Disappointment, Pessimism and the Equity Risk Premia

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Abstract:

We analyze the implications of the introduction of disappointment averse agents on the financial markets. The underlying intuition is that agents take account for the potential disappointment of their decisions, in particular when they invest on the stock market. After having defined the concepts of disappointment aversion, we show that in our framework a disappointment averse agent is pessimistic. We then explore the consequences of disappointment aversion and pessimism on the CAPM and the C-CAPM. We finally study a Lucas asset pricing model that is standard, except that the representative agent is supposed to be disappointment averse. Using a constant marginal utility function, we show that the model can account for both large equity risk premia and low risk-free rates. It may so be viewed as a solution to the equity premium puzzle

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1 Introduction

This paper focuses on the equity premium puzzle highlithed by Mehra and Prescott (1985). That puzzle refers to the seeming inability of standard dynamic asset pricing models to explain the average equity premium observed in the US markets. In particular, they showed that reasonable configurations of the preference parameters[1] embbeded in a Lucas (1978) consumption-based asset pricing model (hereafter CCAPM) cannot produce high enough equity premia. The only way for producing large equity premia is to consider large value (more than 50) for the relative risk-aversion coefficient. But the acceptation of such values as a correct description of the representative consumer behavior leads to an another puzzle, namely the riskfree rate puzzle identified by Weil (1989).

The CCAPM is the classic model for asset pricing in a dynamic framework. The level of equity risk in that model is based on the covariance of asset returns with per capital consumption. The equity premium puzzle comes from the relative low value of that latter covariance found in the data. Therefore, producing a sufficiently high intertemporal marginal rate of substitution requires very large values for the risk aversion parameter.

The model used by Mehra and Prescott is based on three main assumptions.[2] Comments on these assumptions can be found in the excellent literature survey of Kocherlakota (1996). The different attempts to solve this puzzle have focused on relaxing one or the other of these three assumptions.

[1] The relative risk aversion parameter and the psychological discount factor.

[2] i) Expected Utility Maximizer;
ii) Asset markets are complete;
iii) Asset trading is costless.

Our paper belongs among the ones that try to generalize the preferences of the representative consumer to account for both the equity premium and the risk-free rate puzzles.

There is a large body of works that took the previous way. One of the most successful follows the initial work of Constantinides (1990) and introduces a property of habit persistence in the form of a subsistence level for consumption in the utility function. In the latter, the large variations in the marginal rate of substitution are due to the fact that small changes in consumption generate large changes in consumption net of the subsistence level. Campbell and Cochrane (1999) explore the role of time-varying habit persistence in the explanation of various moments of asset returns. Nevertheless, they still have to assume implausibly risk averse investors to explain the equity premium puzzle.

An another way of thinking is the paper of Cechetti, Lam and Mark (2000). In the latter, they show how the canonical asset pricing model with particular non rational agents, actually agents who are pessimistic, can solve both the equity premium and risk-free rate puzzles. But, in addition to that hypothesis of non rationality of the representative agent, they have to assume that (s)he is not a bayesian learner.

In that paper, we argue that the agents when facing a risky situation, as it would be before investing on the stock market, could feel some disappointment by observing the *ex post* resolution of the uncertainty. We then think that a rational agent takes account for the potential disappointment of his decision.

Preferences that exhibit disappointment aversion have been axiomatized by Gul (1991) to offer a solution to the so-called Allais paradox. This is done by modeling an agent who maximizes a weigthed sum of utility, where the weitghts deviate from the original probabilities so as to reflect disappointment aversion. Until now, few attempts have been made for using that kind of preferences in an asset pricing model. Only Epstein and Zin (1991a) and more recently Ang *et alii* (2004) and Routledge and Zin (2004) use that concept in a financial perspective.

We use a slightly different version of the Gul's model. Indeed, the Gul preferences imply technical difficulties by the need of the ex ante calculation of the certainty equivalent of a lottery. Our framework proposes a way to get rid of those drawbacks. As Gul does, our decision maker is supposed to maximize the expected utility of his terminal wealth but the weights he uses while doing so, are not the original probabilities but transformed probabilities. In our model, the latter probabilities convey directly the property of disappointment aversion.

A very interesting property of our model is that the more disappointment averse the decision maker is, the more pessimistic he is. In that sense, our model may be viewed as a foundation of the Cechetti *et alii* article, since we provide the framework for solving the two puzzles considering both the full rationality of expectations and the existence of bayesian learners.

After having thus defined and decribed our disappointment weighted utility theory, our analysis consists of two steps.

First, we redefine the concepts of risk-aversion and equity premium in the framework of our theory. We highlight a new term that expresses the disappointment aversion. We then explore some theoretical implications in finance. We establish that an appealing justification of using mean and variance as choice criteria (or the use of the CAPM for valuation) is the existence of disappointment averse decision-makers with constant marginal utility. Finally, we show that the Euler equations derived from the intertemporal equilibrium model exhibit an intertemporal marginal rate of substitution that is more volatile than the usual one.[3] As we notice above, that latter property is the principal condition to produce sufficiently high equity premia.

Finally, together with the framework employed by Mehra and Prescott (1985), we show how our model can help explain the low risk-free rate and the large equity premium observed in the U.S. data.

The paper is organized as follows: In Section 2, we present the key features of the model and we examine the consequences of shifting from the original to the new probabilities on the definition and the properties of risk aversion. Section 3 explores the characteristics of an equilibrium on financial markets when agents are disappointment averse, looking at static and intertemporal equilibria in succession. In Section 4, we apply the Mehra and Prescott (1985) methodology to our new Euler equations to evaluate the ability of our theory to solve the equity premium puzzle. Section 5 presents our conclusions.

2 A short description of the disappointment weighted utility theory

The goal of that paper is not to provide a fully axiomatization of our preference functional but to present how our particular model can be useful in

[3] That one obtained by the standard consumption-based asset pricing model with time separable isoelastic preferences.

finance. So, in this section, we only present the necessary elements to carry out the financial implications of our theory.

The concept of disappointment is not new in the literature. Bell (1985), Loomes and Sugden (1986) and more recently Gul (1991) provide some useful models that convey this property. Ang *et alii* (2004) and Routledge and Zin (2004) provide financial applications of the Gul theory. In that kind of models, it is suggested that, when facing a risky prospect, an indivudual forms some prior expectation about that prospect. After the uncertainty is resolved, the individual experiences one particular consequence of the prospect. If that consequence falls short of the prior expectation, the individual may feel some degree of disappointment, whereas if the consequence is better than the prior expectation, the individual may also feel some measure of elation.

The satisfaction an individual feels after a lottery L has been run, may then depend on two elements: the expected utility of the outcome and disappointment (or elation). Disappointment can be modelized by the difference between the *ex ante* expected value of the utility of the lottery and its *ex post* value $(E_L[u(w)]) - u(w))$. Thus, the satisfaction of the considered individual is usually identified to the sum of the expected value of his cardinal utility function $E_L[u(w)]$ and that of a function of disappoinment *i.e.*:

$$U(L) = E_L[u(w)] - D[E_L[u(w)] - u(w)]$$

The model we use is related to the literature about disappointment although it is somewhat different. In this paper, the satisfaction of an individual will be assumed to be the expected value of a cardinal utility of outcome, given that transformed probabilities are used instead of the original ones and that the distortion depends on the importance of disappointment. Hence we shall write:

$$U(L) = E_{T(L)}\left[u(w)\right]$$

where $T(L)$ is the lottery whose cumulative distribution function F is derived from that of lottery L, using a change of probabilities $(dF_{T(L)}/dF_L)$ which will be assumed to depend on disappointment: formally:

$$\frac{dF_{T(L)}}{dF_L} = \gamma_L\left[u(w) - E_L\left[u(w)\right]\right]$$

Following _ and _ (2004), it can be shown that the lottery L_1 will be prefered to the lottery L_2 if and only if:

$$\begin{aligned} & \int_{-\infty}^{+\infty} u(w)\left(1 - A_{\left(E_{L_1}[u(w)]\right)}\left(u(w) - E_{L_1}\left[u(w)\right]\right)\right) dF_{L_1} \\ \leqslant & \int_{-\infty}^{+\infty} u(w)\left(1 - A_{\left(E_{L_2}[u(w)]\right)}\left(u(w) - E_{L_2}\left[u(w)\right]\right)\right) dF_{L_2} \end{aligned} \qquad (1)$$

Under some restrictions about the value of A,[4] the term $\left(1 - A_{(E_L[u(w)])}\left(u(w) - E_L\left[u(w)\right]\right)\right) f(w)$, where f is the density function of the lottery L, can be viewed as a change of probability measure. The term A plays an important role in our theory, since it is one that weights the degree of disappointment or elation. So, it can be interpreted as the absolute **disappointment aversion** of our decision maker.

Therefore, our valuation functional (1) expresses that our individual maximizes the expected utility of his wealth but using transformed probabilities which depend on the level of his disappointment.

[4] A is also positive.

A propety of our model is that disappointment averse decision makers are pessimistic in the sense that the distribution of transformed probabilities is stochastically dominated by the original one. Indeed, in the case of low outcomes, the term $-A_{(E_L[u(w)])}\left(u(w) - E_L\left[u(w)\right]\right)$ is positive, so that the probabilities attached at those particular states of nature are well overweighted. By contrast, the probabilities attached to the high outcomes are under-weigthed, since the previous term become negative.

2.1 Two definitions for risk premia and risk aversion

We now turn to studying risk "in the small" *i.e.* infinitesimal risks. We define as usual the risk premium of an infinitesimal risk as the difference between the expected outcome of the lottery and its certainty equivalent. We are led to the following equality, which is established in Appendix 2:

$$\underbrace{PR\left(\overline{w}\right)}_{TRP} = \underbrace{A(\overline{u(w)})Var\left[\widetilde{w}\right]}_{PRP} - \underbrace{\left(u'\left(\overline{w}\right)/(2u''\left(\overline{w}\right))\right)Var\left(\widetilde{w}\right)}_{COP}$$

$$\begin{aligned} & TOTAL\ RISK\ PREMIUM \\ = \ & DISAPPOINTMENT\ PREMIUM \\ & +CONCAVITY\ PREMIUM\ \ (or\ Arrow\text{-}Pratt\ Premium) \end{aligned}$$

Each of the two premia is proportional to the variance of the lottery under review. When the first premium is considered, the coefficient of proportionality only depends on the tastes of the investor since it is equal to $A(\overline{u(w)})$, which characterizes disappointment aversion. Hence, we can write:

$$TRA\left(\overline{w}\right)=\underbrace{2A(\overline{u(w)})}_{DRA(\overline{w})}+\underbrace{\left(-u^{''}\left(\overline{w}\right)/u^{'}\left(\overline{w}\right)\right)}_{ARA(\overline{w})}$$

$$\begin{aligned}
& TOTAL\ RISK\ AVERSION \\
= & \ DISAPPOINTMENT\ AVERSION \\
& +ARROW-PRATT\ RISK\ AVERSION
\end{aligned}$$

The reason for coefficient 2 to appear is that, according to tradition, we have defined risk aversion as twice the ratio of the premium to the variance of the payoffs.

All the preceding results hold "in the small", *i.e.* for infinitesimal risks. We could now generalize Pratt 's (1964) theorem and cope with risk in the large. To spare space we limit here, as far as finite risks are considered, to selecting Markowitz' point of view, leading to an alternative definition of a risk premium, *i.e.:* the difference between the utility of the expected gain $E_p\left[u\left(\widetilde{w}\right)\right]$ and the utility $U\left(\widetilde{w}\right)$ of the uncertain wealth $\widetilde{w}$ which is but the expected utility of wealth using the transformed probabilities $E_\pi\left[u\left(\widetilde{w}\right)\right]$. Hence:

$$\begin{aligned}
U\left(\widetilde{w}\right) &= E_\pi\left[u\left(\widetilde{w}\right)\right]=E_p\left[\left(1-A_p\left(u(\widetilde{w})-E_p\left[u(\widetilde{w})\right]\right)\right)u\left(\widetilde{w}\right)\right] \\
&= E_p\left[u\left(\widetilde{w}\right)\right]-A_pVar_p\left[u\left(\widetilde{w}\right)\right]
\end{aligned}$$

and, finally:

$$\underbrace{u\left(E_p\left[\widetilde{w}\right]\right)-U\left(\widetilde{w}\right)}_{TOTAL\ PREMIUM} \equiv u\left(E_p\left[\widetilde{w}\right]\right)-E_\pi\left[u\left(\widetilde{w}\right)\right] \tag{2}$$

$$= \underbrace{u\left(E_p\left[\widetilde{w}\right]\right)-E_p\left[u\left(\widetilde{w}\right)\right]}_{MARKOWITZ\ PREMIUM} + \underbrace{A_{\left(E_p\left[u\left(\widetilde{w}\right)\right]\right)} Var_p\left[u\left(\widetilde{w}\right)\right]}_{DISAPPOINTMENT\ PREMIUM}$$

The new premium is the sum of two terms: the first, which is Markowitz' premium, would correspond to the case of a VNM individual. The second corresponds to the behavior of probability transformation. In the case when utility is linear the Markowitz premium vanishes whereas the disappointment reduces to $A_{\left(E_p\left[u\left(\widetilde{w}\right)\right]\right)} Var_p\left[\widetilde{w}\right]$.

2.2 The mean-variance criterion as "dual" theory

A particular case arises when individuals have a constant marginal utility.[5] Although we cannot bring here some empirical evidence, we think it is probably a very frequent situation in the daily life. Indeed, institutional investors are likely to have both a marginal utility close to one and to feel disappointment when they observe their performance. Considering equation (2) and assuming a constant marginal utility, the difference between the utility of expected wealth and its expected utility is:

$$u\left(E_p\left[\widetilde{w}\right]\right)-E_\pi\left[u\left(\widetilde{w}\right)\right] = A_p Var_p\left[u\left(\widetilde{w}\right)\right]$$

One can view this relation as a justification of the utilization of the mean-variance criterion since the equation above can be written as follows:

[5] The VNM axiomatics cannot suitably deal with this case except for admitting the risk neutral assumption of all investors.

$$
\begin{aligned}
U(\widetilde{w}) &= E_\pi[u(\widetilde{w})] = u(E_p[\widetilde{w}]) - A_p Var_p[u(\widetilde{w})] \\
&= E_p[\widetilde{w}] - A_p Var_p[\widetilde{w}]
\end{aligned}
$$

where A is a decreasing function of $E_p[\widetilde{w}]$. Then, $U(\widetilde{w})$ is well an increasing function of $E_p[\widetilde{w}]$. and a decreasing function of $Var_p[\widetilde{w}]$.

3 Market Equilibrium: Some Illustrations

We now address the issue of market equilibrium with disappointment averrse individuals. Our analysis is threefold. We first characterize a static market equilibrium. We then show that the CAPM can be viewed as a model corresponding to the dual case of constant marginal utility and of constant absolute disappointment aversion. Finally, we cope with intertemporal models and we propose new Euler equations.

3.1 Static market equilibrium

We consider the case of one single period securities market. We assume there exists a finite set of states of the world with K elements. The value of each state is revealed to the investors at time $t = 1$. There are $N + 1$ assets exchanged on this market. They are labelled with the superscript n $(n = 0, 1, ..., N)$. S_0^n will denote the time-0 price of security n, and, S_k^n $(k = 1, ..., K)$ the time-1 price of security n in the kth state of world.

We assume that the preferences of our consumer market are represented by an additive and separable utility function, and we suppose he cautiously alters the probabilities. Using α^{-1} as subjective discount factor, his utility function can be written as follows:

$$U(w_0, \widetilde{w}) = u(w_0) + \alpha^{-1} \sum_{k=1}^{K} p_k \left(1 - A\left(E_p\left[u(\widetilde{w})\right]\right)\left(u(w_k) - E_p\left[u(\widetilde{w})\right]\right)\right) u(w_k) \tag{3}$$

To spare space, we assume the existence and the uniqueness of a static equilibrium. Given (3), Appendix 3 provides the equilibrium prices of the market described above, and we get: So, we get:

$$S_0^n = \frac{1}{\alpha u'(w_0)} \begin{pmatrix} E_p\left[u'(\widetilde{w})\widetilde{S}^n\right]\left(1 - A' Var_p\left[u(\widetilde{w})\right]\right) \\ -2ACov_p\left(u'(\widetilde{w})\widetilde{S}^n, u(\widetilde{w})\right) \end{pmatrix} \tag{4}$$

3.2 Constant marginal utility and constant disappointment aversion

We turn to the interesting case of both constant marginal utility ($u(w) = w$) and constant absolute disappointment aversion ($A' = 0$). We can rewrite (4) as follows:

$$S_0^n = \frac{1}{\alpha}\left(E_p\left[\widetilde{S}^n\right] - 2ACov_p\left[\widetilde{S}^n, \widetilde{w}\right]\right)$$

We assume the existence of a risk-free asset. It will be denoted by the superscript ($n = 0$), we then have:

$$1 = \frac{1}{\alpha}(1 + r_0) \Leftrightarrow \alpha = 1 + r_0$$

The expected return of the risky assets ($n > 0$) is given by:

$$E_P\left[\widetilde{r}^n\right] = r_0 + 2ACov_P\left[\widetilde{w}, \widetilde{r}^n\right] \tag{5}$$

At the equilibrium, the risk premium is equal to the product between the risk aversion and the covariance between the rate return of the asset and the income of the consumer. To go on with our analysis, we now suppose there exists a representative consumer. Let us denote M_0 and M, the values of the market portfolio in dates 0 and 1, and θ^n, the agent's asset n holdings at date 0. If the state of world k occurs then the final wealth is:

$$\widetilde{w}_k = \beta\left(W_0 - w_0 - \sum_{j=1}^{N}\theta_0^j S_0^j\right) + \sum_{j=1}^{N}\theta_k^j \widetilde{S}_k^j = M + \beta\left(W_0 - w_0 - M_0\right)$$

If $\tilde{r}^M$ designates the rate of return of the market portfolio, we have $M = M_0\left(1 + \tilde{r}^M\right)$, and, for the representative consumer:

$$Cov_P\left(\widetilde{w}_k, \tilde{r}^n\right) = \sum_{j=1}^{N}\theta_k^j Cov_P\left(\widetilde{S}^j, \tilde{r}^n\right) = Cov_P\left(M, \tilde{r}^n\right) = M_0 Cov_P\left(\tilde{r}^M, \tilde{r}^n\right)$$

Using this result and Equation (5), we can write for all assets:

$$E_P\left[\tilde{r}^n\right] = r_0 + 2AM_0 Cov\left(\tilde{r}^M, \tilde{r}^n\right) \tag{6}$$

And, for the market portfolio:

$$E_P\left[\tilde{r}^M\right] = r_0 + 2AM_0 Var\left[\tilde{r}^M\right] \tag{7}$$

Note that $Cov\left(\tilde{r}^M, \tilde{r}^n\right) = \beta_M^n Var\left[\tilde{r}^M\right]$. Using (6) and (7) leads to the well-known CAPM equation:

$$E_P\left[\widetilde{r}^n\right] = r_0 + \beta_M^n\left(E_P\left[\tilde{r}^M\right] - r_0\right)$$

It is well-known that, in the VNM universe, the CAPM relation can be established if and only if at least one of the two following conditions is verified:

–The asset returns are distributed following by laws fully defined from their two first moments;

–The investors utility functions are quadratic.

Nevertheless, it is well-documented that the second assumption implies a result invalidated by empirical works: the absolute risk aversion in the Arrow-Pratt sense is not an increasing function of wealth. The first assumption leads to consider the normality of the asset returns. But it is contradicted by the empirical observations even if we consider the interesting Lévy-stable laws.

Hence, it is somewhat paradoxical that the CAPM has been so popular, that it remains the reference for any empirical work on the stock markets and that fund managers still continue to use the mean-variance criterion. Such a paradox can be addressed, if one assumes that investors, especially institutional investors, who play the most important role on financial markets, have both a constant marginal utility and a constant absolute disappointment risk aversion; if so, the market equilibrium must be in accordance with the CAPM. This justification of the use of the CAPM appears to us more convincing than that of the one traditionally made, in terms of quadratic utility functions or normal distributions.

3.3 The consumption-based asset pricing model with probabilities alteration:

We can write, using the same labelling as before, the maximization program of the representative consumer, where w_t and W_t respectively represent the consumption and the wealth of the representative agent at time t:

$$MAX \quad E_{\pi,0}\left[\sum_{t=0}^{\infty} \alpha^{-t} u\left(\widetilde{c}_t\right)\right]$$

subject to the budget constraints:

i) $W_t = \sum_{n=1}^{N} \theta_t^n S_t^n + c_t$

ii) $W_{t+1} = \sum_{n=1}^{N} \theta_t^n S_{t+1}^n$

Note that current wealth Wt is a state variable and that current asset holdings θ_t^n are the control variables of this program; we can thus write the following Bellman equation:

$$U\left(W_t, \overrightarrow{S}_t\right) = \underbrace{MAX}_{\overrightarrow{\theta}_t} \left[\begin{array}{c} u\left(W_t - \sum_{n=1}^{N} \theta_t^n S_t^n\right) \\ +\alpha^{-1} E_{p,t}\left[\gamma^p\left(W_{t+1}\right) U\left(W_{t+1}, \overrightarrow{S}_{t+1}\right)\right] \end{array} \right]$$

Appendix 4 provides the resolution of this program. The corresponding Euler equations are:

$$S_t^n = \frac{1}{\alpha u'\left(c_t\right)} \left[\begin{array}{c} E_{p,t}\left[\widetilde{S}_{t+1}^n u'\left(c_{t+1}\right)\right]\left(1 - A' Var_{p,t}\left[u\left(c_{t+1}\right)\right]\right) \\ -2ACov_{p,t}\left(u(c_{t+1}), \widetilde{S}_{t+1}^n u'\left(c_{t+1}\right)\right) \end{array} \right] \qquad (8)$$

4 Disappointment and Pessimism and The Equity Premium Puzzle

In this section we explore the ability of our theory to cope with a well-known puzzle of the financial literature, namely the Equity Premium Puzzle. In a seminal paper, Mehra and Prescott (1985) found that the Lucas model with an expected utility maximizer was not able to account for the average equity premium over the period 1889-1978. Indeed, with realistic values of the parameters, the authors could only produce an average equity premium of 0.35% (instead of 6.18% corresponding to the premium observed in the US data). They also established that the historical equity premium could be achieved by the model considering large relative risk-aversion coefficients (more than 30). However, even if one believes in such values, one cannot get rid of an another puzzle, namely the risk-free rate puzzle, first identified by Weil (1989), since the simulated values of the risk-free rate then seem much too high (more than 15% compared to the historical value of 0.8%).

The main approach used in the literature to solve this puzzle consists of generalizing preferences of the representative consumer to introduce non-separabilities in the state space or in the temporal space. Epstein and Zin (1991b) explore a model using recursive utility functions, while Constantinides (1990) and Campbell and Cochrane (1999) mix in a property of habit formation with the utility function. Except for Epstein and Zin (1990) who use the dual theory of choice proposed by Yaari (1987), few authors have tried to exploit the new theories of decision-making under risk to explain the behavior of real per-capita consumption and asset returns. Up to 1996 as reported by Kocherlakota, none of these approaches have succeeded in solving the two puzzles, even if a study of Benartzi and Thaler (1995) using the concept of *myopic loss aversion*[6] in a timeless framework[7] provided an explanation of the equity premium puzzle considering investors who evaluate annually their portfolio.

Recently, Barberis *et alii* (2000) using a preference representation with both CRRA utility function and a function of wealth with a property of *loss aversion* were able to explain them. However, though that work appears very promising, the preference function they use has not been justified by any axiomatics.

Our theory is very closely related to the article of Cechetti *et alii (2000)*. In their paper, they show that the standard asset pricing model in which agents are pessimistic can solve both the equity premium and the risk-free rate puzzles. To justify that assumption of pessimistic agents, they argue that they are not fully rational. In addition to that "undesirable" property, their framework do not permit agents to be bayesian, in the sense that they can not learn from their past errors.

Our theory provides answers for these two drawbacks. Actually, our axiomatics (See Chauveau et Nalpas, 2000) show that people care about disappointment. The disappointment aversion implies that probabilities of favorable (unfavorable) events are then under(over)-weighted. So, the more disappointment averse an agent is, the more pessimistic he is. Moreover, that property of pessimism is a consequence of a behavior towards risk. So our model can embed both full rationality of expectations and systematic

[6]that mixes the concepts of *loss aversion* and *mental accounting*.

[7]It's due to the property of *loss aversion* for which the utility is no more defined over the consumption plan but only over the wealth, what doesn't allow an intertemporal consumption-based analysis.

pessimism.

We use the Mehra and Prescott (1985) framework to estimate the level of disappointment necessary to produce the observed equity premium and risk-free rate in the United States data. We then calibrate our new Euler equations (8) following the Mehra and Prescott methodology. In order to grasp the only influence of disappointment on equity premia and risk-free rates, we assume the risk neutrality of our representative agent in the Arrow-Pratt sense. So the model displays the same number of parameters as the standard asset pricing model. The relative risk aversion parameter is then replaced by the disappointment aversion one.

The calibration requires two kinds of parameters: (i) those defining the preferences of the representative agent,[8] and (ii) those defining the considered technology of the economy.[9] The latter parameters are selected so that they match the sample values for the US economy over the period 1889-1978.

To avoid complex calculations, we make the traditional assumption that our decision maker has a constant relative disappointment aversion. Figure 1 gives the average simulated equity premia corresponding to the Mehra and Prescott original model in which, as the authors did, we have introduced a leverage effect (θ).[10]. The maximum premium simulated by the Mehra and Prescott model does not exceed 0.7%. This value is much lower than the observed equity premium (6.18%) and it is obtained with an induced risk-free rate close to 4%, which is much higher than that observed (0.8%).

[8] Her subjective discount factor and her relative disappointment aversion coefficient.

[9] The average growth rate of the per capita consumption on non-durables and services, its standard deviation, and its first-order serial correlation.

[10] We have allowed θ to vary in the interval [0,0.975] in order to produce sufficient high levels for the second moment of the simulated equity premia.

In order to compare our results with those of Mehra and Prescott, we have used similar specifications:

i) we impose a maximal value of 4% for the simulated risk-free rate, which produces an interval of [0.96,1] for the subjective discount factor of our model;

ii) We limit the values of the simulated equity premia to those which correspond to a disappointment aversion coefficient less than 10.

The values of the equity risk premia obtained with our model are plotted on Figure 1.

Thus, our model is able to produce both high equity premia and low risk-free rates. It can be considered as a model that solves the equity premium and the risk-free rate puzzles.

Figure 1: Simulated Equity Premia for The Disappointment Weighted Utility Theory

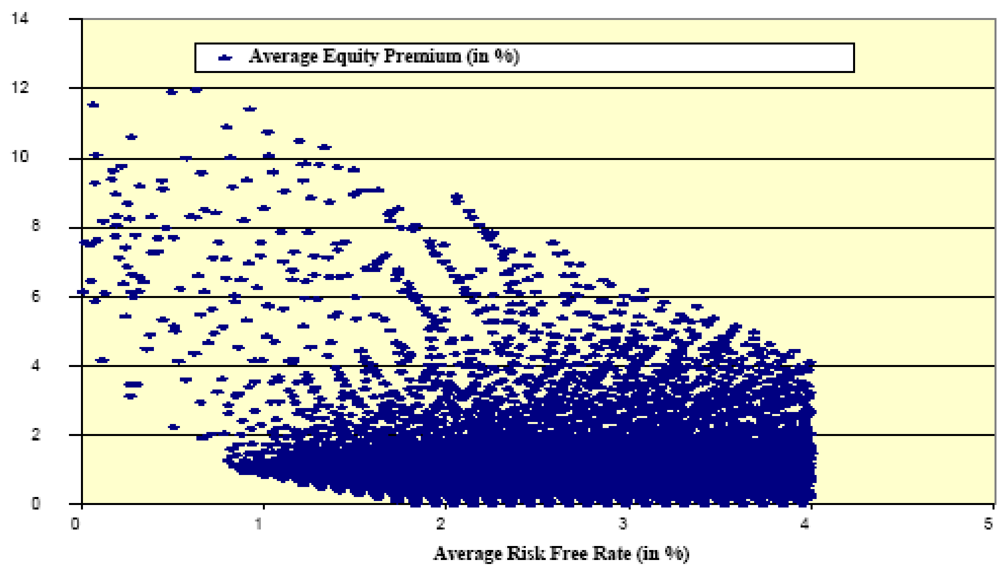

Figure 2:

5 Conclusion

In that paper, we have proposed a preference function based on the concept of disappointment. We have distinguished two different concepts for describing the behavior towards risk in our model, which are represented by the standard risk aversion and the disappointment aversion. The idea underlying our theory is that individuals are disappointment averse, what lead them to over-weight the probabilities of the lower outcomes and under-weight the probabilities of the higher ones, so that they are pessimistic.

Finally, we obtained three main results:

-(i) the risk premium is the sum of the Arrow-Pratt premium, which , in reality, is only a concavity premium, and a disappointment premium characterizing the intensity of the deformation of probabilities; such a result holds in the small and in the large;[11]

-(ii) an "attractive" justification for utilizing the mean-variance criterion, and, consequently for referring to the CAPM is the existence of investors having both a constant marginal utility and a constant absolute disappointment aversion;

-(iii) We have modified the Euler equations of an intertemporal asset-pricing model which from now on consist of the sum of four terms, incorporating the behavior towards disappointment risk. Using both these new Euler equations and the framework of Mehra and Prescott (1985) has enabled us to obtain results that can be viewed as a solution to the equity premium puzzle.

Two final remarks can be made: one could believe that assuming disappointment aversion may challenge the validity of the CAPM in the case of decreasing marginal utility. This is obviously not true since a Taylor development to the second (third, . . .) order of the utility function allows for the 3CAPM (4CAPM, . . .) to prevail. Finally, further empirical analysises shoud be undertaken, in particular, investigations on how our theory can cope with the observed second moments of the equity pemium and risk-free rate as well as the persitence and predictability of excess returns found in the data.

[11] However we have limited ourselves to considering Markowitz' point of view and not Pratt's, for risk in the large.

APPENDIX 1:

Calculation of the risk premium

We write therefore, for an additive risk with usual notations:

$\widetilde{w} = \overline{w} + \widetilde{\varepsilon} \quad , \quad E_p[\widetilde{\varepsilon}] = 0 \quad , \quad E_p[\widetilde{w}] = \overline{w} \quad , \quad Var_p[\widetilde{w}] = Var_p[\widetilde{\varepsilon}] = \sigma^2$

The utility of this lottery is:

$$
\begin{aligned}
U(\widetilde{w}) &= E_p\left[\gamma_p(\widetilde{w})\,u(\widetilde{w})\right] = E_p\left[\left(1 - A_p\left(u(\widetilde{w}) - E_p[u(\widetilde{w})]\right)\right)u(\widetilde{w})\right] \\
&= E_p\left[\begin{array}{c}\left(\begin{array}{c} 1- \\ A_p\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)u'(\overline{w}) + \frac{1}{2}\left(\begin{array}{c}\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2 \\ -Var_p[\widetilde{w}]\end{array}\right)u''(\overline{w})\right) \\ +o\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2\right)\end{array}\right) \\ \left(\begin{array}{c} u(\overline{w}) + \left(\widetilde{w} - E_p[\widetilde{w}]\right)u'(\overline{w}) + \\ \frac{1}{2}\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2 u''(\overline{w}) + o\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2\right)\end{array}\right)\end{array}\right] \\
&= E_p\left(\begin{array}{c} 1- \\ A_p\left(\begin{array}{c}\left(\begin{array}{c} u(\overline{w}) + \left(\widetilde{w} - E_p[\widetilde{w}]\right)u'(\overline{w}) \\ +\frac{1}{2}\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2 u''(\overline{w}) + o\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2\right)\end{array}\right) \\ -A_p\left[\left(\begin{array}{c}\left(\widetilde{w} - E_p[\widetilde{w}]\right)u(\overline{w})\,u'(\overline{w}) + \left(\widetilde{w} - E_p[\widetilde{w}]\right)^2 u'(\overline{w})^2 + \\ \frac{1}{2}\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2 - Var_P[\widetilde{w}]\right)u(\overline{w})\,u''(\overline{w}) \\ +o\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2\right)\end{array}\right)\right]\end{array}\right)\end{array}\right) \\
&= u(\overline{w}) + \frac{1}{2}u''(\overline{w})\,Var_p[\widetilde{w}] - A_p u'(\overline{w})^2\,Var_p[\widetilde{w}] + o\left(\left(\widetilde{w} - E_p[\widetilde{w}]\right)^2\right)
\end{aligned}
$$

So, if we limit to second order terms:

$$U(\widetilde{w}) = u(\overline{w}) - \left(A_p u'(\overline{w}) - \frac{1}{2}u''(\overline{w})\right)Var_p[\widetilde{w}]$$

The certainty equivalent of this lottery is defined by:

$$U(\widetilde{w}) = u(EC(\widetilde{w}))$$

And the risk premium is:

$$PR(\widetilde{w}) = \overline{w} - EC(\widetilde{w})$$

Combining the preceding equations, we get:

$$U(\widetilde{w}) = u(EC(\widetilde{w})) = u(\overline{w} - PR(\overline{w})) = u(\overline{w}) - PR(\overline{w})\, u^{'}(\overline{w}) + o(PR(\overline{w}))$$

And, we finally have:

$$PR(\overline{w}) = A_p Var_p[\widetilde{w}] - \frac{1}{2}\frac{u^{''}(\overline{w})}{u^{'}(\overline{w})} Var_p[\widetilde{w}]$$

APPENDIX 2:

Static maximization program with probabilities alteration

$MAX\ E_\pi \left[U\left(w_0, \widetilde{w}\right)\right]$

subject to the contraints:

i) $w_0 + \theta^0 + \sum\limits_{n=1}^{N} \theta^n S_0^n - W_0 = 0$

$\Rightarrow w_0 = W_0 - \left(\theta^0 + \sum\limits_{n=1}^{N} \theta^n S_0^n\right)$

ii) $w_k - \beta\theta^0 - \sum\limits_{n=1}^{N} \theta^n S_k^n = 0$

$\Rightarrow w_k = \beta\theta^0 + \sum\limits_{n=1}^{N} \theta^n S_k^n \quad , k = 1, ..., K$

Where θ^n is the date 0 holding of the asset n, w_k is the date 1 consumption of the state k.

The Lagrangian function of this program is:

$$\begin{aligned} L &= U\left(w_0, \widetilde{w}\right) - \lambda_0 \left(w_0 + \theta^0 + \sum_{n=1}^{N} \theta^n S_0^n - W_0\right) \\ &\quad - \sum_{k=1}^{K} \lambda_k \left(w_k - \beta\theta^0 - \sum_{n=1}^{N} \theta^n S_k^n\right) \end{aligned}$$

with:

$$\begin{aligned} U\left(w_0, \widetilde{w}\right) &= u(w_0) + \alpha^{-1} \left[\sum_{j=1}^{K} p_j (1 + A\left(E_p\left[u\left(\widetilde{w}\right)\right] - u\left(w_j\right)\right)) u\left(w_j\right)\right] \\ &= u(w_0) + \alpha^{-1} \left[\sum_{j=1}^{K} p_j \gamma_j u\left(w_j\right)\right] \end{aligned}$$

The first order conditions are:

- $\frac{\partial L}{\partial w_0} = u^{'}(w_0) - \lambda_0 = 0 \Rightarrow \lambda_0 = u^{'}(w_0)$

- $\frac{\partial L}{\partial w_k} = \alpha^{-1}\left[p_k \gamma_k u^{'}(w_k) + \sum_{j=1}^{K} p_j u(w_j) \frac{\partial \gamma_j}{\partial w_k}\right] - \lambda_k = 0$

  $\Rightarrow \lambda_k = \alpha^{-1}\left[p_k \gamma_k u^{'}(w_k) + \Gamma(k)\right] \qquad , k = 1, ..., K$

- $\frac{\partial L}{\partial \theta^0} = -\lambda_0 + \left(\sum_{k=1}^{K} \lambda_k\right)\beta = 0 \Rightarrow \sum_{k=1}^{K} \lambda_k = \beta^{-1} u^{'}(w_0)$

- $\frac{\partial L}{\partial \theta^n} = -\lambda_0 S_0^n + \left(\sum_{k=1}^{K} \lambda_k S_k^n\right) = 0 \ , \ n = 1, ..., N$

Combining the third and last equations, we get:

$$S_0^n = \frac{\sum_{k=1}^{K} \lambda_k S_k^n}{u^{'}(w_0)} = \beta^{-1} \frac{\sum_{k=1}^{K} \lambda_k S_k^n}{\sum_{k=1}^{K} \lambda_k} = \beta^{-1} \sum_{k=1}^{K} \mu_k S_k^n$$

$$\text{With} \qquad \mu_k = \frac{\lambda_k}{\sum_{j=1}^{K} \lambda_j} \geq 0 \qquad \text{and} \qquad \sum_{k=1}^{K} \mu_k = 1$$

The equilibrium is defined by the condition that the μ_k are equal to the risk neutral probabilities q_k. Thus, we have the solution of our program:

$$S_0^n = \frac{\sum_{k=1}^{K} \lambda_k S_k^n}{u^{'}(w_0)} = \frac{1}{\alpha u^{'}(w_0)} \sum_{k=1}^{K} \left(p_k \gamma_k u^{'}(w_k) S_k^n + \Gamma(k) S_k^n\right)$$

Since we have:

$$
\begin{aligned}
&\sum_{k=1}^{K}\left(p_k\gamma_k u^{'}\left(w_k\right)S_k^n\right)\\
=\;&\sum_{k=1}^{K}\left(p_k u^{'}\left(w_k\right)S_k^n\left(\left(1-A\left(u(w_k)-E_p\left[u(\widetilde{w})\right]\right)\right)\right)\right)\\
=\;&E_p\left[u^{'}\left(\widetilde{w}\right)\widetilde{S}^n\right]-AE_p\left[\left(u(\widetilde{w})-E_p\left[u(\widetilde{w})\right]\right)u^{'}\left(\widetilde{w}\right)\widetilde{S}^n\right]\\
=\;&E_p\left[u^{'}\left(\widetilde{w}\right)\widetilde{S}^n\right]-ACov_p\left(u^{'}\left(\widetilde{w}\right)\widetilde{S}^n,u(\widetilde{w})\right)
\end{aligned}
$$

And,

$$
\begin{aligned}
\frac{\partial\gamma_j}{\partial w_k} &= p_k u'\left(w_k\right)\left[A'\left(E_p\left[u\left(\widetilde{w}\right)\right]-u\left(w_j\right)\right)+A\right]-\delta_{jk}Au'\left(w_k\right)\\
\Gamma\left(k\right) &= \sum_{j=1}^{K}p_j u\left(w_j\right)\frac{\partial\gamma_j}{\partial w_k}\\
&= p_k u'\left(w_k\right)\left[-A'Var_p\left[u\left(\widetilde{w}\right)\right]+A\right]-Ap_k u\left(w_k\right)u'\left(w_k\right)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\sum_{k=1}^{K}\Gamma\left(k\right)S_k^n &= -A'Var_p\left[u\left(\widetilde{w}\right)\right]\left\{\sum_{k=1}^{K}p_k u'\left(w_k\right)S_k^n\right\}\\
&\quad +A\left\{\sum_{k=1}^{K}p_k u'\left(w_k\right)S_k^n\left(E_p\left[u\left(\widetilde{w}\right)\right]-u\left(w_k\right)\right)\right\}\\
\sum_{k=1}^{K}\Gamma\left(k\right)S_k^n &= -A'E_p\left[u^{'}\left(\widetilde{w}\right)\widetilde{S}^n\right]Var_p\left[u\left(\widetilde{w}\right)\right]-ACov_p\left(u^{'}\left(\widetilde{w}\right)\widetilde{S}^n,u\left(\widetilde{w}\right)\right)
\end{aligned}
$$

So, we get:

$$
S_0^n=\frac{1}{\alpha u'\left(w_0\right)}\begin{pmatrix} E_p\left[u^{'}\left(\widetilde{w}\right)\widetilde{S}^n\right]\left(1-A'Var_p\left[u\left(\widetilde{w}\right)\right]\right)\\ -2ACov_p\left(u^{'}\left(\widetilde{w}\right)\widetilde{S}^n,u\left(\widetilde{w}\right)\right)\end{pmatrix}
$$

APPENDIX 3:

Intertemporal maximization program with probabilities alteration

Consider an economy where $N+1$ assets are traded: the fisrt one is riskless whereas the others are risky. The evolution of the vector $S_t = (S_t^0, S_t^1, ..., S_t^N)$ of the $N+1$ prices is assumed to be described by a multivariate stochastic process. In each period t, the decision maker must choose simultaneously his/her consumption and his/her portfolio whose composition is characterized by the vector$\{\theta_t = (\theta_t^0, \theta_t^1, ..., \theta_t^N)\}$ denoting the quantities of assets held during the th period. We shall denote θ_t' its transpose. In other words, the individual chooses the value for the endogenous state variable in the subsequent period and his/her choice can be described by a sequence $\Theta_T = \{\theta_0, \theta_1, \theta_2, \; ...., \theta_{T-1}, \theta_T\}$ of vectors $\{\theta_t\}$ corresponding to the quantities of assets held during the th period. A *feasible* investment plan can be viewed as a multivariate stochastic process $\{\theta_t\}$ obeying the following constraints :

$$c_t = (\theta_t' - \theta_{t+1}')S_t \geqslant 0 \text{ for } t \in \mathbb{N}_+$$

with the following denominations:

$$\begin{aligned} w_t &= \theta_t' S_t \\ c_t &= w_t - \theta_{t+1}' S_t = (\theta_t' - \theta_{t+1}')S_t \end{aligned}$$

or equivalently:

$$\begin{aligned} w_{t+1} &= \theta_{t+1}' S_{t+1} \\ w_t &= c_t + \theta_{t+1}' S_t \end{aligned}$$

and, conventionnally:

$$c_0 = w_0 - \theta_1' S_0 = (\theta_0' - \theta_1') S_0$$

By assumption, in each period t, the decision maker exhibits the following lifetime utility function:

$$\begin{aligned}
\mathcal{U}_0 &= u(c_0) \\
&+\alpha^{-1} E_p \left[\begin{array}{c} u(c_1)\{1 + A(E_p[u(c_1) \mid \mathcal{F}_0] - u(c_1))\} \\ \mid \mathcal{F}_0 \end{array} \right] \\
&+\alpha^{-2} E_p \left[\begin{array}{c} E_p[u(c_2)\{1 + A(E_p[u(c_2) \mid \mathcal{F}_1] - u(c_1))\} \mid \mathcal{F}_1] \\ \mid \mathcal{F}_0 \end{array} \right] + .. \\
&+\alpha^{-T} E_p \left[..E_p \left[E_p \left[\begin{array}{c} u(c_T)\{1 + A(E_p[u(c_T) \mid \mathcal{F}_{T-1}] - u(c_T))\} \\ \mid \mathcal{F}_{T-1} \end{array} \right] \mid \mathcal{F}_{T-2} \right] \mid .. \mid \mathcal{F}_0 \right]
\end{aligned}$$

or:

$$\begin{aligned}
\mathcal{U}_0 &= u(c_0) + \alpha^{-1} \left(\begin{array}{c} E_p[u(c_1) \mid \mathcal{F}_0] + \\ E_p \left[u(c_1) A \left(\begin{array}{c} E_p[u(c_1) \mid \mathcal{F}_0] \\ -u(c_1) \end{array} \right) \mid \mathcal{F}_0 \right] \end{array} \right) \\
&+\alpha^{-2} \left(E_p \left[\begin{array}{c} E_p[u(c_2) \mid \mathcal{F}_1] + \\ E_p \left[u(c_2) A \left(\begin{array}{c} E_p[u(c_2) \mid \mathcal{F}_1] \\ -u(c_1) \end{array} \right) \mid \mathcal{F}_1 \right] \mid \mathcal{F}_0 \end{array} \right] \right) + .. \\
&+\alpha^{-T} E_p \left[\begin{array}{c} ..E_p \left[\begin{array}{c} E_p[u(c_T) \mid \mathcal{F}_{T-1}] + \\ E_p \left[u(c_T) A \left(\begin{array}{c} E_p[u(c_T) \mid \mathcal{F}_{T-1}] \\ -u(c_T) \end{array} \right) \mid \mathcal{F}_{T-1} \right] \\ \mid \mathcal{F}_{T-2} \end{array} \right] \\ \mid .. \mid \mathcal{F}_0 \end{array} \right]
\end{aligned}$$

or, equivalently:

$$\mathcal{U}_0 = u(c_0) + \alpha^{-1}\left(\begin{array}{c} E_p[u(c_1) \mid \mathcal{F}_0] \\ +E_p\left[E_p\left[u(c_1)\, A\left(\begin{array}{c} E_p[u(c_1) \mid \mathcal{F}_0] \\ -u(c_1)\end{array}\right) \mid \mathcal{F}_0\right] \mid \mathcal{F}_0\right]\end{array}\right)$$
$$+\alpha^{-2}E_p\left[\left(\begin{array}{c} E_p[u(c_2) \mid \mathcal{F}_1] \\ +E_p\left[E_p\left[u(c_2)\, A\left(\begin{array}{c} E_p[u(c_2) \mid \mathcal{F}_1] \\ -u(c_1)\end{array}\right) \mid \mathcal{F}_1\right] \mid \mathcal{F}_1\right]\end{array}\right) \mid \mathcal{F}_0\right] + ..$$
$$+\alpha^{-T}E_p\left[\begin{array}{c} ..E_p\left[\begin{array}{c}\left(\begin{array}{c} E_p[u(c_T) \mid \mathcal{F}_{T-1}] \\ + \\ E_p\left[\begin{array}{c} E_p\left[u(c_T)\, A\left(\begin{array}{c} E_p[u(c_T) \mid \mathcal{F}_{T-1}] \\ -u(c_T)\end{array}\right) \mid \mathcal{F}_{T-1}\right] \\ \mid \mathcal{F}_{T-1}\end{array}\right]\end{array}\right) \\ \mid \mathcal{F}_{T-2}\end{array}\right] \\ \mid .. \mid \mathcal{F}_0\end{array}\right] \quad (9)$$

Now, let[12]*

$$\ominus_t\left(w_t - \theta'_{t+1}S_t\right) = u\left(w_t - \theta'_{t+1}S_t\right) + \alpha^{-1}E_p\left[\begin{array}{c} u\left(\begin{array}{c} w_t \\ -\theta'_{t+1}S_t\end{array}\right) A[.]\left(E_p\left[\begin{array}{c}\left(w_t - \theta'_{t+1}S_t\right) \\ \mid \mathcal{F}_{t-1}\end{array}\right] - \right. \\ \mid \mathcal{F}_{t-1}\end{array}\right.$$
$$with :$$
$$A[.] \equiv A\left[E_p\left[\begin{array}{c}\left(w_t - \theta'_{t+1}S_t\right) \\ \mid \mathcal{F}_{t-1}\end{array}\right]\right]$$
$$\ominus_0(w_0 - \theta'_1 S_0) = u(w_0 - \theta'_1 S_0)$$
$$\theta'_{T+1} = 0 \Rightarrow \ominus_T\left(w_T - \theta'_{T+1}S_T\right) = \ominus_T(w_T)$$

The lifetime utility function then reduces to:

---

[12] with the following convention:

$$\varphi_0(c) = u(c)$$

$$
\begin{aligned}
\mathcal{U}_0(.) &= \ominus_0\left(w_0-\theta_1' S_0\right)+\alpha^{-1} E_p\left[\ominus_1\left(w_1-\theta_2' S_1\right) \mid \mathcal{F}_0\right] \\
&+\alpha^{-2} E_p\left[E_p\left[\ominus_2\left(w_2-\theta_3' S_2\right) \mid \mathcal{F}_1\right] \mid \mathcal{F}_0\right]+ \\
&..+\alpha^{-T} E_p\left[..E_p\left[E_p\left[\ominus_T\left(w_T\right) \mid \mathcal{F}_{T-1}\right] \mid \mathcal{F}_{T-2}\right] .. \mid \mathcal{F}_0\right]
\end{aligned}
$$

and, one period ahead, it reads:

$$
\begin{aligned}
\mathcal{U}_1(.) &= \ominus_1\left(w_1-\theta_2' S_1\right)+\alpha^{-1} E_p\left[\ominus_2\left(w_2-\theta_3' S_2\right) \mid \mathcal{F}_1\right] \\
&+\alpha^{-2} E_p\left[\begin{array}{c} E_p\left[\ominus_3\left(w_3-\theta_4' S_3\right) \mid \mathcal{F}_2\right] \\ \mid \mathcal{F}_1 \end{array}\right]+ \\
&..+\alpha^{-(T-1)} E_p\left[\begin{array}{c} ..E_p\left[\begin{array}{c} E_p\left[\ominus_T\left(w_T\right) \mid \mathcal{F}_{T-1}\right] \\ \mid \mathcal{F}_{T-2} \end{array}\right] \\ .. \mid \mathcal{F}_1 \end{array}\right]
\end{aligned}
$$

We thus, have:

$$
\mathcal{U}_0(.)=\ominus_0\left(w_0-\theta_1' S_0\right)+\alpha^{-1} E_p\left[\mathcal{U}_1(.) \mid \mathcal{F}_0\right]
$$

More generally, we have:

$$
\begin{aligned}
\mathcal{U}_t(.) &= \ominus_t\left(w_t-\theta_{t+1}' S_t\right)+\alpha^{-1} E_p\left[\ominus_{t+1}\left(w_{t+1}-\theta_{t+2}' S_{t+1}\right) \mid \mathcal{F}_t\right] \\
&+\alpha^{-2} E_p\left[E_p\left[\ominus_{t+2}\left(w_{t+2}-\theta_{t+3}' S_{t+2}\right) \mid \mathcal{F}_{t+1}\right] \mid \mathcal{F}_t\right]+ \\
&..+\alpha^{-(T-t)} E_p\left[..E_p\left[E_p\left[\ominus_T\left(w_T\right) \mid \mathcal{F}_{T-1}\right] \mid \mathcal{F}_{T-2}\right] .. \mid \mathcal{F}_t\right]
\end{aligned}
$$

and, finally:

$$
\mathcal{U}_t(.)=\ominus_t\left(w_t-\theta_{t+1}' S_t\right)+\alpha^{-1} E_p\left[\mathcal{U}_{t+1}(.) \mid \mathcal{F}_t\right] \text{ for } t=0, T-1
$$

This last equation exhibits the usual recursive structure of a dynamic programming problem which may be stated with a finite horizon as:

$$\begin{aligned}\underset{\Theta_T}{Max}\,\mathcal{U}_0(.) &= \ominus_0\left(w_0-\theta_1'S_0\right)\\ &+\sum_{t=1}^{T-1}\alpha^{-t}E_p\left[\begin{array}{c}..E_p\left[E_p\left[\begin{array}{c}\ominus_{t+1}\left(w_{t+1}-\theta_{t+2}'S_{t+1}\right)\\ \mid\mathcal{F}_{t-1}\end{array}\right]\mid\mathcal{F}_{t-2}\right]\\ ...\mid\mathcal{F}_0\end{array}\right]\\ &+\alpha^{-T}E_p\left[..E_p\left[E_p\left[\ominus_T\left(w_T\right)\mid\mathcal{F}_{T-1}\right]\mid\mathcal{F}_{T-2}\right]..\mid\mathcal{F}_t\right]\end{aligned}$$

or with an infinite horizon as:

$$\begin{aligned}\underset{\Theta}{Max}\,\mathcal{U}_0(.) &= \ominus_0\left(w_0-\theta_1'S_0\right)\\ &+\sum_{t=1}^{+\infty}\alpha^{-t}E_p\left[\begin{array}{c}..E_p\left[E_p\left[\begin{array}{c}\ominus_{t+1}\left(w_{t+1}-\theta_{t+2}'S_{t+1}\right)\\ \mid\mathcal{F}_{t-1}\end{array}\right]\mid\mathcal{F}_{t-2}\right]\\ ..\mid\mathcal{F}_0\end{array}\right]\\ &*\left(aVar_p\left[g_{t+1}\mid\mathcal{F}_t\right]\right.\end{aligned}$$

given that the last program will make sense and will exhibit a unique solution only under standard restrictive assumptions detailed below. However, if $\{S_t\}$ is a stationary markovian process whose transition probabilities are denominated

$$p_{ij}=\Pr\left(S_{t+1=}S_i\mid S_{t=}S_j\right)$$

which we now assume, we can get rid of the dependency of $\ominus_t$ upon t . Indeed we have:

$$\begin{aligned}\ominus_t\left(w_t-\theta'_{t+1}S_t\right) &= \ominus\left(w_t-\theta'_{t+1}S_t\right)\\ &= u\left(w_t-\theta'_{t+1}S_t\right)\\ &\quad+\alpha^{-1}\sum_{k=1}^{K}p_{k,i(t)}\left\{\begin{array}{c} u\left(w_t-\theta'_{t+1}S_t\right)A.\left[\begin{array}{c}\sum_{j=1}^{K}p_{j,i(t)}u\\ \left(w_t-\theta'_{t+1}S_t\right)\end{array}\right]\\ \left\{\begin{array}{c}\left(\sum_{j=1}^{K}p_{j,i(t)}u\left(w_t-\theta'_{t+1}S_t\right)\right)\\ -u\left(w_t-\theta'_{t+1}S_t\right)\end{array}\right\}\end{array}\right\}\end{aligned}$$

where $i(t)$ is the state of $\{S_t\}$ at time t. As usual , the solution will be obtained using the following usual Bellman equation:

$$\begin{aligned}\underset{\theta_{t+1}}{Max}\mathcal{V}_t\left(w_t,w_{t+1,}...,w_T\right) &= \ominus\left(w_t-\theta'_{t+1}S_t\right)\\ +\alpha^{-1}E_p\left[\mathcal{V}_{t+1}\left(w_{t+1,}...,w_T\right)\mid\mathcal{F}_t\right]\text{ for }t &= 0,1,...\end{aligned}$$

where:and whose first order condition reads:

$$\frac{\partial\mathcal{V}_t}{\partial\theta^n_{t+1}}\left(w_t,.\right)=\frac{\partial\ominus}{\partial\theta^n_{t+1}}\left(w_t-\theta'_{t+1}S_t\right)+\alpha^{-1}E_p\left[\frac{\partial\mathcal{V}_{t+1}}{\partial\theta^n_{t+1}}\left(w_{t+1},.\right)\mid\mathcal{F}_t\right]=0\text{ for }t=0,T-1\text{ and }n=1,N$$

However, it will be more convenient to rewrite the Bellman equation as follows:

$$\begin{aligned}\underset{\theta_{t+1}}{Max}\mathcal{U}_t\left(c_t,c_{t+1},...\right) &= \oplus\left(\theta_t,\theta_{t+1},S_t\right)\\ +\alpha^{-1}E_p\left[\mathcal{U}_{t+1}\left(c_{t+1},c_{t+2},...\right)\mid\mathcal{F}_t\right]\text{ for }t &= 0,1,...\end{aligned}$$

and the corresponding first order condition now reads:

$$\begin{aligned}
\frac{\partial \mathcal{U}_t}{\partial \theta^n_{t+1}} &= \frac{\partial \oplus}{\partial \theta^n_{t+1}} \left(\theta_t, \theta_{t+1}, S_t\right) \\
+\alpha^{-1} E_p \left[\frac{\partial \mathcal{U}_{t+1}}{\partial \theta^n_{t+1}} \mid \mathcal{F}_t \right] &= 0 \text{ for } t = 0, T-1 \text{ and } n = 1, N
\end{aligned}$$

which is equivalent to :

$$\begin{aligned}
0 &= -S^n_t u'\left(c_t\right) + \alpha^{-1} E_p \frac{\partial}{\partial \theta^n_{t+1}} \left[u\left(c_{t+1}\right) A\left[.\right] \left(E_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right] - u\left(c_{t+1}\right)\right) \mid \mathcal{F}_t \right] \\
&\quad + \alpha^{-1} S^n_{t+1} u'\left(c_{t+1}\right) \\
&= -S^n_t u'\left(c_t\right) \\
&\quad + \alpha^{-1} \frac{\partial}{\partial \theta^n_{t+1}} \left\{ \begin{array}{c} A\left[.\right] \left(E_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right]\right)^2 - \\ A\left[.\right] E_p\left[u^2\left(c_{t+1}\right) \mid \mathcal{F}_t\right] \end{array} \right\} + \alpha^{-1} S^n_{t+1} u'\left(c_{t+1}\right) \\
&= -S^n_t u'\left(c_t\right) \\
&\quad + \alpha^{-1} A'\left[.\right] \left\{ \begin{array}{c} \left(E_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right]\right)^2 \\ -E_p\left[u^2\left(c_{t+1}\right) \mid \mathcal{F}_t\right] \end{array} \right\} E_p \left[\frac{\partial}{\partial \theta^n_{t+1}} u\left(c_{t+1}\right) \mid \mathcal{F}_t \right] \\
&\quad + \alpha^{-1} A\left[.\right] \left\{ \begin{array}{c} \frac{\partial}{\partial \theta^n_{t+1}} \left(E_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right]\right)^2 \\ -\frac{\partial}{\partial \theta^n_{t+1}} E_p\left[u^2\left(c_{t+1}\right) \mid \mathcal{F}_t\right] \end{array} \right\} + \alpha^{-1} S^n_{t+1} u'\left(c_{t+1}\right) \\
&= -S^n_t u'\left(c_t\right) - \alpha^{-1} A'\left[.\right] Var_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right] E_p\left[S^n_{t+1} u'\left(c_{t+1}\right) \mid \mathcal{F}_t\right] \\
&\quad + 2\alpha^{-1} A\left[.\right] \left\{ \begin{array}{c} E_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right] E_p\left[S^n_{t+1} u'\left(c_{t+1}\right) \mid \mathcal{F}_t\right] \\ -E_p\left[u\left(c_{t+1}\right) u'\left(c_{t+1}\right) S^n_{t+1} \mid \mathcal{F}_t\right] \end{array} \right\} \\
&\quad + \alpha^{-1} S^n_{t+1} u'\left(c_{t+1}\right)
\end{aligned}$$

and, finally:

$$\begin{aligned}
0 &= \frac{\partial \mathcal{U}_t}{\partial \theta^n_{t+1}} = -S^n_t u'\left(c_t\right) + \alpha^{-1} E_p\left[S_{t+1} u'\left(c_{t+1}\right) \mid \mathcal{F}_t\right] \left(1 - A'\left[.\right] Var_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right]\right) \\
&\quad + 2\alpha^{-1} A\left[.\right] \left\{ E_p\left[u\left(c_{t+1}\right) \mid \mathcal{F}_t\right] E_p\left[S_{t+1} u'\left(c_{t+1}\right) \mid \mathcal{F}_t\right] - E_p\left[u\left(c_{t+1}\right) u'\left(c_{t+1}\right) S_{t+1} \mid \mathcal{F}_t\right] \right\}
\end{aligned}$$

which are the new Euler equations:

$$
\begin{aligned}
S_t^n u'(c_t) &= \alpha^{-1} \left\{ \begin{array}{c} E_p\left[S_{t+1}^n u'(c_{t+1}) \mid \mathcal{F}_t\right] \left(1 - A'[.]\, Var_p\left[u(c_{t+1}) \mid \mathcal{F}_t\right]\right) \\ -2A[.]\, Cov_p\left[u(c_{t+1}), S_{t+1}^n u'(c_{t+1}) \mid \mathcal{F}_t\right] \end{array} \right\} \\
t &= 1, ...n = 0, .., N
\end{aligned}
$$

We have thus established that the Euler equations are necessary conditions for solving $(PROG)$

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