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Does purchasing power parity hold for countries with closer economic relations?\*

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Abstract

This paper provides empirical tests of the efficient-markets view of purchasing power parity (PPP) for countries with closer economic relations. A cross-section approach is adopted in which Australia-New Zealand and USA-Canada data are used. The results are mixed. Interestingly, the standard (classical) procedures reject PPP, but their Bayesian counterparts provide reasonable support for these countries. It remains a challenge to explain this finding.

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## 1. Introduction

The theory of purchasing power parity (PPP) is arguably the oldest and most controversial in the exchange rate literature. It states that prices across countries are arbitrated so that they are the same when expressed in a common currency (absolute PPP), or less strictly, the change in the exchange rate for any two countries' currencies approximates the difference of their respective inflation rates. Historically, this theory has experienced considerable ups and downs. Nevertheless, presumably because of simplicity and practical versatility, PPP has continued to occupy the center stage of contemporary research in international finance<sup>1</sup>. In quantifying the amount of research into PPP during the past three decades, Lan [2002] puts it, '... rather than 'collapsing', PPP research can be described as 'exploding' (p.12). Rogoff [1996], and more recently, Sarno and Taylor [2002], among others, provide excellent survey of the state of play in this regard<sup>2</sup>.

A stylized feature of recent work on PPP is to concentrate on testing the efficient-markets version of PPP (that is, the hypothesis of random walk behaviour in real exchange rates). Originally due to Roll [1979], this version implies that the real exchange rate should follow a random walk process, which is contrary to PPP in the way that there is no long-term equilibrium value to which the real exchange rate tends to return and that these changes are permanent. However, empirical results based on this approach, mainly for major industrialized countries, have so far been very mixed. See Lan [2002] for an excellent review of the existing literature.

The purpose of this paper is to conduct further tests of PPP along the above lines for Australia and New Zealand. For comparison purposes, we also run a parallel set of tests for USA and Canada. Note that these two pairs of countries share considerable similarities in terms of closer bilateral economic relations<sup>3</sup>. Consequently, arbitrage, and hence PPP, should perform better for these countries pair wise. We present the test procedures in the next section, followed by a discussion of empirical results. The last section concludes.

## 2. Time-series techniques

The standard procedure to test whether or not the real exchange rate follows a random walk process induces the following autoregressive model of the real exchange rate:

$$r_t - \mu = \rho(r_t - \mu) + \varepsilon_t \quad (1)$$

where  $r_t$  is the real exchange rate at time  $t$ ,  $\mu$  is the "long-run" equilibrium value of the real exchange rate consistent with PPP, and  $\varepsilon_t \sim N(0, \sigma^2)$  is the serially-uncorrelated error term. In this model, the autoregressive coefficient  $\rho$  is the critical value for the long-run behavior of the real exchange rate. If  $0 < \rho < 1$ , indicates that the real exchange rate tends to move smoothly towards its long-run equilibrium and, hence, the process is stable. This means that in the absence of future shocks, the deviation from the long-run equilibrium,  $r_t - \mu$ , would shrink in the subsequent periods when  $0 < \rho < 1$ . However, if

there is a unit root ( $\rho=1$ ), the deviations from the long-run equilibrium show no tendency to shrink and behavior of the real exchange rate is a random walk.

In the literature on empirical tests of PPP, there are controversies about the testing procedures and the power of the classical tests. One problem occurs when using conventional unit root tests as the Dickey and Fuller [1979] to make statistical inferences about the value of  $\rho$ , as pointed out by Hakkio [1986], in the way that these tests have low power against the alternative hypothesis that the real exchange rate has a sizable autocorrelation coefficient. The existing empirical literature suggests either that the real exchange rate is a random walk, or that it has a sizable autocorrelation coefficient that is statistically very difficult to distinguish from a unit root, given the low power of the statistical tests that have been used [Whitt, 1992]. Hakkio [1986] provides evidence that standard statistical tests have very low power when the autocorrelation coefficient is fairly close to unity and that the power of these tests is often below 20 percent.

Sims [1988] argues that these classical tests are fundamentally biased towards favoring the unit root hypothesis, and as an alternative he proposes a test based on Bayesian posterior odds ratios that are designed to discriminate between a unit root and a large but stationary autocorrelation coefficient. He claims that this bias stems from the discontinuity in the asymptotic distribution theory and disjointed confidence regions, which would encourage unreasonable approaches to inference [Sims, 1988]. The alternative test proposed by Sims [1988] for the presence of unit roots is given by a weighted average of the likelihood function over all points consistent with the null hypothesis, divided by a similar function for the alternative hypothesis, with weights derived from the prior distribution of the parameters. To apply this procedure to equation (1), Sims [1988] proposes a probability  $\alpha$  for a prior distribution for  $\rho$  which spreads between  $0 < \alpha < 1$ , and gives the unit root ( $\rho = 1$ ) probability  $(1 - \alpha)$ . By contrast, Sims and Uhlig [1988] demonstrates that when using classical unit root tests, in the case of an OLS estimated  $\hat{\rho}$  equal to 0.95, the implicit prior treats the values of  $\rho$  around unity as being two to three times more likely than values around 0.90, even though the likelihood function is symmetric around 0.95. And, more disturbingly, the prior implicit in classical tests gives even more weight to values of  $\rho$  above unity than to values around unity, and the weight given to the region above unity increase as  $\hat{\rho}$  approaches unity from below [Whitt, 1992]. To derive the approximate likelihood function, Sims [1988] defines  $T \equiv ([1 - \hat{\rho}] / \sigma_\rho)$ , where  $\sigma_\rho \equiv \sqrt{(\sigma^2 / \sum r_{t-1}^2)}$ , to be the conventional  $t$  statistic for testing  $\rho = 1$ ,  $\phi(T)$  to be the cumulative distribution function for the standard normal distribution evaluated at  $T$ , and  $\theta(T)$  to be its probability density function. Sims then shows that in large samples the posterior odds ratio favors the null hypothesis ( $\rho = 1$ ) if:

$$\frac{(1 - \alpha)\theta(T)}{\sigma_\rho [\alpha\phi(T)]} > 1. \quad (2)$$

The disparity between this criterion and classical hypothesis tests, is not only because of the role of the prior distribution, but also because the presence of  $\sigma_\rho$  in the denominator.

If the null hypothesis ( $\rho = 1$ ) is true,  $\sigma_\rho$  should shrink much faster as sample size increase than when the alternative hypothesis is true. This criterion tends to favor the null hypothesis ( $\rho = 1$ ) when  $\sigma_\rho$  is small. By contrast, likelihood ratio tests fail to use all the information in  $\sigma_\rho$ . Sims [1988] suggests that in actual applications using quarterly economic data, the alternative hypothesis can reasonably be limited to values of  $\rho$  approximately between 0.84 and unity<sup>4</sup>. Therefore, Sims [1988] proposes the following revised criterion where the null hypothesis ( $\rho = 1$ ) is favored if:

$$\gamma > 0 \tag{3}$$

where  $\gamma \equiv 2 \log([1 - \alpha]/\alpha) - \log(\sigma^2) + 2 \log(1 - 2^{[-1/s]}) - T^2$ ,  $s$  is the number of periods per year (for example, four for quarterly data). Smaller values of  $\sigma_\rho$  favor the unit root hypothesis, while larger values of  $T$  favor the alternative hypothesis.

The modeling objective of the Bayesian approach is not to reject or fail to reject a hypothesis, but to determine how probable a hypothesis is relative to other competing hypothesis. The most common method is to calculate posterior odds ratios for various competing hypothesis based on prior and sample information. This gives the researcher the odds in favor of the one hypothesis relative to other competing hypothesis [Ahking, 2002].

### 3. Empirical results

The data used in this paper, all collected from IMF *International Financial Statistics*, consist of quarterly nominal end-of-period exchange rates and consumer price indices<sup>5</sup> covering the period 1986:1 to 2004:2 for United States, Canada, Australia and New Zealand. Year 2000 is the base year for the consumer price indices for all countries. Note that the United States and Canada have experienced free floating exchange rates since the breakdown of the Bretton-Woods regime in 1973, and, Australia's exchange rate became floating in 1983, New Zealand did not exhibit a free floating exchange rate until March 1985. To conduct comparable examinations, we choose the start date of the sample period by allowing some time for the system to mature, and the end period is dictated by the availability of this data.

#### 3.1 Australia and New Zealand

As discussed above, the theory of purchasing power parity implies that the real exchange rate is stationary. To test for stationarity, (augmented) Dickey-Fuller tests for the presence of a unit root are performed. Simultaneously, the Sims test for the presence of a unit root is also carried out for comparison purposes. First, the (augmented) Dickey-Fuller tests are conducted on the real exchange rates between Australia and New Zealand, and the results are given in Table 1 .

Table 1: (Augmented) Dickey-Fuller tests for a unit root in the real exchange rate between Australia and New Zealand using data in levels

		Dickey-Fuller test				
		DF	ADF1	ADF2	ADF3	ADF4
Australia / New Zealand	No trend	-2.2519 (1.6213**)	-2.7149 (1.9445)	-2.5364 (2.0111)	-2.5278 (2.0119)	-2.3573 (1.9663)
	With trend	-2.2981 (1.6222**)	-2.7133 (1.9433)	-2.5620 (2.0162)	-2.5890 (2.0211)	-2.4589 (1.9755)

Note: The critical values for the (A)DF test at the 95% significant level are -2.89 (without trend) and -3.45 (with trend). Durbin-Watson statistic is in parenthesis. The critical values for the DW test at the 95% significant level are 1.58 and 1.64. One asterisk denotes neither rejecting nor accepting.

As can be seen, using this standard test, it is not possible to reject the null hypothesis of a unit root. The Durbin Watson Q-test, shown in parenthesis, is indicating autocorrelation in the lagged residuals using the Dickey-Fuller test. With lags introduced for the augmented Dickey-Fuller tests to eliminate significant autocorrelation in the lagged residuals, the results remain the same indicating that the real exchange rate is  $I(1)$  and requires to be differenced once to achieve stationarity.

We next employ the Sims test procedure, and the results are given in Table 2. The first column in Table 2 is the t-squared which is used as the test statistic. The “Schwarz limit” and the “Small sample limit” are the asymptotic and small sample Bayesian “critical values” for the test statistic. As can be seen, the test statistics are larger than, or very close to, its critical value, indicating that the null hypothesis of a unit root is rejected. Accordingly, contrary to the Dickey-Fuller results, the Sims test indicates that the null hypothesis of a random walk is rejected. The “Marginal alpha” presented in the last column of Table 2, provides a measure of how strong the rejection of the null

Table 2: Sims tests for a unit root in the real exchange rates between Australia and New Zealand using data in levels

		Sims test $\gamma$				
			Squared t	Schwarz limit	Small sample limit	Marginal alpha
Australia / New Zealand	No trend	No lags	5.071	5.559	-0.438	0.2029
		1 lag	5.071	5.559	-0.438	0.2029
		2 lags	7.371	5.532	-0.465	0.0737
		3 lags	6.433	5.411	-0.586	0.1068
		4 lags	6.390	5.306	-0.691	0.1040
	With trend	No lags	5.281	5.535	-0.461	0.1847
		1 lag	5.281	5.535	-0.461	0.1847
		2 lags	7.362	5.505	-0.492	0.0731
		3 lags	6.564	5.387	-0.610	0.0997
		4 lags	6.703	5.287	-0.710	0.0895

Note: Bayesian tests using RATS.

hypothesis is. The “Marginal alpha” is the value of alpha at which the posterior odds for and against the unit root are even. The smaller the value is, the stronger is the data’s rejection of the unit root hypothesis. On an ex post basis, it is possible to calculate the minimum prior probability on the null hypothesis,  $1-\alpha^*$ , that would be necessary in order to force the Sims criterion to favor the null hypothesis. As the last column in Table 2 indicates, the power of the rejection ranges between 0.7971 and 0.9105, which indicates a rejection of the unit root by a comfortable margin. These results from the Sims test are in favor of the long-run PPP. Interestingly, these results are consistent with Whitt [1992], and Manzur and Ariff [1995].

### 3.2 Do we have a similar story for USA and Canada?

The results for Australia and New Zealand above are puzzling. The classical unit root tests reject PPP decisively, while the Bayesian Sims test lends solid support to it. Do we get similar results for USA and Canada? Table 3 presents the ADF results for USA and Canada. As can be seen, these results are very similar to those in Table 1. That is, the USA/Canada real exchange rate contains a unit root.

Table 3: (Augmented) Dickey-Fuller tests for a unit root in the real exchange rate between United States and Canada using data in levels

		Dickey-Fuller test				
		DF	ADF1	ADF2	ADF3	ADF4
United States / Canada	No trend	-0.9021 (1.8365)	-0.9488 (2.0095)	-1.0457 (2.0361)	-1.2974 (2.0068)	-1.4738 (1.8730)
	With trend	-1.9244 (1.8138)	-1.9055 (2.0238)	-2.1750 (2.0812)	-2.5724 (2.0552)	-2.3332 (1.8561)

Note: The critical values for the (A)DF test at the 95% significant level are -2.89 (without trend) and -3.45 (with trend). Durbin-Watson statistic is in parenthesis. The significant points for the DW test at the 95% significant level are 1.58 and 1.64.

We next performed the Sims test on the real exchange rate between United States and Canada, and the results are given in Table 4. As can be seen, the results are significantly different from those in Table 2. Here, the data cannot reject the null hypothesis of unit root, except when a trend term is introduced and we go up to the fourth lag.

Table 4: Sims tests for a unit root in the real exchange rates between Australia and New Zealand using data in levels

		Sims test $\gamma$				
			Squared t	Schwarz limit	Small sample limit	Marginal alpha
United States / Canada	No trend	No lags	0.814	7.326	1.330	0.8381
		1 lag	0.814	7.326	1.330	0.8381
		2 lags	0.900	7.298	1.301	0.8302

		3 lags	1.093	7.269	1.273	0.8139
		4 lags	1.683	7.275	1.278	0.7657
	With trend	No lags	3.703	6.417	0.420	0.4365
		1 lag	3.703	6.417	0.420	0.4365
		2 lags	3.631	6.295	0.298	0.4305
		3 lags	4.731	6.173	0.176	0.2909
		4 lags	6.617	6.085	0.088	0.1326

Note: Bayesian tests using RATS.

## 6. Conclusion

This paper employs both classical ADF and the Bayesian Sims tests to evaluate the random walk hypothesis of the real exchange rate for Australia and New Zealand on one hand, and the USA and Canada on the other. The results are strikingly mixed. The ADF tests are not able to reject the null hypothesis of a unit root for the real exchange rate between Australia and New Zealand. However, the Sims test tends to reject the unit root hypothesis. For the real exchange rate between USA and Canada neither the ADF nor the Sims tests are able to reject the null hypothesis of a unit root. These results corroborate to the idea that PPP puzzle could be beyond a statistical issue of whether the real exchange rate contains a unit root, and consequently, focus needs to be shifted on to the economic sources of deviations from PPP (see Higgins and Zakrajšek, 1999).

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## Endnotes

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<sup>1</sup> The beauty of PPP is perhaps its simplicity and practical versatility: it serves as a benchmark to judge the level of an exchange rate, which, in turn, becomes a useful prediction model for the rates.

<sup>2</sup> The most detailed summary of early work on PPP is Officer (1976). See also Frenkel (1978, 1980), Isard (1977), Kravis and Lipsey (1971), Katseli-Papaefstratiou (1979), Mussa (1979). The first among the many papers uncovering long-term PPP relationships was Manzur (1990), but see also Diebold, Hushted and Rush (1991) and Cheung and Lai (1993).

<sup>3</sup> The similarities in the relationship between United States and Canada and the relationship between Australia and New Zealand are far more than just limited to geographic positions. Both Canada and New Zealand is considered as a little brother to United States and Australia, respectively, in an economic sense. New Zealand to Australia is like what Canada is to USA. As an affluent, high-tech industrial society, Canada closely resembles the United States in its market-oriented economic system, pattern of production, and high living standards. This close cross-border relationship propelled the North American Free Trade Agreement (1994). Similarly, the Australia-New Zealand Closer Economic Relations Trade Agreement (1983), popularly known as the CER Agreement, created conventional free trade in the trans-Tasman area involving the removal of all border restrictions on trade in goods.

<sup>4</sup> Because 0.84 to the power of four, equals the lower bound for annual data (0.5,1); the interval is (0.94,1) for monthly data.

<sup>5</sup> The use of consumer price indices as a measure of a nation's price level has been controversial in the PPP literature. The roots of these controversies are the inclusion of non-traded goods, which can give a wrong impression of one nation's price level. However, most authors favor this broad index, while others favor a narrower index with heavy weight on tradable goods, such as the wholesale price index.