DAILY TRANSACTIONS AND MARKET DEPTH IN SHORT STERLING FUTURES

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ABSTRACT

The objective of the present study is to examine the interplay between information, trading volume and volatility in Short Sterling futures. More specifically, the paper concentrates on the role of liquidity variables as conduits of information arrival and whether such variables could be an exclusive platform of the market's information set. The analytical framework employed, to examine the interaction among those factors, is based on the conditional volatility family of techniques. The approach is well suited as it naturally leads to examine the interaction among volatility and sources of information. In an attempt to identify proxies of information and their role in determining volatility four main conclusions have emerged. First, the empirical findings suggest that both volume and open interest exhibit a positive correlation with volatility. Second, based on the current methodology one can observe the persistence and importance of GARCH effects after accounting for liquidity. Third, both liquidity variables remain significantly exogenous compared with other studies. Finally, although both liquidity variables are found significant, their role as vehicles of transmitting information is proved to be weak with respect to the information itself.

Key Words: Short sterling futures, Information set, Volume, Open interest, GARCH modelling.

JEL classification: C22, C50, G10, G14

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1. INTRODUCTION

Volatility is a natural consequence of trading which occurs through the information arrival in the market and the subsequent reaction of hedgers and/or speculators. Thus, the chain reaction of agents[1] will force prices to reach a post-information equilibrium level. The revision of their expectations and their subsequent action will be solely reflected in the liquidity of the particular market and specifically on the amount of contracts traded. If we think the above process in a continuous time of revising expectation, and since the underlying prime mover is common (i.e. flow of information), one would expect that there is a triangular relation among information, liquidity and volatility. Empirical tests of this relationship have been mainly carried out in the stock market [Crouch (1970), Epps & Epps (1976), Rogalski (1978), Smirlock & Starks (1985), Wood *et al.* (1985), Harris (1986, 1987), Richardson *et al.* (1986), Gallant *et al.* (1992), Richardson & Smith (1994), Kandel & Pearson (1995), Chordia *et al.* (2001)] supporting a positive correlation between price changes and volume[2]. Similar results have been found by Clark (1973), Cornell (1981) in commodity futures, Tauchen & Pitts (1983) in T-bill futures and Grammaticos & Saunders (1986), Fung & Patterson (2001) in currency futures. In general empirical research has identified a link between volume and price changes. While such link remains quite an empirical issue, it is not obvious why this is happening. The theoretical framework provides no definite answer. One difficulty in evaluating such relationship is that it is not obvious what information volume per se provides to the market. Empirical work has examined this information role in a noisy rational expectations framework [Wang (1994), Blume *et al.* (1994)].

The relation among information, volume and volatility is consistent with three competing hypotheses; the sequential information hypothesis [Copeland (1976)], the mixture of distributions hypothesis [Epps & Epps (1976), Harris (1987)] and the information-trading volume model of Blume *et al.* (1994). The mixture of distributions model assumes that equilibrium is immediately obtained in a world where the arrival of information induces trading activity to rebalance portfolios. Consequently, within this framework, the theory posits volatility as an increasing

[1] Obviously their expectation about futures price movements will determine their trading volume. Hedgers will mainly respond in order to secure their future income, while speculators will take advantage should their expectations about futures volatility come true.
[2] The theory and a detailed survey of the literature can be found in Karpoff (1986, 1987).

function of the stochastic mixing variable defined as the information inflow[3]. Nevertheless, the mixture of distributions hypothesis is subject to one limitation, most notably it fails to consider the precision or quality of information. Blume *et al.* (1994) examine this issue whereby a model is developed based on the notion that trading volume plays an informationally important role in an environment where traders receive pricing signals of different quality. Of paramount importance is the assumption that the equilibrium price is non-revealing given that pricing signals alone do not provide sufficient information to ascertain the underlying value. Trading volume is treated as containing information regarding the quality of signals received by traders whereas prices alone do not; thereby, leading to the formulation of a link among trading volume, the quality of information flow and volatility. Finally, the sequential information hypothesis forwarded by Copeland (1976) is based on the notion that prices may not change immediately in response to the arrival of new information. Instead, a scenario is envisaged where information is received by individual traders. Then in response to the signal, their trades lead to a number of incomplete equilibria and final equilibrium is attained when all traders observe the same information set. The sequential response to the arrival of information implies that price volatility is forecastable, based on the knowledge of trading volume. Following this, the implication of the sequential information hypothesis is that the volume-volatility relation is sequential, not contemporaneous[4].

The purpose of this paper is to examine the aforementioned dynamics in interest rate futures. The Short Sterling market has grown significantly over the last decade and represents now one of the most active markets. Changes in interest rates play a profound effect on the economy through their impact on the values and yields of assets and liabilities as well as on a number of other economic variables, particularly those involved in monetary policy, asset pricing, debt management and analysis of contingent claims. In a conditional volatility framework prices deviate from their conditional mean by the arrival of new information. Variability in prices reflects variability of the arrival of new information, with the latter depending on past news. The persistence of past news remains an empirical question. Thus, the

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[3] The motivation behind this model is drawn by the apparent leptokurtosis exhibited in daily price changes that is attributed to the random events of importance to the pricing of the security.

[4] This model is open to at least two criticisms. First, it is the assumption that prohibits traders from learning from the market price as other traders become informed. Second, it is the implication that volume is greatest when all investors agree on the meaning of the information.

paper examines whether liquidity affects the variability of information and consequently the variability in prices beyond any GARCH effects. Another area of interest is whether the stage of market breadth affects the liquidity-volatility relationship and to this extent the study "discriminates" between an early and a mature futures period. In addition to the aforesaid issues, there is no previously work on the UK interest rate futures market. The choice of the UK market differentiates this study as it is given the opportunity to compare and contrast the current set of results against the US literature; as well as enabling it to examine the robustness of previous findings over the last two decades. Therefore, the study intends to fill this gap in the literature and also examines whether the correlation between price variability and liquidity is contemporaneous, or causality is present between volume and volatility. The latter is an interesting relationship since technical analysts would find such interaction quite valuable.

In what follows, section 2 presents the data set used along with the methodology employed. The main body of this paper is in section 3, where the econometric analysis and discussion of the results are presented. Finally, the summary of the findings and some concluding remarks are contained in section 4.

2. DATA SELECTION & METHODOLOGY

In order to investigate the aforementioned issues the data set employed is the continuous time series of the Short Sterling futures contract along with its trading volume and open interest. The Short Sterling contract is the three-month rate traded in the futures market. The current data set consists of daily futures rates and the sample covers the period between December 1982 and December 2001. The Short Sterling market has four different contracts a year, with each one expiring at the third Wednesday of March, June, September and December respectively[5]. The time series consists of rollover nearby futures contracts, since these are the most actively traded. The series have been detrended and adjusted for the non-trading days in the sample period. Table 1 provides various statistics for the unconditional distribution of the short sterling changes and liquidity variables.

[5] Note that these contracts used to expire on the second Wednesday of the delivery month until September 1985 inclusive.

Table 1. Descriptive statistics for futures data.

	Short sterling (Δ)	Volume	Open interest
Mean (μ)	-0.00127	49,885	342,886
t-value ($\mu = 0$)	-0.70329	62.600	68.600
Variance	0.01575	3,051 [a]	120,044 [a]
Skewness	-2.76751	2.1568	0.8190
Kurtosis	65.0859	7.7150	0.5641
Minimum	-2.63	25	1,319
Maximum	1.43	552,954	1,239,695
Q(30) [b]		57,574	142,406
$Q^x(30)$ [c]	138.76		

Δ indicates the change in the short sterling futures rate.
[a] Figure divided by 10^6.
[b] Q = Ljung-Box test for serial correlation for up to 30 lags.
Q test critical value at 5% level: X^2 (30) = 43.77
[c] Q^x test for nonlinearity. The marginal significance level is 0.00000

The data exhibit skewness and have excess kurtosis relative to the normal distribution. The Ljung-Box test, for serial correlation up to 30 lags, is significant at the 5% level of significance indicating that the liquidity variables are highly forecastable. This is similar to the results reported by Lamoureux & Lastrapes (1990) and Sharma *et al.* (1996) for the stock index market. In the view of the existing evidence that excess kurtosis is due to a possible time-varying variance [Akigary *et al.* (1991), Fujihara & Mougoue (1997)], a test for nonlinearity is conducted using the McLeod & Li (1983) method. The test is used to gauge the existence of conditional heteroscedasticity in the short sterling futures market. The particular method is based on the autocorrelation coefficients and the Q^x-statistic for the squared data. The statistic, reported in table 1, is highly significant indicating that the series exhibits some form of conditional heteorscedasticity. This primary justification for the use of conditional volatility processes leads us to the description of the model employed.

Since the flow of information and expectation are not easily quantified, a proxy of these variables could be the action taken in the market by traders and agents. A promising candidate for such analysis would be the trading volume in the futures market as a vehicle of transmitting information. In addition to that, the open interest[6] is also employed for comparison purposes. A distinguishing feature of futures markets is that the number of contracts in existence is endogenously determined at each point in time. Thus, open interest provides an additional measure of trading activity. As volume measures the breadth of the market, open interest is a proxy for market depth since it reflects the willingness and ability of traders to risk capital in the presence of price volatility[7]. If these and other underlying determinants of depth do not change quickly, then a variable constructed from lagged open interest should contain information on current depth [Bessembinder & Seguin (1993)]. The primary hypothesis is whether any liquidity variable (i.e. volume or open interest) can act as a proxy of the information set. As a result, the GARCH (p,q) model employed for this purpose is the following:

$$\Delta F_t = \gamma_0 + u_t \qquad u_t \sim N(0, h_t)$$

$$h_t = a_0 + \sum_{i=1}^{q} a_i u_{t-i}^2 + \sum_{j=1}^{p} b_j h_{t-j} + \delta V_t \tag{1}$$

where F = the short sterling futures rate.

h = the conditional volatility.

V = the liquidity variable (i.e. trading volume or open interest).

Equation (1) constitutes the cornerstone of the present study where $a_1,\ldots..,a_q$ $b_1,\ldots..,b_p$ and a_0 are constant parameters. The parameter a_0 is a measure of volatility acting as floor, which prevents the variance from dropping below that level. The main argument to support the use of the GARCH model is that as information arrives in the market in 'clusters', an immediate effect of clustering returns is observed, which is the main characteristic that is captured by processes of conditional volatility. A mathematical expression of the intuition behind this argument follows. The paper starts with the assumption that n intra-day equilibrium rates are observed, denoted as r, which in turn determine the unexpected change such as $u_t = \sum_{i=1}^{n} r_{it}$. If the stochastic rate of information, n, is large and $r \sim iid\ (0,\sigma^2)$ then the unexpected change will be:

$$u_t \,|n_t \sim N(0, n_t\, \sigma^2) \tag{2}$$

Note that u_t is drawn from a mixture of distributions, where the variance of each distribution depends upon information arrival. Assume, now, that the rate of

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[6] Open interest is the total number of contracts outstanding on each day.

[7] Kyle (1985) suggested that market depth is the order of flow required to move prices by one unit. As the order flow changes, the open interest of the futures contracts will also change endogenously.

information is serially correlated, which can be represented by the following AR(p) process:

$$n_t = c + a_i \sum_{i=1}^{p} n_{t-i} + e_t \qquad (3)$$

where c is a constant, a_i is a parameter and e_t is white noise. If the mixture hypothesis is valid and taking into account the aforementioned formulas, then the persistence in conditional variance can be formulated as:

$$\zeta_t = c\sigma^2 + \sum_{i=1}^{p} a_i \, \zeta_{t-i} + e_t \, \sigma^2 \qquad (4)$$

where $\zeta_t = n_t \, \sigma^2$. This expression of the variance is analogous to that of the conditional volatility processes. Thus, the GARCH model is the ideal candidate to pick up such persistence. Finally, the inclusion of mixing variables in the conditional variance equation is more a test of the relationship between volume and information, rather than a test of volume-volatility interaction. This can be seen from equation (4) where any variation in information will cause a subsequent change in the squared innovations. The interested reader can find the technical description of the maximum likelihood estimation in appendix 1.

3. EMPIRICAL RESULTS

This section looks at the empirical regularities, which are related to the triangular relation volatility-information-volume. As earlier mentioned, any action taken by agents in the market is guided by the information set and consequently by their expectation about the future. Using a likelihood ratio test and after experimentation with up to four lags for each parameter p and q, the GARCH (1,1) representation was found to be the most appropriate. Moreover, under the null hypothesis of conditional normality, the test statistic for the sample kurtosis has an asymptotic normal distribution with mean 3. The residual sample skewness and kurtosis exhibit values of –2.77 and 65.11 respectively, which are well above the ones indicated by normal distribution. In light of the evidence presented above the models are estimated based on a conditional student-t density function[8]. Using both

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[8] The log likelihood function, under the assumption that the residuals follows a conditional student-t density, is given as follows:

$$Log(L) = \sum_{t=1}^{n} \left\{ \log\left(\Gamma\left(\frac{df+1}{2}\right)\right) - \log\left(\Gamma\left(\frac{df}{2}\right)\right) - 0.5 \times \left[\log\left((df-2)h_t\right) - (df+1) \times \log\left(1 + \frac{u_t^2}{h_t(df-2)}\right)\right]\right\}$$

where n is the sample size, df is the degrees of freedom and $\Gamma(\cdot)$ is the gamma function.

the standard GARCH and the augmented model, with the mixing variables in the conditional variance equation, the econometric results are presented in table 2.

Table 2. GARCH estimation for the short sterling.

$$\Delta F_t = \gamma_0 + u_t$$
$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + b_1 h_{t-1} + \delta V_t$$

$\hat{\gamma}_0$	$\hat{a}_0$	$\hat{a}_1$	$\hat{b}_1$	$\hat{\delta}$	$L\ell$	Q_1	Q_2
-0.606[d]	0.483[d]	0.921	0.945	-	8761	7.07	3.61
(-1.98)	(3.27)	(4.38)	(227.8)	-			
Volume							
-0.582[d]	0.914[d]	0.011	0.017	0.092	7866	6.71	1.37
(-2.01)	(30.2)	(5.09)	(1.89)	(37.3)			
Open Interest							
-0.599[d]	0.453[d]	0.089	0.481	0.098	8763	7.22	3.44
(-1.99)	(3.20)	(4.38)	(23.4)	(2.02)			

Asymptotic t -values in parentheses.
$L\ell$ is the function value.
Q = Ljung-Box test, Q_1: $\left(u_t/\sqrt{h_t}\right)$, Q_2: $\left(u_t/\sqrt{h_t}\right)^2$. Critical value at 5% level: X^2 (12) = 21.03
[d] Coefficients multiplied by 10^2.

The results clearly indicate that the liquidity variables along with the GARCH coefficients remain significant in our sample. The estimation shows that there is a positive correlation between volatility and liquidity variables. A second interesting result is that the persistence of variance, as measured by the coefficients $a_1 + b_1$, has been considerably reduced after accounting for the mixing variable. It is also evident that the exclusion of liquidity variables in the model has made the persistence of shocks to last over future horizons. This is not surprising, though, since the empirical plausibility of integrated GARCH models has already been established by the findings that interest rates typically exhibit parameters, which are not in the stationary region [Engle *et al.* (1987), Bollerslev *et al.* (1988), Najand & Yung (1991)]. Thirdly, in our empirical framework volatility is essentially parameterized as a function of information arrival and past volatility (old news)[9], which their coefficients are found highly significant. This is interesting since it shows that liquidity variables do not absorb the information, included in last period's shocks

[9] The news is proxied by the last period's forecast error or unexpected interest rate shock. The conditional lagged volatility can be also expressed as $h_{t-1} = a_0 + a_1 u_{t-2}^2 + \beta_1 h_{t-2}$ which in turn is a function of past news through the "news" coefficient (a_1).

and past volatility, but remain part of the information set. Our results are in contrast with those of Lamoureux & Lastrapes (1990) and partly in line with those of Sharma *et al.* (1996) and Najand & Yung (1991). The latter found significant coefficients for the volume in some subperiods of their sample. On the other hand, Lamoureux & Lastrapes (1990) reveal that with the introduction of volume the GARCH effects disappear and only a small fraction, approximately 13%, of their stock returns exhibit an ARCH effect. Finally, specification tests for the models show insignificant values for the period under consideration. The evaluation is done using the Ljung-Box Q statistic on standardized and squared standardized residuals. The former examines the existence of serial correlation up to 12 lags, while the latter tests whether our model adequately represents the variance's dynamics.

Nevertheless, before any final inferences are drawn there are two things worth noting. First, if rates and the mixing variable are jointly determined, or information is not exogenous, then equation (1) may be not unbiased. Thus, the fact that the contemporaneous volume is employed this may invalidate our estimates. Second, our sample spans the period from the very beginning of the futures market until very recently. That is, one could easily argue that the market has experienced considerable changes and, thus, discriminating between an early and mature period of trading may provide additional and more comprehensive support to our findings. A closer examination of the daily trades indicates that until 1990 the maximum transaction is 120,418 while from 1991 onwards the maximum comes up to 552,954. This is the result of the market moving from an early stage to a mature one, where trading becomes more intense and frequent. Thus, the next step is to estimate GARCH models using two different sample periods. A distinction between early (until 1990) and mature (from 1991) futures period is necessary and this is what the paper turns to next. The results of the estimation process for both periods are presented in table 3.

Table 3. Split sample GARCH estimation for the short sterling.

$$\Delta F_t = \gamma_0 + u_t$$
$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + b_1 h_{t-1} + \delta V_t$$

$\hat{\gamma}_0$	$\hat{a}_0$	$\hat{a}_1$	$\hat{b}_1$	$\hat{\delta}$	$L\ell$	Q_1	Q_2
			Early Period				
0.261[d]	0.515[d]	0.127	0.902	-	2693	10.2	3.75
(2.11)	(2.95)	(4.02)	(66.5)	-			
Volume							
0.247[d]	0.853[d]	0.416	0.336	0.013	2677	9.74	3.04
(1.97)	(6.63)	(4.91)	(7.35)	(7.48)			
Open Interest							
0.272[d]	0.526[d]	0.132	0.299	0.019	2694	10.07	3.82
(2.09)	(2.81)	(3.78)	(63.1)	(6.69)			
			Mature Period				
-0.100[d]	0.395[d]	0.030	0.965	-	6096	5.45	4.10
(-2.13)	(3.47)	(3.84)	(217)	-			
Volume							
-0.118[d]	0.399[d]	0.029	0.220	0.092	5982	6.52	3.27
(-1.99)	(4.34)	(2.89)	(4.15)	(6.11)			
Open Interest							
-0.163[d]	0.371[d]	0.014	0.379	0.091	5643	6.69	12.00
(-2.22)	(3.25)	(2.02)	(2.05)	(9.21)			

Asymptotic t -values in parentheses.
$L\ell$ is the function value.
Q = Ljung-Box test, Q_1: $\left(u_t/\sqrt{h_t}\right)$, Q_2: $\left(u_t/\sqrt{h_t}\right)^2$. Critical value at 5% level: X^2 (12) = 21.03
[d] Coefficients multiplied by 10^2.

It is clear that the GARCH effects remain highly significant in both the early and mature period along with the liquidity variables. It is also evident that the GARCH effects have decreased with the inclusion of the mixing variable in the conditional variance equation. The coefficients on both volume and open interest are smaller in the early period compared to the mature stage. The impact of the liquidity variables on volatility has significantly increased during the period of intense trading. Coefficient analysis, between the two periods, shows that the difference is statistically significant at the 1% level of significance with high t values. Thus, in the mature period the volume does have a bigger impact on volatility, but still smaller compared to the squared unexpected change and past volatility variables. These findings are consistent with our previous results, since the information set has expanded by escalating the importance of volume in determining volatility. Yet, one could easily argue that contemporaneous volume should absorb all information and remove the GARCH effects. The persistence of shocks, however, as measured by a_1 + b_1, has decreased in the mature period but still remains unabsorbed by the market and thus determining volatility.

As earlier mentioned, the other point to which attention must be drawn is that the contemporaneous use of volume may be a problem in our estimation process. It is established by now that in many financial data an equation may be part of a larger system of simultaneous equations. Thus, if volume is depended to this system and correlated with the residuals in the stochastic part of the model, then statistical estimates would probably be inconsistent, since the likelihood function obtained conditional on these variables invalidates the estimation process. This issue is also known as simultaneity bias problem. Harvey (1989) suggests that in order to avoid such problem a lagged value of the variable should be used which automatically is characterized as predetermined. Therefore, the paper proceeds with the estimation of the stochastic equation re-modelled i.e. using the lagged values of the mixing variables. The estimation results for the whole period are presented in table 4.

Table 4. GARCH estimation using lagged volume and open interest.

$$\Delta F_t = \gamma_0 + u_t$$
$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + b_1 h_{t-1} + \delta V_{t-1}$$

$\hat{\gamma}_0$	$\hat{a}_0$	$\hat{a}_1$	$\hat{b}_1$	$\hat{\delta}$	$L\ell$	Q_1	Q_2
Volume							
-0.494[d]	0.142[d]	0.105	0.723	0.081	8779	6.01	3.74
(-1.96)	(3.55)	(4.41)	(5.11)	(3.49)			
Open Interest							
-0.605[d]	0.466[d]	0.090	0.799	0.069	8767	6.79	3.38
(-1.98)	(3.32)	(4.51)	(4.86)	(2.25)			

Asymptotic *t* -values in parentheses.
$L\ell$ is the function value.
Q = Ljung-Box test, Q_1: $\left(u_t/\sqrt{h_t}\right)$, Q_2: $\left(u_t/\sqrt{h_t}\right)^2$. Critical value at 5% level: X^2 (12) = 21.03
[d] Coefficients multiplied by 10^2.

The persistence of GARCH effects is still evident along with the statistical significance of the mixing variables. The empirical findings are in contrast with those of Lamoureux & Lastrapes (1990) since their research suggests that the volume is a poor instrument and, therefore, having little explanatory power. However, they are in line with Najand & Yung (1991), who found significant results for most of the sub-periods between 1984 and 1989. Moreover, the magnitude of the effect of these variables still remains smaller compared with the volatility variables. Finally, the estimation is repeated for the two sub-periods separately, using again the lagged liquidity variables, and the results are presented in table 5.

Table 5. Split sample GARCH estimation using lagged volume and open interest.

$$\Delta F_t = \gamma_0 + u_t$$
$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + b_1 h_{t-1} + \delta V_{t-1}$$

$\hat{\gamma}_0$	$\hat{a}_0$	$\hat{a}_1$	$\hat{b}_1$	$\hat{\delta}$	$L\ell$	Q_1	Q_2
			Early Period				
Volume							
0.420^d	0.433^d	0.321	0.601	0.036	2632	9.91	4.20
(2.35)	(2.97)	(3.21)	(5.57)	(4.99)			
Open Interest							
0.258^d	0.525^d	0.128	0.802	0.037	2694	10.23	3.74
(1.95)	(2.85)	(3.83)	(4.44)	(6.69)			
			Mature Period				
Volume							
-0.318^d	0.999^d	0.097	0.208	0.099	5072	11.4	13.5
(-1.85)	(367)	(2.21)	(3.47)	(5.52)			
Open Interest							
-0.104^d	0.331^d	0.024	0.211	0.098	6103	6.14	7.46
(-1.71)	(3.75)	(4.08)	(6.02)	(3.99)			

Asymptotic *t* -values in parentheses.
$L\ell$ is the function value.
Q = Ljung-Box test, Q_1: $\left(u_t/\sqrt{h_t}\right)$, Q_2: $\left(u_t/\sqrt{h_t}\right)^2$. Critical value at 5% level: X^2 (12) = 21.03
^{d} Coefficients multiplied by 10^2.

The last set of results is in line with the previous ones, suggesting the persistence of volatility factors and the significance of the mixing variables. A closer examination, however, reveals that although the mixing variables remain small; the effect is much bigger in the mature period with a simultaneous decrease in the GARCH estimates. Although a similar comment is made for the contemporaneous volume, comparing table 3 and 5 the effect of the lagged mixing variable is bigger as the proportionate reduction in the GARCH estimates is higher in the last table. Coefficient analysis shows that the difference between the two mixing variables in the early and mature period is significant, at the 1% level of significance, with high *t* values. Using a lagged liquidity variable the results are more consistent with the fact that volume plays an important role as a proxy for the information arrival. Past shocks, as measured by $a_1 + b_1$, remain the same for both equations in the early period and reduce noticeably in the mature period. Moreover, a comparison with the standard conditional volatility model (table 3 excluding the mixing variable) reveals that volatility shocks do not persist over time and are considerably smaller in size. This could be attributed to the fact that information is absorbed quicker by the market now. One could also argue that this is the result of the integration of financial markets or markets have become more efficient where agents incorporate their errors (or yesterday's news) in their actions reflected through their trading positions.

4. CONCLUSION

The empirical evidence presented in this paper shows that Short Sterling movements can be described by a conditional heteroscedastic process, which allows serial correlation in the second moment. Liquidity variables, such as trading volume and open interest, have a significant positive effect on rate variability. More specifically, the results suggest that the volume-volatility relationship is positive, but volume has not removed the GARCH effects indicating that the latter remain part of the information set. The volume found to have a significant coefficient by either using a contemporaneous or lagged value of it. These results come in contrast with previous studies, which either found a statistical significant coefficient of the contemporaneous variable only and no GARCH effects, or found significant coefficient of the lagged volume only and persistence of the GARCH effect.

Furthermore, both volume and open interest proved to be exogenous at least with the current sample and methodology employed. The findings further indicate that the liquidity variables may not be a good proxy of information arrival or information itself. The latter could be further explored as an avenue for future research where one could assume the existence of noise traders, which in turn implies that although volume contains information, this may be irrelevant to volatility. Finally, one could argue that volume provides information about the dispersion and quality of information signals, rather than representing the information signal itself.

APPENDIX 1

The Maximum Likelihood Estimation Procedure

Because the variance of u_t depends upon the unobservable past values of u, the h function and, hence, the likelihood function has to be generated recursively. The average log likelihood function for a sample of T observations, excluding a constant term, is:

$$L\ell_T = T^{-1} \sum_{t=1}^{T} l\ell_t \tag{5}$$

Where the log likelihood term for each entry t takes the form:

$$l\ell_t = -\tfrac{1}{2}[\ln h_t + (R_t - X_t' \gamma)\, h_t^{-1}] \tag{6}$$

Differentiating with respect to the variance parameter $w' = (a_0, a_1, ..., a_q, b_1, ..., b_p, \delta)$, we obtain:

$$\frac{\partial\, l\ell_t}{\partial\, w} = \tfrac{1}{2} h_t^{-1} \frac{\partial\, h_t}{\partial\, w}\left(\frac{u_t^2}{h_t} - 1\right) \tag{7}$$

$$\frac{\partial^2\, l\ell_t}{\partial\, w\, \partial\, w'} = \left(\frac{u_t^2}{h_t} - 1\right) \frac{\partial}{\partial\, w'} \left\langle \tfrac{1}{2} h_t^{-1} \frac{\partial\, h_t}{\partial\, w} \right\rangle - \tfrac{1}{2} h_t^{-2} \frac{\partial\, h_t}{\partial\, w} \frac{\partial\, h_t}{\partial\, w'} \frac{u_t^2}{h_t} \tag{8}$$

where

$$\frac{\partial\, h_t}{\partial\, w} = v_t + \sum_{i=1}^{p} b_i \frac{\partial\, h_{t-i}}{\partial\, w} \tag{9}$$

$$h_t = w\, v' \qquad\qquad v' = (1,\, L^1 u_t^2, \ldots\ldots, L^q u_t^2,\, L^1 h_t, \ldots\ldots, L^p h_t,\, g)$$

It is obvious the inclusion of the recursive part in equation (9). Since it is necessary for the recursive estimation to have pre-sample values ($t \leq 0$) for h_t and u_t, it is simple to obtain $T^{-1} \sum_{i=1}^{T} u_t^2$.

The differentiation with respect to the mean parameters yields:

$$\frac{\partial\, l\ell_t}{\partial\, \gamma} = u_t\; X_t\; h_t^{-1} + \tfrac{1}{2} h_t \frac{\partial\, h_t}{\partial\, \gamma}\left(\frac{u_t^2}{h_t} - 1\right) \tag{10}$$

$$\begin{aligned}\frac{\partial^2\, l\ell_t}{\partial\, \gamma\, \partial\, \gamma'} &= -h_t^{-1} X_t X_t' - \tfrac{1}{2} h_t^{-2} \frac{\partial\, h_t}{\partial\, \gamma} \frac{\partial\, h_t}{\partial\, \gamma'} \left(\frac{u_t^2}{h_t}\right) \\ &\quad - 2 h_t^{-2} u_t X_t \frac{\partial\, h_t}{\partial\, \gamma} + \left(\frac{u_t^2}{h_t} - 1\right) \frac{\partial}{\partial\, \gamma'} \left\langle \tfrac{1}{2} h_t^{-1} \frac{\partial\, h_t}{\partial\, \gamma} \right\rangle\end{aligned} \tag{11}$$

where

$$\frac{\partial h_t}{\partial \gamma} = -2\sum_{i=1}^{q} a_i \; X_{t-i} \; u_{t-i} + \sum_{j=1}^{p} b_j \; \frac{\partial h_{t-j}}{\partial b} \tag{12}$$

Again the single difference with the simple ARCH(q) regression model is the inclusion of the recursive part in equation (12). An iterative procedure will be used to obtain maximum likelihood estimates, and second order efficiency. Let s^i denote the parameter estimates after the i iteration. Successive values of these parameters are estimated as follows:

$$s^{i+1} = s^i + c_i \left(\sum_{t=1}^{T} \frac{\partial l\ell_t}{\partial s} \frac{\partial l\ell_t}{\partial s'} \right)^{-1} \sum_{t=1}^{T} \frac{\partial l\ell_t}{\partial s} \tag{13}$$

The c_i is a variable step length chosen to maximise the likelihood function and the $\partial l\ell_t / \partial s$ is evaluated at s^i. The aforementioned algorithm is the Berndt, Hall, Hall & Hausman (1974) or usually called BHHH[10] and the recursive terms in equation (9) and equation (12) make the procedure very complicated. A detailed analysis of this method is beyond the scope of this research.

[10] However, other algorithms are also available like the Broyden, Fletcher, Goldfarb & Shanno -BFGS, Davidon, Fletcher & Powell -DFP, and SIMPLEX. The SIMPLEX is a sophisticated type of search algorithm, which does not require derivatives; while the rest, require twice-differentiable formulas. The major disadvantage of the former is that it cannot provide standard errors for the estimated parameters.

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