The Valuation of Greenhouse Gas (GHG) Emissions Allowances

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Abstract

This paper identifies, first, the important price drivers of marginal allowance prices in a pure allowance trading environment (without the use of abatement technologies): companies' profitability, emissions intensities, and the correlation between electricity and allowance prices. Second, it shows that the use of alternative greenhouse gas (GHG) abatement technologies changes marginal allowance prices significantly. – The fact that marginal allowance prices are different for installations with divergent emissions intensities and alternative GHG abatement technologies creates an ideal environment for allowance trading. In other words, this theoretically derived price behavior is in absolute accordance with the goals of the Kyoto Protocol.

Keywords: GHG abatement technology; emissions allowances; greenhouse gases (GHG); risk; valuation

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1. Introduction

1.1 Preliminaries

Greenhouse gases (GHG) in general and CO_2 in particular are said to be the major cause of global warming. Therefore, by way of the Kyoto Protocol several countries have agreed to reduce greenhouse gas emissions via an innovative mechanism: trading of so-called emissions allowances. The idea behind this particular design of GHG reduction is convincing: it is believed that markets know best at what price and with which abatement technology to reduce GHG emissions.

A prerequisite of successful emissions trading is, however, that decision makers are able to determine their marginal prices for emissions allowances under different production and abatement technologies. Otherwise, the market mechanism will be inefficient and allowance trading will neither be able to provide information on penalties/rewards for failing to meet/meeting emissions goals (price of GHG emissions) nor be able to discover the best GHG abatement technology.

For that reason, it is the objective of this paper to determine marginal allowance prices under simultaneous consideration of alternative GHG abatement technologies.

To achieve this goal, valuation know-how from the field of finance is applied to this problem of energy economics. More precisely, marginal allowance prices are calculated via so-called utility-based pricing, a methodology that has been developed to compute marginal prices of stocks and financial derivatives (see, e.g., Breeden (1979) or Cox/Ingersoll/Ross (1985)). With the help of this methodology, important price drivers of, and their influence on, marginal allowance prices can be identified.

First, marginal allowance prices depend, in a pure allowance trading environment (without the use of abatement technologies), on companies' profitability (the higher the profitability, the lower marginal allowance prices, i.e., negative influence on marginal allowance prices), emissions intensities (positive influence), and the correlation between electricity and allowance prices (positive influence). Second, the use of alternative abatement technologies exerts visible influence on marginal allowance prices. On the one hand, the use of GHG abatement technologies in the form of carbon injection reduces the number of price drivers to one: marginal payouts for injections. On the other hand, switching production between two production technologies imposes two price bounds on marginal allowance prices. One price bound is the marginal allowance price when total production happens at installation 1; the other price bound stems from the situation where only installation 2 is used for production. In the event both installations produce electricity, marginal allowance prices behave linearly between these two bounds and are an increasing function of installation 2's output. Finally, a spot market for electricity is considered because the spot market for electricity serves as both a procurement and a sales market and, thus, exerts influence on GHG reduction goals. Spot market transactions lead to just one bound for marginal allowance prices, a bound based on the fact that electricity production at any installation cannot be negative, i.e., there is a maximum for the amount of electricity purchased on the spot market. However, electricity sales on the spot market can be (nearly) arbitrarily high. Outside this price bound, marginal allowance prices are a negative linear function of spot market transactions.

This paper distinguishes itself from the literature in one significant aspect: to the best of my knowledge, it is the first attempt to value emissions allowances under risk and under simultaneous consideration of alternative GHG abatement technologies.

The overview of the relevant literature from the field of energy economics in Springer/Varilek (2004) and, in particular, in Springer (2003, p. 532 n.) points toward a wide range of allowance prices (mean price over all simulation studies: \$9/t CO_2 and standard deviation of \$5.14/t CO_2 in a global trading environment; mean of \$26.94/t CO_2 and standard deviation of \$20.31/t CO_2 in an Annex B country trading environment). Moreover, Rettberg (2004, p. 2) reports a drop of allowance prices on the Chicago Climate Exchange CCX from \$13 at the beginning of 2004 to \$6 in December 2004. Both observations can be interpreted as an indication that allowance prices will be highly risky if emissions trading starts in the European Union in 2005. Despite this evidence, the energy economics literature has, until now, neglected risk. Rubin (1996) discusses the role of banking and borrowing emissions allowances in a world under certainty. Schleich et al. (2002, p. 67 n.) and Fraunhofer (2003, p. 128) apply present value formulas under certainty (or – at best – risk neutrality) to determine the price of emissions allowances relative to abatement cost. Under risk, however, neither of these simple present value formulas will work nor will alternative GHG abatement technologies be perfect substitutes.

1.2 Methodology

1.2.1 Marginal price

Emissions reductions via allowance trading require companies, or, to be more precise, installations (power plants, etc.), to participate in allowance trading. However, there are two prerequisites to successful emissions trading: first, installations must have a general interest in emissions trading and, second, some installations must purchase whereas others must sell emissions allowances.

Both prerequisites depend on allowance prices and, thus, can be characterized with the help of what will be called marginal allowance price. Define the marginal allowance price of a particular installation as that price at which the owner-operator of that installation is indifferent between participating in or refraining from allowance trading. Then, whenever real market prices of emissions allowances are above the marginal allowance price of one installation, this installation will sell; in the opposite event, it will buy emissions allowances.

For a more detailed analysis, the definition of marginal allowance prices must be further refined. Being indifferent between participating in and refraining from allowance trading means that the owner-operator holds an allowance position that covers his obligation each time GHG limits must be met, but at other points in time he does not trade (optimal transaction size equals zero). That is, in the event the installation needs additional emissions allowances to cover GHG limits, the owner-operator buys exactly the needed quantity; in the opposite event, any surplus is sold. However, the owner-operator does not speculate or hedge with emissions allowances at intermediate points in time, nor does he carry forward any surplus of emissions allowances from one period to the next. Formally, marginal allowance prices can be calculated with the help of so-called utility-based pricing, a methodology developed to compute marginal prices of stocks and financial derivatives (see, e.g., Breeden (1979) or Cox/Ingersoll/Ross (1985)). Using this methodology, marginal allowance prices are obtained as follows. First, differentiate the objective function of the owner-operator with respect to the number of emissions allowances. Second, set the number of emissions allowances equal to zero (optimal transaction size equals zero). Third, solve the necessary condition with respect to the price of emissions allowances.

Two facets must be taken into account when judging this methodology. First, this procedure looks complicated compared to the Black/Scholes (1973) approach of derivative pricing, which determines the price of a derivative as the price of a portfolio that duplicates the derivative's payoff by continuously trading the derivative's underlying and the riskless asset. However, GHG emissions do not have a market price. Therefore, the duplication approach is not applicable to emissions allowances. Second, the concept of marginal prices in no panacea – it has limitations. On the one hand, this method can determine whether or not installations participate in emissions trading, but it cannot reveal the extent of participation, i.e., how many emissions allowances installations should buy and sell. To answer this question, allowance prices must be assumed as exogenously given; based on that assumption, the optimum number of emissions allowances becomes accessible.[1] On the other hand, this allowance valuation approach requires that the number of emissions allowances must be specified exogenously because decision makers without governmental power cannot set both price and quantity of an asset. This paper takes this limitation into consideration by setting transaction sizes equal to zero, which is not as arbitrary as it might seem since a transaction size of zero separates successful from unsuccessful emissions trading.

1.2.2 Model setup

Two forces drive selection of the valuation model's setup in general and, in particular, the choice between discrete- and continuous-time valuation models. The valuation model must be able to cope with major institutional details of allowance trading, a requirement that would seem to favor discrete-time models because they are able to easily deal with features such as free allocation of emissions allowances at certain points in time, GHG constraints at the end of each year, and so forth. On the other hand, explicit solutions for allowance prices are desirable to enable researches to make economic interpretations and for real-time trading via fast computations for companies. Unfortunately, discrete-time valuation models often cannot be solved in explicit form (see, e.g., Breeden (2004)), so in this respect continuous-time models are preferable. Finally, the valuation model must be somewhat robust with respect to the distribution of future allowance prices, a requirement that once again makes the choice of time-discrete models superior. For such models can work with more general and less specified distributional

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[1] This is done in a companion paper.

assumptions, a feature that assumes additional importance because it is not yet clear whether the market for emissions allowances will be liquid enough to allow for continuous-time trading (see the related example of Joskow/Schmalensee/Bailey (1998), who report liquidity frictions in the early years of the U.S. sulfur dioxide market).

To deal with these conflicting requirements, I have chosen a compromise framework that is outlined by the following assumptions.

Assumption 1: Objective function of the owner-operator of an energy-supply company

The owner-operator of an energy-supply company, i.e., a potential candidate for participation in allowance trading, has μ-σ-preferences over terminal wealth at time T. T denotes the end of a trading period (e.g., April 30, 2007 for the first trading period or April 30, 2012 for the first Kyoto commitment (second trading) period, see National Allocation Plan (2003), p. 31). μ-σ-preferences allow for explicit solutions of valuation problems even in discrete time. – I do not want to base mean-variance calculus on an expected utility framework, but instead follow that strand of the literature (see, e.g., Löffler (2001), p. 57, Nielsen (1990), p. 226) that regards μ-σ-preferences as a preference relation of its own. In summary, the objective function of the owner-operator of an energy-supply company reads:

$$\Phi \equiv E\{W_T\} - \frac{a}{2} \cdot var(W_T) \qquad (1)$$

where W_T denotes wealth of the owner-operator at time T, E{.} expected value based on information available at time 0 (the begin of the trading period, e.g. January 1, 2005 or January 1, 2008), and var(.) variance (based on information available at time 0). a is the preference parameter, which weighs mean and variance.

Assumption 2: Production environment

For the sake of simplicity, it is assumed that there is one homogenous good, namely, electricity. Moreover, electricity output is fixed and can be sold completely; in other words, output is not a decision variable of the owner-operator. Such assumptions are not completely unrealistic – for example, consider the situation in which a local energy supplier is obligated to provide electricity for its “hometown.” Finally, the production facility is already existent, i.e., no investment in production technologies is required.

Assumption 3: Allowance trading

Each installation receives a free initial allocation of emissions allowances at times 0, 1, ..., and T – 1 to equip the installation with a basis for trading. This allocation happens annually in equal proportions rather than once for the entire period (see National Allocation Plan (2003), p. 28) and might contain special allocations for early action, combined heat and power, and so forth (see National Allocation Plan (2003), p. 38). At times 1, 2, ..., and T, each installation must meet its GHG limits (see National Allocation Plan (2003), p. 44).[2] It is important to note, however, that both initial allocation and GHG limits are defined on the installation, not the company, level (see Article 4 Directive 2003/87/EC). In addition, the owner-operator can trade emissions allowances at arbitrary points in time within the EU (see Article 2 paragraph 1 Directive 2003/87/EC and National Allocation Plan (2003), p. 4). Although it is possible to transfer a potential surplus of allowances from one year to another within one trading period (so-called banking, see Article 2 Directive 2003/87/EC and National Allocation Plan (2003), p. 42), in Germany banking is forbidden between trading periods (e.g., from 2005-2007 to 2008-2012, see National Allocation Plan (2003), p. 44). Finally, the market for emissions allowances is competitive (no market participant has monopoly powers), and there

[2] In reality, initial allocation and GHG limits overlap slightly within trading periods since GHG limits must be met by April 30 and initial allocation for the next period happens on February 28 (see Article 11 paragraph 4 in connection with Article 12 paragraph 2 Directive 2003/87/EC).

are no technological externalities in the sense of DeAngelo (1981, p. 22): electricity prices are exogenous and not a function of allowance prices.

Assumption 4: Riskless borrowing and lending is possible at interest rate r – the term structure is assumed to be flat and interest rates constant through time

The owner-operator can use the riskless asset to balance his budget constraint, i.e., to cover potential finance needs or to invest any surplus. To keep the model simple, it is also assumed that emissions allowances are the only risky assets available for investment purposes.

2. Emissions allowances: basic valuation results

2.1 Wealth dynamics

The owner-operator has the following budget constraint at time 0:

$$W_0 + \overline{N}_{C,0} \cdot P_{C,0} = N_{C,0} \cdot P_{C,0} + N_{f,0} \qquad (2)$$

where W_0 denotes the owner-operator's initial wealth, $\overline{N}_{C,0}$ initial allocation of emissions allowances at time 0, $N_{C,0}$ the number of emissions allowances bought or sold at time 0, $P_{C,0}$ the allowance price at time 0, and $N_{f,0}$ the number bought (investment) or sold (loan) of the riskless asset at time 0. The price of the riskless asset is assumed to be 1.

In words, the owner-operator uses his initial wealth and the wealth from the initial allocation to invest in emissions allowances and the riskless asset.

At time τ ($0 < \tau < 1$), the owner-operator can restructure his holdings in emissions allowances for speculative or hedging reasons.[3] This investment decision yields the following wealth at time τ:

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[3] Restructurings in allowance holdings are, generally speaking, not restricted to just one restructuring at time τ. However, adding several trading opportunities does not change the structure of the result, but merely complicates the calculations. Therefore, this paper focuses on just one date for intermediate re-

$$W_\tau = N_{C,0} \cdot P_{C,\tau} + N_{f,0} \cdot (1+r)^\tau \quad (3)$$

where r denotes the riskless rate.

Reinvesting wealth at time τ, i.e., restructuring positions in allowances and the riskless asset, yields:

$$W_\tau = N_{C,\tau} \cdot P_{C,\tau} + N_{f,\tau} \quad (4)$$

which translates into the following wealth at time t = 1 (before reinvestment, but after balancing the GHG constraint):

$$W_1 = x_{1,1} \cdot \overline{N}_{P,0} \cdot P_{E,1} - N_{C,1} \cdot P_{C,1} + N_{f,\tau} \cdot (1+r)^{1-\tau} \quad (5)$$

where x denotes the spread between payoffs from electricity sales and payouts for electricity production, $\overline{N}_{P,0}$ the fixed production output, $P_{E,1}$ the random electricity price at time 1, and $P_{C,1}$ the random allowance price at time 1.

Wealth at time 1 consists of three parts. First, the excess payoff of electricity sales over payouts for production, which is at the same time a measure of the installation's profitability – since the installation already exists (see Assumption 1), no initial investment for production is needed –. Second, the payoff from transactions in emissions allowances to meet GHG limits at time 1, and, third, the payoff from transactions in the riskless asset.

Budget, intermediate, and terminal wealth equations are fairly standard for a normal multiperiod investment problem although the investment vehicle, emissions allowances, is not. However, what distinguishes the decision problem of allowance trading from a normal investment problem is the following GHG constraint at time 1:

$$e_{1,1} \cdot \overline{N}_{P,0} = \overline{N}_{C,0} + N_{C,1} + N_{C,\tau} \quad (6)$$

where $e_{1,1}$ denotes the so-called emissions intensity of installation 1 at time 1. Accord-

structurings.

ing to equation (6), emissions intensities can be stochastic as well as time dependent. Stochastic emissions intensities might result from direct measurement of GHG emissions as opposed to determinations based on fuel input, which yields ex ante known emissions. Both approaches are admissible (see National Allocation Plan (2003), p. 25).

The GHG constraint (6) demands that total GHG emissions caused by the production of electricity output $\overline{N}_{P,0}$ at installation 1 must be covered with emissions allowances. Emissions allowances stem from initial allocation at time 0 modified by the number of allowances the owner-operators owns after transactions at time τ (banking) and transactions at time 1 that balance the GHG constraint. These transactions involve either the purchase of allowances to avoid borrowing or the sale of any surplus allowances. Payouts for buying emissions allowances to avoid borrowing are prerequisite for electricity production such as, for example, sources of energy. Cash inflows from selling emissions allowances, on the other hand, are due to the definition of marginal allowance prices (the GHG constraint is to meet exactly, see Section 1.2.1). – With respect to the time-dependent initial allocation, the GHG constraint (6) is slightly more general than the National Allocation Plan, which assumes annual allocations in equal proportions (see National Allocation Plan (2003), p. 28). With respect to the timeframe of the GHG constraint, equation (6) is slightly different from the National Allocation Plan, according to which (National Allocation Plan (2003), p. 30 n.), initial allocations for the next year happen by February 28, whereas the GHG limit must be met by April 30 (a time difference of two months, with one exception: in Germany, emissions allowances cannot be transferred between 2005-2007 and the first commitment period (2008-2012); see National Allocation Plan (2003), p. 44). The GHG constraint (6) uses a time difference of zero, but allows for intermediate trading at time τ and in that fashion captures the idea of a time difference; in reality, trading can take place between February 28 and April

30.

Combining equations (2) through (7) and repeating this procedure for every time between 1 and T yields[4] the following terminal wealth at time T:

$$W_T = \sum_{t=1}^{T} \overline{N}_{P,t-1} \cdot \left(x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t}\right) \cdot (1+r)^{T-t} \tag{8}$$

$$+ \sum_{t=1}^{T} \overline{N}_{C,t-1} \cdot \left(P_{C,t} + (1+r) \cdot P_{C,t-1}\right) \cdot (1+r)^{T-t}$$

$$+ \sum_{t=1}^{T} N_{C,t-1} \cdot \left(P_{C,t-1+\tau} \cdot (1+r)^{1-\tau} - (1+r) \cdot P_{C,t-1}\right) \cdot (1+r)^{T-t}$$

$$+ \sum_{t=1}^{T} N_{C,t-1+\tau} \cdot \left(P_{C,t} - (1+r)^{1-\tau} \cdot P_{C,t-1+\tau}\right) \cdot (1+r)^{T-t}$$

$$+ W_0 \cdot (1+r)^T$$

According to equation (8), the owner-operator’s terminal wealth consists of four components: first, the compounded cumulated gains and losses from production (first term of line 1) and the payouts for emissions allowances that are needed to cover the GHG emissions of $\overline{N}_{P,t-1}$ (t = 0, 1,..., T – 1) under the assumption that all[5] allowances to meet the GHG constraint are bought or sold at time t (second term of line 1); second, the increase in wealth caused by initial allocations at times 0 to T – 1 (second term); third, compounded cumulated gains and losses from trading in emissions allowances at times 0, τ, ..., T – 1 + τ (third and fourth terms); and, finally, compounded initial wealth (fifth term).

[4] See Appendix A.1 for a derivation where the following modifications are needed to cope with this scenario: $\mathbf{N}_{P_2} \equiv \mathbf{0}$ and $\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2} \equiv \overline{\mathbf{N}}_C$.

[5] That is, there is no initial allocation of allowances and no intermediate allowance trading.

2.2 Valuation results

The decision problem from which marginal allowance prices will be derived reads as follows:

$$\underset{N_{C,0},N_{C,\tau},\ldots,N_{C,T-1+\tau}}{\text{Max}} \quad \Phi \equiv E\{W_T\} - \frac{a}{2} \cdot \text{var}(W_T) \tag{9}$$

with W_T as defined in equation (8).

The numbers of emissions allowances at each time t (t = 0, τ, 1, 1 + τ, ..., T – 1 + τ) are the only decision variables available to maximize the preference functional (9). More precisely, at time 0, the owner-operator settles on a strategy for the number of emissions allowances needed from time 0 to time T – 1 + τ. A strategy encompasses a complete conditional plan, i.e., the optimum number of allowances at all times t dependent on the state that will occur at every time t. For example, the owner-operator determines the optimum number of allowances $N_1(S_i)$ in the event of state i (S_i) at time 1 and $N_2(S_i, S_j)$ at time 2 assuming that S_i occurred at time 1 and S_j at time 2, and so forth.

Calculating marginal allowance prices for this scenario yields:[6]

$$P_{C,0} = \frac{1}{(1+r)^T} \cdot E\{P_{C,T}\} - a \cdot \frac{1}{(1+r)^T} \cdot \overline{\mathbf{N}}_P^T \, COV_{PC} \mathbf{1} - a \cdot \frac{1}{(1+r)^T} \cdot \overline{\mathbf{N}}_C^T COV_{\overline{C}C} \mathbf{1} \tag{10}$$

where ^{T} denotes transposition of a vector or matrix (this symbol, however, should not be confused with discounting over T periods as in $\frac{1}{(1+r)^T}$), and $\overline{\mathbf{N}}_i$ ($i \in \{P,C\}$) the vector of the amount of production or initial allocation of allowances at times 0, 1,..., T – 1. COV_{PC} is the covariance matrix between $Z_{C,t-1+\tau} \equiv \left(P_{C,t} - (1+r)^{1-\tau} \cdot P_{C,t-1+\tau}\right) \cdot (1+r)^{T-t}$ as well as $Z_{C,t} \equiv \left(P_{C,t-1+\tau} \cdot (1+r)^{1-\tau} - (1+r) \cdot P_{C,t-1}\right) \cdot (1+r)^{T-t}$ and $\left(x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t}\right) \cdot (1+r)^{T-t}$ for t = 1,

[6] See Appendices A.3 and A.4 for a derivation where the same modification as in footnote 4 must be

2,..., T, and $COV_{\bar{C}C}$ is the covariance matrix between $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ and $\left(P_{C,t} + (1+r) \cdot P_{C,t-1}\right) \cdot (1+r)^{T-t}$ for t = 1, 2,..., T.

In the special case of serially uncorrelated cash flows, emissions intensities, and spreads, equation (10) simplifies to:[7]

$$P_{C,0}^{ref,unc} = \frac{1}{(1+r)^T} \cdot E\{P_{C,T}\} \tag{11}$$

$$- a \cdot \frac{1}{(1+r)^T} \cdot cov\left(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}, P_{C,T}\right) \cdot \overline{N}_{P,T-1}$$

$$- a \cdot \frac{1}{(1+r)^T} \cdot var\left(P_{C,T}\right) \cdot \overline{N}_{C,T-1}$$

This is significantly more simple than the general case (10) for several, easily understood reasons. First, cash flows, emissions intensities, and spreads are serially uncorrelated, and, second, emissions allowances do not offer intermediate cash flows such as dividends, meaning that there is no intertemporal risk. – Add to this the third fact, namely, that allowance trading is possible and, thus, wealth at time T depends only on allowance positions that will be established at time T – 1 + τ. In other words, in this special case, only the most recent holdings and prices of electricity output and emissions allowances are relevant to valuation.

2.3 *Interpretation of marginal allowance prices*

According to equations (10) and (11), the marginal allowance price is determined by two fundamental components: the discounted expected allowance price at time T and a risk correction. This risk correction consists of risk of terminal wealth weighted with the risk preference parameter a of the owner-operator. – So far, the pricing formula coincides with the one for arbitrary cash flows in a μ-σ-world (see the analogy to, e.g.,

made.

7 This can be seen immediately from equation (A.26).

Cox/Ingersoll/Ross (1985), p. 374 if equations (10) and (11) are reformulated as risk premiums $E\{P_{C,T}\} - (1+r)^T \cdot P_{C,0}$). – However, what particularizes this valuation formula to emissions allowances is the definition of risk. There is risk from interactions between electricity production and allowance holdings on the one hand and, on the other hand, risk from interactions between initial allocation and intermediate allowance positions. The first risk is captured by the covariance between cash flows from electricity production and allowance prices (second line of equations (10) and (11)), the second risk by the covariance between cash flows of allowance positions stemming from initial allocation and intermediate allowance positions (third line of equations (10) and (11)). Since both electricity production and initial allocations are positive, the signs of the covariance terms determine whether marginal allowance prices lie above or below their discounted expected allowance price at time T ($\frac{1}{(1+r)^T} \cdot E\{P_{C,T}\}$). – Obviously, risk at every point in time is relevant to pricing; however, only the expected value at time T matters. This is because expected values at time τ, 1, ..., $\tau - 1 + \tau$ are a result of pricing, i.e., their values are model endogenous, not model exogenous such as variances and covariance.[8]

Although equations (10) and (11) describe important aspects of marginal allowance prices, they do not completely characterize marginal allowance prices because additional price bounds for emissions allowances need to be taken into consideration.

The lowest allowance price during a trading period equals zero, which will occur if GHG constraints are not binding. In addition, emissions allowances do not cause cash outflows – initial allocation is free (see National Allocation Plan (2003), p. 28) and there are no other costs, for example, there are no disposal costs. According to arbitrage theory (see, e.g., Dybvig/Ross (1992)), a nonnegative cash flow must have a nonnega-

8 See Appendix A.4 for the formal implementation of this verbal description.

tive price and a cash flow of zero must have a price of zero. Hence, zero is a lower price bound for emissions allowances.

The upper price consists of the penalty for failing to meet GHG constraints, which is 40 EUR between 2005-2007 (see Article 16 paragraph 4 Directive 2003/87/EC) and 100 EUR between 2008-2012 (see Article 16 paragraph 3 Directive 2003/87/EC). Thus, no rational buyer of emissions allowances will pay more than 40 (100) EUR for emissions allowances to meet GHG constraints at times 1, 2,..., T. In this connection, the price bound holding at times 1, 2,..., T has to be translated into a price bound at time 0. Allowance trading during the first period can deal only with the GHG constraint at time 1 because initial allocation at time 1, 2,..., T – 1 and trading between time 1 and time T can alter allowance positions and, hence, the potential violation of the GHG constraint at time 2,..., T. Therefore, the relevant penalty from time 0 is the one for violating the GHG constraint at time 1. No-arbitrage considerations again guarantee that a maximum allowance price of 40 (100) EUR at time 1 translates into an upper bound for allowance prices of the present value of 40 (100) EUR, i.e., $\frac{40}{1+r}$ ($\frac{100}{1+r}$) EUR.[9]

In summary, marginal allowance prices have a lower bound at zero, a higher bound at $\frac{40}{1+r}$ ($\frac{100}{1+r}$) EUR, and follow between these bounds equations (10) or (11).

For more insight into the behavior of marginal allowance prices (10) or (11) within the price bounds in general and, in particular, to obtain economic intuition regarding covariances' signs, a more detailed interpretation of marginal allowance prices is needed. Because of the simple structure of the special case, equation (11) is a good point of de-

[9] This reasoning remains valid even if initial allocation for the next year occurs on February 28 and the GHG limit for the current year must be met by April 30 because intermediate trading between February 28 and April 30 can cause a violation of the GHG constraint at April 30. Only if the GHG limit violation is assumed not to occur before time T (see National Allocation Plan (2003), p. 44), the upper bound for marginal allowance prices will read $\frac{40}{(1+r)^T}$ or $\frac{100}{(1+r)^T}$.

parture.

A positive covariance between allowance price $P_{C,T}$ and cash flow from production $x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}$ means that allowance positions increase the risk of terminal wealth and a risk deduction is justified. This risk deduction will be the higher, the higher the spread x and the lower the emissions intensity e. This contrary effect of both price drivers on marginal allowance prices can be explained as follows. A higher spread signifies, on the one hand, higher uncertain cash flows of the installation. Higher stochastic cash flows automatically imply higher risk[10] under positive covariances and, therefore, a higher risk deduction (risk-based explanation). On the other hand, the higher the spread, the more profitable the installation. In other words, highly profitable installations have enough buffer to pay for emissions allowances at the time the GHG constraint must be met. For that reason, the owner-operator of the installation will not be willing to pay a high price for the opportunity to purchase or sell emissions allowances in advance (demand-based explanation). – These interrelations are different for the emissions intensity. The higher the emissions intensity, the lower are the installation's cash flows. A lower overall cash flow, however, means lower total risk and, thus, smaller risk deductions (risk-based explanation). The demand-based explanation develops as follows. Since the installation is required to supply a fixed amount of electricity, it cannot adjust its production to counter the negative effects of high emissions intensities. Consequently, emissions allowances are more valuable to installations with higher emissions intensities than they are to those with lower emissions intensities.

In the event of negative covariances between cash flows from electricity production and allowance prices, emissions allowances act like a hedge against terminal wealth fluctuations. Therefore, the higher the spread, the higher the installation's cash flow and the

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[10] To further illustrate this fact, simply consider the variance of one stock compared to the variance of 10 units of the same stock, which equals 100 times the variance of one stock.

higher the hedge (risk-based explanation). On the other hand, a hedge vehicle is of higher value to an installation's owner-operator the greater the cash flow risk is; hence, the hedge vehicle becomes more valuable to the owner-operator with increasing spread (demand-based explanation). An analogue reasoning shows that higher emissions intensities lead to a decrease of hedge potential and, thus, to a lower price of emissions allowances.

One final question arises in connection with the first covariance: Which is more realistic – a positive or a negative covariance between cash flows from production and allowance prices? Since allowance trading has not yet started, this question cannot be answered empirically. However, a positive relation seems to be probable following the argumentation of the National Allocation Plan (2003, p. 44), which forecasts no problems with GHG constraints in 2005 and 2006, but that some effort will be needed to meet the constraint in 2007 (remember, in Germany emissions allowances cannot be transferred between 2005-2007 and the first commitment period, 2008-2012; see National Allocation Plan (2003), p. 44).

The second covariance, covariance between cash flows from initial allocation and intermediate allowance holdings, calls for a price deduction (risk-based explanation) in the special case as variance of allowance prices is positive. The demand-based explanation is even more intuitive. The higher an installation's initial allocation, the lesser the GHG constraint is binding, and emissions allowances become less valuable for this particular installation.

These results concerning the special case of equation (11) will need to be modified slightly if the general case of equation (10) is considered since intertemporal risk interrelations enter the valuation. This is because not only covariances between cash flows at time T, but also covariances between cash flows at every time between time τ and T, must be taken into account; formally, all these covariance are added up ($COV_{PC}\mathbf{1}$ and $COV_{\bar{C}C}\mathbf{1}$). These intertemporal risk interrelations influence covariances between cash flows from electricity production and allowance prices in two ways. On the one hand, look at the covariances $\operatorname{cov}\left(x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t},\ P_{C,t+\tau} \cdot (1+r)^{1-\tau} - (1+r) \cdot P_{C,t}\right)$ and $\operatorname{cov}\left(x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t},\ \ P_{C,t} - (1+r)^{1-\tau} \cdot P_{C,t-1+\tau}\right)$. The random variable $P_{C,t}$ enters the first co-variance term with two negative signs and the second one with a negative and a positive sign. The economic reason behind this is that the first covariance term encompasses the situation where electricity is sold combined with a purchase of allowances ($x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t}$) and allowances are bought at time t ($-(1+r) \cdot P_{C,t}$); the second co-variance deals with a sale of electricity combined with a purchase of allowances and a sale of allowances at time t ($P_{C,t}$). It is easy to understand that the relation between two selling transactions has a sign opposite to the one of a buying and a selling transaction. On the other hand, there are covariances between cash flows from production and al-lowance holdings at times 1,..., T – 1 (in addition to the covariance at time T $\operatorname{cov}\left(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T},\ \ P_{C,T} - (1+r)^{1-\tau} \cdot P_{C,T-1+\tau}\right)$). – Intertemporal risk interrelations are even more pronounced for the second risk factor, the covariance between cash flows from initial allocation and intermediate allowance holdings. First, $P_{C,t}$ enters $\operatorname{cov}\left(P_{C,t} + (1+r) \cdot P_{C,t-1},\ P_{C,t} - (1+r)^{1-\tau} \cdot P_{C,t-1+\tau}\right)$ with a different sign than $\operatorname{cov}\left(P_{C,t} + (1+r) \cdot P_{C,t-1},\ P_{C,t+\tau} \cdot (1+r)^{1-\tau} - (1+r) \cdot P_{C,t}\right)$; the first covariance term involves allowance sales from ini-tial allocation ($P_{C,t} + (1+r) \cdot P_{C,t-1}$) and allowances sales from trading ($P_{C,t}$), whereas the second covariance combines sale of allowances from initial allocation with purchase of allowances at time t ($-(1+r) \cdot P_{C,t}$). Second, the general sign of the covariance between allowance prices at different times, e.g., $\operatorname{cov}\left(P_{C,t}, P_{C,t-1}\right)$ or $\operatorname{cov}\left(P_{C,t}, P_{C,t-1+\tau}\right)$, is as yet un-known because allowance trading hasn't started. A positive correlation would mean that high allowance prices at one time lead to even higher allowances prices at later times, a negative covariance induces the contrary behavior. A negative correlation, however, seems to be unlikely in a market that is not dominated by speculation, which follows from the National Allocation Plan (2003, p. 44), which forecasts no problems with GHG constraints in 2005 and 2006, but that some effort will be needed to meet the constraint in 2007 (again, remember that in Germany emissions allowances cannot be transferred between 2005-2007 and the first commitment period, 2008-2012; see National Allocation Plan (2003), p. 44).

Although a negative correlation seems unlikely, it cannot be excluded. This means that covariances between cash flows from initial allocation and intermediate allowance holdings may lead to an increase in marginal allowance prices; obviously, allowance prices at earlier times must contain a hedge against allowance prices at later times. Therefore, risk-based and demand-based explanations call for a higher marginal allowance price.

Three final remarks will conclude this section on marginal allowance prices in a pure allowance trading environment.

First, it is highly probable that $E\{P_{C,T}\}$ will be larger than zero even though emissions allowances will expire after time T and become worthless – (in Germany they cannot be transferred between 2005-2007 and the first commitment period, 2008-2012; see National Allocation Plan (2003), p. 44). Because $E\{P_{C,T}\}$ measures the expected allowance price at the last time the GHG limit had to be met; expirations will occur only after this price has been determined**.**

Second, consider regulated electricity markets where the electricity price is nonstochastic. Under this scenario and under the additional assumption of nonstochastic spreads and emissions intensities, equation (11) simplifies to:

$$P_{C,0} = \frac{1}{(1+r)^T} \cdot E\{P_{C,T}\} - a \cdot \frac{1}{(1+r)^T} \cdot \operatorname{var}(P_{C,T}) \cdot \left(\overline{N}_{C,T-1} - e_{1,T} \cdot \overline{N}_{P,T-1}\right) \qquad (12)$$

that is, the variance of allowance prices is the only risk driver. Regardless of the size of $\operatorname{var}(P_{C,T})$, marginal allowance prices depend on the relation between emissions allowances obtained via initial allocation $\overline{N}_{C,T-1}$ and emissions allowances actually needed due to current emissions $e_{1,T} \cdot \overline{N}_{P,T-1}$. If $\overline{N}_{C,T-1}$ exceeds $e_{1,T} \cdot \overline{N}_{P,T-1}$, a price discount obtains (the GHG constraint can be met easily and the installation does not have a strong demand for intermediate allowance positions: demand-based explanation; the initial allocation increases the risk of the owner-operator's wealth: risk-based explanation), in the contrary event, a price rise results. Interestingly, for the special case that initial allocation at time T – 1 exactly matches emissions at time T, the risk term vanishes and allowances are priced as in a risk-neutral world.

Third, equations (10) and (11) demonstrate that marginal allowance prices do indeed support allowance trading. Marginal allowance prices are different for installations with divergent emissions intensities. Hence, trading can take place because installations with higher emissions intensities will likely purchase, and those with lower emissions intensities will probably sell, emissions allowances, a behavior that fully matches the spirit of the Kyoto Protocol. – Moreover, there are other sources of trading that are not directly related to the Kyoto Protocol: heterogenous expectations and different risk preference parameters. Owner-operators with higher risk preference parameters will have lower marginal allowance prices under positive covariances (higher marginal allowance prices under negative covariances) and thus be more inclined to sell (buy) emissions allowances than owner-operators with lower risk preference parameters. Following a suggestion by Löffler (2001, p. 61) and particularizing the risk preference parameter $a \equiv \frac{1}{W_0}$, it becomes clear that owner-operators of small companies have higher risk preference parameters and hence will more likely sell emissions allowances than will owner-operators of larger installations.

3. Emissions allowances and alternative GHG abatement technologies

So far, the role of emissions allowances as market-based penalties/rewards for exceeding/remaining below planned GHG emissions has been discussed. However, the Kyoto Protocol aims not only to determine prices of GHG, but also to find the best abatement technology via allowance trading. Therefore, this section deals with the influence of three alternative GHG abatement technologies on marginal allowance prices: an explicit abatement technology, the switching between two installations with different emissions intensities, and the use of the spot market for electricity.

3.1 Explicit abatement technologies (abatement technology in the narrower sense)

“Explicit abatement technology in the narrower sense” means that GHG emissions are reduced with the help of a technical device where the original installation continues to be used for electricity production. One such abatement technology, which will be available in the near future, is integrated gasification and combined cycle with capture and sequestration (IGCC) (see, e.g., Manne/Richels (2004), p. 607 or Kurosawa (2004), p. 680). The captured GHG is transported and injected, e.g., into depleted gas wells or the ocean.

3.1.1 Wealth dynamics

As in the reference case of Section 2 (pure allowance trading environment without abatement technology), it is assumed that the installation already exists, i.e., no initial investment for the production facility is needed. However, the injection of GHG causes known payouts that depend on the amount of GHG injected. The owner-operator must decide at time t – 1 on the amount $N_{A,t-1}$ that will be injected at time t. Injecting occurs simultaneously with production: GHG have to exist before they can be captured and injected. The decision on the amount to be injected must be made before the injecting itself occurs because the injection company needs to know in advance the amount that will be injected so that it can make appropriate preparations.

Based on these expositions, the GHG constraint reads:

$$e_{1,1} \cdot \overline{N}_{P,0} - N_{A,0} = \overline{N}_{C,0} + N_{C,1} + N_{C,\tau} \tag{13}$$

where $N_{A,0}$ denotes the amount of GHG that has been decided upon at time 0 to be injected at time 1.

Assuming payouts for injections are due at the time the injections occur and are nonlinear in the amount of GHG injected (see Kurosawa (2004), p. 680), wealth at time t = 1 (before reinvestment, but after balancing the GHG constraint) holds as follows:

$$W_1 = x_{1,1} \cdot \overline{N}_{P,0} \cdot P_{E,1} - N_{C,1} \cdot P_{C,1} + N_{f,\tau} \cdot (1+r)^{1-\tau} - K(N_{A,0}) \tag{14}$$

where $K(N_{A,0})$ denotes payouts for injections.

Proceeding from time 1 to time T, one obtains:[11]

$$W_T = W_T^{ref} + \sum_{t=1}^{T} \left(N_{A,t-1} \cdot P_{C,t} - K(N_{A,t-1})\right) \cdot (1+r)^{T-t} \tag{15}$$

where the last term of equation (15) describes the wealth effects of GHG injections compared to the reference case of pure allowance trading as shown in equation (8). In other words, it consists of the cash flow consequences of GHG injection in the form of fewer allowance purchases (more allowance sales) at time t and additional payouts for GHG injections.

3.1.2 Valuation results

The decision problem from which the valuation results will be derived reads as follows:

$$\underset{\substack{N_{C,0},N_{C,\tau},\ldots,N_{C,T-1+\tau},\\ N_{A,0},\ldots,N_{A,T-1}}}{\text{Max}} \quad \Phi \equiv E\{W_T\} - \frac{a}{2} \cdot \text{var}(W_T) \tag{16}$$

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[11] See Appendix A.1 for a derivation where the following formal modifications have to be made: the second "installation" equals the abatement technology, i.e., $x_{2,t} \equiv x_{1,t} - \frac{K(N_{A,t-1})}{N_{A,t}} \cdot \frac{1}{P_{E,t}}$ and $e_{2,t} \equiv e_{1,t} - 1$.

with W_T as defined in equation (15).

That is, the owner-operator can choose the number of emissions allowances at each time t (t = 0, τ, 1, 1 + τ, ..., T – 1 + τ) as well as the number of GHG injections at times 0, 1,..., T – 1 so as to maximize the preference functional of equation (16). To be more precise, the owner-operator determines a strategy for both decision variables, i.e., a complete conditional plan.

Calculating marginal allowance prices for this scenario yields:[12]

$$P_{C,0} = P_{C,0}^{ref} - a \cdot \frac{1}{(1+r)^T} \cdot \mathbf{N}_A^T COV_{AC} \mathbf{1} \tag{17}$$

where $P_{C,0}^{ref}$ denotes the marginal allowance price of the reference case of equation (10) and COV_{AC} is the covariance matrix between $P_{C,t} \cdot (1+r)^{T-t}$ and $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ for t = 1, 2,..., T.

For the special case of serially uncorrelated cash flows, emissions intensities, and spreads one obtains:

$$P_{C,0} = P_{C,0}^{ref,unv} - a \cdot \frac{1}{(1+r)^T} \cdot var(P_{C,T}) \cdot N_{A,T-1} \tag{18}$$

where $P_{C,0}^{ref,unv}$ denotes the marginal allowance price of the reference case of equation (11).

Equations (17) and (18) define the range of marginal allowance prices under GHG injection.

The amount of GHG injected must be nonnegative (minimal $N_{A,t-1} = 0$ for t = 1, 2,..., T) because it is impossible to inject a negative amount. Therefore, one price bound is the marginal allowance price without GHG injections, i.e., the reference price of equation (10) or (11). The other price bound stems from the situation where all GHG emissions

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12 See Appendices A.3 and A.4 for a derivation using the proper values for $x_{2,t}$ and $e_{2,t}$.

are injected ($N_{A,t-1} = e_t \cdot N_{P,t-1}$ for t = 1, 2,..., T). In other words, GHG injections eliminate emissions and, thus, their direct price influence completely. According to equations (17) and (18), marginal allowance prices are a linear function of the amount injected[13] between these two bounds.

To determine whether the derived price bounds are upper or lower bounds and to compare allowance prices under GHG injections (equations (17) and (18)) with those of the reference case in equations (10) and (11) (pure allowance trading environment without abatement technology), equations (17) and (18) must be analyzed in more detail. Since $\mathbf{N}_A$ must be positive, marginal allowance prices in the special case of equation (18) lie below marginal prices of the reference case; equation (11) constitutes an upper bound, the extreme event of total injection a lower price bound. The reason for this price behavior is intuitive. The abatement technology makes the GHG constraint easier to meet, hence emissions allowances become less valuable to the owner-operator (demand-based explanation). Simultaneously, cash flows at time T will be higher because fewer allowances need to be bought thanks to the abatement technology. Thus, total risk is higher, which justifies a price discount (risk-based explanation). – Things are slightly more complicated in the general case of equation (17) because emissions allowances distinguish themselves by intertemporal risk connections from those of the special case. For that reason, marginal allowance prices under a positive (negative) covariance between $P_{C,t} \cdot (1+r)^{T-t}$ and $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ lie below (above) the ones of the reference case. A positive covariance adds additional risk compared to the special case of serially uncorrelated cash flows, emissions intensities, and spreads; a negative covariance induces hedge aspects. Nevertheless, the same fundamental price bounds obtain as in the special case.

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13 In a side note, observe that equations (17) and (18) hold irrespective of whether GHG injections are infinitely divisible.

Finally, to describe marginal allowance prices fully under GHG injections, the price bounds of the reference case have to be taken into account, i.e., the lower bound at zero and the upper bound at $\frac{40}{1+r}$ ($\frac{100}{1+r}$) EUR. This means that marginal allowance prices under GHG injections are a positive linear function of the amount injected and are located between max{0, allowance price with full injection } and min{ $\frac{40}{1+r}\left(\frac{100}{1+r}\right)$, (10)} or min{ $\frac{40}{1+r}\left(\frac{100}{1+r}\right)$, (11)}.

So far, a positive linear relationship between marginal allowance prices and the amount injected has been discovered. This result should not be confused with the statement that marginal allowance prices will be a linear function of the (nontrivial) price drivers spread and emissions intensity. To analyze their relation to marginal allowance prices, rely on the optimum amount of injections. For the special case of serially uncorrelated cash flows, emissions intensities, and spreads, equation (18) simplifies to:[14]

$$P_{C,0} = \frac{1}{(1+r)^T} \cdot \frac{\partial K(N_{A,T-1})}{\partial N_{A,T-1}} \qquad (19)$$

Or, in words, the marginal allowance price equals the present value of marginal payouts for injections. In particular, the owner-operator's risk aversion, the risk of the emissions allowances, and the initial allocation of emissions allowances do not seem to influence marginal allowance prices. – This is because GHG injections and emissions allowances are substitutes with respect to the GHG constraint. Therefore, the marginal payout for injectinos ($\frac{\partial K(N_{A,T-1})}{\partial N_{A,T-1}}$) is the benchmark for the price of emissions allowances.

This interpretation of equation (19) prompts a warning. Since marginal payouts for

[14] See Appendix A.6 for a derivation.
The calculations for the general case are omitted because they do not yield such an instructively compact formula. This is not really surprising. Since the general case involves intertemporal risk connections, abatement technology and allowance trading are no longer perfect substitutes.

GHG injections are a nonlinear function of the amount injected, the owner-operator of the installation has to know his optimal injections to determine marginal allowance prices. However, $N_{A,T-1}$, according to equation (A.30), is a function of the owner-operator's risk aversion, the risk of emissions allowances, and the initial allocation. In other words, a naïve interpretation of equation (19) is deceiving except for one payout function: $K(N_{A,T-1})$ is linear in $N_{A,T-1}$.

3.2 Switching between two production technologies

Switching between two[15] production technologies means that the owner-operator of an energy-supply company has two installations available for producing electricity. This flexibility also allows the owner-operator two ways of meeting GHG constraints: allowance trading and/or increasing the output of the installation with the lower emissions intensity.

3.2.1 Wealth dynamics

As in the reference case of Section 2 (pure allowance trading environment without abatement technology), it is assumed that both installations already exist, i.e., no initial investment for production facilities is needed. However, because total output remains fixed, a production constraint has to be added to the decision problem: total production is split between production at installations 1 and 2; formally:

$$\overline{N}_{P,0} = N_{P_1,0} + N_{P_2,0} \tag{20}$$

where $N_{P_i,0}$ (i = 1, 2) denotes electricity production at installation i.

Equation (20) clarifies that the production of only one installation is decision variable; the other is determined via the production constraint. To ensure that switching between both installations is due to GHG emissions only, it is assumed that both installations

[15] Switching between more than two installations does not produce substantially different results, but contains by far more lengthy derivations. For that reason, this case is omitted.

have a capacity high enough to produce $\overline{N}_{P,0}$; otherwise, production switching could be due to the lack of capacity, instead of lower GHG emissions, of one installation.

Since there are two installations, two GHG constraints need to be considered (see Article 4 in connection with Annex I Directive 2003/87/EC as long as combustion installations have a rated thermal input exceeding 20 MW).

The GHG constraint for installation 1 is:

$$e_{1,1} \cdot \left(\overline{N}_{P,0} - N_{P_2,0}\right) = \overline{N}_{C_1,0} + N_{C_1,1} + N_{C_1,\tau} \tag{21}$$

and the GHG constraint for installation 2 is:

$$e_{2,1} \cdot N_{P_2,0} = \overline{N}_{C_2,0} + N_{C_2,1} + N_{C_2,\tau} \tag{22}$$

where $\overline{N}_{C_i,0}$ denotes initial allocation for installation i (i = 1, 2) at time 0, and $N_{C_i,t}$ is the number of emissions allowances traded for installation i at time t (t = τ, 1).

However, trading in emissions allowances does not distinguish between allowances from installation 1 and those from installation 2 (see Article 2 paragraph 1 Directive 2003/87/EC and National Allocation Plan (2003), p. 4). This means that allowance trading can be centralized within a company as long as the central trading department has allocated to each installation enough allowances to meet its own GHG constraint. Therefore, it becomes possible to focus on total transactions in emissions allowances instead of transactions for each installation. In particular, there is only the following GHG constraint:[16]

$$e_{1,1} \cdot \left(\overline{N}_{P,0} - N_{P_2,0}\right) + e_{2,1} \cdot N_{P_2,0} = \overline{N}_{C_1,0} + \overline{N}_{C_2,0} + N_{C,1} + N_{C,\tau} \tag{23}$$

The GHG constraint of equation (23) implies, however, that switching production does not lead to a reduction of initial allocation for each installation. According to the Na-

[16] In other words, there will be no difference between GHG constraints at the installation or company level if there is a market for emissions allowances. Without a market for emissions allowances, the two GHG constraints are clearly different. An installation that produces a small amount of GHG can hedge an installation that produces a huge amount of GHG under GHG constraints at the company level, but not under GHG constraints at the installation level. Emissions allowances reestablish this

tional Allocation Plan (2003, p. 32), an installation's initial allocation will be reduced proportional to the capacity utilization of the installation if its annual emissions are lower than 60% of its average annual emissions during the reference period 2000-2002.

Combining transactions in emissions allowances and initial allocations for installations 1 and 2 as in the GHG constraint (23), i.e., using total transactions and total initial allocation, budget equation at time 0, wealth from intermediate allowance trading, and wealth at time 1 (before reinvestment, but after balancing the GHG constraints) are no different compared to the reference case. Thus, these intermediate steps are omitted and one immediately obtains:[17]

$$W_T = W_T^{ref} + \sum_{t=1}^{T} N_{P_2,t-1} \cdot \left(\left(x_{2,t} - x_{1,t}\right) \cdot P_{E,t} - \left(e_{2,t} - e_{1,t}\right) \cdot P_{C,t}\right) \cdot (1+r)^{T-t} \qquad (24)$$

The last term of equation (24) describes the wealth effects of switching between two installations relative to the reference case of pure allowance trading (see equation (8)) (if $\overline{\mathbf{N}}_{C1} + \overline{\mathbf{N}}_{C2} = \overline{\mathbf{N}}_{C}$ is assumed). It stems from the fact that switching production to installation 2 creates cash flows that are based on spreads and emissions intensities of installation 2 instead of those of installation 1.

3.2.2 Valuation results

The decision problem from which the valuation results will be derived is formulated as follows:

$$\underset{\substack{N_{C,0},N_{C,\tau},\ldots,N_{C,T-1+\tau},\\ N_{P_2,0},\ldots,N_{P_2,T-1}}}{\text{Max}} \Phi \equiv E\{W_T\} - \frac{a}{2} \cdot \text{var}(W_T) \qquad (25)$$

with W_T as defined in equation (15).

That is, the owner-operator can choose the number of emissions allowances at each time t (t = 0, τ, 1, 1 + τ, ..., T – 1 + τ) as well as the output produced at installation 2 at times

hedging opportunity.

[17] See Appendix A.1 for a derivation.

0, 1,..., T – 1 so as to maximize the preference functional (16). To be more precise, the owner-operator determines a strategy for both decision variables, i.e., a complete conditional plan.

Appendices A.3 and A.4 show that marginal allowance prices for this scenario read:

$$P_{C,0} = \quad P_{C,7}^{ref} - a \cdot \frac{1}{(1+r)^T} \cdot \mathbf{N}_{P_2}^T COV_{P_2 C} \mathbf{1} \tag{26}$$

where $COV_{P_2 C}$ denotes the covariance matrix between $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ and $\left(\left(x_{2,t} - x_{1,t}\right) \cdot P_{E,t} - \left(e_{2,t} - e_{1,t}\right) \cdot P_{C,t}\right) \cdot (1+r)^{T-t}$ for t = 1, 2,..., T.

For the special case of serially uncorrelated cash flows, emissions intensities, and spreads one obtains:

$$P_{C,0} = \quad P_{C,0}^{ref,unc} - a \cdot \frac{1}{(1+r)^T} \cdot cov\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}, P_{C,T}\right) \cdot N_{P_2,T-1} \tag{27}$$

From equations (26) and (27) it is obvious that there are price bounds for allowance prices under switching production.

The amount produced at either installation must be nonnegative because it is impossible to produce a negative amount. Therefore, one price bound is the marginal allowance price when total production happens at installation 1 ($N_{P_2,t-1} = 0$ for t = 1, 2,..., T), the other price bound stems from the situation where all production is produced at installation 2 ($N_{P_2,t-1} = \overline{N}_{P,t-1}$ for t = 1, 2,..., T). Both price bounds can be calculated with equations (10) or (11) because these equations describe the behavior of marginal allowance prices in an environment with just one installation. Moreover, according to equations (26) and (27), between these two bounds marginal allowance prices are a positive linear function of the output of installation 2.

To determine whether the identified price bounds are upper or lower bounds, and to compare allowance prices under switching production (equations (26) and (27)) with those of the reference case (equations (10) and (11)) (pure allowance trading environment without abatement technology), equations (26) and (27) need to be analyzed in more detail. Since $N_{P_2,T-1}$ must be positive, marginal allowance prices in the special case of equation (27) depend on the sign of the difference between $\mathrm{cov}(x_{2,T} \cdot P_{E,T} - e_{2,T} \cdot P_{C,T}, P_{C,T})$, i.e., covariance between cash flows from electricity production at installation 2 and allowance prices, and $\mathrm{cov}(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}, P_{C,T})$, i.e., covariance between cash flows from electricity production at installation 1 and allowance prices. If this difference is positive, $P_{C,0}^{ref,unc}$ will be an upper bound for marginal allowance prices under switching production. This interpretation is easy to understand. A positive difference means that risk has increased by producing output at installation 2. Higher risk, however, calls for a price discount. A negative difference signifies reduced risk by switching production (partially) to installation 2; a lower price discount follows. In that event, $P_{C,0}^{ref,unc}$ will be a lower bound for marginal allowance prices under switching production. In addition, the risk-based explanation clarifies that a dominant production installation, i.e., an installation with both a higher spread and a lower emissions intensity, cannot guarantee an unambiguous influence on marginal allowance prices. To see this, consider a higher risk due to production at installation 2, i.e., a positive sign of $\mathrm{cov}((x_{2,T} - x_{1,T}) \cdot P_{E,T} \quad - (e_{2,T} - e_{1,T}) \cdot P_{C,T}, P_{C,T})$. Assuming a positive sign of $\mathrm{cov}(P_{E,T}, P_{C,T})$ and nonstochastic spreads and emissions intensities, the positive sign of the above covariance will be true if

$$x_{2,T} > x_{1,T} + (e_{2,T} - e_{1,T}) \cdot \frac{\mathrm{var}(P_{C,T})}{\mathrm{cov}(P_{E,T}, P_{C,T})} \tag{28}$$

with $\frac{\mathrm{var}(P_{C,T})}{\mathrm{cov}(P_{E,T}, P_{C,T})}$ 1/regression coefficient of the regression of $P_{E,T}$ on $P_{C,T}$.

Obviously, there are parameter constellations possible where $e_{2,T}$ is smaller than $e_{1,T}$, and $x_{2,T}$ larger than $x_{1,T}$, but equation (28) is nevertheless violated. Thus, the ambiguous effect of a dominant production installation on marginal allowance prices is proven.

The demand-based explanation of marginal allowance prices adds further illustrative insights. Assume that the owner-operator produces some electricity at installation 2. A smaller marginal allowance price with increasing output of installation 2 signifies that the owner-operator feels less pressure to purchase allowances, which could be due to the fact that installation 2 has a lower emissions intensity. Marginal allowance prices that rise with an increasing output at installation 2 make emissions allowances more valuable to the owner-operator the higher installation 2's output is. One possible explanation is that emissions allowances allow for switching to a more profitable installation, albeit one with a higher emissions intensity that makes GHG constraints more difficult to meet.

Since the general case of equation (26) merely contains intertemporal covariance terms, the results of the special case can be transferred seamlessly.

Finally, to describe marginal allowance prices fully under switching production, the price bounds of the reference case have to be taken into account, i.e., the lower bound at zero and the upper bound at $\frac{40}{1+r}$ $(\frac{100}{1+r})$ EUR. This means that marginal allowance prices under switching production are a positive linear function of installation 2's output and are located between max{0, (10) or (11) calculated based on total output produced at installation 1} or max{0, (10) or (11) calculated based on total output produced at installation 2} and min{ $\frac{40}{1+r}\left(\frac{100}{1+r}\right)$, (10) or (11) calculated based on total output produced at installation 2} or min{ $\frac{40}{1+r}\left(\frac{100}{1+r}\right)$, (10) or (11) calculated based on total output produced at installation 1}.

So far, a positive linear relationship between marginal allowance prices and the amount produced at installation 2 has been discovered. This result should not be confused with the statement that marginal allowance prices will be a linear function of the (nontrivial) price drivers spread and emissions intensity. To analyze their relation to marginal allowance prices, rely on the optimum output of installation 2 in equations (A.20) and (A.27) and integrate it into equations (26) and (27). Although the resulting formulas are too lengthy to be set out here, they clearly show a nonlinear effect of price drivers spread and emissions intensity on marginal allowance prices.

3.3 Transactions on the spot market for electricity

Spot market transactions de-couple the amount of electricity sold from the amount of electricity produced, a fact that makes spot market transactions of interest for the analysis of marginal allowance prices for two reasons. First, the spot market for electricity serves as both a procurement and a sales market. As such, the spot market influences GHG constraints and thus can be regarded as an abatement technology in the broader sense. Second, since spot market transactions can either be positive (purchase on the spot market) or negative (sale on the spot market), the owner-operator does not have to produce a given output at installation 1 alone. In other words, spot market transactions allow for the analysis of marginal allowance prices in an environment where the level of production at installation 1 can be chosen freely.

3.3.1 Wealth dynamics

As in the reference case of Section 2 (pure allowance trading environment without abatement technology), it is assumed that installation 1 already exists, i.e., no initial investment for production facilities is needed. However, since total output remains given, a production constraint must be added to the decision problem: total production is split between production at installation 1 and spot market transactions; formally:

$$\overline{N}_{P,0} = N_{P_1,0} + N_{S,0} \tag{29}$$

where $N_{S,0}$ denotes the amount bought or sold on the spot market.

Equation (29) clarifies that only spot market transactions are decision variables at time 0; the amount produced at installation 1 is determined via the production constraint. This is because the production plan (spot market transactions and production at installation 1 that balances the production constraint (29)) needs to be determined at time 0 to be able to start producing and have energy ready for sale at time 1.

In addition, spot market transactions affect GHG constraints. Purchasing electricity on the spot market makes GHG constraints easier to fulfill because less physical production and, thus, less GHG emissions occur than planned.[18] Producing more electricity and selling the surplus on the spot market makes GHG constraints harder to meet. With these effects of spot market transactions in mind, the GHG constraint is formulated as follows:

$$e_{1,1} \cdot \left(\overline{N}_{P,0} - N_{S,0}\right) = \overline{N}_{C,0} + N_{C,1} + N_{C,\tau} \tag{30}$$

Finally, spot market transactions influence wealth at time 1 in two ways. First, so that electricity production at installation 1 is able to balance the production constraint (29), spot market transactions must occur at the same time that electricity output is available – at time 1 but not at time 0 or intermediate times. Second, spot market transactions are a different sort of good compared to "normal" electricity sales. Therefore, spot market prices can be different from prices for "normal" electricity sales. Using both impacts of spot market transactions, wealth at time 1 (before reinvestment, but after balancing the GHG constraints) reads:

$$\begin{aligned} W_1 = \; & x_{1,1} \cdot \left(\overline{N}_{P,0} - N_{S,0}\right) \cdot P_{E,1} + N_{S,0} \cdot \left(\left(P_{E,1} - P_{S,1}\right) - \left(x_{1,1} \cdot P_{E,1} - e_{1,1} \cdot P_{C,1}\right)\right) \\ & + \overline{N}_{C,0} \cdot P_{C,1} + N_{C,\tau} \cdot P_{C,1} + N_{f,\tau} \cdot (1+r)^{1-\tau} \end{aligned} \tag{31}$$

where $P_{S,1}$ denotes electricity spot market prices at time 1.

[18] Similar to the GHG constraint in the event of switching production (equation (23)), the GHG constraint (29) implies that purchasing electricity on the spot market does not lead to a reduction of initial allocation for the installation.

Proceeding by induction, terminal wealth reads:[19]

$$W_T = W_T^{ref} + \sum_{t=1}^{T} N_{S,t-1} \cdot \left(P_{E,t} - P_{S,t} - \left(x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t}\right)\right) \cdot (1+r)^{T-t} \qquad (32)$$

The last term of equation (32) describes the wealth effects of the abatement technology compared to the reference case of equation (8). It stems from the fact that spot market transactions create cash flows that are based on the price difference between “normal” electricity and spot market prices instead of on the difference between spreads and emissions intensities of installation 1.

3.3.2 Valuation results

The decision problem from which the valuation results will be derived reads as follows:

$$\underset{\substack{N_{C,0}, N_{C,\tau}, \ldots, N_{C,T-1+\tau}, \\ N_{S,0}, \ldots, N_{S,T-1}}}{\text{Max}} \quad \Phi \equiv E\{W_T\} - \frac{a}{2} \cdot \text{var}(W_T) \qquad (33)$$

with W_T as defined in equation (32).

That is, the owner-operator can choose the number of emissions allowances at each time t ($t = 0, \tau, 1, 1 + \tau, \ldots, T - 1 + \tau$) as well as the output purchased or sold on the spot market at times 0, 1,..., T – 1 so as to maximize the preference functional (33). To be more precise, the owner-operator determines a strategy for both decision variables, i.e., a complete conditional plan.

Appendices A.3 and A.4 show that marginal allowance prices for this scenario are:

$$P_{C,0} = P_{C,0}^{ref} - a \cdot \frac{1}{(1+r)^T} \cdot \mathbf{N}_S^T COV_{SC} \mathbf{1} \qquad (34)$$

where COV_{SC} denotes the covariance matrix between $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ and $\left(P_{E,t} - P_{S,t} - \left(x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t}\right)\right) \cdot (1+r)^{T-t}$ for t = 1, 2,..., T.

[19] See Appendix A.1 for a derivation where the following modifications are needed to cope with this scenario: $\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2} = \overline{\mathbf{N}}_C$, $x_{2,t} \equiv 1$, $N_{P_2,t-1} \equiv N_{S,t-1}$, and $e_{2,t} \equiv \frac{P_{S,t}}{P_{C,t}}$.

For the special case of serially uncorrelated cash flows, emissions intensities, and spreads one obtains:

$$P_{C,0} = \quad P_{C,0}^{ref,unc} - a \cdot \frac{1}{(1+r)^T} \cdot \mathrm{cov}\left(P_{E,T} - P_{S,T} - \left(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}\right), P_{C,T}\right) \cdot N_{S,T-1} \qquad (35)$$

Since spot market transactions can be either positive or negative, there will be just one bound for marginal allowance prices, a sharp contrast to the situations under GHG injections and switching production. The only bound comes from the fact that production at installation 1 cannot be negative (minimal $N_{P_1,t-1} = 0$ for t = 1, 2,..., T), which means that there is a maximum for $N_{S,t-1}$ (for t = 1, 2,..., T): the fixed electricity production will be completely purchased on the spot market, i.e., $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T). However, electricity sales on the spot market can be arbitrarily high as long as installation 1’s output is high enough to meet the production constraint (29). Below or above this bound, marginal allowance prices are a linear function of spot market transactions $N_{S,t-1}$ (for t = 1, 2,..., T) (see equations (34) and (35)).

The next question is whether this bound is a lower or higher bound and how marginal allowance prices under spot market transactions (equations (34) and (35)) compare to those of the reference case (equations (10) and (11)) (pure allowance trading environment without abatement technology). As only $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T) and, thus, a purchase of electricity leads to a price bound, marginal allowance prices in the special case of equation (35) depend on the sign of the difference between $\mathrm{cov}\left(P_{E,T} - P_{S,T}, P_{C,T}\right)$, i.e., covariance between cash flows from electricity sales acquired via spot market transactions and allowance prices, and $\mathrm{cov}\left(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}, P_{C,T}\right)$, i.e., covariance between cash flows from electricity production at installation 1 and allowance prices. If this difference is positive, marginal allowance prices at $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T) will be a lower bound for marginal allowance prices under spot market transactions. Hence, marginal allowance prices increase from the point $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T) linearly with decreasing spot market transactions, i.e., the less electricity is produced or the more electricity is sold on the spot market. This is easily understood: a positive difference means that risk will be maximal if the fixed electricity output is purchased on the spot market. Buying less or even selling electricity on the spot market reduces risk and, thus, calls for an increase in marginal allowance prices (lower bound). A negative difference signifies minimal risk by purchasing the fixed electricity output on the spot market. Buying less or even selling electricity via the spot market increases risk and a price discount follows (upper bound), i.e., marginal allowance prices decrease from the point $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T) linearly with decreasing spot market transactions. Thereby, the marginal allowance price is located below (above) the marginal allowance price of the reference case $P_{C,0}^{ref,unc}$ for a positive (negative) covariance as long as $N_{S,t-1} > 0$ (for t = 1, 2,..., T).

The demand-based explanation of this behavior of marginal allowance prices adds further illustrative insights. Assume that the owner-operator purchases less electricity on the spot market than $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T) or even sells electricity on the spot market. A higher allowance price obviously means that purchases on spot markets are too expensive for the owner-operator. Thus, he either increases electricity production at installation 1 so that he needs to purchase less electricity on the spot market or he even sells electricity on the spot market; both scenarios make GHG constraints more difficult to meet and emissions allowances become more valuable. A lower allowance price combined with purchasing less than $\overline{N}_{P,t-1} = N_{S,t-1}$ (for t = 1, 2,..., T) or even selling electricity can occur because the increased production at installation 1 is so profitable that the owner-operator will have enough buffer to pay for emissions allowances at the time GHG constraints must be met. For that reason, he is not willing to pay a high price for the opportunity to purchase or sell allowances in advance.

Since the general case of equation (34) merely contains several intertemporal covariance terms, the results of the special case can be transferred seamlessly.

Finally, to describe marginal allowance prices fully under spot market transactions, the price bounds of the reference case need to be taken into account, i.e., the lower bound at zero and the upper bound at $\frac{40}{1+r}$ $(\frac{100}{1+r})$ EUR. This means that marginal allowance prices under spot market transactions are a negative linear function of $N_{S,t-1}$ (for t = 1, 2,..., T) and lie between max{0, (34) or (35) evaluated at $\overline{N}_{P,t-1} = N_{S,t-1}$} and min{$\frac{40}{1+r}\left(\frac{100}{1+r}\right)$, (34) or (35) evaluated at $\overline{N}_{P,t-1} = N_{S,t-1}$}.

So far, a negative linear relationship between marginal allowance prices and spot market transactions has been discovered. This result should not be confused with the statement that marginal allowance prices will be a linear function of the (nontrivial) price drivers spread and emissions intensity. To analyze their relation to marginal allowance prices, rely on the optimum amount of spot market transactions and integrate them into the price determined by equations (34) and (35).[20] Although the resulting formulas are too lengthy to be set out here, they clearly show a nonlinear effect of price drivers spread and emissions intensity on marginal allowance prices.

5. Conclusion

This paper started from the observation that a prerequisite of successful emissions trading is that companies are able to determine their marginal prices for emissions allowances under different production and abatement technologies. Otherwise, the market mechanism will be inefficient and allowance trading will neither be able to provide in-

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[20] Thereby, the optimum spot market transaction becomes accessible from equations (A.20) and (A.27)

formation on penalties/rewards for failing/meeting emissions goals (price of GHG emissions) nor be able to discover the best GHG abatement technology.

This paper derived two results. First, marginal allowance prices, in a pure allowance trading environment (without the use of abatement technologies), depend on companies' profitability (the higher the profitability, the lower the allowance price, i.e., negative influence on allowance prices), emissions intensities (positive influence), and the correlation between electricity and allowance prices (positive influence). Second, the use of alternative abatement technologies exerts visible influence on marginal allowance prices.

The fact that marginal allowance prices, i.e., prices at which owner-operators of installations are indifferent between participating in and refraining from allowance trading, are different for installations with divergent emissions intensities and alternative GHG abatement technologies creates an ideal environment for allowance trading. Allowance trading can be implemented successfully because installations with low emissions intensities and/or low abatement costs will likely sell allowances, whereas installations with high emissions intensities and/or high abatement costs will probably buy emissions allowances. – This theoretically derived price behavior is in absolute accordance with the two goals of the Kyoto Protocol: the determination of penalties/rewards for GHG emissions and the discovery of the best abatement technology.

Appendix

A.1 Terminal wealth

Budget constraint:

$$W_0 + \overline{N}_{C_1,0} + \overline{N}_{C_2,0} = N_{C,0} \cdot P_{C,0} + N_{f,0} \qquad \text{(A.1)}$$

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using $\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2} = \overline{\mathbf{N}}_C$, $x_{2,t} \equiv 1$, $N_{P_2,t-1} \equiv N_{S,t-1}$, and $e_{2,t} \equiv \frac{P_{S,t}}{P_{C,t}}$.

GHG constraint:

$$e_{1,1} \cdot \left(\overline{N}_{P,0} - N_{P_2,0}\right) + e_{2,1} \cdot N_{P_2,0} = N_{C,1} + \overline{N}_{C_1,0} + \overline{N}_{C_2,0} + N_{C,\tau} \tag{A.2}$$

Wealth at time t = 1 for the owner-operator (before reinvestment, but after balancing the GHG constraint) reads:

$$W_1 = \overline{N}_{P,0} \cdot x_{1,1} \cdot P_{E,1} + N_{P_2,0} \cdot \left(x_{2,1} - x_{1,1}\right) \cdot P_{E,1} - N_{C,1} \cdot P_{C,1} + N_{f,\tau} \cdot (1+r)^{1-\tau} \tag{A.3}$$

Since from the budget equation follows:

$$N_{f,0} = W_0 + \left(\overline{N}_{C_1,0} + \overline{N}_{C_2,0}\right) \cdot P_{C,0} - N_{C,0} \cdot P_{C,0} \tag{A.4}$$

and from equations (3) and (4):

$$N_{f,\tau} = N_{C,0} \cdot P_{C,\tau} + N_{f,0} \cdot (1+r)^{\tau} - N_{C,\tau} \cdot P_{C,\tau} \tag{A.5}$$

that is,

$$N_{f,\tau} = N_{C,0} \cdot \left(P_{C,\tau} - (1+r)^{\tau} \cdot P_{C,0}\right) - N_{C,\tau} \cdot P_{C,\tau} \tag{A.6}$$

$$+ W_0 \cdot (1+r)^{\tau} + \left(\overline{N}_{C_1,0} + \overline{N}_{C_2,0}\right) \cdot P_{C,0} \cdot (1+r)^{\tau}$$

one obtains for wealth at time 1 (before reinvestment, but after balancing the GHG constraint):

$$W_1 = \overline{N}_{P,0} \cdot \left(x_{1,1} \cdot P_{E,1} - e_{1,1} \cdot P_{C,1}\right) \tag{A.7}$$

$$+ \left(\overline{N}_{C_1,1} + \overline{N}_{C_2,1}\right) \cdot \left(P_{C,1} + (1+r) \cdot P_{C,0}\right)$$

$$+ N_{C,0} \cdot \left(P_{C,\tau} \cdot (1+r)^{1-\tau} - (1+r) \cdot P_{C,0}\right)$$

$$+ N_{C,\tau} \cdot \left(P_{C,1} - (1+r)^{1-\tau} \cdot P_{C,\tau}\right)$$

$$+ W_0 \cdot (1+r)$$

$$+ N_{P_2,0} \cdot \left(\left(x_{2,1} - x_{1,1}\right) \cdot P_{E,1} - \left(e_{2,1} - e_{1,1}\right) \cdot P_{C,1}\right)$$

Proceeding by induction yields equation (24).

A.2 Objective function particularized with the definition of terminal wealth (15)

The mean of terminal wealth reads:

$$E_0\{W_T\} = \overline{\mathbf{N}}_P^T\, E_0\{\mathbf{Z}_{P_1}\} + \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right)^T E_0\{\mathbf{Z}_{\overline{C}}\} + \mathbf{N}_C^T E_0\{\mathbf{Z}_C\} + W_0 \cdot (1+r)^T + \overline{\mathbf{N}}_{P_2}^T\, E_0\{\mathbf{Z}_{P_2}\} \tag{A.8}$$

where ^{T} denotes transposition of a vector or matrix (this symbol, however, should not be confused with discounting over T periods as in $\frac{1}{(1+r)^T}$), $\overline{\mathbf{N}}_i$ ($i \in \{P, P_2, C_1, C_2\}$) the vector of the amount of production or initial allocation of allowances at times 0, 1,..., T – 1, Z_i ($i \in \{P_1, P_2, \overline{C}\}$) the payoffs from production or initial allocation with $Z_{P_1,t} \equiv (x_{1,t} \cdot P_{E,t} - e_{1,t} \cdot P_{C,t}) \cdot (1+r)^{T-t}$, $Z_{P_2,t} \equiv ((x_{2,t} - x_{1,t}) \cdot P_{E,t} - (e_{2,t} - e_{1,t}) \cdot P_{C,t}) \cdot (1+r)^{T-t}$, and $Z_{\overline{C},t} \equiv (P_{C,t} + (1+r) \cdot P_{C,t-1}) \cdot (1+r)^{T-t}$

The vector **N** is the trading vector, i.e., it contains the numbers of emissions allowances bought or sold at times 0, τ, 1, 1+ τ,..., T – 1, T – 1 + τ with cash flows $Z_{C,\tau}$, $Z_{C,1}$,..., $Z_{C,T-1+\tau}$, and $Z_{C,T}$ where $Z_{C,t-1+\tau} \equiv (P_{C,t-1+\tau} \cdot (1+r)^{1-\tau} - (1+r) \cdot P_{C,t-1}) \cdot (1+r)^{T-t}$ and $Z_{C,t} \equiv (P_{C,t} - (1+r)^{1-\tau} \cdot P_{C,t-1+\tau}) \cdot (1+r)^{T-t}$; these cash flows are collected in the vector $\mathbf{Z}_C$.

The variance of terminal wealth is:

$$\begin{aligned} \mathrm{var}(W_T) = {} & \overline{\mathbf{N}}_P^T \mathbf{\Omega}_P \overline{\mathbf{N}}_P + \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right)^T \mathbf{\Omega}_{\overline{C}} \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right) + \mathbf{N}_C^T \mathbf{\Omega}_C \mathbf{N}_C \\ & + 2 \cdot \overline{\mathbf{N}}_P^T \mathrm{COV}_{P\overline{C}} \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right) + 2 \cdot \overline{\mathbf{N}}_P^T \mathrm{COV}_{PC} \mathbf{N}_C \\ & + 2 \cdot \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right)^T \mathrm{COV}_{\overline{C}C} \mathbf{N}_C + \mathbf{N}_{P_2}^T \mathbf{\Omega}_{P_2} \mathbf{N}_{P_2} \\ & + 2 \cdot \mathbf{N}_{P_2}^T \mathrm{COV}_{P_2 P} \overline{\mathbf{N}}_P + 2 \cdot \mathbf{N}_{P_2}^T \mathrm{COV}_{P_2 \overline{C}} \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right) + 2 \cdot \mathbf{N}_{P_2}^T \mathrm{COV}_{P_2 C} \mathbf{N}_C \end{aligned} \tag{A.9}$$

The components of the variance of terminal wealth can be explained as follows:

$$\mathrm{var}\left(\sum_{t=1}^{T} N_{P_2,t-1}\cdot\left(\left(x_{2,t}-x_{1,t}\right)\cdot P_{E,t}-\left(e_{2,t}-e_{1,t}\right)\cdot P_{C,t}\right)\cdot(1+r)^{T-t}\right)=\mathbf{N}_{P_2}^{T}\mathbf{\Omega}_{P_2}\mathbf{N}_{P_2} \qquad \text{(A.10)}$$

where Ω_{P_2} denotes the variance/covariance matrix of $Z_{P_2,t}$ (t = 1, 2,..., T), i.e.,

$$\Omega_{P_2}=\begin{pmatrix} \mathrm{var}(Z_{P_2,1}) & \mathrm{cov}(Z_{P_2,1};Z_{P_2,2}) & \cdots \\ \mathrm{cov}(Z_{P_2,2};Z_{P_2,1}) & \mathrm{var}(Z_{P_2,2}) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

In an analogous way, the variance/covariance matrices $\mathbf{\Omega}_{\overline{C}}$ based on $Z_{\overline{C},t}$ and $\mathbf{\Omega}_{P}$ based on $Z_{P_1,t}$ can be defined. Since $\mathbf{\Omega}_{C}$ covers variance/covariance relations between $Z_{C,t-1+\tau}$ and $Z_{C,t}$, it is useful to write it down explicitly:

$$\Omega_{C}=\begin{pmatrix} \mathrm{var}(Z_{C,\tau}) & \mathrm{cov}(Z_{C,\tau};Z_{C,1}) & \mathrm{cov}(Z_{C,\tau};Z_{C,1+\tau}) & \cdots \\ \mathrm{cov}(Z_{C,1};Z_{C,\tau}) & \mathrm{var}(Z_{C,1}) & \mathrm{cov}(Z_{C,1};Z_{C,1+\tau}) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

The next component of the variance of terminal wealth reads:

$$\mathrm{COV}\left(\sum_{t=1}^{T} N_{P_2,t-1}\cdot\left(\left(x_{2,t}-x_{1,t}\right)\cdot P_{E,t}-\left(e_{2,t}-e_{1,t}\right)\cdot P_{C,t}\right)\cdot(1+r)^{T-t};\right. \qquad \text{(A.11)}$$

$$\left.\sum_{t=1}^{T} \overline{N}_{P,t-1}\cdot\left(x_{1,t}\cdot P_{E,t}-e_{1,t}\cdot P_{C,t}\right)\cdot(1+r)^{T-t}\right)=\mathbf{N}_{P_2}^{T}\mathrm{COV}_{P_2P}\overline{\mathbf{N}}_{P}$$

where COV_{P_2P} denotes the covariance between $Z_{P_2,t}$ and $Z_{P_1,t}$ (t = 1, 2,..., T), i.e.,

$$\mathrm{COV}_{P_2P}=\begin{pmatrix} \mathrm{cov}(Z_{P_2,1};Z_{P_1,1}) & \mathrm{cov}(Z_{P_2,1};Z_{P_1,2}) & \cdots \\ \mathrm{cov}(Z_{P_2,2};Z_{P_1,1}) & \mathrm{cov}(Z_{P_2,2};Z_{P_1,2}) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

In a similar fashion, the covariance matrices $\mathrm{COV}_{P_2\overline{C}}$ (based on the covariances between $Z_{P_2,t}$ and $Z_{\overline{C},t}$ for t = 1, 2,..., T) and $\mathrm{COV}_{P\overline{C}}$ (based on the covariances between $Z_{P_1,t}$ and $Z_{\overline{C},t}$ for t = 1, 2,..., T) can be defined.

Only $COV_{P_2 C}$ deserves further illustration because of the somewhat more complex structure of $Z_{C,t-1+\tau}$ and $Z_{C,t}$. It reads:

$$COV\left(\sum_{t=1}^{T} N_{P_2,t-1}\cdot\left(\left(x_{2,t}-x_{1,t}\right)\cdot P_{E,t}-\left(e_{2,t}-e_{1,t}\right)\cdot P_{C,t}\right)\cdot(1+r)^{T-t};\right. \quad (A.12)$$

$$\sum_{t=1}^{T} N_{C,t-1}\cdot\left(P_{C,t-1+\tau}\cdot(1+r)^{1-\tau}-(1+r)\cdot P_{C,t-1}\right)\cdot(1+r)^{T-t}$$
$$\left.+\sum_{t=1}^{T} N_{C,t-1+\tau}\cdot\left(P_{C,t}-(1+r)^{1-\tau}\cdot P_{C,t-1+\tau}\right)\cdot(1+r)^{T-t}\right)=\mathbf{N}_{P_2}^{T} COV_{P_2 C}\mathbf{N}_C$$

where $COV_{P_2 C}$ denotes the covariances between $Z_{P_2,t}$ and $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ (t = 1, 2,..., T), i.e.,

$$COV_{P_2 C}=\begin{pmatrix} cov\left(Z_{P_2,1};Z_{C,\tau}\right) & cov\left(Z_{P_2,1};Z_{C,1}\right) & cov\left(Z_{P_2,1};Z_{C,1+\tau}\right) & \cdots \\ cov\left(Z_{P_2,2};Z_{C,\tau}\right) & cov\left(Z_{P_2,2};Z_{C,1}\right) & cov\left(Z_{P_2,2};Z_{C,1+\tau}\right) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

COV_{PC} (based on the covariances between $Z_{P_1,t}$ and $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ for t = 1, 2,..., T) and $COV_{\overline{C}C}$ (based on the covariances between $Z_{\overline{C},t}$ and $Z_{C,t-1+\tau}$ as well as $Z_{C,t}$ for t = 1, 2,..., T) are defined analogously.

A.3 Necessary conditions

Differentiation of the objective function with respect to $\mathbf{N}_C$ and evaluation of the derivative at $\mathbf{N}_C = \mathbf{0}$ (remember, the marginal allowance price is defined as the price where the owner-operator restrains from allowance trading) yields:

$$\left.\frac{\partial\Phi}{\partial\mathbf{N}_C}\right|_{\mathbf{N}_C=\mathbf{0}}=\mathbf{0}=\ E_0\left\{\mathbf{Z}_C\right\}-a\cdot\overline{\mathbf{N}}_P^{T} COV_{PC}-a\cdot\left(\overline{\mathbf{N}}_{C_1}+\overline{\mathbf{N}}_{C_2}\right)^{T} COV_{\overline{C}C}-a\cdot\mathbf{N}_{P_2}^{T} COV_{P_2 C} \quad (A.13)$$

$$\left.\frac{\partial\Phi}{\partial\mathbf{N}_{P_2}}\right|_{\mathbf{N}_C=\mathbf{0}}=\mathbf{0}=\ E\left\{\mathbf{Z}_{P_2}\right\}-a\cdot\Omega_{P_2}\mathbf{N}_{P_2}-a\cdot COV_{P_2 P}\overline{N}_P-a\cdot COV_{P_2\overline{C}}\left(\overline{\mathbf{N}}_{C_1}+\overline{\mathbf{N}}_{C_2}\right) \quad (A.14)$$

More precisely, the owner-operator determines a strategy for both decision variables, i.e., a complete conditional plan. Therefore, the necessary conditions read in a more explicit way: $\frac{\partial \Phi}{\partial N_{C,1}(S_i)}$, $\frac{\partial \Phi}{\partial N_{C,2}(S_i,S_j)}$, $\frac{\partial \Phi}{\partial N_{P_2,1}(S_i)}$ etc.

From this necessary condition it becomes possible to obtain the desired marginal allowance price.

A.4 Price equation: general case

To derive marginal prices from the necessary conditions, proceed as follows.

First, have a look at the necessary conditions for $N_{C,0}$, i.e., the first row of the vector (equation (A.13):

$$\left.\frac{\partial \Phi}{\partial N_{C,0}}\right|_{\mathbf{N}_C=\mathbf{0}} = 0 = \quad E_0\left\{\left(P_{C,\tau}\cdot(1+r)^{1-\tau} - (1+r)\cdot P_{C,0}\right)\cdot(1+r)^{T-1}\right\} \tag{A.15}$$

$$- a\cdot \mathrm{cov}\left(Z_{C,\tau}, \sum_{t=1}^{T} Z_{P_1,t}\cdot \overline{N}_{P,t-1}\right)$$

$$- a\cdot \mathrm{cov}\left(Z_{C,\tau}, \sum_{t=1}^{T} Z_{\overline{C},t}\cdot\left(\overline{N}_{C_1,t-1}+\overline{N}_{C_2,t-1}\right)\right) - a\cdot \mathrm{cov}\left(Z_{C,\tau}, \sum_{t=1}^{T} Z_{P_2,t}\cdot N_{P_2,t-1}\right)$$

From equation (A.15) one obtains:

$$P_{C,0} = \frac{1}{(1+r)^T}\cdot E_0\left\{P_{C,\tau}\cdot(1+r)^{1-\tau}\right\} \tag{A.16}$$

$$- a\cdot\frac{1}{(1+r)^T}\cdot\left[\mathrm{cov}\left(Z_{C,\tau}, \sum_{t=1}^{T} Z_{P_1,t}\cdot \overline{N}_{P,t-1}\right)\right.$$

$$\left. + \mathrm{cov}\left(Z_{C,\tau}, \sum_{t=1}^{T} Z_{\overline{C},t}\cdot\left(\overline{N}_{C_1,t-1}+\overline{N}_{C_2,t-1}\right)\right) + \mathrm{cov}\left(Z_{C,\tau}, \sum_{t=1}^{T} Z_{P_2,t}\cdot N_{P_2,t-1}\right)\right]$$

In other words, the desired marginal allowance price at time 0 is a function of the mean of the allowance price at time τ $E_0\left\{P_{C,\tau}\cdot(1+r)^{1-\tau}\right\}$. This value can, in a second step, be calculated from the necessary condition for $N_{C,\tau}$, i.e., the second row of the vector in equation (A.13):

$$\left.\frac{\partial \Phi}{\partial N_{C,\tau}}\right|_{\mathbf{N}_C=\mathbf{0}} = 0 = \quad E_0\left\{\left(P_{C,1} - (1+r)^{1-\tau} \cdot P_{C,\tau}\right) \cdot (1+r)^{T-1}\right\} \tag{A.17}$$

$$- a \cdot \text{cov}\left(Z_{C,1}, \sum_{t=1}^{T} Z_{P_1,t} \cdot \overline{N}_{P,t-1} \right)$$

$$- a \cdot \text{cov}\left(Z_{C,1}, \sum_{t=1}^{T} Z_{\overline{C},t} \cdot \left(\overline{N}_{C_1,t-1} + \overline{N}_{C_2,t-1}\right)\right) - a \cdot \text{cov}\left(Z_{C,1}, \sum_{t=1}^{T} Z_{P_2,t} \cdot N_{P_2,t-1} \right)$$

which yields:

$$E_0\left\{(1+r)^{1-\tau} \cdot P_{C,\tau}\right\} = E_0\left\{P_{C,1}\right\} \tag{A.18}$$

$$- a \cdot \frac{1}{(1+r)^{T-1}} \cdot \left[\text{cov}\left(Z_{C,1}, \sum_{t=1}^{T} Z_{P_1,t} \cdot \overline{N}_{P,t-1} \right) \right.$$

$$\left. + \text{cov}\left(Z_{C,1}, \sum_{t=1}^{T} Z_{\overline{C},t} \cdot \left(\overline{N}_{C_1,t-1} + \overline{N}_{C_2,t-1}\right)\right) + \text{cov}\left(Z_{C,1}, \sum_{t=1}^{T} Z_{P_2,t} \cdot N_{P_2,t-1} \right) \right]$$

From equation (A.18) it become obvious that expected values of future allowance prices at times τ, 1, 1 + τ,..., T – 1 + τ are derived model endogenously, i.e., that they are a result of the valuation and not specified model exogenously.

Using the necessary conditions for $N_{C,t}$ at different times t, proceeding by induction leads to:

$$P_{C,0} = \frac{1}{(1+r)^T} \cdot E\left\{P_{C,T}\right\} \tag{A.19}$$

$$- a \cdot \frac{1}{(1+r)^T} \cdot \overline{\mathbf{N}}_P^T \, \text{COV}_{PC} \mathbf{1} - a \cdot \frac{1}{(1+r)^T} \cdot \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right)^T \text{COV}_{\overline{C}C} \mathbf{1}$$

$$- a \cdot \frac{1}{(1+r)^T} \cdot \mathbf{N}_{P_2}^T \text{COV}_{P_2 C} \mathbf{1}$$

In a third step, the optimum values for $\mathbf{N}_{P_2}$ must be inserted into price equation (A.19). From the necessary conditions (A.14), one obtains:

$$\mathbf{N}_{P_2} = \frac{1}{a}\Omega_{P_2}^{-1}E\{\mathbf{Z}_{P_2}\} - \Omega_{P_2}^{-1}COV_{P_2 P}\overline{\mathbf{N}}_P - \Omega_{P_2}^{-1}COV_{P_2 \overline{C}}\left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right) \quad (A.20)$$

Substituting optimum output for installation 2 (equation (A.20)) into price equation (A.19) yields:

$$P_{C,0} = \frac{1}{(1+r)^T} \cdot \left(E\{P_{C,T}\} - E\{\mathbf{Z}_{P_2}\}^T \Omega_{P_2}^{-1} COV_{P_2 C}\mathbf{1}\right) \quad (A.21)$$

$$- a \cdot \frac{1}{(1+r)^T} \cdot \overline{\mathbf{N}}_P^T \left(COV_{PC}^T \mathbf{1} - COV_{P_2 P}^T \Omega_{P_2}^{-1} COV_{P_2 C}\mathbf{1}\right)$$

$$- a \cdot \frac{1}{(1+r)^T} \cdot \left(\overline{\mathbf{N}}_{C_1} + \overline{\mathbf{N}}_{C_2}\right)^T \left(COV_{C\overline{C}}^T \mathbf{1} - COV_{P_2 \overline{C}}^T \Omega_{P_2}^{-1} COV_{P_2 C}\mathbf{1}\right)$$

A.5 Price equation: special case

The special case deals with the situation of serially uncorrelated cash flows, emissions intensities, and spreads.

The calculations that are necessary to develop the allowance price of the special case from the general price equation (equation (A.19)) can be illustrated with the help of $COV_{P_2 C}\mathbf{1}$. Calculating the first row of $COV_{P_2 C}$, it becomes obvious that all covariances equal zero besides $cov(Z_{P_2,1}; Z_{C,1})$ and $cov(Z_{P_2,1}; Z_{C,1+\tau})$. To be more precise, it holds:

$$cov(Z_{P_2,1}; Z_{C,1}) = cov(Z_{P_2,1}; P_{C,1}) \cdot (1+r)^{T-1} \quad (A.22)$$

and

$$cov(Z_{P_2,1}; Z_{C,1+\tau}) = -cov(Z_{P_2,1}; P_{C,1} \cdot (1+r)) \cdot (1+r)^{T-2} \quad (A.23)$$

Computing $COV_{P_2 C}\mathbf{1}$, i.e., summing over the covariances of the first row, means that both covariances cancel out. This is true for every time except T because there is no cash flow at time T + τ that has a compensating covariance. Therefore, it holds:

$$cov(Z_{P_2,1}; Z_{C,T}) = cov(Z_{P_2,1}; P_{C,T}) \quad (A.24)$$

and, finally:

$$\mathrm{COV}_{P_2C}\mathbf{1} = \begin{pmatrix} 0 \\ \vdots \\ \mathrm{cov}\left(Z_{P_2,1}; P_{C,T}\right) \end{pmatrix} \quad \text{(A.25)}$$

Applying an analogue reasoning to the other covariances of price equation (A.19), one obtains:

$$\begin{aligned} P_{C,0} = & \frac{1}{(1+r)^T} \cdot E\{P_{C,T}\} \quad \text{(A.26)} \\ & - a \cdot \frac{1}{(1+r)^T} \cdot \mathrm{cov}\left(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{E,T}, P_{C,T}\right) \cdot \overline{N}_{P,T-1} \\ & - a \cdot \frac{1}{(1+r)^T} \cdot \mathrm{var}\left(P_{C,T}\right) \cdot \left(\overline{N}_{C_1,T-1} + \overline{N}_{C_2,T-1}\right) \\ & - a \cdot \frac{1}{(1+r)^T} \cdot \mathrm{cov}\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}, P_{C,T}\right) \cdot N_{P_2,T-1} \end{aligned}$$

To express the allowance price without reference to optimum output of installation 2 at time T – 1 ($N_{P_2,T-1}$), proceed as follows. $N_{P_2,T-1}$ can be obtained by extracting the last (fourth) row of the vector (equation (A.14)):

$$\begin{aligned} N_{P_2,T-1} = & \frac{1}{a} \frac{\left(x_{2,T} - x_{1,T}\right) \cdot E\{P_{E,T}\} - \left(e_{2,T} - e_{1,T}\right) \cdot E\{P_{C,T}\}}{\mathrm{var}\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}\right)} \quad \text{(A.27)} \\ & - \frac{\mathrm{cov}\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}, x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}\right)}{\mathrm{var}\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}\right)} \cdot \overline{N}_{P,T-1} \\ & - \frac{\mathrm{cov}\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}, P_{C,T}\right)}{\mathrm{var}\left(\left(x_{2,T} - x_{1,T}\right) \cdot P_{E,T} - \left(e_{2,T} - e_{1,T}\right) \cdot P_{C,T}\right)} \cdot \left(\overline{N}_{C_1,T-1} + \overline{N}_{C_2,T-1}\right) \end{aligned}$$

A.6 Optimum amount injected

Taking into consideration the fact that:

$$\left.\frac{\partial E_0\{W_T\}}{\partial N_{A,t-1}}\right|_{\mathbf{N}_C = \mathbf{0}} = \sum_{t=1}^{T} \left(P_{C,t} - \frac{\partial K\left(N_{A,t-1}\right)}{\partial N_{A,t-1}} \right) \cdot (1+r)^{T-t} \equiv \sum_{t=1}^{T} Z_{A,t} - MK\left(N_A\right) \quad \text{(A.28)}$$

optimal GHG injections (interior solution) read:

$$\mathbf{N}_A = \frac{1}{a}\Omega_A^{-1}E\{\mathbf{Z}_A - MK(N_A)\} - \Omega_A^{-1}COV_{P_2 P}\overline{\mathbf{N}}_P - \Omega_A^{-1}COV_{A\overline{C}}\overline{\mathbf{N}}_C \tag{A.29}$$

where Ω_A denotes the variance/covariance matrix of $P_{C,t} \cdot (1+r)^{T-t}$ for t = 1, 2,..., T.

In the special case of serially uncorrelated cash flows, emissions intensities, and spreads, equation (A.29) simplifies to:

$$\begin{aligned} \operatorname{var}(P_{C,T}) \cdot N_{A,T-1} = {} & \operatorname{cov}(x_{1,T} \cdot P_{E,T} - e_{1,T} \cdot P_{C,T}, P_{C,T}) \cdot \overline{N}_{P,T-1} \\ & + \operatorname{var}(P_{C,T}) \cdot \overline{N}_{C,T-1} \\ & - \frac{1}{a} \cdot E_0\{P_{C,T}\} + \frac{1}{a} \cdot \frac{\partial K(N_{A,T-1})}{\partial N_{A,T-1}} \end{aligned} \tag{A.30}$$

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