The value of tax shields and the risk of the net increase of debt

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Abstract

The value of tax shields in a world with no leverage cost is the tax rate times the current debt, plus the tax rate times the present value of the net increases of debt. This expression is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company.

The value of tax shields depends only on the nature of the stochastic process of the net increase of debt; it does not depend on the nature of the stochastic process of the free cash flow.

For perpetual debt, the value of tax shields is the debt times the tax rate. When the company is expected to repay the current debt without issuing new debt, the value of tax shields is the present value of the interest times the tax rate, discounted at the required return to debt.

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1. Introduction

There is no consensus in the existing literature regarding the correct way to compute the value of tax shields. Most authors think of calculating the value of the tax shield in terms of the appropriate present value of the tax savings due to interest payments on debt. Myers (1974) proposes that the tax savings be discounted at the cost of debt, while Harris and Pringle (1985) propose that they be discounted at the cost of capital for the unlevered firm. Reflecting this lack of consensus, Copeland et al. (2000, p. 482) claim that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct." In this paper, I show that a consistent way to estimate the value of the tax savings is by thinking of them not as the present value of a set of cash flows, but as the difference between the present values of two different sets of cash flows: flows to the unlevered firm and flows to the levered firm.

I show that the value of tax shields in a world with no leverage cost is the tax rate times the debt, plus the tax rate times the present value of the net increases of debt. This expression is the difference between the present values of two different cash flows, each with its own risk: the present value of taxes for the unlevered company and the present value of taxes for the levered company.

For perpetual debt, the value of tax shields is equal to the tax rate times the value of debt. When the company is expected to repay the current debt without issuing new debt, Myers (1974) applies, and the value of tax shields is the present value of the interest times the tax rate, discounted at the required return to debt. For constant growth companies, and under certain assumptions, the value of tax shields in a world with no leverage costs is the present value of the debt times the tax rate times the required return to the unlevered equity, discounted at the required return to the unlevered equity (Ku).

The paper also shows that some commonly used methodologies for calculating the value of tax shields, including Harris and Pringle (1985), Miles and Ezzell (1980), and Ruback (2002), are incorrect for growing perpetuities.

The paper is organized as follows. Section 2 follows a new method to prove that the value of tax shields in a world without leverage costs is equal to the tax rate times the value of debt (DT), plus the tax rate times the present value of the net increases of debt. Section 2 also applies this general result to specific situations, and derives the relation between the required return on assets and the required return on equity for perpetuities in a world without leverage costs. The corresponding relation between the beta of the levered equity, the beta of the unlevered equity, and the beta of debt is also derived. Section 3 revises and analyzes the existing financial literature on the value of tax shields. Most of the existing approaches, including Harris and Pringle, Miles and Ezzell, and Ruback, yield inconsistent results regarding the present value of the net increases of debt. Finally, Section 4 concludes.

2. Value of tax shields and the stochastic process of net debt increases

The present value of debt (D) plus that of the equity (E) of the levered company is equal to the value of the unlevered company (Vu) plus the value of tax shields due to interest payments (VTS):

$$E_t + D_t = Vu_t + VTS_t \qquad \textbf{(1)}$$

In the literature, the value of tax shields defines the increase in the company's value as a result of the tax saving obtained by the payment of interest. If leverage costs do not exist, Eq. (1) could be stated as follows:

$$Vu_t + Gu_t = E_t + D_t + G_{Lt} \qquad \textbf{(2)}$$

where Gu is the present value of the taxes paid by the unlevered company and G_L is the present value of the taxes paid by the levered company. Eq. (2) means that the total value of the unlevered company (left-hand side of the equation) is equal to the total value of the levered company (right-hand side of the equation). Total value is the enterprise value (often called the value of the firm) plus the present value of taxes. Please note that Eq. (2) assumes that expected free cash flows are independent of leverage. When leverage costs do exist, the total value of the levered company is lower than the total value of the unlevered company. A world with leverage cost is characterized by the following relation:

$$Vu_t + Gu_t = E_t + D_t + G_{Lt} + \text{Leverage Cost} > E_t + D_t + G_{Lt} \qquad \textbf{(3)}$$

Leverage cost is the reduction in the company's value due to the use of debt.

From (1) and (2), it is clear that VTS is

$$VTS_t = Gu_t - G_{Lt} \qquad \textbf{(4)}$$

Note that the value of tax shields is not the present value (PV) of tax shields. It is the difference between the PVs of two flows with different risk: the PV of the taxes paid by the unlevered company (Gu) and the PV of the taxes paid by the levered company (G_L).

It is quite easy to prove that the relation between the profit after tax of the levered company (PAT_L) and the equity cash flow (ECF) is:

$$ECF_t = PAT_{Lt} - \Delta NFA_t - \Delta WCR_t + \Delta D_t \quad \textbf{(5)}$$

Where:

ΔWCR_t = Increase of Working Capital Requirements in period t.

ΔNFA_t = Increase of Net Fixed Assets in period t.

$\Delta D_t = D_t - D_{t-1}$ = Increase of Debt in period t.

The relation between the free cash flow (FCF) and the profit after tax of the unlevered company (PATu) is:

$$FCF_t = PATu_t - \Delta NFA_t - \Delta WCR_t \quad \textbf{(6)}$$

T is the tax rate. As the relation between the profit after tax (PAT) and the profit before tax (PBT) is PAT = PBT (1-T), the taxes paid every year by the unlevered company ($Taxes_U$) are

$$Taxes_{Ut} = T \cdot PBTu_t = [T/(1+T)] \ (FCF_t + \Delta NFA_t + \Delta WCR_t) \quad \textbf{(7)}$$

And the taxes paid by the levered company are:

$$Taxes_{Lt} = T \cdot PBT_t = [T/(1+T)] \ (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \quad \textbf{(8)}$$

$PV_0[\cdot]$ is the present value operator. The present value in t=0 of equations (7) and (8) are:

$$Gu_0 = [T/(1+T)] \ (Vu_0 + PV_0[\Delta NFA_t + \Delta WCR_t]) \quad \textbf{(9)}$$

$$G_{L0} = [T/(1+T)] \ (E_0 + PV_0[\Delta NFA_t + \Delta WCR_t] - PV_0[\Delta D_t]) \quad \textbf{(10)}$$

Equation (9) means that the present value of the taxes paid by the unlevered company (Gu) is the present value of the taxes paid every year ($Taxes_U$). The value of the unlevered equity (Vu_0) is: $Vu_0 = PV_0[FCF_t] = PV_0[E\{FCF_t\};\ Ku]$. $E\{\cdot\}$ is the expected value operator.

Equation (10) means that the present value of the taxes paid by the levered company (G_L) is the present value of the taxes paid every year ($Taxes_L$). The value of the levered equity (E_0) is: $E_0 = PV_0[ECF_t] = PV_0[E\{ECF_t\};\ Ke]$.

Following equation (4), the value of tax shields is the difference between (9) and (10):

$VTS_0 = Gu_0 - G_{L0} = [T/(1+T)]\ (Vu_0 – E_0 + PV_0[\Delta D_t])$ **(11)**

As, according to equation (1), $Vu_0 – E_0 = D_0 - VTS_0$,

$VTS_0 = [T/(1+T)]\ (D_0 - VTS_0 + PV_0[\Delta D_t])$. And the value of tax shields is:

$\mathbf{VTS_0 = T\cdot D_0 + T\cdot PV_0[\Delta D_t]}$ **(12)**

We do not know which are the correct values of Gu and G_L, but we know the value of the difference, provided we can value $PV_0[\Delta D_t]$, the present value of the net debt increases.

Equation (12) shows that the value of tax shields depends *only* upon the nature of the stochastic process of the net increase of debt, and does not depend upon the nature of the stochastic process of the free cash flow[1]. The problem of equation (12) is how to calculate $PV_0[\Delta D_t]$, which requires knowing the appropriate discount rate to apply to the expected value of the increase of debt.

Equation (12) is also derived in Appendix 1 using pricing kernels.

Now, we apply (12) to specific situations.

2.1. Perpetual debt

If the debt is a constant perpetuity (a consol), $PV_0[\Delta D_t] = 0$, and

$VTS_0 = T\cdot D_0$ (13)

This result is far from being a new idea. Brealey and Myers (2000), Modigliani and Miller (1963), Taggart (1991), Copeland et al. (2000), Fernández (2004) and many others report it. However, the way of deriving it is new. Most of these papers reach this result by arguing that the appropriate way of computing the value of the tax shield is to consider a certain flow DT multiplied by some measure of cost of funds, α, and then discounting that flow at the same rate α. At first glance, α could be

[1] If the nominal value of debt (N) is not equal to the value of debt (D), because the interest rate (r) is different from the required return to debt flows (Kd), equation (12) is: $VTS_0 = T\cdot D_0 + T\cdot PV_0[\Delta N_t]$. The relationship between D and N is: $D_0 = PV_0[\Delta N_t] + PV_0[N_t \cdot r_t]$.

[4] We use Kd so as not to complicate the notation. It should be Kd_t, a different rate following the yield curve. Using Kd we may also think of a flat yield curve.

anything, related or unrelated to the company that we are valuing. Modigliani and Miller (1963) and Sick (1990) argue that α is the risk-free rate (R_F). Myers (1974) assumes that α is the cost of debt (Kd) and says that the value of tax shields is the present value of the tax savings (D·Kd·T) discounted at the cost of debt (Kd). Fernández (2004) argues that α is the required return to unlevered equity (Ku).

2.2. Debt of one-year maturity but perpetually rolled-over

As in the previous case, $E\{D_t\} = D_0$, but the debt is expected to be rolled-over every year. The appropriate discount rate for the cash flows due to the existing debt is Kd.[4] Define K_{ND} as the appropriate discount rate for the new debt that must be obtained every year, then:

Present value of obtaining the new debt every year[5] = D_0 / K_{ND}

Present value of the principal repayments at the end of every year[6] = $D_0 (1+K_{ND}) / [(1+Kd)\ K_{ND}]$

$PV_0[\Delta D_t]$ is the difference of these two expressions. Then:

$$PV_0[\Delta D_t] = - D_0 (K_{ND} - Kd) / [(1+Kd)\ K_{ND}] \tag{14}$$

If $K_{ND} = Kd$, then $PV_0[\Delta D_t] = 0$

In a constant perpetuity ($E\{FCF_t\} = FCF_0$), it seems reasonable that, if we do not expect credit rationing, $K_{ND} = Kd$, which means that the risk associated with the repayment of the current debt and interest (Kd) is equivalent to the risk associated with obtaining an equivalent amount of debt at the same time (K_{ND}).

2.3. Debt is proportional to the Equity value

This is the assumption made by Miles and Ezzell (1980) and Arzac and Glosten (2004), who claim that if $D_t = L \cdot E_t$, then the value of tax shields for perpetuities growing at a constant rate g is:

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[5] Present value of obtaining the new debt every year = $D/(1+K_{ND}) + D/(1+K_{ND})^2 + D/(1+K_{ND})^3 + \ldots$ because $D = E\{D_t\}$, where D_t is the new debt obtained at the end of year t (beginning of t+1).

[6] The present value of the principal repayment at the end of year 1 is $D/(1+Kd)$.
The present value of the principal repayment at the end of year 2 is $D/[(1+Kd)(1+K_{ND})]$.
The present value of the principal repayment at the end of year t is $D/[(1+Kd)(1+K_{ND})^{t-1}]$.
Because $D = E\{D_t\}$, where D_t is the debt repayment at the end of year t.

$$VTS_0 = \frac{D_0 Kd\, T}{(Ku - g)} \frac{(1+Ku)}{(1+Kd)} \qquad (15)$$

Substituting (15) in (12), we get:

$$PV_0[? D_t] = D_0 \frac{(Kd - Ku) + g(1 + Kd)}{(Ku - g)(1 + Kd)} \qquad (16)$$

For the no growth case (g = 0), equation (16) is:

$PV_0[\Delta D_t] = D\ (Kd\text{-}Ku) / [Ku(1+Kd)] < 0.$

Comparing this expression with equation (14), it is clear that Miles and Ezzell imply that $K_{ND} = Ku$. (16) is zero for g = (Ku-Kd) / (1+Kd) and negative for smaller growth rates. There is not much economic sense in this expression.

Furthermore, to assume $D_t = L \cdot E_t$ is not a good description of the debt policy of any company because:

1. If the company pays a dividend Div_t, simultaneously the company should reduce debt in an amount $\Delta D_t = - L \cdot Div_t$
2. If the equity value increases, then the company should increase its debt, while if the equity value decreases, then the company should reduce its debt. If the equity value is such that $L \cdot E_t >$ Assets of the company, then the company should hold excess cash only for the sake of complying with the debt policy.

2.4. Debt increases are as risky as the free cash flows

Then the correct discount rate for the expected increases of debt is Ku, the required return to the unlevered company. In the case of a constant growing perpetuity,

$PV_0[\Delta D_t] = g \cdot D_0 / (Ku\text{-}g),$

And the VTS is:

$$VTS_0 = T \cdot Ku \cdot D_0 / (Ku\text{-}g) \qquad (17)$$

For g = 0, equations (17) and (13) are equal.

Equation (17) is equal to equation (28) in Fernández (2004), although the way of deriving it is different.

2.5. The company is expected to repay the current debt without issuing new debt.

In this situation, the appropriate discount rate for the negative ΔD_t (because they are principal payments) is Kd, the required return to the debt. In this situation, Myers (1974) applies:

$$PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; Kd]$$

And the VTS is:

$$VTS_0 = D \cdot T + T \cdot PV_0[E\{\Delta D_t\}; Kd] \tag{18}$$

For perpetual debt, equations (18), (17) and (13) are equal.

For a company that is expected to repay the current debt without issuing new debt, the value of the debt today is: $D_0 = PV_0[E\{D_{t-1}\} \cdot Kd - E\{\Delta D_t\}; Kd]$.

Substituting this expression in (51), we get the Myers (1974) formula:

$$VTS_0 = PV_0[T \cdot E\{D_{t-1}\} \cdot Kd; Kd]$$

3. Value of net debt increases implied by the alternative theories

There is a considerable body of literature on the discounted cash flow valuation of firms. This section addresses the most salient papers, concentrating particularly on those papers that propose alternative expressions for the value of tax shields (VTS). The main difference between all of these papers and the approach proposed above is that most previous papers calculate the value of tax shields as the present value of the tax savings due to the payment of interest. Instead, the correct measure of the value of tax shields is the difference between two present values: the present value of taxes paid by the unlevered firm and the present value of taxes paid by the levered firm. We will show how these proposed methods result in inconsistent valuations of the tax shields.

Modigliani and Miller (1958, 1963) study the effect of leverage on firm value. Their famous Proposition 1 states that, in the absence of taxes, the firm's value is independent of its debt, i.e., $E + D = Vu$, if $T = 0$. In the presence of taxes and for the case of a perpetuity, but with zero risk of bankruptcy, they calculate the value of tax shields by discounting the present value of the tax savings due to interest payments on risk-free debt at the risk-free rate (R_F), i.e., $VTS = PV[E\{D \cdot T \cdot R_F\}; R_F] = D \cdot T$. As indicated above, this result is the same as our Eq. (16) for the case of perpetuities, but it is neither correct nor applicable for growing perpetuities. Modigliani and Miller explicitly ignore the issue of the riskiness of the cash flows by assuming that the probability of bankruptcy was always zero.

Myers (1974) introduces the APV (adjusted present value) method in which the value of the levered firm is equal to the value of the firm with no debt plus the present value of the tax savings due to the payment of interest. Myers proposes calculating the VTS by discounting the expected tax savings (D·Kd·T) at the cost of debt (Kd). The argument is that the risk of the tax savings arising from the use of debt is the same as the risk of the debt. The value of tax shields is

$$VTS = PV[E\{D \cdot Kd \cdot T\}; Kd]. \qquad (19)$$

This approach has also been recommended in later papers in the literature, such as Luehrman (1997). On section 2.5 we have shown that this expression is correct only when the company is expected to repay the current debt without issuing new debt.

Harris and Pringle (1985) propose that the present value of the tax savings due to the payment of interest should be calculated by discounting the expected interest tax savings (D·Kd·T) at the required return to unlevered equity (Ku), i.e.,

$$VTS = PV[E\{D \cdot Kd \cdot T\}; Ku] \qquad (20)$$

Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets (Ku). Furthermore, Harris and Pringle believe that "the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves" (p. 242). Ruback (1995, 2002), Kaplan and Ruback (1995), Brealey and Myers (2000, p. 555), and Tham and Vélez-Pareja (2001), this last paper following an arbitrage argument, also claim that the appropriate discount rate for tax shields is Ku, the required return to unlevered equity. Ruback (2002, p. 91) also shows that the relation between the beta of the levered equity (β_L), the beta of the unlevered equity (βu), and the beta of debt (βd) consistent with (20) is

$$\beta_L = \beta u + (\beta u - \beta d)\ D / E. \qquad \mathbf{(21)}$$

Therefore, all of Ruback's results (the relation between β_L and βu using no taxes, and Ku being the appropriate discount rate for capital cash flows) come from his method of estimating VTS, which is the same as that of Harris and Pringle.

The enterprise value (E + D) according to Fernández (2004) is

$$E + D = Vu + PV[E\{D \cdot Ku \cdot T\}; Ku] = PV[E\{FCF + D \cdot Ku \cdot T\}; Ku] \quad \textbf{(22)}$$

Fernández (2004) also shows that the relation between the beta of the levered equity (β_L), the beta of the unlevered equity (βu), and the beta of debt (βd) consistent with (22) is

$$\beta_L = \beta u + (\beta u - \beta d)\ D\ (1\text{-}T)\ /\ E. \quad \textbf{(23)}$$

A large part of the literature argues that the value of tax shields should be calculated differently depending on the debt strategy of the firm. A firm that wishes to keep a constant D/E ratio must be valued differently from a firm that has a preset level of debt. Miles and Ezzell (1980) indicate that for a firm with a fixed debt target (i.e., a constant [D/(D+E)] ratio), the correct rate for discounting the tax savings due to debt is Kd for the first year and Ku for the tax savings in later years. Lewellen and Emery (1986) also claim that this is the most logically consistent method. Although Miles and Ezzell do not mention what the value of tax shields should be, this can be inferred from their equation relating the required return to equity with the required return for the unlevered company in Eq. (22) in their paper. This relation implies that

$$VTS = PV[E\{D \cdot T \cdot Kd\}; Ku]\ (1 + Ku)/(1 + Kd) \quad \textbf{(24)}$$

Inselbag and Kaufold (1997) and Ruback (2002) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by the Myers (1974) formula. However, if the firm targets a constant debt/value ratio, the value of the tax shields should be calculated according to Miles and Ezzell (1980). Finally, Taggart (1991) proposes to use Miles and Ezzell (1980) if the company adjusts to its target debt ratio once a year, and Harris and Pringle (1985) if the company adjusts to its target debt ratio continuously.

Damodaran (1994, p. 31) argues that if all the business risk is borne by the equity, then the formula relating the levered beta (β_L) to the asset beta (βu) is $\beta_L = \beta u + (D/E)\ \beta u\,(1 - T)$. This formula is exactly the formula in Eq. (23), assuming that $\beta d = 0$. One interpretation of this assumption is (Damodaran, 1994, p. 31) that "all of the firm's risk is borne by the stockholders (i.e., the beta of the debt is zero)." In some cases, it may be reasonable to assume that the debt has a zero beta. But then, as assumed by Modigliani and Miller (1963), the required return to debt should be the risk-free rate. This relation for the levered beta appears in many finance books and is widely used by many consultants and investment bankers as an attempt to include some leverage cost in the valuation: for a given risk of the assets (βu), this formula results in a higher β_L (and consequently a higher Ke and a lower equity value) than Eq. (23). In general, it is hard to accept that the debt has no risk and that the return on the debt is uncorrelated with the firm's return on assets. From Damodaran's expression for β_L it is easy to deduce the relation between the required return to equity and the required return to assets, i.e., $Ke = Ku + (D / E)\ (1 - T)\ (Ku - R_F)$. Although Damodaran does not mention what the value of tax shields should be, his formula relating the levered beta to the asset beta implies that the value of tax shields is:

$$VTS = PV[Ku;\ D \cdot T \cdot Ku - D\ (Kd - R_F)\ (1 - T)]. \tag{25}$$

Finally, a common way of calculating the levered beta with respect to the asset beta (often used by consultants and investment banks) is the following (see, e.g., Ruback 1995, p. 5):

$$\beta_L = \beta u\ (1 + D/E) \tag{26}$$

It is obvious that given the same value for βu, a higher β_L (and a higher Ke and a lower equity value) is obtained than according to Eq. (23) or Damodaran (1994). Formula (26) relating the levered beta to the asset beta implies that the value of tax shields is $VTS = PV[E\{D \cdot T \cdot Kd - D(Kd - R_F)\};\ Ku]$. Given its widespread use in the industry, we will call this method the Practitioners' method.

Given the large number of alternative methods existing in the literature to calculate the value of tax shields, Copeland, Koller, and Murrin (2000, p. 482) assert that "the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct." They further conclude, "We leave it to the reader's judgment to decide which approach best fits his or her situation."

We propose three ways to compare and differentiate among the different approaches. One way is to calculate the value of tax shields for level perpetuities according to the different approaches. A second way is to check the implied present value of the net increases of debt in each of the different approaches. A third way is to check the implied relation between the unlevered and levered cost of equity in each of the different approaches. The levered cost of equity should always be higher than the cost of assets (Ku), since equity cash flows are riskier than the free cash flows.

Table 1 summarizes the implications of these approaches for the value of tax shields in level perpetuities. Table 1 shows that only four out of the eight approaches compute the value of tax shield in perpetuities as DT. The other four approaches imply a lower value of tax shields than DT.

From equation (12) the present value of the increases of debt is:

$PV_0[\Delta D_t] = (VTS_0 - T \cdot D_0) / T$

Applying this equation to the theories mentioned, we may construct the predictions that each of these theories have for $PV_0[\Delta D_t]$. These predictions are reported in Table 2. $PV_0[\Delta D_t]$ for level perpetuities should be zero. That is the case only in Modigliani-Miller (1963), Myers (1974) and Fernández (2004).

As we have already argued, Myers (1974) should be used when the company will not issue new debt; Fernández (2004) when the company expects to issue new debt in the future; and Modigliani-Miller may be applied only if the debt is risk-free.

Table 3 summarizes the implications for the relation between the cost of assets (Ku) and the cost of equity (Ku) in growing perpetuities. Table 3 shows that not all of the approaches also satisfy the relation between the cost of equity and the cost of assets. The Modigliani and Miller (1963) and Myers (1974) approaches do not always give a higher cost of equity than the cost of assets. Myers obtains Ke lower than Ku if the value of the tax shields is higher than the value of debt. This happens when $D \cdot T \cdot Kd / (Kd - g) > D$, that is, when the growth rate is higher than the after-tax cost of debt: $g > Kd (1 - T)$. Please note also that in this situation, as the value of tax shields is higher than the value of debt, the equity (E) is worth more than the unlevered equity (Vu). This hardly makes any economic sense. Modigliani and Miller provide the inconsistent result of Ke being lower than Ku if $g > R_F (1 - T)$.

4. Conclusions

This paper shows that the value of tax shields is:

$$VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t] \quad (12)$$

The critical parameter for calculating the value of tax shields is the present value of the net increases of debt. It may vary for different companies, but in some special circumstances it may be calculated.

If the debt is a constant perpetuity (a consol), $PV_0[\Delta D_t] = 0$, and $VTS_0 = T \cdot D_0$

If the company is expected to repay the current debt without issuing new debt, the appropriate discount rate for the negative ΔD_t (because they are principal payments) is Kd, the required return to the debt. In this situation, Myers (1974) applies: $PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; Kd]$. And the VTS is $VTS_0 = D\,T + T \cdot PV_0[E\{\Delta D_t\}; Kd]$.

If the correct discount rate for the increases of debt is Ku, the required return to the unlevered company: $PV_0[\Delta D_t] = PV_0[E\{\Delta D_t\}; Ku]$. In the case of a constant growing perpetuity, $PV_0[\Delta D_t] = g \cdot D_0 / (Ku - g)$, and $VTS_0 = T \cdot Ku \cdot D_0 / (Ku - g)$.

The paper also shows that discounting the expected tax shields at the required return to unlevered equity, as suggested by Harris and Pringle (1985), Miles and Ezzell (1980), and Ruback (2002), is inconsistent

Table 1
Comparison of value of tax shields (VTS) in perpetuities.
Only three out of the seven approaches correctly compute the value of the tax shield in perpetuities as DT.
The other four theories imply a lower value of the tax shield than DT.

Theories	VTS	VTS in perpetuities
Correct method	$D \cdot T + T \cdot PV_0[\Delta D_t]$	DT
Damodaran (1994)	$PV[E\{D \cdot T \cdot Ku - D(Kd - R_F)(1-T)\}; Ku]$	< DT
Practitioners	$PV[E\{D \cdot T \cdot Kd - D(Kd - R_F)\}; Ku]$	< DT
Harris-Pringle (1985), Ruback (1995)	$PV[E\{D \cdot T \cdot Kd\}; Ku]$	< DT
Myers (1974)	$PV[E\{D \cdot T \cdot Kd\}; Kd]$	DT
Miles-Ezzell (1980)	$PV[E\{D \cdot T \cdot Kd\}; Ku] (1+Ku) / (1+Kd)$	< DT
Modigliani-Miller (1963)	$PV[E\{D \cdot T \cdot R_F\}; R_F]$	DT
Fernández (2004)	$PV[E\{D \cdot T \cdot Ku\}; Ku]$	DT

Ku = unlevered cost of equity

Kd = required return to debt

T = corporate tax rate

D = debt value

R_F = risk-free rate

$PV[E\{D \cdot T \cdot Ku\}; Ku]$ = present value of the expected value of $D \cdot T \cdot Ku$ discounted at the rate Ku

Table 2

Present value of the increases of debt implicit in the most popular formulae for calculating the value of tax shields. Constant growing perpetuities at a rate g

	$PV_0[\Delta D_t]$ for constant growing perpetuities at a rate g	**$PV_0[\Delta D_t]$ if g=0**
Damodaran (1994)	$\frac{g \cdot D_0}{Ku-g} - \frac{D_0(Kd-R_F)}{Ku-g}\frac{(1-T)}{T}$	$-\frac{D_0(Kd-R_F)}{Ku}\frac{(1-T)}{T}$
Practitioners	$\frac{g \cdot D_0}{Ku-g} - \frac{D_0(Ku-Kd)}{Ku-g} - \frac{D_0(Kd-R_F)}{(Ku-g)T}$	$-\frac{D_0(Ku-Kd)}{Ku} - \frac{D_0(Kd-R_F)}{Ku\ T}$
Harris and Pringle (1985), Ruback (1995)	$\frac{g \cdot D_0}{Ku-g} - \frac{D_0(Ku-Kd)}{Ku-g}$	$-\frac{D_0(Ku-Kd)}{Ku}$
Myers (1974)	$g \cdot D_0/(Kd-g)$	0
Miles and Ezzell (1980)	$\frac{g \cdot D_0}{Ku-g} - \frac{D_0(Ku-Kd)}{(Ku-g)(1+Kd)}$	$-\frac{D_0(Ku-Kd)}{Ku(1+Kd)}$
Modigliani-Miller (1963)	$g \cdot D_0/(R_F-g)$	0
Fernández (2004)	$g \cdot D_0/(Ku-g)$	0

Table 3

Comparison of the relation between Ke (levered cost of equity) and Ku (unlevered cost of equity) for growing perpetuities.

The approaches of Modigliani and Miller (1963) and Myers (1974) do not always result in a higher cost of equity (Ke) than cost of assets (Ku). Myers (1974) obtains Ke lower than Ku if the value of tax shields is higher than the value of debt. This happens when the growth rate (g) is higher than the after-tax cost of debt, i.e., g > Kd (1 – T). Modigliani and Miller (1963) also provide the inconsistent result of Ke being lower than Ku if the value of tax shields is higher than D [Ku – Kd (1 – T)] / (Ku – g). This happens when leverage, the tax rate, the cost of debt, or the market risk premium are high.

Theories	Ke (levered cost of equity)	Ke<Ku
Damodaran (1994)	$Ke = Ku + (D/E)(1 - T)(Ku - R_F)$	No
Practitioners	$Ke = Ku + (D/E)(Ku - R_F)$	No
Harris-Pringle (1985), Ruback (1995)	$Ke = Ku + (D/E)(Ku - Kd)$	No
Myers (1974)	$Ke = Ku + (D - VTS)(Ku - Kd)/E$	**Yes,** if $g > Kd(1 - T)$
Miles-Ezzell (1980)	$Ke = Ku + \frac{D}{E}(Ku - Kd)\left[1 - \frac{T\,Kd}{1 + Kd}\right]$	No
Modigliani-Miller (1963)	$Ke = Ku + \frac{D}{E}[Ku - Kd(1 - T) - (Ku - g)\frac{VTS}{D}]$ *	**Yes,** if $g > R_F(1-T)$
Fernández (2004)	$Ke = Ku + (D/E)(1 - T)(Ku - Kd)$	No

* Valid only for growing perpetuities

D = debt value

E = equity value

g = growth rate

Kd = required return to debt

R_F = risk-free rate

T = corporate tax rate

VTS = value of tax shields

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Appendix 1. Derivation of the VTS formula using the pricing kernel

I assume that the price today (P_0) of an asset that pays a random amount CF_t at time t is the sum of the expectation of the product of CF_t and M_t, the pricing kernel for time t cash flows:

$$P_0 = \sum_1^\infty E\{M_t\ CF_t\}$$

E{} is the expected value operator.
The value of the unlevered equity (Vu) and the value of the equity are:

$$Vu_0 = \sum_1^\infty E\{M_t\ FCF_t\}$$

$$E_0 = \sum_1^\infty E\{M_t\ ECF_t\}$$

Applying the same valuation methodology to equations (7) and (8)

$$Taxes_{Ut} = T \cdot PBTu_t = [T/(1+T)]\ \ (FCF_t\ + \Delta NFA_t + \Delta WCR_t) \quad \textbf{(7)}$$

$$Taxes_{Lt} = T \cdot PBT_t = [T/(1+T)]\ \ (ECF_t + \Delta NFA_t + \Delta WCR_t - \Delta D_t) \quad \textbf{(8)}$$

we get: $GL_0 = \sum_1^\infty E\{M_t\ TaxesL_t\}$ and $Gu_0 = \sum_1^\infty E\{M_t\ Taxes_{Ut}\}$

Taking into consideration that VTS = Gu – G_L, we get:

$$VTS_0 = \frac{T}{1-T}\left[\sum_1^\infty E\{M_t\ FCF_t\} - \sum_1^\infty E\{M_t\ \Delta D_t\} - \sum_1^\infty E\{M_t\ ECF_t\}\right] =$$

$$= \frac{T}{1-T}\left[Vu_0 - E_0 - \sum_1^\infty E\{M_t\ \Delta D_t\}\right]$$

As, following equation (1), Vu – E = D – VTS, we get:

$$VTS = \frac{T}{1-T}\left[D - VTS - \sum_1^\infty E\{M_t\ \Delta D_t\}\right]$$

$VTS = TD - T\sum_1^\infty E\{M_t\ \Delta D_t\}$, which is equation (12).

Appendix 2. Main valuation formulas
Market value of the debt = Nominal value

Theories	VTS	Ke
Correct	$VTS_0 = T \cdot D_0 + T \cdot PV_0[\Delta D_t]$	
Damodaran (1994)	$PV[E\{D \cdot T \cdot Ku - D\ (Kd - R_F)\ (1-T)\};\ Ku]$	$Ke = Ku + (D/E)\ (1-T)\ (Ku - R_F)$
Practitioners	$PV[E\{D \cdot T \cdot Kd - D\ (Kd - R_F)\};\ Ku]$	$Ke = Ku + (D/E)\ (Ku - R_F)$
Harris-Pringle (1985), Ruback (1995)	$PV[E\{D \cdot T \cdot Kd\};\ Ku]$	$Ke = Ku + (D/E)\ (Ku - Kd)$
Myers (1974)	$PV[E\{D \cdot T \cdot Kd\};\ Kd]$	$Ke = Ku + (D - VTS)\ (Ku - Kd)\ /E$
Miles-Ezzell (1980)	$PV[E\{D \cdot T \cdot Kd\};\ Ku]\ (1+Ku)\ /\ (1+Kd)$	$Ke = Ku + \frac{D}{E}\ (Ku - Kd)\left[1 - \frac{T\ Kd}{1 + Kd}\right]$
Modigliani-Miller (1963)	$PV[E\{D \cdot T \cdot R_F\};\ R_F]$	$Ke = Ku + \frac{D}{E}[Ku - Kd(1-T) - (Ku - g)\frac{VTS}{D}]$ *
Fernández (2004)	$PV[E\{D \cdot T \cdot Ku\};\ Ku]$	$Ke = Ku + (D/E)\ (1-T)\ (Ku - Kd)$ (21)

Theories	$ß_L$	WACC
Damodaran (1994)	$ß_L = ßu + \frac{D(1-T)}{E} ßu$	$Ku\left(1 - \frac{DT}{E+D}\right) + D\frac{[Kd - R_F](1-T)]}{E+D}$
Practitioners	$ß_L = ßu + \frac{D}{E} ßu$	$Ku - D\frac{R_F - Kd\ (1-T)}{E+D}$
Harris-Pringle (1985), Ruback (1995)	$ß_L = ßu + \frac{D}{E}(ßu - ßd)$	$Ku - \frac{D\ Kd\ T}{E+D}$
Myers (1974)	$ß_L = ßu + \frac{D - VTS}{E}(ßu - ßd)$	$Ku - \frac{VTS(Ku - Kd) VTS + DTKd}{E+D}$
Miles-Ezzell (1980)	$ß_L = ßu + \frac{D}{E}(ßu - ßd)\left[1 - \frac{T\ Kd}{1 + Kd}\right]$	$Ku - D\frac{Kd\ T}{E+D}\ \frac{1+Ku}{1+Kd}$
Modigliani-Miller (1963)	$ß_L = ßu + \frac{D[Ku - Kd(1-T)] - VTS(Ku - g)}{E\ P_M}$ *	$Ku - \frac{(Ku - g)VTS}{E+D}$ *
Fernández (2004)	$ß_L = ßu + \frac{D(1-T)}{E}(ßu - ßd)$	$Ku\left(1 - \frac{DT}{E+D}\right)$

* Valid only for growing perpetuities

Theories	$WACC_{BT}$
Damodaran (1994)	$Ku + D\frac{Kd - R_F - T(Ku - R_F)}{E+D}$
Practitioners	$Ku + D\frac{Kd - R_F}{E+D}$
Harris-Pringle (1985), Ruback (1995)	Ku
Myers (1974)	$Ku - \frac{VTS(Ku - Kd)}{E+D}$
Miles-Ezzell (1980)	$Ku - \frac{D\ T\ Kd}{(E + D)}\ \frac{(Ku - Kd)}{(1 + Kd)}$
Modigliani-Miller (1963)	$Ku - \frac{(Ku - g)VTS - TDKd}{E+D}$ *
Fernández (2004)	$Ku - \frac{DT(Ku - Kd)}{(E + D)}$