The Active Management of Distressed Debt

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Abstract

In case of default event, information is needed to determine whether the firm is economically viable or not. If the response is negative, liquidation must be undertaken whatever the deepness of the financial default. On the contrary, if the firm is thought to be economically viable, several approaches exist to organise a quick recovery among which the infusion of equity or the debt-equity swap. This paper studies these alternatives in light of a contingent claim approach. One also explores the need of a monitoring process beyond the first default event.

Keywords: Debt Restructuring, Default, Liquidation, Monitoring, Recovery, Uncertainty, Viability.
Code JEL: G3, G33, G34.

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Introduction

Many approaches exist to restructure the debt of firms facing financial distress. One finds the debt rescheduling, i.e. the extension of the debt's maturity accompanied or not by a reduction of interest payments, the forgiveness of due and promised payments and the distribution of equity securities to creditors known as the debt-equity swap. These different approaches have been considered separately in the academic literature (Roe (1983), Longstaff (1990) and James (1995) among many others[1]). All of them are specific scenarios of the debt restructuring. One refers to John (1993) for a general account on this area.

Empirically, Gilson and *al*. (1990) documents the reorganization process by focusing on the terms of the successful restructurings undertaken between 1978 and 1987. They find that 48.8 percent of the successful restructuring firms extend the maturity of their debt, 72.5 percent reduce the interest or the principal by forgiving the overdue or future promised payments and a similar percentage of 73.8 percent distribute some equity securities to creditors. Evidences show that the maturity extension and the distribution of equity concern half of the restructured bank debt whereas only a very small fraction, 6.7 percent, of the publicly traded debt is rescheduled. By contrast, the debt-

[1] For instances, the debt maturity extension has been studied by Longstaff (1990) who shows that it has value for both parties. In case of liquidation, the equity is worth nothing (unless the debtors bet on an improbable deviation from the absolute priority rule). For their part, creditors have some reasons for granting delays. Longstaff (1990) argue that "[because of] *the positive liquidation costs, the bondholders always prefer to extend the maturity of the defaulting bonds rather than instigate bankruptcy proceedings*". Chen, Weston and Altman (1995) recall that lengthening the maturity of all or a portion of the debt enhances the probability of repayment. One can add that the debtholders may be exposed to a so tightened network of firms that they fear an infectious propagation into the network and a contagion of defaults into their debt portfolios.

equity swap is rather frequent for this latter debt since "*holders of publicly traded debt are given equity securities 86.7 percent of time*".

In case of default on a payment of coupon, interest or principal, the management of the distressed claims by the debtholders can be either passive or active. Whatever, it is crucial for the creditor to gather information in order to take in the near future the right decisions. The way they will think of restructuring strongly depends on the nature of the distress. In particular, it is determinant to assess the economic viability of the firm and whether problems are more financial than operational in nature. Kahl (2002) has pointed out that financial distress is only an imperfect indicator of economic viability so it can take some time to figure out what is the economic future of the company.

If the economic viability is established, a recovery process is credible; this should be organized as soon as possible. This requires a plan of reorganization that is nothing else that an exchange of existing (and distressed) financial claims for a new basket of claims, including cash if some creditors are supposed to be partially or fully reimbursed. If not, a liquidation procedure must be undertaken. It is important to note that any infusion of liquidity by investors is a positive signal on the economic viability of the firm[2]. So this is an important incentive for reorganization. In some cases, it is not possible immediately to get a precise idea of the economic viability of the firm. Uncertainty prevails. Creditors are then forced to behave in an undecided way. They will most probably extend the maturity of their debt to wait and see.

This paper analyzes methods for debt restructuring in lights of the contingent claim analysis. One assumes that the creditors have a significant

bargaining power in the debt restructuring; they can optimise their positions. This assumption is however strongly weakened by the degree of information they hold. One follows Kahl (2002) who suggests that learning about the viability of the firm is a time-consuming process. A certain period of time is needed after the default time, denoted T, to make an intensive diagnosis and allow stakeholders to make up their minds about firm future economic viability.

At the end of the diagnosis period, say at date T_1, creditors may still face uncertainty about the firm economic viability. The optimal strategy for them is to extend the maturity of their debt to preserve their existing options. These options are first to be paid back in the future i.e. take benefit from the firm's recovery and second to liquidate the firm's assets if necessary. Extending the debt maturity also give creditors more time to learn about the real economic efficiency of the firm.

If, instead, the economic viability is concluded, a plan of reorganization is highly profitable to make the recovery of the firm not only plausible but rapid. On the one hand, existing or outside investors may refund the firm to invest in the industrial activities or to partially reimburse the debt or both. By doing so, they simultaneously offer a positive signal to the debtholders and significantly contribute to reinforce the decision to go on the firm. On the other hand, debtholders can recap the distressed firm by a debt-equity swap. The partial or total forgiveness is here compensated by the distribution of equity. Both reorganization processes imply the reduction of the total amount of debt in the

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[2] These investors are either the former equityholders, instutional investors or hedge funds investing in distressed situations (see Hotchkiss and Mooradian (1997) for further discussion). Such a distinction is not necessary in what follows.

capital structure. Both enable the firm to invest without debt overhang. In the latter case, however, the liquidation option owned by creditors is cancelled.

At time T_1, if it is thought that the firm is not viable, the firm should be liquidated. The liquidation value depends not only on the degree of obsolescence and the specificity of the assets of the firm but also on the conditions prevailing in the secondary market[3]. By their nature, the value of some assets may sharply decrease in a near future. In some cases, however, the immediate liquidation can induce very high liquidation costs. Some kind of controlled extension may therefore be granted just to improve the liquidation value of the firm assets. In what follows, one uses indifferently the realization rate and the liquidation cost. The realization rate is one minus the liquidation cost.

Whatever the non-liquidation process retained, this paper argues that a monitoring of the firm after the date T_1 period appears a critical feature for the restructuring. To a certain extent, the absence of a monitoring device has rendered the first default a surprise. Debtholders will certainly monitor the firm more closely to react more promptly to a second default.

The rest of the paper is organized as follows. Section 1 presents the framework and depicts a typical scenario at the time of default. The debt extension solution is analysed when uncertainty remains. Section 2 considers the cases of clearly viable firms for which reorganization is desirable to make possible the recovery. This section examines requirements from equity-holders to refund the firm and the opportunity for debt-holders to swap equity for debt. Section 3 then turns to the cases of clearly unviable firms for which

debtholders will refuse to keep on the business. If economic viability is not credible, the firm's assets have to be liquidated. Sometimes, the assets are specific and the associated realization rate may be time-varying. To handle with this, one suggests some specific strategy of immediate or postponed liquidation. Section 4 motivates the need of a monitoring process beyond the first default event in every situation. Section 5 offers some concluding remarks.

1. The framework

This section presents the framework we use. We consider a perfect and complete financial market in lines with Black, Scholes (1973) and Merton (1974). Trading takes place continuously and short positions are possible. There are no tax, nor transaction cost, nor agency cost, nor (for the moment) bankruptcy or liquidation cost. There exists a riskless asset paying a known interest rate denoted *r*.

Let it be a risky levered firm with a simple capital structure consisting of equity and a single issue of discount bonds with maturity T and face value F_1. The firm asset's value is denoted *V* and, under the risk neutral measure, its process is correctly described by the dynamics :

$$dV = rVdt + \sigma VdW \qquad (1)$$

where *W* is a Brownian motion and σ denotes the firm volatility. The payoff function at time F_1 for the equity holders is that of a call option ($\max(V_T - F_1, 0)$). In absence of bankruptcy costs, the one for the debtholder is $\min(F_1, V_T)$ where V_T also stands for the liquidation value of the assets. If there are bankruptcy costs, the liquidation value is then strictly lower than the assets'

[3] Maksimovic and Phillips (1998) note that the level of demand and the industry conditions, in

value of the defaulting firm. Hereafter, one denotes by β the percentage realization of the firm's assets in case of liquidation ($0 < \beta < 1$). βV is the value of the liquidated assets and $(1-\beta)V$ stands for the total liquidation costs.

A) Typical scenarios at the default time

If they face a default at time T, creditors do certainly not have any perfect information about the economic viability of the distressed firm. A certain period of time is needed to clarify the perspective of the firm. From T to T_1, an intensive diagnosis phase must take place and the analysis should ideally lead to an optimal scenario. If, despite these efforts, the creditors at T_1 are unable to figure out what is the economic viability of the firm, then:

1. Lenders can decide the extension of maturity of their debt. Because they intimately believe that the financial distress is not an accurate signal of economic efficiency, they need more time to discover the true economic characteristics of the firm. Operationally, they only exchange their defaulting debt for new claims that have extended maturity.

If the firm is judged as economically viable, the debt and capital structure are restructured to solve the financial distress:

2. Lenders can require equityholders to contribute to the restructuring and infuse some liquidity in the firm before accepting to reschedule the debt.
3. Lenders can require equityholders to contribute to the restructuring and reimburse part of their due payment before accepting to reschedule the rest.
4. Lenders can exchange part or the total of their defaulting debt (i.e. due or promised payments) for new equity. This “equity for debt” swap assumes

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particular the industry-wide excess capacity, affect the liquidation process.

however the willingness and the possibility for debtholders to become equityholders. Legal issues may limit this scenario. See James (1995) for an in-depth discussion.

If the firm is judged as economically unviable, the scenario is straightforward:

5. Lenders decide either to promptly precipitate the liquidation of the firm's assets if they fear some rapid devaluation (like obsolescence) or to postpone liquidation for a better sale of the firm's assets[4].

In all the cases, it will be necessary after default to check the performances of the firm so

6. Lenders follow one of the previous scenario by simultaneously imposing a strict monitoring device in the future.

All these scenarios generate quite different private negotiations. Anticipating *ex ante* the future situation, they may be arranged and combined. Some of them are rather similar from a financial viewpoint. They merit additional comments. The second and third scenarios are just the same as the first except that the debtholders need some signal from the equityholders. Scenario 2 and 3 are equivalent from the equityholders viewpoint since their wealth is overall disminished by a given amount. Because the contribution of the equityholders may be invested in some operational issue, the debt reimbursement is not an *all or nothing* ultimatum. The following section questions how much is worth this contribution. Scenario 4 suggests that debtholders may accept to exchange their claims and become equityholders. If the swap is fair, then they will get a value

[4] The improvement of the liquidation value may have some global connotation in the sense that the overall value of the liquidated firm is not from a broad viewpoint just the sum of its liquidated tangible assets. The total value of the tangible assets you can sold separately on the

that should not be different from the one they would receive from a partial reimbursement. This is a kind of reimbursement in nature. If this is not the case, either the debtholders or the equityholders have suffered from the negotiation. The fifth scenario concerns liquidation which is viewed here as a dynamic process. The sixth scenario recalls that a monitoring procedure will be set up by lenders to supervise the recovery progress or the liquidation after the first default.

B) The net gain of the restructuring and the wealth of equityholders

Our setting assumes that the creditors have a significant bargaining power. They can choose a restructuring method in lights of the returns they anticipate. The NPV evaluation provides a standard approach for them to decide. Let's introduce the net gain functions of the restructuring process by noting:

$$H = NPV\left(restructuring\right)$$

The net present value is equal to the present value of the claim implicitly received in the plan of reorganization minus the amount abandoned by restructuring. This amount is the liquidation value of the firm's assets. For short, one has:

$$H = PV\left(restructuring\right) - \beta V_{T_1}.$$

The non liquidation of the firm then affects differently the wealth of equityholders according to the chosen restructuring approach. In all cases, they receive a new financial claim in exchange of their distressed one. Because the value of this distressed claim is zero in case of default, a direct consequence of the restructuring decision is that it can offer a positive wealth to equityholders.

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second-hand market is often lower than the global value of the firm including both the human capital and the intangible assets (even they are insufficient to cover the due payment).

If, however, new common shares are distributed to creditors, the issuance dilutes prebankruptcy interests. Outside investors are sometimes solicited to invest in the distressed firm. They then take a management and a control position in the considered company so as to influence its investment and operating policies.

C) Extending the debt maturity

After the default event, creditors may face uncertainty about the firm economic viability at T_1. So they will wait and see i.e. just extend the maturity of their debt until a future date $T_2 = T_1 + \tau$. The extension of maturity improves their possibility to obtain supplementary knowledge concerning the future of the firm. Moreover, by keeping their claims unchanged, debtholders preserve their entire interest and intervention opportunities. It enables them to liquidate the firm assets later (if the firm does not improve its performance and re-enters financial distress) or to benefit from an eventual recovery.

Whatever, the creditors exchange at T_1 the known payoff βV_{T_1} for a contingent claim that pays $\beta V_{T_1+\tau}$ at $T_1 + \tau$ if $V_{T_1+\tau}$ is lower than F_1 and F_1 otherwise. Debtholders have an incentive do so if the associated net gain function is positive. This net gain function denoted $H\left(V_{T_1}, F_1, T_2\right)$ is given by discounting the payoff under the risk neutral measure Q:

$$H\left(V_{T_1}, F_1, T_2\right) = e^{-r(T_2-T_1)} E_{T_1}^{Q}\left[\beta V_{T_2} 1_{V_{T_2}<F_1} + F_1 1_{V_{T_2}\geq F_1}\right] - \beta V_{T_1}.$$

Under the Black, Scholes and Merton's setting, we obtain:

$$H\left(V_{T_1}, F_1, T_2\right) = -\beta V_{T_1} + \beta V_{T_1} N\left[-d_{1,V_{T_1}/F_1}\left(T_2 - T_1\right)\right] + F_1 e^{-r(T_2-T_1)} N\left[d_{2,V_{T_1}/F_1}\left(T_2 - T_1\right)\right] \quad (2)$$

where $d_{1,x}(t) = \dfrac{\ln x + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$ and $d_{2,x}(t) = d_{1,x}(t) - \sigma\sqrt{t}$. This function $H\left(V_{T_1}, F_1, T_2\right)$ is concave with respect to the maturity T_2 so there exists an

optimal extension period. The optimal length of time is obtained at date T_1 by computing:

$$\tau(V_{T_1}) = \arg\max_{t\in[0,\infty[} H(V_{T_1}, F_1, t) \tag{3}$$

and the optimal extended maturity by $T(V_{T_1}) = \arg\max_{t\in[T_1,\infty[} H(V_{T_1}, F_1, t-T_1)$. Both depend on the severity of default $\frac{V_{T_1}}{F_1}$, the realization rate and the firm value. Simulations of Longstaff (1990) show that the optimal length $\tau(V_{T_1})$ is strictly increasing as the firm value at T_1 gets lower. It is therefore an one-to-one function of V_{T_1}.

One has already claimed that the non immediate liquidation of the firm affects the wealth of equityholders. This is also verified in the specific context of the maturity extension. By rescheduling their claim, debtholders implicitly deliver a new claim to equityholders. This new claim is nothing else than a standard call option written on the firm assets and whose expiration is the optimal extension date $T(V_{T_1})$. They receive the price of this new claim which is $Eq_{T_1}^{BSM}(V_{T_1}, F_1, T(V_{T_1})-T_1)$[5].

2. Debt and Capital restructuring

When the firm is considered as economically viable at time T_1, a prompt recovery of the firm may require significant restructuring of the firm. The recovery process may indeed be rather limited by the debt overhang. The debt

[5] In the framework of Longstaff (1990), debtholders reschedule their debt as soon as the associated net gain function is positive. No matter is the length of the extension period. No need to appreciate further the future economic viability of the firm, no need either to reorganize the firm. His setting assumes complete information and, to a certain extent, an automatic restructuring process. By contrast, our context considers that information on the firm viability is sparse or uncertain and that this is an incentive to reschedule and to "wait and see"…

overhang leads to a myopic investment policy and can burden the economic viability of the firm. So, it can be non optimal for creditors to simply extend their debt because it prevents the firm to recover as quickly as possible, forbid it to invest and benefit from its investment growth opportunities. Many restructuring schemes can take place; most of them aim at lowering the debt ratio.

A) A contribution from equityholders[6]

Considering the economic viability of the firm as given, a contribution from equityholders may significantly influence the financial or the operating restructuring of the firm. The contribution signals their beliefs in the economic viability of the firm too. If the automatic rescheduling of the debt is viewed as a necessary medecine it can be insufficient to facilitate a quick recovery of the distressed firm. Because the bargaining power of creditors is supposed to be strong, they can require equityholders to concede some substantial contribution to restructure either the financing structure or the business portfolio. The threat for these latter is that debtholders swap equity for their debt and take over the control of the firm. Because continuation is worth for existing equityholders, they will not rationally refuse this proposal but only to a certain extent. Overall one assumes that: there are situations where the (inside or outside) shareholders, eager to run the firm, are constrained to inject an amount of money A after the default time for continuation. More practically, they invest either in the business of the firm or to redeem part of the debt.

Re-investing in the firm or paying back to debtholders for partially reimbursement has several consequences. First, either the firm asset value at T_1 is increased or the debt is lowered. In the former case, the firm asset value

[6] There exist many kinds of subjective motivations for the firm owners to avoid the bankruptcy at any price. E.g., some of them are perhaps the original entrepreneurs. However we follow in this section a pure objective analysis.

grows to be $\tilde{V}_{T_1} = V_{T_1} + A$ and this variable can be written $\tilde{V}_t = \frac{V_{T_1} + A}{V_{T_1}} V_t$ for any future date $t > T_1$. In the latter case, the remaining due face value becomes: $F_1 - A \equiv F_2$. In both cases, the leverage ratio beyond the default time is lowered and the probabilities of recovery and complete repayment of the debt in the future is increased. Second, the contribution lowers the wealth of equityholders. Third, it has a direct impact on the net gain function of debtholders.

If the amount A is injected in the firm, the net gain function is defined by the equation :

$$H\left(V_{T_1}, F_1, T_2; A\right) = e^{-r(T_2 - T_1)} E_{T_1}^{Q}\left[\beta \tilde{V}_{T_2} 1_{\tilde{V}_{T_2} < F_1} + F_1 1_{\tilde{V}_{T_2} \geq F_1}\right] - \beta V_{T_1}$$

where $V_{T_1} \in \left[K, K^{'}\right]$. If instead the contribution A serves to reimburse the debtholders partially, the net gain function is described by:

$$H\left(V_{T_1}, F_1, T_2; A\right) = e^{-r(T_2 - T_1)} E_{T_1}^{Q}\left[\beta V_{T_2} 1_{V_{T_2} < F_1 - A} + \left(F_1 - A\right) 1_{V_{T_2} \geq F_1 - A}\right] - \beta V_{T_1} + A$$

for $V_{T_1} \in \left[K, K^{'}\right]$. Under the Black, Scholes and Merton setting, we then obtain respectively:

$$\begin{aligned} H\left(V_{T_1}, F_1, T_2; A\right) = & -\beta V_{T_1} + \beta V_{T_1} N\left[-d_{1,(V_{T_1}+A)/F_1}\left(T_2 - T_1\right)\right] \\ & + F_1 e^{-r(T_2 - T_1)} N\left[d_{2,(V_{T_1}+A)/F_1}\left(T_2 - T_1\right)\right] \end{aligned} \tag{4}$$

and

$$\begin{aligned} H\left(V_{T_1}, F_1, T_2; A\right) = & A - \beta V_{T_1} + \beta V_{T_1} N\left[-d_{1,V_{T_1}/(F_1 - A)}\left(T_2 - T_1\right)\right] \\ & + \left(F_1 - A\right) e^{-r(T_2 - T_1)} N\left[d_{2,V_{T_1}/(F_1 - A)}\left(T_2 - T_1\right)\right] \end{aligned} \tag{5}$$

Graphs in Figure 1 show the effects of the equityholders contribution on the net gain function. The left graph plots H for a contribution A invested in the firm whereas the right one considers that A is devoted to a partial reimbursement of the debt. A ranges from 1 to 10. In both graphs, the lowest line is associated

with the lowest contribution. Other parameters value are $V_{T_1} = 40$, $\sigma = 20\%$, $F_1 = 50$, $r = 10\%$.

Insert Figure 1. about here.

Both graphs show that the contribution increases the net gain function. The participation of stockholders is an important factor. A contribution dedicated to the partial reimbursement of the face value slightly increases the total wealth of the debtholders (compared to the right situation). This alternative does not however change dramatically the shape of the function H.

In view of the Figure 1, debtholders may prefer a partial reimbursement to liquidity infusion and will seek to maximize the amount A they receive. This is not however so simple. The amount A is indeed not totally exogenous because equityholders will refuse a too large contribution. From their viewpoint, they will decline any plan of reorganization where they are supposed to offer more than the value of the claim they implicitly receive.

Let's denote by $\mathbf{C}$ the value of the claim stockholders receive in default. In the context of a maturity extension, the new claim is a standard call option ($\mathbf{C} = Eq^{BSM}$). The net present value of the restructuring for the stockholders is given by:

$$NPV_{eq}(restructuring) = \mathbf{C} - A$$

Stockholders don't accept to contribute more than the value they get means that they consent to the plan of reorganization only if their own net present value is positive. It is important to see that the amount A intricately depends on the way stockholders will contribute. If the amount is injected in the firm for a given delay $T(V_{T_1})$, the maximum value denoted $\tilde{A}$ verifies :

$$\mathbf{C}\left(V_{T_1} + \tilde{A}, F_1, T\left(V_{T_1}\right) - T_1\right) - \tilde{A} = 0\,. \tag{6}$$

If the contribution serves to reimburse partially the debtholders, the maximum value denoted $\tilde{\tilde{A}}$ is :

$$\mathbf{C}\left(V_{T_1}, F_1 - \tilde{\tilde{A}}, T\left(V_{T_1}\right) - T_1\right) - \tilde{\tilde{A}} = 0\,. \tag{7}$$

Simulations could show that $\tilde{A} > \tilde{\tilde{A}}$. Stockholders will contribute at a higher level if they are allowed to invest in the firm and not redeem part of the debt. This result should incite debtholders to let the stockholders invest in the firm. This contrasts with the conclusion of the Figure 1. Overall, the arbitrage for debtholders is not straightforward. As a final remark, if the maximum amount A is offered by the equityholders, they are not better off by the restructuring. So there exist scenarios under which the restructuring has no consequence on the current wealth of equityholders. Under a specific scenario, the generated wealth is entirely captured by the debtholders since they are partially repaid.

B) The equity swap procedure

If the debtholders are the only stakeholders aware of the firm viability, they can plan a reorganization to exchange their claim for equity i.e. to run a debt-equity swap. Instead of keeping their claims, the creditors convert their debt into equity and suppress debt overhang. They can also permit some new long term investment projects to take place (Kahl (2002)) and attract some new funding. Overall, the firm will avoid myopic investment behaviour. This strategy allows a quick end full recovery of the viable firm and enables it to realise its growth opportunities.

In a debt-equity swap, the debtholders will change themselves in equityholders, at least for part of their stakes, and we could say that debtholders behave like convertible security holders who exercise their right at a zero call price for

equity. Thus, some active management could characterise the behaviour of the lenders as soon as they get some bargaining power and some degrees of freedom.

Since one has assumed that debtholders have a significant bargaining power, they can receive from 1% to 100% of the new equity in exchange of the amount of debt they forgive. Under an extreme scenario, the debtholders can acquire the whole equity and preserve parts of their debt. By doing so, they capture the whole net present value of the restructuring.

3. The dynamics of the liquidation process

In severe economic distresses, the value of the firm at time T_1 can be very low or not. In every case, if the economic perspectives are desperate, debtholders will refuse the continuation. An efficient resolution of corporate distresses should lead to liquidate firms that should be liquidated. It has already been claimed that a certain period of time should be devoted to intensive diagnosis to clarify the true nature of the firm. If creditors have sufficient information about the non economic viability of the distressed firm, then liquidation must take place. This liquidation can be immediate or postponed depending on the conditions prevailing on the secondary markets and the specificity of the assets. A high level of specialization of the firm's assets means that the liquidation costs are not negligible (see Franks and Torous (1989), Weiss (1990)) and that the knowledge of the second-hand market becomes a crucial issue, in particular if they are not very liquid.

A) An immediate liquidation

If the firm's assets are very liquid and the market conditions propitious to this, liquidation should be undertaken immediately. Using the realization rate β as a proxy for these conditions, this means that β at time T_1 must be close to

unity. One assumes that *there exists a minimum realization rate* $\beta_{\min}$ *over which the liquidation is undertaken immediately*.

B) A postponed liquidation

Postponing liquidation may increase the realization rate of the liquidated assets because the firm may look for the better way to sell its assets so as to get the lowest realization costs possible. In some cases, the whole firm may also be sold in one part. Ideally, as time passes, the realization rate increases to a maximum (β^*) or, equivalently, the replacement costs decrease to a minimum ($1-\beta^*$).One assumes *that the realization rate* $\beta(t)$ *is time-dependent. increasing and that it converges to* β^**, as time goes through infinity*. Since $\beta(T_1)$ is small and lower than $\beta_{\min}$ - the minimum realization rate, the increasing property of β incites debtholders to let the firm survive in order to appreciate the liquidated value.

To see this, let's assume that β is well described by:

$$\begin{cases} \beta(T_1) = \beta_1 \\ d\beta(t) = a\left(\beta^* - \beta(t)\right)dt \end{cases}$$

whose solution is : $\beta(t) = \beta^*\left(1-e^{-a(t-T_1)}\right) + \beta_1 e^{-a(t-T_1)}, t > T_1$. Generalization to the stochastic case is a straightforward exercise. One has: $\beta(T_1) < \beta_{\min}$. The parameter a models the rising knowledge of the second-hand market: the larger a is, the faster the realization rate grows to β^*. Under the risk neutral measure Q, the associated net gain function is given by:

$$H\left(V_{T_1}, F_1, T_2; \beta(t)\right) = e^{-r(T_2-T_1)} E_{T_1}^{Q}\left[\beta(T_2) V_{T_2} 1_{V_{T_2} < F_1} + F_1 1_{V_{T_2} \geq F_1}\right] - \beta(T_1) V_{T_1}.$$

So we obtain under the Black, Scholes and Merton setting:

$$\begin{aligned} H\left(V_{T_1}, F_1, T_2; \beta(t)\right) = & -\beta(T_1) V_{T_1} + \beta(T_2) V_{T_1} N\left[-d_{1, V_{T_1}/F_1}(T_2 - T_1)\right] \\ & + F_1 e^{-r(T_2-T_1)} N\left[d_{2, V_{T_1}/F_1}(T_2 - T_1)\right] \end{aligned} \qquad (8)$$

The former constant realization rate β has just been replaced by its time-varying but deterministic values: $\beta(T_1)$ and $\beta(T_2)$.

Figure 2 shows that the increasing property of the realization rate has a major impact on the wealth expected by debtholders. One plots $H \equiv H(\beta(T_1), a)$ as a function of T_2 for different value of $\beta(T_1)$ and a. The left graph keeps $\beta(T_1)$ equal to 5% and varies a from 10% to 100% by 10% (with 1% and 95%). The lowest line stands for $H(5\%,1\%)$. The right graph keeps the a equal to 50% and ranges $\beta(T_1)$ from 0% to 90% by 10% (with 2.5% and 5%). The lowest line stands for $H(90\%,50\%)$. Other parameters are identical for the two graphs, these are: $V = 40$, $\sigma = 20\%$, $F_1 = 50$, $r = 10\%$ and $\beta^* = 90\%$.

Insert Figure 2. about here.

Figure 2 indicates that the net gain function significantly increases, as a grows. It almost doubles for the chosen range of a. This increasing feature is confirmed by the positivity of the first derivative of H with respect to a. The second derivative of H with respect to a is negative meaning that it is vain to develop a closed to perfect knowledge of the second-hand market. The right graph sheds light on the specific impacts of $\beta(T_1)$. The highest is $\beta(T_1)$, the smallest is the net gain function. Other way writing, the lowest is the liquidation cost, the fewest is the incentive to extend.

4. A monitoring device after the restructuring

Once there is a first default, it is more than probable that, during the restructuring procedure, debtholders want to monitor the firm more closely with the objectives to secure their capital, and, if necessary, to react promptly to a second default. In fact, monitoring the firm is justified for slightly different

motivations. When debtholders just extends the debt maturity, they monitor the firm because of the uncertainty they face about the economic viability. In other case, they monitor the firm to go with liquidation or the complex restructuring they undertake (recap with equityholders contribution, debt-equity swap).

The monitoring can be considered in our framework by introducing a monitoring threshold V_B and by assuming that, when the firm asset value reaches this barrier, debtholders can force stockholders to bankrupt the firm. So, once declared, the second default leads to an immediate liquidation of the firm assets[7]. To sum up, one assumes that *debtholders supervise the firm with the help of a monitoring barrier denoted* V_B. *If the firm value reaches the barrier* V_B, *this causes an "early" bankruptcy and the immediate liquidation of the firm assets. The assets are then sold and the debtholders are repaid with, eventually, some specific realization costs with rate* $(1-\beta_{V_B})$[8]. Because of its monitoring meaning, the monitoring barrier is assumed deterministic. For instance, its level is a constant or a function of the renegotiated face value of the debt. The level of the monitoring threshold depends on the considered approach.

The net gain function is once again the discounting value of the different payoffs received by the debtholders minus the value given up at time T_1. If the monitoring barrier is not reached during the granted period (if $\forall \tau \in \left[T_1, T_2\right]$, $V_\tau > V_B$), the debtholders will, at T_2, be fully repaid (if $V_{T_2} > F_1$) or they will

[7] This barrier may take the form of a covenant on the future cash-flows of the firm in lines of Anderson and Sundaresan (1996). It serves the role of an "early default" threshold in the spirit of the one considered in Black-Cox (1976) and Longstaff and Schwartz (1995). An important difference however is that it is used beyond a first default event.

receive the firm assets (if $V_{T_2} \le F_1$). If the barrier is attained during the period ($\exists \tau \in \left[T_1, T_2\right], V_\tau \le V_B$), then there is a second default and a precipitated bankruptcy. The assets are immediately liquidated at the amount $\beta_{V_B} V_B$. This amount can be received either immediately or latter (and we assume that this future date is the extended maturity T_2) causing an alternative. More formally, one has:

$$H(V_B) = e^{-r(T_2-T_1)} E_{T_1}^{Q}\left[\beta(T_2) V_{T_2} 1_{\{V_B < V_{T_2} < F_1; \forall \tau \in [T_1,T_2], V_\tau > V_B\}} + F_1 1_{\{F_1 < V_{T_2}; \forall \tau \in [T_1,T_2], V_\tau > V_B\}}\right] + \beta_{V_B} V_B R - \beta(T_1) V_{T_1}$$

where $\beta_{V_B} V_B R$ stands for the value received in case of a second default. Denoting Q the risk neutral probability and $\tilde{Q}$ the equivalent probability which used *V* as numéraire, one can write:

$$\begin{aligned} H(V_B) = & -\beta(T_1) V_{T_1} \\ & + \beta(T_2) V_{T_1} \tilde{Q}\left[V_B < V_{T_2} < F_1; \forall \tau \in [T_1, T_2], V_\tau > V_B\right] \\ & + F_1 e^{-r(T_2-T_1)} Q\left[F_1 < V_{T_2}; \forall \tau \in [T_1, T_2], V_\tau > V_B\right] \\ & + \beta_{V_B} V_B R \end{aligned}$$

If the amount $\beta_{V_B} V_B$ is paid at T_2, then $R = e^{-r(T_2-T_1)} Q[\exists \tau \in [T_1, T_2], V_\tau \le V_B]$. If it is paid immediately, then $R = \int_{T_1}^{T_2} e^{-rt} h(t) dt$, where h is the standard first hitting time density[9]. Within a Black-Scholes-Merton, this expression can be computed analytically. One finds:

---

[8] β_{V_B} is a constant realization rate in case of subsequent default. Nothing prevents one to assume a time-varying realization rate and eventually a further postponed liquidation.

[9] One may recognize here for the first expression one minus the rebate of down-and-in barrier options and for the second one the rebate of down and out barrier options.

$$
\begin{aligned}
H(V_B) = & -\beta(T_1)V_{T_1} + \beta_{V_B} V_B R \\
& + \beta(T_2)V_{T_1}\left\{\left(N\left[d_{1,V_{T_1}/F_2}\right] + \left(\frac{V_B}{V_{T_1}}\right)^{2\lambda} N\left[d_{1,V_B^2/(F_2 V_{T_1})}\right]\right)\right. \\
& \left. -\left(N\left[-d_{1,V_{T_1}/V_B}\right] + \left(\frac{V_B}{V_{T_1}}\right)^{2\lambda} N\left[d_{1,V_B/V_{T_1}}\right]\right)\right\} \\
& + F_1 e^{-r(T_2-T_1)}\left(N\left[d_{2,V_{T_1}/F_2}\right] - \left(\frac{V_B}{V_{T_1}}\right)^{2\lambda-2} N\left[d_{2,V_B^2/(F_2 V_{T_1})}\right]\right)
\end{aligned}
\tag{9}
$$

where, depending on the way the debtholders recover the value of the liquidated assets either at maturity or straight, one has respectively:

$$
R = \begin{cases} N\left[-d_{2,V_{T_1}/V_B}\right] + \left(\dfrac{V_B}{V_{T_1}}\right)^{2\lambda-2} N\left[d_{2,V_B/V_{T_1}}\right] \\ \left(\dfrac{V_B}{V_{T_1}}\right)^{2\lambda-1} N\left[d_{1,V_B/V_{T_1}}\right] + \dfrac{V_{T_1}}{V_B} N\left[-d_{1,V_{T_1}/V_B}\right] \end{cases}
\tag{9'}
$$

Here, $\lambda\sigma^2 = r + \frac{1}{2}\sigma^2$, $d_{1,x} = \dfrac{\ln x + \lambda\sigma^2(T_2 - T_1)}{\sigma\sqrt{T_2 - T_1}}$ and $d_{2,x} = d_{1,x} - \sigma\sqrt{T_2 - T_1}$.

The figure 3 illustrates the effect of introducing a monitoring barrier. One plots debtholders' net gain function H as a function of T_2 for different level V_B. The monitoring barrier V_B ranges from 80% of the firm assets value $V(T_1)$ to 99% with 85%, 90% and 95%. The continuous line represents a monitoring barrier equal to $80\% \times V(T_1)$. The left graph sets the realization rate to a constant value (50%). On the right one, everything is the same except that the realization rate is assumed to be time-varying. The realization rate converges to $\beta^* = 90\%$ at a speed $a = 0.75$. Other parameters are $V_{T_1} = 40$, $\sigma = 20\%$, $F_1 = 50$, $r = 10\%$ and $\beta_{V_B} = \beta(T_1) = 50\%$.

Insert Figure 3. about here.

Both graphs of figure 3 display the same results except that the function H is greater for the time-varying realization rate. Both graphs show that the net gain remains a concave function. As a result, there still exists an optimal extension period. Moreover, both graphs give evidence that the net gain strictly decreases as the monitoring barrier gets higher. When the monitoring barrier is high, the probability of a precipitated liquidation in the near future is high; for very high level of V_B, the liquidation is just postponed. Since, in both cases, $\beta_{V_B} = \beta(T_1)$, the obtained liquidated value is not much different. By contrasts, when the monitoring barrier is low, the extension may "save" the firm and the debtholders may be fully repaid. Because of the convexity of H, there exists an optimal extension period and the optimization procedure suggested by Longstaff (1990) can be easily adapted. For illustration, the figure 4 plots the optimal extension periods τ in presence of a monitoring device. The monitoring barrier is expressed with respect to the firm's assets value at date T_1. The constant realization rate $\beta(T_1) = \beta$ ranges from 50% to 90% (10% by 10%), the continuous line representing $\beta(T_1) = 50\%$.

Insert Figure 4. about here.

The graph shows that the optimal extension period tends to diminish as the monitoring barrier increases but also strictly decreases as the realization rate gets higher for a given monitoring barrier. Large values for the monitoring barrier greatly affect the duration of the extra period. This suggests that the monitoring device cannot be considered independently from the expected realization rate, or more precisely, that the way debtholders choose the monitoring barrier should be intimately related to what they anticipate about the realization rate.

The monitoring device impacts on the debtholders net gain function and the decision to extend. It has also some consequences on the way the liabilities are priced. For instance, the function **C** which describes the equity price in equations (6) and (7) has to be reconsidered. In presence of a monitoring device, this is nothing else than the price of a down-and-out call option. Nevertheless, this issue depends on the definitive scenario adopted at T_1.

5. Concluding remarks

This paper considers the active management of distressed debt by studying different approaches for debt restructuring and liquidation in lights of the contingent claim analysis. Our analysis assumes a significant bargaining power of creditors significantly lowered by the information held at the default time. We emphasize that the degree of information about the economic viability is an important determinant for deciding to liquidate or go on business an for choosing the restructuring methods. In case of uncertainty, the automatic rescheduling lauded by Longstaff (1990) is attractive. Otherwise, different approaches are possible and the maturity extension is only a partial solution. In fact, something must happen after the default either recovery or liquidation. If the firm is economically viable, debt restructuring is profitable If not, liquidation must take place either immediately or postponed to maximize the value of the liquidated assets. We introduce a contribution from equityholders and study classical approaches such as the debt-equity swap. We finally justify the need for a monitoring device beyond a first and surprising default. We show how these elements modify the wealth and the behaviour of stakeholders and point out a scenario under which the decision to restructure has no consequence on the wealth of equityholders. Under this specific scenario, the wealth generated is entirely captured by debtholders. The present article can be viewed as a first step in the direction of a strategic behaviour of the creditors within a contingent claim analysis. Future research should deal with the presence of several creditors and that of different types of debts.

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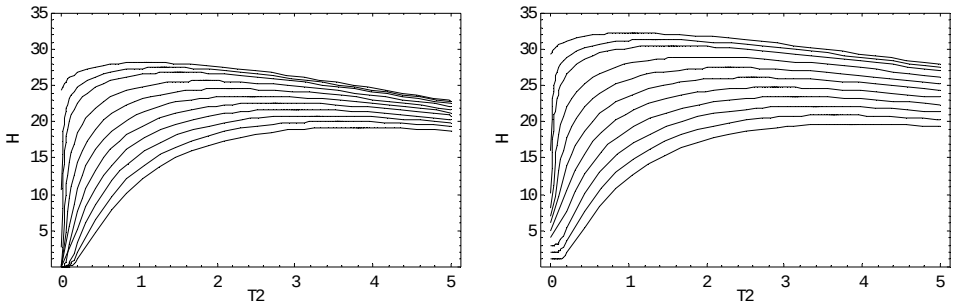
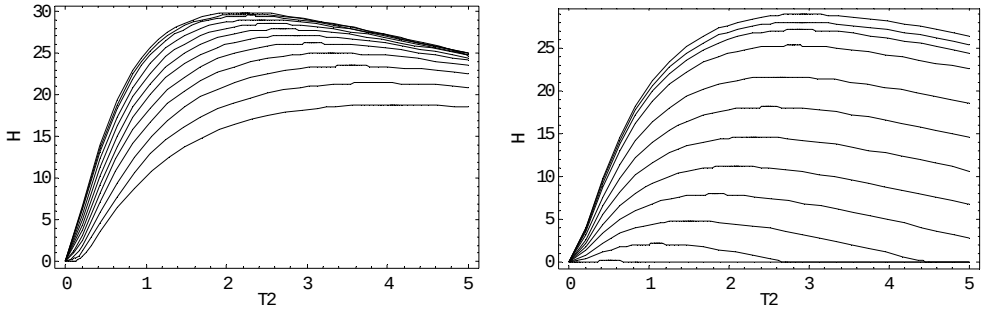
Figure 1: The effect of the equityholders contribution on the debtholders net gain function



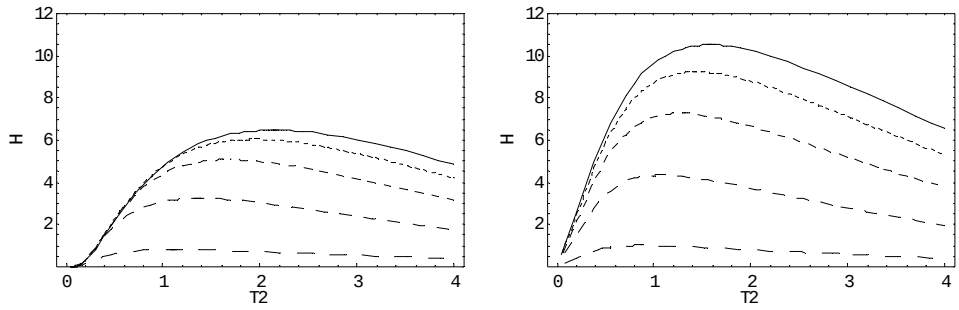
The left graph plots H for a contribution A invested in the firm. The right graph plots H for contribution A devoted to a partial reimbursement of the debt. The value of the contribution A ranges from 1 to 10 with in addition 9.5 from the bottom to the top. Other parameters are $V_{T_1} = 40$, $\sigma = 20\%$, $F_1 = 50$, $r = 10\%$.

Figure 2: The net gain function and the realization rate parameters: $H(\beta(T_1), a)$



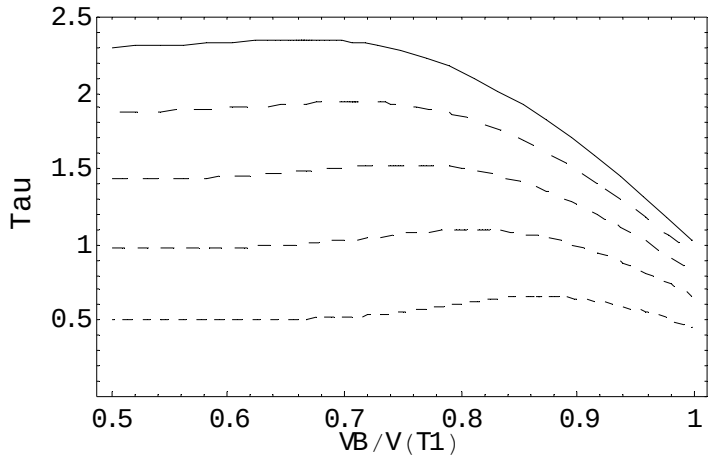
On the left graph, $\beta(T_1) = 5\%$, a ranges from 10% to 100% by 10% with in addition 1% and 95% and the lowest line stands for $H(5\%,1\%)$. On the right graph : $a = 50\%$, $\beta(T_1)$ ranges from 0% to 90% by 10% with in addition 2.5% and 5% and The lowest line stands for $H(90\%,50\%)$. Other parameters are $V_{T_1} = 40$, $\sigma = 20\%$, $F_1 = 50$, $r = 10\%$ and $\beta = 90\%$. For ease of representation, useless negative values have been omitted.

Figure 3: The net gain function and the monitoring barrier



Both graphs plot the function $H(T_2)$ as a function of T_2 for different level of the monitoring barrier V_B. V_B ranges from 80% of the firm assets value $V(T_1)$ to 90%. The continuous line represents $V_B = 80\% \times V(T_1)$. On the left graph, the constant realization rate is set to $\beta(T_1) = 50\%$. On the right graph, it is time-varying and converges to $\beta^* = 90\%$ at the speed $a = 0.75$. In both cases, $\beta_{V_B} = \beta(T_1)$. Other parameters are $V_{T_1} = 40$, $\sigma = 20\%$, $F_1 = 50$ and $r = 10\%$.

Figure 4: The optimal extension period and the monitoring barrier



This graph plots optimal extension periods as a function of the monitoring barrier for different values of β ranging from 50% (the continuous line) to 90% (the dotted one) 10% by 10%. Other parameters are $V_{T_1} = 40$, $\sigma = 20\%$, $F_1 = 50$ and $r = 10\%$.