# Is There a Latent Factor in Stock Returns?

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**Abstract**

*The measurement problems encountered while trying to exhibit the influence of market risk factor on asset returns may be numerous. It seems then difficult to highlight the unique common latent factor underlying stock return evolutions in the market. So far, excess return relationships are mainly and broadly considered. Moreover, basic and common studies require a market factor proxy (i.e., market portfolio benchmark). The chosen proxy usually impacts related results (see Roll [1977]). To bypass such problems, we resort to Kalman filtering methodology to exhibit the common latent factor underlying stock market returns. Of course, when this one exists...*

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**JEL Codes:** C32,D8.

## 1 Introduction

The stock return puzzle has a long story. Indeed, many authors attempted to explain the global evolution of stock returns. Formerly, Sharpe

(1963,1964), among others, established that stock returns depend on both a market factor as well as an idiosyncratic factor of risk. Such a dependence is usually described by some linear-type relationship. Later, the influence of market variables (e.g., risk free rate, term spread, and yield curve slope) as well as idiosyncratic factors on stock returns is exhibited (see Fama & French [1989,1992], Campbell [1987], Harvey [1989], Breen *et al.* [1989], and Ferson & Harvey [1981,1999]). Specifically, Banz (1981), Berk (1995) and Kothari *et al.* (1995) show the importance of firm size whereas Bhandari (1988) underlines the leverage effect on asset returns. Chan *et al.* (1991) explain stock returns with book-to-market features while Merton (1987) and Amihud & Mendelson (1989) exhibit the informational impact (i.e., news arrival in the market) on stock returns and related liquidity. Recently, Malkiel & Xu (2002,2003) focus on idiosyncratic risk and volatility in asset return. In the same line, Campbell *et al.* (2001) exhibit the importance and significance of idiosyncratic risk in asset returns. They find that though idiosyncratic volatility has highly grown over time, stock return global volatility remains driven by market volatility (i.e., global common trend).

However, such linear relationship between stock returns and both market factors as well as idiosyncratic factors suffers from many measurement problems (e.g., heteroskedasticity and autocorrelation; see Fama [1965,1976], Blattberg & Gonedes [1974], and Affleck-Graves & McDonald [1989] among others) leading to a biased explanation of stock return evolution. To solve such problems, some authors resort to specific econometric tools or methods. For example, Shanken (1992) and Jagannathan & Wang (1996) propose a GMM methodology solving the error-in-variables problem. Differently, Ahn & Gadarowski (2000) propose an estimation method, which is robust to conditional heteroskedasticity as well as autocorrelations in asset returns. Recently, Barnes & Hughes (2002) propose a quantile regression methodology (see Buchinsky [1998]), which is robust to error-in-variables bias, omitted variables bias, sensitivity to outliers, and non-normal error distributions. Those authors find results that lead to a rejection of both the unconditional single-factor CAPM and the conditional multi-factor CAPM. More recently, Koutmos & Knif (2002) use conditional time-varying distributions (i.e., GARCH modeling) to assess the influence of systematic risk on stock returns. They consider given market stock indices, and exhibit stationary mean-reverting beta CAPM parameters with a four-day persistence degree. Differently, Gençay *et al.* (2003) use multiscaling wavelet techniques to estimate stock return beta parameters while using the S&P 500 index as a market portfolio (i.e., systematic risk factor proxy).

Given existing literature, it seems sometimes hard to exhibit the existence of one significant common latent factor in asset returns. Moreover, a market index is always required to proxy the actual market factor of risk. The quality of the chosen market benchmark impacts the accuracy as well as quality or reliability of related measurements (see Roll [1977]). To bypass such problems and open questions, we resort to a robust econometric method to exhibit the latent factor of risk common to any asset in the market. For this purpose, we employ a Kalman filtering methodology, which allows to leave the market factor of risk undetermined (i.e., endogenous to the estimation process). Our paper is then organized as follows. Section 2 introduces the Kalman filter and related EM estimation. Section 3 introduces the data under consideration as well as their statistical properties while section 4 employs Kalman econometric method under our financial framework. Specifically, we consider both US and French data samples. Further investigation is undertaken in section 5 while investigating a common component in both French and US common latent factors. Finally, section 6 draws some concluding remarks and open points for future research.

# 2 Econometric framework

We expose therein the usefulness of the chosen econometric framework, namely Kalman filter, given our working setting as well as related advantages. Then, we introduce the general econometric estimation process.

## 2.1 Principle and motivations

The Kalman filter (see Kalman [1960], Harvey [1989a,b], Meinhold & Singpurwalla [1983], Brown & Hwang [1997], Wikle & Cressie [1999], and Cressie & Wikle [2002]) is commonly employed for short term forecasting as well as time series analysis or estimation. This simple econometric method is known to be optimal (i.e., unbiased and minimum error variance algorithm) and robust. The principle is to establish the state, or equivalently, linear dynamic of a given system, and to link such a dynamic to available or observed information about the system at each point of time. Solving such a dependence structure depends on the initial sate of the system (i.e., detailed or accurate information about the initial state of the system is required).

Kalman filter is a state-space model describing a system's state as well as its evolution over time. Incidentally, a state-space representation allows for incorporating unobserved variables (i.e., state variables), which are estimated with the observable model (i.e., observed variables or measures). Specifically, Kalman filter is a recursive linear predictor-corrector filter, which minimizes the expected square error between the system's state and corresponding estimate(s) (i.e., quadratic minimization algorithm). For Gaussian random variables, Kalman filter represents the optimal linear predictor and estimator. For non-Gaussian variables, Kalman filter estimator is the best one among the linear estimator class. The main interest of this econometric methodology is its ability to forecast a system's state through past, present, and future. In general, observed measures are functions of state variables (i.e., state of the system) insofar as measures are disturbed by a random noise called measurement noise. Hence, Kalman filter attempts to estimate state variables given disturbed observations about the system. Such a forecasting process relies on two sets of equations. The first set of equations is time-updating, and forecasts the system's current state as well as the related error covariance matrix over the next time step. The second set of equations is measure-updating, and corrects the errors committed in the first set of equations (see Chui & Chen [1987]). For this purpose, second order moments of equation noises (i.e., state and measurement noises) are required.

Finally, Kalman filtering methodology exhibits five advantages (see Lemoine & Pelgrin [2003] among others). First, measure uncertainty is recursively taken into account. Second, ex ante information is taken into account when this one exists. Third, this econometric method can be applied to stationary as well as non-stationary data. Fourth, state and measurement noises can be non-Gaussian. Finally, time-varying estimates are enabled.

In this paper, we use Kalman methodology for filtering purpose; namely we look for the best proxy of the current system's state given past and present observations.

## 2.2 General framework

We introduce here the general modeling framework where state variables are assumed to follow a first order Markovian process. Namely, we consider following linear measurement and state equations:[1]

$$Y_t = Z_t X_t + D_t + \varepsilon_t \tag{1}$$

$$X_t = A_t X_{t-1} + C_t + R_t \eta_t \tag{2}$$

where $Y_t$ is a $N \times 1$ vector of observations (i.e., measure variables); $Z_t$ is a $N \times k$ measurement sensitivity matrix; $X_t$ is a $k \times 1$ vector of state variables;[2] $D_t$ is a $N \times 1$ vector related to exogenous known variables; $\varepsilon_t$ is a $N \times 1$ measurement white noise; $A_t$ is a $k \times k$ state transition matrix; $C_t$ is a $k \times 1$ vector related to exogenous known variables; $R_t$ is a $k \times g$ matrix; $\eta_t$ is a $g \times 1$ state white noise; and finally $t$ is current time ranging from 1 to $T$ (i.e., multivariate time series with $T$ observations).

We first assume that state and measurement noises follow normal distributions and are independent. Second, initial values of state variables and white noises are independent[3] (i.e., causal and invertible state-space model) while initial values of state variables follow a normal law. Namely, we consider:[4]

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \backsim \mathcal{N}\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q_t & \mathbf{0} \\ \mathbf{0} & H_t \end{bmatrix} \right) \tag{3}$$

$$X_0 \backsim \mathcal{N}\left(m_0, P_0\right) \tag{4}$$

where $m_0$ and $P_0$ are known expectation and covariance matrix parameters of dimensions $k \times 1$ and $k \times k$ respectively. The recursive nature of Kalman filter implies that state and measure variables are functions of the initial system's state, past state errors, past measurement errors, and exogenous variables. Hence, Kalman principle is to estimate state variables at each time $t$ conditional on observed variables (i.e., measure variables) until time $t$. Specifically, minimizing realized square errors on state variables requires five steps in the estimation process. These five steps are divided into an updating and a forecasting stage as follows:

$$E_{t-1}\left[X_t\right] = A_t E_{t-1}\left[X_{t-1}\right] + C_t \tag{5}$$

---

[1] For any given observed variable, there exist several possible state-space representations.

[2] $Z_t X_t$ is considered as a signal at current time $t$.

[3] Namely, we assume that $E\left[\varepsilon_t \eta_t'\right] = E\left[\varepsilon_t X_0'\right] = E\left[\eta_t X_0'\right] = 0$.

[4] At initial time $t_0 = 0$, hidden variables $X_0$ (i.e., latent common and idiosyncratic factors, or equivalently, unobserved factors) are Gaussian.

$$Var_{t-1}[X_t] = A_t P_{t-1} A_t' + R_t Q_t R_t' \tag{6}$$
$$E_{t-1}[Y_t] = Z_t E_{t-1}[X_t] + D_t \tag{7}$$
$$v_t = Y_t - E_{t-1}[Y_t] \tag{8}$$
$$F_t = Z_t Var_{t-1}[X_t] Z_t' + H_t \tag{9}$$
$$E_t[X_t] = E_{t-1}[X_t] + K_t v_t \tag{10}$$
$$P_t = (\boldsymbol{I}_k - K_t) \times Var_{t-1}[X_t] \tag{11}$$
$$K_t = Var_{t-1}[X_t] Z_t' F_t^{-1} \tag{12}$$

where $E_t[\cdot]$ and $Var_t[\cdot]$ are expectation and covariance operators conditional on available information set at time $t$; $P_t = Var_t[X_t]$ is the mean quadratic error on $Z_t$; $K_t$ is the Kalman gain matrix; $\boldsymbol{I}_k$ is the identity matrix of dimension $k$; $Z_t'$ is the transposition of matrix $Z_t$; $F_t$ is the covariance matrix of $v_t$; $F_t^{-1}$ is the inverse matrix of $F_t$; and $v_t$ is an innovation process. Notice that $E_{t-1}[X_t]$ and $Var_{t-1}[X_t]$ are the best estimates of $X_t$ and $P_t$ conditional on available information set at time $t-1$. Analogously, $E_t[X_t]$ is an optimal estimate of $X_t$ given available information and observations at time $t$. Moreover, relations (11) and (6) are covariance matrix equations, namely Ricatti equations allowing for the computation of Kalman gain series. Relations (10) and (11) deal with state estimate and related covariance matrix updating. Relations (5) and (6) concern forecasting (i.e., time updating). Relation (12) is the gain matrix update; incorporating this matrix in relation (11) increases the estimation accuracy of $E_t[X_t]$ relative to $E_{t-1}[X_t]$. Indeed, the state error covariance matrix represents a state uncertainty estimate. By the way, $Var_{t-1}[X_t]$ is an ex ante covariance matrix while $Var_t[X_t]$ is an ex post covariance matrix. And, $Var_{t-1}[X_t]$ is the mean quadratic error of forecast $E_{t-1}[X_t]$.

Kalman filter requires to specify starting values for state variables (i.e., initial guess) and to replace unknown matrices with their estimates. Given starting values, unknown matrix parameters are estimated while maximizing $Y_t$ log-likelihood. For this purpose, we assume that $Y_t$ follows a multivariate Gaussian distribution conditional on $X_t$ as well as past values of both $X_t$ and $Y_t$. Specifically, the log-likelihood under normality assumptions writes:

$$\ell_t = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln|F_t| - \frac{1}{2}v_t' F_t^{-1} v_t \tag{13}$$

where $|F_t|$ is the determinant of matrix $F_t$. General setting leads to a non stationary (i.e., time-varying) estimation framework whereas we get a stationary case when $Z_t$, $D_t$, $H_t$, $A_t$, $C_t$, $R_t$, and $Q_t$ do not depend on time.

Table 1: Asset denomination

| **France** | **USA** |
|---|---|
| ACCOR | AT & T |
| ALCATEL ALSTOM | DJIA |
| AXA | DOW JONES |
| BOUYGUES | FORD MOTOR |
| L'OREAL | INTL.BUS.MACH. (IBM) |
| MICHELIN | MERRILL LYNCH |
| PEUGEOT SA | MICRON TECH. |
| SBF120 | MICROSOFT |
| TOTAL FINA ELF SA | WALT DISNEY |

# 3 Data and properties

We introduce the data we consider as well as a preliminary statistical analysis.

## 3.1 Data sets

We consider two different data sets (i.e., two different country analyses). The first set concern 8 French stock prices and one French stock index price ranging from 01/02/1997 to 07/12/2001, namely 1139 observations per series (see table 1 where DJIA is the Dow Jones Average Industrial index and SBF120 is a diversified French stock index). The second set concern 8 US stock prices and one US stock index price ranging from 01/02/1997 to 07/12/2001, namely 1142 observations per series. We also consider the global set of 18 asset prices also ranging from 01/02/1997 to 07/12/2001, namely 1111 observations per series after adjusting for non-working day differences.

As we are interested in the common latent component underlying asset return evolutions, we compute asset returns on a continuous basis as follows:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \approx \frac{S_t - S_{t-1}}{S_{t-1}} \tag{14}$$

where $S_t$ is the asset price at time $t$. Hence, we consider 1138 return observations for French assets, 1141 return observations for US assets, and finally 1110 return observations for the merged global data set.

## 3.2 Statistical profiles

We consider asset returns on a percentage basis. Our three return data sets exhibit some key statistical features (see tables 2, 3 and 17[5]).

Roughly, speaking French and US stock returns are far from being normally distributed. Those returns exhibit both asymmetry and skewness features (i.e., skewed probability distributions). Related probability distributions exhibit fatter tails than Gaussian ones (see related skewness). Moreover, their positive kurtosis profile is quite very heterogeneous among stock returns. The most volatile French stock returns (i.e., in terms of distance between extreme values, or equivalently, minimum and maximum values) are Alcatel, Axa and Total ones whereas the most volatile US stock returns are Ford Motor and IBM ones. Same conclusions apply to table 17, except that IBM is no more a highly volatile stock return whereas Dow Jones stock return becomes very volatile. Moreover, Kendall's correlation coefficients between asset returns for each financial market exhibit a strong positive link. Indeed, considering each market separately, the obtained non-linear correlation coefficients are significant at a 1% bilateral test level (see tables 4 and 5).

# 4 Econometric study

We expose and explain the relevant version of Kalman filtering given our framework as well as related econometric results.

## 4.1 Model

We only observe stock return data whereas each stock return is driven by both a common latent risk factor as well as an idiosyncratic risk factor. Hence, each stock evolution depends on two unobserved variables (see Sharpe [1963,1964]). Consequently, we employ Kalman statistical method to describe the dynamics of both latent and idiosyncratic factors insofar as we have incomplete knowledge about the relevant phenomenon underlying those dynamics (i.e., hidden statistical variables).

For each stock return $i$, we set the following dependence structure:

$$R_t^i = \beta^i M_t + e_t^i \tag{15}$$

[5] This table is exposed in the appendix.

Table 2: French asset return statistics

| | $i$ | Mean | Stand. Dev. | Skewness | Excess kurtosis | Min. | Max. | Median | 1st quartile* | 3rd quartile* |
|---|---|---|---|---|---|---|---|---|---|---|
| Accor | 2 | 0.0909 | 2.4188 | -0.0983 | 2.1469 | -14.7809 | 10.7692 | 0.0617 | -1.3622 | 1.4590 |
| Alcatel | 3 | 0.0704 | 3.6228 | -1.9734 | 28.5615 | -48.4564 | 14.4352 | 0.0000 | -1.7712 | 2.0603 |
| Axa | 4 | 0.1537 | 2.4601 | 5.3827 | 98.2156 | -9.9142 | 45.1151 | 0.1328 | -1.0800 | 1.2579 |
| Bouygues | 5 | 0.1642 | 2.9474 | 0.0745 | 2.6521 | -17.1909 | 14.0123 | 0.0000 | -1.4303 | 1.7247 |
| L'Oréal | 6 | 0.0909 | 2.4025 | 0.0530 | 1.0562 | -10.0285 | 9.2622 | 0.0000 | -1.4438 | 1.5263 |
| Michelin | 7 | 0.0004 | 2.3254 | -0.0588 | 2.2524 | -11.3445 | 11.8360 | 0.0000 | -1.2826 | 1.2242 |
| Peugeot | 8 | 0.1106 | 2.2944 | -0.2363 | 3.4715 | -16.3259 | 10.4635 | 0.0000 | -1.1462 | 1.3547 |
| SBF120 | 1 | 0.0671 | 1.3292 | -0.2569 | 1.2001 | -5.3336 | 5.9459 | 0.0861 | -0.6712 | 0.9181 |
| Total | 9 | 0.1813 | 3.1092 | 10.3303 | 230.6055 | -13.1709 | 70.5330 | 0.1002 | -1.3007 | 1.5753 |

* Upper bound of the quartile.

Table 3: US asset return statistics

| | $i$ | Mean | Stand. Dev. | Skewness | Excess kurtosis | Min. | Max. | Median | 1st quartile* | 3rd quartile* |
|---|---|---|---|---|---|---|---|---|---|---|
| AT & T | 6 | 0.0096 | 2.8911 | 0.2564 | 9.9905 | -23.2620 | 22.1301 | 0.0000 | -1.6201 | 1.4453 |
| DJIA | 1 | 0.0209 | 2.4023 | -0.0922 | 5.1539 | -16.9523 | 14.2029 | 0.0000 | -1.3585 | 1.2820 |
| Dow Jones | 8 | 0.0426 | 1.2074 | -0.3804 | 3.1491 | -7.4549 | 4.8605 | 0.0650 | -0.6428 | 0.7991 |
| Ford Motor | 9 | 0.0145 | 1.8777 | -1.4286 | 17.3635 | -21.4531 | 8.7601 | 0.0000 | -0.9509 | 0.9558 |
| IBM | 2 | 0.0142 | 2.6071 | -2.7651 | 43.1666 | -38.6561 | 10.6264 | 0.0000 | -1.2807 | 1.3866 |
| Merrill Lynch | 7 | 0.0749 | 2.5578 | -0.1300 | 4.7890 | -16.8916 | 12.3665 | 0.0936 | -1.4402 | 1.4534 |
| Micron Tech. | 4 | 0.1081 | 3.1734 | 0.1772 | 1.3117 | -12.2978 | 14.0477 | 0.0000 | -1.8919 | 2.0379 |
| Microsoft | 5 | 0.1180 | 4.8077 | 0.1636 | 0.9410 | -19.1160 | 21.7202 | 0.0000 | -3.0511 | 3.0750 |
| Walt Disney | 3 | 0.1204 | 2.7225 | -0.1585 | 4.6285 | -16.9577 | 17.8692 | 0.0290 | -1.4069 | 1.6963 |

* Upper bound of the quartile.

Table 4: Kendall's correlation matrix for French asset returns

| | $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 1 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Accor | 2 | 1.0000 | 0.2042 | 0.1961 | 0.1128 | 0.1802 | 0.1790 | 0.1965 | 0.3012 | 0.1731 |
| Alcatel | 3 | | 1.0000 | 0.2576 | 0.2684 | 0.2296 | 0.1809 | 0.1776 | 0.5007 | 0.1795 |
| Axa | 4 | | | 1.0000 | 0.1464 | 0.2965 | 0.2229 | 0.2136 | 0.4364 | 0.1838 |
| Bouygues | 5 | | | | 1.0000 | 0.1423 | 0.0867 | 0.1426 | 0.3304 | 0.1225 |
| L'Oréal | 6 | | | | | 1.0000 | 0.2127 | 0.2038 | 0.4302 | 0.1779 |
| Michelin | 7 | | | | | | 1.0000 | 0.1971 | 0.2886 | 0.1396 |
| Peugeot | 8 | | | | | | | 1.0000 | 0.3059 | 0.1772 |
| SBF120 | 1 | | | | | | | | 1.0000 | 0.3424 |
| Total | 9 | | | | | | | | | 1.0000 |

Table 5: Kendall's correlation matrix for US asset returns

| | $i$ | 6 | 1 | 8 | 9 | 2 | 7 | 4 | 5 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|
| AT & T | 6 | 1.0000 | 0.2509 | 0.1602 | 0.1167 | 0.1539 | 0.1942 | 0.1189 | 0.1588 | 0.1528 |
| DJIA | 1 | | 1.0000 | 0.2702 | 0.3273 | 0.3686 | 0.4232 | 0.1831 | 0.3381 | 0.3056 |
| Dow Jones | 8 | | | 1.0000 | 0.1570 | 0.1280 | 0.1974 | 0.0849 | 0.1247 | 0.1448 |
| Ford Motor | 9 | | | | 1.0000 | 0.1259 | 0.1997 | 0.0785 | 0.1648 | 0.1167 |
| IBM | 2 | | | | | 1.0000 | 0.2167 | 0.2210 | 0.2864 | 0.1496 |
| Merrill Lynch | 7 | | | | | | 1.0000 | 0.1529 | 0.2743 | 0.2382 |
| Micron Tech. | 4 | | | | | | | 1.0000 | 0.2365 | 0.0986 |
| Microsoft | 5 | | | | | | | | 1.0000 | 0.1765 |
| Walt Disney | 3 | | | | | | | | | 1.0000 |

$$M_t = M_{t-1} + w_t \tag{16}$$

where $e_t^i$, and $w_t$ are independent Gaussian white noises such that $e_t^i$ incorporates the related idiosyncratic risk factor of asset return $R_t^i$; and $M_t$ is the market factor of risk underlying any stock return $R_t^i$. The only observed variables are asset returns $R_t^i$ where $i \in \{1, ..., N\}$ with $N$ being 9, 9 and 18 respectively for French, US, and global merged asset data samples. Moreover, $t \in \{1, ..., T\}$ with $T$ being 1138, 1141 and 1110 respectively for French, US, and global merged data sets. Such a framework can easily be translated into a state-space representation. Namely, the previous system of equations rewrites:

$$\begin{pmatrix} R_t^1 \\ \vdots \\ R_t^N \end{pmatrix} = \begin{pmatrix} \beta^1 \\ \vdots \\ \beta^N \end{pmatrix} \cdot M_t + \begin{pmatrix} e_t^1 \\ \vdots \\ e_t^N \end{pmatrix} \tag{17}$$

$$M_t = c_M \; M_{t-1} + w_t \tag{18}$$

where (17) corresponds to measurement equation (1) and (18) corresponds to state equation (2). Hence, we get $Y_t = \begin{bmatrix} R_t^1 & \cdots & R_t^N \end{bmatrix}'$, $X_t = M_t$, $D_t = \mathbf{0}$, $\varepsilon_t = \begin{bmatrix} e_t^1 & \cdots & e_t^N \end{bmatrix}'$, $k = 1$, $C_t = \mathbf{0}$, $R_t = I_k = 1$, $\eta_t = w_t$, $g = k$, $Z_t = \begin{bmatrix} \beta^1 & \cdots & \beta^N \end{bmatrix}'$, and $A_t = c_M$. To sum up, we consider the following linear stat-space model:

$$Y_t = Z_t X_t + \varepsilon_t \tag{19}$$

$$X_t = A_t X_{t-1} + \eta_t \tag{20}$$

We assume that initial conditions $m_0$ and $P_0$ about the system are unknown, and elements of $Q_t$ do not depend on $P_t$. Moreover, we state $Q_t = \sigma_M^2$ such that $P_0 \neq Q_t$, and :

$$H_t = \begin{pmatrix} \sigma_1^2 & 0 & \cdots & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & 0 & 0 & \sigma_{N-1}^2 & 0 \\ 0 & \cdots & 0 & 0 & \sigma_N^2 \end{pmatrix} \tag{21}$$

Therefore, our specification requires to estimate $\beta^1, \cdots, \beta^N$ (i.e., measurement equation), $M_0$ (i.e., the initial state of the system $X_0$), $H_t$ (i.e., a diagonal[6] covariance matrix composed of $N$ elements), $P_0$, $c_M$ and $\sigma_M$ (i.e., 1 element of covariance matrix $Q_t$). Hence, our linear system requires to estimate $2N+4$ parameters.

## 4.2 Econometric results

We achieve our state-space model estimation for both French and US assets while employing a Broyden-Fletcher-Goldfarb-Shanno-type optimization method[7] (i.e., for log-likelihood maximization). The estimates we get are displayed in tables (6) and (7); and we find $P_t^{France} = 4.5128 \times 10^{-12}$ and $P_t^{US} = 0.2113$ whatever time $t$. Moreover, the accuracy level we set to compute relative gradients is $10^{-6}$.
For both markets, the variance of the common latent component[8] appears to be significant in our state-space formulation. Strikingly, the common latent factor's coefficient $c_M$ is positive and significant on the French market whereas it appears to be negative and insignificant on the US market. By the way, the starting value of the common latent factor is significant only for the US market. However, $H_t$ covariance matrix[9] as well as beta coefficients are generally significant for the two financial markets under consideration. Recall that beta coefficients represent the impact of the common latent factor on asset returns. Considering both financial markets, beta coefficients are all positive, which indicates that asset returns are market driven. Moreover, these coefficients are above unity for all French asset returns as well as IBM, Micron Tech., Microsoft and Merrill Lynch asset returns, those assets magnifying therefore market fluctuations. Moreover, Alcatel stock return amplifies nearly three times market fluctuations. Differently, all the remaining asset returns exhibit beta coefficients below unity, absorbing then market impact. We also get the following statistical profile for the common latent factor $M_t$ inherent to each financial market under consideration (see tables 8 and 9).

Both latent common factors exhibit asymmetric (i.e., leptokurtic) as well as non-normal features. However, we notice structural differences between those

[6] We explicitly assume that stock returns are only correlated through their common latent component.

[7] Non-linear maximization problem.

[8] This one is assumed to remain constant over time.

[9] This covariance matrix is also assumed to be constant over time.

Table 6: Kalman estimates for French asset returns

| **Parameters** | **Estimate** | **Gradient** | **Std. Dev.** | **T-Student** |
|---|---|---|---|---|
| $\beta^1$ | 1.5677 | 0.0003 | 0.3262 | 4.8056 |
| $\beta^2$ | 1.3508 | 0.0007 | 0.2905 | 4.6500 |
| $\beta^3$ | 2.7563 | 0.0020 | 0.5785 | 4.7649 |
| $\beta^4$ | 1.4940 | 0.0006 | 0.3218 | 4.6431 |
| $\beta^5$ | 1.7118 | -0.0010 | 0.3549 | 4.8230 |
| $\beta^6$ | 1.7152 | -0.0003 | 0.3615 | 4.7449 |
| $\beta^7$ | 1.1700 | 0.0006 | 0.2523 | 4.6374 |
| $\beta^8$ | 1.3099 | -0.0014 | 0.2804 | 4.6724 |
| $\beta^9$ | 1.4182 | -0.0010 | 0.3070 | 4.6197 |
| $\sigma_1$ | 0.0000 | 0.0011 | 0.0106 | -0.0003 |
| $\sigma_2$ | 2.1317 | 0.0096 | 0.0447 | 47.7151 |
| $\sigma_3$ | 2.7667 | 0.0022 | 0.0578 | 47.8603 |
| $\sigma_4$ | 2.1136 | -0.0036 | 0.0443 | 47.7081 |
| $\sigma_5$ | 2.5695 | -0.0011 | 0.0538 | 47.7305 |
| $\sigma_6$ | 1.9131 | -0.0006 | 0.0401 | 47.6949 |
| $\sigma_7$ | 2.1026 | -0.0099 | 0.0441 | 47.7049 |
| $\sigma_8$ | 2.0100 | 0.0020 | 0.0421 | 47.7183 |
| $\sigma_9$ | 2.8723 | 0.0003 | 0.0602 | 47.6959 |
| $Q_t$ | 0.8465 | 0.0000 | 0.1776 | 4.7655 |
| $P_0$ | 0.6548 | 0.0048 | 1.5079 | 0.4342 |
| $M_0$ | 8.9016 | 0.0000 | 11.3236 | 0.7861 |
| $c_M$ | 0.0725 | 0.0001 | 0.0128 | -5.6475 |

Table 7: Kalman estimates for US asset returns

| **Parameters** | **Estimate** | **Gradient** | **Std. Dev.** | **T-Student** |
|---|---|---|---|---|
| $\beta^1$ | 0.7864 | 0.0030 | 0.1038 | 7.5742 |
| $\beta^2$ | 1.0414 | -0.0039 | 0.1395 | 7.4654 |
| $\beta^3$ | 0.7811 | 0.0072 | 0.1095 | 7.1313 |
| $\beta^4$ | 1.1960 | 0.0012 | 0.1822 | 6.5654 |
| $\beta^5$ | 1.0495 | -0.0075 | 0.1413 | 7.4269 |
| $\beta^6$ | 0.8219 | 0.0011 | 0.1179 | 6.9742 |
| $\beta^7$ | 1.4912 | -0.0085 | 0.1973 | 7.5588 |
| $\beta^8$ | 0.5277 | 0.0108 | 0.0769 | 6.8653 |
| $\beta^9$ | 0.7915 | 0.0058 | 0.1113 | 7.1081 |
| $\sigma_1$ | 0.4642 | -0.0041 | 0.0326 | 14.2237 |
| $\sigma_2$ | 2.0897 | 0.0028 | 0.0467 | 44.7560 |
| $\sigma_3$ | 2.1318 | -0.0018 | 0.0462 | 46.1350 |
| $\sigma_4$ | 4.5001 | -0.0002 | 0.0966 | 46.6022 |
| $\sigma_5$ | 2.2828 | 0.0087 | 0.0514 | 44.4024 |
| $\sigma_6$ | 2.6457 | 0.0004 | 0.0568 | 46.5429 |
| $\sigma_7$ | 2.3685 | 0.0012 | 0.0560 | 42.2666 |
| $\sigma_8$ | 1.7222 | 0.0075 | 0.0369 | 46.6931 |
| $\sigma_9$ | 2.3531 | -0.0056 | 0.0506 | 46.4712 |
| $Q_t$ | 1.4182 | -0.0062 | 0.1862 | 7.6187 |
| $P_0$ | 0.9001 | 0.0000 | 2.8521 | -0.3156 |
| $M_0$ | 0.9004 | 0.0054 | 0.0524 | 17.1845 |
| $c_M$ | -0.0057 | -0.0029 | 0.0093 | -0.6142 |

Table 8: Statistics for common latent factors in asset returns

| | **France** | **USA** |
|---|---|---|
| Mean | 0.0428 | 0.0513 |
| Stand. Dev. | 0.8482 | 0.0397 |
| Skewness | -0.2569 | -0.3396 |
| Excess kurtosis | 1.2001 | 3.2187 |
| Min. | -3.4021 | -8.1372 |
| Max. | 3.7927 | 5.5226 |
| Median | 0.0549 | 0.0659 |
| 1st quartile* | -0.4300 | -0.7245 |
| 3rd quartile* | 0.5862 | 0.8474 |

* Upper bound of the quartile.

Table 9: Correlations of common latent factors with asset returns

| | **France** | | **USA** | |
|---|---|---|---|---|
| $i$ | Kendall's Tau | Spearman's Rho | Kendall's Tau | Spearman's Rho |
| 1 | 1.0000 | 1.0000 | 0.8543 | 0.9690 |
| 2 | 0.3024 | 0.4320 | 0.4218 | 0.5892 |
| 3 | 0.5017 | 0.6830 | 0.3438 | 0.4878 |
| 4 | 0.4347 | 0.6046 | 0.2336 | 0.3370 |
| 5 | 0.3322 | 0.4724 | 0.4087 | 0.5758 |
| 6 | 0.4293 | 0.5905 | 0.2933 | 0.4212 |
| 7 | 0.2832 | 0.4100 | 0.4977 | 0.6828 |
| 8 | 0.3093 | 0.4451 | 0.2962 | 0.4286 |
| 9 | 0.3450 | 0.4895 | 0.3333 | 0.4730 |

two market factors. Indeed, the US market factor return exhibits fatter tails as well as bigger variation bounds (i.e., extreme values) than the French one. As expected, non-linear correlation coefficients between common latent factors and related asset returns are positive. Strikingly, we notice a perfect correlation between the French market factor return and SBF120 stock index return. Analogously, the US market factor return is highly correlated with the DJIA return. At a first glance, we conclude that SBF120 French stock index captures the common latent component inherent to the French financial market in terms of market risk changes (see Gatfaoui [2005]). Differently, though the high previous correlation coefficients, the DJIA US stock index does not capture the whole of market risk changes that are peculiar to the US financial market. Moreover, we attempt to assess the efficiency of the systematic risk factor while explaining stock return evolutions. For this purpose, we realize a set of regressions that are introduced in the appendix (see tables 18 and 19). Results exhibit the general inefficiency of the US systematic risk factor except for AT & T stock return. Namely, the systematic risk factor encompasses the whole information describing the evolution of AT & T stock return. However, such a market factor is generally insufficient to explain the whole evolution of US stock returns, underlining then the significance of the idiosyncratic (i.e., unsystematic) risk component (see Campbell *et al.* [2001]). Analogously, the French market risk factor is inefficient, and fails to explain the whole evolution of French asset returns. Further investigation while comparing French and US financial markets requires to consider all asset returns on the same date scale. Such an investigation is undertaken in the next section.

# 5 Further investigation

We consider returns on a percentage basis, and on the same date scale (i.e., after adjusting for non-working days in both French and US countries). First, we display all the statistic profiles on the new time scale. Second, we attempt to extract some common component from the latent common factors inherent to the French and US markets. Finally, we describe briefly the link between this new global common component, and both the French and US market factor ones.

## 5.1 Statistical profiles

Estimating again our state-space model over the same time scale (and on each market separately) leads to the estimates, which are displayed in tables (11) and (10) with $P_t^{France} = 2.9461 \times 10^{-12}$ and $P_t^{US} = 0.3615$ whatever time $t$.

Adjusting for non-working dates in both countries changes slightly our previous estimate results.[10] We have globally the same behavior as the results introduced in the previous section except for some specific details. First, among high beta coefficients, only Merrill Lynch asset return's beta remains above unity, amplifying then market movements. Second, the initial state of the US market factor $M_0$ is no more significant while $c_M$ coefficient becomes significant here. Moreover, $Q_t$ variance is higher than previously (i.e., a 46.56% increase).
In the French case, though coefficient estimates changes slightly in level, the same conclusions as in the previous section apply. Namely, all asset returns amplify significantly market movements.

We obtain the following statistical profile for the common latent factor $M_t$ peculiar to each financial market under consideration (see table 12).

[10] Removing some data is equivalent to remove some information, and such a loss of information impacts slightly our results.

Table 10: Kalman estimates for US asset returns

| **Parameters** | **Estimate** | **Gradient** | **Std. Dev.** | **T-Student** |
|---|---|---|---|---|
| $\beta^1$ | 0.6019 | -0.0001 | 0.0466 | 12.9205 |
| $\beta^2$ | 0.7909 | -0.0024 | 0.0634 | 12.4727 |
| $\beta^3$ | 0.5944 | -0.0096 | 0.0533 | 11.1424 |
| $\beta^4$ | 0.9268 | 0.0009 | 0.0937 | 9.8957 |
| $\beta^5$ | 0.7791 | 0.0091 | 0.0472 | 16.4934 |
| $\beta^6$ | 0.6180 | 0.0116 | 0.0367 | 16.8373 |
| $\beta^7$ | 1.1459 | -0.0070 | 0.0848 | 13.5077 |
| $\beta^8$ | 0.4012 | 0.0010 | 0.0405 | 9.9033 |
| $\beta^9$ | 0.6117 | -0.0002 | 0.0592 | 10.3301 |
| $\sigma_1$ | 0.4601 | 0.0008 | 0.0345 | 13.3241 |
| $\sigma_2$ | 2.1203 | -0.0020 | 0.0478 | 44.3131 |
| $\sigma_3$ | 2.1634 | -0.0008 | 0.0476 | 45.4959 |
| $\sigma_4$ | 2.2991 | 0.0010 | 0.0994 | 45.9180 |
| $\sigma_5$ | 4.5650 | 0.0048 | 0.0523 | 43.9870 |
| $\sigma_6$ | 2.6583 | -0.0028 | 0.0580 | 45.8704 |
| $\sigma_7$ | 2.3959 | 0.0028 | 0.0576 | 41.6293 |
| $\sigma_8$ | 1.7401 | -0.0048 | 0.0377 | 46.1082 |
| $\sigma_9$ | 2.3793 | 0.0035 | 0.0521 | 45.6791 |
| $Q_t$ | 1.8838 | -0.0008 | 0.1344 | 14.0128 |
| $P_0$ | 0.5004 | 0.0000 | 16.5527 | 0.0302 |
| $M_0$ | 0.5005 | 0.0023 | 19.4252 | 0.0258 |
| $c_M$ | -0.0033 | 0.0057 | 0.0009 | -3.6362 |

Table 11: Kalman estimates for French asset returns

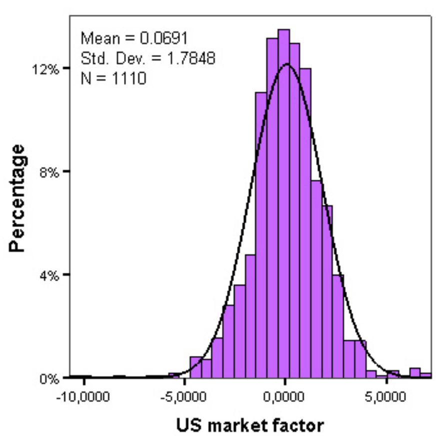
| **Parameters** | **Estimate** | **Gradient** | **Std. Dev.** | **T-Student** |
|---|---|---|---|---|
| $\beta^{10}$ | 1.6604 | 0.0082 | 0.4308 | 3.8542 |
| $\beta^{11}$ | 1.4138 | 0.0018 | 0.3768 | 3.7524 |
| $\beta^{12}$ | 2.9518 | -0.0063 | 0.7780 | 3.7938 |
| $\beta^{13}$ | 1.5750 | -0.0018 | 0.4169 | 3.7782 |
| $\beta^{14}$ | 1.8353 | 0.0043 | 0.4835 | 3.7963 |
| $\beta^{15}$ | 1.8058 | -0.0066 | 0.4700 | 3.8417 |
| $\beta^{16}$ | 1.2540 | 0.0033 | 0.3208 | 3.9090 |
| $\beta^{17}$ | 1.3712 | 0.0015 | 0.3587 | 3.8230 |
| $\beta^{18}$ | 1.4985 | 0.0038 | 0.3971 | 3.7736 |
| $\sigma_{10}$ | 0.0000 | -0.0005 | 0.0198 | -0.0001 |
| $\sigma_{11}$ | 2.1644 | -0.0011 | 0.0460 | 47.0964 |
| $\sigma_{12}$ | 2.8350 | -0.0028 | 0.0602 | 47.0895 |
| $\sigma_{13}$ | 2.1382 | 0.0029 | 0.0454 | 47.1005 |
| $\sigma_{14}$ | 2.6103 | 0.0011 | 0.0554 | 47.1075 |
| $\sigma_{15}$ | 1.9252 | 0.0002 | 0.0409 | 47.0702 |
| $\sigma_{16}$ | 2.1233 | 0.0006 | 0.0451 | 47.0818 |
| $\sigma_{17}$ | 2.0347 | 0.0005 | 0.0432 | 47.1166 |
| $\sigma_{18}$ | 2.9125 | 0.0006 | 0.0618 | 47.1101 |
| $Q_t$ | 0.8110 | -0.0015 | 0.2107 | 3.8495 |
| $P_0$ | 0.7106 | 0.0051 | 1.6835 | 0.4221 |
| $M_0$ | 8.8777 | 0.0000 | 12.2015 | 0.7276 |
| $c_M$ | 0.0687 | -0.0003 | 0.0312 | 2.2009 |

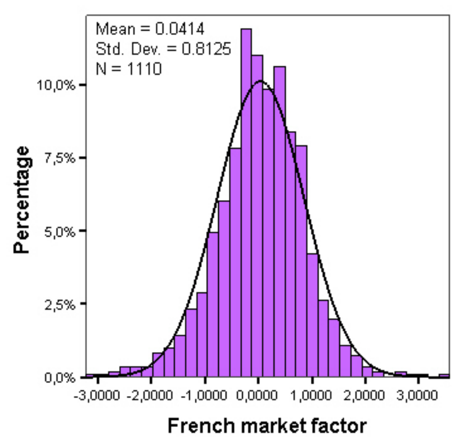
Table 12: Statistics for common latent factors in asset returns (same time scale)

| | **France** | **USA** |
|---|---|---|
| Mean | 0,0414 | 0,0691 |
| Stand. Dev. | 0,8125 | 1,7848 |
| Skewness | -0,1951 | -0,3436 |
| Excess kurtosis | 1,1508 | 3,0918 |
| Min. | -3,2123 | -10,7063 |
| Max. | 3,5810 | 7,2395 |
| Median | 0,0494 | 0,0829 |
| 1st quartile* | -0,4206 | -0,9741 |
| 3rd quartile* | 0,5546 | 1,1309 |

* Upper bound of the quartile.

The same conclusions as the former section apply here. Briefly, common latent French and US market factors are leptokurtic, the US market factor being more left-asymmetric and having fatter tails than the French one. As a rough guide, we also translate these results into graphs while plotting related histograms as well as related Gaussian distributions (i.e., Normal densities with corresponding moments of French and US market factor returns).




Previous histograms exhibit the higher impact of losses in both French and US financial markets (i.e., higher negative returns in absolute value as compared to their positive counterparts). The magnitude of observed losses (i.e.,

Table 13: Correlations of common latent factors with asset returns (same time scale)

| | **France** | | **USA** | |
|---|---|---|---|---|
| Return $i$ | Kendall's Tau | Spearman's Rho | Kendall's Tau | Spearman's Rho |
| 1 | 1.0000 | 1.0000 | 0.8611 | 0.9720 |
| 2 | 0.3012 | 0.4295 | 0.4183 | 0.5856 |
| 3 | 0.5007 | 0.6810 | 0.3447 | 0.4879 |
| 4 | 0.4364 | 0.6067 | 0.2366 | 0.3415 |
| 5 | 0.3304 | 0.4697 | 0.4030 | 0.5681 |
| 6 | 0.4302 | 0.5921 | 0.2937 | 0.4216 |
| 7 | 0.2886 | 0.4180 | 0.4971 | 0.6818 |
| 8 | 0.3059 | 0.4406 | 0.2941 | 0.4252 |
| 9 | 0.3424 | 0.4850 | 0.3388 | 0.4806 |

absolute value of negative market factor returns) is higher for the US financial market. The same conclusion holds for the magnitude of observed positive market returns. To get a view about the link between market factors and corresponding asset returns, we consider related non-linear correlation coefficients (see table 13).

As expected, we get the same results as in the previous section. Namely, correlation coefficients are all positive, and mean that asset returns are market driven whatever the financial market under consideration (see Campbell *et al.* [2001]). For further investigation, we focus on a potential common component in both French and US common latent factors (i.e., French and US market factors).

## 5.2 Systemic component

We ask the question of how to characterize some potential link prevailing between French and US financial markets. Specifically, we look for a relationship between French and US common latent factors. As a first step, we consider their Kendall and Spearman correlation coefficients, which are respectively $\tau = 0.2834$ and $\rho = 0.4053$. Hence, we exhibit clearly some positive link between these two components. Therefore, at a systemic level

Table 14: Kalman estimates for systemic component in asset returns

| Parameters | Estimate | Gradient | Std. Dev. | T-Student |
|---|---|---|---|---|
| $b_1$ | 2.3888 | 0.0157 | 0.6710 | 3.5602 |
| $b_2$ | 1.2464 | 0.0155 | 0.3551 | 3.5098 |
| $\sigma_1$ | 1.4249 | -0.0065 | 0.0620 | 22.9762 |
| $\sigma_2$ | 0.5884 | 0.0111 | 0.0377 | 15.6252 |
| $Q_t$ | 0.4391 | -0.0317 | 0.1166 | 3.7654 |
| $P_0$ | 1.0150 | 0.0621 | 0.0898 | 0.0001 |
| $B_0$ | 3.2020 | -0.0173 | 2.8342 | 1.1298 |
| $c_B$ | 0.2160 | 0.0301 | 0.0466 | 4.6308 |

of consideration, we attempt to extract a common component in French and US market factors. Such a component may result from business cycle effect, macroeconomic risk or financial integration effect on these two financial markets for example. To this end, we assume that:

$$M_t^{US} = b_1 B_t + e_t^1 \tag{22}$$

$$M_t^{France} = b_2 B_t + e_t^2 \tag{23}$$

$$B_t = c_B B_{t-1} + \eta_t \tag{24}$$

where $B_t$ is the systemic component (i.e., a component common to French and US financial markets); $M_t^{France}$ and $M_t^{US}$ are French and US market factors; $(e_t^i)$, $\eta_t$ and $c_B$ as in the previous section. The state-space formulation of such a specification is then:

$$\begin{pmatrix} M_t^{US} \\ M_t^{France} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \cdot B_t + \begin{pmatrix} e_t^1 \\ e_t^2 \end{pmatrix} \tag{25}$$

$$B_t = c_B B_{t-1} + \eta_t \tag{26}$$

with $N = 2$, $k = g = 1$. Recall that we only observe market factors $M_t^{France}$ and $M_t^{US}$ from which we try to infer a common systemic component $B_t$ (i.e., unobserved state variable). The related estimates we get while applying a Kalman methodology are displayed in table 14 with $P_t = 0.0808$.

As expected from correlation coefficients, estimates exhibit a positive link between systemic factor and both French and US market factor returns. Moreover, coefficient $c_B$ is positive and significant. In the same way, variance

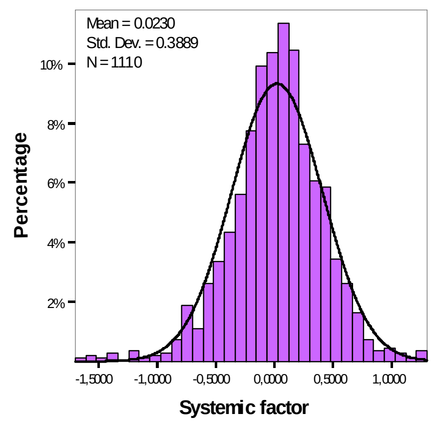
Table 15: Statistics for systemic factor in asset returns

| | $B_t$ |
|---|---|
| Mean | 0.0207 |
| Stand. Dev. | 0.3488 |
| Skewness | -0.3636 |
| Excess kurtosis | 1.3965 |
| Min. | -1.5214 |
| Max. | 1.1667 |
| Median | 0.0326 |
| 1st quartile | -0.1761 |
| 3rd quartile | 0.2323 |

Table 16: Correlations of market and systemic factor returns

| Market return | Kendall's Tau | Spearman's Rho |
|---|---|---|
| $M_t^{US}$ | 0.5560 | 0.7405 |
| $M_t^{France}$ | 0.7185 | 0.8902 |

parameter $Q_t$ is significant. Finally, the statistical profile of the systemic component is summarized in table 15.
Analogously to French and US market factors, the systemic factor exhibits non-normal and asymmetric features. Specifically, this component is left-skewed and exhibits fatter tails than the Normal probability distribution. Systemic skewness is higher than market skewness both in France and USA. However, systemic kurtosis lies between French and US ones. Finally, we end our study with a quick statistical profile of the obtained systemic factor. Indeed, a brief correlation analysis is displayed in table 16. Then, we plot the related histogram.
As expected, the correlation coefficients we get exhibit a positive and strong link between systemic factor and both French and US market factors. Hence, French and US financial markets tend to evolve in the same direction (i.e., same structural changes). However, the French market is more sensitive to structural changes than the US one (i.e., higher correlation with the systemic component).
The previous histogram illustrates clearly the statistical profile of systemic factor return. As a rough guide, we also plot the corresponding Gaussian distribution function. Namely, the probability distribution of the systemic factor




return is obviously non-normal, fat-tailed as well as left-skewed. Indeed, the magnitude of negative systemic returns is higher than the magnitude of their positive counterparts.

# 6 Concluding remarks

In this paper, we investigate the existence of a common latent component in asset returns. Our study concerns both the French and US financial markets, and is undertaken in two steps.

First, given that we only observe asset returns, we resort to Kalman filtering methodology to infer some knowledge about the unobservable French and US market factors. This estimation method requires to translate our investigation into a state-space representation. The results we get are powerful in the sense that we find strong evidence of a common latent factor in both financial markets. Moreover, such factors exhibit tail and asymmetric features analogously to their related respective asset returns.

Second, we further investigate a potential link or a potential common component in our two market factors while considering a systemic level. For this purpose, we resort again to Kalman filtering method to infer knowledge about the unobserved systemic component. Such an approach is useful to capture macroeconomic effects and economic as well as financial links between countries. Our results are also strong here and exhibit a strong positive link between the systemic factor and both French and US market factors. However, we chose a linear framework to undertake our study and investigate potential common links between asset returns. Current markets suggest the existence of non-linear link between asset returns. So, some extensions may be undertaken in the lens of non-linearity patterns, and could perhaps lead to even stronger results.

Future research should therefore attempt to apply improved versions of Kalman methodology. Indeed, extensions have been proposed to allow for relaxing required initial conditions as well as to account for missing data (see Rao [2001]). Moreover, given known non-linear market characteristics (see Gourieroux & Jasiak [2001] among others), extended Kalman filter (i.e., EKF) and iterated extended Kalman filter (i.e., IEKF) allow for accounting for non-linear system features (see Jaswinsky [1970], Maybeck [1979,1982], Chui & Chen [1987], and Julier & Uhlmann [1998]).

# 7 Appendix

We expose here computational details as well as complementary explanations and statistics.

## 7.1 Statistical profiles

For example, table (17) presents the statistical profiles of both French and US stock returns on the same time scale.

## 7.2 Efficiency of French and US market factors

We assess here the efficiency of the obtained systematic risk factors (i.e., market factors of risk) for each financial market under consideration. Namely, we consider the following regression for each stock $i$, for each financial market, and for time $t$ in $\{1, ..., T\}$:

$$R_t^i - M_t = \lambda_i \, M_t + u_t^i \tag{27}$$

where $\lambda_i$ a constant regression coefficient, and $(u_t^i)$ is a Gaussian noise. Hence, testing for the efficiency of the systematic risk factor consists of testing whether $\lambda_i$ is significantly zero for each stock return. This is equivalent

Table 17: French and US asset return statistics (same time scale)

| | $i$ | Mean | Stand. Dev. | Skewness | Excess kurtosis | Min. | Max. | Median | 1st quartile* | 3rd quartile* |
|---|---|---|---|---|---|---|---|---|---|---|
| Accor | 11 | 0.0932 | 2.4491 | -0.0908 | 2.0994 | -14.7809 | 10.7692 | 0.0614 | -1.3637 | 1.4603 |
| Alcatel | 12 | 0.0721 | 3.7140 | -1.9344 | 27.1277 | -48.4564 | 14.4352 | 0.0000 | -1.7712 | 2.0700 |
| Axa | 13 | 0.1575 | 2.4875 | 5.3428 | 96.2590 | -9.9142 | 45.1151 | 0.1107 | -1.0933 | 1.2997 |
| Bouygues | 14 | 0.1683 | 3.0020 | 0.1215 | 2.6075 | -17.1909 | 14.0123 | 0.0000 | -1.4388 | 1.7331 |
| L'Oréal | 15 | 0.0932 | 2.4195 | 0.0535 | 1.0333 | -10.0285 | 9.2622 | 0.0000 | -1.4583 | 1.5281 |
| Michelin | 16 | 0.0004 | 2.3554 | -0.0540 | 2.2005 | -11.3445 | 11.8360 | 0.0000 | -1.2998 | 1.2604 |
| Peugeot SA | 17 | 0.1134 | 2.3175 | -0.2112 | 3.4553 | -16.3259 | 10.4635 | 0.0000 | -1.1562 | 1.3811 |
| SBF120 | 10 | 0.0688 | 1.3484 | -0.1951 | 1.1508 | -5.3336 | 5.9459 | 0.0819 | -0.6963 | 0.9201 |
| Total Fina Elf SA | 18 | 0.1859 | 3.1517 | 10.2020 | 223.8976 | -13.1709 | 70.5330 | 0.0575 | -1.3258 | 1.5837 |
| AT & T | 6 | 0.0099 | 2.9021 | 0.3304 | 9.6706 | -23.2620 | 22.1301 | 0.0000 | -1.6330 | 1.4661 |
| DJIA | 1 | 0.0438 | 1.2229 | -0.3677 | 3.0016 | -7.4549 | 4.8605 | 0.0608 | -0.6492 | 0.8093 |
| Dow Jones | 8 | 0.0149 | 1.8971 | -1.4312 | 17.0840 | -21.4531 | 8.7601 | 0.0000 | -0.9598 | 0.9864 |
| Ford Motor | 9 | 0.0146 | 2.6436 | -2.7074 | 41.9896 | -38.6561 | 10.6264 | 0.0000 | -1.2917 | 1.3975 |
| IBM | 2 | 0.0770 | 2.5903 | -0.1285 | 4.6693 | -16.8916 | 12.3665 | 0.1026 | -1.4513 | 1.4926 |
| Merrill Lynch | 7 | 0.1111 | 3.2231 | 0.1488 | 1.3390 | -12.2978 | 14.0477 | 0.0000 | -1.8949 | 2.0523 |
| Micron Tech. | 4 | 0.1213 | 4.8859 | 0.1404 | 0.8554 | -19.1160 | 21.7202 | 0.0000 | -3.0678 | 3.0864 |
| Microsoft | 5 | 0.1238 | 2.7249 | -0.0395 | 3.8832 | -15.6310 | 17.8692 | 0.0423 | -1.4497 | 1.6989 |
| Walt Disney | 3 | 0.0215 | 2.4359 | -0.0728 | 4.9720 | -16.9523 | 14.2029 | 0.0000 | -1.3777 | 1.3666 |

* Upper bound of the quartile.

to test whether the market factor summarizes the whole information that describes asset return evolutions in each considered financial market. Related results are displayed in tables 18 and 19.

We also performed the standard regressions of $R_t^i$ on $M_t$ for each financial market under consideration, and found good explanatory powers, and positive and highly significant regression coefficients for both financial markets. These regression coefficients are far below unity for stock returns, and close or equal to unity for stock index returns. Moreover, the explanatory powers indicate that systematic risk factors fail generally to explain the whole evolution of asset returns (except for stock index returns). To spare space, we do not report related results, which are available upon request of course.

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Table 18: French asset return regressions

| | $i$ | $R^2$ | Adjusted $R^2$ | Durbin Watson Stat. | $F(1, 1137)$ | $\lambda_i$ | Student $t$ |
|---|---|---|---|---|---|---|---|
| Accor | 2 | 0.0191 | 0.0183 | 2.1176 | 22.1902 | 0.1384 | 4.7106 |
| Alcatel | 3 | 0.2250 | 0.2244 | 1.7856 | 330.1784 | 0.4744 | 18.1708 |
| Axa | 4 | 0.0379 | 0.0370 | 1.8181 | 44.7578 | 0.8946 | 6.6901 |
| Bouygues | 5 | 0.0524 | 0.0516 | 1.9317 | 62.8793 | 0.2289 | 7.9296 |
| L'Oréal | 6 | 0.0915 | 0.0907 | 2.0659 | 114.5326 | 0.3025 | 10.7020 |
| Michelin | 7 | 0.0047 | 0.0038 | 1.9083 | 5.3550 | 0.0685 | 2.3141 |
| Peugeot | 8 | 0.0168 | 0.0160 | 1.8042 | 19.4851 | 0.1298 | 4.4142 |
| SBF120 | 1 | 1.0000 | 1.0000 | 1.9776 | $4.3000\times10^{17}$ | 1.0000 | $6.5\times10^{8}$ |
| Total | 9 | 0.0151 | 0.0142 | 1.9311 | 17.3733 | 0.1227 | 4.1681 |

Table 19: US asset return regressions

| | $i$ | $R^2$ | Adjusted $R^2$ | Durbin Watson Stat. | $F(1, 1137)$ | $\lambda_i$ | Student $t$ |
|---|---|---|---|---|---|---|---|
| AT & T | 6 | 0.0017 | 0.0009 | 2.0744 | 1.9984 | -0.0418 | -1.4136 |
| DJIA | 1 | 0.2759 | 0.2753 | 1.9095 | 434.4350 | -0.5253 | -20.8431 |
| Dow Jones | 8 | 0.0946 | 0.0938 | 2.1528 | 119.1421 | -0.3076 | -10.9152 |
| Ford Motor | 9 | 0.0044 | 0.0036 | 2.3769 | 5.0947 | -0.0667 | -2.2571 |
| IBM | 2 | 0.0116 | 0.0107 | 1.9692 | 13.3583 | 0.1076 | 3.6549 |
| Merrill Lynch | 7 | 0.1358 | 0.1351 | 2.0458 | 179.1805 | 0.3685 | 13.3858 |
| Micron Tech. | 4 | 0.0101 | 0.0093 | 1.9812 | 11.6760 | 0.1007 | 3.4170 |
| Microsoft | 5 | 0.0107 | 0.0099 | 1.8885 | 12.3704 | 0.1036 | 3.5172 |
| Walt Disney | 3 | 0.0066 | 0.0057 | 2.1442 | 7.5540 | -0.0811 | -2.7485 |

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