Improving the asset pricing ability of the Consumption-Capital Asset Pricing Model?

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Abstract

This paper compares the asset pricing ability of the traditional consumption based capital asset pricing model to models from two strands of literature attempting to improve on the poor empirical results of the C-CAPM. One strand is based on the intertemporal asset pricing model of Campbell (1993, 1996) and Campbell and Vuolteenaho (2004). The model takes the traditional C-CAPM as its starting point, but substitutes all references to consumption out, as empirical consumption data is assumed to be error ridden.

The other strand to be investigated is based on the premise that the C-CAPM is only able to price assets conditionally as suggested by Cochrane (1996) and Lettau and Ludvigson (2001b). The unconditional C-CAPM is rewritten as a scaled factor model using the approximate log consumption-wealth ratio *cay*, developed by Lettau and Ludvigson (2001a), as scaling variable.

The models are estimated on US data and the resulting pricing errors are compared using average pricing errors and a number of composite pricing error measures. The conditional C-CAPM and the two beta I-CAPM of Campbell and Vuolteenaho (2004) result in pricing errors of approximately the same size, both average and composite. Thus, there is no unambigous solution to the pricing ability problems of the C-CAPM. Models from both the alternative literature strands are found to outperform the traditional C-CAPM on average pricing errors. However, when weighting pricing errors by the full variance-covariance matrix of returns or the moment matrix of returns, the traditional C-CAPM actually outperforms the models from both the two new litterature strands.

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I Introduction

The consumption-based capital asset pricing model (C-CAPM) introduced by Lucas (1978), Breeden (1979), and Grossman and Shiller (1981), determines asset risk by the covariance of the asset's return with marginal utility of consumption. However, empirical investigations have lent little support to the relations obtained from the model. Tests of the C-CAPM have led to rejection of the model as well as unrealistic parameter estimates resulting in the establishment of the so-called "equity premium puzzle" (Hansen and Singleton (1983), Mehra and Prescott (1985), Kocherlakota (1996)). This in spite of the fact that the model is of an intertemporal nature, in the spirit of the I-CAPM of Merton (1973). In fact, the model is found to be outperformed by the static CAPM (Mankiw and Shapiro (1986)) and unrestricted multifactor models, when it comes to explaining cross sectional asset returns.

Despite the empirical failures of the consumption based model, the economic intuition underlying the model is so intuitively appealing that it would be a mistake to dismiss it completely. It also has the property that models such as the CAPM and the APT, can be mapped into the framework as special cases, as pointed out by Cochrane (2001). The relation linking the marginal utility of consumption to asset returns still holds, but additional assumptions are made enabling other variables to be used in place of consumption.

So if the model can't be dismissed, why is it failing empirically? This paper looks at two strands of literature addressing the poor empirical findings of the C-CAPM. One is the I-CAPM of Campbell (1993, 1996) and Campbell and Vuolteenaho (2004). This is an intertemporal model, based on the same framework as the C-CAPM, but rephrased without reference to consumption data. The other strand looks at the conditional pricing ability of the C-CAPM in a scaled factor setup, as in Cochrane (1996), Ferson and Harvey (1999), and Lettau and Ludvigson (2001b).

These models take two different directions, but with the common goal of improving on the empirical performance of the C-CAPM. The question is, do they succeed? This paper will compare the asset pricing ability of the traditional C-CAPM with that of these alternative models as well as evaluating the relative pricing ability of the two new model strands. In addition, we include estimates of the static CAPM and the Fama and French (1993) three factor model, to give an idea of the pricing error level of the consumption based framework compared to other well known asset pricing models.

The first strand to be investigated is based on the argument of Campbell (1993), that the poor empirical performance of the C-CAPM may be due to problems inherent in the empirical consumption data used to test the model, rather than with the theoretical assumptions underlying. Firstly, aggregate consumption data are measured with error and are time-aggregated (Grossman (1987), Wheatley (1988), and Breeden et al. (1989)). Secondly, the consumption of asset-market participants may be poorly proxied by aggregate consumption (Mankiw and Zeldes (1991)).

To address these issues Campbell (1993) develops an intertemporal model, which uses the same building blocks as the C-CAPM, but makes no references to consumption data. Using a first order Taylor expansion of the intertemporal budget constraint of the representative investor and combining it with a log-linear Euler equation, one is able to express unanticipated consumption as a function of expectational revisions in current and future returns to wealth, thereby eliminating all references to consumption. The model can thus be estimated empirically without running into consumption data issues usually faced when testing the C-CAPM. In Campbell and Vuolteenaho (2004), the model is rewritten in a two-beta notation and is found to give an explanation for the size and value anomalies found when estimating traditional asset pricing models such as the CAPM and C-CAPM. Campbell and Vuolteenaho (2004) compare the pricing errors of their model to the CAPM and find these to be significantly lower in the two beta model. The pricing ability is not compared to that of the traditional C-CAPM with consumption data.

The other strand of literature treated in this paper looks at the conditional pricing ability of the C-CAPM in a scaled factor setup. That is, it is assumed that the cause of the poor empirical performance of the C-CAPM is that the model is in fact only able to price assets conditionally. This would allow for recent empirical evidence of time variation in expected returns. By estimating the models conditionally, we can incorporate time-varying risk premia into the models. Lettau and Ludvigson (2001b) find that the conditional version of the C-CAPM, using the approximate log consumption-wealth ratio *cay* as scaling variable, outperforms the unconditional C-CAPM. The scaled consumption factor has significant pricing ability, and the model results in much lower average pricing errors than those found from traditional models. It performs about as well as the three factor model of Fama and French (1993).

Both strands of literature have had some success in explaining the empirical anomalies the C-CAPM fails to fit. But how do the models compare to each other? Is one of the model strands unequivocally better than the other at fitting the historical data? Do we have a clear cut empirical replacement for the C-CAPM? To answer these questions we first look at the statistical significance of the pricing errors resulting from the various models. In order to look into the relative pricing ability, we compute a number of pricing error measures which allow us to compare the pricing ability across models. Both the average squared pricing errors and a composite pricing error, created by weighting pricing errors by the variance of the respective asset returns, are investigated. These measures will give us an idea of the economic magnitude of the pricing errors of the models. Finally, the distance measure of Hansen and Jagannathan (1997) is also computed.

Estimation of the traditional asset pricing models undertaken in this paper supports the findings of previous research. The CAPM and C-CAPM result in insignificant coefficient estimates and high pricing errors. The three-factor Fama-French model results in a high R^2, however the risk price on the market return beta is negative and unstable over subsamples.

Turning to the two new model strands, our empirical results are less supportive of previous findings. Unlike Lettau and Ludvigson (2001), we find no significance of the scaled consumption factor in the conditional version of the C-CAPM. This coefficient is very sensitive to the time period over which the model is estimated. Eliminating the final two years of observations in our time-series results in the risk price of scaled consumption becoming significantly positive.

There is generally no distinct difference in the pricing ablity of the conditional C-CAPM and the I-CAPM of Campbell and Vuolteenaho. Despite the lack of significant coefficients, average pricing errors from both models are lower than those of the traditional C-CAPM. However, when weighting pricing errors by the full variance-covariance matrix or the moment matrix of asset returns, the traditional C-CAPM outperforms the models from both the new literature strands. The pricing improvement of the new models is thus not consistent across pricing error measures.

The paper is structured in the following manner. Section II will present the different asset pricing models to be considered. Section III describes the empirical estimation techniques used. The data is described in section IV, empirical results in section V and finally in section VI we conclude.

II The Models

In the absence of arbitrage, a stochastic discount factor M_{t+1} exists, such that any asset return $R_{i,t+1}$ obeys the following relation

$$1 = E_t \left[M_{t+1} R_{i,t+1} \right] \tag{1}$$

$R_{i,t+1}$ is the gross return on asset i from time t to $t+1$, M_{t+1} is the stochastic discount factor or pricing kernel, and E_t is the conditional expectation operator. The question we face is, how is the stochastic discount factor to be expressed?

The economic argument of the consumption based asset pricing model (C-CAPM) is that M_{t+1} should be a measure of the marginal rate of substitution. In order for an agent to invest in a given asset at time t, the expected return at time $t+1$ must compensate for the consumption possibilities given up at time t. In the C-CAPM, the stochastic discount factor is thus given by

$$M_{t+1} = \delta \frac{u'(C_{t+1})}{u'(C_t)}. \tag{2}$$

δ is the subjective rate of time preference discount factor, $u(\bullet)$ is the utility function, and C_t denotes consumption. Despite the strong underlying economic intuition, the empirical performance of this model has been poor. If we don't wish to disregard the model completely, we have to look at what factors may be causing the empirical problems. Are there some deviations between the simple economic theory presented in (1) and (2) and the empirical estimates. If we look at where problems could arise, three places spring to mind. Firstly, to estimate the C-CAPM we must choose an operational form of the utility function. Many utility functions have been suggested, but the traditional C-CAPM is based on the power utility function. This may however be an inaccurate description of the utility function of agents and hence the model may be failing on these grounds. Secondly, the functional forms of (2) traditionally associated with the C-CAPM assume constant risk premia. Evidence of time-variation in expected returns, on the contrary, makes it desirable to allow for time-variation in the risk premia of the model. The problem with the C-CAPM may thus be, that in actuality it only holds conditionally. Finally, estimation of the C-CAPM requires the use of an empirical measure of the consumption of the marginal investor c_t. This is most often proxied by some measure of aggregate consumption in the economy, often based on expenditures on goods and services. Any deviations between the theoretical consumption measure and the empirical data may be resulting in the poor empirical performance of the C-CAPM.

Attempts have been made to rectify the empirical struggles of the C-CAPM, by developing factor models with SDF's that form good proxies for the right hand side of (2), and take the three problems described above into consideration. In this paper we will look at whether these attempts help or harm the pricing ability of the C-CAPM.

The models treated in the following can all be mapped into the linear factor model framework. In this case, M_{t+1} is quantified as a linear combination of a number of factors determined by the underlying theory of the given model. Let $\mathbf{F}'_{t+1} = \left[1, \mathbf{f}'_{t+1}\right]$, where $\mathbf{f}'_{t+1}$ is the vector of factors included in the model. The stochastic discount factor is given by

$$M_{t+1} = \mathbf{c}\mathbf{F}_{t+1} \tag{3}$$

where $\mathbf{c} = [\alpha, \mathbf{b}']$ and $\mathbf{b}$ is the vector of coefficients on the variable factors of the model. As noted by Cochrane (2001), all factor models are in reality specializations of the consumption-based model. Some additional assumptions are made, allowing marginal utility growth to be replaced by other economically relevant factors, such that the right hand side of (2) is proxied by the right hand side of (3).

In order to estimate the factor models cross-sectionally, we rewrite them in a beta representation. Inserting (3) into the general pricing equation (1), taking unconditional expectations and applying a variance decomposition results in the following cross-sectional multifactor model[1]

$$E\left[R_{i,t+1} - R_{f,t+1}\right] = \boldsymbol{\beta}_i'\boldsymbol{\lambda} \tag{4}$$

$$\boldsymbol{\beta}_i \equiv cov\left(\mathbf{f}, \mathbf{f}'\right)^{-1} cov\left(\mathbf{f}, R_{i,t+1}\right) \tag{5}$$

$$\boldsymbol{\lambda} \equiv -E\left[R_{f,t+1}\right] cov\left(\mathbf{f}, \mathbf{f}'\right)\mathbf{b} \tag{6}$$

$R_{f,t+1}$ is the risk-free rate of return, for which it holds that $R_{f,t+1} = \frac{1}{E(M_{t+1})}$.

We have now introduced the general linear factor model framework and next it is time to look at which factors are included in the specific models studied in

[1]The derivation follows that of Cochrane (1996).

this paper.

II.1 Traditional asset pricing models

As noted in the previous section, the stochastic discount factor of the C-CAPM is based on the marginal rate of substitution of consumption. To estimate the model, one in principal needs to decide how to model the utility function. Often power utility is applied. However extensions using the more general Epstein-Zin-Weil utility and habit based functions have also been introduced. To avoid being constrained by the choice of utility function, it is assumed that M can be proxied by a linear function of log consumption growth c_{t+1}

$$M_{t+1} \approx a + b\Delta c_{t+1} \tag{7}$$

This approximation imposes few restrictions on the functional form of the investors utility. As is evident, (7) is a linear factor model with log consumption growth as the sole factor. It is also referred to as the log-linear C-CAPM. Traditionally, the parameters a and b are taken to be time invariant, which will also be the case for the base model of this paper. From (4) we find the cross-sectional asset pricing model, in beta representation

$$E\left[R_{i,t+1} - R_{f,t+1}\right] = \beta_{i,\Delta c}\lambda_{\Delta c} \tag{8}$$

This is the model that forms the basis of our investigation. A simple equation which relates asset returns to the consumption growth beta[2].

Although the focus of this paper is on the C-CAPM, and attempts to improve on its poor empirical performance, we also include two other well known asset pricing models. This allows us to get a sense of the level of pricing errors we are experiencing in the consumption based setup, compared to other models traditionally estimated in the literature. The two models are the CAPM and the three factor Fama-French (1993) model.

The SDF of the CAPM with time invariant parameters can be written as

$$M_{t+1} \approx a + bR_{m,t+1} \tag{9}$$

where $R_{m,t+1}$ is the return on the aggregate market. In beta representation we have

$$E\left[R_{i,t+1} - R_{f,t+1}\right] = \beta_{i,R_m}\lambda_{R_m} \tag{10}$$

[2]For the traditional time-separable power utility function with constant relative risk aversion $u\left(C_t\right) = \frac{C_t^{1-\gamma}-1}{1-\gamma}$, which implies $M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$. We can use a first-order Taylor expansion of M_{t+1} around $C_{t+1} = C_t$ to rewrite the SDF as $M_{t+1} \approx \delta(1 - \gamma\Delta c_{t+1})$. As we are estimating the model using excess returns, the mean of the SDF is not defined and we therefore set $\delta = 1$ and $E(M_{t+1}) = 1$. The cross-sectional model thus becomes $E\left[R_{i,t+1} - R_{f,t+1}\right] = \gamma var(\Delta c_{t+1})\beta_{i,\Delta c}$

As Cochrane (2001) points out, the CAPM is in fact contained in the C-CAPM as a special case, adding additional motivation for the introduction of the CAPM.

The Fama-French model is slightly different from the other models investigated, in that it is empirically, not theoretically, driven. The factors are chosen based on patterns observed in data, rather than being derived from an underlying economic theory, although Fama and French (1996) argue that the model can be seen as a three-factor version of Merton's (1973) I-CAPM or the APT-model of Ross (1976). The three factors of the model are the return on the aggregate market $R_{m,t+1}$, known from the CAPM, the return on the "small minus big" portfolio (SMB), and the return on the "high minus low" portfolio.

$$M_{t+1} \approx a + b_1 R_{m,t+1} + b_2 SMB_{t+1} + b_3 HML_{t+1} \tag{11}$$

SMB and HML are constructed in Fama and French (1993), and are based on 6 portfolios of US stocks sorted on size and the ratio of book equity to market equity (BE/ME) of the assets. SMB is the difference in returns between the small and big stock portfolios, sorted by size. HML is the difference in returns on the high- and low-BE/ME portfolios.

Finally, we estimate a two factor model which combines the factors of the C-CAPM and the CAPM

$$M_{t+1} \approx a + b_1 R_{m,t+1} + b_2 \Delta c_{t+1} \tag{12}$$

The motivation for this model, should become evident when the I-CAPM of Campbell and Vuolteenaho (2004) is introduced. In order to develop that model, Campbell (1993) bases his derivations on a C-CAPM with Epstein-Zin-Weil (EZW) utility. A log-linearization of the EZW C-CAPM results in a two factor model of the form presented in (12). Hence, when we compare the I-CAPM to the original consumption based asset pricing framework, it makes sense to use the functional form on which the I-CAPM is based.

II.2 Conditional models

The models presented so far, have all been assumed to price assets unconditionally. However, the cause of the empirical failure of the C-CAPM may be that the model is in fact only able to price assets conditionally. In recent years there has been increasing evidence indicating predictability in excess stock returns. Predictability implies that expected returns can vary over time. This variation in investors' expectations of asset returns may be due to time varying risk premia. The risk premia can become state dependent if agents require a higher risk premium to invest in stocks in times of recession for example as proposed by Campbell and Cochrane (1999). The traditional C-CAPM and CAPM do not allow for such time variation in risk premia and Lettau and Ludvigson (2001b) suggest this to be a reason for the empirical failure of the models.

To model time-variation in risk premia, we need to let the weights on the factors in the pricing kernel become time dependent

$$M_{t+1} = a_t + \mathbf{b}_t \mathbf{f}_{t+1}. \tag{13}$$

In order to estimate this model, Cochrane (1996) and Ferson and Harvey (1999) show that we can scale the factors in the SDF with the vector of instruments $\mathbf{z}_t$ containing time t information about the state of the economy, allowing us once more to estimate the model unconditionally. In the following we will assume the state of the economy to be described by a single state variable z_t.

First, model the parameters as linear functions of the instrument z_t

$$\begin{aligned} a_t &= \gamma_0 + \gamma_1 z_t \\ \mathbf{b}_t &= \boldsymbol{\eta}_0' + \boldsymbol{\eta}_1' z_t \end{aligned}$$

The pricing kernel with time varying coefficients can then be rewritten as

$$\begin{aligned} M_{t+1} &= a_t + \mathbf{b}_t \mathbf{f}_{t+1} \\ &= (\gamma_0 + \gamma_1 z_t) + (\boldsymbol{\eta}_0' + \boldsymbol{\eta}_1' z_t)\, \mathbf{f}_{t+1} \\ &= \gamma_0 + \gamma_1' z_t + \boldsymbol{\eta}_0' \mathbf{f}_{t+1} + \boldsymbol{\eta}_1' \left(z_t \mathbf{f}_{t+1} \right) \end{aligned} \tag{14}$$

and we are back in the unconditional framework with time invariant coefficients. For the consumption based model this results in

$$M_{t+1} = a_t + b_t \Delta c_{t+1} = \gamma_0 + \gamma_1 z_t + \eta_0 \Delta c_{t+1} + \eta_1 \left(\mathbf{z}_t \Delta c_{t+1} \right) \tag{15}$$

Equivalently, if we substitute $R_{m,t+1}$ into the above equation instead of Δc_{t+1} we obtain the conditional CAPM.

In matrix notation

$$\mathbf{M}_{t+1} = \mathbf{c}' \overline{\mathbf{F}}_{t+1} \tag{16}$$

with $\overline{\mathbf{F}}_{t+1} = \left[1, z_t, \mathbf{f}_{t+1}', \mathbf{f}_{t+1}' z_t\right]' = \left[1, \overline{\mathbf{f}}_{t+1}'\right]$, $\overline{\mathbf{f}}_{t+1} = \left[z_t, \mathbf{f}_{t+1}', \mathbf{f}_{t+1}' z_t\right]'$, where $\mathbf{f}_{t+1}$ is a $k \times 1$ vector of k factors, $\mathbf{c} = [\gamma_0, \mathbf{b}']'$ where γ_0 is a scalar and $\mathbf{b} = [\gamma_1, \boldsymbol{\eta}_0', \boldsymbol{\eta}_1']$.

Analogously to the case without scaling variables, inserting these terms into the general pricing equation (1), taking unconditional expectations and applying a variance decomposition results in the following cross sectional multifactor model in beta representation

$$\begin{aligned} E\left[R_{i,t+1} - R_{f,t+1}\right] &= \boldsymbol{\beta}_i' \boldsymbol{\lambda} \\ \boldsymbol{\beta}_i &\equiv cov\left(\overline{\mathbf{f}}, \overline{\mathbf{f}}'\right)^{-1} cov\left(\overline{\mathbf{f}}, R_{i,t+1}\right) \\ \lambda &\equiv -E\left[R_{f,t+1}\right] cov\left(\overline{\mathbf{f}}, \overline{\mathbf{f}}'\right) \mathbf{b} \end{aligned} \tag{17}$$

where $\boldsymbol{\beta}$ is the vector of regression coefficients stemming from regressing returns

$R_{i,t+1}$ on $\overline{\mathbf{F}}_{t+1}$, λ is a free parameter vector and $R_{f,t+1}$ is the return on the zero-beta portfolio or risk-free rate of return.

In order to estimate conditional factor models, we need to choose a vector $\mathbf{z}_t$ of scaling variables. The conditional models state that the coefficients on the factors in the SDF of the factor model are dependent on the investors information set at time t. Hence the scaling variables need to describe the state of the business cycle at time t. As it would be impossible to include all information in the investors information set in an empirical estimation, we need to find variables that summarize all relevant effects. We also need to take into account the tractability of empirical estimation when choosing the scaling variables. To avoid an explosion in the number of parameters to be estimated, relative to the length of the time-series of data used in this paper, we limit the number of scaling variables to one.

Lettau and Ludvigson (2001b) suggest using the variable *cay* as the scaling variable in the conditional factor models. *cay* can be described as a proxy for the log consumption-aggregate wealth ratio and it may be used to forecast excess stock market returns. It is calculated as $cay_t = c_t - \omega a_t - (1-\omega)\, y_t$, where c_t is consumption, a_t is asset wealth, and y_t is labour income. ω is the average share of asset wealth in total wealth. The three variables are assumed to be cointegrated and ω is computed as a cointegrating coefficient. Lettau and Ludvigson (2001a, 2005) show that cay_t is able to forecast excess stock returns, better than traditional forecasting variables such as p/d and p/e ratios at short to intermediate horizons. Hence it makes a good choice as conditioning instrument. Hodrick and Zhang (2001) also use this variable to test conditional factor pricing models.

II.3 The Campbell I-CAPM

One of the major problems in estimating the C-CAPM is the quality of the empirical consumption data needed to estimate the model. If there is a large divide between the theoretical measure of consumption growth of the model and the empirical data, then it is natural to expect poor empirical results for the model. Not because the model as such is faulty, but merely due to data issues.

Campbell (1993) suggests a way out of this problem, by substituting consumption out of the C-CAPM, but still keeping the model tractable for empirical estimation. Under the assumption of homoskedasticity and joint lognormality of asset returns and consumption, Campbell shows that

$$\begin{aligned} cov_t\left[r_{i,t+1}, \Delta c_{t+1}\right] &= cov_t\left[r_{i,t+1}, r_{m,t+1} - E_t r_{m,t+1}\right] \\ &\quad + (1-\psi)\, cov_t\left[r_{i,t+1}, (E_{t+1} - E_t)\sum_{j=1}^{\infty}\rho^j r_{m,t+1+j}\right] \end{aligned}$$

where $r_{i,t+1}$ is the log return on asset i, $r_{m,t+1}$ is the log market return, ψ is the elasticity of intertemporal substitution. Thereby, one is able to transform a log linearized C-CAPM into a cross-sectional asset pricing model, making no references to consumption:

$$\begin{aligned} E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} &= \theta \frac{V_{ic}}{\psi} + (1-\theta) V_{im} && (18) \\ &= \gamma V_{im} + (\gamma - 1) V_{ih} && (19) \\ V_{ii} &\equiv var_t \left[r_{i,t+1} \right] \\ V_{ic} &\equiv cov_t \left[r_{i,t+1}, \Delta c_{t+1} \right] \\ V_{im} &\equiv cov_t \left[r_{i,t+1}, r_{m,t+1} - E_t r_{m,t+1} \right] \\ V_{ih} &\equiv cov_t \left[r_{i,t+1}, (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \right] \end{aligned}$$

(18) states the log-linearized C-CAPM, assuming Epstein-Zin-Weil utility, and (19) introduces the Campbell I-CAPM, in which all references to consumption growth have been eliminated. γ is the coefficient of relative risk aversion and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. The model states that the excess return on asset i is determined by a weighted average of the asset return's covariance with the current market return and the return covariance with news about future market returns.

Campbell and Vuolteenaho (2004) develop the model further and rewrite it in beta representation as a two factor intertemporal model. Starting with the basic loglinear approximate decomposition of asset returns from Campbell and Shiller (1988), the following expression obtains:

$$\begin{aligned} r_{i,t+1} - E_t r_{i,t+1} &= (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{i,t+1+j} && (20) \\ &\quad - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{i,t+1+j} \\ &\equiv N_{i,CF,t+1} - N_{i,DR,t+1} \end{aligned}$$

where $r_{i,t+1}$ is the log return on asset i, $d_{i,t+1}$ is the log dividend on asset i, and ρ is a discount coefficient[3]. The identity (20) states that unexpected returns are linked to changes in expected cash flows or changes in expected discount rates. Increases in expected cash flows imply positive unexpected returns today. Increases in the expected future discount rate, on the other hand, have a negative effect on current returns. If future discount rates rise, we must discount cash flows by a higher rate thus resulting in a downward revision in prices today and thereby returns. This downward revision will however be reversed in the future as increases in future discount rates also imply improved future investment

[3] ρ is the average ratio of the stock price to the sum of the stock price and the dividend. ρ will be fixed at 0.987 in the empirical estimates of this paper.

opportunities. Unlike shocks stemming from cash flow revisions, the return shocks stemming from revisions in forecasts of discount rates are thus of a transitory nature.

Let the return $r_{m,t+1}$ be given by the aggregate market return, $N_{m,CF,t+1} \equiv (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{m,t+1+j}$, and $N_{m,DR,t+1} \equiv (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j}$. The unrestricted SDF of Campbell and Vuolteenaho (2004) is

$$M_{t+1} = a + b_1 N_{m,CF,t+1} + b_2 N_{m,DR,t+1} \tag{21}$$

In order to determine $N_{m,CF,t+1}$ and $N_{m,DR,t+1}$ empirically, a VAR approach is used and estimated with OLS. $\mathbf{s}_t$ is a K-element state vector. The first element of $\mathbf{s}_t$ is the market return. The remaining elements are variables relevant in forecasting future stock index returns. All variables have been demeaned as the constants, that would otherwise arise, just capture the linearization constraints. It is assumed that $\mathbf{s}_t$ follows a first-order VAR

$$\mathbf{s}_{t+1} = \mathbf{A}\mathbf{s}_t + \boldsymbol{\epsilon}_{t+1}. \tag{22}$$

This is not restrictive, since higher order VAR systems can be written in companion form. The VAR methodology enables us to express multiperiod forecasts of future returns in the following manner

$$E_t \mathbf{s}_{t+1+j} = \mathbf{A}^{j+1} \mathbf{s}_t. \tag{23}$$

Define $\mathbf{e1}$ as a K-element vector with first element one and the remaining elements zero. This vector is used to pick out the return on the market from the state vector $\mathbf{s}_t$. The discounted sum of forecast revisions in returns on the market can now be found as

$$\begin{aligned} N_{m,DR,t+1} &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{m,t+1+j} \\ &= \mathbf{e1}' \sum_{j=1}^{\infty} \rho^j \mathbf{A}^j \boldsymbol{\epsilon}_{t+1} \\ &= \mathbf{e1}' \rho \mathbf{A} \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} \boldsymbol{\epsilon}_{t+1} \\ &\equiv \mathbf{e1}' \boldsymbol{\Lambda} \boldsymbol{\epsilon}_{t+1}, \end{aligned} \tag{24}$$

where $\mathbf{I}$ is the $K \times K$ identity matrix. Since $r_{m,t+1} - E_t r_{m,t+1} = \mathbf{e1}' \boldsymbol{\epsilon}_{t+1}$

$$\begin{aligned} N_{m,CF,t+1} &= r_{m,t+1} - E_t r_{m,t+1} + N_{m,DR,t+1} \\ &= \left(\mathbf{e1}' + \mathbf{e1}' \boldsymbol{\Lambda} \right) \boldsymbol{\epsilon}_{t+1} \end{aligned} \tag{25}$$

The Campbell (1993) I-CAPM of (19) can now be restated in terms of the two factors $N_{m,CF,t+1}$ and $N_{m,DR,t+1}$ derived by Campbell and Vuolteenaho (2004).

$$E_t r_{i,t+1} - r_{f,t+1} + \frac{V_{ii}}{2} = \gamma cov_t \left[r_{i,t+1}, N_{m,CF,t+1}\right] - cov_t \left[r_{i,t+1}, N_{m,DR,t+1}\right] \quad (26)$$

Finally, we need to rephrase the model of (26) in a beta representation. Define two beta terms based on $N_{m,CF,t+1}$ and $N_{m,DR,t+1}$[4]

$$\beta_{i,CF_m,t} \equiv \frac{cov_t\left(r_{i,t+1}, N_{m,CF,t+1}\right)}{var_t\left(N_{m,CF,t+1}\right)} = \frac{\sigma_{i,CF_m,t}}{\sigma^2_{CF_m,t}} \quad (27)$$

$$\beta_{i,DR_m,t} \equiv \frac{cov_t\left(r_{i,t+1}, -N_{m,DR,t+1}\right)}{var_t\left(N_{m,DR,t+1}\right)} = \frac{-\sigma_{i,DR_m,t}}{\sigma^2_{DR_m,t}} \quad (28)$$

Substitute these expressions into (26) and we obtain the following cross-sectional asset pricing model

$$E_t r_{i,t+1} - r_{f,t+1} + \frac{\sigma^2_{i,t}}{2} = \gamma\sigma^2_{CF_m,t}\beta_{i,CF_m,t} + \sigma^2_{DR_m,t}\beta_{i,DR_m,t} \quad (29)$$

By taking unconditional expectations and rewriting the left hand side of the relation in simple expected returns form, $E\left[R_{i,t+1} - R_{f,t+1}\right]$, we get the two beta I-CAPM of Campbell and Vuolteenaho (2004)

$$E\left[R_{i,t+1} - R_{f,t+1}\right] = \gamma\sigma^2_{CF_m}\beta_{i,CF_m} + \sigma^2_{DR_m}\beta_{i,DR_m} \quad (30)$$

The model will also be estimated in an unconditional, unrestricted version:

$$E\left[R_{i,t+1} - R_{f,t+1}\right] = \lambda_{CF}\beta_{i,CF_m} + \lambda_{DR}\beta_{i,DR_m} \quad (31)$$

Finally, in the spirit of Hodrick and Zhang (2001) we also estimate a linear SDF with a constant and those variables included in the state vector $\mathbf{s}_t$ of Campbell and Vuolteenaho (2004) as factors. This is not an intertemporal model as the I-CAPM, but a factor model in the spirit of the APT model. The factors used are not innovations, but pure factors and it has none of the parameter restrictions imposed on the Campbell and Vuolteenaho model. A comparison with this model will tell us, if any improvements the I-CAPM results in over the C-CAPM are a result of the merit of the theory and techniques underlaying Campbell and Vuolteenaho (2004) or if a simple model containing the state variables of their VAR does equally well. When comparing the models, we must take into account the fact that the pure factor model contains more free parameters than the Campbell and Vuolteenaho model. We should therefore not be surprised to see some improvement in the pricing ability when using this model, even if the restrictions of the I-CAPM are valid.

[4] The beta definition is slightly different than that rapported in Campbell & Vuolteenaho (2004). Campbell & Vuolteenaho define their betas relative to the variance on the total market return instead of the variance of N_{CF} and N_{DR} respectively. As the variance of N_{CF} and N_{DR} are both invariant across time and estimation portfolios, this change of definition will not affect the pricing ability of the model.

III Estimation technique

To estimate the β and λ parameters of (17) a number of econometric methodologies could be applied. This paper uses the approach suggested by Fama and MacBeth (1973). The method is advantageous in this study due to the small number of time series observations relative to the number of cross sectional portfolios treated. The dataset consists of 200 quarterly observations and 25 asset return portfolios. Instead of estimating the models with the Fama-MacBeth methodology, one could estimate the models by GMM. However, to get stable GMM estimates we would most likely have to reduce the number of portfolios investigated. The ratio of moment restrictions to time series observations, would simply be too high with all 25 portfolios. Fama-MacBeth estimation is akin to 1. stage GMM with an identity matrix as weighting matrix. All 25 portfolios investigated are given equal importance when attempting to fit the model to the data. Alternatively one could use GMM with the optimal matrix of Hansen (1982) and iterate. This would mean placing different weights on the various portfolios dependent on the variance of the returns. We would like the models treated here to be able to price all 25 Fama-French portfolios equally well, which such an approach would not take into account. One of the main problems with the traditional models has been their inability to price the extreme portfolios. Small stocks and value stocks have historically realized higher returns than predicted by the betas of the traditional CAPM and C-CAPM. So to take an econometric approach that allows the models to place varying weights on these portfolios, would eliminate some of the effects we are trying to investigate. We want to see how well the different models price these specific 25 portfolios.

The Fama-MacBeth estimation technique for a factor model is as follows. First, run time-series regressions of portfolio excess returns on the factors of the respective models to find estimates of $\boldsymbol{\beta}$.

$$R_t^{ei} = a_i + \boldsymbol{\beta}_i' \mathbf{f}_t + \varepsilon_{i,t}, \qquad t = 1, 2, ..., T \text{ for each } i. \tag{32}$$

This gives us a vector of beta estimates for each asset portfolio. Now run one cross-sectional regression for each time period of excess returns on the time series regression betas

$$R_t^{ei} = \boldsymbol{\beta}_i' \boldsymbol{\lambda}_t + \alpha_{i,t}, \qquad i = 1, 2, ..., N \text{ for each } t. \tag{33}$$

The Fama-MacBeth estimates $\boldsymbol{\lambda}$ and α_i are then found as the time series average of the parameters estimated in the cross-sectional regressions and the residuals resulting.

$$\widehat{\boldsymbol{\lambda}} = \frac{1}{T}\sum_{t=1}^{T} \widehat{\boldsymbol{\lambda}}_t \qquad \widehat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T} \widehat{\alpha}_{i,t}$$

Standard errors of the parameter estimates are obtained in the following manner

$$\sigma^2\left(\widehat{\boldsymbol{\lambda}}\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\widehat{\boldsymbol{\lambda}}_t - \widehat{\boldsymbol{\lambda}}\right)^2 \tag{34}$$

$$cov\left(\widehat{\alpha}\right) = \frac{1}{T^2}\sum_{t=1}^{T}\left(\widehat{\alpha}_{i,t} - \widehat{\alpha}_i\right)\left(\widehat{\alpha}_{i,t} - \widehat{\alpha}_i\right)' \tag{35}$$

Cochrane (2001) shows that the parameter estimates from the Fama-MacBeth procedure will be equivalent to those found from a pure cross-sectional OLS estimate, given time-invariant $\boldsymbol{\beta}_i$ and estimation errors $\varepsilon_{i,t}$ which are uncorrelated over time. However, the OLS distribution assumes that the right-hand variables in $\boldsymbol{\beta}$ are constant. This is not the case in the Fama-MacBeth regression as we are estimating $\boldsymbol{\beta}$ in the time-series regression. Hence, we need a correction for the sampling error in $\boldsymbol{\beta}$. Shanken (1992) shows that the correction can be made using a multiplicative term given by $\left(1 + \boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right)$. $\boldsymbol{\Sigma}_f$ is the variance-covariance matrix of the factors. The resulting corrected standard errors are given by

$$\begin{aligned}
\sigma^2\left(\widehat{\boldsymbol{\lambda}}_{SH}\right) &= \frac{1}{T}\left[\left(\boldsymbol{\beta}'\boldsymbol{\beta}\right)^{-1}\boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta}\left(\boldsymbol{\beta}'\boldsymbol{\beta}\right)^{-1}\left(1 + \boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right) + \boldsymbol{\Sigma}_f\right] \\
&= cov\left(\widehat{\boldsymbol{\lambda}}\right)\left(1 + \boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right) - \frac{1}{T}\boldsymbol{\Sigma}_f\left(\boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right) \qquad (36) \\
cov\left(\widehat{\alpha}_{SH}\right) &= \frac{1}{T}\left(\mathbf{I}_N - \boldsymbol{\beta}\left(\boldsymbol{\beta}'\boldsymbol{\beta}\right)^{-1}\boldsymbol{\beta}'\right)\boldsymbol{\Sigma}\left(\mathbf{I}_N - \boldsymbol{\beta}\left(\boldsymbol{\beta}'\boldsymbol{\beta}\right)^{-1}\boldsymbol{\beta}'\right)\left(1 + \boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right) \\
&= cov\left(\widehat{\alpha}\right)\left(1 + \boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right) \qquad (37)
\end{aligned}$$

$\boldsymbol{\Sigma}$ is the residual covariance matrix from the time-series regression, $\mathbf{I}_N$ is the $N \times N$ identity matrix. Jagannathan and Wang (1998) find that Fama-MacBeth standard errors may not overstate the precision of the estimated coefficients when conditional heteroskedasticity is present. For this reason we also present uncorrected standard errors.

To test for zero pricing errors we can run the test

$$\left(1 + \boldsymbol{\lambda}'\boldsymbol{\Sigma}_f^{-1}\boldsymbol{\lambda}\right)^{-1}\widehat{\alpha}'cov\left(\widehat{\alpha}\right)^{-1}\widehat{\alpha}\,\tilde{}\,\chi^2_{N-K} \tag{38}$$

where $\widehat{\alpha}$ is the vector of pricing errors from the Fama-MacBeth procedure, N is the number of assets, and K is the number of factors in the model[5].

In addition to just testing whether the pricing errors resulting from the various models estimated in the paper are statistically different from zero, we

[5] Due to singularity of the covariance matrix of pricing errors, we use a Penrose Moore pseudo inversion. The singularity can be seen by noting that $\left(\mathbf{I}_N - \boldsymbol{\beta}\left(\boldsymbol{\beta}'\boldsymbol{\beta}\right)^{-1}\boldsymbol{\beta}'\right)$ in (36) is idempotent and hence singular.

also want to be able to compare the magnitude of the pricing error across models. Firstly we report average squared pricing errors. In this case, pricing errors from all portfolios investigated are thus given equal weighting. We also compute a composite pricing error given by

$$\widehat{CE} = \left[\widehat{\alpha}'\mathbf{\Omega}^{-1}\widehat{\alpha}\right]^{\frac{1}{2}} \tag{39}$$

where $\widehat{\alpha}$ is the vector of estimated residuals from the cross-sectional regression and $\mathbf{\Omega}^{-1}$ is the variance-covariance matrix of asset returns. Here the weight given to the pricing error of each portfolio is dependent on the precision with which the average returns on that portfolio are measured. As is evident, the weighting matrix in the CE measure is invariant between the different factor models. This allows comparison of the magnitude of the pricing errors across models. There are concerns with the accuracy of the estimate of the full variance-covariance matrix of asset returns given the high number of asset portfolios relative to time-series observations. Hence we also report composite pricing errors based on a diagonal variance matrix. The diagonal elements of the matrix contain the variance of returns and the remaining elements are set to zero.

Finally, the Hansen-Jagannathan distance measure is also reported. This is computed as

$$\widehat{HJ} = \left[\widehat{\alpha}'\mathbf{E}\left(\mathbf{RR}'\right)^{-1}\widehat{\alpha}\right]^{\frac{1}{2}} \tag{40}$$

So in this case we weight the vector of estimated residuals from the cross-sectional regression by the moment matrix of asset returns to achieve a measure of model pricing ability. Hansen and Jagannathan (1997) show that this measure can be interpreted as the maximum pricing error pr. unit payoff norm.

To calculate asymptotic standard errors of the four pricing error measures we follow the delta method. First construct the time series

$$\nu_t = \boldsymbol{\alpha}_t'\widehat{\boldsymbol{\zeta}} \tag{41}$$

where $\boldsymbol{\alpha}_t$ is the vector of pricing errors from the model at time t. $\widehat{\boldsymbol{\zeta}}$ is the sample estimate of $\boldsymbol{\zeta}$, defined as follows for each pricing error measure.

For the Hansen-Jagannathan measure

$$\boldsymbol{\zeta} = E\left(\mathbf{R}_t\mathbf{R}_t'\right)^{-1}\widehat{\boldsymbol{\alpha}} \tag{42}$$

For the average pricing error measure

$$\boldsymbol{\zeta} = \left(\mathbf{i}'\mathbf{i}\right)^{-1}\widehat{\boldsymbol{\alpha}} \tag{43}$$

where $\mathbf{i}$ is a $N \times 1$ vector of ones. N is the number of assets over which the pricing error is computed. For the composite pricing error measure with the full variance-covariance matrix of asset returns $\mathbf{\Omega}_F$ as weighting matrix

$$\boldsymbol{\zeta} = \boldsymbol{\Omega}_F^{-1}\widehat{\boldsymbol{\alpha}} \tag{44}$$

Finally, for the composite pricing error measure with the diagonal variance matrix $\boldsymbol{\Omega}_D$ as weighting matrix

$$\boldsymbol{\zeta} = \boldsymbol{\Omega}_D^{-1}\widehat{\boldsymbol{\alpha}} \tag{45}$$

The sample mean of ν_t for each $\boldsymbol{\zeta}$ equals the sample estimate of the respective pricing error measure squared, $\widehat{PE}^2$. Let s^2 be the variance of the sample mean of ν_t. From the delta method it then follows that the asymptotic standard error of the estimated pricing error measure $\widehat{PE}$ is consistently estimated by $\frac{s}{2\widehat{PE}}$.

IV Data

This paper estimates the models described on US data at a quarterly frequency for the time period running from the 1. quarter of 1952 to the 4. quarter of 2001. For the return on the stock market portfolio the return on the CRSP value weighted stock index (NYSE/AMEX/NASDAQ) is used. The risk-free rate is obtained as the return on T-bills with three months maturity, taken from CRSP. Consumption growth is based on seasonally adjusted, real pr. capita, quarterly expenditure on nondurables and services, taken from the Bureau of Economic Analysis, U.S. Department of Commerce.

The factors N_{CF} and N_{DR}, which form the basis of the CV model, are based on a VAR model using the same state variables as those used in Campbell and Vuolteenaho (2004)[6]. The state variables used for the main estimation will be the log excess return on the market portfolio, the yield spread between long-term and short-term bonds, the smoothed price-earnings ratio from Shiller (2000), and the small-stock value spread. The value spread is based on data from the website of Kenneth French and is defined as the difference between the log book-to-market ratio of small value and small growth stock. The N_{CF} and N_{DR} estimates are based on VAR estimations over the full sample of Campbell and Vuolteenaho (2004) which is 1929-2001. The four factors are also used to estimate a simple linear factor model.

In a recent paper Chen and Zhao (2005) estimate the I-CAPM of Campbell and Vuolteenaho using a variety of alternative state variables in the VAR model. The paper shows the cash flow and discount rate betas to be sensitive to changes in the composition of the state variable vector. However, only when the number of state variables is significantly increased is an improvement in the adjusted R^2 of the cross sectional regression observed. We therefore keep the state variable vector of Campbell and Vuolteenaho (2004) when estimating the models cross sectionally and comparing the pricing ability of this model to the C-CAPM and conditional models.

[6]The data is available on the webpage of Vuolteenaho: http://post.economics.harvard.edu/faculty/vuolteenaho/papers.html

The conditioning variable used in the conditional factor models is the *cay* variable developed in Lettau and Ludvigson (2001a). The data is available on the website of Sydney Ludvigson[7]. The scaling variable is demeaned as in Lettau and Ludvigson (2001b).

For the cross-sectional asset pricing model estimation, equity return data is based on the excess returns of 25 portfolios sorted by the Fama and French (1993) factors. These are returns on US stocks (NYSE/AMEX/NASDAQ) sorted into 25 portfolios. The portfolios are constructed on the basis of size and the book equity to market equity ratio (BE/ME) quantiles. The portfolio returns are available on the website of Kenneth French[8]. Returns are measured in excess of the 3-month T-bill rate. Data on the SMB and HML factors of the 3 factor Fama-French model are also taken from the website of Kenneth French.

V Empirical Results

The assets used for the empirical cross-sectional estimations of this paper are the 25 Fama-French portfolios. As noted in the previous section, these consist of returns on US stocks grouped into 25 portfolios based on a sorting by size and BE/ME.

Summary statistics are shown in table 1. The portfolios show a clear pattern of increasing average returns as we move from growth to value stocks, within a size quantile. Growth stocks are defined as stocks with a low BE/ME ratio, whereas value stocks have high BE/ME ratios. Growth stocks have good future prospects expressed in the form of high stock prices and low current returns. The opposite is the case for value stocks, thus giving rise to the pattern of rising average returns when moving from portfolios of growth stocks to value stocks. In the small stock case we go from an annualized return of 4.8% for growth stocks to 14.3% for value stocks. The standard deviation of growth stocks are higher than those of value stocks. The least volatile portfolios are those in the mid quantiles, based on the BE/ME sorting. This is where the largest concentration of stocks is placed, whereas the extreme portfolios are based on relatively few cross sectional observations. Apart from the five growth portfolios, there is a general tendency for falling average returns when moving from small to large stock portfolios. For value stocks average annualized returns fall from 14.3% for small stocks to 9% for large stocks. For the growth portfolios, the pattern is slightly different. Unlike the other portfolios, we observe a tendency towards rising average returns when moving from small to large stock portfolios.

The question is now, how well the various models fit these return patterns.

The following sections present results from the cross sectional Fama-MacBeth model estimations. We use excess returns on the 25 Fama-French portfolios. Models are estimated both without a constant, i.e. with the zero-beta rate restricted to equal the T-bill return, and with a constant, allowing the zero-beta

[7] http://www.econ.nyu.edu/user/ludvigsons/

[8] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

rate to be freely estimated. When estimated without a constant, we are asking the model to fit the unconditional equity premium, in addition to fitting across the 25 stock return portfolios.

V.1 Traditional models

The second stage of estimation gives us the λ coefficients of the cross-sectional models. In table 2 we present estimates of the base case C-CAPM, the traditional CAPM, and the Fama-French three factor model.

The models generally do a poor job of explaining the equity premium. The constant is significantly different from zero, when included, even though we are estimating the models on excess returns. In this case, the theory behind all the models would predict the constant to be zero.

Estimation of the CAPM with unrestricted zero-beta rate results in a negative coefficient on the market return beta. This is one of the classic problems seen in empirical estimates of the model. Unlike that predicted by theory, this implies that assets with high return covariance with the market give lower excess returns, than assets with low market betas. The coefficient is insignificant and the low R^2 emphasizes the poor performance of the static CAPM, as has also been found in previous studies (Fama and French (1992), Lettau and Ludvigson (2001b)). When we restrict the zero-beta rate to equal the risk-free rate, a significant positive market return beta coefficient results. This stems from the aggregation of the coefficient estimate on the constant in our unrestricted model and the market return beta term. As the market return beta structure is relatively flat across average portfolio returns, this term behaves almost as a constant in the cross sectional regression. Hence, when the zero beta rate is restricted to equal the risk free rate, much of that which was previously captured by the intercept term is compounded into the market return beta term.

The C-CAPM performs marginally better than the static CAPM. Looking at the consumption beta coefficient estimates in table 3 we see a relatively high degree of variation across the 25 portfolios, in line with the average return pattern observed in table 1. However, we also observe relatively high standard errors on these estimates, indicating time variation in the realized beta pattern. These patterns are compounded into the cross sectional model estimates. The adjusted R^2 for the unrestricted zero-beta model rises to 13%, compared to 7% for the CAPM, though the consumption coefficient is statistically insignificant. Only when the zero-beta rate is restricted to equal the risk-free rate, does consumption growth obtain a significant positive coefficient. Thus, even without imposing a structure on the model in the form of a specific utility function, we observe poor empirical performance. Only when we look at the pricing errors of the C-CAPM without a constant and with the Shanken correction can we not reject the hypothesis of zero pricing errors. This none rejection is mainly due to the large Shanken correction factor imposed.

Estimates of the risk aversion coefficient γ, based on the linearized C-CAPM with power utility derived in section II.1, are 117 for the unrestricted zero-beta model and 351 for the restricted zero-beta case. These are extremely high, highlighting the well known equity premium puzzle of the C-CAPM. The results are in line with similar estimates of Cochrane (1996).

The third model of the table can be seen as a log linearized version of the C-CAPM with Epstein-Zin-Weil (EZW) utility. This model contains the market return factor of the CAPM and the consumption growth factor of the C-CAPM. It is included, because it is this version of the C-CAPM that forms the basis of the Campbell and Vuolteenaho (2004) model, to be estimated shortly. The parameter estimates are similar to those found in the previous models. We still obtain a negative coefficient on the market return, when the zero-beta rate is unrestricted. However, the consumption growth factor becomes significantly positive in both cases. There is a rise in the adjusted R^2 to 43% in the unrestricted case. Again, we can compute estimates of γ following equation (18). The EZW C-CAPM results in a γ of 98 and -117 for the unrestricted and restricted zero-beta models respectively. So we still observe values of the risk aversion parameter which go against those predicted by theory.

For the Fama-French model, the pattern for the market return mirrors that found in the CAPM, with a λ estimate of -1.1196. The additional factors of the Fama-French model are thus not able to explain the negative market return coefficient, when the zero-beta rate is allowed to vary freely. However the HML factor is consistently significantly positive and the adjusted R^2 has risen to around 65%. This is contrary to evidence from Lettau and Ludvigson (2001b), where a positive coefficient is estimated for the market return beta. There is a slight difference in the time periods on which our model estimates and those of Lettau and Ludvigson (2001b) are based. If we instead estimate the Fama-French model on our data, but using the time period from the third quarter of 1963 to the third quarter of 1998, corresponding to that of Lettau and Ludvigson (2001b), we obtain estimates similar to those found in their paper with a positive coefficient estimate on the market return beta of 1.3198. The coefficient estimate on the market return of the Fama-French model thus appears to be extremely unstable. In fact, we need only add four to five quarters of data to the Lettau and Ludvigson subsample, in either the preceding or succeeding period, to go from a positive coefficient estimate to a negative coefficient estimate. Fama and French (1992) also find a negative coefficient on the market return beta when estimating a similar 3 factor model including size and BE/ME factors.

The basis for this pattern can easily be found if we take a quick look at the market return beta values computed in our first stage estimates with the full sample. The market return betas across the 25 test portfolios are presented in table 3. As is evident, there is very little variation in the beta values across assets. The estimated value is close to 1 for all assets. When we come to estimating the cross sectional regression the market beta regressor will mimic the features of a constant regressor with value 1. In our unrestricted zero beta model, the pure constant will capture the intercept value of the series. As there is only very little variation left in the market return beta, the estimated cross sectional coefficient becomes insignificant and unstable across subperiods. If we restrict the zero beta rate to equal the risk-free return, the market return beta steps in and acts almost as a constant. This is exactly the same pattern as that observed for the CAPM. If we take this to the extreme and completely eliminate the market return factor from the Fama-French model, the adjusted R^2 actually rises compared to the three factor Fama-French model[9].

V.2 I-CAPM

This section treats estimates of the Campbell and Vuolteenaho I-CAPM in both its restricted and unrestricted form as stated in table 5. Additionally, a simple model based on the factors included in the VAR of the I-CAPM is estimated.

To start with, we look at estimates of the two betas of the I-CAPM for all 25 portfolios as presented in table 4. If we look at the beta pattern across assets, we find that the discount rate beta is higher for growth than value stocks. On the contrary, the cash flow beta is higher for value than growth stocks. Given our expectation of higher risk premia on cash flow shocks than discount rate shocks, these patterns would be in-line with the empirical observation of higher average returns on value than growth stocks. For both beta there is a pattern of higher values for small stocks than large stocks, thereby explaining the tendency for small stocks to have higher average returns than large stocks. These patterns imply that the cross-sectional model should be better at fitting asset return variation across the 25 portfolios than the CAPM and C-CAPM, which tend to show a relatively flat beta structure across the size and BE/ME sorted portfolios.

Turning now to the cross sectional model estimates of table 5, the first general tendency we observe, across all three models, relates to the effect of allowing the zero-beta rate to vary freely versus restricting it to equal the risk-free rate. The coefficient on the constant, when this is included, is insignificant in all cases, resulting in only small differences between the two model versions. The models thus appear relatively good at handling the equity premium. Only for the restricted CV model is there an observable difference in the R^2 and pricing errors between the restricted and unrestricted zero-beta rate versions. The unrestricted CV model obtains an adjusted R^2 of 54%, which is much higher than the 9% observed for the traditional C-CAPM.

If we look at the coefficient estimates on the risk factors of the I-CAPM, the risk price on the cash flow beta is positive and significantly different from zero. It is also much higher than that placed on the discount rate beta. This is the case both when the risk price on the discount rate beta is freely estimated and when it is restricted to be equal to the variance on N_{DR}. In fact, when the risk price on the discount rate beta is freely estimated a negative, but highly insignificant, coefficient is obtained. Campbell and Vuolteenaho (2004) refer to this pattern as the story of the good beta and the bad beta. Namely, that it is risk associated with cash flows, which is priced highest by investors. Investors require higher excess returns on stocks with a high cash flow beta or "bad beta", as shocks to cash-flows are of a permanent nature in contrast to the transitory behavior of discount rate shocks. As we observed above, the pattern of beta

[9]estimates are available from the author on request.

estimates across the 25 Fama-French portfolios, indicates that the value and size puzzles found previously in the literature, are in fact not puzzles at all. The excess returns on value stocks over growth stocks is due to a higher exposure of the value stocks to the cash flow beta, for which investors require a higher return premium. Similarly, small stocks have higher exposures to both cash flow and discount rate betas than large stocks, thereby giving a risk based explanation for the excess returns observed hereon.

The risk price estimates are similar to those found by Campbell and Vuolteenaho, but with slightly different magnitudes. This is due to our use of traditional beta estimates as factors in the cross-sectional model, where Campbell and Vuolteenaho use betas measured relative to the market return variance. In both the restricted and unrestricted case with a constant, we cannot reject the hypothesis of zero pricing errors based on Shanken standard errors.

The risk aversion coefficient γ can be derived from the lambda estimates of the restricted I-CAPM, following equation (30). When the model is estimated with a constant γ is 31 and for the case with the zero-beta rate restricted to equal the riskfree rate γ is 16. These estimates are in line with those found by Campbell and Vuolteenaho at a quarterly data frequency. Compared to the γ of the C-CAPM computed previously, the risk aversion coefficient has been greatly reduced. However, it is still above the levels generally referred to as theoretically sound.

The final two columns of table 5 present estimates of the VAR factor model. Unlike the CAPM and Fama-French models, the beta on the market return is positive in this case, as predicted by the theory. We observe that the value spread factor is not statistically different from zero in the model. This despite the high importance placed on this variable by Campbell and Vuolteenaho (2004). The beta of the factor, from the first stage estimate, does however seem to vary greatly across asset portfolios[10]. When the zero-beta rate is freely estimated, only the term yield factor is significant. We do find an adjusted R^2 of 0.82. The low degree of significance in the coefficient estimates on the factors included suggests that this may be more due to the high number of free factors in this model, rather than an actual ability of the factors to explain the asset return patterns found.

V.3 Conditional models

The final set of models to be estimated, are conditional versions of the CAPM and C-CAPM. The models are estimated as scaled factor models using the log consumption-wealth ratio proxy *cay* as scaling factor and results are presented in table 6[11].

[10]Beta estimates from the first stage of Fama-MacBeth estimation are available from the author upon request.

[11]The conditional models have also been estimated with alternative conditioning variables. Estimates of the conditional C-CAPM using the yield spread, the log dividend price ratio, a default risk premium, and the price-output variable of Rangvid (2005) as conditioning variables all result in similar conclusions.

Across all the models, the time-varying component of the intercept has little power in explaining average excess returns. However, the coefficient on the constant is statistically significant for all three models. This indicates that even when we allow for time variation in the factor coefficients the zero-beta rate does not equal the risk-free rate.

Contrary to the findings of Lettau and Ludvigson, we find no significance of the scaled consumption factor in the conditional C-CAPM, when the zero-beta rate is freely estimated. The explanation for this divergence in results can, once again, be found in the sample period used. Lettau and Ludvigson's estimates are based on a sample ending in the third quarter of 1998, whereas our sample runs through the fourth quarter of 2001. If we exclude the last two years of data from our sample, we obtain a highly significant positive coefficient on the scaled consumption factor, as was the finding of Lettau and Ludvigson (2001). The scaled consumption factor is unable to capture the developments of the stock market in the first years of the 21^{st} century.

When the zero-beta rate is restricted to equal the risk-free rate, the scaled factor does become significant. In this case we find a significant positive coefficient on scaled consumption growth. The pattern is similar to that observed in the unconditional model, where a significant positive coefficient on consumption growth was found.

Compared to the unconditional C-CAPM the adjusted R^2 of the conditional model has gone from 0.09 to 0.53 for the unrestricted zero-beta model. So we do see a large improvement in the ability of the model to explain the observed asset returns over that of the traditional C-CAPM. The improvement is similar to that found when estimating the two beta model of Campbell and Vuolteenaho (2004).

The conditional CAPM performs much better than the unconditional model. For the CAPM, the scaled factor is significantly positive. It thus appears, that part of the problems of the static CAPM can be solved by allowing time-variation in the market return coefficient.

The conditional EZW model with a constant only results in significant coefficient estimates for the constant. The remaining parameters are found to be insignificant and it results in a lower adjusted R^2 than the conditional C-CAPM.

V.4 Pricing errors

In addition to looking at the credibility of the coefficient estimates of the models, we want to measure the pricing ability by investigating the magnitude of the pricing errors resulting from the empirical estimates. To look at the comparative magnitude of pricing errors across models, four numbers are presented in table 7. These are the square root of average squared pricing errors, the Hansen-Jagannathan distance measure, and two measures of variance weighted pricing errors. The weighting matrices are the full variance-covariance matrix of portfolio returns and a diagonal matrix of the variances of portfolio returns.

If we look at the magnitude of pricing errors across the traditional models, the Fama-French model results in the lowest average pricing errors. The average pricing error is 0.32 and 0.34 in the unrestricted zero-beta and restricted zero-beta case respectively. Average pricing errors are slightly smaller for the C-CAPM than for the CAPM. For the unrestricted zero-beta case the C-CAPM has an average pricing error of 0.56 and the CAPM has 0.58. The improvement in pricing ability resulting from moving from the static CAPM to the intertemporal C-CAPM is thus marginal, in accordance with previous empirical findings. In all cases, the restricted zero-beta versions of the models perform worse than the unrestricted case. On the one hand this is to be expected, as we have more free parameters in the unrestricted case. On the other hand, if we are to follow the theory underlying the models, the constant should be zero. So the fact that the models perform better with a constant indicates a failure of the underlying theory. When weighting pricing errors by portfolio variances, using the diagonal matrix results in exactly the same comparative pattern as simple average pricing errors. With the full variance-covariance matrix, the CAPM has lower pricing errors than the C-CAPM.

Looking at the I-CAPM of Campbell and Vuolteenaho (CV), the restricted CV model performs slightly worse than the unrestricted version. The unrestricted model results in an average pricing error of 0.39 in both the case when a constant is included and when the zero-beta rate is constricted to equal the risk-free rate. For the restricted model average pricing errors are 0.42 and 0.49 with unrestricted and restricted zero-beta rate respectively. This is to be expected given the additional free parameter in the unrestricted model. The relative pricing ability of the two models is consistent across pricing error measures.

The final set of models presented are the conditional models. For these we observe lower pricing errors, across all measures, for the conditional C-CAPM than the conditional CAPM, when a constant is included. Average pricing errors for the conditional C-CAPM are 0.38 and 0.42 for the conditional CAPM. As was the case for the unconditional models, when the constant is eliminated, the CAPM outperforms the C-CAPM.

Next we compare the pricing ability of the two new asset pricing strands to each other. First we will look at models where a constant has been included. The average squared pricing error of the conditional EZW C-CAPM is slightly smaller than that of both the restricted and unrestricted CV-model. So when giving equal weight to the pricing errors from the 25 portfolios, the conditional EZW C-CAPM outperforms the Campbell and Vuolteenaho setup. The conditional C-CAPM, that is the model without the market return factor, also has slightly lower average squared pricing errors than the CV-models. When pricing errors are weighted by the diagonal variance matrix of returns, the pattern is similar to that found using an equal weighting. Using the moment matrix as in the Hansen-Jagannathan distance measure shows a slightly different pattern. Now the best performance is achieved using the unrestricted CV-model, followed by the conditional C-CAPM and EZW C-CAPM. The highest H-J measure is obtained from the restricted CV-model. The same pattern is found when comparing pricing errors using the full variance-covariance matrix of returns as weighting matrix.

Next we look at models where we restrict the zero-beta rate to equal the risk-free rate of return by working without a constant in our cross-sectional models. Based on average squared pricing errors, the unrestricted CV-model outperforms the conditional C-CAPM and EZW C-CAPM. The restricted CV-model has lower pricing errors than the conditional C-CAPM, but not the conditional EZW C-CAPM. The same pattern is observed when weighting pricing errors by the diagonal variance matrix of returns. When using the full variance-covariance matrix, the restricted and unrestricted CV-models both clearly outperform the conditional C-CAPM and EZW C-CAPM. If we evaluate the pricing ability of the model by weighting pricing errors by the moment matrix as suggested by the Hansen-Jagannathan distance measure, the same pattern is observed.

All in all, based on the four pricing error measures investigated here, there is no clear pattern of one of the two alternative strands dominating the other. Which model offers the best pricing ability is dependent on which pricing error measure we investigate. And even then, the differences are marginal.

Finally, we compare the pricing ability of the traditional models to the new models, to see if any improvement over the pricing ability of the C-CAPM has been achieved. If we focus on average pricing errors these fall from 0.56 for the unconditional C-CAPM to around 0.4 for the conditional C-CAPM and CV models, in the unrestricted zero-beta rate case. This is the measure focused on by Lettau and Ludvigson (2001b) when comparing the pricing ability of a conditional and unconditional C-CAPM. We thus find the same support for price improvement as is the case in their paper. Similarly, if we compare pricing ability by looking at the composite pricing errors weighted by the diagonal variance matrix of asset returns, we find improved pricing ability from the conditional C-CAPM and the I-CAPM of Campbell and Vuolteenaho, over the traditional C-CAPM and CAPM. This is similar to the findings of Campbell and Vuolteenaho (2004), who compare their model to the CAPM using this pricing error measure. However, when looking at composite pricing errors using the full variance-covariance matrix there is only a small difference in the pricing ability of the new models and the unconditional C-CAPM. In fact, when weighting pricing errors by the moment matrix of asset returns in the H-J distance measure, the unconditional C-CAPM has lower pricing errors than both the conditional C-CAPM and the two CV-models.

The overall picture resulting from the comparison of the two new literature strands is thus, that they both offer improved pricing ability over the traditional C-CAPM when looking at average pricing errors. However, this improvement in pricing ability is not consistent across pricing error measures. When using the H-J distance measure, the C-CAPM outperforms the new models and we are thus no further in solving the empirical asset pricing problems of the consumption based literature. The Fama-French three factor model is still the model that results in the lowest pricing errors, across all error measures.

V.5 Pricing ability

To give a more visual description of the pricing capability of the models investigated, the graphs presented in figure1, figure 2, figure 3, and figure 4 show plots of average realized excess returns and the average excess returns obtained from the respective models. Realized returns are plotted along the horizontal axis and estimated returns along the vertical axis. If a given model were to fit the empirical data perfectly, all observations would lie along the 45° line.

From the plots of the four traditional models in figure1 and figure 2, it is clear that the Fama-French model has the best performance. The points are relatively close to the 45° line and there are no extreme outliers. It is also obvious that the CAPM fits the data poorly. In fact, the portfolios with the lowest realized excess returns obtain the highest estimated excess returns from the CAPM. These are the small growth portfolios. The extreme outliers below the 45° line are also from the small size quantile, but are on the other end of the BE/ME spectrum as value stocks. This is the well documented value-effect, which the static CAPM is unable to handle. The C-CAPM does slightly better, but is still unable to fit the realized excess return of the small growth portfolio. As the CAPM documents, there is little pricing ability in the market return factor. Including this along with the consumption growth factor, as is done with the EZW C-CAPM, thus results in plots closely mimicking the C-CAPM.

The intertemporal models of Campbell and Vuolteenaho perform somewhat better than the traditional C-CAPM and CAPM, as can be seen in figure 3. In the unrestricted case particularly, realized and estimated excess returns line up relatively well. The restricted CV model faces some of the same problems as the static CAPM. Estimated excess returns for the small growth and small value stock portfolios are very close, especially when the zero-beta rate is restricted to equal the risk-free rate. This contradicts the empirical observations of a realized annualized excess return of 4.8% for the small growth portfolio and 14.3% in the small value portfolio case. Both the CV-models have problems fitting the small growth portfolio situated as the left most point on both plots. The models predict higher returns than the empirically realized excess return of 1.2% per quarter. The best fit is found when estimating the four factor model that includes the VAR factors of the Campbell and Vuolteenaho I-CAPM. The plots of portfolio returns lie very close to the 45° line and there is no value or size effect issues. As could be expected, the more free factors, the better model fit results.

The final models investigated are the conditional CAPM and C-CAPM. As can be seen from figure 4, there is still some dispersion in the plots around the 45° line. In comparison to the unconditional models however, a clear improvement can be noted. Especially in the case of the CAPM. The conditional C-CAPM fits the small growth portfolio slightly better than the CV-models. The conditional EZW C-CAPM without a constant also fits this portfolio relatively well, but on the other hand has problems with the small value portfolio located at the right most point on the figure. There thus still seems to be some problems in fitting the value premium.

VI Concluding remarks

The consumption based capital asset pricing model (C-CAPM) has had an important place in the finance research literature over the past 25 years, despite its poor empirical performance. The reason it has maintained the interest of the academic world, is the simplicity and intuitive appeal of the theory underlying the model. We are thus not interested in dismissing the model completely. Instead focus is on the assumptions made to get from the basic idea of the priced risk of an asset being determined by the asset's covariance with the marginal utility of consumption, to the risk measures estimated empirically. Can the empirical problems of the C-CAPM be solved by revising these assumptions?

A number of alternative models attempting to improve on the pricing ability of the C-CAPM have been developed. Two strands of the literature are investigated in this paper. The intertemporal CAPM of Campbell and Vuolteenaho (2004) and the conditional C-CAPM of Cochrane (1996) and Lettau and Ludvigson (2001b).

The first model is based on the assumption that empirical consumption data is a poor proxy for the measure of consumption referred to in the theoretical C-CAPM. If the deviations between the theoretical and empirical measures of consumption are large, these data issues may be the root of the poor empirical performance of the C-CAPM. From this observation Campbell and Vuolteenaho (2004) develop a two beta intertemporal model, based on the same framework as the C-CAPM, but without reference to consumption. Instead the asset risk premium is determined by a cash-flow and a discount-rate beta.

The second set of models investigated, focus on the conditional pricing ability of the C-CAPM. The reasoning behind this setup is recent empirical evidence of time variation in expected returns. If the cause of this is time-varying risk premia, then we must incorporate it into the model. This can be achieved by restating the C-CAPM conditionally, in a scaled factor model setup. The approximate log consumption-wealth ratio *cay* of Lettau and Ludvigson (2001a) is used as conditioning variable.

The traditional C-CAPM, as well as the CAPM and the well known three factor Fama-French model are estimated on quarterly US data. These are then compared to estimates of the models from the two alternative strands of literature, to investigate whether any quantifiable improvements have been made in the empirical asset pricing ability. The assets priced are the 25 size and BE/ME sorted Fama-French portfolios.

The empirical results of this paper underline previous research showing the poor performance of the C-CAPM as well as the static CAPM. The estimated coefficients are insignificant or of the wrong sign, compared to that predicted by the underlying theory. The C-CAPM has slightly lower average pricing errors than the CAPM, but is still unable to explain the value and size effects.

Compared to the C-CAPM the intertemporal model of Campbell and Vuolteenaho (2004) has much higher R^2 and lower average pricing errors. The two betas of the model do a better job than the traditional models of fitting the unconditional equity premium, even though only the rsik price of the cash-flow beta is significant. However, there are still some problems in matching the realized returns on the extreme small growth stock portfolio. Especially when we restrict the price of discount-rate risk to equal the sample variance of the discount rate.

When scaling the consumption growth factor of the C-CAPM to obtain an estimate of the conditional C-CAPM, a similar pattern is observed. With an unrestricted zero-beta rate, average pricing errors fall from 0.55 for the unconditional C-CAPM to 0.4 for the conditional model. However, neither the consumption growth nor the scaled consumption growth factor are statistically significant, imploring one to be cautious when interpreting the improved pricing ability found.

Unlike previous research into the pricing improvement of these two new model strands, we delve futher into the pricing ability of the models estimated by looking at a number of weighted pricing error measures. Comparing the resulting observations for the two new model strands can only lead one to the conclusion, that there is no clear cut winner. None of the models from the alternative literature strands perform significantly better than the others. In fact, which strand results in the lowest weighted pricing errors, is very much dependent on the weighting matrix chosen. Generally the Campbell and Vuolteenaho (2004) framework has the lowest pricing errors when using a full variance-covariance matrix of returns or the moment matrix. The picture becomes more blurred when looking at average squared pricing errors or errors weighted by the diagonal variance matrix of returns.

The most damning evidence against the two new model strands is found when looking at pricing errors weighted by the full variance-covariance matrix of asset returns or the moment matrix of the returns. Based on these two pricing error measures we actually find that the traditional C-CAPM outperforms the new models.

On the basis of these observations it thus seems wise to continue the search, as it were, if we wish to improve on the pricing ability of the consumption-based asset pricing model.

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Table 1: Summary statistics for 25 Fama-French portfolio returns
The table presents average quarterly excess returns and standard errors for the 25 Fama-French portfolios. The time series data run from the 1. quarter of 1952 through the 4. quarter of 2001. Returns are measured in excess of the 3-month T-bill rate. Standard deviations are in parentheses.

	Growth	2	3	4	Value	Value-Growth
Small	1.1888	2.5099	2.6682	3.3000	3.5871	2.3983
	(15.5616)	(13.3429)	(11.8208)	(11.3130)	(12.2051)	
2	1.4924	2.2801	2.8382	3.0223	3.2880	1.7956
	(14.0166)	(11.7393)	(10.3388)	(10.2094)	(10.9666)	
3	1.7851	2.3729	2.3846	2.8640	3.0160	1.2309
	(12.5797)	(10.2430)	(9.5581)	(9.3997)	(10.3032)	
4	1.9518	1.8021	2.4781	2.6302	2.8606	0.9088
	(11.4247)	(9.5816)	(8.8473)	(8.8734)	(10.3074)	
Large	1.7223	1.7071	1.9978	2.0347	2.2393	0.5170
	(9.0670)	(8.0063)	(7.2724)	(7.8221)	(8.6240)	
Large-Small	0.5335	-0.8028	-0.6704	-1.2653	-1.3478	

Table 2: Unconditional models. Excess returns. With and without constant.

The table presents $\boldsymbol{\lambda}$ estimates from the cross-sectional Fama-MacBeth regressions $E\left[R_{i,t+1} - R_{f,t+1}\right] = \boldsymbol{\beta}_i'\boldsymbol{\lambda}$. The test assets are the 25 size and BE/ME sorted portfolios of Fama and French. Returns are measured in excess of the risk free rate. R_m is the return on the CRSP value weighted stock index and Δc denotes consumption growth. Standard errors are presented in parentheses. The top set are uncorrected Fama-MacBeth standard errors and below are standard errors modified with the Shanken (1992) correction. Alpha and alpha-Shanken are χ^2 tests of the hypothesis of zero-pricing errors. A star (*) denotes significant at a 5% level.

	C-CAPM		CAPM		EZW C-CAPM		Fama-French		2 factor Fama-French	
constant	1.6390		3.3280		3.1284		2.9508		2.1558	
	(0.5762)*		(0.9317)*		(0.9357)*		(1.2870)*		(0.5164)	
	(0.6545)*		(0.9366)*		(1.3455)*		(1.3331)*		(0.5322)	
R_m			-0.8332	2.0682	-0.8088	1.8972	-1.1196	1.7297		
			(1.0754)	(0.6174)*	(1.0769)	(0.6057)*	(1.3899)	(0.5812)*		
			(1.0795)	(0.6201)*	(1.4307)	(0.6492)*	(1.4313)	(0.5824)*		
Δc	0.2467	0.7367			0.4600	0.5480				
	(0.1882)	(0.2135)*			(0.1663)*	(0.1648)*				
	(0.2131)	(0.4011)			(0.2367)*	(0.2544)*				
SMB							0.4283	0.5058	0.4417	2.0266
							(0.4156)	(0.4164)	(0.4163)	(0.5424)*
							(0.4165)	(0.4186)	(0.4172)	(0.5649)*
HML							1.2949	1.3701	1.3160	1.2043
							(0.4343)*	(0.4345)*	(0.4343)*	(0.4351)*
							(0.4360)*	(0.4382)*	(0.4357)*	(0.4395)*
R^2	0.1332	-0.4360	0.0734	-0.8443	0.4305	-0.3730	0.7185	0.6690	0.7148	-1.3050
R^2_{adj}	0.0955	-0.4360	0.0331	-0.8443	0.3788	-0.4327	0.6783	0.6389	0.6889	-1.4052
alpha	71.25*	97.77*	69.51*	92.38*	68.79*	90.78*	59.67*	75.90*	62.55*	94.04*
alpha-Shanken	55.22*	27.23	68.79*	86.74*	33.27	37.24*	55.62*	65.39*	58.89*	79.11*

Table 3: Beta estimates from traditional models
The table presents time-series estimates of the consumption growth beta of the C-CAPM and the market return beta of the Fama-French 3 factor model for the 25 size and BE/ME sorted portfolios of Fama and French.

$\widehat{\beta}_{\Delta c}$	Growth	2	3	4	Value	Value-Growth
Small	4.5895	5.3624	3.7071	3.9913	4.3175	-0.2720
	(2.3877)	(2.0308)	(1.8115)	(1.7288)	(1.8650)	
2	3.1068	2.9144	3.4219	2.9458	3.8994	0.7926
	(2.1593)	(1.8061)	(1.5825)	(1.5671)	(1.6755)	
3	2.9968	2.4610	2.8261	2.7729	3.3448	0.4512
	(1.9364)	(1.5766)	(1.4665)	(1.4422)	(1.5777)	
4	2.3813	2.2463	2.0008	2.5116	3.9361	1.5548
	(1.7611)	(1.4752)	(1.3627)	(1.3625)	(1.5715)	
Large	2.6319	1.5746	2.0922	1.9735	3.2097	0.5778
	(1.3916)	(1.2348)	(1.1163)	(1.2032)	(1.3159)	
Large-Small	-1.9576	-3.7878	-1.6149	-2.0178	-1.1078	
$\widehat{\beta}_{Rm}$	Growth	2	3	4	Value	Value-Growth
Small	0.9998	0.9929	0.9061	0.9128	0.9975	-0.0023
	(0.0520)	(0.0343)	(0.0327)	(0.0281)	(0.0301)	
2	1.0776	1.0059	0.9649	0.9934	1.0467	-0.0309
	(0.0336)	(0.0285)	(0.0262)	(0.0261)	(0.0255)	
3	1.0803	1.0065	0.9924	1.0107	1.0329	-0.0474
	(0.0292)	(0.0282)	(0.9924)	(0.0301)	(0.0318)	
4	1.0522	1.0271	1.0098	1.0119	1.1042	0.0520
	(0.0280)	(0.0336)	(0.0311)	(0.0303)	(0.0403)	
Large	1.0505	1.0008	0.9209	1.0188	1.0646	0.0141
	(0.0224)	(0.0269)	(0.0309)	(0.0287)	(0.0370)	
Large-Small	0.0507	0.0079	0.0148	0.1060	0.0671	

Table 4: Beta estimates from Campbell model

The table presents time-series estimates of the cash flow and discount rate beta of the Campbell and Vuolteenaho (2004) I-CAPM for the 25 size and BE/ME sorted portfolios of Fama and French.

$\widehat{\beta}_{CF}$	Growth	2	3	4	Value	Value-Growth
Small	0.9986	1.0504	1.0495	1.1404	1.2779	0.2793
	(0.2260)	(0.1869)	(0.1709)	(0.1613)	(0.1869)	
2	0.9407	0.9848	1.1201	1.1724	1.2754	0.3347
	(0.1786)	(0.1537)	(0.1316)	(0.1393)	(0.1596)	
3	0.8067	1.0760	1.1878	1.2546	1.3058	0.4991
	(0.1448)	(0.1205)	(0.1174)	(0.1235)	(0.1493)	
4	0.9563	1.0727	1.2481	1.2319	1.3603	0.4040
	(0.1181)	(0.1045)	(0.1002)	(0.1031)	(0.1456)	
Large	0.6866	0.9042	0.9602	1.0988	1.0079	0.3213
	(0.0773)	(0.0823)	(0.0855)	(0.0991)	(0.1297)	
Large-Small	-0.312	-0.1462	-0.0893	-0.0416	-0.2700	

$\widehat{\beta}_{DR}$	Growth	2	3	4	Value	Value-Growth
Small	1.4726	1.2857	1.1078	1.0600	1.0744	-0.3982
	(0.0869)	(0.0718)	(0.0657)	(0.0620)	(0.0718)	
2	1.4273	1.1724	1.0287	0.9700	0.9935	-0.4338
	(0.0687)	(0.0591)	(0.0506)	(0.0535)	(0.0613)	
3	1.3350	1.0584	0.9510	0.8907	0.9213	-0.4137
	(0.0556)	(0.0463)	(0.0451)	(0.0475)	(0.0574)	
4	1.2460	1.0113	0.8911	0.8874	0.9323	-0.3137
	(0.0454)	(0.0402)	(0.0385)	(0.0396)	(0.0559)	
Large	1.0325	0.8595	0.7305	0.7484	0.7591	-0.2734
	(0.0297)	(0.0316)	(0.0329)	(0.0381)	(0.0498)	
Large-Small	-0.4401	-0.4262	-0.3773	-0.3116	-0.3153	

Table 5: Unconditional models. Excess returns. With and withour constant. The table presents $\boldsymbol{\lambda}$ estimates from the cross-sectional Fama-MacBeth regressions $E\left[R_{i,t+1} - R_{f,t+1}\right] = \boldsymbol{\beta}_i' \boldsymbol{\lambda}$. The test assets are the 25 size and BE/ME sorted portfolios of Fama and French. Returns are measured in excess of the risk-free rate. R_m is the return on the CRSP value weighted stock index, N_{CF} and N_{DR} are the two I-CAPM factors developed by Campbell and Vuolteenaho (2004), TY is the bond yield spread, PE is the price earnings ratio, and VS is the value spread. Standard errors are presented in parentheses. The top set are uncorrected Fama-MacBeth standard errors and below are standard errors modified with the Shanken (1992) correction. Alpha and alpha-Shanken are χ^2 tests of the hypothesis of zero-pricing errors. A star (*) denotes significant at a 5% level.

	CV-unrestricted		CV-restricted		VAR factors	
constant	-0.3556		-1.7015		0.9538	
	(1.4144)		(1.1359)		(1.3318)	
	(1.8788)		(1.0114)		(2.3704)	
R_m					0.9101	1.7891
					(1.4440)	(0.5904)*
					(2.4273)	(0.6312)*
N_{CF}	2.7391	2.5481	3.1254	1.5932		
	(0.8819)*	(0.5463)*	(0.8429)*	(0.6167)*		
	(1.1549)*	(0.6792)*	(1.0871)*	(0.3983)*		
N_{DR}	-0.2130	-0.3531	0.6828	0.6828		
	(1.1387)	(0.8083)				
	(1.4237)	(0.9264)				
TY					0.7376	0.7192
					(0.1786)*	(0.1919)*
					(0.3118)*	(0.3549)*
PE					0.2029	0.2529
					(0.1083)	(0.0745)*
					(0.1895)	(0.1350)
VS					-0.0055	-0.0210
					(0.0238)	(0.0304)
					(0.0400)	(0.0552)
R^2	0.5765	0.5735	0.5027	0.3314	0.8530	0.8284
R^2_{adj}	0.5380	0.5549	0.4811	0.3314	0.8236	0.8038
alpha	62.08*	69.82*	68.05*	70.45*	46.63*	56.72*
alpha-shanken	35.19	42.14*	36.68	57.68*	14.72	16.01

Table 6: Conditional models. Excess returns. With and without constant. The table presents $\boldsymbol{\lambda}$ estimates from the cross sectional Fama-MacBeth regressions $E\left[R_{i,t+1} - R_{f,t+1}\right] = \boldsymbol{\beta}_i' \boldsymbol{\lambda}$. The test assets are the 25 size and BE/ME sorted portfolios of Fama and French. Returns are measured in excess of the risk-free rate. $R_{m,t+1}$ is the return on the CRSP value weighted stock index and Δc_{t+1} denotes consumption growth. cay_t is the lagged consumption-wealth ratio proxy, used as scaling variable in the conditional models. Standard errors are presented in parentheses. The top set are uncorrected Fama-MacBeth standard errors and below are standard errors modified with the Shanken (1992) correction. Alpha and alpha-Shanken are χ^2 tests of
the hypothesis of zero-pricing errors. A star (*) denotes significant at a 5% level.

	Conditional C-CAPM		Conditional CAPM		Conditional EZW C-CAPM	
constant	2.8670		1.9016		2.2007	
	(0.9091)*		(0.8472)*		(0.8290)*	
	(1.218)*		(1.2083)		(1.0315)*	
$\widehat{cay}_t$	-0.1696		-0.9892		-0.2154	
	(0.3555)		(0.4852)*		(0.3366)	
	(0.4697)		(0.6861)		(0.4138)	
$R_{m,t+1}$			-0.0378	1.8666	-0.2101	1.8465
			(1.0139)	(0.6190)*	(0.9979)	(0.6136)*
			(1.3231)	(0.6505)*	(1.1667)	(0.6432)*
$\widehat{cay}_t * R_{m,t+1}$			0.0503	0.0770	0.01739	0.0472
			(0.0195)*	(0.0194)*	(0.0187)	(0.0236)*
			(0.0270)*	(0.0249)*	(0.0227)	(0.0310)
Δc_{t+1}	0.0802	-0.0029			0.0913	0.1502
	(0.1786)	(0.1965)			(0.1132)	(0.1627)
	(0.2375)	(0.3590)			(0.1388)	(0.2153)
$\widehat{cay}_t * \Delta c_{t+1}$	0.0043	0.0125			0.0029	0.0042
	(0.0025)	(0.0036)*			(0.0023)	(0.0023)
	(0.0034)	(0.0065)*			(0.0028)	(0.0031)
R^2	0.5943	0.2217	0.5014	0.3620	0.6093	0.5094
R^2_{adj}	0.5363	0.1879	0.4301	0.3342	0.5065	0.4393
alpha	69.40*	92.30*	69.05*	91.48*	66.73*	88.11*
alpha-shanken	38.68*	27.12	33.95	52.63*	43.10*	49.42*

Table 7: Pricing Errors

The table presents four measures of pricing errors for the models estimated: C-CAPM, EZW C-CAPM, CAPM, Fama-French 3 factor, conditional C-CAPM, conditional EZW C-CAPM, conditional CAPM, Campbell and Vuolteenaho I-CAPM, restricted and unrestricted, and the VAR factor model. The pricing error measures are composite pricing errors, measured with both a full and a diagonal variance-covariance matrix, the square root of average squared pricing errors, and the Hansen-Jagannathan distance measure. The first column for each model represents the case where a constant is included in the model and the second column represents the case where no constant is included. Asymptotic, Newey-West corrected, standard errors are in parentheses.

	C-CAPM		EZW C-CAPM		CAPM		Fama-French		VAR factors	
pricingerror-full	0.6210	0.8304	0.6643	0.7523	0.5925	0.6799	0.5533	0.6177	0.5180	0.5555
	(0.0372)	(0.0480)	(0.0433)	(0.0442)	(0.0331)	(0.0375)	(0.0348)	(0.0384)	(0.0414)	(0.0392)
pricingerror-diag	0.2355	0.3139	0.2203	0.2986	0.2843	0.3480	0.1451	0.1581	0.1115	0.1180
	(0.0297)	(0.0339)	(0.0265)	(0.0343)	(0.0387)	(0.0365)	(0.0113)	(0.0110)	(0.0089)	(0.0143)
Average pricing error	0.5598	0.7205	0.4538	0.7046	0.5788	0.8166	0.3190	0.3459	0.2305	0.2491
	(0.0698)	(0.0741)	(0.0485)	(0.0765)	(0.0743)	(0.0811)	(0.0272)	(0.0278)	(0.0192)	(0.0385)
H-J dist	0.5391	0.7594	0.6144	0.6686	0.5277	0.5695	0.4908	0.5382	0.4865	0.5214
	(0.0352)	(0.0491)	(0.0437)	(0.0440)	(0.0297)	(0.0314)	(0.0305)	(0.0337)	(0.0431)	(0.0386)
	Conditional C-CAPM		Conditional EZW C-CAPM		Conditional CAPM		CV-unrestricted		CV-restricted	
pricingerror-full	0.6337	0.7711	0.6450	0.7395	0.7502	0.7750	0.6109	0.5984	0.6593	0.5959
	(0.0386)	(0.0521)	(0.0416)	(0.0459)	(0.0608)	(0.0498)	(0.0461)	(0.0419)	(0.0487)	(0.0399)
pricingerror-diag	0.1792	0.2375	0.1717	0.1920	0.1904	0.2178	0.1789	0.1816	0.1814	0.2006
	(0.0179)	(0.0222)	(0.0147)	(0.0165)	(0.0194)	(0.0260)	(0.0173)	(0.0236)	(0.0167)	(0.0275)
Average pricing error	0.3830	0.5305	0.3758	0.4212	0.4246	0.4803	0.3913	0.3927	0.4240	0.4917
	(0.0311)	(0.0563)	(0.0307)	(0.0319)	(0.0393)	(0.0493)	(0.0370)	(0.0409)	(0.0445)	(0.0679)
H-J dist	0.5758	0.6997	0.5797	0.6625	0.6821	0.6923	0.5526	0.5400	0.5942	0.5145
	(0.0368)	(0.0510)	(0.0394)	(0.0430)	(0.0602)	(0.0471)	(0.0447)	(0.0394)	(0.0466)	(0.0348)

Figure 1: Pricing performance of traditional models

In the figures below, returns for the 25 Fama-French portfolios from the C-CAPM and CAPM are presented. Realized sample average excess returns are on the x-axis and predicted excess returns are along the y-axis. Black squares represent estimates for the models with the zero-beta rate restricted to equal the risk-free rate and white squares give estimates where the zero-beta rate is freely estimated.

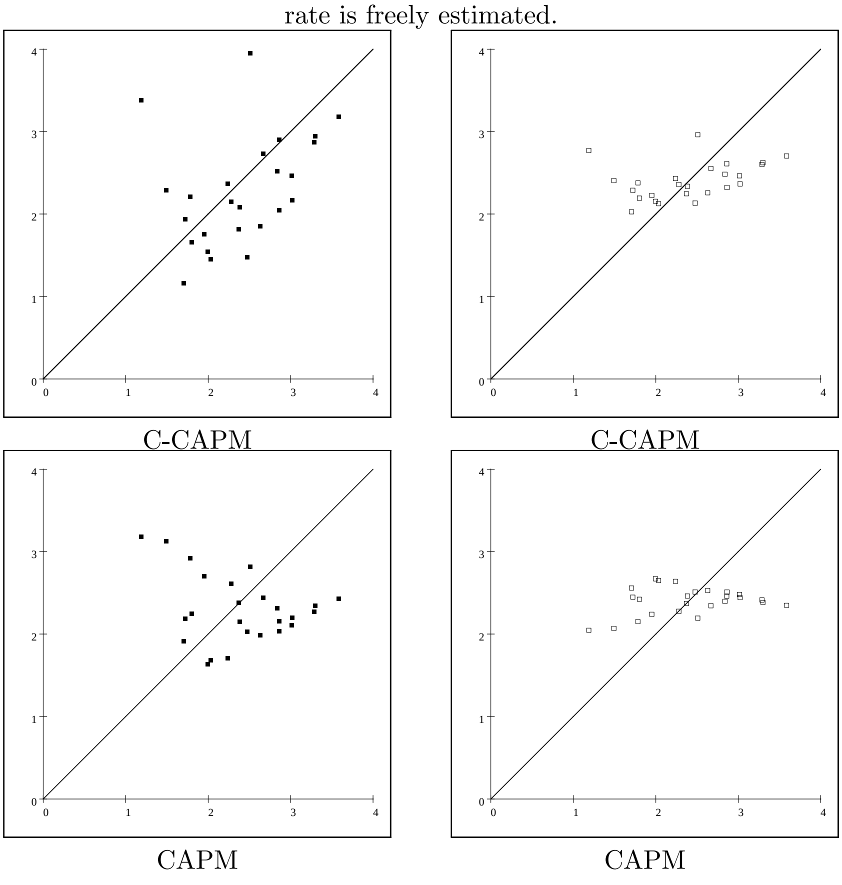
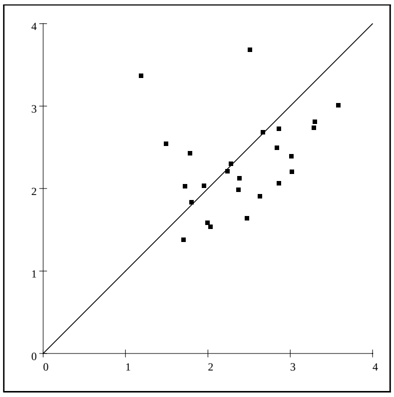
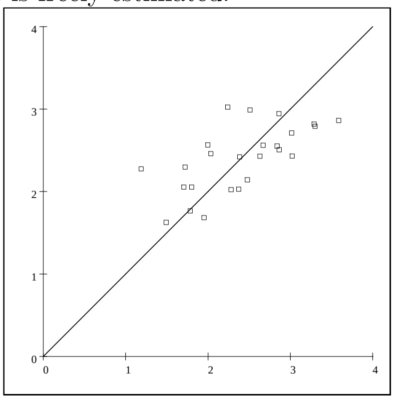


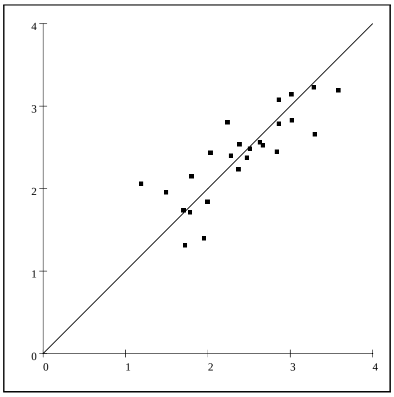
Figure 2: Pricing performance of traditional models
In the figures below, returns for the 25 Fama-French portfolios from the EZW C-CAPM and Fama-French model are presented. Realized sample average excess returns are on the x-axis and predicted excess returns are along the y-axis. Black squares represent estimates for the models with the zero-beta rate restricted to equal the risk-free rate and white squares give estimates where the zero-beta rate is freely estimated.



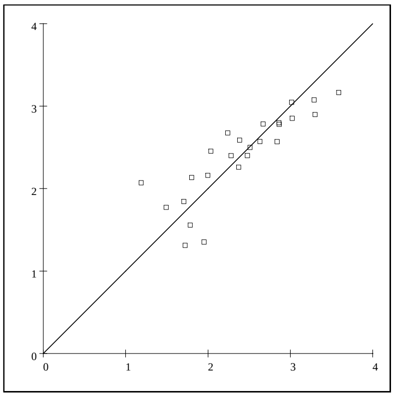
EZW C-CAPM



EZW C-CAPM

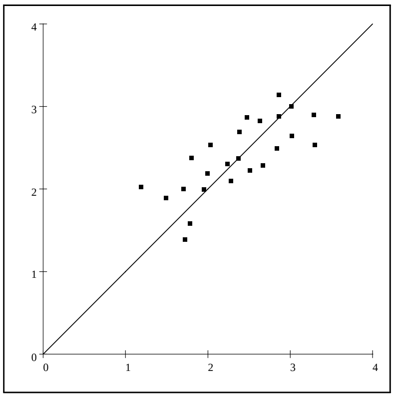


Fama-French

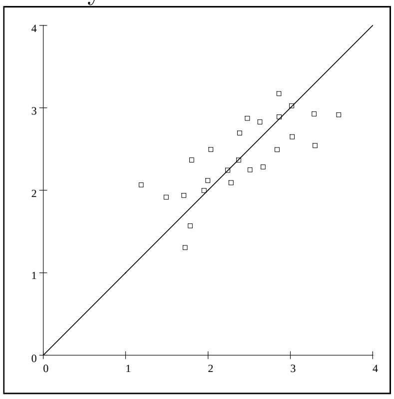


Fama-French

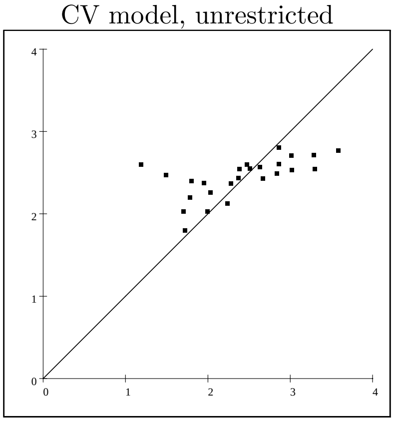
Figure 3: Pricing performance of I-CAPM
In the figures below, returns for the 25 Fama-French portfolios from the Campbell and Vuolteenaho models are presented. Realized sample average excess returns are on the x-axis and predicted excess returns are along the y-axis. Black squares represent estimates for the models with the zero-beta rate restricted to equal the risk-free rate and white squares give estimates where the zero-beta rate is freely estimated.



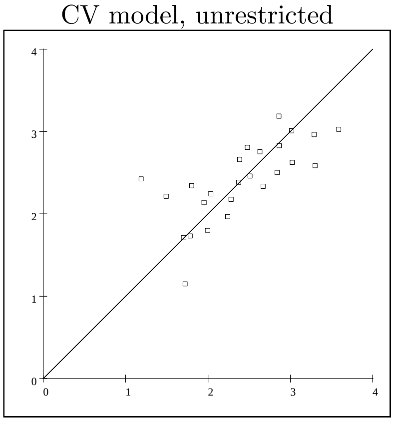
CV model, unrestricted



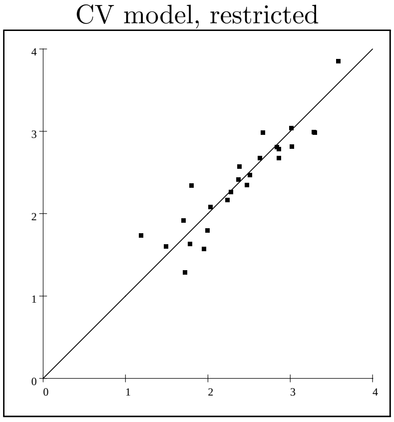
CV model, unrestricted



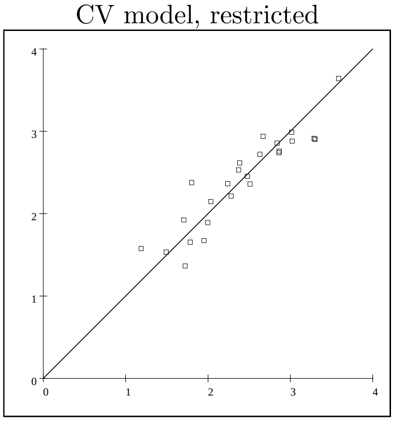
CV model, restricted



CV model, restricted

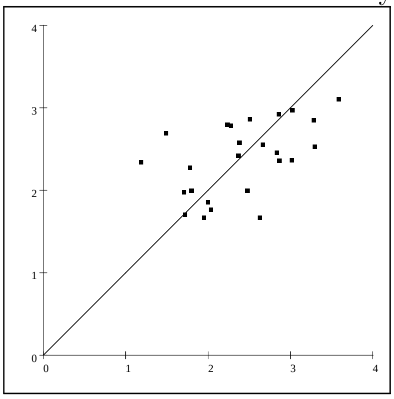


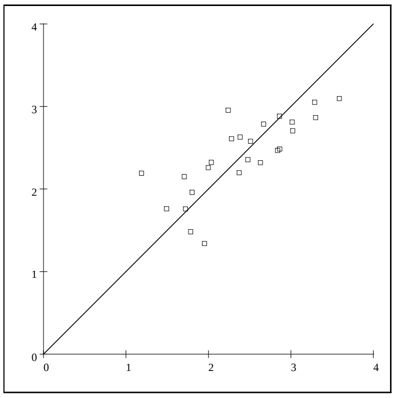
VAR Factor model



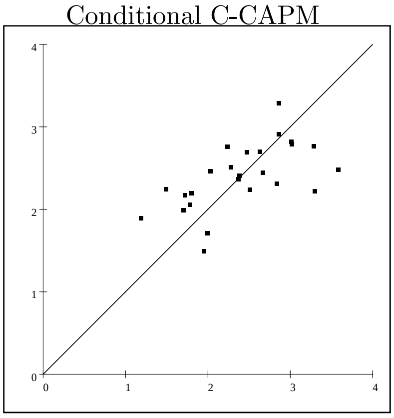
VAR Factor model

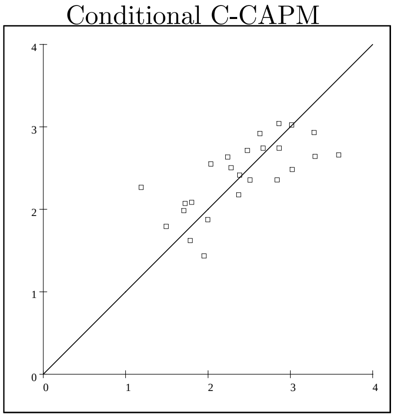
Figure 4: Pricing performance of conditional models
In the figures below, returns for the 25 Fama-French portfolios from the conditional models are presented. Realized sample average excess returns are on the x-axis and predicted excess returns are along the y-axis. Black squares represent estimates for the models with the zero-beta rate restricted to equal the risk-free rate and white squares give estimates where the zero-beta rate is freely estimated.



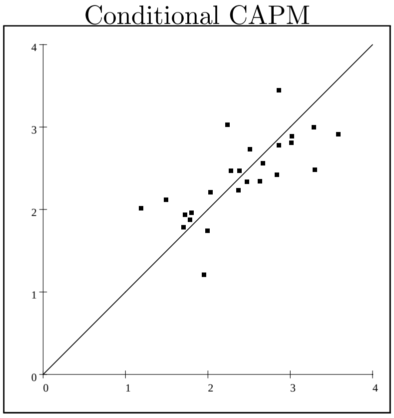


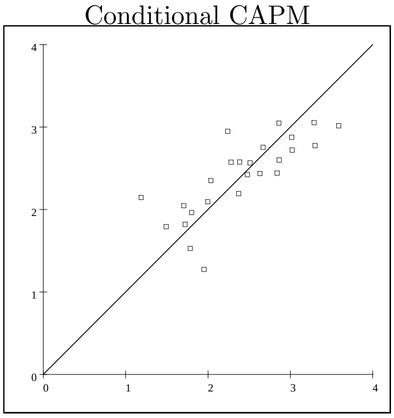
Conditional C-CAPM | Conditional C-CAPM





Conditional CAPM | Conditional CAPM





Conditional EZW C-CAPM | Conditional EZW C-CAPM