Testing for Nonlinearity & Modeling Volatility in Emerging Capital Markets: The Case of Tunisia

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Abstract

Capital market efficiency of emerging markets has been investigated widely in recent years, but to-date the empirical results remain inconclusive because most empirical studies fail to consider the institutional features of emerging markets and employ efficiency tests which have been developed for highly liquid markets of developed countries. Furthermore, these studies use empirical tests that are designed to detect linear structure in financial time series. However, recent developments in econometrics of financial markets show evidence of nonlinear relationships in asset returns in developed markets. Given the defining characteristics of emerging capital markets, nonlinearity is most likely to be even more evident in these developing markets compared to developed ones. Using BDS test, the present paper rejects the random walk hypothesis (RWH) for the Tunisian Stock Market (TSE). Despite the multitude of economic and financial reforms, the rejection of the RWH seems to be the result of substantial non-linear dependence and not to non-stationarity in the returns series, which in turn implies a GARCH modeling. Results from Hsieh test show that the source of nonlinearity structure is multiplicative, not additive. Further investigations suggest the use of a FIEGARCH model to cope with the evidence of high volatility persistence and long memory in the conditional variance. Our empirical results also show that despite a high leverage in the TSE index the leverage parameter is insignificantly different from zero. Finally, we argue that the common assumptions of constant variance and Gaussian returns underlying the theory and practice of option pricing, portfolio optimization and value-at-risk (VaR) calculations are simply invalid for emerging markets.

JEL classification: G14; G15

Keywords: Random Walk, BDS test, Nonlinear Dynamics, Conditional Volatility

1. Introduction

Adhering to the original work of the French mathematician Bachelier (1900), and the seminal papers of Samuelson (1965) and Fama (1970), efficient stock market prices should obey a random walk model and always fully reflect all available and relevant information. Successive share price changes are therefore independent and identically distributed (henceforth *i.i.d*). As a result, future share prices are unpredictable and fluctuate only in response to the random flow of news. Since these seminal works, an extensive literature has appeared to test the efficiency of developed and emerging financial markets. Until quite recently, the vast majority of these studies has supported the efficient market hypothesis (henceforth EMH).[1]

Employing traditional statistical tests such as autocorrelation tests, most empirical tests of the EMH have looked into the linear predictability of future share price changes. If the later turn out to be uncorrelated then the EMH is accepted and the stock market in question is deemed informationally efficient, and if they are found to be serially correlated, the EMH is rejected and the market is considered inefficient. However, recent studies point out the existence of spurious autocorrelation in returns data caused by some institutional factors (see, for instance, McInish and Wood, 1991). Furthermore, applications of nonlinear dynamics and chaos theory to economic and financial series find evidence of non-linearity structure (see, for example, Hsieh, 1991). Therefore, failing to test for nonlinearity could lead to incorrect acceptance or rejection of the EMH. Indeed, share price movements can appear unpredictable when using linear models, but they are forecastable under non-linear models, at least over short time spans. Thus, testing for weak form efficiency using only linear procedures may not be appropriate.

Since absence of linear dependence does not necessarily mean independence, but merely a lack of linear autocorrelation (Granger and Anderson, 1978; and Sakai and Tokumaru, 1980), studies of the random walk hypothesis or the *i.i.d* hypothesis should use tests capable of detecting both linear and nonlinear dependencies. The implications of rejecting the *i.i.d* hypothesis go beyond the issue of market efficiency. Evidence of non-linearity is continuingly reshaping our traditional views of modeling asset prices, portfolio and risk management, as well as forecasting techniques. For instance, Bera et al (1993) question the ability of the Ordinary Least Square Model in estimating the optimal hedge ratio using futures contracts and find that, compared to ARCH hedge ratios, the conventional model leads to too many or too few short-sellings of future contracts.

Despite the increasing weight of emerging capital markets in the world market and their importance in international portfolio diversification, studies dealing with non-linear dynamics have focused mainly on mature markets, such as those of US, UK, Japan, and Germany.[2] Only a few studies have investigated nonlinearity dynamics in emerging markets and have modeled return-generating processes accordingly. Swell et al. (1993) find evidence of nonlinear dynamics in weekly indices of four emerging Asian markets (Hong Kong, Korea, Singapore, and Taiwan), Japan and US stock markets. Yadav et al (1996) also report non-linearity in daily stock index returns of markets in Hong Kong, Singapore and Japan. Note however that Hong Kong and Singapore are now considered developed rather emergent markets. A second empirical study by Poshakwale and Wood

[1] Fama (1970, 1991, and 2001) provides a thorough survey of this literature.
[2] See, among others, Brock et al (1991), Hseih (1991), Willey (1992), Abhyankar et al (1995) and (1997), Opong et al (1999), Kosfeld and Robé (2001), and Serletis and Shintani (2003).

(1998) reports evidence of non-linearity in the Warsaw Stock Exchange. Poshakwale (2002) also rejects the random walk hypothesis for the Indian stock market and finds evidence of non-linear behavior in daily returns of an equally weighted portfolio of 100 stocks and a sample of the most actively traded stocks at the Bombay Stock Exchange. This paper aims to contribute to the literature of emerging markets by examining, for the first time, the Tunisian stock market.

Since 1986, the Tunisian financial sector has undergone several reforms aimed at increasing the degree of financial liberalization and integration. As in most emerging markets, the financial liberalization has been implemented largely through on-going structural adjustment programs. Consequently the reforms have led to the deregulation of the financial sector, strengthening of the banking system, enhancing of financial innovation and development of securities markets, in particular the Tunisian Stock Exchange. Despite the global economic and financial turbulence over the last decade, the Tunisian economy has successfully maintained a high average growth rate. In 1999, the World Economic Forum ranked Tunisia first for competitiveness on the African continent, and second in 2004 and 34th worldwide.

Due to its crucial role in economic development and in attracting foreign capital flows, the TSE was subject to dramatic changes. The focus has been on enhancing means of trading, clearing and settlement, and reliability of the information disclosure mechanisms. For example, in 1994, the TSE was privatized and its management passed to the Association of Brokerage Houses. In addition, a regulatory entity named Conseil de Marché Financier (CFM) was created, which is equivalent to the US Security Exchange Commission (SEC). Furthermore, taxes on dividend incomes and capital gains were eliminated and foreign ownership of stocks has been allowed up to 49%. In 1996, the TSE adopted a new trading system based on the French SUPERCAC electronic trading system, permitting a high degree of price transparency and real-time price quotations on Reuters. Although, the TSE has shown significant growth since its establishment in 1969, it is small relative to the domestic economy, with a market capitalization of only 12.5% of the GDP in 2003.[3] Still, characterized by low country-risk, negative correlation with major stock markets, high risk-adjusted returns, and no taxes on both dividend incomes and capital gains, the TSE offers significant diversification potential for global investment.[4]

The movement of capital flows from developed markets to developing ones was mainly motivated by the traditional views of low correlation between these two types of markets. However, stock market liberalization has helped the integration of emerging stock markets into world capital markets, which casts doubt on the effectiveness of investing in these markets for risk-minimization reason. Thus, international investors need a deeper understanding of the behaviour of emerging capital markets.

The present study has important implications for academicians, investors and policy makers. First to the best of our knowledge, this paper is the first to examine the Tunisian stock market. In particular, it investigates the presence of non-linear dependencies using a powerful tests capable of detecting both linearity and non-linearity structure in data series. If the *i.i.d* hypothesis is rejected, we examine the nature of non-linearity to see

[3] In 1992, the market capitalization was 5.78% (Source: TSE annual report).

[4] The Emerging Market Factbook provides comprehensive information on the correlation of the TSE with major stock markets such as US, UK and Japan.

whether it is additive or multiplicative, because knowing the source of non-linearity is of great importance when modeling the returns generating process. Finally, given that banks dominate the market capitalization in the TSE, this study examines the leverage effect in the Tunisian stock market.

To summarize, we reject the random walk hypothesis for the Tunisian stock market. The rejection of the *i.i.d* hypothesis is due to the substantial non-linear dependence and not to the non-stationarity in the returns series. Results from Hsieh test suggest that the source of nonlinearity structure is multiplicative, not additive, which in turn implies a GARCH modeling. Further investigations suggest the use of a FIEGARCH model to cope with evidence of high volatility persistence and long memory in the conditional variance. Finally, despite a high leverage in TSE index we find that the leverage parameter is insignificantly different from zero. This is consistent with the growing evidence that shows little or no direct connection between leverage effect and index (or firm) leverage. The remainder of the paper is organized as follows. Section 2 outlines and explains our research methodology. Section 3 describes our data set. Section 4 discusses the empirical findings. Finally, section 5 summarizes the main conclusions.

2. Theory and Research Methodology

2.1. Random Walk Hypothesis

Fama (1970) argues that efficient stock market prices fully reflect all available and relevant information, meaning an absence of excess-profit opportunities. Share price changes are therefore independent and fluctuate only in response to the random flow of news. Trading strategies based on past and current information are useless in generating excess-profit opportunities.[5] This implies a random walk market, where a random walk model best describes stock prices. Following Campbell, Lo and MacKinlay (1997), there are three different versions of the random walk model: Random Walk I, Random Walk II, and Random Walk III. The Random Walk I or *strict white noise process* requires sequences of price changes to be independent and identically distributed. If we assume sequences of price changes to be independent and drop the identically distributed assumption, we get the version of Random Walk II. Finally, the Random Walk III or *white noise process* is obtained by relaxing the independent and the identically distributed assumption.[6]

Harvey (1993) argues that non-linear models may have the white noise property although they are dependent and identically distributed. Given the growing theoretical and empirical studies showing that share price changes are inherently non-linear, evidence of uncorrelated share price changes is not a sufficient condition for a market to be efficient. Therefore, the present paper examines the assumption of *i.i.d* share price changes, which is the most restrictive version of the random walk hypothesis and most appropriate to test the efficient market hypothesis.

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5 Samuelson (1965) also shows that share prices, in an efficient stock market, fluctuate randomly and only in response to the arrival of new information.

6 A white noise process is a sequence of uncorrelated random variables with constant mean and variance: for any $s \neq 0$, $E(\epsilon_t \epsilon_{t-s}) = 0$, and for $s = 0$, $E(\epsilon_t) = 0$, and $E(\epsilon_t \epsilon_s) = \sigma^2_t$

Let P_t be the level of the TSE index at time t, and define $P_t \equiv Ln(P_t)$ as a stochastic process given by the recursive relation:[7]

$$p_t = \mu + p_{t-1} + \omega_t \tag{1}$$

The continuously compounded return for the period t-1 to t is expressed as

$$r_t \equiv \Delta p_t = \mu + \omega_t \tag{2}$$

where μ is the expected price change or drift and ω_t are represents the residuals.

Equation 1 describes the random walk model with a drift. Under the random walk hypothesis, the drift should be insignificantly different from zero, the distribution of returns should be stationary over time $(r_t \sim I(0))$, and the residuals should be *i.i.d* random variables or, in other words, a strict white noise.

First, we examine the stationarity assumption using two powerful unit root tests- the Augmented Dickey Fuller test (Said and Dickey, 1984) and the Philips-Perron test (Philips and Perron, 1988). We apply these tests to price levels as well as to price changes and expect the index prices to be *I(1)* and, therefore, the returns series to be *I(0)*. Then, we estimate Equation 2 with ordinary least squares and test the statistical significance of the drift μ. As mentioned earlier, in an informationally efficient stock market, price changes are serially uncorrelated. To test for serial independence Box and Pierce (1970) suggested the Q-statistic:

$$Q(k) = T\sum_{i=1}^{k} \hat{\rho}_j^{2} \tag{3}$$

Under the null hypothesis of a white noise series, $Q(k)$ is asymptotically distributed $\chi^2(k)$, where T is the sample size and ρ_i is the autocorrelation coefficient for $i > 0$. However, since, in a finite sample, Q-statistic is not well approximated by the $\chi^2(k)$, we will apply the modified Q-statistic of Ljung and Box (1978) to the residuals of Equation 2:

$$MQ(k) = T(T+2)\sum_{i=1}^{k} (T-K)^{-1} \hat{\rho}_i^{2} \tag{4}$$

We should note that testing for linear serial independence of price changes is neither a necessary nor a sufficient condition to accept or reject the random walk hypothesis. If the index returns turn out to be serially correlated, this should not necessarily imply that the Tunisian stock market is inefficient. Spurious autocorrelation may exist due to institutional factors such as non-synchronous trading which may induce price-adjustment

[7] We use the natural logarithm of prices in order to make the process generating the times series to be independent of the actual price levels. Furthermore, p_t has favorable econometric properties in comparison to P_t (see Campbell, Lo and MacKinlay, 1997).

delays into the trading process.[8] Lo and Mackinlay (1990) argue that individual stock prices trading at different frequencies can lead to a spurious positive autocorrelation in market-index returns. As a non-synchronous trading autocorrelation effect is relatively short timed, we should expect autocorrelation to be persistent in daily index returns series. Furthermore, since significant autocorrelations are observed in highly liquid markets characterized by reliable information and sophisticated investors (see Fama, 1965; Amihud and Mendelson, 1987; and McInish and Wood, 1991), we should expect it to be even more evident in thin emerging capital markets, such as that of Tunisia. If price changes turn out to be statistically uncorrelated, it would not necessarily imply efficiency. Market-index returns can be linearly uncorrelated but at the same time non-linearly dependent.

2.2. Non-linearity in Emerging Stock Market Returns

The study of non-linear dynamics and chaos theory has successfully helped describe important phenomena in physics, ecology, biology, meteorology, and chemistry. Given the ability of low-dimension deterministic non-linear processes to mimic random walk behaviour and allow for significant and unpredictable fluctuations such as those seen in stock market crises (e.g. "Black Monday" in October 1987), several authors have been tempted to apply non-linear analysis to economics and financial data (see, Brock and Sayers 1988; Scheinkman and Lebaron, 1989; Peters, 1991; Tata, 1991; Savit, 1988, 1989; Hsieh, 1989 and 1993; Sterlis and Dormaar, 1996; Serletis and Gogas 1999, 2000; and Serletis and Shahmoradi, 2004). In particular, some studies focus on whether these processes can describe the dynamic of stock-market returns. Although results show mixed evidence for chaos behaviour in stock markets, several empirical studies were able to detect non-linearity dynamics.[9]

As Campbell, Lo, and Mackinlay (1997, pp. 467) explain, "*... many aspects of economic behaviour may not be linear. Experimental evidence and casual introspections suggest that investors' attitudes towardss risk and expected return are non-linear. The terms of many financial contracts such as options and other derivative securities are non-linear. In addition, the strategic interactions among market participants, the process by which information incorporates into security prices, and the dynamics of economy-wide fluctuations are all inherently non-linear. Therefore, a natural frontier for financial econometrics is the model of non-linear phenomena*". In fact, several reasons may explain the non-linear behaviour of financial markets. First, market imperfections and some features of market microstructure may lead to delays of response to new information, implying non-linearity in share price changes.[10] For instance, transaction costs may make investors unwilling to respond rapidly to the arrival of new information. In turn, they would rather wait until their expected excess profits (net of transaction cost) are high enough to allow for positive returns. This delay in adjustment may lead to non-

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[8] Other studies such as of Hasbrouck and Ho (1987) explain the existence of autocorrelation by the lagged adjustment of limit-orders price. See also Campbell, Grossman and Wang (1993) for a modeling of autocorrelations in index and stock returns.

[9] For instance, while Vaidunathan and Krecjbiel (1992) and Mayfield and Mizrach (1992) find evidence of chaos behaviour in the S&P 500 index, Abhyankar *et a.l* (1997), and Serletis and Shintani (2003) reject the null hypothesis of low-dimensional chaos and report evidence of nonlinear dependency in S&P 500 and Dow Jones Industrial Average.

[10] Schatzberg and Reiber (1992) suggest that share prices do not always adjust instantaneously to new information.

linearity in share price changes. Considering the institutional features and trading conditions of emerging stock markets, the likelihood of non-linearity in return generation process is even higher than in the mature stock markets. The efficient market hypothesis assumes market participants to behave rationally, in the sense that traders are risk averse, make unbiased forecasts, and respond instantaneously to new information, which in turn, implies linearity in the data generating process. However, investors in emerging stock markets are relatively uninformed and irrational, which may cause non-linear dependencies.[11] Shleifer and Summers (1990) argue that there are two types of investors in the market: rational arbitrageurs or speculators who trade on the basis of reliable information, and noise traders who trade on the basis of imperfect information.[12] Given that a significant number of traders in emerging markets may trade on the basis of imperfect information, share prices are likely to deviate from their equilibrium values. In addition, given the informational asymmetries and lack of reliable information, noise traders in emerging markets may also lean towards delaying their responses to new information in order to assess informed traders reaction, and then respond accordingly.

The theory and empirical evidence of non-linearity in share price changes suggest that the *i.i.d* assumption is a necessity for an appropriate examination of efficiency market hypothesis. Hence, statistical techniques capable of detecting linearity as well as non-linearity in share price changes series need to be used.

2.3. Testing for Non-linearity: The BDS test

To test whether the share price changes are *i.i.d* we use a powerful test originally proposed by Brock, Dechert and Scheinkman (1987) (henceforth BDS) and designed by Brock *et al* (1996). The BDS test is a non-parametric test with the null hypothesis that the series in question are *i.i.d* against an unspecified alternative. The test is based on the concept of correlation integral, a measure of spatial correlation in n-dimensional space originally developed by Grassberger and Procacccia (1983). To be more specific, consider a vector of m histories of the TSE index return,

$$r_t^m = (r_t, r_{t+1}, \ldots, r_{t+m-1}) \tag{5}$$

the correlation integral measures the number of m vectors within a distance of ε of one another. The correlation integral is defined as

$$C_m(\varepsilon, T) = \frac{2}{T_m(T_m - 1)} \sum_{t<s} I_\varepsilon\left(r_t^m, r_s^m\right) \tag{6}$$

where the parameter m is the embedding dimension, T is the sample size, $T_m = T - m + 1$ is the maximum number of overlapping vectors that we can form with a sample of size T, I_ε is an indicator function that is equal to one if $\left\| r_t^m - r_s^m \right\| < \varepsilon$ and equal to zero otherwise. A pair of vectors r_t^m and r_s^m is said to be ε apart, if the maximum-norm $\| \, . \, \|$ is greater or equal to ε. Under the null hypothesis of independently and identically distributed random variables, $C_m(\varepsilon) = C_1(\varepsilon)^m$. Using this relation the BDS test statistic is defined as,

[11] Shiller (1999) argues that investors are often not just irrational but irrational in predictable ways.
[12] See Russell and Torbey (2002).

$$BDS(m,\varepsilon) = \frac{C_m(\varepsilon,T) - [C_1(\varepsilon)]^m}{\sigma_m(\varepsilon,T)/\sqrt{T}} \tag{7}$$

where $\sigma_m(\varepsilon,T)/\sqrt{T}$ is the standard deviation of the difference between the two correlation measures $C_m(\varepsilon,T)$ and $[C_1(\varepsilon)]^m$. For large samples, the BDS statistic has a standard normal limiting distribution under the null of *i.i.d*. If asset price changes are not identically and independent random variables, then $C_m(\varepsilon) > C_1(\varepsilon)^m$.

It is important to note that the BDS test statistic is sensitive to the choice of the embedding dimension *m* and the bound ε. As mentioned by Scheinkman and LeBaron, (1989) if we attribute a value that is too small for ε, the null hypothesis of a random *i.i.d* process will be accepted too often irrespective of it being true or false. As well, it is not safe to choose too large a value for ε. To deal with this problem Brock et al. (1991) suggest that, for a large sample size (T > 500), ε should equal 0.5, 1.0, 1.5 and 2 times standard deviations of the data. As for the choice of the relevant embedding dimension *m*, Hsieh (1989) suggests consideration of a broad range of values from 2 to 10 for this parameter. Following recent studies of Barnett et al. (1995), we implement the BDS test for the range of m-values from 2 to an upper bond of 8.

In general, a rejection of the null hypothesis is consistent with some type of dependence in the returns that could result from a linear stochastic process, non-stationarity, a non-linear stochastic process, or a non-linear deterministic system.[13] According to Hsieh (1991), linear dependence can be ruled out by prior fitting of Akaike Information Criterion (AIC)-minimizing autoregressive moving average (ARMA) model.[14] In addition, since we are using daily data over a relatively short time period, is it safe to argue that for an economically and politically stable country such as Tunisia, non-stationarity is unlikely to be the cause of non-linearity, a hypothesis that will be tested using unit root tests.[15] Therefore, a rejection of the *i.i.d* assumption using filtered data can be the result of a non-linear stochastic process or a non-linear deterministic system. However, BDS test is neither able to distinguish between stochastic and deterministic non-linearity, nor can it discriminate between additive and multiplicative stochastic dependence. Because we are concerned with a stochastic explanation of returns behaviour, the latter issue matters in this case. To determine the source of non-linearity in the returns series we use Hsieh's test.

2.4. Searching for the Source of Non-linearity: The Hsieh Test

As stated earlier, in order to choose an appropriate non-linear model describing the returns series, it is crucial to know the source of non-linearity in the data. Non-linearity can enter through the mean of a return generating process (additive dependence) as in the

[13] The Simulation studies of Brock, Hsieh and LeBaron (1991) show that the BDS test has power against a variety of linear and non-linear processes, including for example GARCH and EGARCH processes.

[14] The Akaike's Information Criterion (Akaike, 1974) is defined as $AIC(k) = n\ln(\sigma_{\varepsilon,k}) + 2k$, where $\sigma_{t,k}$ is an estimate of the error variance in the model.

[15] Non-stationarity is assumed to be mainly the result of structural change, such as policy changes, technological and financial innovation, etc.

case of threshold autoregressive model, or through the variance (multiplicative dependence), as in the case ARCH model proposed by Engle (1982). Non-linearity can be both additive and multiplicative as in the case of GARCH-M model.

Considering the residuals of the prior determined ARMA model, ν_t, Hsieh (1989) expresses the two types of non-linearity in returns series, r_t, as follows:

Additive dependence:

$$\nu_t = \omega_t + f(r_{t-1},...,r_{t-k},\nu_{t-1},...,\nu_{t-k}) \tag{8}$$

Multiplicative dependence:

$$\nu_t = \omega_t f(r_{t-1},...,r_{t-k},\nu_{t-1},...,\nu_{t-k}) \tag{9}$$

where ω_t is an *i.i.d* random variable with zero mean and independent of past r_t's and ν_t 's, and $f(.)$ is an arbitrary non-linear function of r_t's and ν_t 's for finite k.

Multiplicative dependence is characterized by

$$E(\nu_t | r_{t-1},...,r_{t-k},\nu_{t-1},...,\nu_{t-k}) = 0 \tag{10}$$

While additive dependence implies

$$E(\nu_t | r_{t-1},...,r_{t-k},\nu_{t-1},...,\nu_{t-k}) \neq 0 \tag{11}$$

Hsieh designed a third-moment test (known as Hsieh Test) where under the null hypothesis of multiplicative non-linearity, the third-order moment of a whitened return series (ν_t),

$$\rho_{\nu\nu\nu}(i,j) = E(\nu_t \nu_{t-i} \nu_{t-j} / \sigma_\nu^3) \tag{12}$$

equals zero for all $i, j > 0$. If the dependence is additive then $\rho_{\nu\nu\nu}(i,j) \neq 0$ and the null hypothesis is rejected. Hsieh estimates the third-order moment $\rho_{\nu\nu\nu}(i,j)$ by

$$\eta_{\nu\nu\nu}(i,j) = T^{-1} \sum \nu_t \nu_{t-i} \nu_{t-j} \Big/ \left[T^{-1} \sum \nu_t^2\right]^{1.5} \tag{13}$$

Hsieh test is defined as

$$\psi = \sqrt{T} \cdot \eta_{eee}(j,k) / s_r(j,k) \tag{14}$$

where

$$s_r^2(j,k) = T^{-1} \sum \nu_t^2 \nu_{t-i}^2 \nu_{t-j}^2 \Big/ \left[T^{-1} \sum e_t^2\right]^3 \tag{15}$$

Equation 15 defines the consistent estimate of the variance of $\eta_{\nu\nu\nu}(i,j)$, which is asymptotically a standard normal distributed variable. An alternative test for non-linearity is the Tsay test (Tsay, 1986). But, unlike Hsieh test, Tsay test is designed to detect any type of non-linearity and therefore does not differentiate between the two types of non-linearity. Furthermore, a simulation study by Terasvirta (1996) shows that the Tsay test has low power against multiplicative non-linearity

2.5. Modeling Conditional Heteroscedasticity

Although the Hsieh Test provides us with the type of non-linearity underlying the data series, it does not tell what model to choose for the returns generating process. Still, the results of the third-moment test provide the first step towards finding the best non-linear model to fit the data. If the source of non-linearity turns out to be the variance, (a multiplicative dependence) then we should look into ARCH models. Engle (1982) was first to introduce these models which are now very widely used in financial time series modeling. For example the generalized ARCH (GARCH) models, designed by Bollerslev (1986), are very successful in describing certain properties of high frequency financial time series such as excess kurtosis and volatility clustering. Assuming that the returns process is expressed as an autoregressive process of order *k*:

$$r_t = \beta_0 + \sum_{i=1}^{k} \beta_i r_{t-i} + \omega_t \tag{16}$$

Conditional on information set up to time t-1, ω_t is an *i.i.d* random variable with mean 0 and variance σ_t^2 , a GARCH(p,q) model is expressed as follows:

$$\sigma_t^2 = \eta + \lambda(L)\omega_t^2 + \theta(L)\sigma_t^2 \tag{17}$$

where *L* is the lag operator, $\lambda(L) = \sum_{i=1}^{p} \lambda_i L^i$ and $\theta(L) = \sum_{i=1}^{q} \theta_i L^i$.

with constraints:

$$\begin{aligned} &\sum_{i=1}^{p} \lambda_i + \sum_{i=1}^{q} \theta_i < 1 \\ &\eta > 0 \\ &\lambda_i \geq 0 \qquad i = 1,2,...,p \\ &\theta_j \geq 0 \qquad j = 1,2,...,q \end{aligned}$$

It is important to note that the GARCH(p,q) model is a symmetric variance process, in that the sign of the disturbance is ignored. Several empirical studies show that a GARCH(1,1) model expressed as:

$$\sigma_t^2 = \eta + \lambda_1 \omega_t^2 + \theta_1 \sigma_t^2 \tag{18}$$

provides a parsimonious fit for share price changes series (see, for instance, Baillie and Bollerslev ,1989). The sum $\lambda_1 + \theta_1$ measures the persistence of a shock to the variance. It is very frequent to have the value of $\lambda_1 + \theta_1$ close or equal to one, indicating that shocks to the conditional variance will persist over future horizons. A sum of $\lambda_1 + \theta_1$ equals to one implies that the ARMA(1,1) representation of the GARCH(1,1) process has a unit root, and the model becomes the Integrated GARCH (IGARCH) process.

To generalize the above argument, consider τ_t such that $\tau_t = \omega_t^2 - \sigma_t^2$. We can easily express the GARCH model as an ARMA(m,q) process:

$$[1 - \lambda(L) - \theta(L)]\omega_t^2 = \eta + [1 - \theta(L)]\tau_t^2 \tag{19}$$

with m = max(p,q).

Let $\phi(L) = 1 - \lambda(L) - \theta(L)$, if $\phi(L)$ has a unit root then the GARCH model is referred to as the IGARCH model. Note that the GARCH model assumes that the persistence of shocks to the conditional variance will decay exponentially while the IGARCH process supposes that shocks persist infinitely. However, depending on the degree of efficiency of the underlying stock market, shocks persistence lies between these two extreme views. To cope with this problem, the ARMA(m,q) process in Equation 19 is expanded to a FARIMA(m,d,q) model as follows:

$$(1 - L)^{\mathrm{d}}\,\phi(L)\omega_t^2 = \eta + [1 - \theta(L)]\tau_t^2 \tag{20}$$

where *d* is the integration parameter. We can re-write the model above in terms of the conditional variance σ_t^2:

$$[1 - \theta(L)]\sigma_t^2 = \eta + \left[[1 - \theta(L)] - \phi(L)(1 - L)^d\right]\omega_t^2 \tag{21}$$

Note that when d = 0, Equation 21 becomes the usual GARCH process; when d = 1, we get the IGARCH model, and when 0 < d < 1, we will have the fractionally IGARCH (FIGARCH) model of Baillie et al (1996).

Similar to the GARCH process, the FIGARCH model does not allow for leverage effect, which is known also as volatility asymmetry. Discovered by Black (1976), leverage effect means that volatility tends to rise in response to lower than expected returns and to fall in response to higher than expected returns. Many studies have explained the leverage effect with the degree of leverage (see, for example, Christie, 1982). In addition, leverage effect seems to be more persistent in index series compared to individual stocks (Bouchard et al, 2001). Therefore, we use the FIEGARCH of Bollersev and Mikkelsen (1996):[16]

$$[1 - \theta(L)]^d\,\phi(L)\ln\left(\sigma_t^2\right) = \eta + \sum_{j=1}^{q}\left(\theta_j\left|\xi_{t-j}\right| + \rho_j\xi_{t-j}\right) \tag{22}$$

where ξ_{t-j} is the standardized residuals, $\xi_{t-j} = \omega_t / \sigma_t$, and ρ_j measures the leverage-effect in the data series.

[16] Bollerslev and Mikkelsen (1996) argue that FIEGARCH is stationary when the integration parameter is between 0 and 1.

The superiority of FIEGARCH model, in comparison to GARCH and IGARCH, comes from its flexibility. In fact, other than modeling volatility clustering and excess kurtosis, FIEGARCH process is capable of describing high volatility persistence, long memory in the conditional variance, as well as leverage effect, which are features that emerging stock markets are likely to exhibit.

Now let us go back to the implications of Hsieh's test. If the result from the third-moment test shows that the non-linearity dependence is additive then a GARCH-in-mean (GARCH-M) model of Engle et al (1987), would better describe returns series. The particularity of GARCH-M model is that it accounts for risk premium effect by introducing a volatility term into the return equation:

$$r_t = \beta_0 + \sum_{i=1}^{k} \beta_i r_{t-i} + \delta\sigma_t^2 + \omega_t \qquad (23)$$

That is, GARCH-M measures the relationship between risk and returns. An insignificant δ implies that risk does not affect the returns process. Once we select the model that best fits the data, we test for any ARCH effects using the Lagrange Multiplier test (LM) proposed by Engel (1982).[17] If the null hypothesis that the disturbance lacks ARCH effect is accepted, then we employ the BDS test to the standardized residuals of the model to see whether all the non-linearity is accounted for.[18]

3. Data and Descriptive Statistics

Empirical research in non-linear dynamics needs large sample sets. Working with ultra-high frequency data or choosing a long time interval or both can solve this. However, as noted by Hiesh (1991), ultra-high frequency data captures some artificial dependencies, which are caused by market microstructure and are detected easily by the BDS test. On the other hand, long time interval data series can be non-stationary, especially in emerging markets where financial liberalization and deregulation have led to multiple structural breaks in their financial and economic series. To handle this problem, we use the daily closing price of the Tunisian Stock Exchange General Price Index, Tunindex, from January 2, 1998 to April 1, 2004, with a total of 1544 observations, which provides a sample size large enough to fulfill this paper's goals. The data is obtained from Tunisia Stock Exchange. Market index prices are transformed to daily returns $r_t = 100 \cdot \ln(P_t / P_{t-1})$, where P_t and P_{t-1} are prices at date t and t-1 respectively. Tunindex is a weighted average portfolio of the 30 most liquid stocks on the market. The stock market capitalisation is dominated by the banking industry. The heavy presence of banks presents an opportunity to examine the leverage effect in stock index.

Table 1 below provides various descriptive statistics for index returns. The distribution of daily returns are positively skewed. The null hypothesis of skewness coefficient conforming to the normal distribution value of zero is rejected at 1% level. In addition, the null hypothesis of kurtosis coefficient conforming to the normal distribution value of

[17] It is important to mention that to the extent that any non-normality is attributable mainly to excess kurtosis, we expect deviation from normality of returns to diminish when ARCH effects are accounted for.

[18] Brock (1987) proves that BDS test provides the same results whether employed to residuals or raw data in linear models.

three is rejected at 5% level. The daily returns are thus not normally distributed, a conclusion which is confirmed by Jarque-Bera test statistic and the QQ-plot shown in Figure 2.[19] Figure 1 below shows graphs of the log daily market index as well as the daily returns over the sample period. From the later, we can see that large price changes tend to follow large changes, and small changes tend to follow small changes. This is a property of asset prices, called volatility clustering (a type of heteroscedasticity) that TSE index seems to exhibit.

Table 1. Summary statistics for TUNINDEX

Statistic	Rt
Mean (%)	0.017
Median (%)	0.003.
Minimum (%)	-1.81
Maximum (%)	2.87
Standard deviation (%)	0.45
Skewness (s)	0.61*
t-statistics	(9.68)
Kurtosis (κ)	4.19 *
t-statistics	(2.01)
JB	96.16*
Observations	1543

Note: * Significant at the 5% level, JB is the Jarque-Bera test for normality

4. Empirical Results

Table 2 below reports the OLS estimate of the constant (or drift) by estimating Equation (2), together with a JB test statistic. The results suggest that the mean of the return series is insignificantly different from zero, which is consistent with the random walk hypothesis. Note that JB test statistic supports the same conclusion as with the descriptive statistic in Table 1, indicating a departure from normality in return series, a common feature of financial asset returns. As mentioned earlier, under the random walk hypothesis, the distribution of returns should be stationary over time. Furthermore, since structural changes can cause a rejection of the *i.i.d* process, it is important to explore the possible non-stationarity in our data series to see whether we have chosen the right sample interval. In searching for unit roots, we employ the Augmented Dickey-Fuller (ADF) and Philips-Perron (PP) unit root tests on the levels and on the first differences of the TSE daily series. Table 3 below displays the ADF statistics and PP statistics. These findings suggest that the return series is non-stationary in levels and stationary in first differences at 5% level of significance. Although these results are consistent with the random walk hypothesis, we cannot decide on the latter until we explore the dependence structure of the returns series.[20]

[19] The QQ-plot is a scatterplot of the standardized empirical of data series against the quantiles of a standard normal random variable.

[20] The reason is simple; unit root tests are not tests for predictability. They are designed just to investigate whether a series is difference-stationary or trend stationary.

Figure 1. TUNINDEX Daily log Prices and Returns from 01/1998 to 04/2004

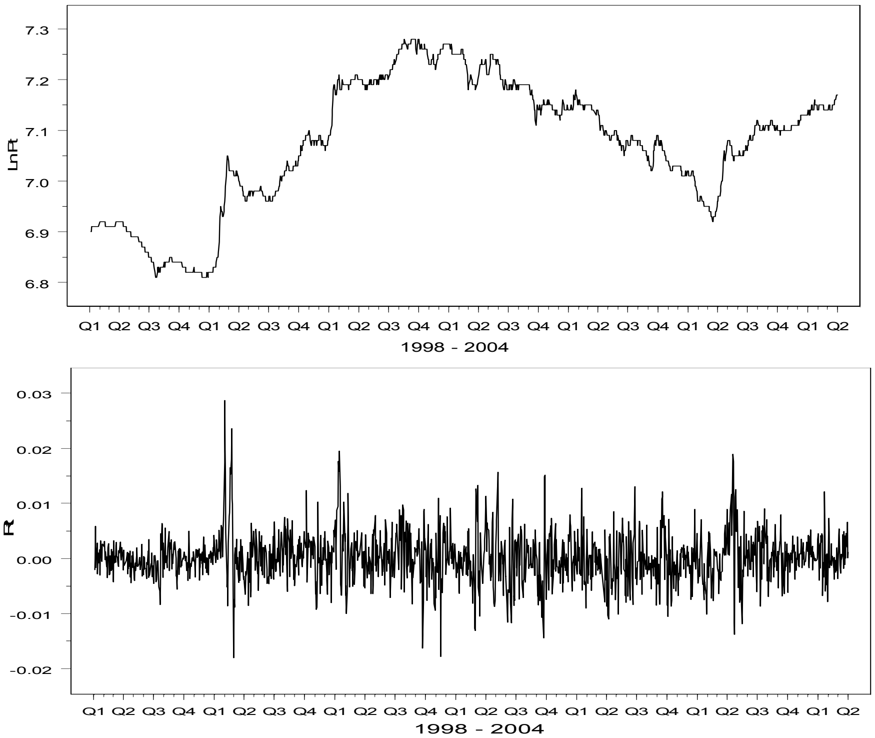
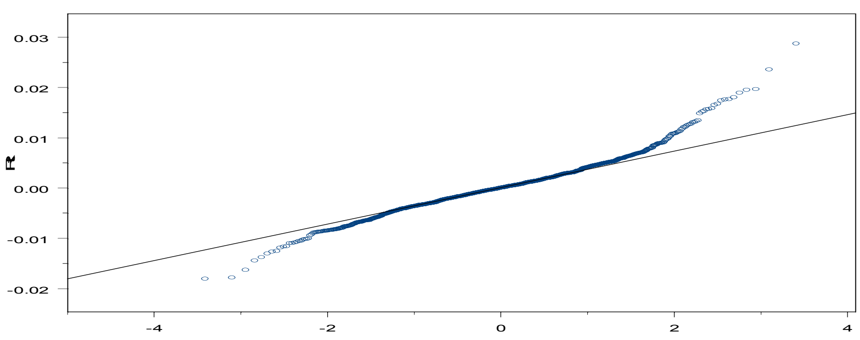


Figure 2. The QQ Plot for the Daily Returns Series



To examine the linear dependence of the returns series, we use the modified Q-statistic of Ljung and Box (1978). Table 4 below provides the autocorrelations coefficients up to lag 40. The results suggest the existence of significant serial autocorrelation at all lags. As mentioned earlier, evidence of a temporal linear relationship can be spurious; therefore, independence assumption should not be ruled out without an extensive examination of the underlying linear as well as non-linear dependence.

To test for the *i.i.d* assumption we employ the BDS test. It is a powerful test frequently used to detect several non-linear structures and to test for the adequacy of a variety of models. Table 5 below reports the BDS statistic for embedding dimension 2 to 8 and for epsilon values starting from 0.5 to 2 times the standard deviation of the returns series. The results strongly reject the null hypothesis of independently and identically distributed index price changes at 5% and 1% significance level. Now that we reject the RWH I, we focus on uncovering the structure of dependency in the returns series. Since the BDS test has a good power against linear as well as non-linear system, we use a filter to remove the serial dependence in the return series and the resulting residuals series are re–tested for possible non-linear hidden structures. We use an autoregressive AR(k) model to take out all the linearity in the series. Empirical studies show that non-synchronous trading causes a deviation of the observed index returns from the true index returns. An advantage of using the residuals of AR(k) model is that it reduces the effect of infrequent trading, which is more pronounced in price indices of thinly traded stock markets. [21]

Table 2. Results of the regression of random walk model with drift

Estimated constant	t-statistic	JB
- 0.000175	- 0.0294	179.99*

Note: * Significance at the 5% level, JB is the Jarque-Bera test for normality

Table 3. Unit root tests

	Ln (Pt)		Rt	
Test statistic	Trend	No Trend	Trend	No Trend
PP	-1.22	-1.29	-25.32 *	-25.32 *
ADF(1)	-0.91	-1.11	-25.34*	-25.35*
ADF(2)	-1.29	-1.35	-20.62*	-20.62*
ADF(3)	-1.31	-1.33	-19.19*	-19.19*
ADF(4)	-1.27	-1.32	-16.98*	-16.98*
ADF(5)	-1.28	-1.33	-15.55*	-15.55*
ADF(6)	-1.27	-1.31	-14.25*	-14.25*

Note: *Significant at 5% level, PP is the Phillips-Perron test, ADF is the augmented Dickey-Fuller test.

Table 4. Test for serial correlation of the daily returns: modified Q-statistic

MQ(5)	MQ(10)	MQ(15)	MQ(20)	MQ(30)	MQ(40)	MQ(50)
324.62*	345.79*	394.93*	400.53*	418.86*	428.19*	435.77*

Note: *MQ(k)* is the modified Q-statistic at lag k, *Significance at the 5% level.

Table 5. BDS test statistic for raw data

m	ϵ/σ		ϵ/σ		ϵ/σ		ϵ/σ	
2	0.5	17.540**	1	17.736**	1.5	18.946**	2	21.489**
3	0.5	22.361**	1	21.000**	1.5	20.807**	2	22.500**
4	0.5	27.092**	1	23.467**	1.5	21.767**	2	22.460**
5	0.5	33.824**	1	26.395**	1.5	23.076**	2	22.756**
6	0.5	43.163**	1	30.132**	1.5	24.536**	2	23.025**
7	0.5	55.453**	1	34.581**	1.5	26.081**	2	23.249**
8	0.5	73.042**	1	39.865**	1.5	27.797**	2	23.558**

Note. m is embedding dimension, ε is the bound, * Significant at the 5% level.,** Significant at the 1% level. The critical values for BDS test are 1.96 for 5% and 2.58 for 1%.

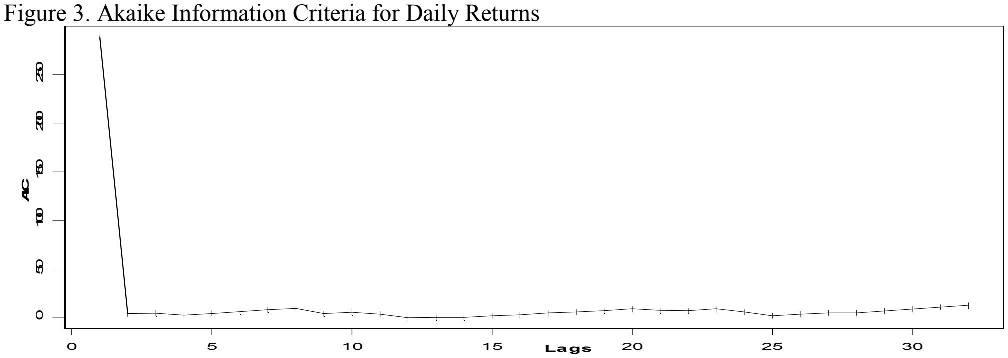
The identification of the AR(k) bases on the lowest AIC. Figure 3 below shows a plot of Akaike's criterion. It starts indexing at 1, but the first element of the AIC component is

[21] To proxy for the true but unobserved index returns Stoll and Whaley (1990) have used the residuals from an ARMA regression.

for order 0. Note that the minimum AIC is at 12, suggesting an autoregressive model of order 12 to fit the returns series. The Modified Q-statistics provided in Table 6 below shows that the residuals of the AR(12) are white noise, suggesting that the model accounts for all the linearity dependence in the series. Figure 4 displays the correlogram of the residuals of the AR(12). The horizontal dashed lines are the upper and lower 2.5% boundaries for rejecting the null hypothesis of zero autocorrelation. All the autocorrelation coefficients, except at lag 23, are small and lie within the horizontal bands, indicating that they do not differ significantly from zero. However, this does not mean that the residuals of the whitened data follow a pure random process. In fact, although there is no evidence of autocorrelation in the residuals of the AR(12) itself, the significant values of McLeod-Li (ML) test statistics suggest that its squared residuals exhibit significant autocorrelation, indicating evidence of non-linear dependencies in the returns series. [22] To confirm the presence of non-linear dependence, we applied the BDS test to the residuals of the whitened series. Although lower than those of Table 5, the BDS statistics displayed in Table 7 strongly reject the *i.i.d* assumption, which gives a clear indication of the existence of non-linear dependencies in returns series. [23] Figure 4 below provides the plot of the autocorrelation coefficients for daily returns, squared daily returns, and residuals and squared residuals from AR(12). Because the squared daily returns and the squared residuals measure the second moments of the series, significant autocorrelations are evidence of time varying conditional heteroskedasticity in the daily returns as well as in the residuals of the AR(12).

Since we can rule out the non-startionarity and linearity as causes of the rejection of the *i.i.d* assumption, we can say that the inherent non-linearity in the TSE index returns is either stochastic or deterministic. However, to date, there is no strong evidence of low-dimension chaos found in the most efficient stock markets on the world. Therefore, it is not possible to argue that chaos could be the cause of non-linearity in thin emerging stock markets. As a result, studies of non-linearity in these markets should focus on how to effectively model the stochastic non-linearity taking into account their particular features.

Figure 3. Akaike Information Criteria for Daily Returns



[22] Following Granger and Anderson (1978) work on squared-residuals as an indicator to detect non-linearity, McLeod and Li (1983) proposed a test which is designed exactly as the quadratic counterpart to the modified Q-statistic of Ljung and Box (1978). McLeod-Li test is used as a diagnostic check for nonlinearity.

[23] Note though that linear filter has attuned the persistence of autocorrelation in the returns series.

Table 6. Test for serial correlation of the daily returns: modified Q-statistic

MQ(5)	MQ(10)	MQ(15)	MQ(20)	MQ(30)	MQ(40)	MQ(50)
0.07	0.25	6.79	10.19	28.36	41.58	46.24
ML (5)	ML(10)	ML(15)	ML(20)	ML(30)	ML(40)	ML(50)
287.29*	330.45*	373.33*	397.84*	412.89*	421.11*	425.67*

Note: *Significance at the 5% level, MQ(k) is the modified Q-statistic at lag k for the AR(11) residuals series, ML(k) is the McLeod-Li test (see McLeod and Li, 1983) at lag k for the AR(11) squared residuals series.

Table 7. BDS statistics for the residuals of the AR(12) model

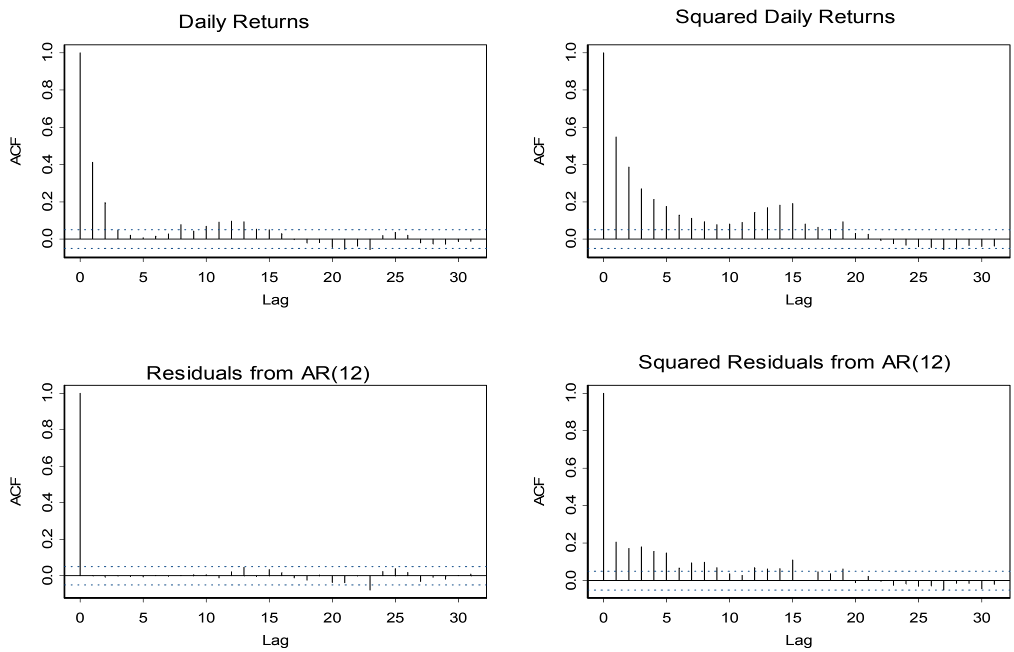
m	ϵ/σ		ϵ/σ		ϵ/σ		ϵ/σ	
2	0.5	13.727**	1	12.685**	1.5	11.345**	2	10.111**
3	0.5	17.177**	1	14.622**	1.5	12.531**	2	11.196**
4	0.5	21.531**	1	17.024**	1.5	13.992**	2	12.329**
5	0.5	27.437**	1	19.616**	1.5	15.42**	2	13.332**
6	0.5	34.019**	1	22.355**	1.5	16.707**	2	14.023**
7	0.5	45.142**	1	25.612**	1.5	17.969**	2	14.605**
8	0.5	62.328**	1	29.642**	1.5	19.274**	2	15.129**

Note. m is embedding dimension, ε is the bound, * Significant at the 5% level.,** Significant at the 1% level. The critical values for BDS test are 1.96 for 5% and 2.58 for 1%.

Although the results from the BDS test strongly support the existence of inherent non-linearity, it does not tell us whether it enters through the mean or variance of the returns series. To uncover the source of non-linear behaviour, we calculate the third-order moment test statistics of Hsieh (1989). None of the values of the approximately normally distributed Hsieh test statistic, reported in Table 8 below, are significant, implying a failure to reject the null hypothesis of multiplicative dependence. This supports the view expressed above that volatility clustering is responsible for the rejection of *i.i.d* in index returns series. Therefore, a GACRH model is most likely to succeed in describing the return generating process than a GACRH-M model.

Given the results of Hsieh's test, we have examined several GARCH (p,q) models. Using the AIC and BIC as tools for model selection, it turns out that a GARCH (1,1) is the best model to fit the data. Table 9 reports the estimation results of a GARCH(1,1) process under the assumption that the innovations follow a normal distribution. The coefficients of the conditional variance equation, λ_1 and θ_1, are significant at 1% level implying a strong support for the ARCH and GARCH effects.

Figure 4. ACF of Daily Returns and Residuals of AR(12) and their Squared Values



The results of the diagnostic tests show that the model is correctly specified. The modified Q-statistics for the standardized residuals and standardized squared residuals are both insignificant, suggesting the chosen GARCH process is successful at modeling the serial correlation structure in the conditional mean and conditional variance. JB and Sharp tests for normality fail to reject the null hypothesis that the standardized residuals are normally distributed. Note, however, that the sum of the parameters estimated by the variance equation is close to one. A sum of λ_1 and θ_1 near one is an indication of a covariance stationary model with a high degree of persistence; and long memory in the conditional variance. $\lambda_1 + \theta_1 =$ 0.989 is also an estimation of the rate at which the response function decays on daily basis. Since the rate is high, the response function to shocks is likely to die slowly. For instance, a month after the initial shock, 72% (or 0.979^{30}) of the impact remains in effect. Even six months later, 14% (or 0.979^{180}) of initial shock remains persistent. The evidence of high volatility persistence and long memory in the GARCH(1,1) model suggests that a FIGARCH (p,d,q) model may be more adequate to describe the data. Nevertheless, in order to examine the leverage effect, we have looked at the FIEGARCH model as well. Table 9 provides the results of parameters estimates of FIGARCH(1,1) and FIEGARCH(1,1), as well as the BIC and AIC for the sake of comparison. Having the lowest BIC and AIC values, the FIEGACRCH seems to be the best model to fit the data. Note, however, that although the market capitalisation of the TSE is dominated by banks the leverage coefficient is insignificant at 5% level, implying that conditional variance of future returns responds similarly to positive and negative shocks.[24] Parameters estimate for FIEGARCH(1,d,1) show that the fractional difference parameter *d* is significant, which confirms the existence of long memory. Notice also that the sum of λ_1 and θ_2 is lower than of the initial GARCH process.

[24] We also examined other type of asymmetric GARCH models, such as PGARCH and GLS-GARCH. The leverage parameter remains insignificant (Results are available upon request from the authors)

We calculate the Lagrange-multiplier (LM) for ARCH effect proposed by Engle (1982). The null hypothesis that the residuals lack ARCH effect is not rejected, which shows that the FIEGARCH has counted for all the volatility clustering in the data.[25] Tests for normality provide opposite results. While the Jacque-Bera test rejects the null hypothesis that the standardized residuals are normally distributed, the Shapiro-Wilk test (Shapiro and Wilk, 1965) fails to do so. To get more decisive conclusion regarding the normality assumption, we look at the QQ-plot given in Figure 5. Deviation in both tails from the normal QQ-line is significant, thus the normality for the residuals may not be suitable. Notably, despite that we have accounted for ARCH effects, evidence of non-normality, although has diminished, may be significant still. In fact, volatility clustering can accounts for some but not all of the fat tail effect observed in returns series. A part of the fat tail effect can also result from the presence of non-Gaussian asset return distributions, such as Student's *t* distribution.

To examine whether the FIAGARCH model has succeeded in capturing all the nonlinear structure in the data, we employ the BDS test to its standardized residuals. A rejection of the *i.i.d* hypothesises will imply that the conditional heteroskedasticity is not responsible for all the nonlinearity in index returns, and there is some other hidden structure in the data. To have a preliminary view of the FIEGARCH modeling capability, we look at the diagnostic plot provided by Figure 5. First, the residuals series of the FIEGARCH seem to behave as *i.i.d* random variables. Secondly, the autocorrelation coefficient for both the standardized residuals and squared standardized residuals show that the AR(12)-FIGARCH model captures all the linear as well non-linear dependencies in the index returns series. Table 10 displays the BDS statistics on the standardized residuals from the FIGARCH process. In line with the observations from Figure 5, the BDS test fails to reject the null hypothesis that the standardized residuals are *i.i.d* random variables at 5% and 1% degree of significance. This confirms that he FIGARCH process indeed captures all the non-linearity in the series, and that the conditional heteroscedasticity is the cause of the non-linearity structure uncovered in the returns series.

To confirm the results from Hsieh test and to examine the intertemporal relationship between expected return and conditional volatility in TSE, we use the GARCH in Mean model. Table 9 gives the estimates of the GARCH-M process. The coefficient (δ) of the conditional volatility is insignificantly different from zero. Thus, conditional volatility is not priced in the Tunisian market, which means absence of risk premium effect. An increase in the conditional variance will not be associated with an increase in the conditional mean, implying that risk neutral investors dominate trading in TSE. This is consistent with results from the Hsieh Test stating that non-linearity in Tunindex daily returns is multiplicative. Our result is also consistent with most of the studies that examined the intertemporal relationship between expected returns and conditional volatility. For example, Theodossiou and Lee (1995) find no relationship between returns and volatility in ten industrialized countries.

Finally, Figure 6 below plots the conditional volatility obtained from the FIEGARCH model. It is obvious from the graph that the conditional variance varies over time. The series is characterized by significant heterescedasticity, which manifests by changes in volatility of TSE index over the period of investigation. From Figure 5 and 6 we can say that the common assumptions of constant variance and Gaussian returns underlying the

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[25] Under the null hypotheses the test statistic $LM = T \cdot R^2 \overset{A}{\sim} \chi^2(p)$, where T is the sample size and R^2 is computed using the estimated residuals.

theory and practice of option pricing, portfolio optimization and value-at-risk (VaR) calculations are simply invalid for emerging markets.

Table 8. Hsieh test of Residuals from AR(12) model

Lags i	Lags j		Lags i	Lags j	
1	1	-0.258	2	4	0.112
1	2	-0.698	2	5	0.325
1	3	0.687	3	3	-0.542
1	4	0.242	3	4	-0.09
1	5	-0.511	3	5	0.147
2	2	0.445	4	4	-0.564
2	3	-0.213	4	5	-0.214

Table 9. Modeling Conditional Heteroscedasticity

	AR(12)-GARCH(1,1)		AR(12)-FIGARCH(1,1)		AR(12)-FIEGARCH(1,1)		AR(12)-GARCH-M(1,1)	
Coefficient		*p-value*		*p-value*		*p-value*		*p-value*
$\beta0$	0.000	0.483	0.001	0.446	0.004	0.321	-0.009	0.046
$\beta1$	0.222	0.000	0.23	0.000	0.223	0.000	0.224	0.000
$\beta2$	0.079	0.005	0.077	0.007	0.076	0.006	0.076	0.006
$\beta3$	0.043	0.062	-0.039	0.087	-0.031	0.134	-0.042	0.064
$\beta4$	0.031	0.138	0.028	0.173	0.039	0.080	0.027	0.164
$\beta5$	0.020	0.225	0.019	0.248	0.01	0.348	0.018	0.255
$\beta6$	0.029	0.147	0.030	0.140	0.038	0.074	0.025	0.180
$\beta7$	0.007	0.394	0.001	0.483	-0.005	0.422	0.005	0.428
$\beta8$	0.060	0.007	0.061	0.008	0.058	0.008	0.059	0.008
$\beta9$	-0.010	0.300	-0.013	0.318	-0.003	0.449	-0.012	0.330
$\beta10$	0.023	0.162	0.02	0.201	0.011	0.315	0.021	0.186
$\beta11$	0.012	0.305	0.013	0.300	0.007	0.383	0.012	0.304
$\beta12$	0.039	0.046	0.043	0.037	0.030	0.090	0.038	0.050
η	0.012	0.000	0.009	0.001	-0.342	0.000	0.012	0.000
λ_1	0.228	0.000	0.098	0.118	0.373	0.000	0.222	0.000
θ_1	0.761	0.000	0.552	0.000	0.699	0.000	0.717	0.000
d	-	-	0.706	0.000	0.327	0.001	-	-
ρ_1	-	-	-	-	0.009	0.348	-	-
δ	-	-	-	-	-	-	0.088	0.181
$\lambda_1 + \theta_1$	0.989	-	-	-	-	-	-	-
AIC	1366	-	1373.1	-	1354.1	-	1366.2	-
BIC	1451	-	1463.9	-	1450.3	-	1457.0	-
LM Test	5.1	0.955	5.49	0.94	6.746	0.874	5.020	0.957
JB	44.75	0.000	44.8	0.000	30.82	0.000	45.3	0.000
S-W	8.59	0.737	7.425	0.828	0.988	0.62	0.987	0.407
MQ(10)	7.87	0.642	9.878	0.451	9.768	0.461	7.275	0.699
MQ(20)	14.65	0.796	17.436	0.625	17.23	0.638	13.86	0.837
MQ(30)	36.96	0.178	36.65	0.188	36.67	0.187	35.82	0.214
ML(10)	4.64	0.914	5.985	0.817	6.152	0.802	4.66	0.913
ML(20)	17.08	0.648	19.112	0.515	19.11	0.515	17.54	0.618
ML(30)	36.62	0.188	40.3	0.099	40.02	0.104	36.43	0.194

Note: $\lambda, \theta, d, \rho, \delta$ are the ARCH, GARCH, integration, leverage and risk premium parameters respectively. S-W is the Shapiro-Wilk test for normality proposed by Shapiro and Wilk (1965). MQ(k) is the modified Q-statistic at lag k for the standardized residuals series. ML(k) is the McLeod-Li test at lag k for the squared standardized residuals series.

Figure 5. FIEGARCH residuals with diagnostic plots

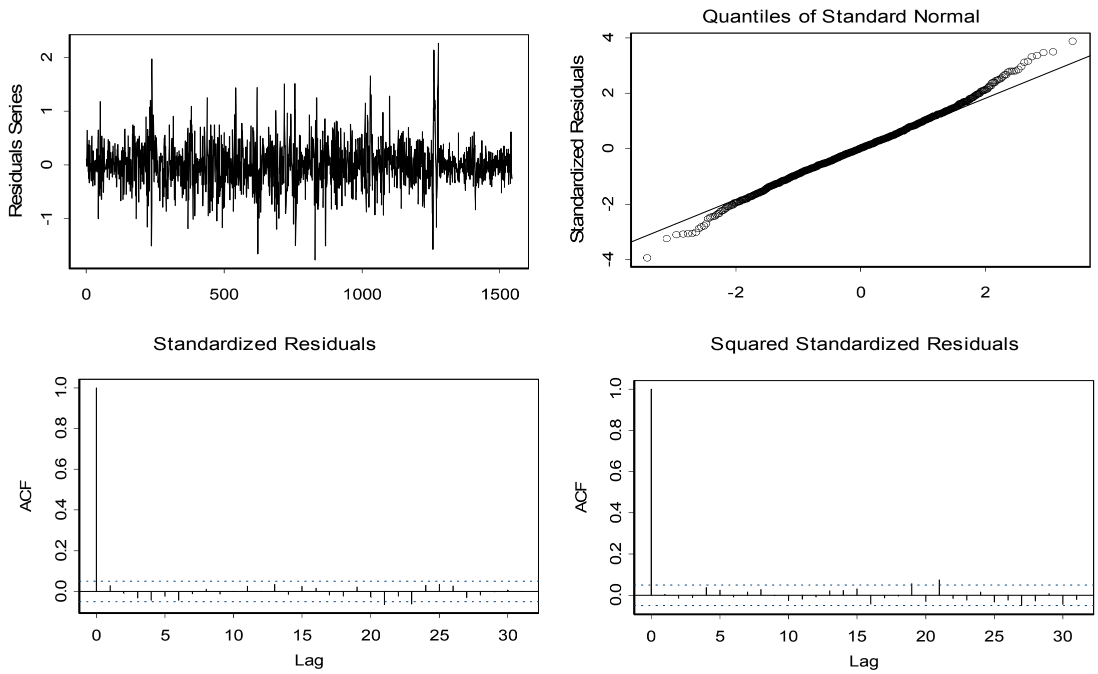
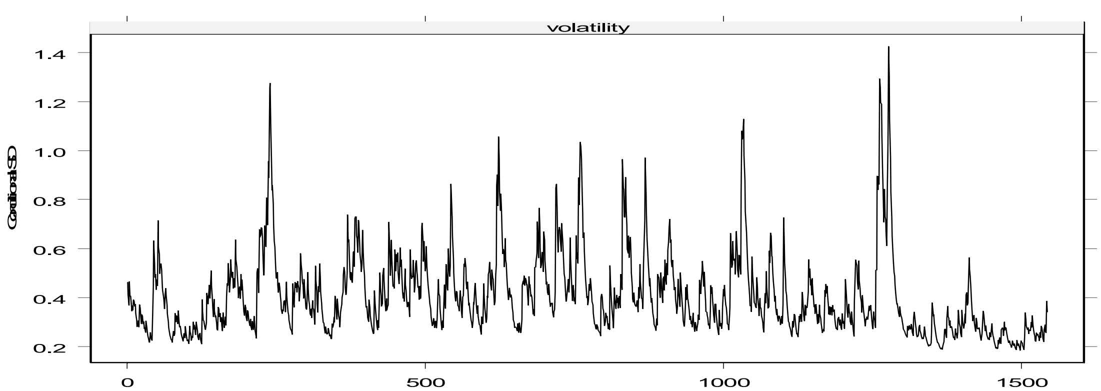


Table 10. BDS Test Statistics for Standardized Residuals from FIEGARCH Model

m	ϵ/σ		ϵ/σ		ϵ/σ		ϵ/σ	
2	0.5	0.709	1	0.664	1.5	0.175	2	-0.118
3	0.5	0.145	1	0.170	1.5	-0.313	2	-0.613
4	0.5	0.172	1	0.141	1.5	-0.222	2	-0.331
5	0.5	0.414	1	0.445	1.5	0.087	2	0.026
6	0.5	0.592	1	0.709	1.5	0.311	2	0.202
7	0.5	0.811	1	0.946	1.5	0.534	2	0.378
8	0.5	1.094	1	1.295	1.5	0.765	2	0.564

Note. Critical values for BDS test are 1.96 for 5% and 2.58 for 1%

Figure 6. Time Varying Volatility of Tunindex Returns



5. Summary and Conclusion

This paper tests for the random walk hypothesis in the daily returns of the TSE index by examining both linear and non-linear dependence. First, the results provide evidence of serial correlation in the series and suggest the rejection of the *i.i.d* hypothesis. To investigate the reason for rejecting the *i.i.d* hypothesis, we filter the data using an autoregression, and then employ the BDS test to the residuals of the selected model. The *i.i.d* hypothesis is rejected again. The results from unit root tests show that the returns of the TSE index are stationary which confirm the presence of non-linear structure in the series. To best model the non-linear dependency, we search for the source of non-linearity and found that it is caused by conditional heteroscedasticity, which is generally modeled with GARCH type model. We found evidence of volatility persistence and long memory in conditional variance. As such, we use of a FIEGARCH model which has successfully accounted for all the non-linearity in the returns series. However, we find no leverage effect in the TSE despite the fact that it is dominated by banks. Furthermore, our results suggest that conditional volatility is not priced in the Tunisian market. Finally, we can say that the common assumptions of constant variance and Gaussian returns underlying the theory and practice of option pricing, portfolio optimization and value-at-risk (VaR) calculations do not hold for emerging markets.

Though the present study rejects the random walk hypothesis for the Tunisian stock market, and finds evidence of non-linearity dependence in the TSE index returns series, the results are not necessarily inconsistent with efficient market hypothesis, simply because non-linearity does not necessarily mean predictability. As noted by Abhyankar et al (1997) the future price changes can be predictable but only with a time horizon too short to allow for excess profits. Furthermore, the relatively high transaction costs in emerging markets and the excess profit from forecasting is likely to be nil if not negative.

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