)}{(1-xy)}\} \geq 0 \qquad \text{(A19)}$$

$$\frac{\partial s}{\partial x} = \frac{(1-xy)(-2\varepsilon)(1-y) + 2\varepsilon(1-x)(1-y)y}{(1-xy)^2} \leq 0 \qquad \text{(A20)}$$

$$\frac{\partial s}{\partial y} = \frac{(1-xy)(-2\varepsilon)(1-x) + 2\varepsilon(1-x)(1-y)x}{(1-xy)^2} \leq 0 \qquad \text{(A21)}$$

Reference:

Admati A. R. and P. Pfleiderer (1988), “A theory of intraday patterns: volume and price variability,” *The Review of Financial Studies* 1, 3-40.

Al-Suhaibani M. and Kryzanowski L. (2001), “Limit vs. market order trading on the Saudi Stock Market,” working paper, Department of Economics, Imam University, Riyadh, Saudi Arabia.

Angle, J (1994), “Market and limit orders,” *Canadian Shareowner* 8, 18-20.

Bagehot, Walter (1971), “The only game in town,” *Financial Analysts Journal* 27, 12-14.

Bae, K.-H., H. Jang and K.S. Park (2003), “Traders’ choice between limit and market orders: evidence from NYSE stocks,” *Journal of Financial Markets* 6, 517-538.

Biais, B., Hillion, P., and Spatt, C. (1995), “An empirical analysis of the limit order book and the order flow in the Paris Bourse,” *Journal of Finance* 50, 1655-1689.

Brock, W. and A. Kleidon, 1992, Periodic market closure and trading volume: a model of intraday bids and asks, *Journal of Economic Dynamics and Control* 16, 451-489.

Chan, K., P. Chung, and H. Johnson, 1995, The intraday behavior of bid-ask spreads for NYSE stocks and CBOE options, *Journal of Quantitative and Financial Analysis* 30, 329-346.

Copeland, T., Galai, D. (1983), “Information effects on the bid-ask spreads,” *Journal of Finance* 38, 1457-1469.

Easley, D., and M. O’Hara (1987), “Price, trade Size and information insecurities markets,” *Journal of Financial Economics* 19, 69-90.

Fany Declerck (2000), “Trading costs on a limit order book: evidence from the Paris Bourse,” working paper, ESA, University de Lille2.

Foucault, T. (1999), “Order flow composition and trading cost in a dynamic limit order market,” *Journal of Financial Markets* 2, 99-134.

Foucault, Kadan, Kandel (2001), “Limit order book as a market for liquidity,” Working Paper, HEC School of Management, Paris; Centre for Economic Policy Research (CEPR).

Foster F. D. and S. Viawanathan (1990), “A theory of the interday variations in volume, variance, and trading costs in securities markets,” *The Review of Financial Studie*s 3, 593-624.

Glosten, Lawrence R (1989), “Insider trading, liquidity, and the role of the monopolist specialist,” *Journal of Business* 62, 211-236.

Glosten, Lawrence R (1994), “Is the electronic open limit order book inevitable?” *Journal of Finance* 49, 1127-1161.

Glosten, L., Milgrom, P. (1985), “Bid, ask and transaction prices in a specialist market with heterogeneously informed traders,” Journal of Financial Economics 14, 71-100.

Hamao, Y., and Hasbrouck, J. (1995), "Securities trading in the absence of dealers: trades and quotes on the Tokyo Stock Exchange," *Review of Financial Studies* 8, 849-878.

Handa, P., and R. Schwartz (1996), "Limit order trading," *Journal of Finance* 51, 1835-1861.

Handa, P., R. Schwartz, A. Tiwari (2003), "Quote setting and price formation in an order driven market," *Journal of Financial Markets* 6, 461-489.

Harris, L. (1998), "Optimal dynamic order submission strategies in some stylized trading problems," working paper, University of Southern California.

Harris, L. and Hasbrouck, J. (1996), "Market versus limit orders: the SuperDOT evidence on order submission strategies," *Journal of Financial and Quantitative Analysis* 31, 213-231.

Hollifield, B., R. A. Miller, and P. Sandas (2001), "Empirical Analysis of Limit Order Markets," working paper, Carnegie Mellon University.

Hollifield, B., R. A. Miller, P. Sandas J. Slive (2002), "Liquidity supply and demand in limit order markets" working paper, Carnegie Mellon University.

Jain, P. J., and G. Joh. (1986), "The dependence between hourly prices and trading volume," working paper, University of Pennsylvania, Wharton School.

Lee, C. M.C., B. Mucklow, and M. Ready, 1993, spreads, depths, and the impact of earnings information: an intraday analysis, *Review of Financial Studies* 6, 345-374.

Lo, A., A. C. Mackinlay and J. Zhang (2001), "Econometric models of limit-order executions," *Journal of Financial Economics* 65, 31-71.

McInish, T. and R. A. Woods,1992, An analysis of intraday patterns in bid/ask spreads for NYSE stocks, Journal of Finance 47, 753-764.

Parlour, C. (1998), “Price dynamics in limit order market,” *Review of Financial Studies* 11, 789-816.

Ranaldo, Angelo (2003), “Order Aggressiveness,” Working paper, Olsen & Associates, Seefeldstrasse 233, 8008 Zurich, Switzerland.

Sandas, P. (2001), “Adverse selection and competitive market making: empirical evidence from a limit order market,” *Review of Financial Studies* 14, 705-734.

Seppi, Duane (1997), “Liquidity provision with limit orders and a strategic specialist,” *Review of Financial Studies* 10, 103-174.

Wood. R. A., T. H. McInish, and J. K. Ord (1985), “An investigation of transaction data for NYSE stocks,” Journal of Finance 40, 723-741.