Risk:
Perceptions and Attitudes

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Abstract

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1. Introduction

Changes in future outlook and risk attitudes may be among more popular explanations of asset price movements used by practitioners, but it would certainly be an overstatement to say that these have been enthusiastically embraced in academic circles. There are signs of change, although these two explanations have fared somewhat differently. Whereas changes in investors' expectations seem to be increasingly accepted as explanation of a wide range of phenomena, the status of changes in risk attitudes seems to be controversial.[1] Practitioners' regular appeals to changes in risk attitudes are, perhaps equally regularly, dismissed by academics.[2] The grounds for dismissal are methodological: changes is risk attitudes amount to relaxing the assumption of constant preferences which is thought to safeguard rigour in research.[3] Furthermore, changes in expectations, through

[1] Recent examples of the use of changes in expectations as explanatory devices include Cecchetti et al. (2000), Danthine et al. (2003), Kurz (1997), Ch. 11, and Melino and Yang (2003). Campbell, Lo, and MacKinley (1997), Ch. 8.4, provide a discussion and additional references. For the links to the business cycle literature, see Beaudry and Portier (2004). Misina (2003) discusses some pitfalls associated with the use of these models.

[2] Changing risk aversion is accepted in the context of the habit persistence; Cf. Campbell, Lo, and MacKinley (1997), Ch. 8.4. However, changes in risk attitude obtained via habit persistence are unlikely to help explain shorter-term fluctuations, since risk-attitudes are related to consumption, a variable that is quite stable over short periods of time.

[3] The key issue is clearly summarized by Arrow (1982): "A fundamental element of rationality ... is, in logicians' language, that of *extensionality*. The chosen element depends on the opportunity set from which the choice is made, independently of how that set is described ... The cognitive psychologists deny that the choice is in fact extensional; the framing of the question affects the answer."

learning for example, are thought to be consistent with individual rationality,[4] while changes in risk attitudes are not.

And yet, there seems to be little reason for either this asymmetric treatment or explanatory dichotomy. The explanatory dichotomy has been breached by the use of changes in individual risk *perceptions* in a variety of contexts, to explain shorter-term developments in financial markets, as part of a mechanism amplifying fluctuations in financial markets, and in accounts of 'irrational exuberance.'[5] In this view, changes in risk perception have impact on individuals' attitude towards risk, as well as their willingness to bear risks. The literature on behavioural foundations of choice under uncertainty has identified a number of factors that influence risk perceptions.[6] Among these, individual assessment of future outcomes has been established as an important determinant.[7] In this way, explanations based on changes in risk perceptions relate individual assessment of outcomes to

[4] Here, individual rationality means axiomatic consistency of individual choices, rather than the question of how individuals form expectations.

[5] For example: "... the decline in longer-term interest rates and diminished perceptions of credit risk in recent months have provided a substantial lift to the market value of nearly all major categories of household assets." (Greenspan (2003)) For examples related to financial cycles and irrational exuberance, see Borio (2003), and Shiller (2000), p. 46, respectively.

[6] See also, Slovic et al (2000) and references cited therein.

[7] Cf. Hirshleifer (2001), pp. 1550-1. For the evidence on the relationship between future outlook and probability assessments see Wright and Bower (1992). These authors also provide a discussion and further references on the impact of mood on risk assessments. More generally, the issue here is the status of the extensionality axiom. Considerable evidence, starting with the early identification of the framing effects, has been accumulated in support of the claim that the extensionality axiom may be violated. See Tversky and Koehler (1994) for further discussion.

their views regarding risks.[8] The key problem is absence of a formal framework that can be used to describe this type of interdependence.

This work seeks to introduce the notion of risk perception within the context of the expected utility theory, while addressing the methodological concerns expressed above. The starting point of the discussion is the formulation

$$U = \sum_{s} \pi_s^i u_s(x),$$

where π_s^i denotes agent i's probability of state s. In this context it is, in principle straightforward to obtain the necessary links within the constant risk-aversion class by specifying that the risk aversion parameter, ρ, depends on individual's expectations $\boldsymbol{\pi}^i = (\pi_1^i, ..., \pi_S^i)$, i.e. $\rho = \rho(\boldsymbol{\pi}^i)$. Assuming that there are σ 'expectations states', i.e. $\boldsymbol{\pi}^i \in \{\boldsymbol{\pi}_1^i, ..., \boldsymbol{\pi}_\sigma^i\}$, in this formulation state-dependence is modelled through variation of risk aversion parameter across these states.

This formulation, while incorporating links between expectations and risk attitudes, fails to meet the methodological objections raised above: preferences here are formally not constant, but state dependent. To deal with this objec-

[8] We do not investigate the sources of revisions in subjective assessments although the cognitive psychology literature identifies a number of patterns: optimistic bias has been well documented in the literature Cf. Weinstein and Klein (2002), Armor and Taylor (2002), and Hirshleifer (2001), p. 1550-1.

tion, the primitive assumption in the analysis is that the subutility function is state-independent, and belongs to the constant risk-aversion class:

$$u_j(\cdot) = u_k(\cdot), \ \forall j, k.$$

This assumption will, within the class of constant risk aversion utility functions, guarantee that individual risk attitudes will be represented by an exogenously specified risk-aversion parameter. Changes in agent i's expectations are represented by changes in their probability distribution over future states, $(\pi_1^i, ..., \pi_S^i)$. The difficulty is that under the assumption of constant subutility, establishing links between the future outlook and individual risk-attitudes seems precluded.

To resolve this problem, we introduce the notion of *implied risk aversion.* This coefficient enables one to relate expectations and risk-attitudes. Moreover, it is possible to characterize the nature of this relationship in qualitative terms. It will be shown that upward revisions in probabilities of good states are associated with lower implied risk aversion; upward revisions of probabilities of bad states are associated with increased implied risk aversion.

The implied risk aversion coefficient can be interpreted, within the class of the constant risk aversion utility functions, as capturing the way individuals *perceive* risks. While the usual Arrow-Pratt measure captures static risk attitudes, the implied risk aversion will provide information regarding changes in perceived risks due to revisions of individual assessments of probabilities of future states. Furthermore, if upward revisions of probabilities of good states can be interpreted as indicators of optimism, and downward revisions as indicators of pessimism, this framework offers a description of interaction between disposition towards future (optimism, pessimism) and individual risk attitudes, via changes in risk perceptions.[9] Changes in disposition towards future affect the way individuals perceive risks, their risk attitude, and their willingness to bear risks. For example, optimism regarding the future will induce individuals to undertake actions that from some earlier point in time would have been considered too risky. In this way, one can capture the anecdotal evidence of investors who justify their increased willingness to bear risk by saying that their risk attitudes have not changed - they simply realized that assets have become less risky.[10]

The presentation is organized as follows. In section 2, the key terms are defined and the main result relating revisions of probabilities to risk attitudes is

[9] Optimistic bias is defined as an upward bias in assignment of probabilities of good states. The bias is established relative to a benchmark. This issue is further discussed in Sec. 3. Cf. Armor and Taylor (2002).

[10] Stories like this seem particularly frequent in periods of prolonged market upturns, such as the one in the second half of the 1990s. Shiller (2000) provides a detailed evidence.

given. In section 3, the results obtained are given an interpretation in terms of the relationship between disposition towards future and risk perceptions. Section 4 illustrates the concepts by means of an example, using a standard asset-pricing model. Suggestions and conclusions are given in the last section.

2. Probability beliefs and risk aversion

Consider the following setting:

- 2 states of nature: 1 - good; 2 - bad
- objective probabilities defined over these states: π_1, π_2.[11]
- individual i with utility function

$$U^i(c) = \pi_1^i u(c_1) + \pi_2^i u(c_2),$$

where π_s^i is the subjective probability belief about state s.

Individuals can revise their beliefs about the likelihood of good and bad states in both directions: upward revisions with respect to good state, $\pi_1^i \geq \pi_1$, indicate that they believe that the good state is more likely than indicated by an objectively

[11] We are not interested at the moment in the origin of these probabilities.

given measure; upward revisions with respect to bad state, $\pi_2^i > \pi_2$, indicates that the bad state is considered more likely than objectively warranted. The problem is to establish the link between these revisions and risk attitudes, and thus arrive at a formal expression of risk perception.

Individual risk-aversion is specified by the sub-utility function $u(\cdot)$. It will be assumed that $u(\cdot)$ is state-independent, which, in this context, means that the exogenously-specified coefficient of risk aversion does not vary across states. It will be demonstrated that revisions in individual beliefs have two effects: the effect on expected payoffs, and the effect on risk attitudes via changes in risk perception. The latter effect is not captured by the risk-aversion coefficient, which is assumed to be constant.

A formal statement is given below. The key result is contained in Proposition 1. To get there, several preliminary steps are needed, and these are summarized in Lemmas 1 and 2, and Definition 1.

Let S denote the (finite-dimensional) set of possible states of the world, and let $\boldsymbol{\pi} = [\pi(s)]$ denote a probability distribution defined on S. Each state has a payoff associated with it. Let $x = \boldsymbol{\pi} \cdot \mathbf{x}$ represent the expected payoff, where $\mathbf{x} = [x(s)]$. X is the space of expected payoffs.

Define S_γ and S_β so that $S = S_\gamma \cup S_\beta$, $S_\gamma \cap S_\beta = \emptyset$, with

$$s \in S_\gamma \text{ iff } x\left(s \in S_\gamma\right) > x\left(s \notin S_\gamma\right).$$

In words, S_γ is the set of payoff-dominant states with the associated probability $\pi\left(s \in S_\gamma\right) \equiv \sum_{s \in S_\gamma} \pi\left(s\right)$. Probability associated with S_β is $\pi\left(s \in S_\beta\right) \equiv \sum_{s \in S_\beta} \pi\left(s\right)$.

Lemma 2.1. *For any* $x \in X$,

$$\frac{\partial x}{\partial \pi\left(s \in S_\beta\right)} < 0,$$

$$\frac{\partial x}{\partial \pi\left(s \in S_\gamma\right)} > 0.$$

Proof: In the Appendix.

Lemma 1 establishes the relationship between expected payoffs and revisions in individual assessment of good and bad states, respectively. Under the usual assumptions on the subutility function, a change in expected payoffs brought about by a change in these assessments will result in a change in the level of utility of the expected payoff. Upward revision of the probability of the good state will lead to an increase in expected payoff and, to an increase in the level of utility of the expected payoff. To compare initial and final state, it is necessary to bring the individual back to the original level of utility. This can be accomplished by notionally changing the risk-aversion parameter. The change in the risk aversion parameter that is necessary to bring the individual to the original level of utility, after a revision in probabilities, is called the *equivalent variation*, denoted EV_ρ.

Definition 2.2. *Let $u = \bar{u}$ denote the level of utility associated with the expected payoff prior to revision in probabilities and let x' denote the expected payoff after the revision of probabilities. Then,*

$$EV_\rho \equiv \left. \frac{\partial \rho}{\partial \pi \left(s \in S_s \right)} \right|_{u=\bar{u}, x=x'}, \; s = \beta, \gamma.$$

Using this definition, implied risk aversion, ρ^ι, can be defined as the value of the risk aversion parameter that, after a change in expected payoff results in the original level of utility:

$$\rho^\iota \equiv \rho + EV_\rho.$$

The concept of equivalent variation and implied risk aversion establish the link between revisions in probability assessments and risk attitudes. The problem is to characterize this relationship in qualitative terms. To accomplish this, one more result, given in the following lemma is needed.

Lemma 2.3. *Let $u(\cdot)$ denote a differentiable utility function of the CRA type. Then, for any utility function in this class*

$$\frac{\partial u}{\partial \rho} > 0.$$

Proof: In the appendix.

This lemma characterizes the relationship between changes in risk-aversion and the level of utility. The result holds for the class of constant risk-aversion utility functions (both CARA and CRRA).

With these results in hand, we are in position to characterize the relationship between revisions in probability assessments and risk attitudes. This is the content of the following proposition.

Proposition 2.4. *Let $x \in X$, and let $\bar{u} = u(x;\rho)$. Then,*

$$\frac{\partial x}{\partial \boldsymbol{\pi}(s \in S_\beta)} < 0 \Rightarrow \left.\frac{\partial \rho}{\partial \pi(s \in S_\beta)}\right|_{u=\bar{u}, x=x'} > 0,$$

$$\frac{\partial x}{\partial \boldsymbol{\pi}(s \in S_\gamma)} > 0 \Rightarrow \left.\frac{\partial \rho}{\partial \pi(s \in S_\gamma)}\right|_{u=\bar{u}, x=x'} < 0.$$

Proof: In the Appendix

This proposition establishes the nature of the relationship between changes in payoffs due to revisions in probability assessments and equivalent variation. The relationship between the two is inverse: an increase in the probability of a bad state will result in an increase in EV_{ρ}. Similarly, an increase in the probability of a good state will result in a decrease in EV_{ρ}.

The effect of revisions in probabilities on implied risk aversion follows directly from this result in conjunction with the definition of implied risk aversion.

Corollary 2.5.

$$\frac{\partial x}{\partial \boldsymbol{\pi}\left(s \in S_{\gamma}\right)} > 0 \Rightarrow \frac{\partial \rho^{\iota}}{\partial \boldsymbol{\pi}\left(s \in S_{\gamma}\right)} < 0.$$

$$\frac{\partial x}{\partial \boldsymbol{\pi}\left(s \in S_{\beta}\right)} < 0 \Rightarrow \frac{\partial \rho^{\iota}}{\partial \boldsymbol{\pi}\left(s \in S_{\gamma}\right)} > 0.$$

Proof: Follows directly from the main proposition and definition of implied risk aversion.

The corollary establishes that

- upward revisions of probability of good state lead to lower implied risk aversion,
- upward revisions of probability of bad state lead to higher implied risk aversion.

The concept of implied risk aversion allows for the possibility of change in individual risk attitudes when individual beliefs about the likelihood of future outcomes change. Moreover, it can be interpreted as capturing changes in individual risk perceptions (through the EV_{ρ} term) in situations in which these are due to changing views of the future. This is accomplished without formally relaxing the assumption of constant preferences, understood as the state-independent subutility function $u(\cdot)$.[12]

3. Disposition towards future and risk perception

The results of the previous section can be given a more precise interpretation by taking a closer look at the revisions of individual assessments discussed there. The objective here is not to relate revisions to a particular underlying cause but to suggest that the revisions investigated above are consistent with the notions of optimism and pessimism. Optimism is a state in which an individual assigns a greater probability to the good state than the one implied by some objective measure. The key feature of optimism is that individual overestimates the probability of a good state. Pessimism manifests itself in overestimation of the probability

[12] It is the assumption of extensionality of choices is implicitly relaxed here. Alternative descriptions of the same event are assumed to influence individual probability assessment and thus lead to different judgements. Cf. Tversky and Koehler (1994), p. 548.

of a bad state. States of optimism and pessimism will be jointly referred to as 'disposition towards future'.

To make these notions operational one needs to specify what is to be considered a 'normal state'. There are two ways this can be accomplished:

- by specifying a benchmark, or
- by comparison with a previous state.

Distinction between these is of some importance. By defining disposition towards future relative to a benchmark, it is possible to focus on trends in disposition changes. Definitions relating the current state to a previous state capture small variations in disposition but may obstruct the identification of trends. In the ensuing discussion, the definitions will be established relative to a benchmark.[13]

Individual i is said to be *optimistic* if

$$\pi^i\left(s \in S_\gamma\right) > \pi\left(s \in S_\gamma\right).$$

where $\pi\left(s \in S_\gamma\right)$ is the probability of a good state associated with some benchmark. In models relying on the assumption of rational expectations, the bench-

[13] Whereas the construction of the benchmark is outside of the scope of the present discussion, a natural candidate is the stationary measure based on the restrictions on individual beliefs. See Kurz (1997), Introduction, for an exposition of the basic ideas.

mark will be represented by the equilibrium process. In practical applications, benchmark can be taken as the relative frequency of visiting a particular state obtained from the past data.[14]

Individual i is said to be *pessimistic* if

$$\pi^i \left(s \in S_\beta \right) > \pi \left(s \in S_\beta \right).$$

In words, individuals are said to display optimism/pessimism if they overweigh the probability of good/bad states.

These definitions, in conjunction with results of the previous section, enable us to establish precise links between disposition towards future and risk perceptions. Changes in disposition towards future affect agents' actions by affecting the way agents perceive risks. Optimism implies lower risk perception whereas pessimism implies that a given situation will be perceived as riskier than before.

The implications for individual behaviour are immediate. In situations in which individuals are optimistic, they will undertake actions that under normal circumstances would not have been undertaken. They will tend to discount the

[14] This allows for the possibility of extreme events. One could think of a bad event whose frequency in the past data is zero. We would say that an individual displays pessimism if he assigns a positive probability to that event, etc.

risks associated with particular types of assets, and will not demand the risk premium that they would normally have demanded in order to hold them. This opens the possibility of bidding up the prices of assets. Similarly, when individuals are pessimistic they demand higher risk premiums from the existing classes of assets. They perceive most assets as riskier than under normal circumstances and may decide to withhold their investment funds.

Note that in the above framework changes in risk attitudes due to changes in disposition towards the future are captured by the *implied* risk aversion coefficient. Changes in risk perception will have an impact on individual actions, even when risk aversion coefficient in the utility function is unchanged. It follows that risk perceptions may have an important role to play in explaining individual behaviour in dynamic settings.

The above analysis does not imply that changes in risk perceptions will necessarily lead to 'irrational exuberance' or similar events. In this work the factors leading to exuberance are not analyzed.[15] The analysis offered demonstrates that these phenomena *can* be captured within the standard framework and that the role of risk perceptions may have been underemphasized.

[15] Shiller (2000) offers an analysis of factors that create environements in which people are susceptible to mass exuberance.

4. Risk perception and asset prices

The relationship between changes disposition towards future, risk perception, and price behaviour is illustrated in this section.

4.1. Model

The model used is a standard consumption-based asset-pricing model, with a representative agent in an exchange economy with a single consumption good. The focus of the exercise is on the analysis of the behaviour of a risky asset. The utility function is a constant relative risk aversion type, given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

The only source of uncertainty is the time-varying nature of payoffs of the risky asset. It is assumed that there are two possible states of the world, $s_t = \{1, 0\}$, good and bad, and that d_1 and d_2 are dividend payments of this asset associated with each state, respectively. Dividends at each date are selected according to the following rule:

$$d_t = \begin{cases} d^h & \text{if } s_t = 1 \\ d^l & \text{if } s_t = 0 \end{cases}.$$

The state tomorrow is drawn so that $\Pr(s_t = 1) = \Pr(s_t = 0) = 0.5$. One can interpret these as the unconditional probabilities of the two states. The associated transition matrix is

$$T = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \tag{4.1}$$

To isolate the effect of changes in market outlook on asset prices, it is assumed that consumption grows at a known constant but positive rate g_c. With these assumptions, the first order conditions result in the following expression for the stock price in each state:

$$p_i^s = \beta \sum_j \pi_{ij} \left(g_c\right)^{-\gamma} \left(p_j^s + d_j\right), \tag{4.2}$$

4.1.1. Agents' disposition towards the future

Agents form beliefs about dividend payments tomorrow and this will affect their demand for equity today. For simplicity it is assumed that agents know the unconditional state probabilities. They have at their disposal past data and are trying to infer something about the future. At each point in time they decide whether to revise their future outlook either upwards or downwards or to leave it unchanged. The exercise performed allows us to relate decisions with respect to revisions to some well-documented attitudes in psychology literature such as overreaction and conservativism.[16] Revisions in any direction will have an impact on asset prices.

4.1.2. Computation of expected returns

The expected returns at any point in time are computed using the expression:

$$R\left(s_{t+1}=i\right)=\frac{p\left(s_{t+1}=i\right)+d\left(s_{t+1}=i\right)}{p\left(s_t=i\right)},$$

for expected returns if the state tomorrow is unchanged, and

$$R\left(s_{t+1}=j\right)=\frac{p\left(s_{t+1}=j\right)+d\left(s_{t+1}=j\right)}{p\left(s_t=i\right)},$$

if the state tomorrow changes. The expected returns at time t are

$$R_{t+1}^{ex}=\pi\left(s_{t+1}=i|s_t=i\right)R\left(s_{t+1}=i\right)+\pi\left(s_{t+1}=j|s_t=i\right)R\left(s_{t+1}=j\right).$$

Equivalent formulas apply when $s_t=j$.

[16] Revisions of the future outlook could be due to arrival of new information, but they could also be due to different interpretation of existing information. While the underlying motives are taken as exogenous here, the mechanism is general enough to accommodate any of the causes.

4.1.3. Computation of implied risk aversion

Benchmark value of the risk-aversion parameter is taken to be an exogenously given value, which is assumed to be related to the risk attitude when states are generated according to (4.1). The associated level of utility is the benchmark level of utility, $\bar{u}$. Revisions of the future outlook result in changes in expected payoffs. For the new value of expected payoffs, implied risk aversion is the value of the risk-aversion parameter that would yield the original level of utility.

4.1.4. Parameter values:

The purpose of the exercise is not to match particular properties of the data but to investigate the impact of changes in future outlook on asset prices. The process generating the states is assumed to be (4.1). Other parameter values are given as follows:

$$\rho = 3, \ \beta = 0.95, \ \ (d_1, d_2) = (2, 0) \, , \ g_c = 1.01.$$

4.2. Results

To see how an agent might come to revise their future outlook, consider the following case: agent has recently observed the following realization of states:

$\{1, 1, 1\}$. The question is whether this will lead to revisions of the future outlook. There are three possibilities, corresponding to different investment styles.

Case 1: No revisions of future outlook

Agent does not attach any significance to the fact that the last three periods were good states. He believes that the process generating states is a coin toss with equal probabilities, and this process does not preclude the possibility of a sequence of three good states. Indeed, the probability of getting three good states in a row under the postulated process is 12.5%. As a consequence of this reasoning, agent does not revise. This reasoning would correspond to a conservative strategy, which discounts the most recent realizations. The price in period 4 remains the same. The implied risk aversion is unchanged.

Case 2: Upward revisions

Agent believes that three consecutive good states are a sign of a possible regime change. He believes that the probabilities assigned by the postulated process are too low, and revises the probability of a future state upwards. He is more confident than warranted by the past data that the good state will be realized tomorrow, and in our terminology, displays optimism. The implied risk aversion decreases, even though the Arrow-Pratt measure of risk aversion remains the same. The investment outlook is perceived as less risky now, and the agent invests, putting an upward pressure on the price. This is illustrated in Figure 1.

FIGURE 1

Figure 1 illustrates two cases which are termed cautious optimism and exuberance. Under cautious optimism agent revises future outlook upwards but in a gradual fashion. The probability of a good state tomorrow is revised from $\pi_{11} = 0.5$ in period 3 to $\pi_{11} = 0.61$ in period 4. The unconditional probability assigned to the good state increases from 0.5 in period 3 to 0.63 in period 6, as implied risk aversion decreases. This leads to a total increase in price of approximately 27 percent.

Under exuberance, the initial revision is from $\pi_{11} = 0.5$ in period 3 to $\pi_{11} = 0.81$ in period 4. The unconditional probability of the good state is revised from 0.5 in period 3 to 0.92 in period 6. Moreover, the agent considers that the continuation of the good state is virtually certain and sets $\pi_{11} = 0.96$ in period 6. This leads to a dramatic increase in prices of approximately 86 percent. Implied risk aversion declines more than in the case of cautious optimism.

Case 3: Downward revisions

Agent believes that it is highly unlikely that the good times will continue. He takes the three consecutive good states to be 'too good to be true' and consequently revises his future outlook downwards. He displays pessimism relative to the postulated process. As a consequence, the investments are perceived as more risky than before and he will decide to sell, or demand higher expected return to be compensated for the higher perceived risk. His implied risk aversion increases. The consequence is that the market prices will fall. This is illustrated in Figure 2.

FIGURE 2

Figure 2 illustrates two cases which are termed gloom and depression. In both cases the agent believes that the present state is unlikely to continue and this is reflected in downward revisions of probability of the good state. Under the gloom case, the unconditional probability of a bad state increases from 0.5 in period 3 to 0.63 in period 6, whereas the probability of staying in a bad state once there increases from 0.5 to 0.71. This leads to a decline in price of approximately 37 percent as the implied risk aversion increases.

Under depression, the results are more dramatic: the unconditional probability of the bad state is set to 0.84 in period 6, and the probability of remaining in the bad state is now 0.91. This results in a drop in price of approximately 70 percent, as the implied risk aversion increases.

4.3. Comments

The above examples illustrate the links between changes in market outlook and asset price movements. A great variety of patterns can be produced in this way. Persistent optimism can create upward movements in asset prices, whereas persistent pessimism will create downward movements. This story is well known as bulls and bears markets, but what is important to emphasize is that the above results are obtained through changes in risk perceptions. Same series of events can be perceived differently at a different points in time, and this gives rise to changing attitudes towards risk. This effect is not captured either by the CRA class of utility functions, nor by the class of utility functions relying on habit persistence.

5. Conclusions

The objective of this work was to offer a formal description of changes in risk perception as an explanatory device, that would bridge the gap between explanations based on changes in expectations and the ones based on changes in risk attitudes, while taking into account the requirement of constant preferences. The concept of equivalent variation was introduced to relate changes in expectations to changes in risk perceptions, and the notion of implied risk aversion was introduced to capture the imact of changing risk perceptions on risk attitudes.

A particular characterization of the relationship between the two was obtained by associating revisions in probability assessments with optimism and pessimism. This interpretation made it possible to describe the relationship between risk attitudes and revisions in probabilities in behavioural terms: increased optimism was associated with lower implied risk aversion, and increased pessimism with higher implied risk aversion.

Although there are similarities between this approach and the approach relying on state-dependent risk-aversion parameter, we think that the approach offered here has some advantages over the former: state-dependency does not in itself offer any explanation of reasons for a change in investors' risk aversion. Under the proposed approach, there is a direct link between revisions in subjective probability assessments and risk perceptions. While this link can be interpreted in terms of disposition towards future and risk perceptions, this interpretation is not exhaustive. Indeed, one could interpret revisions in probability assessments as a way of capturing violations of the extensianality axiom, regardless of the underlying cause. The interpretation suggested in this paper relies on the observation that optimistic individuals tend to downplay risks, whereas pessimism often leads to extreme caution and overweighing of risks. While there is some empirical support for this type of interpretation, the results do not depend on it.

In the analysis presented in the paper, no assumptions were made regarding the reasons for revisions in individual probability assessments. Several possibilities come to mind, all of which can be accommodated within the proposed framework:

- absence of an objective standard determining the impact of current news on future prospects,
- framing effects, or to some other underlying cognitive biases,
- learning limitations. Agents do not believe that what they learn from the past data is the correct statistical data-generating process. Reasons for this could be structural uncertainty, cognitive biases etc.

Regardless of the underlying causes, the example in Section 4 suggests that the impact on asset prices will clearly depend on the extent of these revisions. The example does not specify the mechanics of revisions but one could proceed by using Bayesian updating at least as a benchmark and then investigate the implications for asset prices.[17] Alternatively, rather than specifying the revision mechanism one could impose a priori restrictions on the extent of admissible revisions, given the past history of the data, as in Kurz (1997). Whereas different learning mechanisms may have different quantitative implications for the asset price movements, the basic relationship between disposition towards future, risk perception, and asset price movements analyzed in this work remains intact.

[17] This learning procedure might be of limited use in environments in which complete learning is precluded, however.

6. Appendix

6.1. Proof of Lemma 2.1:

Let $\boldsymbol{\pi} \equiv [\pi(s \in S_\beta), \pi(s \in S_\gamma)]$, and $\mathbf{x} \equiv [\mathbf{x}(s \in S_\beta), \mathbf{x}(s \in S_\gamma)]$, so that

$$x = \begin{bmatrix} x(s=\beta) & x(s=\gamma) \end{bmatrix},$$

where

$$x(s \in S_\beta) = \sum_{s \in S_\beta} \pi(s)\, x(s),$$

and

$$x(s \in S_\gamma) = \sum_{s \in S_\gamma} \pi(s)\, x(s)$$

Then,

$$\begin{aligned} \frac{\partial x}{\partial \pi(s \in S_\beta)} &= x(s \in S_\beta) - x(s \in S_\gamma) \\ &< 0. \end{aligned}$$

The proof of the second part follows from the above.

6.2. Proof of Lemma 2.3:

Case 1: Constant relative risk-aversion (CRRA) case

From the definition of Arrow-Pratt measure of relative risk aversion

$$R_R = -\frac{xu''}{u'}.$$

it follows that, in the CRRA case

$$-\frac{xu''}{u'} = \rho.$$

From here

$$xu'' + \rho u' = 0.$$

This is a second-order Euler differential equation. The solution guess takes the form $u = x^r$. From here

$$xr\left(r-1\right)x^{r-2} + \rho r x^{r-1} = 0,$$

$$x^{r-1}\left[r\left(r-1\right) + \rho r\right] = 0,$$

which implies that

$$r^2 - (1 - \rho)\, r = 0,$$

so that

$$r_1 = 0,\ r_2 = (1 - \rho)\,.$$

The solution then takes the form

$$u = c_1 x^{1-\rho} + c_2.$$

Any CRRA function will take this form. From here

$$\frac{\partial u}{\partial \rho} = c_1 x^{1-\rho} \ln x.$$

It follows that

$$\frac{\partial u}{\partial \rho} > 0 \text{ if } \ln x > 0 \ \Rightarrow x > 1.$$

If the constant $c_2 = -1$, $x > 1$ guarantees a well-behaved utility function. Other properties of utility functions are used to restrict parameter values for c_1.

Case 2: Constant absolute risk-aversion (CARA)

Starting from the definition of the Arrow-Pratt measure of absolute risk aversion $R_A = -\frac{u''}{u'}$, one gets, in case of CARA utility function

$$R_A = -\frac{u''}{u'} = \rho$$

i.e.,

$$u'' + \rho u' = 0. \tag{6.1}$$

Solution to this differential equation is standard. Guess takes the solution takes the form $u = e^{rx}$. Substituting the appropriate derivatives of the guess into (6.1) yields

$$e^{rx}\left(r^2 + \rho r\right) = 0,$$

which implies

$$r_1 = 0,\ r_2 = -\rho,$$

The general solution thus takes the form

$$u = c_1 e^{-\rho x} + c_2.$$

From here

$$\frac{\partial u}{\partial \rho} = c_1\left(-\rho\right)e^{-\rho x}.$$

Then

$$\frac{\partial u}{\partial \rho} > 0 \text{ if } c_1 < 0.$$

Condition $c_1 < 0$ is needed in order for the utility function to be well-behaved.

6.3. Proof of the Main Proposition:

From $u \equiv u(x(\pi); \rho(\pi))$, we have

$$\frac{du}{d\pi(s \in S_s)} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \pi(s \in S_s)} + \frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial \pi(s \in S_s)}\bigg|_{u=\bar{u}}, \ s = \beta, \gamma.$$

$u = \bar{u}$ (by assumption), implies that $\frac{du}{d\pi} = 0$ and

$$\frac{\partial u}{\partial x}\frac{\partial x}{\partial \pi(s \in S_s)} = -\frac{\partial u}{\partial \rho}\frac{\partial \rho}{\partial \pi(s \in S_s)}\bigg|_{u=\bar{u}}, \ s = \beta, \gamma.$$

Given that $\frac{\partial u}{\partial x} > 0$ (positive marginal utility) and $\frac{\partial u}{\partial \rho} > 0$ (Lemma 2.3), it follows that

$$\frac{\partial \rho}{\partial \pi(s \in S_s)} > 0 \text{ when } s = \beta,$$

since by Lemma 2.1, $\frac{\partial x}{\partial \boldsymbol{\pi}(s \in S_\beta)} < 0$.

Similarly,

$$\frac{\partial \rho}{\partial \pi(s \in S_s)} < 0 \text{ when } s = \gamma,$$

since by Lemma 2.1, $\frac{\partial x}{\partial \boldsymbol{\pi}(s \in S_\gamma)} > 0$.

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Figure 1: Upward revisions

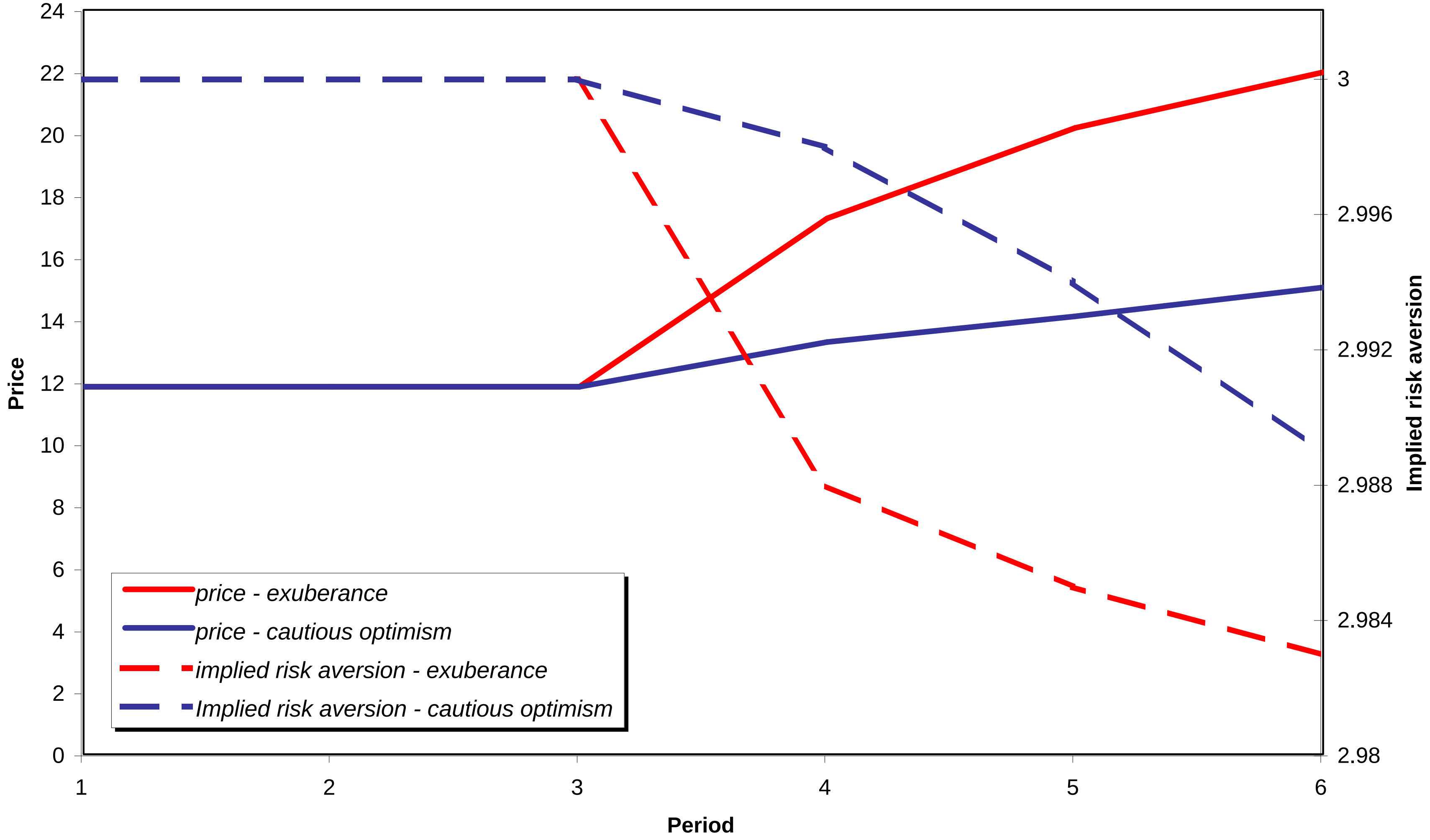

Figure 2: Downward revisions