Could exchange rates just be chaotic?

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Abstract

Economists increasingly focus their analysis of exchange rates on "real" exchange rates. These are not useful for the analysis of daily time series of exchange rates because of the infrequency of the publication of the deflator for the nominal exchange rates –either a CPI or an implicit price deflator. They are especially not useful if the intention of analysis is to provide some forecasting ability.

The presence of central bank intervention in a number of exchange rates which is focused on the restoration of orderly conditions in exchange rate markets creates an expectation that exchange rate time series may not have an underlying structure that may be identifiable in the face of this intervention.

This paper provides a chaotic analysis of nominal exchange rate time series for the New Zealand dollar over the period 1983 to 2004, during which there was no central bank intervention. One of the features that distinguishes our analysis from earlier studies is that the data series length meets the criteria established by Eckmann and Ruelle (1992) which means that the results allow us to make some conclusions as to the actual applicability of chaotic analysis to exchange rates.

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This paper provides a chaotic analysis of nominal exchange rate time series for the New Zealand dollar over the period 1986 to 2004, during which there was no central bank intervention. One of the features that distinguishes our analysis from earlier studies is that the data series length meets the criteria established by Eckmann and Ruelle (1992) which means that the results allow us to make some conclusions as to the actual applicability of chaotic analysis to exchange rates. We then consider the

key elements required to establish a chaotic structure and apply the analysis to the daily $NZ/$US exchange rates over the period March 1985 to March 2004.
We discuss the results and compare them with results for the $C/$US and $A/$US floating rate periods. We draw some conclusions with respect to the relative chaotic structures of the three exchange rates.

Due to the similarity between the volatility of exchange rate movements with the recipe for chaos described by da Silva (1999, at p. 2) :"Starting from an initial deviation a given endogenous variable perpetually fluctuates around equilibrium. As the deviations become too large, the centrifugal force brings the variable back towards equilibrium, while once back to the neighbourhood of equilibrium, the centrifugal force forces it to diverge again." We might well expect a chaotic structure to be present in exchange rate series.
In small open economies such as New Zealand the exchange rate is a crucial asset price. While there is documentation of central bank intervention in the "floating " exchange rates of Australia and Canada, the Reserve Bank of New Zealand has never intervened directly in the currency market since the March 1985 float, although it does adjust its monetary policies in order to influence the currency market.
In March 2004 a government policy change allowed the Reserve Bank of New Zealand to intervene in the foreign exchange market in the future.
The $NZ/$US exchange rate series from 1986 to 2003 is shown in Figure 1. The cyclical variation of the New Zealand exchange rate has been greater since it floated in 1985. However, fluctuation across the cycle in a low inflation environment recently has been a concern to Reserve Bank of New Zealand. Figure 1 shows the NZ$/US$ exchange rate since it floated. The New Zealand dollar depreciated 35% in five years after 1996 and then appreciated 75% within three years. The exchange rate moved to extreme ends within short period which was not supported by the underlying economic situation. The exchange rate series does not show any long-term trend. The series had a sharp turning point close to each US presidential elections after it was fully floated, and the last turning point, not unexpectedly, was on September 11, 2001.

Figure 1 The NZ/$US exchange rate 1986-2004

II

While there have been a number of earlier applications of chaotic analysis to exchange rate time series these have tended to focus on data lengths that breach the minimum data length requirements for this analysis to be applicable found in the physical sciences . note that 3981 observations is the minimum required for this analysis.

In this study we used the daily exchange rate data published by the Reserve Bank of New Zealand which was available from July 1 1986 to 18 March 2004. This provides us with 4390 data points which is greater than the 3981 required to predict a correlation dimension below 4 correctly.

Richards (2000) demonstrated fractal properties are characteristic of foreign exchange markets across a broad range of countries (Australia, Canada, UK, Japan etc.).

Schwartz and Yousefi (2003) investigated low correlation dimension in a number of exchange rates and established low fractal dimension in the DEM/USD, GBP/JPY, GBP/USD and JPY/USD exchange rate series.

In this paper we determine the correlation dimension of the time series using the Grassberger and Procaccia (1984) method. If C (r.m) is the correlation integral that measures the fraction of total numbers of pairs between lagged embedded series (x_i, x_{i+1}, x_{i+2}, .. x_{i+m-1}) and (x_j, x_{j+1}, x_{j+2} , .. x_{j+m-1}) such that the difference between them is no more than r, then the correlation dimension is defined as

$$D_{c = \lim(r\to} 0)\log C (r,m)/\log (r)$$

In this limit equation C (r,m) is the correlation integral calculated for each embedded dimension by increasing the distance r. The Grassberger and Procaccia (1984), is the most frequently used, and considered by physical scientists as the best method, to measure the fractal dimension of the attractor (Barnett and Chen, 1988).This process increases the embedding dimension so that the attractor is completely unfolded once the value of the specific invariant saturates. A low correlation dimension (less than 5) is classified as a chaotic dynamic, since a higher dimension is virtually untraceable with current computing capability.

To test the New Zealand dollar exchange rate for deterministic chaos we first investigate the correlation dimension which is a necessary but not sufficient condition for the identification of a chaotic structure . Next we search for the second condition necessary to complete the identification of chaotic structure, that of a positive Lyapunov exponent.. A main source of chaos is the high sensitivity of the system output to the initial values of input and slight changes of system parameter values. Two trajectories on the attractor that initially separate slightly from each other will diverge exponentially in one direction while converging to each other exponentially in another direction (Gollub, 2000). However, the solution paths will remain within a bounded set if the dynamic system is dissipative. By measuring separation of nearby points in phase space, existence of chaos may be examined. The divergence can be measured by the Lyapunov exponents. The number of Lyapunov exponents is equal to the dimension of the underlying dynamic system. A positive exponent measures the exponential divergence of nearby points and a negative exponent measures the exponential convergence. Chaos exists only when the largest Lyapunov exponent is positive..

Mathematically, the Lyapunov exponent (Gulick, 1992) is developed to measure the sensitivity to initial conditions of the underlying dynamics. In a relaxed definition of chaos, the existence of a positive Lyapunov is enough to indicate the existence of underlying chaotic dynamics. However, a lot of data series, including random series, have positive Lyapunov exponents. Hence, a positive Lyapunov alone is not enough to prove the existence of chaotic dynamics. It needs to be combined with the correlation dimension results..

III

In this section we present the results of our chaotic analysis of the $NZ/$US exchange rate series. In Figure 2 we show the evolution of the correlation dimension since 1985.

The correlation dimension of $NZ/$US exchange series has not changed since its floating of exchange rate in March 1985. Though it is not low-dimensional chaotic, it is less than 5 and the system is solvable.

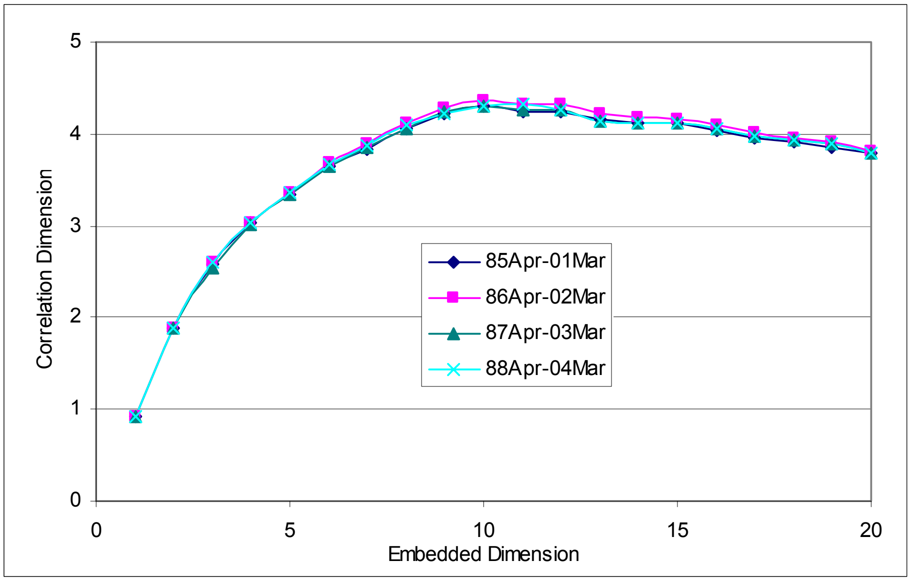

Figure 2 Evolution of the correlation dimension

In Figure 3 we compare the $NZ/$US exchange series with other cross country exchange rates. The results shows that complexity of $NZ/$US exchange series is less compared to other series

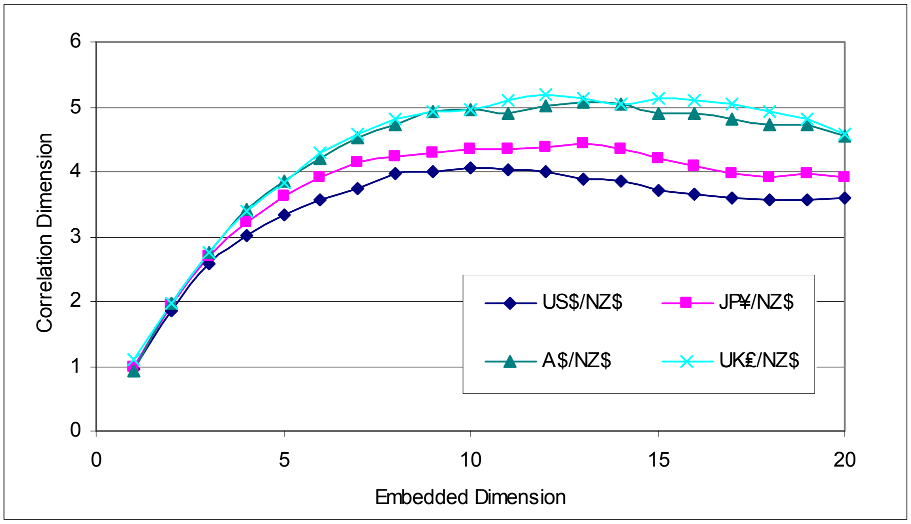

Figure 3 Correlation dimension of NZ exchange rate series.

With respect to the Lyapunov exponent, a positive exponent measures the exponential divergence of nearby points. A BDS statistic of less than 5 is an alternative measure of nonlinearity in time series. The Lyapunov exponent and BDS statistics for above four rate series are given in Table 1.

	NZ$/US$	NZ$/JP¥	NZ$/A$	NZ$/UK£
Lyapunov Exponent	0.132	0.135	0.193	0.190
BDS Statistics	3.331	3.651	4.263	4.255
Kurtosis	4.39	3.166	3.819	3.876

Table 1 The Lyapunov exponents , BDS statistics and kurtosis for $NZ series.

As can be seen from Table 1 The NZ$/US$ series has the lowest BDS statistics which is consistent with the lowest correlation dimension found. The kurtosis of the different series are also greater than 3 in all the cases.
We have found

- a low correlation dimension and
- a positive Lyapunov exponent

which allows us to conclude that the $NZ/$US exhibits deterministic chaos.

Finally we compare in Figure 4 these results with similar analysis of the \$C/\$US and \$A/\$US exchange rate series for the period 1988 to 2003. In both the Canadian and Australian cases the respective central banks have intervened in the exchange markets over the period considered. This gives us a good comparison between these currencies and the "nil intervention" approach of the reserve Bank of New Zealand.

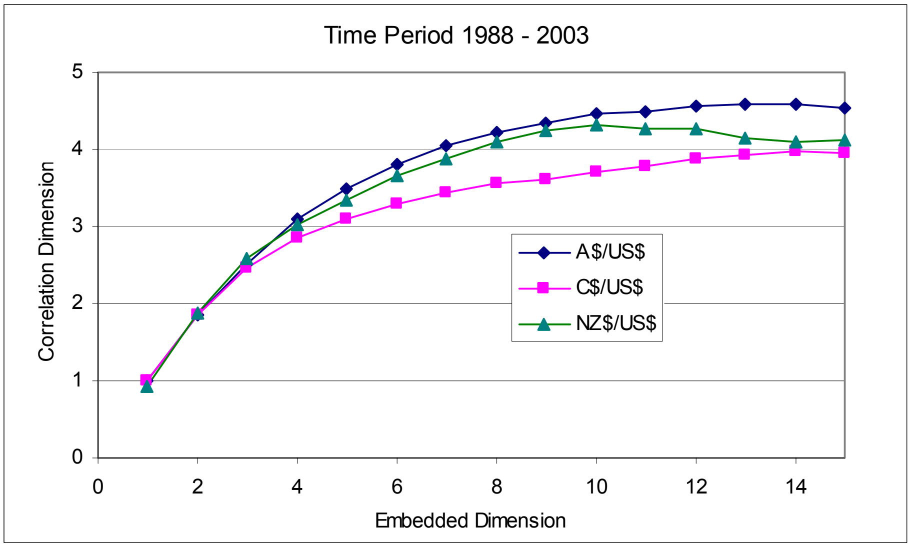

Figure 4 Correlation dimensions: \$A/\$US \$C/\$US; \$NZ/\$US

Figure 4 shows the correlation dimension of \$NZ/\$US exchange rate compared with \$C/\$US and \$A/\$US exchange rate series for data of 1988 to 2003. These results show that there is no significance difference between the NZ\$/US\$ series which had no intervention and the other two series in which there has been a history of intervention.

IV

In this study we have investigated the correlation dimension of the 'nil intervention' NZ\$/US\$ exchange rate series and found that low dimensional deterministic structure exists in the system. We have also found a positive Lyapunov exponent which confirms that the structure of the time series could reasonably be described as chaotic, We compared the results with A\$/US\$ and C\$/US\$ exchange rate series where a history of intervention has been reported by the respective central banks. Our results show that there is no significant difference between correlation dimensions .and that all three series exhibit positive Lyapunov exponents. We are able to conclude from this analysis that exchange rates just might be chaotic.

We also conclude that it is unlikely that the planned Reserve Bank of New Zealand intervention in the exchange rate market will be sufficient to disturb the underlying structure or reduce variability of the exchange rate.

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