Life-Cycle Asset Allocation with Annuity Markets: Is Longevity Insurance a Good Deal?

by

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Abstract

We show how an individual with uninsurable labor income, borrowing constraints, and Epstein-Zin utility function optimally spreads her financial wealth across stocks, bonds and life-annuities over the life-cycle when the finite investment horizon is stochastic. In spite of asymmetric mortality beliefs, public pensions, and bequest motives, we find that the individual starts out with annuitizing 18.59 percent of her accumulated financial wealth five years prior to retirement and continues gradually annuitizing thereafter. Our welfare analysis shows that the presence of an annuity market increases the individual's welfare by 14.14 percent at age 80 and by 30.07 percent at age 90.

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I Introduction

Nowadays, the burden of old-age provision has to be increasingly borne by individuals as nations worldwide experience a shift from troubled public pay-as-you-go to privately funded pension systems. In addition, employers are moving from professionally managed defined benefit to defined contribution plans for their employees. Thus, personal financial planning over the entire life cycle is more and more becoming the sole responsibility of the individual. The reasonable accumulation and decumulation of savings has become a life-task. Faced with this onerous task, the individual has to determine how to consume and how to spread life-savings across different asset classes. Throughout her work life, the individual has to build up a sufficient capital stock for retirement. During retirement, she has to spread her accumulated wealth over the remaining years of life. She has to assume the role of a risk manager for three main risks in order to ensure stable consumption over her lifetime and to guarantee bequest if desired: labor income risk, capital market risk and longevity risk.

The sharp rise in life expectancies in the past few decades makes the risk of exhausting the retirement capital stock more critical than ever before. Insurance products like life-annuities can hedge this longevity risk away. A life-annuity is a financial contract between an insured person and an insurer "that pays out a periodic amount for as long as the annuitant is alive, in exchange for an initial premium" (*Brown et al.*, 2001). The insurance providers themselves can hedge the guaranteed annuity payments by pooling the longevity risks. From the individual's perspective, the premium paid initially cannot be recovered. This inflexibility is said to be the main disadvantage of annuity purchases compared to flexible withdrawal plans.

Many studies compare life-annuity purchases with traditional asset classes. In an early study, *Yaari* (1965) finds that all assets should be annuitized if the individual is a rational investor without a bequest motive. In his model, the investor is only exposed to longevity risk and all annuities are fairly priced from an actuarial standpoint. *Yaari*'s (1965) result has been subject to extensive research in the public economics and insurance literature. *Brown et al.* (2005) show that the conditions under which the purchase of an annuity is optimal are not as demanding as the ones set out in *Yaari* (1965). If there is no bequest motive and the return on the annuity is greater than that of the reference asset, an individual will fully annuitize her wealth in the presence of a complete market. Partial annuitization may become optimal, if the condition of complete insurance markets is relaxed. If there is a bequest motive for investors, complete annuitization will be suboptimal. *Richard* (1975) was the first to include the uncertainty of the time of death in a continuous life-cycle framework and to extend *Merton*'s (1971) model to include instantaneous life insurance. However, this framework lacks the realism of an actual insurance market because *Richard* (1975) models instantaneous life insurance and annuity demand symmetrically. Most recently, *Blake et al.* (2003), *Milevsky and Young* (2002/2003), *Kapur and Orszag* (1999), *Dushi and Webb* (2004), *Kingston and Thorp* (2005) and *Stabile* (2003) investigated annuitization strategies in a utility framework.

However, most of the above mentioned studies solely focus on the retirement phase. Our main contribution is the introduction of a life-annuity market to a realistic life-cycle framework. We provide insight into how a prudent investor should optimally spread her wealth across bonds, stocks, and constant real life-annuities. Our model incorporates uninsurable income during work life, borrowing constraints, and stochastic time of death. The individual's asset allocation and savings decisions are driven by three motives. The first is the precautionary savings motive due to the uninsurable risky income. The investor we consider wants to save and invest her assets in such a way that she can hedge adverse developments in her income stream. This motive was first described by *Deaton* (1991) and *Carroll* (1997). The second is the retirement savings motive because pension income after retirement is lower than the labor income in the preceding accumulation phase. In order to smooth consumption over time and to cushion the drop of income due to retirement, the individual consumes less when she is young in order to consume her savings when she is old. These two motives can be summarized by the consumption smoothing motive. The third motive is the bequest motive because the individual might gain utility from leaving estate to her heirs.

The life-cycle asset allocation and consumption model we use is of the discrete time type and is – except for the annuity market – similar to the models used in the recent life-cycle literature, e.g. *Bertaut and Haliassos* (1997), *Campbell et al.* (1999), *Davis and Willen* (2000), *Gomes and Michaelides* (2003, 2005), *Cocco, Gomes, and Maenhout* (2005), *Dammon, Spatt, and Zhang* (2004), *Cocco* (2004), *Yao and Zhang* (2005). We assume that the individual has Epstein-Zin utility (*Epstein and Zin*, 1989) and can realize utility from bequeathing her heirs.

An important implication of our life-cycle model is that the investor is in control and has the flexibility to spend annuity payouts from previously purchased annuities. She can use them either to consume, to purchase bonds and stocks or even to purchase additional annuities. This flexibility was neglected in recent studies such as *Blake et al.* (2003), *Milevsky and Young* (2002), *Kingston and Thorp* (2005) and *Stabile* (2003) which assumed that annuity payouts can be used for consumption purposes only.

The focus of those studies is on the optimal stochastic and deterministic switching time to an annuity. In our model the investor can gradually purchase annuities at any age and is not restricted to the decision whether to switch completely to an annuity at a certain point in retirement. Previous studies looking at gradual investment in life-payout annuities include *Kapur and Orszag* (1999) and *Milevsky and Young* (2003). However, these authors set their optimization problem up as a continuous time model with time-additive CRRA preferences, focusing merely on the retirement period. In contrast, we can show how an individual chooses her asset allocation and annuity purchases during her life in a realistically calibrated life-cycle framework. Our model demonstrates that decisions in the retirement phase cannot be separated from decisions in the accumulation phase.

We carried out a sensitivity analysis for (1) risk aversions, (2) the strength of bequest motives, (3) mortality asymmetry and (4) public pensions. Our findings indicate a very important role of the risk aversion parameter for determining the annuity demand. Even individuals with a low risk aversion purchase annuities at very high ages but to a lesser extent. Our analysis shows a strong relationship between the strength of the bequest motive and asset allocation. We still find a substantial demand for annuities in the presence of bequest motives. We also allow the individual to value expected utility via a subjective force of mortality, while the annuity is priced by using an annuitant mortality table. We show that these asymmetries of mortality beliefs can contribute to explaining why individuals who believe themselves to be less healthy than average are less likely to buy annuities. However, the effect of mortality asymmetry has a small impact on annuity demand. Although preexisting public pensions have the same payout structure as annuities, we still find individuals purchasing annuities. In a final welfare analysis, we verify that a substantial demand for annuities goes hand in hand with a considerable equivalent increase in financial wealth due to the presence of annuity markets. Especially for senior individuals annuity markets imply a considerable equivalent increase in financial wealth ranging from 14.14 percent at age 80 up to 30.07 percent at age 90.

The remainder of this article is organized as follows. In section II, we describe the investor's optimization problem, the model calibration, and the numerical optimization method. In section III, we show the results for our base-line case with and without annuity markets. Section IV continues with a robustness analysis and section V with a welfare analysis. Section VI concludes the article.

II. The Model

A. Preferences

The model is time discrete with $t = 0,\ldots,T+1$, where t is the adult age of the individual and can be calculated as actual age minus 19. The individual lives up to T years. The individual has a subjective probability p_t^s that she survives until $t+1$ given that she is alive at t. Furthermore, the individual has Epstein-Zin utility defined over a single non-durable consumption good. Let C_t be the consumption level and B_t the bequest at time t. Then Epstein-Zin preferences as in *Epstein and Zin* (1989) are described by

$$V_t = \left\{ \left(1-\beta p_t^s\right) C_t^{1-1/\psi} + \beta E_t \left[p_t^s V_{t+1}^{1-\rho} + \left(1-p_t^s\right) k \frac{\left(B_{t+1}/k\right)^{1-\rho}}{1-\rho} \right]^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}, \tag{1}$$

where ρ is the level of relative risk aversion, ψ is the elasticity of intertemporal substitution, β is the discount factor and k the strength of the bequest motive. Since $p_T^s = 0$ (1) reduces in T to

$$V_T = \left\{ C_T^{1-1/\psi} + \beta E_T \left[k \frac{\left(B_{T+1}/k\right)^{1-\rho}}{1-\rho} \right]^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}},$$

which gives us the terminal condition for V_T.

B. Labor Income Process

During the accumulation phase ($t \leq K$) the individual earns uninsurable, real labor income Y_t. Being consistent with *Cocco et al.* (2005) the process of labor income follows

$$Y_t = \exp(f(t)) P_t U_t,$$
$$P_t = P_{t-1} N_t.$$

$f(t)$ is a deterministic function of age to recover the hump shape of income stream. P_t is a permanent component with innovation N_t and U_t is a transitory shock. The logarithms of N_t

and U_t are normal distributed with means zero and with volatilities σ_N, σ_U respectively. The shocks are assumed to be uncorrelated. In retirement ($t > K$) we assume for the sake of simplicity that the individual receives constant pension payments $Y_t = \varsigma \exp(f(K))P_K$ after retirement, where ζ is the constant replacement ratio. Clearly, it might be worthwhile determining the retirement age K and labor supply endogenously. This question is beyond the scope of this analysis since we focus on the asset allocation decision.

C. Incomplete Annuity Market

The individual can invest in an incomplete insurance market by purchasing constant real payout life-annuities. A life-annuity is a financial contract between an individual and an insurer "that pays out a periodic amount for as long as the annuitant is alive, in exchange for an initial premium" (*Brown et al.*, 2001). The insurance providers themselves can hedge the guaranteed annuity payments by pooling the longevity risks of many annuitants. Contrary to liquid investments, the initial premium cannot be recovered by the individual later on. The actuarial premium PR_t of a life-annuity with payments L is given by:

$$PR_t = L \cdot a_t ,$$

where a_t is the annuity factor for an individual with adult age t is

$$a_t = (1+\delta)\sum_{s=1}^{\infty}\left(\prod_{u=t}^{t+s} p_u^a\right)R_f^{-s} ,$$

where p_u^a are the survival probabilities used by the life-annuity provider and δ is the expense factor. Thus, the annuity factor is the expense factor times the sum of the discounted expected payouts.

Annuities define an asset class with certain return characteristics because payments are conditional on survival. The return on capital of those who die is allocated to the living members of a cohort. The survivors' return from the one-period annuity is $R_f / p_t > R_f$.

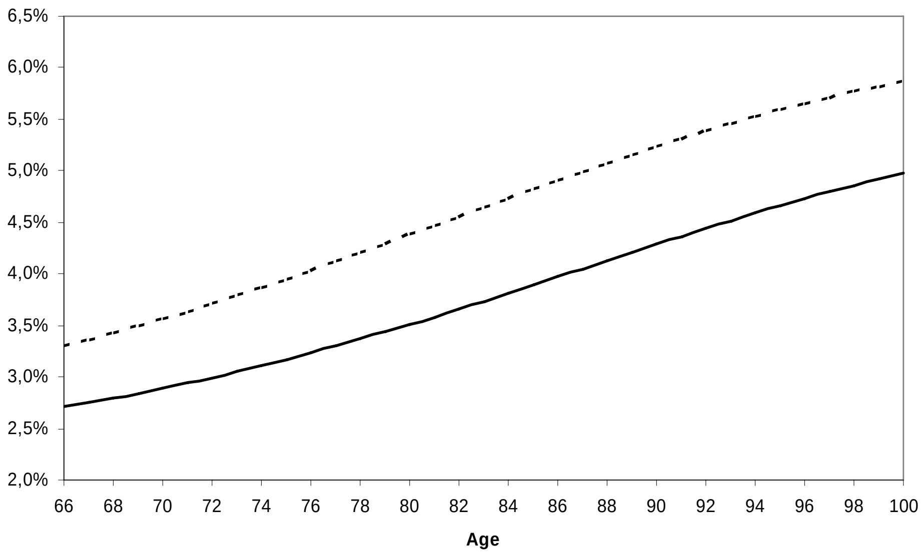

Figure 1. Implied longevity yield.

The solid line depicts the implied longevity yield of a female annuitant for ages over 65. The dashed line shows the implied longevity yield of a male annuitant who bought the annuity at age 65.

The resulting incremental return the annuitant receives above the interest rate is called mortality credit. The older the individual, the lower the survival probability p_t, the higher is the incremental return. The mortality credit neglects the initial loss of the whole lump sum paid to the insurance company.

The implied longevity yield (figure 1), an internal rate of return, accounts for the loss of the initial premium. One way to compute the implied longevity yield of an annuity purchase between two points in time is to solve the following equation as derived in *Milevsky* (2005):

$$a_{t+\Delta t} - \left(a_t - \frac{1}{\xi} \right) e^{\xi \Delta t} - \frac{1}{\xi} = 0 \, .$$

The longevity yield ξ is the constant rate that has to be earned on a portfolio in order to be as well off, after a period of Δt, as if the individual purchased an annuity initially. The equation assumes that the investor confines herself to a self-annuitization strategy in order to accumulate sufficient wealth to purchase an annuity after a period of Δt, where the term self-annuitization refers to the drawdown scheme that replicates the payout structure of a fixed annuity.

Figure (1) shows the longevity yield for both male and female annuitant who purchased a life-annuity at age 65. The implied longevity yield increases every year the annuitant survives and outlives her peers. Clearly, the male annuitant has a higher implied longevity yield because of his lower survival probabilities. In turn, this means that life annuities are a completely different asset depending on the investor's sex and individual survival probabilities.

D. Capital Market

The individual can invest in the two traditional financial assets: riskless bonds and risky stocks. The real bond gross return denotes R_f, and the real risky stock return in t *is* R_t. The risky return is lognormal distributed with expected return μ and volatility σ. Let ϕ_n (ϕ_u) denote the correlation between the stock returns and the permanent (transitory) income shocks.

E. Mortality

To give mortality a functional form we use the Gompertz law for the sake of convenience and because of its widespread use in the insurance and finance literature. Using Gompertz law allows us to model the asymmetry between the insurer's view on mortality and the annuitant's beliefs about her own health in a simple and consistent way. The functional form of the subjective force of mortality λ^s and the force of mortality for computing annuity premiums λ^a are then specified by

$$\lambda_t^i = \frac{1}{b}\exp\left(\frac{t-m^i}{b^i}\right),\ i = a,s.$$

Parameters m^i and b^i determine the shape of the force of mortality function. The survival probabilities can now be expressed as follows:

$$p_t^i = \exp\left(-\int_0^1 \lambda_{t+s}^i ds\right) = \exp\left(-\exp\left[\frac{t-m^i}{b^i}\right]\cdot\left(\exp\left[\frac{1}{b^i}\right]-1\right)\right).$$

Additionally, we model the subjective force of mortality as linear transformations of the force of mortality used for annuity pricing to incorporate asymmetric mortality beliefs. Then we get for the subjective force of mortality and the subjective probabilities:

$$\lambda_t^s = \nu\lambda_t^a,\ p_t^s = \exp(-\nu)p_t^a.$$

F. Wealth Accumulation

At each point in time the investor has to make a decision how to spread cash on hand W_t across bonds, stocks, annuities, and consumption. Therefore, the budget constraint is

$$W_t = M_t + S_t + PR_t + C_t, \tag{2}$$

where $M_t + S_t$ denote the value of financial wealth, M_t is the absolute wealth amount invested in bonds and S_t the amount invested in stocks. PR_t is the amount that the investor pays for an annuity and C_t is consumption. The individual's cash on hand in $t + 1$ is given by

$$W_{t+1} = M_t R_f + S_t R_{t+1} + L_{t+1} + Y_{t+1},$$

where $M_t R_f + S_t R_{t+1}$ denote the next period value of financial wealth, L_{t+1} is the sum of annuity payments which the investor gets from previously purchased annuities and Y_{t+1} is her labor income. The sum of annuity payments follows the process

$$L_{t+1} = L_t + PR_t / a_t,$$

where L_t is the sum of all annuity payments from annuities purchased before t and PR_t / a_t is the annuity payment purchased in t. In $t + 1$ the investor has to make a new decision how to spread her cash on hand W_{t+1} across bonds, stocks, annuities, and consumption. At this point we want to highlight our assumption that the investor is not restricted to use annuity payouts for consumption purposes only, as in *Blake et al.* (2003), *Milevsky and Young* (2002), *Kingston and Thorp* (2005) and *Stabile* (2003). The investor has full flexibility in spending the annuity payouts. They can be used to consume, to purchase bonds or stocks or even to purchase additional annuities. Additionally, we impose borrowing constraints:

$$M_t, S_t, PR_t \geq 0_t, \tag{3}$$

since we do not allow the investor to borrow against future labor income and to sell life-annuities. Hence, from the individual's perspective, the premium paid initially cannot be recovered. If she dies, her bequest B_t will be given by the remaining financial wealth $B_t = M_{t-1} R_f + S_{t-1} R_t$.

G. The Numerical Solution of the Optimization Problem

The problem of the individual is to choose in each year how much she consumes, saves in stocks and bonds, and how much she invests in life-annuities. Thereby she maximizes (1) under consideration of the budget and short-selling restrictions (2) and (3). The optimal policy depends on four state variables: the permanent income P_t, cash on hand W_t and annuity payouts from previously purchased annuities L_t and age. As an analytic solution to this problem does not exist, we use dynamic programming techniques to maximize the value function by backward induction.

First of all, the curse of dimensionality (*Bellman*, 1961) can be partly mitigated by reducing the state space by one state variable. We exploit the scale independence of the optimal policy if we rewrite all variables using lower-case letters as ratios of the permanent income component P_t.

The goal function (1) can then be rewritten as

$$v_t(w_t, l_t, t) = \left\{ \left(1-\beta p_t^s\right) c_t^{1-1/\psi} + \beta E_t \left[p_t^s v(w_{t+1}, l_{t+1}, t+1)^{1-\rho} + \left(1-p_t^s\right) k \frac{\left(b_{t+1}/k\right)^{1-\rho}}{1-\rho} \right]^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}, \quad (4)$$

where the only state variables are normalized cash on hand and normalized annuity payouts. The evolution of the state variables and restrictions are then given by

$$\begin{aligned}
& w_t = m_t + s_t + pr_t + c_t && \forall t \\
& m_t, s_t, pr_t \geq 0 && \forall t \\
& w_{t+1} = \left[m_t R_f + s_t R_{t+1}\right]\left(N_{t+1}\right)^{-1} + l_{t+1} + \exp\left(f(t+1)\right) U_{t+1} && \forall t < K \\
& l_{t+1} = \left[l_t + PR_t / a_t\right]\left(N_{t+1}\right)^{-1} && \forall t < K \\
& w_{t+1} = m_t R_f + s_t R_{t+1} + l_{t+1} + \varsigma \exp\left(f(K)\right) && \forall t \geq K \\
& l_{t+1} = l_t + PR_t / a_t && \forall t \geq K.
\end{aligned}$$

We solve the problem in a three-dimensional state space by backward induction. The continuous state variables normalized wealth w_t and normalized annuity payouts l_t have to be discretized and the only discrete state variable is age t. For each grid point we calculate the optimal policy and the value of the value function. Thereby the expectation operator in (4) is computed by resorting to gaussian quadrature integration and the optimization is done by numerical constrained minimization. We derive the policy functions $s(w,l,t)$, $m(w,l,t)$, $pr(w,l,t)$, $c(w,l,t)$ and the value function $v(w,l,t)$ by cubic-splines interpolation.

H. Calibration of Parameters in the base-line case

The following specifications of parameters define the analyzed base-line case. The individual life-span ranges from age 20 to age 100 (T = 81) at most. Retirement starts at age 65 (K = 46). Hence, work life is 45 years long while the maximum length of the retirement phase is 36 years. The preference parameters are set to standard values found in the life-cycle literature (e.g. *Gomes and Michaelides*, 2005): coefficient of relative risk aversion ρ = 5, elasticity of intertemporal substitution ψ = 0.2, discount factor β = 0.96, and bequest weight

k = 2. Since the empirical evidence on bequest motives is somewhat ambiguous (e.g., *Bernheim et al.*, 1985, and *Hurd*, 1987), we display results for various degrees of bequest strength.

The deterministic age-dependent labor income function $f(t)$ is taken from *Cocco et al.* (2005). The functional dependence reproduces a hump shaped income profile. Like *Gomes and Michaelides* (2005) we select volatility parameters for individuals with high school education but without college education and set them to $\sigma_u = 0.15$ and $\sigma_n = 0.1$ which is in line with the estimates found by *Gourinchas and Parker* (2002). The replacement ratio including accumulated pensions from Social Security but excluding voluntary annuitization is set to 68.2 percent as currently estimated by *Cocco et al.* (2005). Since we expect a very strong relationship between the replacement ratio and the optimal life annuity allocation we do some sensitivity analysis below the current figure of 68.2 percent. Thus, we are able to analyze scenarios of sinking public pensions to various degrees.

We set the real interest rate R_f to 2 percent, the equity premium $\mu - R_f$ to 4 percent and stock volatility σ to 18 percent which is in line with the recent life-cycle literature. The correlation between the stock returns and the transitory (permanent) income shocks ϕ_n (ϕ_u) is zero.

The expense factor δ is set to 7.3 percent for female annuitants. This factor is taken from the 1995 annuity value per premium dollar computed on an after tax basis by *Mitchell et al.* (1999). We refer the interested reader to this article for a greater discussion of the explicit and implicit costs related to annuities. Applying nonlinear least squares we fit the Gompertz force of mortality to two discrete mortality tables: the *1996 US Annuity 2000 Aggregate Basic* and the *2000 Population Basic* mortality table. The least square method gives us the following parameters for the *1996 US Annuity 2000 Basic (female)* table m_f^a = 90.51, b_f^a = 8.73 respectively. For the *2000 Population Basic* mortality table we compute parameters for females m_f^a =86.85, b_f^a = 9.98 respectively. While the first discrete mortality table is used for annuity pricing, the second mortality table describes the individual's subjective mortality beliefs in the base-line case. In the later analysis of asymmetric mortality beliefs we also consider a case with lower than population survival probabilities. To double the force of mortality we set the parameter v to two.

III. Results with Annuity Market and without Annuity Market for the Base-line Case

A. Presence of Annuity Markets.

A.1. Policy Functions. For our base-line case of an average US female with a high school degree, the policy functions show how the individual is influenced in her decision making by each state variable and age as well. Figure (2) depicts the optimal consumption level $c(w,l,t)$, annuity purchases $pr(w,l,t)$, bond holdings $m(w,l,t)$ and stock investments $s(w,l,t)$ in four separate graphs. Policies are conditional on surviving to a specific age. The policy starts at age 20 and ends at age 100. We set the state variable annuity payments l to zero when plotting figure (2). This means that no annuities have been purchased before.

The topology of the consumption policy in graph (A) of figure (2) is almost flat for most of the age-wealth states except for slight increases with higher wealth levels and surges for a very old individual until age 100. The individual consumes only a small part of wealth on hand during most of her life. This is because the individual seeks to cushion short run adverse developments in the income stream and especially the drop in retirement income relative to labor income by saving financial wealth. Furthermore, she wants to leave a certain amount of financial wealth to bequeath her heirs. If the individual turns very old and has a lot of cash on hand, the retiree will start consuming more of her financial wealth since her mortality becomes very high and longevity risk less critical. Due to her bequest motive she never consumes her whole financial wealth and always reserves a certain amount for bequest.

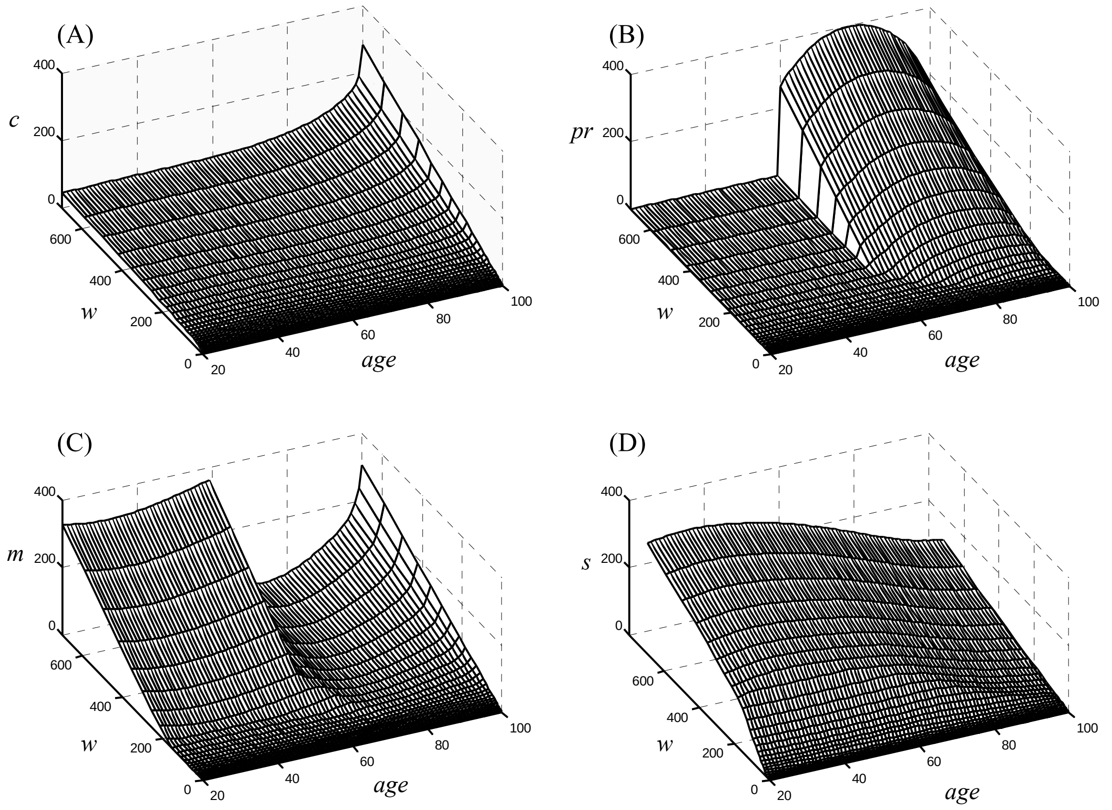

Figure 2. Optimal policy space.

Optimal policy functions for (A) consumption level, (B) annuity purchases, (C) bond investment, and (D) stock investment. The x-axis represents the individual's age, the y-axis the level of normalized cash on hand.

Thus, she aims at achieving a precision landing in terms of consumption and financial wealth according to her bequest motive.

Stock investments in graph (D) swing up as the level of financial wealth rises for any given age while in general stock investments decrease with age. With stock holdings decreasing both bonds and annuities become more important over the remaining life-cycle. This result is in line with recent life-cycle literature and with recommendations made by practitioners as well as policy makers. The reason is that the young individual is over-invested in her human capital which is the present value of labor and pension income. Even though labor income is risky and uninsurable, bonds are considered as a closer substitute for human capital than stocks during work life because the implicit discounting of future income is more considered than its volatility. During the retirement phase human capital represents the present value of the riskless pension income. Then, human capital is an implicit annuity holding because it perfectly resembles its payout structure. Her human capital decreases with age and hence the implicit holdings in bonds and annuities as well. In turn, wealth is increasingly composed of explicit holdings of the latter two assets.

Considering graph (B) of figure (2), the reader can infer the policy functions for new annuity purchases. Even though the recent retirement literature regarding annuities and common wisdom suggests treating payout life-annuities as a vehicle to realize consumption after the individual retires, we find that she actually wants to substantially purchase annuities from age 60 on for most states of financial wealth. The lower the financial wealth, the later the individual starts to buy annuities. If financial wealth is sufficiently high annuity purchases will rise until the female retiree becomes 80 years old and will start decreasing thereafter. Yet, the individual never buys annuities if financial wealth remains very low. The reason is that bonds and stocks are preferred to annuities because the individual has a bequest motive and her pension income crowds out the annuity demand. Interestingly, if the level of financial wealth rises, the demand for annuities will surge relative to bonds. In the last period, at age 100, the individual does not purchase annuities anymore because she cannot survive another period in our model and annuity payouts cannot be transferred to her heirs. Comparing graph (B) and graph (C) the reader can clearly see that annuity purchases are realized at the expense of bond savings. From age 60 on, the increasing mortality credit and the need to hedge longevity risk make annuities more attractive relative to bond savings until age 80. At the end of the life-cycle, the investor reduces her life annuity purchases and shifts back to bonds in order to be able to leave bequest for her heirs. Bond investments again become more attractive relative to annuities at the very end of the life-cycle because the bequest motive becomes stronger and longevity risk is less critical. With mortality being especially high at the end of the life-cycle, the mortality credit itself is still not high enough to avoid a decrease in life-annuity purchases and an increase in bond investments. For the case with previously purchased annuities ($l > 0$) the shape of the annuity policy is similar except that the amount of new annuity purchases would decrease.

A2. Life-Cycle Profiles and Asset Allocation. In order to compute the expected life-cycle profile, we resorted to Monte Carlo methods. We simulated 100,000 life cycles for the base-line case scenario to compare the expected consumption, wealth, income, annuity purchase, and annuity payout path.

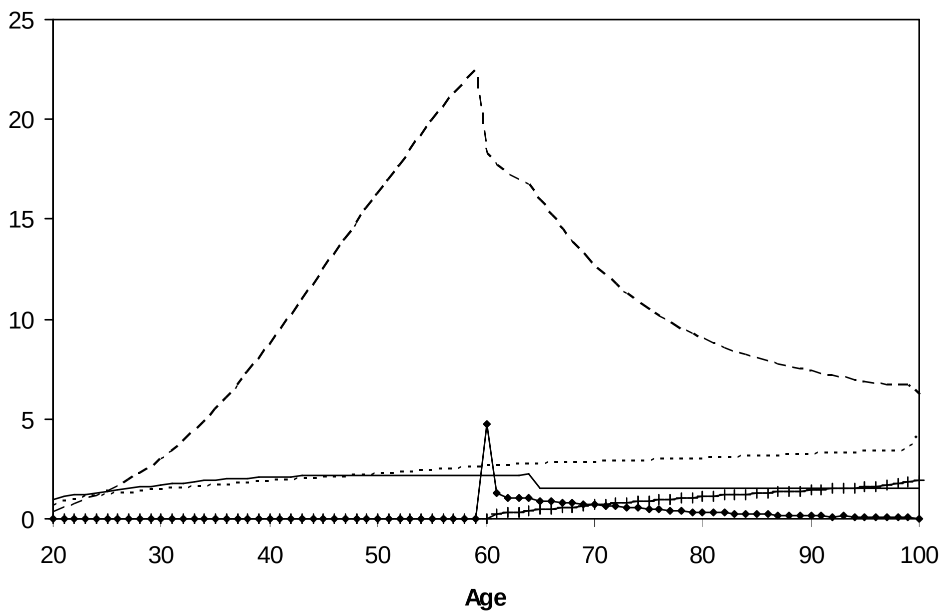

Figure 3. Expected life-cycle profile with annuities.

The dashed line depicts the expected financial wealth path. The dotted line is the expected consumption path. The solid line reflects the expected labor and pension income. The solid line with asterisks represents annuity purchases, and the line with crosses annuity payouts.

Clearly, the income profile is hump shaped. First her income increases then it slightly backslides. At the beginning of retirement, her last income is replaced by a pension payment that is exactly 68.2 percent of her previous labor income. While there is a sharp drop in income, the consumption path remains smooth. We find that the female investor saves from her labor income until she turns 50 years old in expectation. Thereafter, she already starts divesting in order to realize consumption before the actual retirement begins (please see consumption-income-ratio in table (1)). Even so, her financial wealth increases until she reaches age 60 and it peaks at 11.73 times the labor income (compare table (1)). Up to this point she withdraws just from capital gains. Financial wealth and bequest potential remain at a substantial level until age 100. The first time the investor is expected to purchase annuities is age 60. She uses 18.59 percent of her cash on hand in order to buy annuities (compare table (1)). Throughout retirement, she continues annuitizing part of her wealth in expectation.

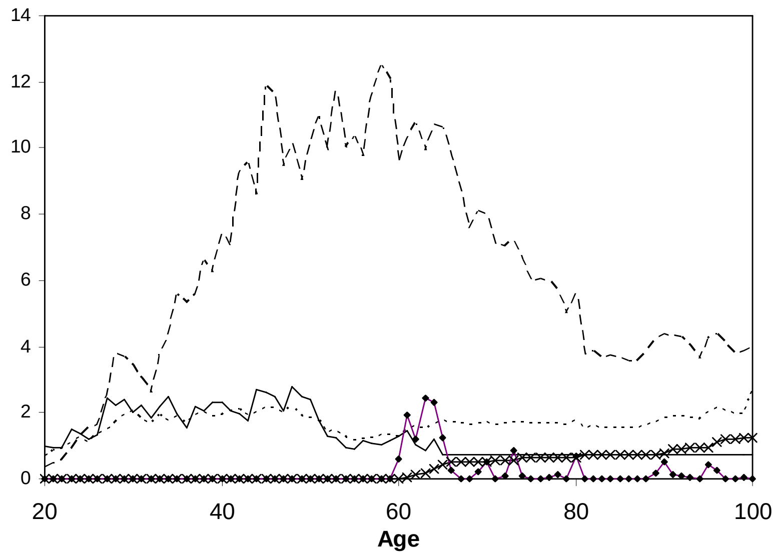

Figure 4. One trajectory of the individual's life-cycle.

The dashed line depicts the financial wealth path. The dotted line is the consumption path. The solid line reflects the labor and pension income. The solid line with asterisks represents annuity purchases and the line with crosses annuity payouts.

Sample paths also show that annuitization is pursued step by step over time (compare the example trajectory of figure (4)). Switching strategies (*Blake et al.* (2003), *Milevsky and Young* (2002), *Kingston and Thorp* (2005) and *Stabile* (2003)) are therefore generally suboptimal in our model.

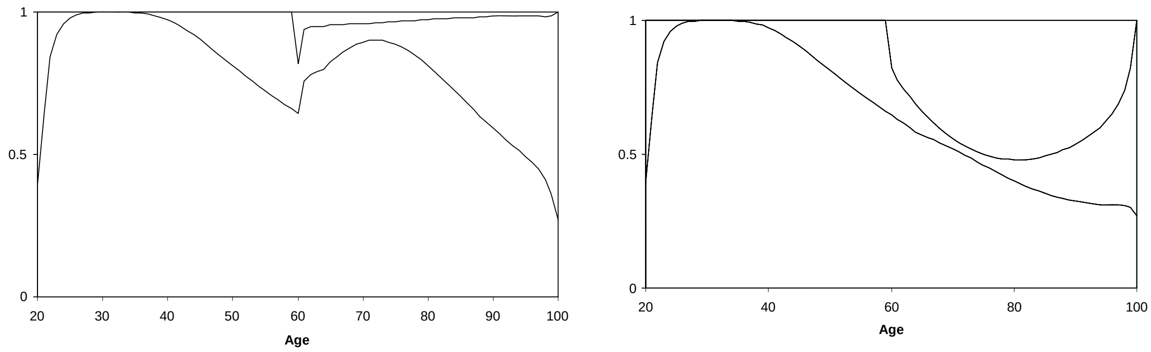

Figure 5. Expected asset allocation.

Left graph: the upper right cut area depicts the purchases of new annuities relative to the sum of stock holdings, bond holdings and annuity purchases. The middle area shows the fraction of bonds, the bottom area reflects the fraction of stocks. Right graph: the upper right cut area depicts the present value of annuities relative to all investment holdings (stocks, bonds and annuity present value). The middle area shows the fraction of bonds, the bottom area is the fraction of stocks.

The savings behavior in our model also suggests that the division of the life-cycle into work life a.k.a. accumulation phase and retirement a.k.a. decumulation phase is not fully adequate since disinvesting and annuitization can occur prior to retirement.

Expected asset allocations are given in figure (5). The left hand graph highlights new annuity purchases relative to the sum of stock holdings, bond holdings and annuity purchases. For ages over sixty the individual buys annuities with initially high and then continuously decreasing purchases. The right hand graph displays the annuity fraction not as newly purchased annuities, but as the present value of all annuities bought. For ages over sixty the asset allocation shifts from bonds to annuities in expectation. This substitution effect is in line with the policy functions given in figure (2). From age 60 on, annuities become more attractive relative to bonds because the mortality credit is now high enough to compensate the individual for the inflexibility drawbacks of annuities.

Table 1

Base case simulation results

	Wealth	Contribution and Withdrawals			Annuity Purchases		Asset Allocation			Consumption		
Age	Cash on Hand-Income-Ratio	Contribution-Cash on Hand Ratio	Contribution-Income-Ratio	Contribution over Income and Annuity Payouts	Annuity Purchases-Cash on Hand Ratio	Annuitized Withdrawal Fraction	Fraction of Stock Investments	Fraction of Bond Investments	Fraction of the Present Value of Annuities	Consumption-Cash on Hand Ratio	Consumption-Income-Ratio	Consumption over Income and Annuity Payouts
25	1.88	0.06	0.11	0.11	0%	0%	97.98%	2.02%	0%	46.99%	0.89	0.89
30	2.62	0.05	0.14	0.14	0%	0%	99.97%	0.03%	0%	32.91%	0.86	0.86
35	3.67	0.04	0.14	0.14	0%	0%	99.78%	0.22%	0%	23.41%	0.86	0.86
40	5.04	0.02	0.11	0.11	0%	0%	97.31%	2.69%	0%	17.67%	0.89	0.89
45	6.66	0.01	0.05	0.05	0%	0%	90.45%	9.55%	0%	14.21%	0.95	0.95
50	8.42	0.00	-0.02	-0.02	0%	0%	81.43%	18.57%	0%	12.14%	1.02	1.02
55	10.18	-0.01	-0.11	-0.11	0%	0%	72.69%	27.31%	0%	10.90%	1.11	1.11
60	11.73	-0.20	-2.35	-2.35	18.59%	92.95%	64.74%	17.43%	17.83%	10.19%	1.19	1.19
65	13.33	-0.08	-1.11	-0.85	4.38%	52.39%	57.23%	8.99%	33.78%	13.84%	1.85	1.42
70	11.26	-0.08	-0.87	-0.60	4.13%	53.17%	52.10%	3.81%	44.09%	16.81%	1.89	1.30
75	9.87	-0.07	-0.65	-0.41	3.29%	49.72%	45.97%	4.06%	49.97%	19.77%	1.95	1.21
80	8.95	-0.05	-0.48	-0.28	2.48%	45.90%	40.09%	7.86%	52.05%	22.44%	2.01	1.16
85	8.44	-0.04	-0.37	-0.20	1.71%	38.98%	35.46%	13.78%	50.76%	24.68%	2.08	1.13
90	8.12	-0.04	-0.29	-0.15	1.14%	31.37%	32.44%	21.26%	46.30%	26.61%	2.16	1.10
95	7.94	-0.03	-0.24	-0.12	0.90%	29.18%	31.15%	31.13%	37.72%	28.19%	2.24	1.08

This table reports the summary results of the Monte-Carlo simulation for the base-line case. The parameters for the base-line case are given in section III A1. All reported figures in this table show averages which are based on 100,000 simulated life-cycles. The table reports the cash on hand-income-ratio; contribution-cash on hand-ratio; contribution-income-ratio; contribution over income and annuity payouts; annuity purchases-cash on hand-ratio; annuitized withdrawal fraction; fraction of stock investments; fraction of bond investments; fraction of present value of annuities; consumption-cash on hand-ratio; consumption-income-ratio; consumption over income and annuity payouts.

Surprisingly, as the individual gets older, the weight of the annuity present value reduces while the fraction of bond holdings increases, even though the individual continues purchasing new annuities. The higher the age, the lower becomes the annuitant's survival probability and hence the value of already purchased annuities. This decrease in the value of annuity holdings outweighs the new annuity purchases. At a very high age bonds are more attractive relative to annuities and stocks because longevity risk becomes smaller and the impact of the bequest motive stronger.

Until age 40 the fraction of stocks is around 100 percent at young ages and monotonically decreases to 26.95 percent. This result is again in line with the previous life-cycle literature and recommendations of policy makers promoting decreasing stock exposures.

B. Absence of Annuity Markets.

B.1. Policy Functions. The first graph of figure (6) shows the optimal consumption policy that can be easily compared to the case with annuity markets because of the similarity between the optimal consumption rules. But the policy recommends consuming less at the end of the life-cycle than in a situation with annuity markets. This result does not come as a surprise since the key insight from investing in annuities is to realize life-long streams of consumption. Without annuity markets longevity risk prevails while bequest and exhaustion of financial wealth becomes critical.

The policy for stock investments appears similar to the case with annuity markets. Again, shrinking human capital is responsible for the decreasing stock exposure over time. The individual cannot fall back on annuity payouts and has to rely on bonds to mitigate longevity risk. The risk of running out of funds and the possibility of not meeting the bequest motive can be more effectively reduced by purchasing bonds instead of stocks.

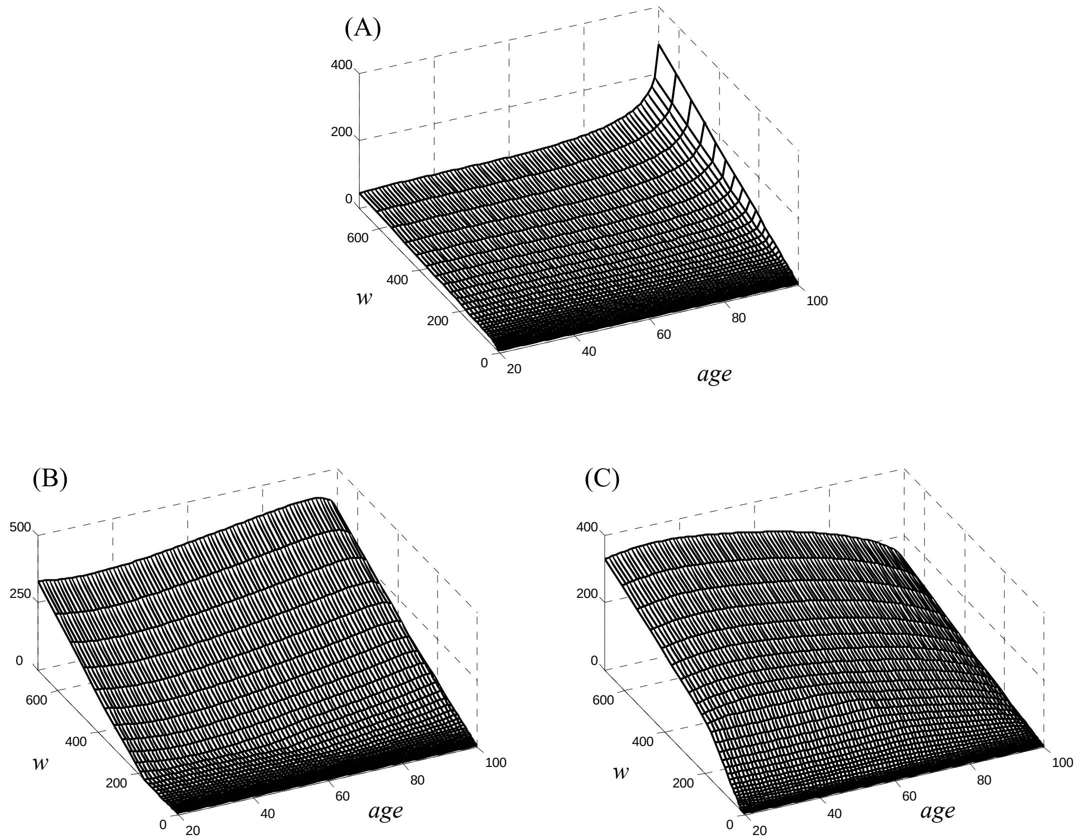

Figure 6. Optimal policy space.

Optimal policy functions for (A) consumption level, (B) bond investment, and (C) stock investment. The x-axis represents the individual's age. The y-axis-represents the level of normalized cash on hand.

B.2. Life-Cycle Profiles and Asset Allocation. The left hand graph of figure (7) highlights the much higher financial wealth when the investor is about to retire compared to the case with annuity markets. Actually, the financial wealth peaks at age 64 when it is 28.7 times the initial income compared to the peak (25.65 times the initial income) at age 60 with annuity markets. The individual needs higher financial wealth to mitigate part of the longevity risk and the related risk of leaving no bequest for the heirs. Unlike the annuity case the absolute level of consumption decreases at the end of the life-cycle.

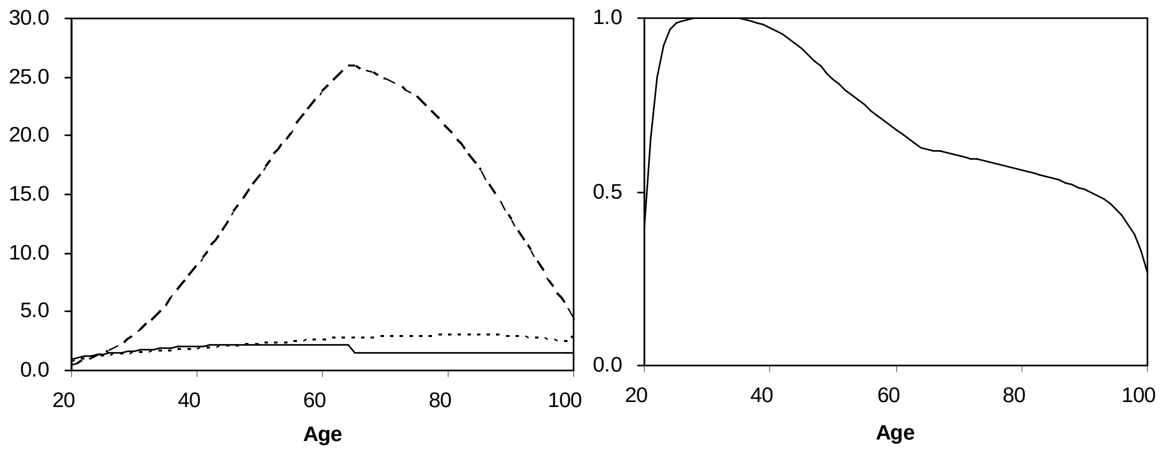

Figure 7. Expected life-cycle profile and asset allocation without annuity market.

Left graph: the dashed line depicts the expected financial wealth path. The dotted line is the expected consumption path. The solid line reflects the expected labor and pension income. Right graph: the uppermost cut area depicts the investments in bonds. The bottom area is the investment holdings in stocks.

The right graph of figure (7) displays the unconditional mean asset allocation in equities and bonds. As in the case with annuity markets the expected fraction invested in stocks is around 100 percent for individuals until age 40, and then decreases continuously down to 27 percent.

IV. Optimal Expected Life-cycle Annuity Investments for Alternative Cases

A. Risk Aversion

Varying the level of risk aversion dramatically changes the optimal asset allocation of the individual. We find that the less risk-averse individual hardly buys annuities in expectation. Only at age 88 is she willing to buy a small amount of annuities. Expected financial wealth of less risk-averse individuals is also relatively high compared to the base-line case. This result does not stem from high contributions but rather from a high fraction of stocks in financial wealth. A higher degree of risk aversion than in the base-line case goes hand in hand with a very high demand for annuities which e.g. at age 60 amounts to 67 percent of financial wealth.

Low risk aversion ($\rho = 2$) | Moderate risk aversion ($\rho = 5$) | High risk aversion ($\rho = 10$)

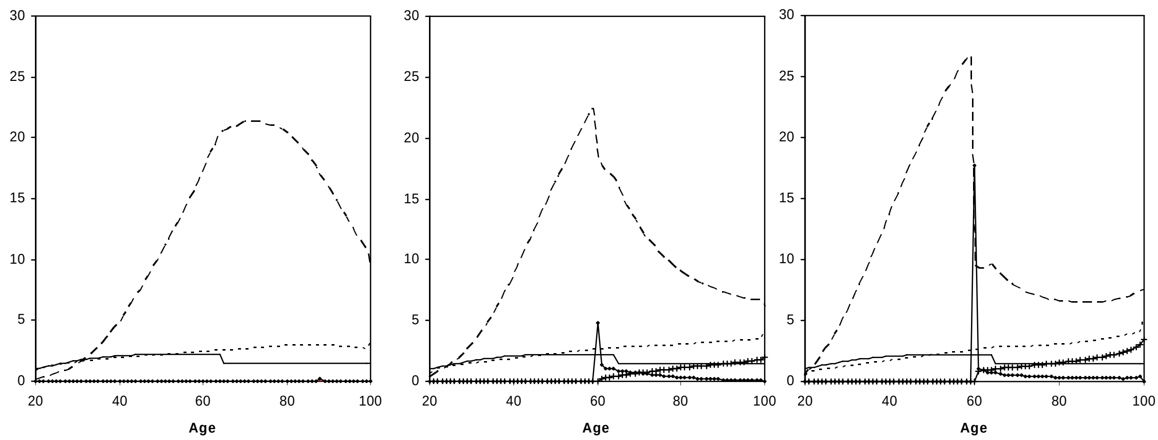

Figure 8. Life-cycle profiles and annuity purchases for parameters of relative risk aversion.

The left hand graph displays a low risk aversion of ρ = 2 while the middle graph reflects a moderate level (ρ = 5). The high level of risk aversion (ρ = 10) is displayed in the right hand graph. Dashed lines are financial wealth. Dotted lines reflect consumption, solid lines income, and the solid lines with asterisks reflect the annuity purchases. The lines with crosses show the annuity payouts.

Also the desire for precautionary savings is much higher than in the base-line case. Strikingly, the high risk-averse investor uses cash flows from annuities and public pensions to reinvest them into financial wealth mainly consisting of bonds to ensure leaving sufficient bequest to her heirs.

B. Implications of Bequest Motives

Empirical studies such as *Kotlikoff and Summers* (1981) found that almost 80 percent of the total accumulated wealth in the United States is due to intergenerational transfers. This stylized, empirical fact raises the question as to whether bequests are accidental or intentional. The literature on intentional bequests distinguishes between altruistic and strategic bequest motives as opposite ends of the spectrum.

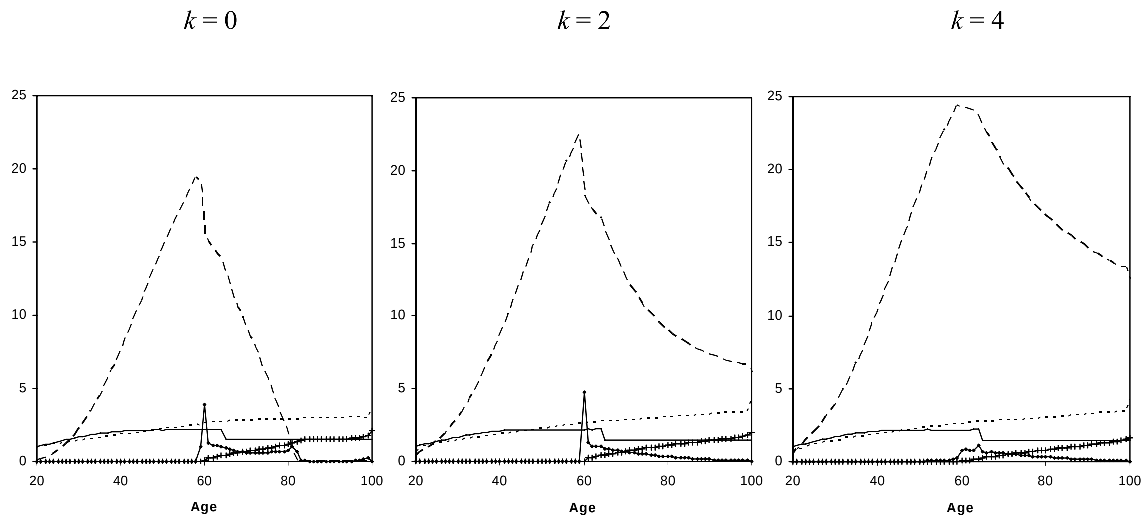

Figure 9. Life-cycle profiles and annuity purchases for different bequest motives.

The left hand graph shows the case in which the individual does not have any bequest motive at all. The middle graph displays the base-line case, while the case of the strong bequest motive is shown in the right graph. Dashed lines are financial wealth. Dotted lines reflect consumption, solid lines income, and the solid lines with asterisks reflect the annuity purchases. The lines with crosses show the annuity payouts.

For instance, *Abel and Warshawsky* (1988) study the altruistic bequest motive in a reduced form and find a joy of giving parameter that is of a substantial magnitude. *Bernheim et al* (1985) analyze the strategic bequest motive and discover empirical evidence. By contrast, *Hurd* (1987) does not find any evidence of bequest motives because the pattern of asset decumulation is similar among different household sizes. In addition, *Hurd* (1989) can support his prior findings by showing that the nature of most bequests is accidental because the date of death is uncertain to an individual. Since the results of these studies seem somewhat ambiguous, we present cases with varying bequest motives in our model.

The left graph of figure (9) illustrates the case in which the individual has no bequest (k = 0). We find that expected financial wealth is the lowest in this case compared to the cases with k = 2 and k = 4. At age 83 she has exhausted her financial wealth completely. This also means that she won't have anything to bequeath thereafter. From that age on she stops purchasing annuities and exclusively uses public pension income as well as annuity income from previously purchased annuities for consumption purposes only. If she survives until age 97 she starts purchasing annuities again because the mortality credit of annuities is extraordinarily high due to the small survival probabilities. In this way she boosts consumption possibilities conditional on her survival.

In cases with bequest, she never exhausts her financial wealth completely. She always keeps a certain liquid capital stock in bonds and stocks on hand in order to guarantee bequest in case she dies. The higher the bequest motive, the more the individual saves in stocks and bonds (dashed line in figure (9)). She prefers liquid financial wealth to annuity payments that last a life-time and cannot be transferred to her heirs. Our results support common wisdom that annuity purchases are especially preferable to other asset classes if the individual has no bequest motive. With decreasing bequest motive, the individual increases the weight of annuities purchases relative to the size of the asset portfolio and purchases more annuities over time. However, even with moderate or high bequest motives there is a remarkable demand for annuities. Surprisingly, the absolute demand for annuities at age 60 is higher in the case in which the individual has a moderate bequest motive compared to the case with no bequest motive. The reason is that the individual without a bequest motive already starts to purchase annuities at age 59.

C. Asymmetry in Mortality Beliefs

We use the discrete mortality *1996 US Annuity 2000* table for pricing annuities. Survival probabilities entering the utility function as well as the computation of the annuity premium are identical for the base-line case. Applying nonlinear least square we fit the Gompertz force of mortality to two discrete mortality tables: the *1996 US Annuity 2000 Aggregate Basic* and the *2000 Population Basic* mortality table. Figure (10) shows the fitted conditional survival probabilities for females from the time they are born to the age of 110.

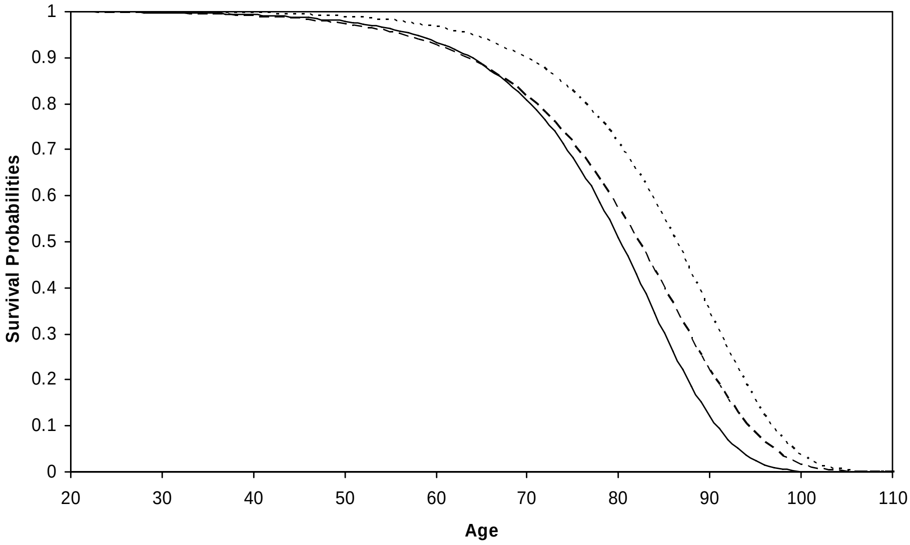

Figure 10. Fitted survival probabilities.

The dotted line shows the conditional survival probabilities according to the fitted US Annuitant 2000 Gompertz law. The dashed line depicts the survival probabilities, if the Gompertz law is fitted to the 2000 population basic mortality table. The solid line is a linear transformation (v = 2) of the Gompertz law for the US Annuitant 2000 mortality table.

Survival probabilities for the transformation (v = 2) range for most ages below the 2000 population basic mortality table. US Annuitant 2000 survival probabilities are by far higher than the 2000 population basic survival probabilities and in particular higher than the transformation (v = 2). Insurance companies calculate annuity premiums from higher survival probabilities as a result of the adverse selection process since individuals who believe themselves to be healthier than average are more likely to buy more annuities (e.g. *Brugiavini* (1993)). The magnitude of asymmetry in mortality beliefs is reflected by the implicit costs the annuitant has to bear when she purchases an annuity. The higher the asymmetry in mortality beliefs, the higher are the implicit costs of annuities from the view of the individual.

1996 US Annuity 2000

Aggregate Basic 2000 Population Basic Doubled force of mortality

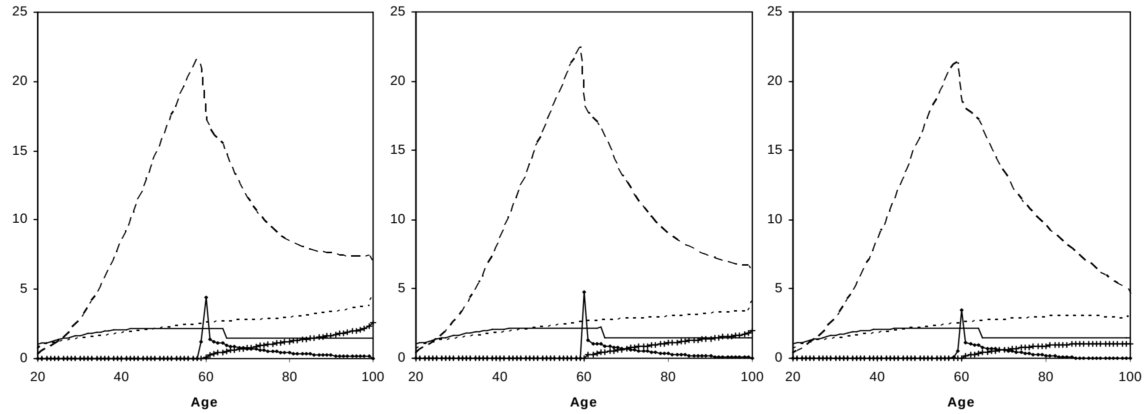

Figure 11. Life-cycle profiles and annuity purchases for different subjective survival probabilities.

The left graph displays the case in which survival probabilities are identical to the underlying mortality 1996 US Annuity 2000 Aggregate Basic table used for annuity pricing. The middle graph reflects the base-line case with 2000 population basic probabilities. The right hand graph shows the case when the force of mortality is twice as high as in the Annuitant 2000 table. Dashed lines are financial wealth. Dotted lines reflect consumption, solid lines income, and the solid lines with asterisks reflect the annuity purchases. The lines with crosses show the annuity payouts.

These higher implicit costs make the purchase of annuities more unattractive because of reduced mortality credits resulting in lower annuity demand. However, even a female with a doubled force of mortality buys considerable amounts of annuities since she is still willing to accept high premiums to hedge longevity risk.

D. Different Levels of Public Pensions

As benefits from public pensions are identical to payout structures of life-annuities, the latter product is undoubtedly a perfect substitute for public pensions. The obvious difference is in the way of funding the future pension payments.

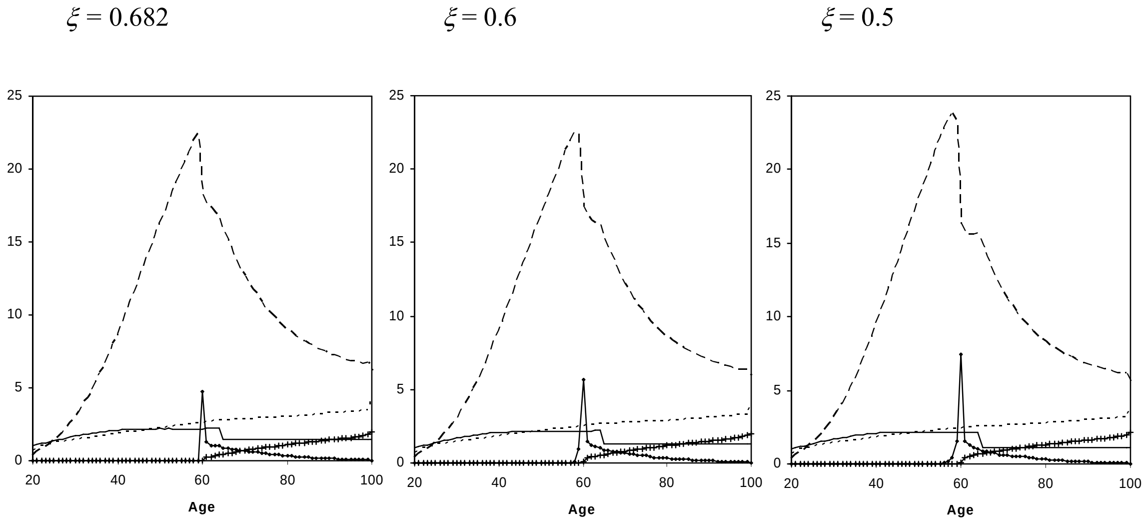

Figure 12. Life-cycle profiles and annuity purchases for different replacement ratios.

The right and the middle graph display the case in which public pensions are cut to a replacement ratio of 50 and 60 percent, respectively. The left hand graph displays the base-line case. Dashed lines are financial wealth. Dotted lines reflect consumption, solid lines income, and the solid lines with asterisks reflect the annuity purchases. The lines with crosses show the annuity payouts.

Most public pension systems are based on an inter-generational contract whereby the generation of Social Security contributors finances the generation of public pension beneficiaries. The pay-as-you-go public pension systems are running into trouble since longer life-expectancies and lower birth rates lead to decreasing ratio of contributors to beneficiaries. Realizing the circumstances, we analyze two cases in which the replacement ratio ζ of our model is cut from 68.2 percent first to 60 percent and then to 50 percent. We assume that the labor income process remains the same to reflect constant Social Security taxes and decreasing public pension payments. On the contrary, life-annuity payouts are funded by the beneficiary herself. Once she pays the annuity premium to the insurance company she receives annuity payouts until she passes away. Figure (12) shows the crowding-in effect into the annuity markets. As anticipated there is a substantial increase in expected annuity purchases when public pensions are cut. This means that the individual wants a substitute for public pension cuts. Direct investments in bonds and stocks increase to a moderate extent. For example in the case $\xi = 0.5$ the individual at age 60 is expected to purchase 56.8 percent more annuities than in the base-line case while investments in bonds and stocks rise moderately by 2.7 percent.

V. Welfare Analysis

The substantial demand for annuities suggests that considerable utility gains can be generated through the presence of annuity markets. We do a welfare analysis similar to *Mitchell et al.* (1999). In our analysis we first compute the expected utility of individuals living in a world with and without access to annuity markets separately. Of course, the expected utility is always higher for individuals who can voluntarily purchase annuities. Then we compute the equivalent increase in financial wealth for every age in order to measure the expected utility gains in monetary units. The equivalent increase in financial wealth refers to the compensation an individual requires to achieve the same utility level in a world without annuity markets as in the presence of them. Therefore, we equate the expected utility values of individuals with and without access to annuity markets by raising the financial wealth of individuals in the no-annuity case. For the base-line case, both graphs in figure (13) depict the equivalent increase in financial wealth due to the presence of annuity markets from the very beginning on.

Figure (13) shows that individuals who can buy annuities realize equivalent increases in financial wealth every year of their lifetime. Even at young ages individuals gain from annuity markets because they anticipate the indirect utility gains emerging at the end of their life-cycle. Especially for old individuals annuity markets imply a considerable wealth increase from 14.14 percent (or 3.36 times the first income) at age 80 up to 30.07 percent (or 3.22 times the initial income) at age 90. The presence of annuity markets allows the individual to finance consumption and bequest more effectively than in the case without annuities.

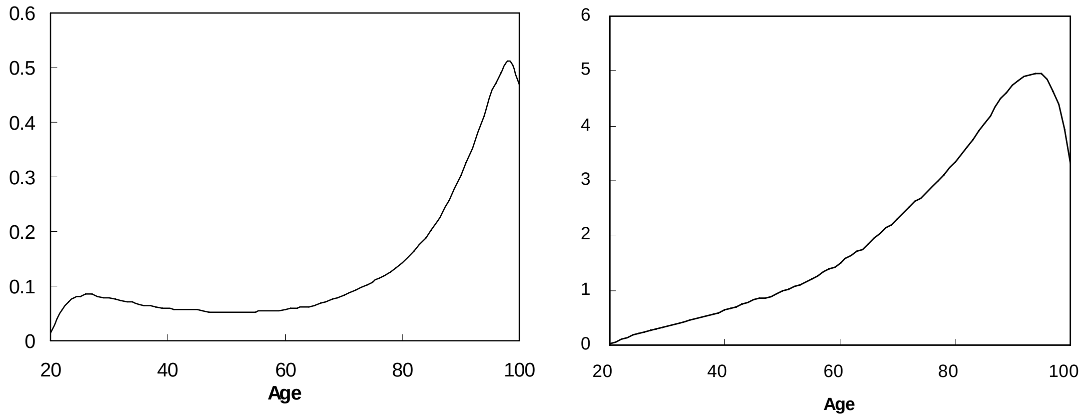

Figure 13. Equivalent relative and absolute increase in financial wealth.

Left graph: the solid line displays the percentage increase in financial wealth which the individual – who cannot buy annuities – needs in order to achieve the same utility as the individual who can buy annuities. Right graph: the solid line displays the increase in financial wealth as multiple of initial income which an individual without access to annuity markets requires in order to achieve the same level of utility as the individual who can buy annuities.

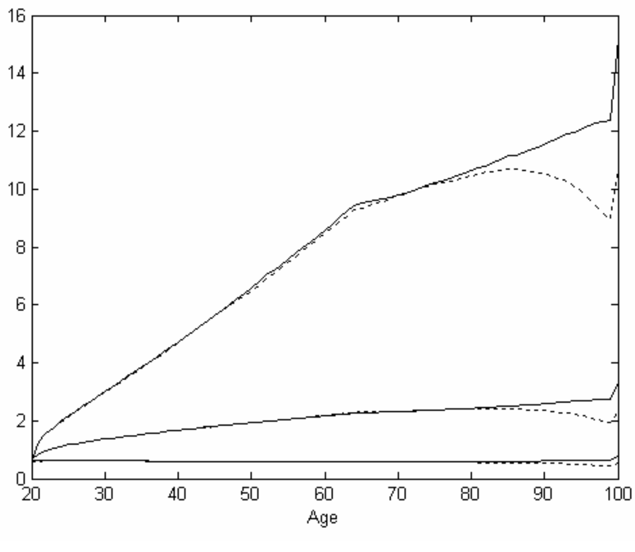

Figure 14. Consumption Percentiles with and without Annuity Markets.

Solid lines show the 1, 50 and 99 percentile of the consumption distribution in the case with annuity markets and dotted lines the percentiles for the case without annuity markets.

Table 2

Equivalent increase in financial wealth (percentage points)

	Age			
Cases:	60	70	80	90
Base-line case	5.69	8.43	14.14	30.07
Low risk aversion (ρ = 2)	0.00	0.00	0.42	1.02
High risk aversion (ρ = 10)	12.87	15.78	23.08	37.38
No bequest motive (k = 0)	9.54	12.79	16.51	31.16
High bequest motive (k = 4)	3.06	4.64	8.80	19.64
Bad survival probabilities (ν = 2)	3.97	6.09	10.77	26.19
Good survival probabilities ($p^s = p^a$)	7.19	9.73	15.42	30.23
Lowest pension income (ζ = 0.5)	6.87	8.75	14.18	30.19
Lower pension income (ζ = 0.6)	6.14	8.45	14.29	30.13

This table reports welfare gains in the presence of annuity markets for the different types of individuals we considered. Welfare gains are computed as the equivalent percentage increase in financial wealth an individual without access to annuity markets would need in order to attain the same expected utility as in the case with annuity markets. The computation is done for age 60, 70, 80 and 90. We assume that individuals have acted optimally until the specific year.

This stems from the fact that she can profit from consumption of life long annuity payments and hedge the longevity risk away. Another way of looking at it is to understand the effect of the longevity yield that increases with the holding period of life-annuities because the individual outlives her peers.

The equivalent increase in financial wealth can be attributed to advantages in consumption possibilities gained from the presence of the annuity markets. Figure (14) demonstrates that without annuity markets the individual's distribution of consumption is decreasing when she gets very old because the individual's financial wealth shrinks whereas the bequest motive is more significant. However, in the case with annuity markets the individual purchases annuities in a way that the consumption distribution does not decrease.

We also calculated the equivalent increases in financial wealth for all cases given in section IV by comparing the utility with that of individuals who do not have access to annuity markets. Table (2) shows that for all cases the pattern of equivalent increases in financial wealth over the life-cycle is similar to the one in the base-line case. Even in the cases with high public pensions, high bequest motives and high degree of asymmetric mortality beliefs annuity markets deliver substantial equivalent increases in financial wealth.

VI. Conclusion

This article introduces incomplete annuity markets to the life-cycle literature and in turn life-cycles to the insurance literature. Life-cycle modeling in this context becomes necessary as separate analyses of the accumulation and decumulation period can be misleading for the investor because decisions in both phases are intrinsically tied to each other.

Our analysis offers insights into the individual's demand for life annuities and the way she gains utility from annuity markets over the life-cycle. We find that the individual has demand for both flexible withdrawal possibilities from financial wealth and inflexible annuities. Strikingly, individuals start with high annuity purchases before retiring and continue annuitizing gradually and slowly over the remaining life-time. Hence, it is shown that switching strategies cannot be optimal. Computations of equivalent wealth increases show that life-annuities are indeed a good deal for the whole spectrum of individuals considered except for those with low risk aversion. This is somewhat surprising, since we considered individuals with already high pension income, strong bequest motives, and asymmetric mortality beliefs. The individual can realize significant increases in equivalent financial wealth if she outlives most of her peers and benefits from the related mortality credit. Annuity payouts enable her to hedge longevity risk as much as they contribute to enjoying a stable consumption stream during the whole retirement period.

Future work suggests itself. First, the suboptimality of annuity switching strategies is a research project we are currently working on, since these strategies have been considered in many other articles lately. Secondly, it is worthwhile accounting for alternative longevity insurance products such as deferred, equity-linked annuities, and life insurances. Thirdly, tax considerations must be incorporated to analyze how the asset allocation varies when taxable and tax deferred accounts are introduced. Finally, interest rate risk is an important risk factor determining the investment opportunity set and hence the individual's consumption, which makes the individual willing to hedge against adverse interest-rate developments.

REFERENCES

Abel, A.B. and M. Warshawsky, 1988, "Specification and the Joy of Giving: Insights from Altruism," *The Review of Economics and Statistics*, 70, 145--149.

Bellman, R., 1961, "Adaptive Control Processes: A Guided Tour," Princeton University Press, Princeton, New Jersey.

Bernheim, D.B., A. Shleifer and L.H. Summers, 1985, "The Strategic Bequest Motive," *The Journal of Political Economy*, 93, 1045--1076.

Bertaut, C. and M. Haliassos, 1997, "Precautionary Portfolio Behaviour from a Life Cycle Perspective," *Journal of Economic Dynamics and Control*, 21, 1511--1542.

Blake, D., A.J. Cairns, and K. Dowd, 2003, "Pension Metrics 2: Stochastic Pension Plan Design during the Distribution Phase," *Insurance: Mathematics and Economics*, 33, 29--47.

Brown, J.R., T. Davidoff and P.A. Diamond, 2005, "Annuities and Individual Welfare," forthcoming *American Economic Review*.

Brown, J.R., O.S. Mitchell, J.M. Poterba, and M.J. Warshawsky, 2001, "The Role of Annuity Markets in Financing Retirement," *MIT Press,* Cambridge, M.A.

Brugiavini, A., 1993, "Uncertainty Resolution and the Timing of Annuity Purchases", *Journal of Public Economics*, 50, 31--62.

Campbell, J.Y., J.F. Cocco, F.J. Gomes and P.J. Maenhout, 1999, "Investing Retirement Wealth? A Life-Cycle Model," Eds.: Risk Aspects of Social Security Reform, *University of Chicago Press*, Chicago.

Carroll, C., 1997, "Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis." *Quarterly Journal of Economics*, 112, 61--156.

Cocco, J.F., 2005, "Portfolio Choice in the Presence of Housing," *Review of Financial Studies*, 18, 535--567.

Cocco, J.F., F.J. Gomes, and P.J. Maenhout, 2005, "Consumption and Portfolio Choice over the Life Cycle," *Review of Financial Studies*, 18, 491--533.

Dammon, R.M., Spatt, C.S., and Zhang, H.H., 2004, "Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing," *Journal of Finance*, 59, 999--1039.

Davis, S. J., and P. Willen, 2000, "Occupation-level Income Shocks and Asset Returns: Their Covariance and Implication for Portfolio Choice," working paper, Graduate School of Business, University of Chicago, Chicago.

Deaton, A., 1991, "Saving and Liquidity Constraints," *Econometrica*, 59, 1221--1248.

Dushi, I. and A. Webb, 2004, "Household Annuitization Decisions: Simulations and Empirical Analysis," *Journal of Pension Economics and Finance*, 3, 109--143.

Epstein, L. and S. Zin, 1989, "Substitution, Risk Aversion and Temporal Behaviour of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937--969.

Gomes, F. and A. Michaelides, 2003, "Portfolio Choice with Internal Habit Formation: A Life Cycle Model with Uninsurable Labor Income Risk," *Review of Economic Dynamics*, 6, 729--766.

Gomes, F. and A. Michaelides, 2005, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, 60, 869--904.

Gourinchas, P.-O. and J. A. Parker, 2002, "Consumption over the Life Cycle," *Econometrica*, 70, 47--89.

Hurd, M.D., 1987, "Savings of the Elderly and Desired Bequests," *American Economic Review*, 77, 298--312.

Hurd, M.D, 1989, "Mortality Risk and Bequests," *Econometrica*, 57, 779--813.

Kapur, S. and M. Orszag, 1999, "A Portfolio Approach to Investment and Annuitization During Retirement," mimeo, Birbeck College, University of London, London, UK.

Kingston, G. and S. Thorp, 2005, "Annuitization and Asset Allocation with HARA Utility," *Journal of Pension Economics and Finance*, 4, 225--248.

Kotlikoff, L.J. and L.H. Summers, 1981, "The Role of Intergenerational Transfers in Aggregate Capital Accumulation," *Journal of Political Economy*, 89, 706--732.

Merton, R. C., 1971, "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3, 373--413.

Milevsky, M.A., 2005, "The Implied Longevity Yield: A Note on Developing an Index for Life Annuities," Journal of Risk and Insurance, 72, 302--320.

Milevsky, M.A. and V. R. Young, 2002, "Optimal Asset Allocation and the Real Option to Delay Annuitization," working paper. Schulich School of Business, York University, Toronto, Canada.

Milevsky, M.A. and V. R. Young, 2003, "Annuitization and Asset Allocation," working paper. Schulich School of Business, York University, Toronto, Canada.

Mitchell, O.S., J.M. Poterba, M. J. Warshawsky, and J.R. Brown, 1999, "New Evidence on the Money's Worth of Individual Annuities," *American Economic Review*, 89, 1299--1318.

Richard, S. F., 1975, "Optimal Consumption, Portfolio and Life Insurance Rules for an Uncertain Lived Individual in a Continuous Time Model," *Journal of Financial Economics*, 2, 187--203.

Stabile, G., 2003, "Optimal timing of the Annuity Purchases: A Combined Stochastic Control and Optimal Stopping Problem," working paper, Universita degli Studi di Roma, Rome, Italy.

Yaari, M.E., 1965, "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer." *Review of Economic Studies*, 32, 137--150.

Yao, R. and H.H. Zhang, 2005, "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints," *Review of Financial Studies*, 18, 197--239.