Forecasting Portuguese Stock Market Volatility

Ricado Pereira*

(Comments Welcome)

This Version:

August 2004

* Lecturer in Finance, Moderna University of Porto, Dep. de Organização e Gestão de Empresas, Rua Amália Rodrigues, 37, 4 dto, 4715-338 Braga, Portugal, Phone: +351-919488275, ramgp@clix.pt

Forecasting Portuguese Stock Market Volatility

Abstract:

In this paper we apply volatility forecasting models based on past measures of risk and some ARCH class models to within-week volatility forecast of Portuguese Stock Market, using different measures of volatility and comparing them through the use of both symmetric and asymmetric error statistics. The results are interpreted at the light of assets and models' features. Since usually the papers about this topic only deal and provide empirical evidence about mature and liquid financial markets and about market indices, the motivation of this study is to improve the knowledge about the volatility forecasting models that drive volatility in emerging capital markets, at the level of individual stocks.

The results follow the ones observed by other authors and we can say that the best forecasting model depends on the evaluation measure used. Notwithstanding, the numbers pointed out to smooth superiority of ARCH class models, principally when using RMSE and MME(U), which means that ARCH class models normally over-predicts volatility. It is also worth noting the excellent performance of volatility forecasting models based on semi-standard deviation, even when compared with ARCH class models. This is probably the most original aspect of this study.

JEL: C51, C52, C53, G12

Keywords: Risk Measures, Volatility Forecasting Models, Symmetric and Asymmetric Error Statistics

1. Introduction

Volatility is one of the most important inputs of several asset pricing models. An accurate estimation of volatility is crucial in the pricing of derivative securities, in capital budgeting (e.g. real options approach), in portfolio selection and financial risk management[1].

Given the importance of volatility forecast, many forecasting models have been developed and empirically applied.

This paper aims to apply several of these volatility forecasting models to the Portuguese Stock Market and compare them using both symmetric and asymmetric error statistics. The results will be interpreted at the light of assets and models' features.

Following the seminal work of Mandelbrot (1963) and Fama (1965), many researchers have found that empirical distribution of stock returns is non-normal (e.g. Peiró[2] (1999), Aparício and Estrada (2001)). These authors found that (i) it is more peaked and with fatter tails than the normal distribution; (ii) it is skewed; and (iii) the variance of stock returns is not constant over time or exhibit volatility clustering. For these reasons, the variance of returns is not an appropriate measure of risk. In 1982, Engle has characterized the changing variances by an Autoregressive Conditional Heteroskedasticity (ARCH) model and Bollerslev (1986) introduced the Generalized ARCH (GARCH), which is more flexible than the previous one. These two papers represent a rupture with the traditional treatment of volatility and, latter, several other papers were published transforming them in a manner that allows them to incorporate some stylized facts about financial market volatility.

Besides this, most measures of dispersion make no distinction between positive or negative returns and it is know that investors only dislike downside volatility (which is related with the skewness of stock returns distribution). This implies that, for example, skewness or upside and downside risk is relevant and must be integrated in the asset pricing models. Aparício and Estrada (2001) expose the conditions for the markets' return distributions being normal. The authors reject the normality of daily stock returns of thirteen European securities markets. Given this, a measure that combines in a single one the information provided by the variance and skewness is needed. This measure could be the semivariance (or semi-standard deviation) which was already

---

[1] Since the first Basle Accord (and reinforced by the second one) many financial institutions are obligated to make volatility forecasting because they have to constitute capital reserves of at least three times that of value-at-risk.

[2] Peiró (1999) refers several authors that propose different statistical distributions for price changes of financial assets. All these distributions reflect the high kurtosis existent in the empirical distribution of returns (more peaked and with fatter tails than the normal distribution).

been defended by Markowitz (1952). Aparício and Estrada (2001) propose the semi-standard deviation as a measure of risk when the asset's distribution of returns is skewed and show that mean-semi standard deviation behavior is an approximately correct criterion to maximize the expected utility function. We also apply the forecasting models to semi-standard deviation.

The remaining sections are organized as follows. Section 2 reviews some research papers that apply volatility forecasting models. Section 3 provides a description of the data and methodology used in this study. In the section 4, the results are presented and a discussion of the empirical evidence is provided. Section 5 summarises and concludes.

2. Literature Review

Engle (1993) and Aydemir (1998) offer a detailed exposition of several time series models for estimating and modelling volatility.

Despite the popularity and theoretical support of ARCH class models, their forecasting power over the simpler ones is, by no means, consensual. In this section, we are going to present some papers that underpin this reality.

Taylor (1987) is one of the first to test time series volatility forecasting models and uses DM/$ futures prices. The author, employing Root Mean Square Error (RMSE), finds that a weighted average of present and past high, low and closing prices is the best volatility forecasting model.

Akgiray (1989) finds evidence in favour of a GARCH (1,1) model (over ARCH (2), Exponential Weighted Moving Average (EWMA) and Historical Volatility (HIS)), especially in periods of high volatility, when applied to U.S. data. The author uses traditional symmetric error statistics (Mean Error (ME), RMSE, Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE)).

Dimson and Marsh (1990) apply five different types of forecasting models (random walk (RW), Moving Average (MA), HIS, Exponential Smoothing (ES) and Regression models) and recommend the last two.

Pagan and Schwert (1990) conclude that for U.S. stock market, from 1834 to 1937, Exponential GARCH (EGARCH) is the best volatility forecasting model (principally when compared with nonparametric models).

Tse (1991) applies EWMA, HIS, ARCH and GARCH models to the Japanese stock market and finds that EWMA is the best model. Tse and Tung (1992) adopt the same in the Singaporean stock market and obtain the same results.

Brailsford and Faff (1996) analyse the predictive power of several forecasting models (RW, HIS, MA, ES, EWMA, Regression and GARCH class models) in monthly Australian stock market volatility. The authors, working with symmetric and asymmetric error statistics, find that GJR-GARCH (Glosten, Jagannathan and Runkle (1993)) is the best model, but the results are very sensible to the used error statistics.

Franses and Van Dijk (1996) study the forecasting power of RW and some GARCH class models, when applied to European stock indices, and conclude that QGARCH is the best model. Curiously, given the previous results, GJR-GARCH model is not recommended.

McMillan, Speight and Gwilym (2000) analyse the predictive power of several GARCH class, RW, HIS, MA, ES, EWMA and Regression models, when applied to FTSE 100 and FT All Share Indices. The authors use symmetric and asymmetric error statistics in the evaluation and conclude that RW, MA and ES dominate the other models.

Poon and Granger (2002), in a superior review about forecasting volatility in financial markets, provide some useful insights to compare studies about this topic. The authors say that the conclusions of these studies depends strongly on the used error statistics, the sampling schemes (e.g. rolling fixed sample estimation or recursive expanding sample estimation), the period and assets studied. Poon and Granger (2002) also argue that is important to interpret the results at the light of assets' features. For instance, volatility estimative for assets that were confronted with financial shocks or that have volatility mean reversion behaviour is more reliable if came out from ARCH class models than from the "simpler" ones, because these models do not handle those facts. ARCH class models will provide better volatility estimation when there are no changes in volatility level because they assume variance stationarity. If changes in volatility level are observed the simpler models, namely the exponential smoothing models, turn out to be preferred.

3. Data Description and Methodology

3.1. Data Description

The data used in this study consists on daily returns (where the returns are computed by natural logarithm differences) from the Portuguese Stock Index PSI 20 and a sample of fifteen stocks traded in this market (these stocks represents 96% of the this market index). The data are obtained from Datastream and summary statistics about these stocks and PSI 20 are provided in Table 1. Returns used throughout the article are daily returns, ranging from the first transaction day until 18th of May 2004.

For each asset, the period from the first transaction day to the last day of September 1998 is used as the initialization set and the period from the first day of October 1998 to the 18th of May 2004 is used as the test set. The error measures are calculated over this period.

3.2. Volatility Measures

Following the work of Balaban, Bayar and Faff (2002), we study weekly volatility forecast, however, unlike them, we employ three measures of volatility:

(i) the within-week standard deviation,

$$\sigma_w = \sqrt{\frac{\sum_{t=1}^{n}(R_{w,t} - \mu_w)^2}{n - 1}} \qquad (1)$$

where σ_w is the within-week standard deviation; $R_{w,\,t}$ is the continuously compounded return on trading day *t*, in week *w*; μ_w is the week *w* mean return; *n* is the number of trading days in a week;

(ii) the within-week semi-standard deviation (Σ_w),

$$\Sigma_w = \sqrt{\frac{\sum_{t=1}^{n}\{\mathrm{Min.}(R_{w,t} - \mu_w ;0)\}^2}{n - 1}} \qquad (2)$$

(iii) and the within-week standard deviation around zero and not week's mean return[3], as it is defined in equation (1). There is no loss of degree of freedom.

3.3. Volatility Forecasting Models

In this study we are going to apply only time series volatility forecasting models and not those based on traded option prices. Additionally, we exclude the nonparametric volatility forecasting models (Pagan and Schwert (1990) argue that these models have a poor performance) and those based on neural networks (given their computational complexity).

3.3.1. Volatility Forecasting Models based on past Measures of Risk

The simplest historical price model is the Random Walk Model (RW). According to this model, the best forecast of the next period volatility is the actual observed volatility.

$$\hat{\sigma}_{w+1} = \sigma_w \tag{3}$$

Extending this concept we have the Historical Volatility Model (HIS). It can be calculated by taking the average of past observed volatilities.

$$\hat{\sigma}_{w+1} = (\sigma_w + \sigma_{w-1} + ... + \sigma_1) / w \tag{4}$$

HIS is suitable when volatility asset behaviour has no volatility clustering and there is variance stationarity.

Moving Average Model (MA-α) is similar to HIS but makes use of the last α observed volatility values.

$$\hat{\sigma}_{w+1} = (\sigma_w + \sigma_{w-1} + ... + \sigma_{w-\alpha+1}) / \alpha \tag{5}$$

We assign the following values for α = 4 and 12. The larger the order of the moving average (α) the greater the smoothing effect. Like HIS, this model does not work very well when the underlying process is not stationary.

Weighted moving average model (WMA-α) is an extension of MA-α, but it gives a different weight to each of α observations.

[3] Poon and Granger (2002) refer that taking deviations around zero, instead of the sample mean, increases volatility forecast accuracy, but it can also result in noisy volatility estimates.

$$\hat{\sigma}_{w+1} = \sum_{i=w-\alpha+1}^{w} \beta_i * \sigma_i \quad (6)$$

where β_I is the weight of each observation; the sum of these weights must be one and we choose a decline of 15% from the newest to the oldest observation. We assume the following values for α = 4 and 12.

The exponential smoothing model (ES) takes the observed volatility for the previous period and adjusts it using the forecast error.

$$\hat{\sigma}_{w+1} = (1-\theta) * \sigma_w + \theta * \hat{\sigma}_w \quad (7)$$

where θ is the smoothing parameter and is restricted to lie between zero and one. We use an analytical procedure to find the value of this parameter, for each asset, by minimizing RMSE (since it is quite consistent across the different error statistics). This model should be used when it is assumed a flat forecast function.

Exponentially weighted moving average model (EWMA-α) is similar to the previous one, but replaces the past observed volatility by the α – week moving average.

$$\hat{\sigma}_{w+1} = (1-\theta) * \sigma_w(\mathrm{MA}-\alpha) + \theta * \hat{\sigma}_w(\mathrm{EWMA}-\alpha) \quad (8)$$

where α = 4 and 12.

Besides these models but still inside volatility forecasting models based on past measures of risk we have regression models that will not be explored in this study. Simple regression models express volatility as a function of its past values and an error term.

$$\hat{\sigma}_{w+1} = \gamma_{1,w} * \sigma_w + \gamma_{2,w-1} * \sigma_{w-1} + \ldots + \varepsilon \quad (9)$$

Regression models are essentially autoregressive. If past volatility errors are included we get an ARMA model for volatility. Introducing a differencing order we get an ARIMA model for volatility.

3.3.2. ARCH Class Models

Unlike the previous models, ARCH class models do not make use of sample standard deviations.

In developing an ARCH model we need to consider two specifications: one for the conditional mean and one for the conditional variance (h_t).

ARCH models were introduced by Engle (1982). For the next models, returns, r_t, have the following process:

$$r_t = c + \varepsilon_t \tag{10}$$

$$\varepsilon_t = \sqrt{h_t}\, e_t \tag{11}$$

$$e_t \sim N(0,1)$$

where h_t follows one of the following ARCH class models. Each of these models tries to capture some tylised facts about financial market volatility such as fat tail distributions, volatility clustering, asymmetry and mean reversion of volatilities. Poon and Granger (2002) refer that correlation among volatility is stronger than that among returns and increases during bear markets. These authors also say that high frequency volatility measures have a long memory property and the autocorrelations of variances stay positive and significantly different from zero for lags up to a thousand or more.

An ARCH (q) means that h_t is a function of *q* past squared returns,

$$H_t = \omega + \sum_{k=1}^{q} \alpha_k * \varepsilon_{t-k}^2 \tag{12}$$

The forecast errors (ε_t) are assumed to be conditionally normally distributed with a zero mean and h_t variance, based on the information set, Ψ_{t-1}, available at time t-1,

$$\varepsilon_t \mid \Psi_{t-1} \sim N(0, h_t)$$

The parameter ω is equal to δV, where V is the long run volatility and δ is the weight given to V. Weights must sum to unity

$$\delta + \sum_{k=1}^{q} \alpha_k = 1$$

In this study we are going to use ARCH (1).

In GARCH (p, q) (Bollerslev (1986)), today's conditional volatility depends on *p* last conditional volatility and on *q* last squared forecast errors

$$h_t = \omega + \sum_{k=1}^{q} \alpha_k * \varepsilon_{t-k}^2 + \sum_{j=1}^{p} \beta_j * h_{t-j} \tag{13}$$

ARCH model is a special case of a GARCH model, in which there are no lagged forecast variances in conditional variance equation. Consistent with GARCH's conditional variance specification, this period's variance is a weighted average of a long term average (the constant), last period forecasted variance (the GARCH term) and the information about the last period realized volatility. This model is coherent with the often observed volatility clustering in financial returns data.

According to GJR-GARCH (1, 1) (Glosten, Jagannathan and Runkle (1993)) the specification for the conditional variance is

$$h_t = \omega + \alpha * \varepsilon_{t-1}^2 + \gamma * D_{t-1} * \varepsilon_{t-1}^2 + \beta * h_{t-1} \tag{14}$$

$$D_{t-1} \begin{cases} 1 \text{ if } \varepsilon_{t-1} < 0 \\ 0 \text{ if } \varepsilon_{t-1} >= 0 \end{cases}$$

In this model, good news (ε_{t-1}<0) and bad news ($\varepsilon_{t-1}\geq 0$) have differential effects on the conditional variance. If $\gamma \neq 0$ the news have an asymmetric impact (γ represents the leverage effect).

Finally, we are going to use the Eviews' adapted version of EGARCH (1, 1) (Nelson (1991)) model. The specification for the conditional variance is

$$\ln(h_t) = \omega + \beta * \ln(h_{t-1}) + \gamma * \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha * \left\{ \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - \left(\frac{2}{\pi}\right)^{0,5} \right\} \tag{15}$$

Because it is done the logarithm of h_t, the leverage effect is exponential and conditional variance is non-negative. The presence of leverage effects can be tested by the hypothesis that γ <0. Chong, Ahmad and Abdullah (1999) refer that, although GARCH models can manage returns' excess kurtosis, they cannot handle with skewness. One advantage of EGARCH model is that it explicitly takes skewed distributions into account.

Poon and Granger (2002) surveyed several other papers that developed and exposed alternative ARCH class models.

3.4. Forecast Evaluation

The reliability of a forecasting study depends largely on evaluation measures. In this paper, we follow Brailsford and Faff (1996) work. So, we employ symmetric and asymmetric error statistics. Symmetric error statistics are ME, MSE (we do not provide empirical evidence about these two measures given the redundancy with the following ones), RMSE and MAPE and we think that they are self explanatory.

Asymmetric error statistics weighted differently under or over predictions. Mean mixed error statistics (MME) are defined as:

$$\mathrm{MME(U)} = E\left\{ \sum_{w=1}^{O} \left| \hat{\sigma}_w - \sigma_w \right| + \sum_{w=1}^{U} \sqrt{\left| \hat{\sigma}_w - \sigma_w \right|} \right\} \tag{16}$$

$$\mathrm{MME(O)} = E\left\{ \sum_{w=1}^{U} \left| \hat{\sigma}_w - \sigma_w \right| + \sum_{w=1}^{O} \sqrt{\left| \hat{\sigma}_w - \sigma_w \right|} \right\} \tag{17}$$

where O and U is the number of over and under predictions, respectively. Given this, MME(U) penalizes the under predictions and MME(O) penalizes the over predictions.

4. Empirical Results

Table 1 shows some descriptive statistics about the analysed data. Assets' daily mean returns are all close to zero (only EDP and SAG have negative daily mean returns) which should imply that within-week standard deviation around zero is a good approximation to traditional within-week standard deviation.

[Insert Table 1]

The kurtosis and Jarque-Bera statistics indicate for all stocks a clear rejection of the normality of daily returns. Returns distribution of BPI, Semapa and PT do not show skewness. According to Ljung-Box test, except for Ibersol, Semapa and Portucel, assets' returns are autocorrelated at a significance level of, at least, 2.5%. The analysis of autocorrelation coefficients (and their statistical significance) allows us to conclude that more than two thirds of assets' returns have an autoregressive process of order 1 (AR(1)). When this happens, ARCH class models are estimated using a stationary AR(1) conditional mean, meaning that the equation return (10) becomes:

$$r_t = c + r_{t-1} + \varepsilon_t$$

Before starting the analysis of the volatility forecasting models' performance, we are going to discuss ARCH class models' specifications (Appendix 1).

Almost all coefficients, in variance equation, are significant at the 1% level. The exceptions are the leverage effect of GJR-GARCH and EGARCH models. For approximately half of the sample, the news do not have an asymmetric impact and, apparently, it cannot be identified any pattern concerning this fact.

Based on the performance of the goodness-of-fit statistics (Log likelihood, Akaike Info Criterion and Schwarz criterion, not presented), EGARCH (1,1) is the best model for all assets. It is also for this model that the null hypothesis of Ljung-Box test is not rejected for the higher number of stocks (thirteen stocks), meaning that, at a significance level of 1%, standardized residuals are identically distributed (iid), as it was supposed to be. For ARCH (1) model, the null hypothesis of that test is not rejected only for nine stocks. Only for the market index and Brisa standardized residuals of all ARCH class models stay autocorrelated.

ARCH (10) LM test reveals ARCH effect in the residuals of some models for some assets. The rejection of the null hypothesis of this test means that variance equation is not well defined. As it can be seen in Appendix 1, there are two facts worthwhile to be pointed out: first, the null hypothesis of this test is rejected, at 1% significance level, for one stock when modelled by EGARCH (1,1) and GJR-GARCH (1,1) models, which means that they capture appropriately the variance's structure of original series of returns; second, only for three stocks, the null hypothesis of this test is rejected, at 1% significance level, when modelled by ARCH (1), which reinforced the conclusions of goodness-of-fit statistics.

For GARCH (1,1) model, the sum of the α and β parameters, to all assets except EDP and SAG, is close to the unity which indicates that long-memory type ARCH process could be appropriated to model daily returns' volatility. This idea is reinforced by Engle's Lagrange Multiplier test (ARCH (q) LM) to standardized residuals of ARCH (1) model. Except for Sonae Indústria and SAG, high values of β parameter in the GARCH (1,1) signifies that a relevant proportion of past volatility carries on to the forward period.

Consistent with GJR-GARCH (1,1), is interesting to note that, exception done to the market index, the leverage effect does not exist in assets with negative skewness. This fact can lead us to ask the following question to Behavioural Finance: Given bad news, why did investors penalize more assets with positive skewness?

After models' estimation, we perform some tests on standardized residuals (Appendix 1). For all models and assets, standardized residuals do not follow a normal distribution (Jarque-Bera test's null hypothesis is rejected) and have high kurtosis. Curto, Reis and Esperança (2004) get the same conclusions and do the reestimation of ARCH class models with Student's t innovations and not with normal ones. After reestimation, the authors observed an improvement in the goodness of fit statistics.

Forecasting volatility models' mean performance is shown through appendix 2 to 6[4]. We are going to focus our attention in the within-week traditional standard deviation and within-week semi-standard deviation, given that error statistics values of the within-week standard deviation around zero are very close to the ones observed for within-week traditional standard deviation. We compare ARCH class models' performance with that of models based on past risk measures, only for within-week traditional standard deviation.

The relative forecast error exhibited through appendix 2 to 5, is obtained by taking the ratio of the error statistics value of a given model with the one of the worst performing model for that asset. Those appendixes also show the ranking of volatility forecasting models' mean performance, given an error statistic, with 1 being the best forecasting model.

RMSE allow us to identify some interesting features. First, the RW model is the worst volatility forecasting model, for all risk measures. Second, ARCH class models (curiously ARCH (1), the worst forecasting model, from this class, according to goodness-of-fit statistics is the best model) provide the best volatility forecast and volatility forecasts are very close one to the other. Third, from volatility forecasting models based on past risk measures, WMA-12 is the best model, followed by EWMA-12 and MA-12. Fourth, even with ARCH class models, RMSE value of the best forecasting model of with-in weekly semi-standard deviation is lower than the comparable one of with-in weekly standard deviation. Probably, this is one of the most curious results because it means that with-in semi-standard deviation is more forecastable than with-in weekly standard deviation, even when we use complex ARCH class models. Thus, we make the following suggestion to mathematicians: why not combining ARCH class models with semi-standard deviation?

The analysis of MAPE error statistics should be done at the light of its serious limitations when the values are inferior to the unity. Nevertheless, this statistics leads us to remarkable changes in conclusions. The most challenging change is the poor ARCH class models' performance. As a matter of fact they are outperformed by WMA, MA and ES models. Despite this, ARCH class models relative performance stay very compact, which means that these models provide very similar forecasts. It is also important to point out the decrease of forecasting power of the models based in with-in weekly semi-standard deviation.

[4] Forecasting volatility models' performance for each stock is available at request.

Turning our attention to MME, the major conclusion is that ARCH class models make over predictions because they perform very well according to MME(U), but very bad according to MME(O). It is also important to notice that, independently of the risk measure used, volatility forecasting models based on shorter past periods (like WMA-4, MA-4, RW, ES) provide more frequently under-predictions and those based on higher past periods have the inverse behaviour. Once again, forecasting models that use with-in weekly semi-standard deviation make more accurate forecasts. Even for MME(U), where ARCH class models provide a substantial improvement to forecast accuracy (when compared to forecasting models based on past risk measures), forecasting models that use with-in weekly semi-standard deviation make very good forecasts.

5. Conclusions

Volatility is an extremely important input to several pricing models in many fields of finance. Given this, academic community over the last decades has been developing several volatility forecasting models. However, the best volatility model or class of models it is by no means consensual. In this study we make an exposition of several volatility forecasting models and apply them to Portuguese stock market. These models can be grouped into two classes: the volatility forecasting models based on past measures of risk and ARCH class models.

Therefore, the main purpose of this study is to analyse the performance of these models, with some risk measures and different evaluation measures (symmetric and asymmetric error statistics).

The results follow the ones observed by other authors and we can say that the best forecasting model depends on the evaluation measure used. Notwithstanding, the numbers pointed out to smooth superiority of ARCH class models, principally when using RMSE and MME(U), which means that ARCH class models normally over-predicts volatility. It is also worth noting the excellent performance of volatility forecasting models based on semi-standard deviation, even when compared with ARCH class models. This is probably the most original aspect of this study.

6. References

Akgiray, V. (1989). Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts, *Journal of Business*, 62, 55-80..

Aparício, Felipe and **Estrada,** Javier, (2001), Empirical Distributions of Stock Returns: European Securities Markets, 1990-95, *European Journal of Finance*, 7, p. 1-21.

Aydemir, A. B. (1998). Volatility Modelling in Finance, in Knight J. and Satchell ed. *Forecasting Volatility in Financial Markets*, Butterworth, 1-46.

Brailsford, T. J. and **Faff** R. W. (1996). An Evaluation of Volatility Forecasting Techniques, *Journal of Banking and Finance*, 20, 419-438.

Balaban, E., **Bayar** A. and **Faff** R. (2002). Forecasting Stock Market Volatility: Evidence from fourteen countries, *Working Paper*, Center for Financial Markets Research, Univ. of Edinburgh.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31, 307-327.

Chong, C. W., **Ahmad**, M. I. and **Abdullah** M. Y. (1999). Performance of GARCH Models in Forecasting Stock Market Volatility, *Journal of Forecasting*, 18, 333-343.

Curto, D., **Reis**, E. and **Esperança** J. P. (2004). Modelling the Volatility in Portuguese Stock Market: a comparative study with German and US market, *Working Paper*.

Dimson, E. and **Marsh** P. (1990). Volatility Forecasting with-out Data-Snooping, *Journal of Banking and Finance*, 14, 399-421.

Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U. K. inflation, *Econometrica*, 50, 987-1008.

Engle, R. F. (1993). Statistical Models for Financial Volatility, *Financial Analysts Journal*, 49, 1, 72-78.

Fama, E. (1965). The Behavior of Stock Market Prices, *Journal of Business*, 38, 34-105.

Franses, P. H. and **Van Dijk** D. (1996). Forecasting Stock Market Volatility using (non-linear) GARCH Models, *Journal of Forecasting*, 15, 3, 229-235.

Glosten L., **Jagannathan** R. and **Runkle** D. E. (1993). On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks, *Journal of Finance*, 48, 1779-1801.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices, *Journal of Business*, 36, 394-419.

Markowitz, H. (1952). Portfolio Selection, *Journal of Finance*, 8, 1, 77-91.

McMillan, D. G., **Speight** A. H. and **Gwilym** O. (2000). Forecasting UK Stock Market Volatility, *Journal of Applied Economics*, 10, 435-448.

Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: a New Approach, *Econometrica*, 59, 2, 347-370.

Pagan, A. R. and **Schwert** G. W. (1990). Alternative Models for Conditional Stock Volatility, *Journal of Econometrics*, 45, 267-290.

Peiró, Amado, (1999), Skewness in Financial Returns, *Journal of Banking and Finance*, 23, p. 847-862.

Poon, Ser-Huang and **Granger** C. (2002). Forecasting Volatility in Financial Markets: a Review, *Working Paper*.

Taylor, S. J. (1987). Forecasting of the Volatility of Currency Exchange Rates, *International Journal of Forecasting*, 3, 159-170.

Tse, Y. K. (1991). Stock Returns Volatility in the Tokyo Stock Exchange, *Japan and the World Economy*, 3, 285-298.

Tse, Y. K. and **Tung** S. H. (1992). Forecasting Volatility in the Singapore Stock Market, *Asia Pacific Journal of Management*, 9, 1-13.

TABLE:

Table 1: Assets Daily Returns' Descriptive Statistics

	PSI20	BCP	BES	BPI	Brisa	Cimpor	Cofina	EDP	Ibersol	J. M.	SAG	Semapa	Sonae Ind.	Sonae SGPS	Portucel	PT
Mean	0.0003	0.0001	0.0004	0.0003	0.0004	0.0004	0.0001	-0.0002	0.0003	0.0005	-0.0005	0.0005	0.0003	0.0006	0.0001	0.0005
Maximum	0.069	0.104	0.096	0.118	0.116	0.092	0.318	0.088	0.198	0.123	0.135	0.104	0.226	0.266	0.130	0.081
Minimum	-0.096	-0.174	-0.124	-0.130	-0.079	-0.135	-0.144	-0.104	-0.116	-0.177	-0.099	-0.126	-0.159	-0.114	-0.109	-0.101
Std. Dev.	0.010	0.016	0.013	0.017	0.017	0.014	0.026	0.017	0.022	0.020	0.016	0.017	0.022	0.023	0.017	0.020
Skewness	-0.629*	-0.414*	0.297*	-0.018	0.357*	-0.421*	2.359*	0.321*	1.447*	-0.274*	0.222*	0.077	0.791*	0.811*	0.436*	-0.092
Kurtosis	10.396*	13.687*	12.773*	9.194*	7.799*	13.750*	27.681*	6.595*	15.689*	11.946*	12.283*	8.805*	14.355*	12.911*	9.500*	5.506*
No. Observ.	2967	2967	2967	2967	1687	2567	1627	1805	1687	2967	1522	2292	2967	2967	2317	2337
Jarque-Bera	6958*	14203*	11850*	4742*	1655*	12435*	42805*	1003*	11907*	9931*	5477*	3221*	16249*	12469*	4152*	615*
LB *Q*(10)	105.77*	37.45*	71.86*	51.69*	137.23*	50.53*	51.56*	22.11**	16.53	77.41*	20.18**	8.04	36.51*	28.0*	15.60	43.30*
ρ1	0.158*	0.096*	0.127*	0.118*	-0.276*	0.125*	0.039	-0.037	0.071*	0.132*	0.012	-0.024	0.094*	0.049*	0.041	0.113*
ρ2	0.031	-0.021	0.038	0.038	-0.025	0.026	-0.069	-0.030	0.030	0.041	0.001	0.006	0.019	0.038	0.005	-0.048
ρ3	0.023	-0.003	0.028	0.010	-0.012	0.000	0.034	-0.039	0.051	0.055	-0.027	0.016	-0.002	0.002	-0.041	-0.011

* Significant at the 1% level; ** Significant at the 2.5% level

Jarque-Bera is the Jarque-Bera test for normality and follows a χ^2 distribution with two degrees of freedom; Jarque-Bera = T* (Skewness^2/6 + (Kurtosis – 3)^2/24). Standard errors of the coefficients of skewness under the null hypothesis of normality where computed as $(6 / n)^{1/2}$, where n is the number of observations. The significance of kurtosis is tested using K = (n/24)(kurtosis – 3)^2, which is $\chi^2(1)$ distributed under the null hypothesis of a kurtosis of 3. LB *Q*(10): is the Ljung-Box test for returns; ρ_i are the estimates of autocorrelation coefficients for returns.

APPENDIXES:

Appendix 1: ARCH Class Models' Specifications

ARCH (1)	**PSI20**		**BCP**		**BES**		**BPI**		**Brisa**		**CIMPOR**		**Cofina**		**EDP**	
C	0.0004	(0.022)	0.0004	(0.153)	0.0005	(0.007)	0.0006	(0.044)	0.0004	(0.279)	0.0005	(0.043)	-0.0012	(0.011)	-0.0001	(0.722)
r(t-1)	0.2155	(0.000)	0.0780	(0.062)	0.1581	(0.000)	0.0146	(0.337)	-0.2710	(0.000)	0.0399	(0.048)			-	-
Variance Equation																
ω	0.0001	(0.000)	0.0001	(0.000)	0.0001	(0.000)	0.0002	(0.000)	0.0002	(0.000)	0.0001	(0.000)	0.0003	(0.000)	0.0002	(0.000)
α	0.4278	(0.000)	0.4832	(0.000)	0.4894	(0.000)	0.2878	(0.000)	0.1411	(0.000)	0.3637	(0.000)	0.5872	(0.000)	0.1438	(0.000)

Stand. Residuals	**PSI20**	**BCP**	**BES**	**BPI**	**Brisa**	**CIMPOR**	**Cofina**	**EDP**
Mean	-0.016	-0.020	-0.012	-0.020	0.001	-0.004	0.036	-0.008
Std. Dev.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Skewness	-0.238	-0.070	0.604	0.016	0.257	0.000	0.865	0.207
Kurtosis	7.138	10.446	11.798	8.155	8.287	10.229	15.263	6.812
Jarque-Bera	2144.3 (0.000)	6854.1 (0.000)	9747 (0.000)	3284.56 (0.000)	1983 (0.000)	5587.3 (0.000)	10397 (0.000)	1105.6 (0.000)
LB *Q* (10)	36.655 (0.000)	13.75 (0.185)	13.099 (0.218)	26.949 (0.003)	31.755 (0.000)	7.821 (0.646)	29.284 (0.001)	23.728 (0.008)
ARCH (10) LM	265.81 (0.000)	54.51 (0.000)	60.42 (0.000)	59.76 (0.000)	12.94 (0.227)	79.33 (0.000)	78.98 (0.000)	18.95 (0.041)

ARCH (1)	**Ibersol**		**J. M.**		**SAG**		**Semapa**		**Sonae Ind.**		**Sonae SGPS**		**Portucel**		**PT**	
C	-0.0010	(0.017)	0.0006	(0.108)	-0.0006	(0.121)	0.0005	(0.106)	-0.0006	(0.070)	0.0005	(0.207)	0.0000	(0.909)	0.0005	(0.201)
r(t-1)	-0.1798	(0.000)	0.1436	(0.000)	-	-	-	-	0.0080	(0.579)	0.0604	(0.007)	-	-	0.1163	(0.000)
Variance Equation																
ω	0.0003	(0.000)	0.0003	(0.000)	0.0002	(0.000)	0.0002	(0.000)	0.0003	(0.000)	0.0004	(0.000)	0.0002	(0.000)	0.0003	(0.000)
α	0.4355	(0.000)	0.3104	(0.000)	0.2941	(0.000)	0.2188	(0.000)	0.5353	(0.000)	0.1723	(0.000)	0.3284	(0.000)	0.3025	(0.000)

Stand. Residuals	**Ibersol**		**J. M.**		**SAG**		**Semapa**		**Sonae Ind.**		**Sonae SGPS**		**Portucel**		**PT**	
Mean	0.035		-0.005		0.007		-0.012		0.022		0.005		0.007		-0.006	
Std. Dev.	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Skewness	0.650		-0.112		0.437		0.094		0.076		1.026		0.412		-0.067	
Kurtosis	10.386		11.961		14.186		8.453		12.161		15.693		8.738		5.435	
Jarque-Bera	3951	(0.000)	9930	(0.000)	7984	(0.000)	2843	(0.000)	10373	(0.000)	20430	(0.000)	3244	(0.000)	578.95	(0.000)
LB *Q* (10)	36.324	(0.000)	19.995	(0.029)	18.346	(0.049)	6.9532	(0.730)	29.442	(0.001)	14.744	(0.142)	13.956	(0.175)	17.549	(0.063)
ARCH (10) LM	42.34	(0.000)	34.79	(0.000)	20.46	(0.025)	40.14	(0.000)	63.72	(0.000)	38.28	(0.000)	37.37	(0.000)	153.51	(0.000)

GARCH (1,1)	**PSI20**		**BCP**		**BES**		**BPI**		**Brisa**		**CIMPOR**		**Cofina**		**EDP**	
C	0.0004	(0.004)	0.0002	(0.282)	0.0005	(0.001)	0.0005	(0.088)	0.0005	(0.191)	0.0003	(0.080)	-0.0005	(0.233)	-0.0001	(0.774)
r(t-1)	0.1722	(0.000)	0.0845	(0.002)	0.0777	(0.000)	0.0650	(0.002)	-0.2548	(0.000)	0.0266	(0.188)	-	-	-	-
Variance Equation																
ω	0.0000	(0.000)	0.0000	(0.000)	0.0000	(0.000)	0.0000	(0.000)	0.0000	(0.000)	0.0000	(0.000)	0.0000	(0.000)	0.0000	(0.000)
α	0.1342	(0.000)	0.1631	(0.000)	0.0486	(0.000)	0.1604	(0.000)	0.0358	(0.000)	0.0808	(0.000)	0.2350	(0.000)	0.0971	(0.000)
β	0.8671	(0.000)	0.8343	(0.000)	0.9505	(0.000)	0.7202	(0.000)	0.9358	(0.000)	0.9027	(0.000)	0.7213	(0.000)	0.7356	(0.000)

Stand. Residuals	**PSI20**		**BCP**		**BES**		**BPI**		**Brisa**		**CIMPOR**		**Cofina**		**EDP**	
Mean	-0.015		-0.013		-0.012		-0.017		-0.003		0.014		0.011		-0.014	
Std. Dev.	1.000		1.000		1.001		1.000		0.999		1.000		1.001		1.000	
Skewness	-0.100		-0.060		0.642		0.188		0.230		0.319		0.242		0.189	
Kurtosis	6.349		8.818		12.173		7.866		8.842		10.243		8.801		6.375	
Jarque-Bera	1391.3	(0.000)	4184.4	(0.000)	10603	(0.000)	2944.1	(0.000)	2412.4	(0.000)	5653.1	(0.000)	2296.9	(0.000)	867.6	(0.000)
LB *Q* (10)	44.436	(0.000)	12.768	(0.237)	24.358	(0.007)	17.585	(0.062)	32.388	(0.000)	8.0221	(0.627)	20.192	(0.027)	15.273	(0.122)
ARCH (10) LM	10.07	(0.434)	10.30	(0.415)	11.44	(0.324)	3.72	(0.959)	3.76	(0.957)	6.10	(0.807)	4.54	(0.920)	11.66	(0.309)

GARCH (1,1)	**Ibersol**		**J. M.**		**SAG**		**Semapa**		**Sonae Ind.**		**Sonae SGPS**		**Portucel**		**PT**	
C	-0.0001	(0.806)	0.0006	(0.065)	-0.0008	(0.023)	0.0007	(0.023)	-0.0004	(0.185)	0.0008	(0.008)	0.0001	(0.654)	0.0007	(0.021)
r(t-1)	-0.0155	(0.549)	0.0998	(0.000)	-	-	-	-	0.0173	(0.478)	0.0607	(0.006)	-	-	0.1139	(0.000)
Variance Equation																
ω	0.0000	(0.000)	0.0000	(0.003)	0.0001	(0.000)	0.0000	(0.000)	0.0001	(0.000)	0.0000	(0.105)	0.0000	(0.000)	0.0000	(0.000)
α	0.1533	(0.000)	0.2038	(0.000)	0.2780	(0.000)	0.1759	(0.000)	0.4616	(0.000)	0.0510	(0.000)	0.1966	(0.000)	0.0565	(0.000)
β	0.8524	(0.000)	0.6937	(0.000)	0.5591	(0.000)	0.7168	(0.000)	0.4708	(0.000)	0.9474	(0.000)	0.7435	(0.000)	0.9423	(0.000)

Stand. Residuals	**Ibersol**		**J. M.**		**SAG**		**Semapa**		**Sonae Ind.**		**Sonae SGPS**		**Portucel**		**PT**	
Mean	-0.007		-0.006		0.015		-0.015		0.008		-0.016		0.003		-0.009	
Std. Dev.	1.000		1.000		1.001		1.000		1.000		1.005		1.000		1.000	
Skewness	0.327		-0.366		-0.246		0.173		0.069		0.441		0.616		0.198	
Kurtosis	8.322		16.894		12.322		8.805		10.072		8.425		8.508		4.412	
Jarque-Bera	2019.6	(0.000)	23922	(0.000)	5525.8	(0.000)	3229.7	(0.000)	6183	(0.000)	3732.9	(0.000)	3075.5	(0.000)	209.39	(0.000)
LB *Q* (10)	16.754	(0.080)	18.94	(0.041)	10.776	(0.375)	7.3978	(0.687)	27.281	(0.002)	16.631	(0.083)	9.0788	(0.525)	12.245	(0.269)
ARCH (10) LM	35.02	(0.000)	2.85	(0.985)	4.31	(0.932)	4.92	(0.896)	6.04	(0.812)	10.44	(0.403)	9.20	(0.513)	20.23	(0.027)

EGARCH (1,1)	**PSI20**		**BCP**		**BES**		**BPI**		**Brisa**		**CIMPOR**		**Cofina**		**EDP**	
C	0.0003	(0.032)	0.0004	(0.046)	0.0006	(0.000)	0.0005	(0.069)	0.0006	(0.116)	0.0006	(0.003)	-0.0008	(0.022)	-0.0006	(0.133)
r(t-1)	0.1907	(0.000)	0.0981	(0.000)	0.0842	(0.000)	0.0691	(0.000)	-0.2654	(0.000)	0.0320	(0.090)	-	-	-	-
Variance Equation																
ω	-0.4890	(0.000)	-0.5354	(0.000)	-0.2708	(0.000)	-0.9414	(0.000)	-0.7055	(0.000)	-0.3981	(0.000)	-0.8939	(0.000)	-1.4042	(0.000)
α	0.2544	(0.000)	0.2679	(0.000)	0.1560	(0.000)	0.2087	(0.000)	0.1265	(0.000)	0.1869	(0.000)	0.4053	(0.000)	0.1882	(0.000)
γ	-0.0429	(0.000)	-0.0280	(0.353)	-0.0333	(0.000)	-0.0637	(0.000)	-0.0380	(0.001)	0.0236	(0.000)	0.0218	(0.068)	-0.1052	(0.000)
β	0.9681	(0.000)	0.9593	(0.000)	0.9810	(0.000)	0.9045	(0.000)	0.9253	(0.000)	0.9692	(0.000)	0.9206	(0.000)	0.8461	(0.000)

Stand. Residuals	**PSI20**		**BCP**		**BES**		**BPI**		**Brisa**		**CIMPOR**		**Cofina**		**EDP**	
Mean	0.001		-0.026		-0.012		-0.018		-0.013		-0.010		0.030		0.012	
Std. Dev.	1.000		1.000		1.001		1.000		1.001		1.000		1.001		1.000	
Skewness	0.024		0.056		0.787		0.128		0.188		0.164		0.206		0.214	
Kurtosis	6.047		8.871		13.259		7.665		9.073		10.762		8.705		6.696	
Jarque-Bera	1147.6	(0.000)	4262	(0.000)	13312	(0.000)	2697.3	(0.000)	2600.7	(0.000)	6453	(0.000)	2217.8	(0.000)	1041.1	(0.000)
LB *Q* (10)	40.630	(0.000)	10.969	(0.360)	23.528	(0.009)	17.4	(0.066)	25.771	(0.004)	7.2776	(0.699)	19.445	(0.035)	15.968	(0.101)
ARCH (10) LM	12.66	(0.243)	11.62	(0.311)	12.73	(0.239)	6.93	(0.732)	5.79	(0.833)	10.07	(0.434)	6.99	(0.726)	14.24	(0.162)

EGARCH (1,1)	**Ibersol**		**J. M.**		**SAG**		**Semapa**		**Sonae Ind.**		**Sonae SGPS**		**Portucel**		**PT**	
C	-0.0003	(0.405)	0.0005	(0.365)	-0.0011	(0.002)	0.0006	(0.026)	-0.0006	(0.010)	0.0013	(0.000)	-0.0001	(0.753)	0.0007	(0.026)
r(t-1)	-0.0580	(0.024)	0.1032	(0.000)	-	-	-	-	0.0632	(0.007)	0.0815	(0.001)	-	-	0.1290	(0.000)
Variance Equation																
ω	-1.0995	(0.000)	-1.4603	(0.000)	-1.8244	(0.000)	-1.2509	(0.000)	-2.3677	(0.000)	-0.1252	(0.012)	-1.3387	(0.000)	-0.1945	(0.000)
α	0.4769	(0.000)	0.3364	(0.000)	0.3880	(0.000)	0.3087	(0.000)	0.6108	(0.000)	0.0989	(0.000)	0.3740	(0.000)	0.1321	(0.000)
γ	0.0053	(0.676)	-0.0181	(0.641)	0.0236	(0.083)	-0.0389	(0.000)	-0.0505	(0.000)	-0.0074	(0.623)	-0.0198	(0.062)	-0.0121	(0.055)
β	0.9013	(0.000)	0.8450	(0.000)	0.8138	(0.000)	0.8746	(0.000)	0.7518	(0.000)	0.9924	(0.000)	0.8689	(0.000)	0.9879	(0.000)

Stand. Residuals	**Ibersol**		**J. M.**		**SAG**		**Semapa**		**Sonae Ind.**		**Sonae SGPS**		**Portucel**		**PT**	
Mean	-0.006		-0.002		0.029		-0.012		0.025		-0.033		0.016		-0.008	
Std. Dev.	1.000		1.000		1.000		1.000		1.000		1.008		1.000		0.999	
Skewness	0.189		-0.126		-0.398		0.204		0.082		0.605		0.623		0.229	
Kurtosis	7.640		16.664		12.933		8.538		10.525		10.149		8.255		4.657	
Jarque-Bera	1522.4	(0.000)	23082	(0.000)	6296.9	(0.000)	2945	(0.000)	7001.1	(0.000)	6497.4	(0.000)	2816	(0.000)	287.73	(0.000)
LB *Q* (10)	20.109	(0.028)	18.198	(0.052)	10.97	(0.360)	6.5319	(0.769)	18.81	(0.043)	16.198	(0.094)	9.8827	(0.451)	12.448	(0.256)
ARCH (10) LM	25.08	(0.005)	2.25	(0.994)	4.04	(0.946)	6.78	(0.746)	3.19	(0.977)	22.20	(0.014)	8.53	(0.577)	22.78	(0.012)

GJR-GARCH (1,1)	**PSI20**	**BCP**	**BES**	**BPI**	**Brisa**	**CIMPOR**	**Cofina**	**EDP**
C	0.0003 (0.044)	0.0001 (0.610)	0.0004 (0.012)	0.0002 (0.556)	0.0003 (0.372)	0.0004 (0.071)	-0.0004 (0.326)	-0.0004 (0.262)
r(t-1)	0.1774 (0.000)	0.0855 (0.001)	0.0814 (0.000)	0.0716 (0.001)	-0.2568 (0.000)	0.0259 (0.201)	- -	- -
Variance Equation								
ω	0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.000)
α	0.1039 (0.000)	0.1391 (0.000)	0.0376 (0.000)	0.0848 (0.000)	0.0306 (0.001)	0.0858 (0.000)	0.2438 (0.000)	0.0351 (0.002)
γ	0.0509 (0.000)	0.0428 (0.426)	0.0380 (0.000)	0.1096 (0.000)	0.0429 (0.002)	-0.0119 (0.189)	-0.0167 (0.481)	0.1356 (0.000)
β	0.8693 (0.000)	0.8365 (0.000)	0.9423 (0.000)	0.7644 (0.000)	0.8834 (0.000)	0.9040 (0.000)	0.7196 (0.000)	0.7344 (0.000)

Stand. Residuals	**PSI20**	**BCP**	**BES**	**BPI**	**Brisa**	**CIMPOR**	**Cofina**	**EDP**
Mean	-0.001	-0.003	0.000	0.005	0.004	0.010	0.008	0.004
Std. Dev.	1.000	1.000	1.001	1.000	1.000	1.000	1.001	1.000
Skewness	-0.019	0.005	0.792	0.228	0.220	0.317	0.229	0.211
Kurtosis	6.293	8.993	12.535	7.438	9.130	10.230	8.722	6.485
Jarque-Bera	1340.6 (0.000)	4438.1 (0.000)	11544 (0.000)	2459.9 (0.000)	2653.5 (0.000)	5631.5 (0.000)	2234 (0.000)	927.01 (0.000)
LB *Q* (10)	44.399 (0.000)	13.17 (0.214)	22.337 (0.013)	16.943 (0.076)	26.929 (0.003)	7.9975 (0.629)	19.835 (0.031)	15.848 (0.104)
ARCH (10) LM	8.08 (0.621)	10.12 (0.430)	12.58 (0.248)	4.52 (0.921)	3.17 (0.977)	6.08 (0.809)	4.44 (0.925)	13.95 (0.175)

GJR-GARCH (1,1)	**Ibersol**	**J. M.**	**SAG**	**Semapa**	**Sonae Ind.**	**Sonae SGPS**	**Portucel**	**PT**
C	-0.0002 (0.623)	0.0004 (0.174)	-0.0008 (0.051)	0.0005 (0.119)	-0.0006 (0.062)	0.0006 (0.058)	0.0001 (0.871)	0.0006 (0.049)
r(t-1)	-0.0147 (0.565)	0.0979 (0.000)	- -	- -	0.0220 (0.366)	0.0605 (0.005)	- -	0.1144 (0.000)
Variance Equation								
ω	0.0000 (0.000)	0.0000 (0.000)	0.0001 (0.000)	0.0000 (0.000)	0.0001 (0.000)	0.0000 (0.105)	0.0000 (0.000)	0.0000 (0.000)
α	0.1406 (0.000)	0.1721 (0.000)	0.3024 (0.000)	0.1411 (0.000)	0.3882 (0.000)	0.0375 (0.004)	0.1797 (0.000)	0.0505 (0.000)
γ	0.0278 (0.050)	0.0594 (0.000)	-0.0557 (0.078)	0.0755 (0.000)	0.1433 (0.000)	0.0286 (0.084)	0.0316 (0.081)	0.0190 (0.019)
β	0.8546 (0.000)	0.6983 (0.000)	0.5666 (0.000)	0.7130 (0.000)	0.4806 (0.000)	0.9469 (0.000)	0.7444 (0.000)	0.9380 (0.000)

Stand. Residuals	Ibersol		J. M.		SAG		Semapa		Sonae Ind.		Sonae SGPS		Portucel		PT	
Mean	0.000		0.002		0.008		-0.003		0.021		-0.002		0.009		-0.003	
Std. Dev.	1.000		1.000		1.000		1.000		1.000		1.005		1.000		1.000	
Skewness	0.390		-0.315		-0.304		0.217		0.148		0.485		0.644		0.217	
Kurtosis	8.401		16.740		12.252		8.497		10.046		8.447		8.610		4.473	
Jarque-Bera	2092	(0.000)	23379	(0.000)	5451.4	(0.000)	2903.7	(0.000)	6146.1	(0.000)	3783.6	(0.000)	3198.6	(0.000)	229.56	(0.000)
LB *Q* (10)	16.941	(0.076)	19.904	(0.030)	10.56	(0.393)	7.1341	(0.713)	28.154	(0.002)	17.313	(0.068)	9.2701	(0.507)	12.226	(0.270)
ARCH (10) LM	35.34	(0.000)	2.84	(0.985)	4.45	(0.925)	5.11	(0.884)	5.78	(0.833)	10.37	(0.409)	8.88	(0.543)	18.72	(0.044)

Appendix 2: Volatility Forecasting Models' Mean Performance (RMSE)

	RMSE Mean (σ)			RMSE Mean (μ^2)			RMSE Mean (Σ)		
	Value	Relative	Rank	Value	Relative	Rank	Value	Relative	Rank
RW	0.01165	1.000	13	0.01169	1.000	9	0.00822	1.000	9
HIS	0.01016	0.872	12	0.01025	0.876	8	0.00711	0.864	8
MA-4	0.01006	0.864	11	0.01007	0.861	7	0.00708	0.861	7
MA-12	0.00965	0.828	7	0.00972	0.831	3	0.00679	0.826	3
WMA-4	0.00999	0.857	9	0.00998	0.854	6	0.00703	0.854	6
WMA-12	0.00952	0.817	5	0.00955	0.816	1	0.00670	0.814	1
ES	0.00999	0.857	9	0.00982	0.840	5	0.00690	0.839	5
EWMA-4	0.00967	0.830	8	0.00973	0.832	4	0.00684	0.832	4
EWMA-12	0.00961	0.825	6	0.00969	0.828	2	0.00678	0.824	2
ARCH(1)	0.00837	0.718	1	-	-	-	-	-	-
GARCH(1,1)	0.00879	0.755	3	-	-	-	-	-	-
EGARCH(1,1)	0.00864	0.741	2	-	-	-	-	-	-
GJR-GARCH(1,1)	0.00880	0.755	4	-	-	-	-	-	-

Appendix 3: Volatility Forecasting Models' Mean Performance (MAPE)

	MAPE Mean (σ)			MAPE Mean (μ^2)			MAPE Mean (Σ)		
	Value	Relative	Rank	Value	Relative	Rank	Value	Relative	Rank
RW	0.75016	0.830	8	0.68906	0.816	7	0.77607	0.864	8
HIS	0.86354	0.956	12	0.84425	1.000	9	0.89826	1.000	9
MA-4	0.69277	0.767	4	0.65349	0.774	3	0.71622	0.797	3
MA-12	0.71219	0.788	5	0.67770	0.803	6	0.73906	0.823	6
WMA-4	0.68645	0.760	1	0.64522	0.764	1	0.70958	0.790	1
WMA-12	0.69199	0.766	3	0.65409	0.775	4	0.71689	0.798	4
ES	0.69124	0.765	2	0.64760	0.767	2	0.71351	0.794	2
EWMA-4	0.71377	0.790	6	0.67192	0.796	5	0.73505	0.818	5
EWMA-12	0.72666	0.804	7	0.69094	0.818	8	0.75154	0.837	7
ARCH(1)	0.90332	1.000	13	-	-	-	-	-	-
GARCH(1,1)	0.79096	0.876	10	-	-	-	-	-	-
EGARCH(1,1)	0.79677	0.882	11	-	-	-	-	-	-
GJR-GARCH(1,1)	0.79028	0.875	9	-	-	-	-	-	-

Appendix 4: Volatility Forecasting Models' Mean Performance (MME(U))

	MME(U) Mean (σ)			MME(U) Mean (μ^2)			MME(U) Mean (Σ)		
	Value	Relative	Rank	Value	Relative	Rank	Value	Relative	Rank
RW	0.04337	1.000	13	0.04263	1.000	9	0.03586	1.000	9
HIS	0.03813	0.879	12	0.03783	0.887	8	0.03144	0.877	8
MA-4	0.03794	0.875	11	0.03748	0.879	7	0.03122	0.871	7
MA-12	0.03610	0.832	8	0.03580	0.840	5	0.02979	0.831	4
WMA-4	0.03777	0.871	10	0.03724	0.874	6	0.03110	0.867	6
WMA-12	0.03590	0.828	7	0.03542	0.831	3	0.02960	0.825	3
ES	0.03675	0.847	9	0.03567	0.837	4	0.02999	0.836	5
EWMA-4	0.03557	0.820	6	0.03539	0.830	2	0.02942	0.821	2
EWMA-12	0.03544	0.817	5	0.03512	0.824	1	0.02933	0.818	1
ARCH(1)	0.02470	0.570	1	-	-	-	-	-	-

GARCH(1,1)	0.02536	0.585	4	-	-	-	-	-	-
EGARCH(1,1)	0.02501	0.577	2	-	-	-	-	-	-
GJR-GARCH(1,1)	0.02529	0.583	3	-	-	-	-	-	-

Appendix 5: Volatility Forecasting Models' Mean Performance (MME(O))

	MME(O) Mean (σ)			MME(O) Mean (μ^2)			MME(O) Mean (Σ)		
	Value	Relative	Rank	Value	Relative	Rank	Value	Relative	Rank
RW	0.04474	0.766	2	0.04412	0.877	1	0.03680	0.905	2
HIS	0.04939	0.845	9	0.05032	1.000	9	0.04068	1.000	9
MA-4	0.04479	0.767	3	0.04446	0.884	3	0.03694	0.908	3
MA-12	0.04666	0.799	6	0.04646	0.923	6	0.03871	0.951	6
WMA-4	0.04458	0.763	1	0.04415	0.877	2	0.03670	0.902	1
WMA-12	0.04593	0.786	5	0.04567	0.908	5	0.03806	0.935	5
ES	0.04583	0.784	4	0.04531	0.900	4	0.03774	0.928	4
EWMA-4	0.04719	0.808	7	0.04664	0.927	7	0.03890	0.956	7
EWMA-12	0.04755	0.814	8	0.04733	0.941	8	0.03932	0.967	8
ARCH(1)	0.05843	1.000	13	-	-	-	-	-	-
GARCH(1,1)	0.05549	0.950	10	-	-	-	-	-	-
EGARCH(1,1)	0.05663	0.969	12	-	-	-	-	-	-
GJR-GARCH(1,1)	0.05563	0.952	11	-	-	-	-	-	-

Appendix 6: Summary of Volatility Forecasting Models' Performance

	Risk Measure - σ				Risk Measure - μ^2				Risk Measure - Σ			
	RMSE	MAPE	MME(U)	MME(O)	RMSE	MAPE	MME(U)	MME(O)	RMSE	MAPE	MME(U)	MME(O)
RW	13	8	13	2	9	7	9	1	9	8	9	2
HIS	12	12	12	9	8	9	8	9	8	9	8	9
MA-4	11	4	11	3	7	3	7	3	7	3	7	3
MA-12	7	5	8	6	3	6	5	6	3	6	4	6
WMA-4	9	1	10	1	6	1	6	2	6	1	6	1
WMA-12	5	3	7	5	1	4	3	5	1	4	3	5
ES	9	2	9	4	5	2	4	4	5	2	5	4
EWMA-4	8	6	6	7	4	5	2	7	4	5	2	7
EWMA-12	6	7	5	8	2	8	1	8	2	7	1	8
ARCH(1)	1	13	1	13	-	-	-	-	-	-	-	-
GARCH(1,1)	3	10	4	10	-	-	-	-	-	-	-	-
EGARCH(1,1)	2	11	2	12	-	-	-	-	-	-	-	-
GJR-GARCH(1,1)	4	9	3	11	-	-	-	-	-	-	-	-