THE ACCURACY OF TIME-VARYING BETAS AND THE CROSS-SECTION OF STOCK RETURNS

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Abstract

This paper provides new evidence about two questions that have been investigated separately in the literature so far. It compares the accuracy of time-varying betas estimated with different techniques and assesses their impact on the results of cross-sectional tests of the CAPM. Tests are performed with monthly data from US industry portfolio over the period 1980-2005. The modeling techniques considered are the rolling regressions, GARCH models, the Kalman filter, the SCHWERT and SEGUIN model, a macroeconomic variables model and an asymmetric beta model. Our results indicate that in times-series tests, the Kalman filter with a beta being specified as a random walk provides the most accurate results. Moreover, these betas provide supportive evidence on the validity of the conditional CAPM as they are statistically related to the cross-section of stock returns. All others specifications of betas, including the widely used rolling regressions, do not produce a significant beta-return relationship.

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1 Introduction

The Capital Asset Pricing Model (CAPM) of SHARPE (1964), LINTNER (1965) and BLACK (1972) shows that beta should be the only determinant of expected stock returns. Since the model is developed in a one-period setting, the beta is assumed to be constant. However, empirical implementation of the model can only be done in a multi-period setting and therefore some assumptions must be made about the temporal behavior of the systematic risk measure. In the vast majority of empirical studies the beta is assumed to be constant over a defined period of time.

This is in contradiction with the early evidence of BLUME (1971) that finds that beta is time-varying and with the results of numerous papers that document beta instability on various markets, (see for example, FABOZZI and FRANCIS (1977), BOS and NEWBOLD (1984) or BROOKS and FAFF (1998)). A series of alternative models have been proposed in the literature to capture the time-varying behavior of the beta. FAMA and MACBETH (1973) propose a rolling regression approach to estimate the beta. They assume that beta is constant during short time intervals while FABOZZI and FRANCIS (1977) propose a beta that is dependent on the state of the market (up or down). SCHWERT and SEGUIN (1990) investigate whether the market volatility has an impact on the beta. Their results are conclusive for the American market but the validity of this model for various international markets is questioned by KOUTMOS, LEE and THEODOSSIOU (1994). BRAUN, NELSON and SUNIER (1995) use a bivariate EGARCH model to estimate a beta influenced asymmetrically by the market's returns. FERSON and HARVEY (1999) examine whether macroeconomic variables play a role in the temporal evolution of the beta. Their results are interesting in the sense that, first, the beta is influenced by the variables and hence is time-varying, and secondly, when these lagged variables are included in the FAMA and FRENCH (1993) three factors models, they find strong evidence against it. Consequently, this model is able to explain unconditional expected returns but not the dynamic process of the expected returns. Next to these methods, econometrics models have been widely used to try to explain the stochastic evolution of the beta. Among them, we can cite GARCH type models and the Kalman filter approach. The later is an algorithm which recursively estimate beta series from a set of priors and is presented as a state space model. If some of these models provide significant results, no consensus has been reach to explain the stochastic evolution of the beta. Furthermore, the data's frequency plays also a role in the stochastic process followed by the beta, as explain by CHANG and WEISS (1991). They find that when the beta is estimated over a short time interval, it follows an autoregressive process but as the time interval lengthens, the process becomes a random walk.

Despite the wealth of alternative specification for time-varying betas, only a few papers compare the accuracy of these estimation methods. Among them FAFF, HILLIER and HILLIER (1998) compare the models for the British market. They found the Kalman Filter, with an observation equation formulated as a random walk, to be the more accurate approach versus a GARCH model and the SCHWERT and SEGUIN model (thereafter SS model) to estimate the beta. BROOKS, FAFF, and MACKENZIE (1998) test the same models for the Australian market and come to the same conclusion. On the same market, GROENEWOLD and FRASER (2000) conclude that various models, based on time trends and some macroeconomic variables, to forecast the beta are not more accurate than the standard rolling regression. LI (2001) found a stochastic volatility model to fit the best the beta's evolution followed by a GARCH model when an out of sample evaluation is made and by the Kalman filter when the test is in sample. EBNER and NEUMANN (2005) evaluate a rolling regression, a random walk Kalman filter and a flexible least square model for individual German stocks. Their results support the later model by improving considerably the accuracy of the beta estimations. In spite of being widely used by the practitioners, and in academic research as well, the rolling regression is even worse than the constant beta estimated by OLS. To the best of our knowledge, there is no paper addressing this issue on the US market, except BRAUN, NELSON and SUNIER (1995), who just compare their EGARCH model with the rolling regression. Their results support the former method. The first objective of this paper is therefore to fill this gap by comparing a wide range of beta estimation methods, developed in the literature, for the US market.

All the tests described so far are time-series tests that evaluate the accuracy of the various beta modeling approaches, using a mean square error (thereafter MSE) or a mean absolute error (thereafter MAE) criterion. These tests are evidence about the method which is the most accurate in a market model. However, they do not provide information about the main role of the beta, which is to act as the main determinant of stock returns. The second (and main) objective of this paper is to investigate the importance of these various methods from the perspective of conditional CAPM tests, using a cross-sectional methodology. This is important since tests of the CAPM (or any other asset pricing model) usually consider just one method of beta estimation. FAMA and FRENCH (1992) for instance only consider the rolling regression and reject statistically the link between the beta and the return. The results could be different with others estimation methods. To the best of our knowledge this has not been done so far in the literature and is therefore the main contribution of our paper.

The remainder of this paper is structure as follow: section 2 describes the different specification of the beta and the test methodology in time-series and in cross-section. The models that we consider in this paper are the rolling regression, a GARCH model, the Kalman filter with an autoregressive observation equation, the SS model, a macroeconomic variables model, an asymmetric beta model. The constant beta of the market model is used as a benchmark. Section 3 provides a description of the data while section 4 presents the empirical results. Section 5 concludes.

2 Empirical framework

2.1 Time varying beta models

2.1.1 The constant beta of the market model

This model is used as a benchmark. It considers the beta to be constant over the whole period:

$$r_{it} = \alpha_i + \beta_i \cdot r_{mt} + \varepsilon_{it} \text{ with } \varepsilon_{it} \text{ being a i.i.d process} \qquad (1)$$

where r_{it} is the simple return in excess of the risk free rate of the portfolio i in time t, r_{mt} is the excess simple return of the market in time t, β_i is the constant beta of the portfolio i and ε_{it} a disturbance vector. This beta is defined as:

$$\beta_i = \frac{Cov(r_{it}, r_{mt})}{Var(r_{mt})} \qquad (2)$$

The beta could identically be obtained by the estimation of equation (1) by OLS. The other models will be also estimated according this equation, but with a time-varying beta. The implication of using portfolio instead of individual stocks is that, according to the diversification principle, the portfolio return is fully explained by its beta and the excess market return. The term ε_{it} is only a disturbance term and not the specific return of the portfolio *i* which has been eliminated in the diversification process when portfolios are formed. The use of portfolios also improves the quality of the beta's estimation.

2.1.2 The rolling regression

This method has been used, among others, by FAMA and MACBETH (1973). It supposes that betas are constant over short time intervals, usually 5 years. Each month a regression of the market model is carried out using the last 60 observations. For each beta, only 1 observation on 60 is new and therefore this overlapping problem lead to autocorrelation in the beta time-series. GROENEWOLD and FRASER (2000) investigate this issue by using non-overlapping sub-periods and conclude that this approach doesn't change their results. As a consequence, we don't consider this issue in this paper.

2.1.3 The GARCH errors model

A problem with equation (1) is that errors are not normally, identically and independently distributed, which lead to a bias in the estimation of this equation by OLS. An approach to overcome this problem could be the use of a GARCH (1,1) model to descibe the disturbance term:

$$r_{it} = \mu_{it} + \upsilon_{it} \quad \text{with } \upsilon_{it} \sim N(0, \sigma^2_{it}) \tag{3}$$

$$r_{mt} = \mu_{mt} + \upsilon_{mt} \quad \text{with } \upsilon_{mt} \sim N(0, \sigma^2_{mt}) \tag{4}$$

where μ_{it} is the conditional mean of the portfolio *i* returns in time *t* and respectively for the market return, υ_{it} a disturbance term and σ^2_{it} the conditional variance which is define as:

$$\sigma^2_{it} = a_i + b_i \upsilon^2_{it-1} + c_i \sigma^2_{it-1} \tag{5}$$

$$\sigma^2_{mt} = a_m + b_m \upsilon^2_{mt-1} + c_m \sigma^2_{mt-1} \tag{6}$$

This formulation implies that the conditional variance depend on the past squared residual (υ^2_{it-1}), associated with the ARCH coefficient (b_i) and the past conditional variance (σ^2_{it-1}) associated with the GARCH coefficient (c_i). The former coefficient could be interpreted as the news coefficient and the later as the old news about volatility. The higher they are, the more the shocks are persistent but the sum of both have to be less than unity to have a finite unconditional variance. The conditional covariance is computed as:

$$Cov(r_{it}, r_{mt}) = \rho_{im} \sqrt{\sigma^2_{it} \cdot \sigma^2_{mt}} \tag{7}$$

where ρ_{im} is the correlation coefficient, between the excess return of portfolio *i* and the market, which is suppose to be constant over time. Then these betas can be estimated by:

$$\beta_{it}^{GARCH} = \frac{Cov(r_{it}, r_{mt})}{\sigma_{mt}^2} \tag{8}$$

2.1.4 The Kalman filter

Instead of calculating conditional variances first, as the former model, the Kalman filter algorithm estimates directly time-varying betas in the framework of a state-space model. This approach distinguishes between known (portfolios and market returns) and unknown (the betas) variables as well as measurement and transition equations. The former equation describes how known variables are generated by the unknown variables and the residuals:

$$r_{it} = \alpha_i + \beta_{it}^{Kal} \cdot r_{mt} + \varepsilon_{it} \qquad \text{(Measurement equation)} \tag{9}$$

where α_i is a constant for each portfolio *i*. The next step is to specify the transition equation which describes the stochastic process followed by the unknown variable, which is the beta, according to its lags and innovations. In this paper we choose to use an AR (1) process:

$$\beta_{it}^{Kal} = \phi_i \beta_{it-1}^{Kal} + e_{it} \qquad \text{(Transition equation)} \tag{10}$$

where φ_i is an autoregressive coefficient. By defining the beta in this way, we let the data and the algorithm choose which stochastic process is the most appropriate to describe the time dependent process of the beta. Indeed, if the autoregressive coefficient, $\varphi_{i,}$ is not statistically different from 1, the process will be a random walk, if it lies between 0 and 1, it follow an AR(1) process and if it is not statistically different from 0, it is a random coefficient model. The estimation of the transition equation by the Kalman filter algorithm gives us 2 different beta time-series. The first one is the filtered and the second one is the smoothed series. The former is estimated by using only the information available at time t and the later smoothes the series once all the estimation is done. The later method need the information of the entire sample and therefore is suitable only for particular purpose like determining a normal return in an academic framework.

2.1.5 The SCHWERT and SEGUIN model

This model developed by SCHWERT and SEGUIN (1990) assumes that stocks respond differently to variations of the market volatility, according to their size. As a consequence, the beta should also depend on the market volatility. They define it as:

$$\beta_{it}^{SS} = \beta_i + \frac{\delta_i}{\sigma_{rmt}^2} \tag{11}$$

where the first component of the beta (β_i) is constant while the second term (δ_i/σ^2_{rmt}) is time-varying and depends on the market volatility. The coefficient δ_i measures the sensitivity of portfolio *i* returns to a variation of the market volatility, σ^2_{rmt}. If the sensitivity coefficient is not statistically significant, this beta and the constant beta defines in equation (2) are equal. The market volatility is obtained by the GARCH (1, 1) model in equation (6). To estimate the coefficients, we insert this beta definition in the market model and the following regression is carried out:

$$r_{it} = \alpha_i + \beta_i \cdot r_{mt} + \frac{\delta_i \cdot r_{mt}}{\sigma^2_{rmt}} + \varepsilon_{it} \tag{12}$$

2.1.6 The macroeconomics variables model

This model has been proposed by FERSON and HARVEY (1999). In this approach, the betas depend on a set of macroeconomic variables, which are supposed to describe the economic cycle. They are: the difference between the yield of a ten years and a one year government bond (TERM), the dividend yield of the US market (DIV), the spread between Moody's Baa and Aaa corporate bond (JUNK) and the return of a one month treasury bill (T-BILL). To generate the beta, we use the lagged value of the variables and we define it as:

$$\beta_{it}^{macro} = b_{0i} + b_{1i} \cdot Z_{t-1} \tag{13}$$

Where the coefficient b_{oi} and b_{1i} are constant and Z_{t-1} is the vector of the lagged macroeconomic variables. The temporal instability of these betas results from their dependency to the lagged variables. The constancy of the coefficient b_{1i} implies that the betas are a constant linear function of the variables. To estimate the coefficient in (13), we carry out the following market model regression:

$$r_{it} = \alpha_i + \left(b_{0i} + b_{1i} \cdot Z_{t-1}\right) \cdot r_{mt} + \varepsilon_{it} \tag{14}$$

2.1.7 The asymmetric beta model

This model, developed by FABOZZI and FRANCIS (1977), supposes that the beta of stocks or portfolios could be influenced by the state of the market. The beta is computed as:

$$\beta_{it}^{asym} = \beta_{0i} + \beta_{1i} D_t \tag{15}$$

where D_t is a dummy variable which takes the value of 1 if the market is defined as an up market (r_{mt} is non negative) and 0 otherwise. The coefficient β_{1i} measures the differential effect of an up market on the beta. According to this beta specification and to estimate the coefficients of equation (15), the market model is redefined as:

$$r_{it} = \alpha_i + \beta_{0i} r_{mt} + \beta_{1i} D_t r_{mt} + \varepsilon_{it} \tag{16}$$

with $D_t = 1$ if $r_{mt} \geq 0$
$D_t = 0$ if $r_{mt} < 0$

According to this specification, the betas can only take 2 different values. They are equal if the coefficient β_{1i} is not significant and the beta is therefore time constant.

2.2 Test methodology

2.2.1 Time-series test

This first test estimate the accuracy of each of the previous beta modeling techniques using the mean squared error criterion (MSE criterion) of in-sample return forecasts where:

$$r^*_{jit} = \beta_{jit} r_{mt} \tag{17}$$

where r^*_{jit} is the forecasted excess return and β_{jit} is the beta of portfolio i in time t using the j^{th} beta modeling technique (j = 1,…, 8). Then we compute the forecasting error e_{jit}:

$$e_{jit} = r_{it} - r^*_{jit} \tag{18}$$

and we can compare the accuracy of each beta technique by computing the MSE criterion:

$$MSE_j = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} e^2_{jit}}{NT} \tag{19}$$

where N is the number of portfolios (35) and T is the number of time periods (120). The most accurate beta model is the one which provides the smallest MSE. This test methodology is widely used in the literature (e.g. by BRAUN, NELSON and SUNIER (1995), FAFF, HILLIER and HILLIER (1998) or GROENEWOLD and FRASER (2000)). However, this test informs only on which method of beta estimation is the most appropriate in the framework of the market model. It does not prove statistically the relationship between a portfolio returns and its beta. To test the existence of this link and thus the validity of the CAPM, a cross-sectional methodology is necessary.

2.2.2 FAMA MACBETH methodology

This CAPM test methodology developed by FAMA and MACBETH (1973) is one of the most widely used in the literature. This test is executed in two steps. First the following regression is carried out for each beta estimation method:

$$r_{it} = \gamma_{jot} + \gamma_{j1t}\beta_{jit} + \eta_{jit} \tag{20}$$

where β_{jit} are used as an explanatory variable and γ_{jot} and γ_{j1t} are the parameters to estimate. The coefficient γ_{j1t} can be interpreted as the theoretical market excess return in time t according to the j^{th} beta estimation method. We estimate this regression using two different assumptions about the distribution of the disturbance term η_{jit}. First, it is supposed to have a zero mean and to be independent across all portfolios. Therefore, the OLS is an efficient estimation method. The second assumption is the presence of contemporaneous correlation. That means that the correlations between disturbances from the regressions of different portfolios, but at the same time, are not zero. In this case the OLS method is not efficient anymore. In order to overcome this problem, we use a seemingly unrelated regressions equations (SURE) system. This is a method used to pool time-series and cross-sections data, thus the model in equation (20) can be estimated as a whole. The estimator used in practice is the Zellner's seemingly unrelated regression estimator, which is defined as:

$$\hat{\beta} = \left[X'(\hat{\Sigma}^{-1} \otimes I)X\right]^{-1} X'(\hat{\Sigma}^{-1} \otimes I)Y \qquad (21)$$

where $\hat{\beta}$ is the (480x1) coefficient vector containing all the estimated γ_{ot} and γ_{1t}, X is a (8400x480) matrix gathering all the β_{it} , $\hat{\Sigma}$ is the estimated covariances (8400x8400) matrix and Y is the (8400x1) return vector containing the returns of all portfolios at all the periods. Note that this model is estimated for each of the 8 models, however to clarify the notation of the equation (21), subscripts are not added.

The second step is to test if the average of the coefficients γ_{j1t} is statistically significant and positive. That would prove the link, on average, between the beta (estimated by the j^{th} method) and the return of the portfolio. If the previous test is satisfactory, by rejecting the null hypothesis, we can test whether this average coefficient is equal to the average realized excess market return. That would prove that the regressed coefficients are, in average, equal to the observed market risk premium. The last part of the test is to check if the coefficient γ_{jot} is on average not different from zero. That would mean that there is no other common factor being able to explain the cross-sections of the returns. The first test is carried out for each of the 8 beta estimation model and whether the null hypothesis is not rejected, the 2 remaining tests are not carried out. As we know, the influence of the beta estimation method in the test of the conditional CAPM has not been analyzed and this is supposed to be the most important contribution of this paper.

3 Data description

For the empirical part of this paper, we use industry portfolios provided by Thomson Financial DataStream corresponding to the FTSE level 4 classification for the American market with a monthly frequency. That is to say we have 35 portfolios. The use of portfolios instead of individual stocks aims to improve the accuracy of the beta estimation. Moreover, the use of portfolios implies that returns are fully explained by their beta and the excess market return, the specific risk being eliminated by the diversification process. The value weighted "US Total Market" index, also provided by Thomson Financial DataStream, is used for the market returns and the three months US government bond is used as the risk free rate. Portfolios and the market index returns include dividend payment. The sample covers the period from January 1980 to January 2005, the 5 first years being reserved for prior betas estimations.

4 Empirical results

4.1 Descriptive statistics

The first part of this section is dedicated to the presentation of the portfolios and their returns. A description, as well as the descriptive statistics, is provided in table 1. Average monthly excess returns range from -1.14% for the steel and others metals sector to 1.84% for the tobacco industry with a market average at 0.69%. Unsurprisingly, only the returns of 2 portfolios the aerospace and defense and the electricity sectors, are normally distributed. All others fail the Jarque-Bera normality test.

Table 1: Portfolios descriptives statistics

Portfolio	*Sector*	*Average return*	*volatility*	*Jarque-Bera*
1	Mining	0.57%	10.50%	**182.61**
2	Oil & Gas	0.43%	5.22%	**33.15**
3	Chemicals	0.57%	5.52%	**142.65**
4	Construction	0.69%	6.60%	**70.15**
5	Forestry & Paper	0.53%	6.82%	**42.86**
6	Steel & Metals	-1.14%	8.38%	**48.54**
7	Aerospace & Defence	1.01%	5.73%	**75.13**
8	Diversified Industries	0.93%	5.55%	**124.47**
9	Electrical equipment	1.51%	6.57%	**89.40**
10	Machinery	1.38%	6.18%	**131.10**
11	Auto & Parts	0.95%	6.46%	**17.71**
12	Textile	1.24%	6.21%	**120.53**
13	Beverages	1.47%	5.68%	**39.50**
14	Food production	1.33%	4.73%	**22.96**
15	Health	1.50%	4.77%	**28.73**
16	Personal care	1.42%	5.08%	**105.53**
17	Pharma & Biotech	1.50%	5.33%	4.06
18	Tobacco	1.89%	8.34%	**53.32**
19	General retail	1.50%	6.62%	**36.67**
20	Leisure & Hotels	1.51%	6.78%	**32.97**
21	Media & Entertainment	1.03%	5.46%	**82.66**
22	Support Services	0.96%	5.65%	**49.93**
23	Transport	1.02%	5.59%	**118.08**
24	Food and drug retail	1.11%	5.22%	**17.98**
25	Telecom services	1.01%	5.77%	**67.01**
26	Electricity	0.99%	4.39%	2.02
27	Other utilities	0.93%	5.52%	**71.99**
28	Information tech. & Hardware	1.25%	8.68%	**14.64**
29	Software & computer services	1.79%	8.24%	**11.05**
30	Banks	1.45%	5.79%	**25.49**
31	Insurance	1.31%	5.23%	**58.98**
32	Life insurance	1.48%	5.70%	**17.77**
33	Investment	0.37%	6.21%	**69.35**
34	Real estate	1.30%	5.44%	**56.85**
35	Other finance	1.63%	6.62%	**42.86**
Market	Us total market	0.69%	4.26%	**95.13**

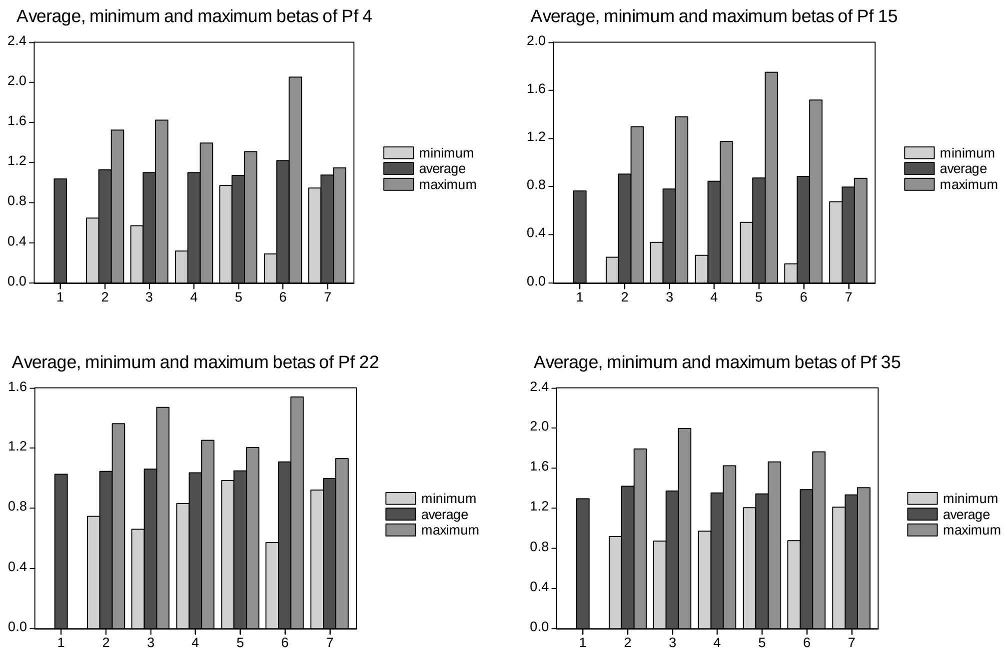
Note: Returns are in excess of the risk free rate. Average monthly returns and volatility are computed for the whole period, from 1980 to 2005. The Jarcque-Bera statistic, based on the skweness and kurtosis, is used to test if the returns are normally distributed. The statistic is distributed as χ^2 with 2 degrees of freedom. Statistics in bold are significant at the conventional 5% level and therefore reject the normality null hypothesis.

4.2 Estimation of time-varying beta models

4.2.1 Overview

Before to present details on the estimation of the specific models developed in section 2.1, we provide, in the table 2, the average betas for the 35 portfolios according to each of the 7 models. It can be seen that average beta according to the various models are quite close to the constant beta of the market model in equation (2). Nevertheless, the betas move widely around their mean. This can be seen on the following figure presenting the mean, the minimum and the maximum beta for some of the portfolios used in this study.

Figure 1 : Average, minimum and maximum betas of various portfolios



Note: the 7 methods used to estimate the betas are by order: the constant beta of the market model, the rolling regression, the GARCH (1, 1), the Kalman AR (1) smoothed, the SS beta, the macro variables beta and the asymmetric beta.

This figure[1] illustrate well the fact that constant beta is a good estimation of the mean of the various time-varying beta models. However, extreme values depart widely from their mean. The figure presented in appendix 1 illustrates the betas estimated according to the various models for each portfolio. The purpose of this figure is not to show precisely the path followed by the beta estimated by one particular method for a portfolio, but rather to show the general behavior of the dynamic beta. It can be seen that the various methods generate very different beta in their temporal evolution. Indeed, they even fluctuate in an opposite way over short time intervals and the beta volatility is also very different according to the estimation models. This issue confirms the importance of comparing the accuracy of the various beta estimation approaches and finding a significant variable in the beta estimation is clearly not enough to define it as a "good" beta.

[1] To save space, we only present 4 portfolios but the results are qualitatively identical for the others.

Table 2: Average beta

Pf	β^{Cst}	β^{RR}	β^{GARCH}	β^{Kal}	β^{SS}	β^{Macro}	β^{Asym}
1	0.445	0.461	0.484	0.303	0.504	0.346	0.302
2	0.651	0.656	0.676	0.647	0.647	0.662	0.641
3	0.915	0.999	0.967	0.936	0.989	1.013	0.914
4	1.037	1.130	1.099	1.099	1.072	1.217	1.075
5	0.999	1.066	1.053	0.993	0.997	1.054	1.026
6	1.159	1.101	1.208	1.155	1.148	1.198	1.167
7	0.875	0.937	0.885	0.875	0.896	1.027	0.866
8	0.950	0.928	0.954	0.944	0.950	0.941	0.949
9	1.314	1.239	1.302	1.268	1.261	1.276	1.347
10	1.063	1.099	1.097	1.065	1.093	1.118	1.065
11	1.017	0.985	1.075	1.015	1.001	1.037	1.007
12	1.111	1.174	1.153	1.140	1.119	1.241	1.114
13	0.773	0.835	0.798	0.821	0.839	0.929	0.811
14	0.606	0.680	0.632	0.655	0.684	0.755	0.636
15	0.765	0.903	0.781	0.843	0.872	0.885	0.797
16	0.701	0.795	0.727	0.747	0.778	0.845	0.726
17	0.824	0.871	0.900	0.849	0.876	0.869	0.845
18	0.761	0.846	0.824	0.805	0.896	0.917	0.827
19	1.185	1.179	1.243	1.170	1.130	1.193	1.239
20	1.269	1.248	1.316	1.263	1.247	1.264	1.250
21	1.067	1.007	1.071	1.041	1.047	1.037	1.082
22	1.026	1.046	1.061	1.036	1.049	1.109	0.998
23	0.938	1.063	1.021	0.995	0.997	1.098	0.911
24	0.742	0.779	0.770	0.775	0.776	0.873	0.781
25	0.895	0.783	0.882	0.895	0.887	0.849	0.896
26	0.340	0.353	0.353	0.386	0.997	0.454	0.366
27	0.647	0.609	0.649	0.641	0.698	0.668	0.559
28	1.517	1.335	1.480	1.493	1.423	1.431	1.513
29	1.469	1.401	1.432	1.429	1.376	1.326	1.489
30	0.978	1.030	1.006	1.016	1.020	1.060	0.980
31	0.782	0.870	0.816	0.883	0.859	0.947	0.825
32	0.835	0.940	0.859	0.888	0.924	0.972	0.876
33	1.030	0.958	1.059	1.035	1.013	1.048	1.036
34	0.750	0.898	0.813	0.839	0.836	0.885	0.721
35	1.298	1.418	1.373	1.352	1.341	1.388	1.333

Note: This table presents average beta generated by the various models discussed in section 2.1 for the period 1985 to 2005. The models are: β^{Cst} = constant beta of the market model, β^{RR} = beta from the rolling regression, β^{GARCH} = GARCH (1,1) beta, β^{Kal} = Kalman AR (1) beta, β^{SS} = SCHWERT and SEGUIN beta, β^{Macro} = macroeconomic variables beta and β^{Asym} = asymmetric beta.

4.2.2 The GARCH error model

The estimation of the coefficients, whose results are provided in appendix 2, by this model are not very satisfactory. The estimation of the conditional variance from equation (5) and (6) results only in 20 significant ARCH coefficients (b_i) and in 30 significant GARCH coefficients (c_i). These results are probably due to the monthly frequency used in this paper and to the relative short time interval. This is not surprising, considering that ARCH effects are more likely to occur in a higher frequency. In comparison, FAFF, HILLIER, HILLIER (2000), using a daily frequency, find all the coefficients of their GARCH (1, 1) model to be significant at the 1% level. Our model seems to be badly specified for a few portfolios, which have no significant coefficient and a sum of both of them is very small and even negative for the portfolio 17. Besides this exception, all meet the condition to have a finite unconditional variance.

4.2.3 The Kalman filter

As explain in section 2.1, this method provides 2 beta series for each portfolio, a filtered and a smoothed one. Table 3 presents the estimation of the autoregressive coefficient from equation (10). The value of this coefficient will determine the best stochastic structure of the time varying beta in the framework of the Kalman filter algorithm. For the filtered beta all these coefficients are not statistically different from unity, except for the first portfolio[2]. As a consequence, the beta follows a random walk of the form:

$$\beta_{it}^{Kal} = \beta_{it-1}^{Kal} + e_{it} \tag{22}$$

When we consider the smoothed beta, the results are very similar. Only the beta of 4 portfolios (including portfolio 1) follow an autoregressive process, with a coefficient, φ_i, very close to unity but still statistically different. On the 31 coefficients remaining, 27 are first order integrated and 4 are second order integrated. That means that the beta needs to be differentiated 2 times to be stationary. This is probably due to the smoothing algorithm used by the Kalman filter. However, to overcome the problems raised by this issue, we set the coefficient at unity for these 4 portfolios. According to the Kalman filter estimation, the beta follows in a great majority a random walk process and as a consequence, the beta is not predictable by its lagged values and it fluctuates randomly from period to period.

4.2.4 The SCHWERT and SEGUIN model

The estimation of the parameters β_i and δ_i from equation (12) do not really support this beta specification. The results are presented in appendix 3. The coefficient δ_i is significant only for 11 portfolios. Moreover, its size is very small, in the order of 0.004. These poor results arise probably from the use of industry portfolios because the sensitivity coefficient δ_i is supposed to depend on the size of the stock or portfolio.

[2] Every beta estimation method gives very poor results for this portfolio.

Table 3: Kalman filter autoregressive coefficient and integration order

Pf	$\beta^{Filtered}$ *φi*	*Integration order*	$\beta^{Smoothed}$ *φi*	*Integration order*
1	0.6596	0	0.3312	0
2	0.9997	1	1.0000	1
3	0.9996	1	0.9998	1
4	0.9982	1	0.9968	1
5	0.9997	1	0.9997	0
6	1.0000	1	1.0000	2
7	0.9995	1	0.9996	1
8	0.9999	1	0.9999	0
9	0.9981	1	0.9976	1
10	0.9999	1	0.9998	0
11	1.0000	1	1.0000	2
12	0.9985	1	0.9997	1
13	0.9924	1	0.9900	1
14	0.9970	1	0.9968	1
15	0.9978	1	0.9984	1
16	0.9934	1	0.9916	1
17	0.9984	1	0.9992	1
18	0.9967	1	0.9971	1
19	0.9994	1	0.9997	1
20	0.9996	1	0.9993	1
21	0.9999	1	0.9998	1
22	0.9998	1	0.9997	1
23	0.9990	1	0.9970	1
24	0.9978	1	0.9977	1
25	0.9833	1	0.9994	1
26	0.9863	1	0.9858	1
27	0.9997	1	1.0000	2
28	0.9993	1	1.0000	2
29	0.9999	1	0.9994	1
30	0.9990	1	0.9998	1
31	0.9962	1	0.9941	1
32	0.9991	1	0.9986	1
33	0.9994	1	1.0000	1
34	0.9982	1	0.9974	1
35	0.9976	1	0.9995	1

Note: The autoregressive coefficient φ_i is obtained by the estimation of the transition equation in the Kalman state space model: $\beta_{it} = \varphi_i\beta_{it-1} + e_{it}$. The sample covers the period 1980-2005. The first 5 years are taken in account because the Kalman filter could result in inaccurate estimations in the early period of the sample. The standard Dickey-Fuller test is conducted to evaluate the integration order of the beta time series. The integration order is the number of differentiations needed to obtain a stationary series. A zero order integrated series is stationary.

4.2.5 The Macroeconomic variables model

First, we check the ability of the lagged macroeconomic variables to explain the portfolios returns. The results are presented in appendix 4. Their predictive capacity is very variable. For instance, the dividend yield of the US market (DIV) is significant for 21 portfolios returns but the JUNK variable is only significant for 1 portfolio. However the regressions R^2 are rather small, ranging from 0.01 to 0.07. When forming size and book to market portfolios, FERSON and HARVEY (1999) find R^2 slightly higher, from 0.08 to 0.15. Table 4 presents the results of the estimation of equation (14) which gives the coefficient necessary to generate the betas. The ability of this model to capture the evolution of the time-varying beta is very different for each portfolio. The R^2 ranges from 0.2 for the electricity sector to 0.78 for the electrical equipment sector. Unsurprisingly, the market is the variable which has the most significant coefficients (26) followed by the dividend yield (20). On the other side, the Term variable has only 5 significant coefficients. It is interesting to note that the portfolios returns and the betas are not influenced by the lagged variables in the same way.

4.2.6 The asymmetric beta model

The estimation of the coefficient β_{1i} from equation (16), which measures the differential effect of an up market on the beta, does not support this asymmetric beta specification. Only 2 coefficients are significant[3]. The results of this model arise probably of the monthly frequency chosen; a higher frequency could potentially lead to different conclusions.

4.2.7 Synthesis of the various time-varying beta

A major issue raised by our results of the estimation of the various models in this section is the importance of the chosen frequency and the way portfolios are computed. That influences greatly the results and the validation of the various models. For example, the beta could be asymmetric in a weekly frequency but not in monthly frequency. However, in the purpose of comparing the various time-varying beta models, we have to use the same specification for all the models. Furthermore, we do not expect the models which are not suitable for our sample specification (i.e. the SS model and the asymmetric beta model) to provide very good results in the next section dedicated to the comparison of the various beta models.

4.3 Empirical comparison of the time-varying beta models

4.3.1 Time-series test

The time-series test, using the MSE criterion, is widely employed in the literature. The results are presented in table 5[4]. On average, the Kalman filter, providing the filtered beta series, gives the smallest error. In second position and with a very close MSE, we find the smoothed series of the Kalman filter, followed by the macroeconomic variables model and the rolling regression. The others models, i.e. the GARCH model, The SS model and the asymmetric beta model, do not beat the constant beta from the market model. This is not surprising, considering that these later beta specifications are not suited for our monthly frequency or our

[3] To save space, the results are not presented in this paper.
[4] This test has also been done with the mean absolute error criterion without affecting the results.

Table 4: Regressions of the macroeconomic variables beta equations

Pf	*Market* β_{0i}	*T-Bill* β_{1i}	*Div* β_{2i}	*Junk* β_{3i}	*Term* β_{4i}	R^2
1	**-1.94**	**529.08**	**-72.65**	1121.49	**909.89**	0.07
2	*0.63*	-45.73	-0.65	262.13	22.93	0.30
3	**0.71**	-16.24	**29.24**	*-654.14*	124.68	0.57
4	**1.29**	-127.45	**46.44**	**-1137.22**	147.95	0.52
5	**0.95**	-100.97	*32.75*	-191.20	-123.21	0.43
6	**1.20**	-93.88	-0.78	360.68	95.74	0.38
7	**1.11**	3.63	**30.95**	**-1494.61**	*241.40*	0.49
8	0.49	90.57	-2.03	31.18	94.91	0.57
9	**1.24**	71.33	**-26.13**	341.11	109.41	0.78
10	**0.81**	-32.11	20.41	-157.50	47.89	0.58
11	**1.13**	-117.92	15.85	205.60	-145.19	0.48
12	**0.89**	-61.66	**41.72**	**-745.35**	120.48	0.67
13	**1.57**	**-293.57**	**77.29**	**-1189.80**	**-376.89**	0.44
14	**0.69**	**-177.06**	**65.14**	**-908.58**	**-108.84**	0.45
15	**0.63**	-40.97	**44.20**	**-964.67**	59.39	0.59
16	**1.40**	**-269.72**	**73.35**	**-1099.32**	**-360.00**	0.47
17	0.55	-56.50	**25.00**	-42.39	-28.55	0.48
18	0.68	*-266.26*	**60.38**	-167.71	-39.09	0.22
19	0.52	66.21	7.10	151.74	93.94	0.63
20	**0.89**	5.79	2.09	351.51	22.25	0.67
21	**0.49**	41.53	-7.39	**625.04**	66.14	0.74
22	**0.73**	-29.55	*19.86*	-246.56	168.09	0.66
23	**0.99**	18.56	**33.39**	**-1470.03**	**290.64**	0.60
24	0.48	-95.54	**45.29**	-588.32	99.00	0.47
25	**0.99**	-88.91	-6.27	*761.44*	-176.35	0.48
26	0.34	**-168.39**	**33.10**	-145.41	69.44	0.20
27	0.67	-101.88	2.32	438.61	9.79	0.28
28	**1.84**	61.59	**-57.04**	788.54	122.52	0.63
29	**0.89**	*178.24*	**-45.69**	**1054.09**	28.61	0.62
30	**1.20**	-2.52	19.69	**-916.49**	75.78	0.56
31	**1.26**	-99.11	**45.94**	**-1479.58**	76.94	0.49
32	**1.07**	**-77.24**	45.17	**-1214.01**	31.59	0.46
33	**1.43**	-60.41	-10.07	70.25	44.30	0.54
34	0.27	-82.23	**57.48**	*-690.05*	50.09	0.50
35	**1.385**	-35.764	**29.344**	**-794.623**	30.090	0.75

Note: The coefficients are obtained by the estimation of the following equation:
$r_{it} = \alpha_i + \beta_{0i} \cdot r_{mt} + \beta_{1i} \cdot T\text{-}Bill_{t-1} \cdot r_{mt} + \beta_{2i} \cdot Div_{t-1} \cdot r_{mt} + \beta_{3i} \cdot Junk_{t-1} \cdot r_{mt} + \beta_{4i} \cdot Term_{t-1} \cdot r_{mt} + \varepsilon_{it.}$ Values in bold are significant at the 5% level and the ones in italic at the 10% level. The lagged macroeconomic variables are in monthly frequency, except for the dividend yield (Div) in a yearly frequency.

Table 5: MSE criterion

Pf	β^{Cst}	β^{RR}	β^{GARCH}	$\beta^{Kal\ F}$	$\beta^{Kal\ S}$	β^{SS}	β^{Macro}	β^{Asym}
1	10.61	10.30	10.55	**7.31**	*7.37*	10.59	10.21	10.78
2	1.91	1.90	1.95	**1.89**	1.91	1.91	*1.89*	1.91
3	1.44	1.33	1.44	**1.29**	*1.31*	1.41	1.32	1.44
4	2.28	2.20	2.32	**1.89**	*1.96*	2.27	2.09	2.29
5	2.74	*2.64*	2.91	**2.63**	2.66	2.74	2.67	2.76
6	4.85	**4.73**	4.88	4.81	4.84	4.84	*4.75*	4.87
7	1.83	1.76	1.94	*1.68*	1.75	1.82	**1.66**	1.84
8	1.36	**1.31**	1.41	*1.32*	1.34	1.36	1.33	1.36
9	1.04	1.01	1.02	**0.80**	*0.83*	1.02	1.01	1.01
10	1.69	*1.61*	1.68	**1.60**	1.62	1.68	1.61	1.69
11	2.19	**2.14**	2.32	2.18	2.19	2.19	*2.16*	2.20
12	1.50	1.36	1.46	**1.20**	*1.28*	1.50	1.28	1.50
13	2.15	2.04	2.17	1.96	**1.60**	2.11	*1.81*	2.08
14	1.60	1.40	1.60	**1.22**	*1.23*	1.55	1.25	1.55
15	1.23	1.05	1.05	**0.91**	*0.93*	1.14	0.98	1.18
16	1.70	1.53	1.68	**1.19**	*1.23*	1.65	1.39	1.66
17	1.61	1.52	1.60	**1.46**	*1.49*	1.59	1.51	1.58
18	5.99	5.74	5.87	**5.47**	5.55	5.83	*5.53*	5.82
19	1.71	1.65	1.85	**1.59**	*1.62*	1.69	1.64	1.66
20	1.54	1.53	1.59	**1.43**	*1.43*	1.54	1.52	1.57
21	0.81	0.80	0.85	0.79	**0.76**	0.80	*0.78*	0.81
22	1.18	1.12	1.22	*1.09*	1.11	1.17	**1.08**	1.19
23	1.45	1.35	1.45	*1.21*	**1.19**	1.42	1.23	1.47
24	1.70	1.53	1.73	**1.41**	*1.42*	1.69	1.43	1.66
25	1.81	1.80	1.81	**1.37**	*1.73*	1.81	1.76	1.81
26	1.75	1.70	1.78	**1.49**	*1.52*	2.38	1.56	1.71
27	2.26	2.24	2.24	*2.22*	2.26	2.24	**2.20**	2.38
28	3.12	2.80	2.96	**2.57**	3.09	3.06	*2.79*	3.13
29	2.70	2.68	2.84	*2.60*	**2.57**	2.65	2.62	2.68
30	1.57	1.53	1.60	**1.44**	*1.48*	1.55	1.50	1.57
31	1.61	1.54	1.53	*1.28*	**1.27**	1.56	1.41	1.55
32	1.98	1.84	1.90	**1.77**	1.78	1.91	*1.77*	1.91
33	1.84	1.76	1.87	**1.65**	*1.68*	1.83	1.77	1.84
34	1.93	1.59	1.83	**1.39**	*1.42*	1.87	1.49	1.98
35	1.19	1.15	1.17	**0.96**	*1.05*	1.18	1.09	1.15
average	2.22	2.12	2.23	**1.92**	*1.96*	2.22	2.06	2.22
ranking	7	4	8	1	2	5	3	6

Note: The values are multiplied by 1000. Values in bold (italic) mean that the corresponding time-varying beta method provides the smallest (second smallest) MSE for the portfolio. $\beta^{Kal\ F}$ means Kalman filtered and $\beta^{Kal\ S}$ Kalman smoothed.

industry portfolios. If we look at the portfolios level, the superiority of the Kalman filter is not reconsidered. The 2 versions of the Kalman filter are 29 times in first position and 25 times in second position. Both the rolling regression and the macroeconomic variables model are 3 times in first position. The 4 remaining models do not even rank once first or second. Our results supporting the Kalman filter approach are in agreement with the conclusions of studies on others markets, like BROOKS, FAFF and MACKENZIE (1998) on the Australian market or FAFF, HILLIER, HILLIER (1998) on the British market. The link with this latter study is interesting in the sense that they use daily UK industry returns. However, the various sophisticated GARCH models they consider, do not generate better return forecasts, according to the MSE criterion, than the simple and constant beta from the market model. They also find the random walk beta from the Kalman filter to be the most accurate approach to estimate the beta.

However, this time-series test methodology does not prove statistically the relationship between the portfolio return and its beta. It allows only to determine which beta generating method fit the best the market model equation.

4.3.2 Cross-sectional tests

This section investigates whether the specification of the beta has an influence on cross-sectional tests of the CAPM. We examine whether the beta time-series obtained with the various beta estimation models have a significant impact on the results of FAMA and MACBETH (1973) cross-sectional tests. To that purpose, 8 cross-sections tests are performed, one for each beta estimation method. In addition we also use an alternative estimator, i.e. the Zellner estimator, described in equation (21), taking in account the contemporaneous correlation, for the estimation of the coefficients γ_{j1t} and γ_{jot} in equation (20). The results are given in table 6.

Table 6 : Cross-sectional tests

	β^{Cst}	β^{RR}	β^{GARCH}	$\beta^{Kal\ F}$	$\beta^{Kal\ S}$	β^{SS}	β^{Macro}	β^{Asym}
OLS statistic	0.404	0.194	0.164	1.347	**2.066**	0.607	1.369	0.110
SURE statistic	0.580	0.285	0.229	1.564	**2.177**	0.636	*1.717*	0.657

Note: This table provides the statistic of the first part of the test explained in section 2.2.2. Values in bold (italic) are significant at the usual 5% (10%) level. In this case, the average γ_{j1t} is different from zero and that proves statistically the link between the beta of a portfolio and its returns. $\beta^{Kal\ F}$ means Kalman filtered and $\beta^{Kal\ S}$ Kalman smoothed. Note that for the smoothed Kalman filter the beta series for the 4 portfolio (number 6, 11, 27 and 28) which are second order integrated have been replace by series estimated, as well with the Kalman filter, but with an autoregressive coefficient set to unity.

The first part of the test examines whether there is a statistical relationship between the beta of a portfolio and its return. In this case, the null hypothesis should be rejected. First, the only beta estimation method which validates the CAPM (at the usual 5% confidence level) is the smoothed series generated by the Kalman filter. It is interesting to note that the filtered version of the Kalman filter, which was more accurate in the previous time-series test, does not allow to validate empirically the CAPM. However, in the way we estimate the smoothed Kalman beta series, we need the information on the entire sample. It is also clear that the coefficients estimated with the SURE system provide systematically higher t-statistics. Considering the contemporaneous correlation could lead to change the test results. For example the SURE statistic for the macroeconomic variables models is significant at the 10% level, while the OLS statistic is not. All others model do not beat the constant beta in the cross-sections test. These models are, thereof not able to explain the time-varying evolution of the beta. This is particularly important for the rolling regression beta because it is a method widely used in academic to compute the beta, notably by FAMA and FRENCH (1992). Consequently, using another beta estimation approach, in our case the Kalman filter with a random walk beta, could lead to validate empirically the CAPM.

In the second part of the test, we examine whether the CAPM is fully validated by the data. First, we test if the average γ_{1t}, for the smoothed Kalman filter only, is different from the realized excess market risk premium. The test is conclusive if we can not reject the null hypothesis. The OLS and the SURE statistics are very close, 0.24 for the former and 0.22 for the later. Both of them allow us to not reject the null hypothesis and to conclude that the average estimated γ_{1t} does not statistically differ from the realized excess market risk premium. The last part of the test investigates whether the average coefficient γ_{ot} is statistically different from zero. We aim not to reject this test, because that would mean there is no other common factor than the excess market risk premium to explain the cross-sections of our portfolios returns. The results of this test are as well conclusive with an OLS statistic of 0.69 and a SURE statistic of 0.94.

In brief, the conditional CAPM is empirically validated whether the smoothed Kalman filter method is used to estimate the betas. The average γ_{1t} is significant at the 5% usual confidence level and it does not differ from the realized excess market risk premium. Furthermore, the constant term, γ_{ot}, is not statistically different from zero. These conclusions prove the influence of the beta estimation method in the CAPM test, as well as the method chosen to estimate the coefficients in the FAMA MACBETH (1973) regressions. Not considering these issues could lead to reject the CAPM even if it holds.

5 Conclusion

In this paper we shed light on the influence of the choice of the beta estimation method on the tests of the conditional CAPM. The purpose is to compare various beta estimation methods presented in the existing literature. The specification that we investigate are the constant beta of the market model (used as a benchmark), the rolling regression, a GARCH (1, 1) model, the Kalman filter with an autoregressive observation equation, the SS model, a macroeconomic variables model and an asymmetric beta model.

The first part of the section 4 emphasizes the impact of the frequency and the way portfolios are generated on the validity of these models. The SS model, the asymmetric beta model and to a lesser extent the GARCH (1, 1) model are clearly not suited for our monthly frequency, or our industry portfolios. To evaluate which model is the most accurate and therefore describes the best the beta temporal evolution, we use time-series, as well as cross-sectional tests. The first test, using the in-sample MSE criterion, finds that the Kalman filter approach provides the most accurate estimates. Note that this model is estimated with an autoregressive transition equation but the vast majority of the autoregressive coefficients are first order integrated and therefore follow a random walk. These results confirm those previously found in the literature on other markets like the UK or Australia. This is the first important result of this paper since this issue has not been examined on the US market so far.

However the major interest of this paper lies in the investigation of the impact of the beta specification in the cross-sectional tests of the conditional CAPM. Our evidence shows that the Kalman filter with random walk betas is again the best specification from this point of view. Indeed, this is the only of our models which supports the validity of the CAPM. First, the relationship between portfolios returns and their betas is statistically significant and furthermore the constant is, on average, not different from zero, meaning that there is no other common variable able to explain portfolios returns. All others models, including the widely used rolling regression, fail to prove empirically the beta-return relationship. Our results call for more research in this area. In particular it would be interesting to investigate whether the SMB and HML risk factors from FAMA and FRENCH (1993) are still significant when betas are generated by the Kalman filter with a random walk specification. This is left for future research.

References

BLUME M., 1971, On the assessment of risk, *The Journal of Finance*, 26, 1-10.

BLUME M., 1975, Betas and their regression tendencies, *The Journal of Finance*, 30, 786-795.

BOS T., P. NEWBOLD, 1984, An empirical investigation of the possibility of systematic stochastic in the market model, *The Journal of Business*, 57, 35-41.

BRAUN, P.A., D.B. NELSON and A.M. SUNIER, 1995, Good news, bad news, volatility and betas, *The Journal of Finance,* 50, 1575-1603.

BROOKS, R.D., FAFF, R. W. and J.H.H. LEE, 1992, The form of time variation of systematic risk: some australian evidence, *Applied Financial Economics*, 2, 191–98.

BROOKS R., FAFF R., M. MACKENZIE, 1998, Time varying beta risk of Australian industry portfolios: a comparison of modelling techniques, *Australian Journal of Management*, 23.

CAMPBELL J., T. VUOLTEENAHO, 2004, Bad beta, good beta, *American Economic Review*, 94, 1249-1275 .

CHO Y., R. ENGLE, 2000, Time varying betas and asymmetric effects of news: empirical analysis of blue chip stocks, *NBER Working Papers,* 7330.

EBNER M., T. NEUMANN, 2005, Time-varying betas of German stock returns, *Financial Market and Portfolio Management*, 19, 29-46.

FABOZZI F., J. FRANCIS, 1977, Stability tests for alphas and betas over bull and bear market conditions, *The Journal of Finance*, 32, 1093-1099.

FAFF R., R. BROOKS, 1998, Time varying beta risk for Australian industry portfolios: An exploratory analysis, *The Journal of Business Finance and Accounting, 25, 721-745.*

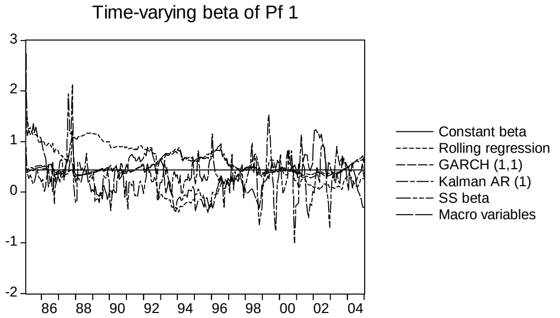
FAFF R., HILLIER D., J. HILLIER, 2000, Time varying beta risk: An analysis of alternative modelling techniques, *The Journal of Business Finance and Accounting*, 27, 523-554.

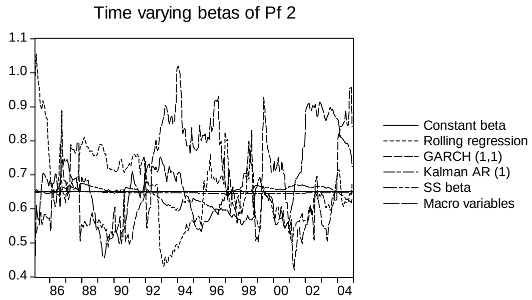
FAMA E., K. FRENCH, 1992, The cross-section of expected stock returns, *The Journal of Finance*, 47, 427-465.

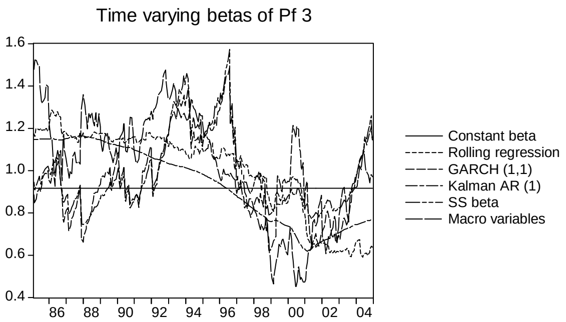
FAMA E., K. FRENCH, 1993, Common risk factors in the returns of stocks and bonds, *The Journal of Financial Economics,* 33, 3-56.

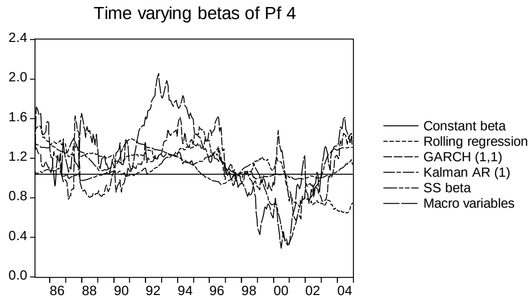
FAMA E., J. MACBETH, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy*, 81, 607-636.
FERSON W., C. HARVEY, 1991, The variation of economic risk premium, *The Journal of Political Economy*, 99, 385-415.
FERSON W., C. HARVEY, 1999, Conditioning variables and the cross section of stock returns, *The Journal of Finance*, 4, 1325-1360.
GROENEWOLD N., P. FRASER, 2000, Forecasting beta: How well does the "five-year rule of thumb" do?, *The Journal of Business Finance and Accounting*, 27, 953-982.
KOUTMOS G., LEE, U. & THEODOSSIOU, P. 1994, Time varying betas and volatility persistence in international stock markets, *The Journal of Economics and Business*, 46, 101–12.
LINTNER J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *The Review of Economics and Statistics*, 47, 13-37.
REYES M., 1999, Size, time-varying beta, and conditional heteroscedasticity in UK stock returns, *Review of Financial Economics*, 8.
SHARPE W., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *The Journal of Finance*, 19, 425-442.
SCHWERT G.W., P. SEGUIN, 1990, Heteroskedasticity in stock returns, *The Journal of Finance*, 45, 1129-1155.
LI X., 2001, On unstable beta risk and its modelling techniques for New Zealand industry portfolio, *Massey University Working Paper NZ.*

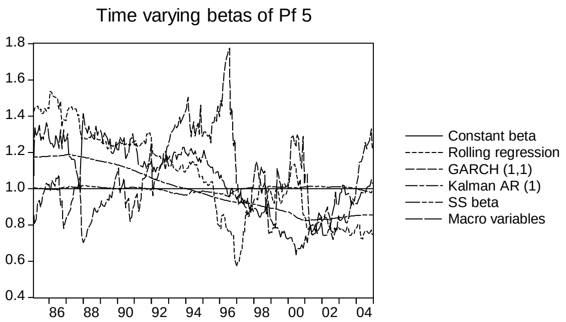
Appendix 1: Evolution of the time-varying beta according the various methods

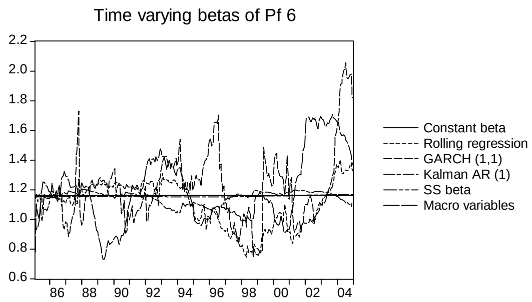


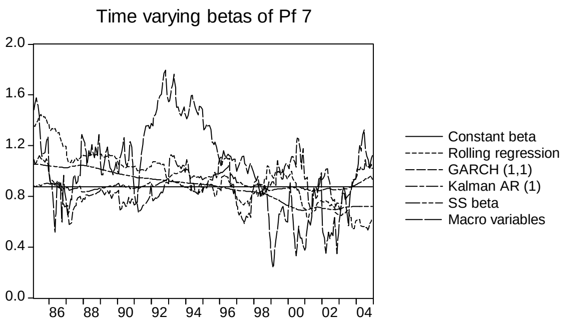


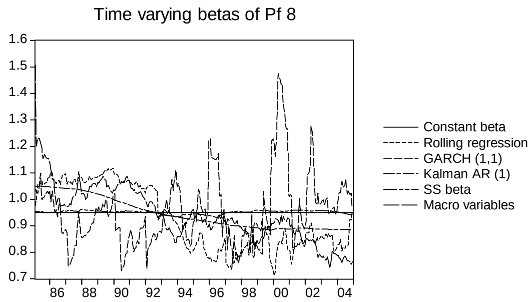




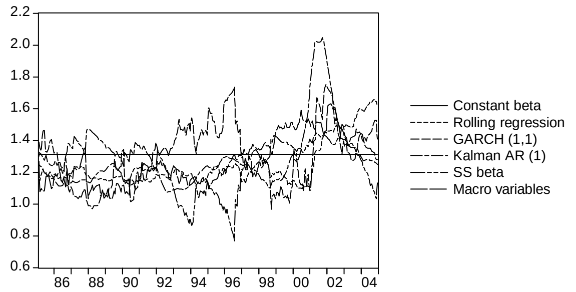




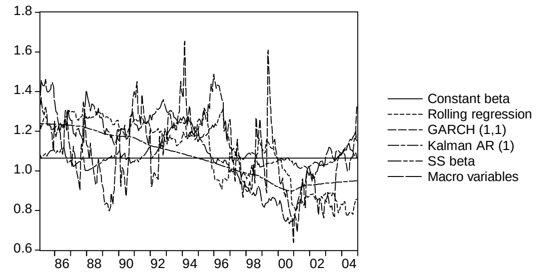




Time varying betas of Pf 9



Time varying betas of Pf 10



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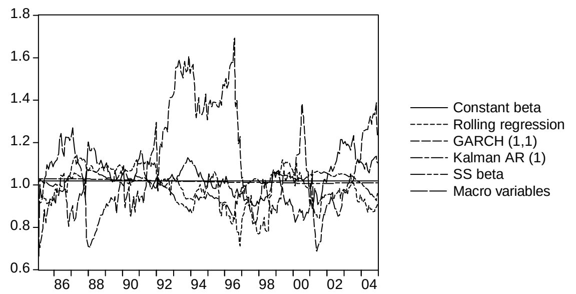

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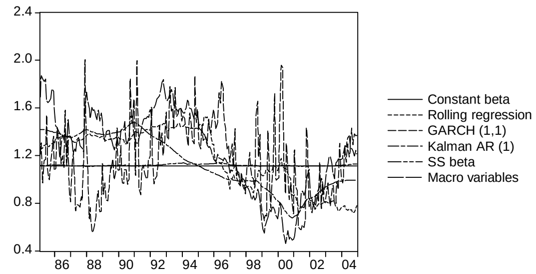

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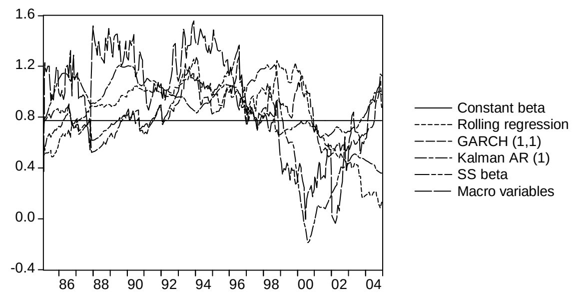
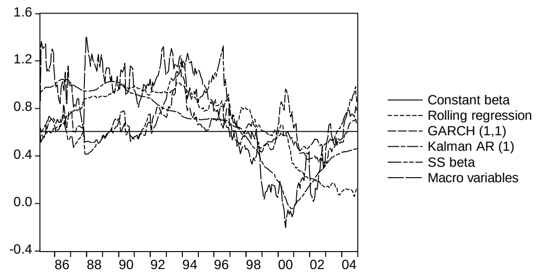


Time varying betas of Pf 14



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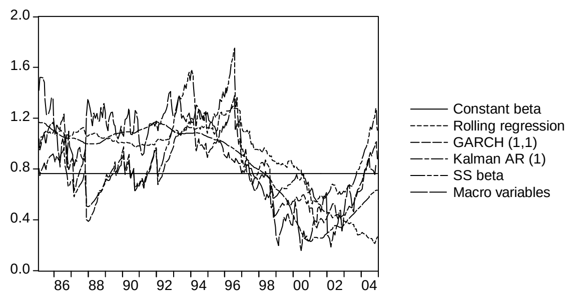

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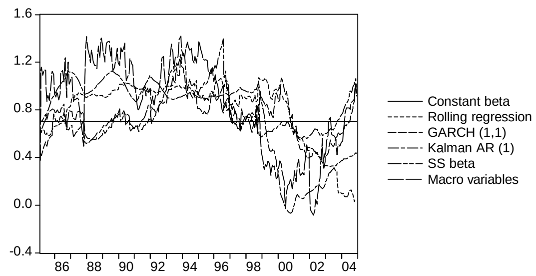
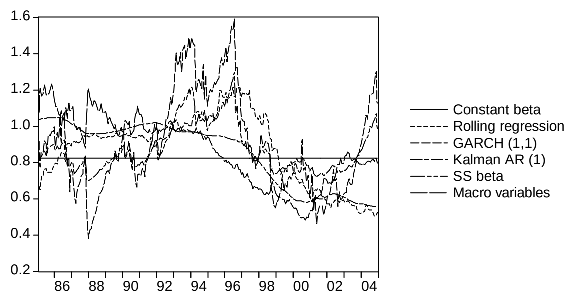
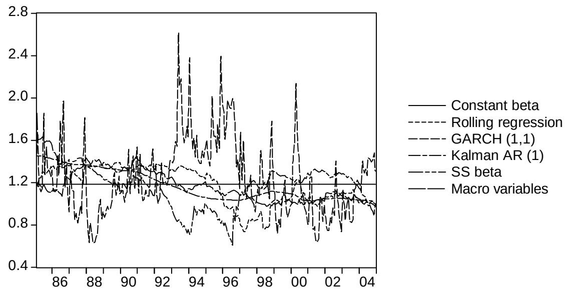


Time varying betas of Pf 17

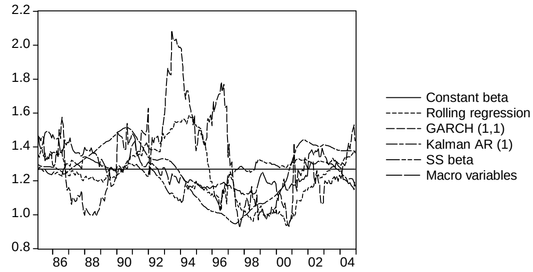


Time-varying betas of Pf 18

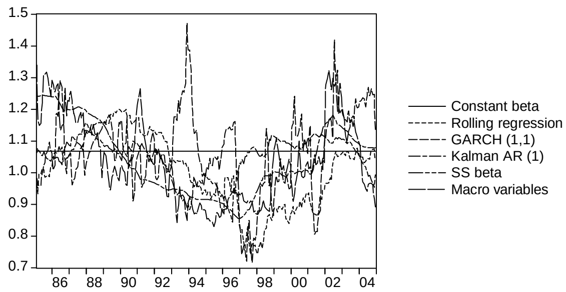
Time varying betas of Pf 19



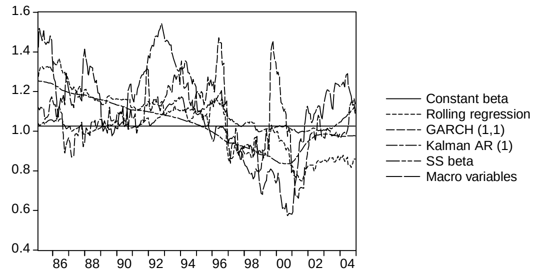
Time varying betas of Pf 20



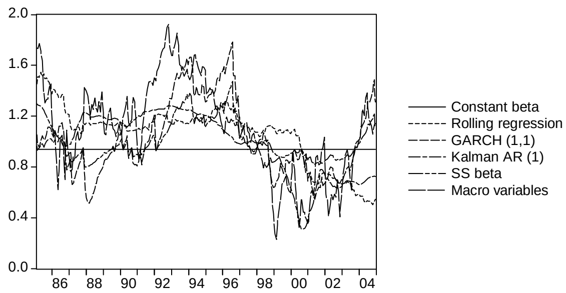
Time varying betas of Pf 21



Time varying betas of Pf 22

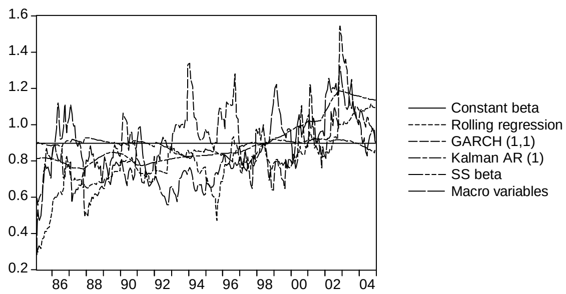


Time varying betas of Pf 23



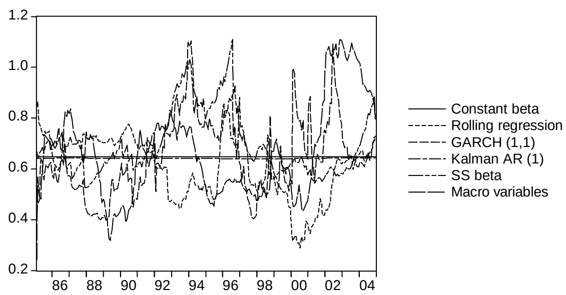
Time varying betas of Pf 24

Time varying betas of Pf 25

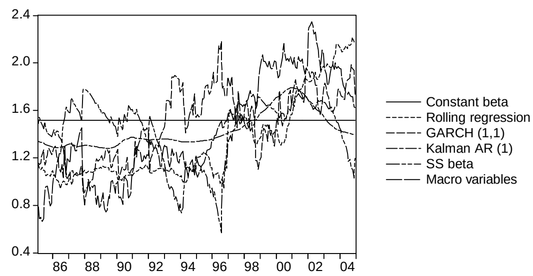


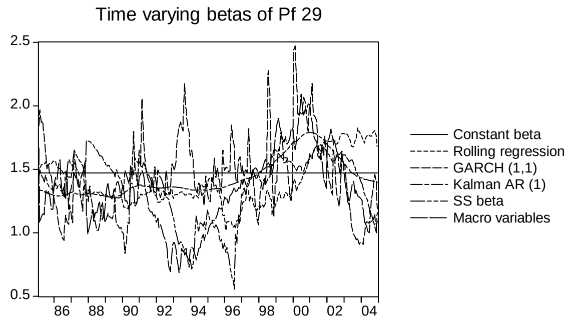
Time varying betas of Pf 26

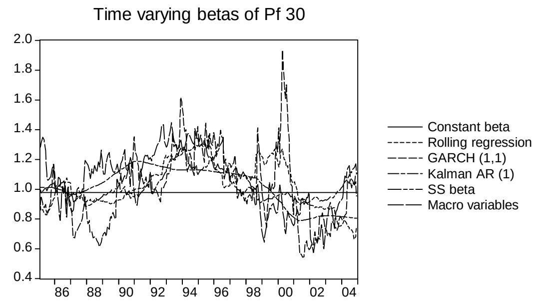
Time varying betas of Pf 27

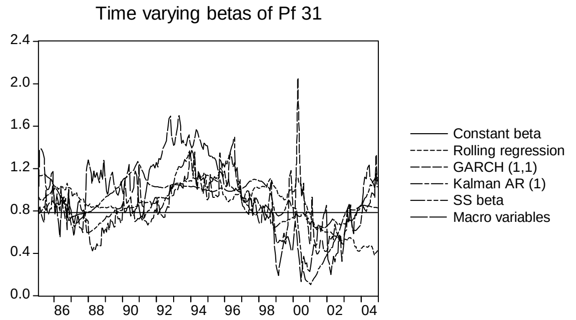


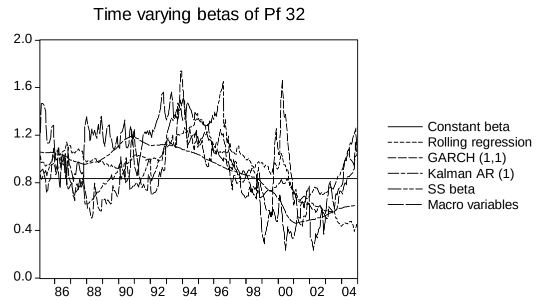
Time varying betas of Pf 28

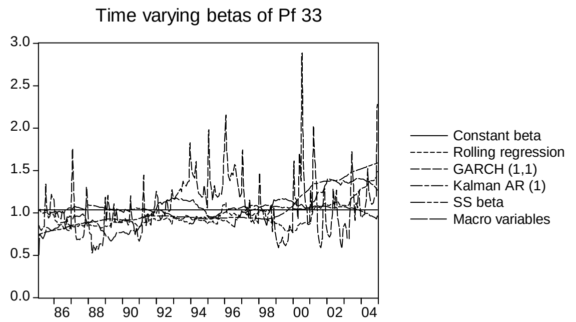


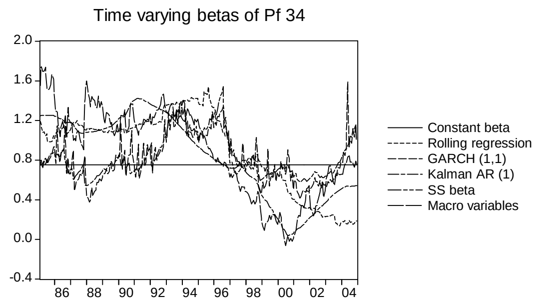


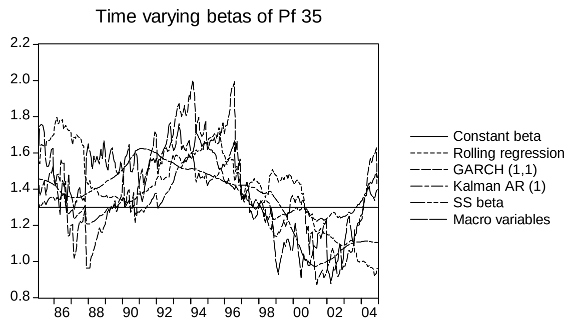












Note: The asymmetric beta is not represented on this figure because it can only take 2 different values and therefore, it is not suitable for a time-series graph.

Appendix 2: Conditional variance coefficients for the GARCH (1, 1) model

Pf	*ARCH* *Coefficient (b_i)*	*GARCH* *Coefficient (c_i)*	*Sum* *(b_i+c_i)*	*Correlation* ρ_{im}
1	0.057	**0.785**	0.842	0.187
2	0.103	**0.843**	0.946	0.545
3	0.048	**0.907**	0.955	0.726
4	0.107	**0.780**	0.887	0.689
5	**0.062**	**0.889**	0.951	0.640
6	**0.085**	**0.894**	0.979	0.605
7	**0.102**	**0.875**	0.977	0.667
8	**0.101**	**0.883**	0.984	0.747
9	0.108	**0.845**	0.953	0.875
10	**0.174**	**0.773**	0.948	0.752
11	0.071	**0.872**	0.943	0.688
12	**0.428**	0.161	0.589	0.783
13	0.079	**0.879**	0.958	0.596
14	**0.101**	**0.861**	0.961	0.560
15	-0.018	**1.006**	0.988	0.701
16	0.111	**0.836**	0.947	0.605
17	0.050	-0.475	-0.424	0.675
18	0.059	**0.827**	0.886	0.399
19	0.362	0.224	0.586	0.784
20	0.100	**0.832**	0.932	0.818
21	**0.101**	**0.887**	0.988	0.854
22	**0.104**	**0.856**	0.961	0.794
23	0.046	0.224	0.270	0.733
24	**0.097**	**0.898**	0.996	0.621
25	**0.139**	**0.805**	0.944	0.677
26	**0.118**	**0.824**	0.942	0.338
27	**0.124**	**0.823**	0.947	0.512
28	**0.104**	**0.857**	0.961	0.764
29	**0.222**	**0.747**	0.968	0.779
30	**0.203**	**0.672**	0.875	0.738
31	0.279	**0.373**	0.652	0.653
32	**0.202**	**0.626**	0.828	0.640
33	**0.433**	0.220	0.653	0.725
34	**0.201**	-0.037	0.164	0.603
35	0.059	**0.892**	0.951	0.858
marché	**0.119**	**0.871**	0.990	1

Note: This table show the estimation results of the equation (5) for the portfolios and (6) for the market over the whole period (1985-2005). In the first two columns, values in bold are significant at the usual 5% level. To have a finite unconditional variance, the sum of the two coefficients, presented in the third column, must be smaller than unity.

Appendix 3: SS beta regression coefficients

Pf	***δ_i x 10^{-4}***	β_i	***Pf***	***δ_i x 10^{-4}***	β_i
1	3.39	0.27	19	-3.89	**1.40**
2	-0.48	**0.68**	20	-1.63	**1.36**
3	**4.33**	**0.69**	21	-1.51	**1.15**
4	1.77	**0.95**	22	1.15	**0.97**
5	-0.32	**1.02**	23	3.53	**0.75**
6	-0.99	1.22	24	1.97	**0.64**
7	1.10	**0.82**	25	-0.69	**0.94**
8	-0.16	**0.96**	26	3.53	**0.75**
9	**-3.66**	**1.52**	27	3.05	**0.49**
10	1.65	**0.98**	28	**-6.33**	**1.86**
11	-1.22	**1.09**	29	**-6.12**	**1.80**
12	0.18	**1.11**	30	2.42	**0.85**
13	3.94	**0.57**	31	**4.72**	**0.53**
14	**4.79**	**0.35**	32	**5.42**	**0.55**
15	**6.52**	**0.42**	33	-1.37	**1.11**
16	**4.59**	**0.46**	34	**5.22**	**0.47**
17	3.12	**0.66**	35	2.40	**1.17**
18	**8.28**	0.32			

Note: The coefficients are estimated according to equation (12) over the whole period, from 1985 to 2005. The coefficients in bold are significant at the usual 5% level. They are multiplied by 10^4.

Appendix 4: Regression of the macroeconomic variables on the portfolios returns

Pf	***T-Bill***	***Div***	***Junk***	***Term***	R^2
1	-11.49	1.17	43.81	-9.58	0.02
2	*-10.26*	*1.45*	-0.33	*-18.42*	0.02
3	**-12.24**	**2.04**	10.86	*-18.36*	0.03
4	-11.83	1.70	13.94	-14.26	0.02
5	-13.86	1.80	25.47	-20.20	0.02
6	-14.24	1.00	29.00	-23.56	0.02
7	-6.92	1.41	-14.46	-10.88	0.01
8	-9.47	**2.08**	-14.98	*-18.80*	0.04
9	*-13.60*	**2.16**	-10.96	-26.55	0.03
10	**-13.36**	*1.83*	11.15	*-18.63*	0.02
11	**-15.44**	**2.40**	-1.86	*-22.52*	0.03
12	**-13.88**	**2.31**	20.08	*-21.15*	0.04
13	-5.84	**1.77**	2.92	-15.09	0.04
14	-3.80	**1.26**	2.70	-10.02	0.03
15	-7.65	**1.69**	-4.52	**-18.66**	0.04
16	*-10.86*	**2.08**	-4.19	*-18.44*	0.03
17	-8.18	**1.99**	-6.87	**-25.17**	0.07
18	-10.21	1.70	28.84	**-30.41**	0.03
19	-11.21	*1.99*	9.73	*-21.93*	0.02
20	**-14.47**	**2.76**	6.18	*-22.26*	0.04
21	**-12.42**	**1.98**	1.24	**-23.04**	0.03
22	**-12.01**	**2.87**	-23.09	-16.12	0.06
23	-9.72	**1.80**	-1.87	-14.94	0.02
24	*-10.33*	**2.07**	-10.11	**-22.83**	0.04
25	**-13.60**	**2.91**	*-33.63*	**-23.17**	0.06
26	2.76	0.27	-17.34	1.72	0.02
27	-7.68	**1.90**	**-41.07**	*-16.76*	0.05
28	**-22.23**	**3.40**	-32.58	**-33.17**	0.03
29	**-18.26**	**3.28**	-14.23	**-30.25**	0.03
30	-1.76	0.73	-2.15	-3.12	0.01
31	2.07	0.50	-16.40	0.09	0.02
32	1.52	0.34	-8.32	0.24	0.01
33	**-16.93**	**2.69**	0.27	*-21.73*	0.04
34	-2.50	0.50	16.09	1.58	0.02
35	-6.53	1.43	-3.49	-14.06	0.01
marché	**-11.94**	**2.12**	-10.07	**-19.28**	0.04

Note: The coefficients are obtained by the estimation of the following equation:
$r_{it} = \alpha_i + \delta_i \cdot T\text{-}Bill_{t-1} + \lambda_i \cdot Div_{t-1} + \kappa_i \cdot Junk_{t-1} + \gamma_i \cdot Term_{t-1} + \varepsilon_{it}$. Values in bold are significant at the 5% level and the ones in italic at the 10% level. The lagged macroeconomic variables are in monthly frequency, except for the dividend yield (Div) in a yearly frequency.