CCAPM, Wealth Shock, and Stock Market Anomalies

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Abstract

Capturing all non-financial wealth risks, we investigate how the expected and unexpected disposal non-financial wealth changes named wealth shock create a demand shock that is the extra of normal demand without it in the conditional CCAPM framework. These shocks affect the realized and expected returns of risky assets in the equilibrium and demand a wealth risk premium. These wealth risks provide a consumption-based theoretical and economical understanding for the pricing of value, size, momentum, liquidity risk, and the unexpected market illiquidity, the new issue puzzle, the negative predictability of aggregated new issues, and Equity Premium Puzzle in the stock markets. We present a simple testable model in which our wealth risk proxy explains a good portion of the equity excess return and the Fama-French 25 size and book-to-market portfolios excess returns. There is an equilibrium price for each risky asset even though there are heterogeneous wealth risks across the economy.

Keywords: consumption-based, wealth risk, demand shock, liquidity shock, risk premium, size, value, growth, momentum, dynamic equilibrium

JEL Classification: G11, G12, G15

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Introduction

The traditional conditional Consumption-based Capital Asset Pricing Model (CCAPM), a variation of Lucas (1978), provides powerful economic intuitions in understanding the exchange between the marginal substitution of consumption-based utility and risky asset return. It fails to explain stock market anomalies and phenomena as explained by Campbell and Cochrane (2000). The most striking concern is that the implied risk version from stock market return is unacceptably high with the smooth consumption growth. This is known as the equity premium puzzle of Mehra and Prescott (1985). Economists have made various efforts to rescue the elegant CCAPM. Abel (1990) and Constantinides (1990) introduce habits into utility. Campbell and Cochrane (1999) derive a consumption surplus growth as a latent variable that has higher variability than the consumption growth and reduce the implied risk aversion. This paper makes an economic extension to CCAPM by considering the expected and unexpected changes of non-financial wealth which we call wealth shock. It creates extra demand for risky assets in the market equilibrium, and affects their returns. Our wealth shock proxy accounts for 18% of the equity premium from 1948 to 2004. The risk aversion in our model can analytically only 40% what traditional CCAPM implied after considering the consumption surplus ratio in Campbell and Cochrane (1999).

To demonstrate our enhanced wealth shock stochastic discount factor can consistent price different stock market portfolios, we show that our proxy consistently have correct wealth risk premium for Fama-French 25 size and book-to-market portfolios. In our economy, we treat these wealth changes as exogenous which are time-varying and not generated from investment returns and we will not make decision on the allocation of non-financial wealth changes in the CCAPM framework. There are human capital shocks, technological shocks and other externality that can affect the financial and non-financial wealth state and level according to Endogenous Growth Models in Lucas (1988), Romer (1990) and Xie (1991). The real business cycle[1] of our economy implies that financial wealth fluctuates over time in various economic states. We should expect that the time varying disposal non-financial wealth changes affect our demands for risky assets and their realized and expected return. The unexpected changes create a system risk in the financial market other than the consumption risk. This wealth risk is significant to investors. For an example, any unexpected increase in our wealth level will allow us to invest more in the financial market and create higher demand whilst any decrease in our non-financial wealth will shrink our demand for financial asset as we need to withdraw capital from financial market to finance our consumption.

Our time-varying wealth shock is different from the consumption habit that Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999) introduced, where the consumption habit is in the utility and related the idea of “catching up with the Joneses”, and pro-cyclical. Our wealth shock risk premium is counter-cyclical the expected market returns. The hidden volatility of consumption growth captured by the consumption surplus ratio increase the denominator of the explicit equation of the risk aversion in CCAPM. The wealth risk premium come from any proportional wealth increase or wealth decrease that is not generated from investment. The aggregated non-financial or non-tradable wealth shocks in this paper include human capital shocks, technological shocks, property appreciation or depreciation adjust for mortgage payment as consumption and its expected return as the inflation rate, newly discovered natural resources, and other wealth created by other

[1] Long and Plosser (1991), Farmer and Guo (1994), and King and Rebelo (2000) present and discuss the real business cycle of the economy.

uninsurable shocks or transformed from other economy. Property is our wealth holding to flight against inflation in our economy.

Various efforts have been made to investigate the wealth effect in financial market. Setting expected return on labor income is constant over time and viewing the change in labor income as the return in human capital and part of market portfolio, Jagannathan and Wang (1996) show that the conditional CAPM performs better than traditional CAPM. Viceira (2001) showed that the idiosyncratic labor income risk makes optimal portfolio choice different between retired investors and non-retired investors. Lettau and Ludvigson (2001) presented that the time-varying consumption to wealth ratio is a strong predictor for stock and bonds under traditional CCAPM framework as this ratio captures higher volatility than the consumption growth. Assuming consumption financed by labor income and dividend streams in the investor's endowment and all financial assets are unconditionally identical, Santos and Veronesi (2006) show that the time-varying ratio of labor income to consumption as its financing share to our consumption affects the covariance of consumption growth and stock returns and improve the CAPM. Menzly, Santos, and Veronesi (2004) show that the effects of time-varying preference and the expected dividend growth have opposite implied relation between dividend yields and expected stock return when they assume total income equal to total consumption in equilibrium. Their finding provides an understanding why the observed weak predictability of dividend yield on expected stock returns and dividend growth. All these efforts show that the wealth to consumption ratios are time-varying and have effect on risky asset returns.

In this paper, we investigate how the expected and unexpected disposal non-financial wealth changes affect the demands for risky assets and their returns. In addition, these efforts have not investigated the implied effects of wealth changes on cross-sectional stock market anomalies from consumption-based economic approach. In the market equilibrium, we find that the expected and unexpected wealth changes create the expected and unexpected increase or decrease demands for financial assets. They have a impact on the aggregate market as a whole and also differently affect various equity assets base on their characteristics that are the size as the aggregate supply, the growth, and its realized return. When other things are equal, stocks with smaller supply (size) are more sensitive to any given level demand or demand changes, and face higher wealth shock risk. Investors will demand higher expected return as they are highly exploded to wealth shocks. For simplicity and without a lost of generality, we assume the stock return follows Gordon's growth model and substitute it into the demand expression. We find that the last period expected growth of stock reduce its sensitivity to wealth shocks and have lower exposure to wealth risk. Investors will demand higher return from stocks with stocks with lower growth. These phenomena are called the size and value effect in the financial market as documented by Fama and French (1992, 1993 and 1995). Our wealth risks provide economical and theoretical understanding on this that consumption risk cannot capture.

We then introduce a new supply (size) of risky assets or seasoned equity offering into the market and show that the increased supply reduces the risky assets' return sensitivity to demand and its shocks, and that the aggregated new supply has a negative predictive power on the market portfolio return. This simple demand and supply relationship help us understand the new issue puzzle documented by Loughran and Ritter (1995) and the negative predictability of aggregate new issue found by Baker and Wurgler (2000). We also investigate whether the current period realized return of risky asset has any effect on expected return and find that it increases the asset's sensitivity to demand and demand shocks. This fact implies that the asset with higher realized return will face higher demand shock risk in next period and should demand higher expected return. This provides a consumption-based economic understanding of the stock momentum phenomena documented by Jegadeesh and Titman (1993).

In a real economy, it is a general belief that there are multiple investors who have heterogeneous wealth level, wealth shocks, and risk aversions, and who decides their own optimal consumption levels. Each investor's labor income can be known to or expected by others in the market. However, each investor does not know the others' investment wealth level and non-investment wealth shocks as they are confidential. Wang (1994) shows that disperse expectations and demands among investors generate turnover and volume in the market. We present an equilibrium price for each risky asset under cross-sectional equilibrium when there are multiple heterogeneous investors and assets, and argue that the wealth risk is priced across stocks. The importance of the demand for risky assets stressed by our testable model economically allies with the empirical finding of Chordia, Subrahmanyam and Anshuman (2001), where dollar trading volume dominates stock returns over and above market, size, value, and momentum factors. Reflecting the expectations of investors, the demand for the risky asset is the first order or fundamental economic aspect of the trading volume and the liquidity of market and assets, the current and future price, and their returns in the market place. The demand for risky assets determines their current price and provides liquidity or trading volume for these assets.

We ally our economic intuition about dollar trading volume with Campbell, Grossman and Wang, (1993) where there is a serial correlation between stock returns and dollar trading volume. We define the liquidity of each risky asset as the return sensitivity of the risky asset to dollar volume or turnover, and the liquidity risk as the return sensitivity of the risky asset to dollar volume or turnover shock that demand shocks created. The turnover or dollar volume is a first order and good indicator of the demand fluctuations in the market. Turnover and volume are widely used to construct the liquidity and market liquidity risk in the Amihud (2002) and the Pastor-Stamburgh (2003). Demand shocks naturally create liquidity shocks and liquidity in the market, and affect assets' returns. The market liquidity shock can be viewed to capture a dimension of the wealth risk generating the unexpected demand shock. This increase or decrease of demand shocks leads to the decreased or increased contemporaneous unexpected illiquidity that has a negative relationship with stock return as found by Amihud (2002). Our analysis provides a consumption-based economic understanding for the pricing of liquidity risk and the effect of unexpected illiquidity.

.We investigate how heterogeneous wealth shocks from investors in a global economy can affect the demands in other countries, and find that exchange rates play a role in home and global demands and their shocks as they did in Asia Financial Crisis. The remainder of the paper is organized into five sections. In section II, we present our economic setting and our testable extended model with the enhanced stochastic discount factor. In section III, we investigate how our wealth risk cross-sectionally affects risky asset return, and provide consumption-based economic understanding of stock market anomalies. Section IV extends the work into a global economy. Section V concludes the paper.

II. The model

A. The economy

We first consider one representative investor, the non-financial wealth and one representative tradable risky asset with gross return R_t at time *t*, which is a value-weighted market portfolio for tradable assets. This non-financial wealth includes labor income, property appreciation adjusted for consumption and inflation, wealth generated by technological shocks including innovations of biotechnology, computer, mechanic, and space technology, and nuclear energy, wealth created by economic and legal reforms eg. in China and Russia, and other non-tradable wealth from exchange rate shocks, wealth transformation from foreign country, and newly discovered natural resources eg. oil and mine fields. The increasing population over time also injects investors and their non-investment wealth into domestic and foreign economies. We later show that there is a market equilibrium price for each risky asset in an economy that has multiple heterogeneous investors and risky assets[2]

The investor's problem is to choose an optimal level of consumption C_t at all time that maximizes all their expected utility. The investor has the investment wealth W_t and the non-investment wealth NW_t expressed as (1a) at time t:

$$NW_t = \sum_{k=1}^{N} FW_{k,t} = L_t + G_t + E_t + T_t + F_t + P_t \ , \qquad (1a)$$

where NW_t is the aggregate non-investment or non-tradable wealth, $L_t = FW_{1,t}$ is the inflation-adjusted labor income or increase in human capital , $G_t = FW_{2,t}$ is wealth from gambling gain/loss, $E_t = FW_{3,t}$ is wealth generated from the economic and/or legal reforms or destructions, $T_t = FW_{4,t}$ is wealth created by technological shocks, , $F_t = FW_{5,t}$ is the other non-investment wealth, and $P_t = FW_{6,t}$ is the property wealth adjusted inflation and

[2] It is a general belief that there are heterogeneous investors in the real economy.

consumption. The return of property is captured in the representative risky asset return as part of the portfolio and is expected to be inflation rate. The inflation and consumption adjusted property appreciation will the unexpected increase in non-financial wealth from property whilst its mortgage payment is captured in the consumption as traditional CCAPM. One can view this P_t as the home equity and can be financed to invest and consume.

For modeling simplicity and without a lost of generality, we make decision on allocating investment and consumption about the changes in non-financial wealth and do not let the original level of unobservable and immeasurable non-financial wealth shifted to investment or financial markets. We also time-varyingly take a proportional ratio for the non-financial wealth to the investment wealth. This proportional simplification has the same line of economic logic as the ratio of habit in the Abel (1990) asset pricing model, the ratio of consumption surplus in the Campbell and Cochrane (1999) habit formation model and the proportional externality shock to human capital and the technological shock to production in the Lucas (1988) and Romer (1990) endogenous growth model. We the following expression for the aggregate time-varying ratio B_t :

$$B_t = \frac{NW_t}{W_t} = \frac{L_t}{W_t} + \frac{G_t}{W_t} + \frac{E_t}{W_t} + \frac{T_t}{W_t} + \frac{F_t}{W_t} + \frac{P_t}{W_t} = l_t + gb_t + e_t + t_t + f_t + p_t \,, \tag{1b}$$

where

$B_t \in \left(B^L, \infty\right), E(B_t) \geq 0, and, E_t(B_{t+\tau}) \in \left(B^L, \infty\right); l_t, t_t \in \left(0, \infty\right), and, gb_t, e_t, f_t, p_t \in \left(B^L, \infty\right);$

$E(l_t), E(t_t) \in \left(0, \infty\right),$ and $E(gb_t) = 0, E(e_t), E(f_t), E(p_t) \in \left(0, \infty\right)$ because their expected values should be positive in the long run. However, their conditional expectations should be different as $E_t(l_{t+\tau}) \in \left[0, \infty\right)$ and $E_t(t_{t+\tau}), E_t(gb_{t+\tau}), E_t(e_{t+\tau}), E_t(f_{t+\tau}), E_t(p_{t+\tau}) \in \left(B^L, \infty\right)$ because they are time-varying. The B^L is set to be -1 in the economy as the lower bound of the ratios because we naturally assume that the representative investor will not live on their borrowings. Allowing negative ratios except the labor and technology wealth ratios, we can capture the above mentioned loss of wealth, due to the negatively impacting events. However, the model would not be affected if we relaxed the assumption that the ratios could be less than -1 when there are multiple heterogeneous investors. In fact, our model's properties become stronger when we allow individual or grouped investors to live on borrowings in the heterogeneous multiple investors' economy and the global economy.

We measure the time-varying wealth shock as $A_t = (1 + B_t)$ that measure the same economic intuition as the wealth shock level. This wealth shock follows the AR (1) processes stated in (1c):

$$A_{t+1} = \overline{A} + \theta A_t + \eta_{t+1}, \tag{1c}$$

$$\eta_t \sim N(0, \sigma_\eta), \sigma_\eta < \infty, \theta \in (0,1), and, E(A_t) = \overline{A} > 1, but, E_t(A_{t+\tau}) \in (0, \infty).$$

This ratio also captures the weight of our non-financial wealth shock to our financial wealth level and its magnitude to our demand for financial wealth.

B. The new budget constraint and expected utility maximization problem

The new wealth constraint is stated as follows:

$$(A_{t+\tau}W_{t+\tau} - C_{t+\tau})R_{t+\tau+1} + NW_{t+\tau+1} = W_{t+\tau+1} + NW_{t+\tau+1} \tag{2}$$

The wealth constraint can be simplified as equation (2b). The representative investor's maximization problem will be as follows:

$$\underset{C_{t+j}}{MAX}\, E_t \sum_{\tau=0}^{\infty} (\beta^\tau U(C_{t+\tau})), \tag{2a}$$

$$\text{subject to } (A_{t+\tau}W_{t+\tau} - C_{t+\tau})R_{t+\tau+1} = W_{t+\tau+1},\ \tau = 0,..,\infty. \tag{2b}$$

The time-varying wealth shock measures the uncertainty of the next period non-investment wealth of the investor. The Lagrangian will be the following (2c):

$$L = E_t \sum_{\tau=0}^{\infty} (\beta^{\tau} U(C_{t+\tau}) + \mu_{t+\tau}((A_{t+\tau} W_{t+\tau} - C_{t+\tau}) R_{t+\tau+1} - W_{t+\tau+1})) \,. \tag{2c}$$

At first, we work with the standard power utility, $U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}, where \;\; \gamma > 1$ [3]. After taking derivative with respect to $C_{t+\tau}$ and $W_{t+\tau+1}$ for all τ , we obtain conditions as follows:

$$U'(C_t) = C_t^{-\gamma} = \mu_t R_{t+1} \tag{3a}$$

$$\mu_t = E_t(\beta \mu_{t+1} A_{t+1} R_{t+2}) \tag{3b}$$

We then derive the explicit dynamic equilibrium Euler equation:

$$1 = E_t(\beta \frac{U'(C_{t+1})}{U'(C_t)} A_{t+1} R_{t+1}) = E_t(\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} A_{t+1} R_{t+1}) \,. \tag{3c}$$

C. The new testable model and wealth risk premium

We can infer the stochastic discount factor in (4) from the central asset pricing formula $1 = E_t\left[M_{t+1} R_{t+1}\right]$ as follows:

$$M_{t+1} = \beta A_{t+1} \frac{U'(C_{t+1})}{U'(C_t)} = \beta A_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \,. \tag{4}$$

The wealth shock enhances the stochastic discount rate and is negatively related to the expected return of the risky asset. The time-varying stochastic discount rate has a negative relationship with the expected return of the risk asset since investors care about maintaining a smooth consumption profile over time. The higher level of disposal non-financial to financial

[3] We can show there is a realized wealth shock return premium for $U(C_t) = \ln(C_t)$ when $\gamma \to 1$ as well.

wealth in the next period the lower return the investor expects from risky asset to finance his next period consumption. Therefore, the wealth shock A_{t+1} must economically and negatively relate to the expected risky asset return. The lower return from an investor's investment will be rationally demanded to smooth their consumption because the investor's financing ability is increased by the higher non-financial wealth and vice versa.

For simplicity and without a lost of generality, we assume that the stochastic discount factor and the risky asset expected return are jointly log normal. We denote conditional operator at time t as follows:

$x_t = \ln(X_t)$, the natural logarithm,

$\bar{x}_{t+1} = E_t(x_{t+1})$, the expectation,

$\sigma^2_{x,t+1} = Var_t(x_{t+1})$, the variance,

$\bar{X}_{t+1} = e^{\bar{x}_{t+1}}$ the exponential expectation,

and $\sigma^2_{x,y,t+1} = COV_t(x_{t+1}, y_{t+1}) = \rho_{x,y}\sigma_{x,t+1}\sigma_{y,t+1}$ the covariance.

We can derive the following risk premium, or excess return, and implied risk aversion as shown in appendix:

obtain the following equations:

Knowing the negative correlation ($\rho_{a,r} < 0$) between wealth shock a_{t+1} and expected return r_{t+1}, we derive the extended testable model as in (6):

Risk Premium:
$$\bar{r}_{t+1} - \bar{r}_{f,t+1} = -\frac{1}{2}\sigma^2_{r,t+1} + (-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1}) + \gamma\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1}, \quad (6)$$

The implied risk aversion:
$$\gamma = \frac{(\bar{r}_{t+1} - \bar{r}_{f,t+1}) + \frac{1}{2}\sigma^2_{r,t+1} - (-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1})}{\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1}} \quad (7)$$

D. The model with habit-formation

Resolving the equity premium puzzle, economists have enrich consumer behavior by considering habits in the utility function. For instance, Abel (1990) introduce ratio habit and Constantinides (1990) consider habit in a difference form in the following utility:

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma} \tag{8}$$

Campbell and Cochrane (1999) extend the habit formation by working with the surplus consumption ratio S_t denoted $S_t \equiv \frac{C_t - X_t}{C_t}$ and the log difference $\Delta s_{t+1} = \ln(S_{t+1}) - \ln(S_t)$.

Incorporating this advance, we extend our testable model with wealth shock and derive the following Euler equation and stochastic discount factor with habit:[4]

$$1 = E_t\left(\beta A_{t+1}\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1}\right) \tag{9}$$

$$M_{t+1} = \beta A_{t+1}\frac{U'(C_{t+1} - X_{t+1})}{U'(C_t - X_t)} = \beta A_{t+1}\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{9a}$$

We then derive the risk premium for risky asset:

$$\begin{aligned} \bar{r}_{t+1} - \bar{r}_{f,t+1} &= -\frac{1}{2}\sigma^2_{r,t+1} - \rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1} + \gamma\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} + \gamma\rho_{\Delta s,a}\sigma_{\Delta s,t+1}\sigma_{r,t+1} \\ &= -\frac{1}{2}\sigma^2_{r,t+1} + \left(-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1}\right) + \gamma\left(\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} + \rho_{\Delta s,r}\sigma_{\Delta s,t+1}\sigma_{r,t+1}\right) \end{aligned} \tag{10}$$

The implied risk aversion becomes: $$\gamma = \frac{\bar{r}_{t+1} - \bar{r}_{f,t+1} + \frac{1}{2}\sigma^2_{r,t+1} - \left(-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1}\right)}{\left(\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} + \rho_{\Delta s,r}\sigma_{\Delta s,t+1}\sigma_{r,t+1}\right)}. \tag{11}$$

[4] We can also derive a extended model when we consider ratio habit introduced by Abel (1990).

The additional consumption risk premium $\rho_{\Delta s,r}\sigma_{\Delta s,t+1}\sigma_{r,t+1}$ also reduce the implied risk aversion through the denominator and helps to explains the smooth consumption through utility preference and precautionary saving. Meanwhile, investor demand risk premium for hearing the wealth shock risk in the market which changes demands for financial assets.

E. The empirical evidence

The wealth risk premium $(-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1})$ reduces the implied risk aversion of the traditional CCAPM. We use the national income deduced from the capital gain from the stock market as our first proxy for the total non-financial wealth changes from 1948 to 2004 as stated in (12). The annual national income is achieved from International Financial Statistic. The annual market value of stock market is achieved from CRSP. The annual equity market return is obtained from the Kenneth French Data Library..

$$A_t = 1 + \frac{NationalIncome - (R_t - 1) * MV_{t-1}}{MV_t}, \tag{12}$$

where the MV is the market capitalization value of stock market at time *t*. This wealth shock proxy is time-varyingly fluctuates as plotted in figure 1.

[Insert Figure 1 here]

Its mean is 2.68 with standard deviation 0.84 with whilst its log has a mean 0.94, volatility 0.30, and a correlation -0.25 with the log equity gross return. The wealth risk premium is 1.16% $(-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1})$-a covariance of -1.16% $(\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1})$ with the log of gross stock market return. This wealth risk premium accounts for 17.95% of the equity premium 6.45% with volatility 15.94%, and reduces the implied risk aversion by 18% in this period. Campbell and Cochrane (1999) show in their calibration that the consumption surplus ratio has larger covariance or risk premium than the consumption growth risk premium. The implied risk aversion in (12b) will be further reduced to lonely 40% of that implied by the traditional CCAPM.

We incorporate the increase of home equity of property into our aggregate wealth shock proxy, which is the increase after adjusted for inflation and consumption. The second proxy is stated as follows:

$$A_t = 1 + \frac{NationalIncome - (R_t - 1) \times MV_{t-1}}{MV_t} + \frac{\Delta realestate - realestate_{t-1} \times (R_f - 1)}{MV_t} \quad (13)$$

This second proxy is stationary as plotted in figure two.

[Insert Figure 2 here]

F *The cross-sectional empirical evidence*

In order to test our wealth risk premium and stochastic discount factor, we find that Fama-French 25 size and book-to-market portfolio uniformly bears correct wealth risk premium. They are all negatively priced by the first proxy as shown in table 1.

[Insert Table 1 here]

We also find that they are negatively priced by the second proxy as well as stated in table 2.

[Insert Table 2 here]

III. The cross sectional effects of wealth shocks in market equilibrium

We investigate how wealth shocks create demand shocks for risky assets and cross-sectionally affect their realized and expected returns in the dynamic equilibrium. The created demand shock for risk assets is the surplus or shortage of the normal demand without wealth shocks. We theoretically analyze the consumption-based economic understanding for the pricing of the value, size, momentum, market liquidity risk and unexpected illiquidity factors in the dynamic equilibrium since these shocks differently affect risky assets according to their characteristics. In our heterogeneous analysis, we assume that the higher aggregated demand generates a higher trading volume of risky assets by taking results from Wang (1994). For simplicity and without a lost of generality, we treat the market return as value-weighted by each risky asset:

$$R_{m,t} = \sum_{i=1}^{M} w_{i,t-1} R_{i,t} \,, \quad w_{i,t-1} = \frac{S_{i,t-1}}{S_{m,t-1}} > 0 \qquad (13)$$

This infers $\frac{\partial R_{m,t}}{\partial R_{i,t}} > 0$.

The market equilibrium condition is that the total demand D_t^A equal to total supply $S_{m,t} = \sum_{i=1}^{M} S_{i,t}$, where M is the number of risky asset in the market. We can derive the current period realized market return $R_{m,t}^A$ at time t in equation (14). The current period size is equal to the last period size of each risky asset times its realized return $R_{i,t}^A$ when there is no additional new supply in the market.

$$D_t^A = A_t W_t - C_t^A = S_{m,t-1} R_{m,t}^A = S_{m,t}^A = \sum_{i=1}^{M} S_{i,t}^A \,. \qquad (14)$$

The realized return R_t^A will be negatively related to the last period supply or the size as follow:

$$R_{m,t}^A = \frac{A_t W_t \left[E_t\left(\beta A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}}}{S_{m,t-1}} \qquad (14a).$$

***Theorem 1**: The realized return of risky asset is positively associated with the current period shock* A_t *and negatively associated with the expected next period shock* $\overline{A}_{t+1}$.

Proof: The investor's optimal consumption and demand for the risky asset are equations (15) when there are no current and next period wealth shocks:

$$C_t^1 = W_t \ (1 - \beta^{\frac{1}{\gamma}} E_t (R_{m,t+1}^{1-\gamma})^{\frac{1}{\gamma}}) \text{ and } D_t^1 = W_t \ - C_t^1 = S_{m,t-1} R_{m,t}^1 = S_{m,t}^1 . \tag{15}$$

The corresponding demand shock ΔD_t^d will be equation (13) minus equation (15) as follows:

$$\Delta D_t^d = A_t W_t \ \left[E_t \left(\beta A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}} - W_t \ \beta^{\frac{1}{\gamma}} E_t (R_{m,t+1}^{1-\gamma})^{\frac{1}{\gamma}} = S_{m,t-1} (R_{m,t}^A - R_{m,t}^1) = S_{m,t-1} R_{m,t}^d . \tag{16}$$

We call this R_t^d the current period realized demand shock premium as follows:

$$R_{m,t}^d = \sum_{i=1}^{M} w_{i,t-1} R_{i,t}^d = \frac{W_t \left(A_t \left[E_t \left(\beta A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}} - \beta^{\frac{1}{\gamma}} E_t (R_{m,t+1}^{1-\gamma})^{\frac{1}{\gamma}} \right)}{\sum_{i=1}^{M} S_{i,t-1}} . \tag{17}$$

This demand shock premium is increasing in wealth shock ($\frac{\partial R_{i,t}^d}{\partial A_t} > 0$) because higher demand drives up the current prices. In contrast, the current period realized return is decreasing in the next period expected wealth shock as $\frac{\partial R_{m,t}^d}{\partial A_{t+1}} < 0$ and $\frac{\partial R_{i,t}^d}{\partial A_{t+1}} < 0$, because $\gamma > 1$. At first glance, this seems puzzling, but it is economically intuitive and logical. This is because the current period consumption is increasing in the expected next period wealth shock (see appendix A). The current period demand shock will decrease in the expected wealth shock.

Theorem 2: *The realized return premium is increasing in the next period expected return when the aggregated demand shock is downwardly-negative, and decreasing in the next period expected return when the aggregated demand shock is upwardly-positive.*

Proof: We define the downwardly-negative demand shock as $\left(A_t \left[E_t \left(A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} \right) \right]^{\frac{1}{\gamma}} - E_t (R_{m,t+1}^{1-\gamma})^{\frac{1}{\gamma}} \right) < 0$ and the upwardly-positive demand shock as $\left(A_t \left[E_t \left(A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma} \right) \right]^{\frac{1}{\gamma}} - E_t (R_{m,t+1}^{1-\gamma})^{\frac{1}{\gamma}} \right) > 0$. They can be $(A_t < 1, or, > 1)$ when $(A_{t+1} = 1)$. As shown in formula (17) and formula (A7b) in Appendix A, the realized demand shock premium will be negative when there is a downwardly-negative demand shock and will increase in the next period expect return as $\frac{\partial R_{i,t}^d}{\partial \overline{R}_{m,t+1}} > 0$ and $\frac{\partial \overline{R}_{m,t+1}}{\partial \overline{R}_{i,t}} > 0$. In this case, the demand for the risky asset is lower than expected or normal. It is economically intuitive and logical that this negative premium will increase in, and alternatively the negative premium will have decreasing absolute value in, the expected return because higher expected return will reduce the tendency to realize its loss or to take its negatively impacted gain earlier than expected.

In contrast, the realized demand shock premium will be positive when there is an upwardly-positive demand shock and be decreasing in the next period expected return due to $\frac{\partial R_{i,t}^d}{\partial \overline{R}_{m,t+1}} < 0$ and $\frac{\partial \overline{R}_{m,t+1}}{\partial \overline{R}_{i,t}} < 0$ by $\gamma > 1$. In this case, the demand for the risky asset is higher than expected, which means that it drives up the current period price. At first glance, it is puzzling that this positive premium is decreasing in the expected return. However, it is economically logical and intuitive because the higher the expected return, the lower the effect of the demand shock according to the asset pricing framework and the theory that the expected return will have already changed and influenced the consumption and/or investment decision. In other words, the normal consumption and the shocked consumption are increasing in the expected return, and the shocked consumption increases faster than the normal consumption. Therefore, the shocked demand of the risky asset increases more slowly than the normal demand because the investor will consume much more than expected while having higher utility with higher consumption. This means that the upwardly-shocked wealth will be allocated more in consumption than in investment because the investor who is expecting a higher rate of return from risky assets can finance smooth consumption in the next period without too much investment. This reasoning is consistent with traditional asset pricing and consumption theory as investors maximize their consumption-based utility function and care about the smoothness of their consumption.

The Size Factor

Theorem 3: *The risky asset's current period realized demand shock premium is negatively related to the last period supply (size) of the risky asset when there is an upwardly-positive demand shock on the risky asset, and is positively related to the last period supply (size) when there is a downwardly-negative demand shock.*

Proof: As shown in equation (17), the positive demand shock realized return premium is decreasing in the asset size of the last period as $\frac{\partial R_{i,t}^{d}}{\partial S_{i,t-1}} < 0$ when there are upwardly-positive demand shocks (R_t^d >0) on the risky asset. In contrast, the realized negative return premium increases or the magnitude of its negative effect decreases in the size of the risky asset. This occurs because the total size or market value of the risky asset decreases the sensitivity of the risky asset to the demand shocks because the larger the original supply, the relatively smaller the demand shock effect. The ratio of the demand shock to the market value is decreasing in the size of the risky asset, and the effect of the demand shock is decreasing in total supply of the risky asset. This reflects what happens in the equity market where the larger the equity, the smaller the drop when there is a crash or burst bubble, and that those small firms perform better than large firms in upward trends or boom markets. Overall, the equity market is generating a higher excess return in the long run, and small stocks historically outperform large stocks.

Theorem 4**:** *The expected return of the risky asset is increasing in the current period shock* A_t *and decreasing in the next period expected shock* $\overline{A}_{t+1}$.

Proof: The explicit expected return under the market equilibrium is stated as follows (see Appendix A for derivations):

$$\overline{R}_{m,t+1} = \overline{A}_{t+1}^{-1}\left[\left(\frac{A_t W_t}{S_{m,t}^A}\right)^{\gamma} \beta e^{0.5\left(\sigma_{a,t+1}^2+\sigma_{r,t+1}^2+\sigma_{a,r,t+1}^2\right)}\right]^{\frac{1}{\gamma-1}} \tag{18a}$$

$$\overline{R}_{m,t+1} = \left[\left[\left(\frac{A_t W_t}{S_{m,t}^A}\right)^{-\gamma} \beta^{-1} - COV_t\left(A_{t+1}^{1-\gamma}, R_{m,t+1}^{1-\gamma}\right)\right] / E_t\left(A_{t+1}^{1-\gamma}\right) e^{\frac{1}{2}(1-\gamma)^2\sigma_{r,t+1}^2}\right]^{\frac{1}{1-\gamma}}, \tag{18b}$$

where $r_{t+1} = \ln(R_{m,t+1}), \overline{r}_{t+1} = E_t(r_{t+1})$ $\overline{A}_{t+1}^{1-\gamma} = e^{(1-\gamma)\overline{a}_{t+1}}$ and $\overline{R}_{m,t+1}^{1-\gamma} = e^{(1-\gamma)\overline{r}_{t+1}}$. We assume that $R_{m,t+1}$ and A_{t+1} are jointly lognormal in (18a) and independently lognormal distributed in (18b)[5]. The expected return of each risky asset $\overline{R}_{i,t+1}$ is negatively related to the last period size

[5] We will use (18b) in the remainder of this paper because it is a general belief that return is lognormal.

of the asset. This is because 1) $\frac{\partial \overline{R}_{m,t+1}}{\partial S_{i,t}^{A}} < 0$ and $\frac{\partial \overline{R}_{m,t+1}}{\partial \overline{R}_{i,t+1}} > 0$ with the properties of $\gamma > 1$ and $R_{m,t+1} = \sum_{i=1}^{N} w_{i,t} R_{i,t+1} (w_{i,t} = S_{i,t} / \sum_{i=1}^{N} S_{i,t} > 0)$, and 2) that the next period expected return will be positively related to the current shock A_t as $\frac{\partial \overline{R}_{t+1}}{\partial A_t} > 0$. This occurs because it takes a higher expected return to attract an investor to invest in risky assets rather than to spend on consumption.

In contrast, a higher level of current consumption will lead to a higher level of next period consumption due to the consumption habit and to the desired smoothness of consumption. The expected return of risky asset is negatively related to the future wealth shock A_{t+1} as $\frac{\partial \overline{R}_{m,t+1}}{\partial A_{t+1}} < 0$ because the investor demands a lower investment return on risky assets while expecting that their next period wealth will increase on top of their investment in the risky asset. Alternatively, the investor will demand a higher investment return when expecting that there will be a downward wealth shock or less wealth incurred from non-financial wealth in the next period. Therefore, the demanded and expected return of the risky asset is affected by the investor's expectation of the uncertainty of the next period wealth shock that leads the current and future demand shocks.

Theorem 5: *The realized and expected returns of the risky asset are always negatively related to the current period supply (size or market value) of risky asset, the newly issued value of shares of risky asset, and the aggregated supply of risky assets. The demand shock return premium of risky assets is This demand and supply relationship provides a theoretically and* *economically understanding for the pricing of the Fama-French size factor, the underperformance of Seasoned Equity Offering (SEO), and the negative predictability of the aggregated supply of risky assets.*

Proof: We introduce the aggregated size or market value of the new supply $\Delta S_{m,t} = \sum_{j=1}^{K} \Delta S_{j,t}$ when there are K risky assets that have newly issued shares in the market at time t. The realized demand shock return and its premium, and the expected return of risky assets will be revised as follows:

Realized Return: $$R_{m,t}^{A} = \frac{A_t W_t \left[E_t\left(\beta A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}} - \sum_{j=1}^{K} \Delta S_{j,t}}{S_{m,t-1}} \tag{19a}$$

Realized Return Premium: $$R_{m,t}^{d} = \frac{W_t \left(A_t\left[E_t\left(\beta A_{t+1}^{1-\gamma} R_{m,t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}} - \beta^{\frac{1}{\gamma}} E_t\left(R_{m,t+1}^{1-\gamma}\right)^{\frac{1}{\gamma}}\right) - \sum_{j=1}^{K} \Delta S_{j,t}}{S_{m,t-1}} \tag{19b}$$

Expected Return: $$\overline{R}_{m,t+1} = \left[\left[\left(\frac{A_t W_t}{S_{m,t}^{A} + \sum_{j=1}^{K} \Delta S_{j,t}}\right)^{-\gamma} \beta^{-1} - COV_t\left(A_{t+1}^{1-\gamma}, R_{m,t+1}^{1-\gamma}\right)\right] / E_t\left(A_{t+1}^{1-\gamma}\right) e^{\frac{1}{2}(1-\gamma)^2 \sigma_{r,t+1}^{2}}\right]^{\frac{1}{1-\gamma}} \tag{19c}$$

The aggregated new supply of risky assets has negative predictive power on the expected market return as $\frac{\partial \overline{R}_{imt+1}}{\partial \Delta S_{m,t}} < 0$. After taking a derivative with respect to $\Delta S_{i,t}$, we know that the current period realized return, the realized demand shock return premium, and the expected return are decreasing in the size of the new supply of risky asset because $\frac{\partial R_{i,t}^{A}}{\partial \Delta S_{i,t}} < 0$, $\frac{\partial R_{i,t}^{d}}{\partial \Delta S_{i,t}} < 0$ and $\frac{\partial \overline{R}_{i,t+1}}{\partial \Delta S_{i,t}} < 0$. After the new issue of risky asset is supplied, the demand shock-to-total supply ratio is reduced. The lesser effect of the demand shock on the current period realized return leads the lower realized demand shock premium. Furthermore, after lagging the period, we can see that the next period realized demand shock premium will be decreasing in the last period size of new supply as an addition of the total size of the risky asset according to Theorem 3. This theoretical property of the model explains the underperformance of Seasoned Equity Offering (SEO) because the reduced demand shock premium can be viewed as the underperformance or the smaller alpha controlled for other market and common factors.

The expected return of each risky asset is negatively related to its current period supply (size) as $\frac{\partial \overline{R}_{m,t+1}}{\partial S_{i,t}} < 0$ and $\frac{\partial \overline{R}_{i,t+1}}{\partial \overline{R}_{m,t+1}} < 0$. When other things are equal, the size of the risky asset negatively affects its current period realized return and the expected return due to their size-reduced sensitivity to wealth and demand shocks. This is because investors will expect a lesser effect of the next period demand shock on the risky asset. Thus, the size factor should be priced because it is cross-sectionally true for each risky asset. We empirically show that Fama French three size portfolios have uniform negative premium and monotonic decreasing value in market capitalization as argued in our theorem.

Theorem 6**:** *The expected return of risky asset is increasing in its current period realized return as it increase its exposure to demand shock and wealth risk. This risk exposure provides a theoretically and economically understanding for the pricing of the Jegadeesh-Titman momentum factor.*

Proof: Substituting budget constraint (2b) into (19c), we have an explicit formula that relates the expected return and the realized return as stated in (20) (Appendix B gives the detail).

$$\overline{R}_{m,t+1} = \left[\left[\left(\frac{A_t \left(\beta A_{t-1} W_{t-1} \left[E_{t-1} \left(\beta A_t^{1-\gamma} R_{m,t}^{1-\gamma} \right) \right]^{\frac{1}{\gamma}} \right) R_{m,t}^A}{S_t^A + \sum_{j=1}^{K} \Delta S_{j,t}} \right)^{-\gamma} \beta^{-1} - COV_t \left(A_{t+1}^{1-\gamma}, R_{m,t}^{1-\gamma} \right) \right] / E_t \left(A_{t+1}^{1-\gamma} \right) e^{\frac{1}{2}(1-\gamma)^2 \sigma_{r,t+1}^2} \right]^{\frac{1}{1-\gamma}} \tag{20}$$

The next period expected return of each risky asset is increasing in the current period realized return as $\frac{\partial \overline{R}_{i,t+1}}{\partial R_{i,t}^A} > 0$. This positive relationship also holds when $R_{i,t} = R_{i,t}^A$. The economic intuition is that investors will naturally expect higher return from past-performing assets because they are more sensitive to the current and next period wealth shocks and demand shocks when other things are equal. Therefore, the momentum factor should be cross-sectionally priced because each risky asset has this property. The model theoretically argues that this positive relationship is not linear and explains the empirical findings of Cooper, Gutierrez and Hameed (2004). It also mathematically suggests that the expected return sensitivity is decreasing in its size, and explains the empirical finding of Lewellen (2002) that small sized equity assets demonstrate stronger momentum. The return sensitivity to value factor also has a momentum factor in the following theorem due to the non-linearity. This also theoretically supports the finding of Lewellen (2002) that stock has stronger momentum within smaller value portfolio.

Theorem 7*: *The growth of risky asset decrease its exposure to wealth risk and provide a theoretically and economically understanding for the Fama-French value factor.

Proof: Without a lost of generality, we assume that the return of risky assets follows Gordon's growth model $R_{t+1} = 1 + \frac{D_{t+1}}{P_t} + g$ (21). After lagging and substituting (21) into (20), we have the following equation (22).

$$\overline{R}_{m,t+1} = \left[\left[\left(\frac{A_t (\beta A_{t-1} W_{t-1} \left[E_{t-1} \left(\beta A_t^{1-\gamma} \left(1 + \frac{DV_{m,t}}{P_{m,t-1}} + g_m \right)_t^{1-\gamma} \right) \right]^{\frac{1}{\gamma}}) R_{m,t}^A}{S_t^A + \sum_{j=1}^{K} \Delta S_{j,t}} \right)^{-\gamma} \beta^{-1} - COV_t \left(A_{t+1}^{1-\gamma}, R_{m,t+1}^{1-\gamma} \right) \right] / \left(E_t \left(A_{t+1}^{1-\gamma} \right) e^{\frac{1}{2}(1-\gamma)^2 \sigma_{r,t+1}^2} \right) \right]^{\frac{1}{1-\gamma}}$$

The expected return of each risky asset is decreasing in its growth rate or its market-to-book ratio because $\frac{DV_{m,t}}{P_{m,t-1}} + g_m = \sum_{i=1}^{M} w_{i,t} \left(\frac{DV_{i,t}}{P_{i,t-1}} + g_i \right)$. In other words, investors demand higher expected returns for low growth or value stocks when other things are equal as $\frac{\partial \overline{R}_{i,t+1}}{\partial g_i} < 0$. Investors naturally price the risky asset with a higher growth rate in the current period price that is driven up by their demands because the growing prospects of risky assets are attractive to investors. The value stocks will need to offer higher expected return to attract the demands of investor. we can mathematically and economically conclude that the value factor should be priced.

***Theorem 8**: There is an equilibrium price for each risky asset under the Cross-Sectional Heterogeneity Market Equilibrium*

Proof: We state the equilibrium price of each risky asset (see Appendix B for details) as

$$P_t^{j,A} = \left(\sum_{i=1}^{N} w_{i,t}^{j} D_{i,t}^{A} \right) / SH_t^{j} = \left(\sum_{i=1}^{N} \left(w_{i,t}^{j} A_{i,t} W_{i,t} \left[E_t \left(\beta A_{i,t+1}^{1-\gamma i} R_{i,t+1}^{p^{1-\gamma i}} \right) \right]^{\frac{1}{\gamma i}} \right) \right) / SH_t^{j} , \qquad (23)$$

where SH_t^j is the number of shares outstanding; and $D_{i,t}^A$ is the demand of investor *i* at time t. From this formula, there will be a cross-sectional equilibrium price even though each investor can have cross-sectional different risk aversion, wealth shocks, and expectation on risky assets.

Theorem 9: *The unexpected wealth decreases reduce demands and dollar trading volume in financial market. The magnitude of the decrease in trading volume is much greater than the magnitude of the decrease in return. This relationship provides a theoretically and economically understanding for the contemporaneous negative relationship between the unexpected market illiquidity and stock returns.*

Proof: In the remainder of this paper, we assume that a higher market aggregated demand for the risky assets generates a higher aggregated trading volume or $\frac{\partial D_t^d}{\partial V_t^d} > 0$ and $\frac{\partial \Delta D_t^d}{\partial \Delta V_t^d} > 0$, and that the aggregated trading volume is a non-linear function of the aggregated demand because the majority of the market turnover reflects the demand for the assets. This function has a second derivative with respect to the aggregated demand. Therefore, the market liquidity will be its sensitivity to the aggregated trading volume. we know $A_t - E_{t-1}(A_t) = \eta_t$, $\frac{\partial A_t}{\partial \eta_t} > 0$, $\frac{\partial R_{i,t}^d}{\partial A_t} > 0$ and $\frac{\partial R_{i,t}^A}{\partial A_t} > 0$,and that the market liquidity $Liq_{m,t} = \frac{\partial R_{m,t}}{\partial V_{m,t}} = \sum_{i=1}^{M} w_{i,t} \frac{\partial R_{i,t}}{\partial V_{i,t}}$, where $V_{m,t} = \sum_{i=1}^{M} V_{i,t}$ is increasing in current period wealth shock $\frac{\partial Liq_t}{\partial A_t} > 0$. The unexpected decrease of wealth shock or the negative η_t creating the unexpected decrease of demand shock will lead to unexpected market illiquidity. The higher unexpected market illiquidity corresponds to a lower realized return and demand shock return premium. According to the Amihud (2002) measure, we know that market illiquidity is $Milliq_t = \frac{1}{M}\sum_{i=1}^{M}\frac{\left|\overline{R}_t^A + R_t^{U,A}\right|}{\left|\overline{V}_t + V_t^U\right|}$ and the unexpected market illiquidity is $UMilliq_t = \frac{1}{M}\sum_{i=1}^{M}\frac{\left|\overline{R}_{i,t}^A + R_{i,t}^{U,A}\right|}{\left|\overline{V}_{i,t} + V_{i,t}^U\right|} - \frac{1}{M}\sum_{i=1}^{M}\frac{\left|\overline{R}_{i,t}^A\right|}{\left|\overline{V}_{i,t}\right|}$, where U represents unexpected components. Both $R_t^{U,A}$ and V_t^U will be negative when η_t is negative and their magnitudes increase in $\left|\eta_t\right|$ while V_t^U has a much higher magnitude than $R_t^{U,A}$. This theoretically explains the finding of Amihud (2002) that contemporaneous unexpected market illiquidity is negatively related to stock return because each risky asset has a lower realized demand shock realized return premium $R_{i,t}^d$ due to the lower demand.

Theorem 10: *The decrease in wealth in the next period will reduce the demands and trading volume in the financial market. The liquidity risk as the return sensitivity to the next period wealth shock captures the next period wealth risk. The return sensitivity to the liquidity risk is the second derivative of the return to wealth shock and increasing in wealth shock. This relationship provides a theoretically and economically understanding the pricing of the* ***market liquidity risk factor****.*

Proof: From equations (14), (17), and (18), we know that the expected return is decreasing in the next period wealth as $\frac{\partial \overline{R}_{i,t+1}}{\partial A_{t+1}} < 0$, and that the next period demand, the next period realized return and demand shock premium are increasing in the next period wealth shock as

$\frac{\partial D_{t+1}^A}{\partial A_{t+1}} > 0$, $\frac{\partial R_{i,t+1}^d}{\partial A_{t+1}} > 0$ and $\frac{\partial R_{i,t+1}^A}{\partial A_{t+1}} > 0$. Assuming the next period volume is increasing in the next period demand and the volume shock is also increasing in the next period demand shock as $\frac{\partial D_{t+1}^A}{\partial V_{t+1}^A} > 0$ and $\frac{\partial \Delta D_t^d}{\partial \Delta V_t^d} > 0$, we also know the next period volume and volume shocks are increasing in the next period wealth shock $\frac{\partial \Delta V_{t+1}^A}{\partial A_{t+1}} > 0$ and $\frac{\partial V_{t+1}^A}{\partial A_{t+1}} > 0$ because $\frac{\partial \Delta D_{t+1}^A}{\partial A_{t+1}} > 0$, $\frac{\partial D_t^A}{\partial A_{t+1}} < 0$.

One important economic intuition that can be derived from this model is that the next period wealth will generate next period demand shocks affecting the next period realized return that were discussed in Theorems 1 and 2. Hence, the uncertainty of the next period wealth or the lower upwardly-positive or downwardly-negative demand shock will create a lower or negative realized premium and a higher risk for the risky asset. The investor will place higher expectations in asset return to compensate for the lower and/or negative next period realized return premium that is associated with the demand shock, which is not priced by other risk factors.

We define the market liquidity risk as $Liqrisk_{m,t} = \frac{\partial \overline{R}_{m,t+1}}{\partial \Delta V_{m,t}} = \sum_{i=1}^{M} w_{i,t} \frac{\partial \overline{R}_{i,t+1}}{\partial \Delta V_{i,t}}$, where $\Delta V_{m,t}$ is the market volume shock. The bigger unexpected next period decrease in demand shocks that are generated by the unexpected smaller value of η_{t+1} in A_{t+1} will lead to the higher market liquidity risk being negative and with a higher absolute value that demands higher expected next period returns as $\frac{\partial \overline{R}_{i,t+1}}{\partial A_{t+1}} < 0$ when other things equal. The demand shock leads to the aggregate market liquidity shock instrumented by turnover because $\frac{\partial \Delta D_{t+1}^{A}}{\partial A_{t+1}} > 0$ and $\frac{\partial \Delta D_{t+1}^{A}}{\partial \Delta V_{t+1}^{A}} > 0$. Knowing $\frac{\partial^2 \overline{R}_{m,t+1}}{\partial A_{t+1}^2} > 0$ and $\frac{\partial^2 \overline{R}_{i,t+1}}{\partial A_{t+1}^2} > 0$, we can naturally argue $\frac{\partial \overline{R}_{i,t+1}}{\partial Liqrisk} > 0$ because the market liquidity risk is caused by the market volume shock gerenrated by wealth shocks. The Pastor and Stamburgh (2003) liquidity measure captures the money flow dimension of liquidity. Their liquidity risk measure is the unexpected market liquidity surplus/shortage or innovation. The next period market liquidity surplus/shortage can be viewed as a proxy for being created by the unexpected increase/decrease demand shock that is led by unexpected higher/smaller A_{t+1}. Therefore, the Wealth Shock Capital Asset Pricing Model theoretically and economically argues the significant positive pricing premium of the market liquidity risk that is found by Pastor and Stamburgh.

Theorem 11**:** *There are various dimensions of the wealth risks cross-sectionally priced in a multi-factor asset pricing model.*

Proof: Economists argue that the returns of risky assets are related to the consumption claims as stated in the presented consumption-based capital asset pricing model. **W**e propose that the wealth risks have various dimensions that are priced across stocks in a multi-factor model as follows:

$$R_{it} = m'_i \tilde{R}_t^m + \sum_{q=1}^{Q} \beta_i^q \tilde{R}_{qt}^a , \tag{23a}$$

$$\beta_i^q = \frac{\mathrm{cov}(R_t^q, R_{it})}{Var(R_{it})} \tag{23b}$$

,where β_i^q is as the sensitivity of the wealth shock risk in the qth-dimension $\tilde{R}_{qt}^a$. The m_t and $\tilde{R}_t^m$ are Lx1 vectors where the L is the number of other pricing factors inclusing the market excess return and other unidentified factors.

IV. The Global Economy

I extend the model into a global economy that has N representative heterogeneous investors and M representative risky assets or market portfolios that are locally diversified, in which each country has a corresponding exchange rate. We set index 1 as the home country or market, and the other index as a foreign country or market, and define the exchange rates as one home country dollar equaling X units of foreign currency as $\$1 = X_{i,t}$ and $X_{1,t} = 1$. The exchange rate return or currency return with respect to the default home country will be $R_{i,t+1}^x = \dfrac{X_{i,t}}{X_{i,t+1}}$. In this economy, each investor naturally has different risk aversion levels and wealth shocks, and the same utility function $U(C_{i,t}) = \dfrac{C_{i,t}^{1-\gamma i} - 1}{1-\gamma i}$ where $\gamma i > 1$. Each investor solves their consumption-based utility maximization problem in their local currency as follows:

$$\underset{C_{i,t+\tau}}{MAX}\ E_t \sum_{\tau=0}^{\infty} (\beta^\tau U(C_{i,t+\tau})), \tag{24}$$

subject to: $(A_{i,t+\tau} W_{i,t+\tau} - C_{i,t+\tau}) R_{i,t+\tau+1}^p = W_{i,t+\tau+1}$, i=1,..,N, $\tau = 0,..,\infty$, (24a) where $R^{p}_{i,t+1} = \sum_{j=1}^{M} w^{j}_{i,t} R^{j}_{i,t+1} R^{j,x}_{i,t+1}$. Having portfolio constraint $\sum_{j=1}^{M} w^{j}_{i,t} = 1$, the investors' weights of each portfolio will be exogenous in the above problem and determined by traditional portfolio theory[6] that is mean-variance dynamic optimization.

Theorem 12*: *The exchange rate of the foreign country will play a role in the home, foreign, and aggregate demand shock for the risky assets, as well as in their realized return and demand shock premium.

Proof: After solving the consumption based optimization problem for each investor, We obtain the demand shocks (25) and the realized return premium that is stated in (26).

The Demand Shocks generated by each investor in local currency

$$\Delta D^{d}_{i,t} = D^{A}_{i,t} - D^{1}_{i,t} = A_{i,t} W_{i,t} \left[E_t \left(\beta A^{1-\gamma}_{i,_{t+1}} R^{p^{1-\gamma_i}}_{i,_{t+1}} \right) \right]^{\frac{1}{\gamma_i}} - W_{i,t} \beta^{\frac{1}{\gamma_i}} E_t \left(R^{p^{1-\gamma_i}}_{i,t+1} \right)^{\frac{1}{\gamma_i}} \qquad (25)$$

The Realized Demand Shock Premium

Home:
$$R^{1,d}_{t} = \left(w^{1}_{1,t} \Delta D^{A}_{1,t} + \sum_{i=2}^{N} \left(w^{1}_{i,t} \Delta D^{A}_{i,t} / X_{i,t} \right) \right) / S^{1,A}_{t-1} \qquad (26a)$$

Foreign:
$$R^{j,d}_{t} = \left(w^{1}_{1,t} \Delta D^{A}_{1,t} X_{j,t} + w^{j}_{j,t} \Delta D^{A}_{j,t} + \sum_{\substack{k=2 \\ k \neq j}}^{N} \left(w^{j}_{i,t} \Delta D^{A}_{i,t} \frac{X_{j,t}}{X_{k,t}} \right) \right) / S^{j}_{t-1} \qquad (26b)$$

The exchange rates play a role in the home, foreign and aggregated demand shocks. The home country realized return is negative or decreasing in the foreign exchange rate as $\frac{\partial R^{1,A}_{t}}{\partial X_{i,t}} \leq 0, i > 1$. This is economically intuitive because the given higher/lower foreign exchange rate[7] will enhance/reduce the total demand of the home risky asset and push down/up the total demand

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[6] Investor's financial wealth can always be normalized to 1. A new way of optimizing the weights of each risky asset as the portfolio optimization problem is beyond the scope of this paper.
[7] The higher the foreign exchange rate the lower the foreign currency value in this economy.

for foreign risky asset when other things are equal, as shown in equation (26a). We also obtain the realized return of each market portfolio as stated in the following.

Home:
$$R_t^{1,A} = \left(w_{1,t}^1 D_{1,t}^A + \sum_{i=2}^{N} w_{i,t}^1 D_{i,t}^A / X_{i,t} \right) / S_{t-1}^{1,A} \tag{26c}$$

Foreign**:**
$$R_t^{j,A} = \left(w_{1,t}^1 D_{1,t}^A X_{j,t} + w_{j,t}^j D_{j,t}^A + \sum_{\substack{k=2 \\ k \neq j}}^{N} \left(w_{i,t}^j D_{i,t}^A \frac{X_{j,t}}{X_{k,t}} \right) \right) / S_{t-1}^{j,A} \tag{26d}$$

The current period realized return of the foreign risky asset increases[8] in its exchange rate as $\frac{\partial R_t^{i,A}}{\partial X_{i,t}} \geq 0$ and decreases in other foreign countries' exchange rates as $\frac{\partial R_t^{i,A}}{\partial X_{j,t}} < 0$ through the same economic reasoning. The demand shock premium of the home/foreign risky asset will be lower/higher in the exchange rate as $\frac{\partial R_t^{1,d}}{\partial X_{i,t}} \leq 0$ and $\frac{\partial R_t^{i,d}}{\partial X_{i,t}} \geq 0$ when the foreign/home country has positive demand shocks. The demand shock premium of the home/foreign risky asset will be higher/lower in the exchange rate as $\frac{\partial R_t^{1,d}}{\partial X_{i,t}} \geq 0$ and $\frac{\partial R_t^{i,d}}{\partial X_{i,t}} \leq 0$ when the foreign/home country has negative demand shocks. These are economically intuitive and logical because the foreign/home positive demand shock effect on the home/foreign country will be lower/higher due to the higher foreign exchange rate or lower foreign currency value. Meanwhile, the negative effect of the negative foreign/home demand shocks on the home/foreign country will be higher/lower as a result of the lower foreign exchange rate or higher foreign currency value. One should remember that the realized return premium and return of each market portfolio is negatively related to its last period size or market value.

[8] We assume that the demand in a market will not be zero, as in reality.

Theorem 13*: *The effect of the size and liquidity risk at the country locally diversified market portfolio level is persistent in the global economy.

Proof: We can derive the following next period expected return close-form formula.

Home**:**

$$\overline{R}^{p}_{1,t+1} = w^{1}_{1,t}\overline{R}^{1}_{t+1} + \sum_{j=2}^{N} w^{j}_{1,t}\overline{R}^{j}_{t+1}\overline{R}^{j,x}_{t+1} = \left[\left[\left(\frac{A_t W_t}{\sum_{j=1}^{M} w^{j}_{1,t} S_{j,t} X^{j}_{1,t}}\right)^{-\gamma i}\beta^{-1} - COV_t\left(A^{1-\gamma i}_{t+1}, R^{1-\gamma i}_{m,t+1}\right)\right] / E_t\left(A^{1-\gamma i}_{t+1}\right) e^{\frac{1}{2}(1-\gamma i)^2\sigma^2_{r,t+1}}\right]^{\frac{1}{1-\gamma i}} \quad (27a)$$

$$\overline{R}^{p}_{i,t+1} = w^{1}_{i,t}\overline{R}^{1}_{t+1} / \overline{R}^{i,x}_{t+1} + w^{i}_{i,t}\overline{R}^{i}_{t+1} + \sum_{\substack{k=2 \\ k \neq i}}^{N}\left(w^{k}_{i,t}\overline{R}^{k}_{t+1}\overline{R}^{k,x}_{i,t+1}\right)$$

Foreign**:**

$$= \left[\left[\left(\frac{A_t W_t}{\sum_{j=1}^{M} w^{j}_{i,t} S_{j,t} X^{j}_{i,t}}\right)^{-\gamma i}\beta^{-1} - COV_t\left(A^{1-\gamma i}_{t+1}, R^{1-\gamma i}_{m,t+1}\right)\right] / E_t\left(A^{1-\gamma i}_{t+1}\right) e^{\frac{1}{2}(1-\gamma i)^2\sigma^2_{r,t+1}}\right]^{\frac{1}{1-\gamma i}} \quad (27b)$$

The next period expected returns of both home and foreign risky assets have the same property with respect to the current and next period wealth shocks and the current period size as the domestic Wealth Shock Capital Asset Pricing Model (see appendix B). We can mathematically and economically argue that the pricing of the global market liquidity risk and size factors is persistent at the country market portfolio level. It should be noted that the global liquidity risk is generated by both countries and complicated by the exchange rate.

A. More on the explanation of the Equity Premium Puzzle

The expected and realized returns of each country's equity market are affected by other countries' wealth and demand shocks. The equity premium in the U.S. market has been pushed higher while the wealth of other nations has increased substantially in the last forty years. In particular, the exponentially increased wealth in China, Southeast Asia, and Eastern Europe has contributed to this equity premium in the past twenty-five years because the equity and fixed-income markets of these regions are not mature and as efficient as the U.S. market. A good portion of their capital or wealth is invested in the largely mature and safer US market as a historical fact. Given economic globalization and the good performance of U.S. economy in creating increasing labor income, property appreciation, and other non-tradable wealth through the information technology revolution in the 1980s and 1990s together with the wealth shocks and demand shocks from these countries and other developed countries pushed up the realized equity market return in the past twenty years for any acceptable risk aversion level that is implied by CCAPM or suggested by equations (7), (10), and (12). This property in the global economy strengthens the power of the Wealth Shock Capital Asset Pricing Model to provide theoretical insights for solving the Equity Premium Puzzle in the U.S. market.

B. Discussion on Asian Financial Crisis

The expected returns of the risky asset in the home/foreign country decrease/increase in the expected exchange rate return based on the home view are $\frac{\partial \overline{R}_{t+1}^{1}}{\partial \overline{R}_{t+1}^{i,x}} > 0, or, \frac{\partial \overline{R}_{t+1}^{1}}{\partial X_{i,t+1}} < 0$ and $\frac{\partial \overline{R}_{t+1}^{i}}{\partial \overline{R}_{t+1}^{i,x}} < 0, or, \frac{\partial \overline{R}_{t+1}^{i}}{\partial X_{i,t+1}} > 0$. This is economically intuitive and logical because the higher/lower expected exchange rate return or foreign currency value will increase/decrease the attractiveness of the foreign risky asset having relatively higher expected returns on the home currency unit. Investors will demand a much higher return on the home risky asset to be attracted to invest into it. This will drive the current price down as the discount rate (expected return) increases for any unchanged fundamentals. The foreign country's local expected return will decrease in its currency value because the expected increasing currency value will drive up its attractiveness and drive relatively down the local currency expected return for any given demanded expected return. Hence, the current period price and the realized return will increase due to the decreased discount rate (expected return) for any unchanged fundamentals.

It should be noted that the current period realized return also increases in foreign and local demand and that its sensitivities to the current exchange rate and the expected exchange rate are in opposite directions because the current exchange rate and demand are known and the expected exchange rate reflects the expected portfolio return in the investors' currency. The fluctuating exchange rate can be viewed as a change in the investment opportunity set, which should be priced according to the Intertemporal Asset Pricing Model of Merton (1973). This property can reasonably explain the rapid decrease of equity prices during the Asian financial crisis, which was caused by the exchange rate collapse because the expected dramatically depreciating currencies in the region rapidly drove down the current period price and return for the local equity assets to reach a much higher expected local currency return to attract foreign and domestic investment. The foreign demand for risky assets in East Asia decreased when foreign investors expected that the currency values were decreasing due to their selling and withdrawing or converting currency into their home currency. This effect was amplified because local demand had a relatively low ratio to foreign demand in Asian-ex-Japan markets. Furthermore, the fact that the fundamentals in the region were very uncertain and vulnerable caused the highly negative aggregated demand shock for the region's equity assets which led the highly negative realizing premium and put very high pressure on the extremely high expected return. The declines of the equity price had been accelerating. Actually, the recent extremely high performance of the equity market in the affected countries proves that the extremely high expectation put on these risky assets by the investors during that period was very rational.

V. Conclusion

We consider all disposal non-financial wealth changes, named as the wealth shock, which have expected and unexpected components. These changes create demand shock in the financial market, and introduce a systematic wealth risk that demands a risk premium. We construct two proxies and find that they uniformly and negatively priced Fama-French 25 size and book-to-market portfolios. This wealth risk also explains a good portion of the equity premium puzzle and Fama-French 25 portfolios. It can reduce 18% of the implied risk aversion in traditional CCAPM.

We extend our model in capturing the consumption surplus ratio in Campbell and Cochrane (1999), and find that our wealth risk premium co-exit with the consumption surplus risk premium. They jointly reduce the unreasonably high implied risk aversion documented by Mehra and Prescott (1985). This empirically shows the importance of our wealth shock. This stochastic wealth shock captures labor income, wealth coming from real estate appreciation/depreciation because housing expenses are part of consumption, gains/losses from gambling as entertainment consumption, wealth generated by technological shocks, wealth increased due to fundamental legal and economic system reforms, wealth destruction due to world war, civil war, and natural disasters such as earthquakes and the outbreak of diseases, and wealth that is transferred from foreign investors.

These wealth shocks create demand shocks in the market for risky assets. Risky assets are sensitive to these current and future wealth and demand shocks and have a realized return premium associated with these wealth and demand shocks. Investigating how wealth and demand shocks affect the realized and expected returns of risky assets in the dynamic equilibrium, we found that our testable model provide a consumption-based theoretical and economical understanding for a wide variety of dynamic asset pricing market anomalies and phenomena. They include the pricing of value, size, momentum, and market liquidity risk factors, the contemporaneous negative relationship between unexpected illiquidity and stock return, the underperformance of Seasoned Equity Offering, and the negative predictability of aggregated new issues. In the global economy, our model economically enhances its power and credibility in providing theoretical insights to solve the Equity Premium Puzzle and explaining the market anomalies and phenomena. We rely on our future study for other empirical study.

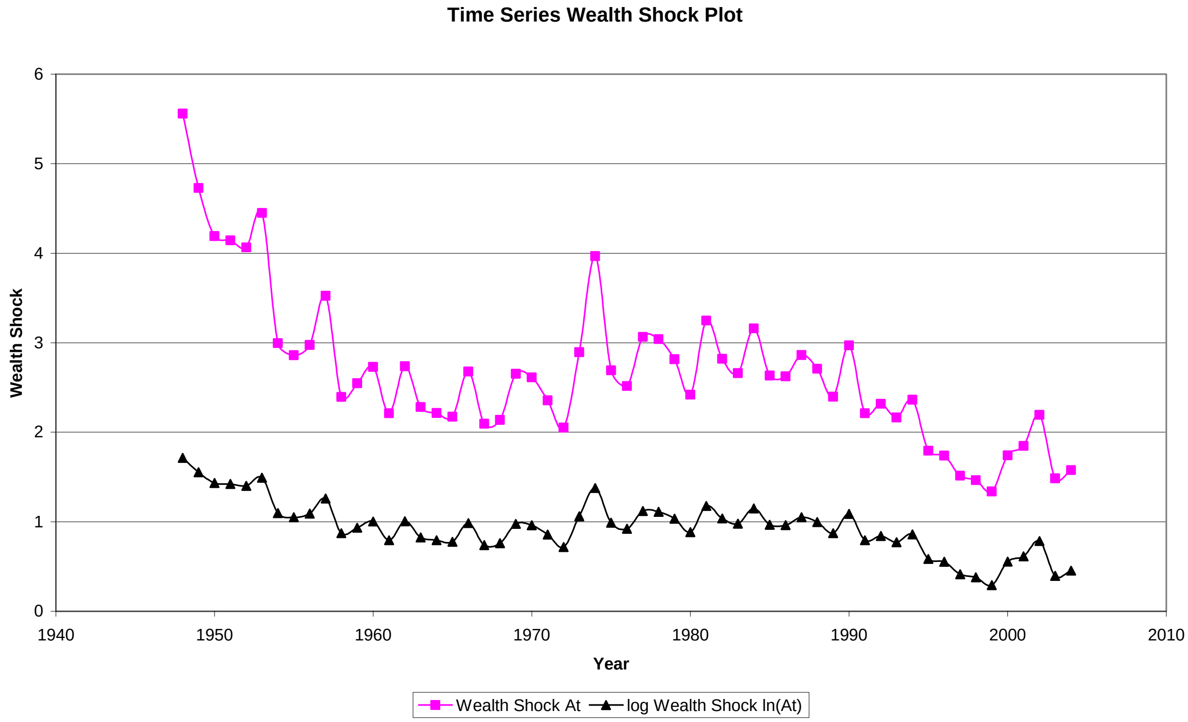
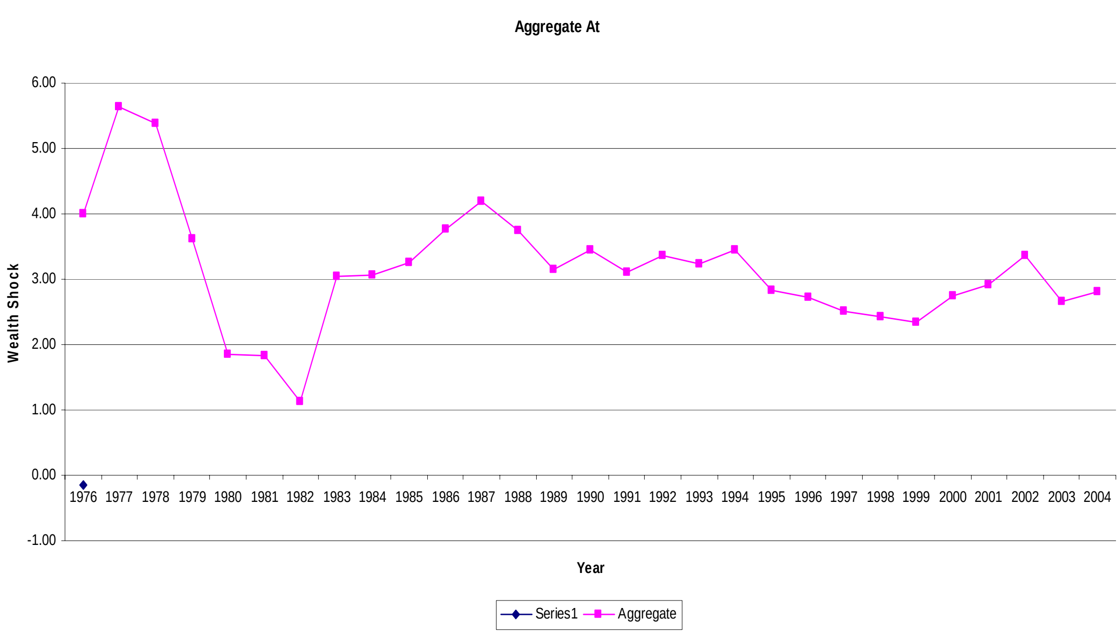


Figure 1: Time Series Wealth Shock Proxy Plot: The total non-investment wealth is proxy as the national income deducted from the capital gain from the stock market and the corresponding wealth shock is $A_t = 1 + \frac{NationalIncome - (R_t - 1) * MV_{t-1}}{MV_t}$, where MV is the market value of stock market at time t.

Figure 2: The time series relationship between the log equity return and the log wealth shock. The x-axis is the natural log of equity market return and the y-axis is the natural log of the wealth shock proxy. The wealth shock proxy is $A_t = 1 + \frac{NationalIncome - (R_t - 1) \times MV_{t-1}}{MV_t} + \frac{\Delta realestate - realestate_{t-1} \times (R_f - 1)}{MV_t}$, where MV_t is the market value of stock market at time t.



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Appendix A: Derivations and Proofs under Homogeneity

A1: Risk Premium of risky asset under standard power utility function.

The Log expension of Euler Equation:

$$1 = e^{\bar{m}_{t+1}+\bar{r}_{t+1}+\frac{1}{2}VAR_t\left(m_{t+1}+r_{t+1}\right)}, \text{ or } \bar{m}_{t+1}+\bar{r}_{t+1}+\frac{1}{2}VAR_t\left(m_{t+1}+r_{t+1}\right)=0 \qquad .(5)$$

Substituting the discount factor formula, we have the expected risky asset return and risk free asset return .

Risky Asset:

$$\bar{r}_{t+1} = -\ln(\beta) - \bar{a}_{t+1} + \gamma\Delta\bar{c}_{t+1} - \frac{1}{2}\sigma^2_{r,t+1} - \frac{1}{2}\gamma^2\sigma^2_{\Delta c,t+1} - \frac{1}{2}\sigma^2_{a,t+1} - \rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1} + \rho_{\Delta c,a}\gamma\sigma_{a,t+1}\sigma_{\Delta c,t+1} + \gamma\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} \qquad \text{(A1a)}$$

Risk Free Asset:

$$\bar{r}_{f,t+1} = -\ln(\beta) - a_{t+1} + \gamma\Delta\bar{c}_{t+1} - \frac{1}{2}\gamma^2\sigma^2_{\Delta c,t+1} - \frac{1}{2}\sigma^2_{a,t+1} + \gamma\rho_{\Delta c,a}\sigma_{a,t+1}\sigma_{\Delta c,t+1} \qquad \text{(A1b)}$$

Risk Premium:

$$\bar{r}_{t+1} - \bar{r}_{f,t+1} = -\frac{1}{2}\sigma^2_{r,t+1} + (-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1}) + \gamma\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1}, \qquad \text{(A1c)}$$

***The Ratio Habit*:**

We consider the internal ratio habit denoted X_t at time t and the investor's utility function as $U(C_t, X_t) = \frac{(C_t / X_t)^{\gamma} - 1}{1-\gamma}$. The time-varying stochastic discount factor and asset pricing formula are as follows:

$$M_{t+1} = \beta A_{t+1}\frac{U'(C_{t+1}/X_{t+1})}{U'(C_t/X_t)} = \beta A_{t+1}\left(\frac{C_{t+1}/X_{t+1}}{C_t/X_t}\right)^{-\gamma} \text{ and } 1 = E_t\left(\beta A_{t+1}\left(\frac{C_{t+1}/X_{t+1}}{C_t/X_t}\right)^{-\gamma} R_{t+1}\right) \qquad \text{(A2)}$$

Following Abel (1990) and Campbell, Lo, and Mackinlay (1997) to simplify the habit $X_t = C_{t-1}^k$ [9] as one lag of consumption, we have the same risk premium formula as (6) whilst the risk free rate formula will change to the following

$$\bar{r}_{f,t+1} = -\ln(\beta) - \bar{a}_{t+1} + \gamma\Delta\bar{c}_{t+1} - k(\gamma-1)\Delta c_t - \frac{1}{2}\gamma^2\sigma^2_{\Delta c,t+1} - \frac{1}{2}\sigma^2_{a,t+1} + \gamma\rho_{\Delta c,a}\sigma_{a,t+1}\sigma_{\Delta c,t+1}. \tag{A3}$$

The Difference Habit

Risky Asset:
$$\begin{aligned}\bar{r}_{t+1} &= -\ln(\beta) - \bar{a}_{t+1} + \gamma\Delta\bar{c}_{t+1} + \gamma\Delta\bar{s}_{t+1} - \frac{1}{2}\sigma^2_{r,t+1} - \frac{1}{2}\gamma^2\sigma^2_{\Delta c,t+1} - \frac{1}{2}\gamma^2\sigma^2_{\Delta s,t+1} \\ &- \frac{1}{2}\sigma^2_{a,t+1} - \rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1} + \rho_{\Delta c,a}\gamma\sigma_{a,t+1}\sigma_{\Delta c,t+1} + \rho_{\Delta s,a}\gamma\sigma_{a,t+1}\sigma_{\Delta s,t+1} \\ &+ \gamma\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} + \gamma\rho_{\Delta s,r}\sigma_{\Delta s,t+1}\sigma_{r,t+1} - \rho_{\Delta s,\Delta c}\gamma^2\sigma_{\Delta c,t+1}\sigma_{\Delta s,t+1}\end{aligned} \tag{A4a}$$

Risk Free Rate:
$$\begin{aligned}\bar{r}_{f,t+1} &= -\ln(\beta) - \bar{a}_{t+1} + \gamma\Delta\bar{c}_{t+1} + \gamma\Delta\bar{s}_{t+1} - \frac{1}{2}\sigma^2_{r,t+1} - \frac{1}{2}\gamma^2\sigma^2_{\Delta c,t+1} - \frac{1}{2}\gamma^2\sigma^2_{\Delta s,t+1} \\ &- \frac{1}{2}\sigma^2_{a,t+1} + \rho_{\Delta c,a}\gamma\sigma_{a,t+1}\sigma_{\Delta c,t+1} + \rho_{\Delta s,a}\gamma\sigma_{a,t+1}\sigma_{\Delta s,t+1} - \rho_{\Delta s,\Delta c}\gamma^2\sigma_{\Delta c,t+1}\sigma_{\Delta s,t+1}\end{aligned} \tag{A4b}$$

Risk Premium:
$$\begin{aligned}\bar{r}_{t+1} - \bar{r}_{f,t+1} &= -\frac{1}{2}\sigma^2_{r,t+1} - \rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1} + \gamma\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} + \gamma\rho_{\Delta s,a}\sigma_{\Delta s,t+1}\sigma_{r,t+1} \\ &= -\frac{1}{2}\sigma^2_{r,t+1} + \left(-\rho_{r,a}\sigma_{a,t+1}\sigma_{r,t+1}\right) + \gamma\left(\rho_{\Delta c,r}\sigma_{\Delta c,t+1}\sigma_{r,t+1} + \rho_{\Delta s,r}\sigma_{\Delta s,t+1}\sigma_{r,t+1}\right)\end{aligned} \tag{A4c}$$

A2: The optimal consumption level

I restate the investor's maximization problem and budget constraint as the following:

$$\underset{C_{t+j}}{MAX}\; E_t \sum_{j=0}^{\infty} (\beta^j U(C_{t+j})) \tag{A5}$$

$$\text{subject to: } (A_{t+j}W_{t+j} - C_{t+j})R_{t+j+1} = W_{t+j+1}. \tag{A5b}$$

Solving optimization problem, we obtain

[9] Detail can be found in Campbell, Lo, and Mackinlay (1997).

$$1 = E_t\left(\beta\left(\frac{\phi A_{t+1}W_{t+1}}{\phi A_t W_t}\right)^{-\gamma} A_{t+1}R_{t+1}\right) = E_t\left(\beta(1-\phi)^{-\gamma} A_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right), \quad \text{(A6)}$$

or $$(1-\phi) = \left[E_t\left(\beta A_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}} \quad . \quad \text{(A6)}$$

We have the following optimal solution for consumptions $C_t, and \phi$:

$$\phi = 1 - \left[E_t\left(\beta A_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}}, \text{ and } C_t^A = A_t W_t \ (1 - \left[E_t\left(\beta A_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{\gamma}}) . \quad \text{(A7)}$$

I expand the expectation operator

$$\begin{aligned} &E_t\left(\beta A_{t+1}^{1-\gamma} R_{t+1}^{1-\gamma}\right) \\ &= \beta\left[E_t\left(A_{t+1}^{1-\gamma}\right)E_t\left(R_{t+1}^{1-\gamma}\right) + COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma}\right)\right] \\ &= \beta\left[\overline{A}_{t+1}^{1-\gamma} e^{\frac{1}{2}(1-\gamma)^2\left(\sigma_{a,t+1}^2+\sigma_{r,t+1}^2\right)} \overline{R}_{t+1}^{1-\gamma} + COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma}\right)\right] \end{aligned} \quad \text{(A8)}$$

and denote $\overline{R}_{t+1} = e^{\bar{r}_{t+1}}, and, \bar{r}_{t+1} = \ln(R_{t+1})$. We can see that the current period of consumption is increasing[10] in the expected next period return ($\frac{\partial C_t}{\partial \overline{R}_{t+1}} > 0$) as the investor will spend more money in current period consumption and can invest less when the investment will generate higher returns to achieve the same level of capital gain and/or return for future consumption. However, the current period of consumption will increase in the expected next period wealth shock ($\frac{\partial C_t}{\partial \overline{A}_{t+1}} > 0$) as $R_{t+1} > 0$. This is economically intuitive because the investor will consume more and invest less current wealth in the current period when he expecting upward or increasing wealth from outside of their investment in the next period. The investor does not need to have a higher capital or investment gain and/or return for the next period to finance

[10] This is consistent with empirical data that equity market return is positively correlated with consumption in the U.S. market and global markets.

their next period smooth consumption and investment because we can assume that the risky asset's next period return $\overline{R}_{t+1} > Rf \geq 0$ without losing generality. After all, the investor's utility is increasing in their consumption level.

A3: The Expected Return of Risky Assets under Market Equilibrium

I restate the demand for risky assets under the market equilibrium condition as follows:

$$D_t^A = A_t W_t - C_t^A = S_{t-1} R_t^A = S_t^A . \tag{A9}$$

Substituting (A5) into (A6) and assuming that $R_{t+1}^{1-\gamma}$ and $A_{t+1}^{1-\gamma}$ are not jointly lognormal, we can obtain the following equation (A10).

$$1 = \left(\frac{A_t W_t}{S_t^A}\right)^{\gamma} \beta \left[E_t\left(A_{t+1}^{1-\gamma}\right) E_t\left(R_{t+1}^{1-\gamma}\right) + COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma}\right)\right] \tag{A10a}$$

Knowing that $E_t\left(R_{t+1}^{1-\gamma}\right) = e^{(1-\gamma)\bar{r}_{t+1} + \frac{1}{2}(1-\gamma)^2 \sigma_{t+1}^2} = \overline{R}_{t+1}^{1-\gamma} e^{\frac{1}{2}(1-\gamma)^2 \sigma_{t+1}^2}$ (A10b), where $r_{t+1} = \ln(R_{t+1})$, $\bar{r}_{t+1} = E_t(r_{t+1})$ and $\overline{R}_{t+1}^{1-\gamma} = e^{(1-\gamma)\bar{r}_{t+1}}$, we have the explicit formula for the expected return for risky assets in the following equation:

$$\overline{R}_{t+1} = \left[\frac{\left[\left(\frac{A_t W_t}{S_t^A}\right)^{-\gamma} \beta^{-1} - COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma}\right)\right]}{E_t\left(A_{t+1}^{1-\gamma}\right) e^{\frac{1}{2}(1-\gamma)^2 \sigma_{r,t+1}^2}}\right]^{\frac{1}{1-\gamma}} = \left[\frac{\left[\left(\frac{A_t W_t}{S_t^A}\right)^{-\gamma} \beta^{-1} - COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma}\right)\right]}{\overline{A}_{t+1}^{1-\gamma} e^{\frac{1}{2}(1-\gamma)^2\left(\sigma_{r,t+1}^2 + \sigma_{a,t+1}^2\right)}}\right]^{\frac{1}{1-\gamma}} .$$

(A10c)

The Equation (A10a) will change to (A10d) and the explicit formula for the expected return for risky assets will be (A10e) when we assume that $R_{t+1}^{1-\gamma}$ and $A_{t+1}^{1-\gamma}$ are jointly lognormal.

$$1 = \left(\frac{A_t W_t}{S_t^A}\right)^{\gamma} \beta e^{(1-\gamma)(\bar{a}_{t+1} + \bar{r}_{t+1}) + 0.5\left(\sigma_{a,t+1}^2 + \sigma_{r,t+1}^2 + \sigma_{a,r,t+1}^2\right)} \tag{A10d}$$

$$\overline{R}_{t+1} = \left[\left(\frac{A_t W_t}{S_t^A} \right)^{\gamma} \beta e^{(1-\gamma)(\overline{a}_{t+1}) + 0.5\left(\sigma_{a,t+1}^2 + \sigma_{r,t+1}^2 + \sigma_{a,r,t+1}\right)} \right]^{\frac{1}{\gamma-1}}$$

$$= \overline{A}_{t+1}^{-1} \left[\left(\frac{A_t W_t}{S_t^A} \right)^{\gamma} \beta e^{0.5\left(\sigma_{a,t+1}^2 + \sigma_{r,t+1}^2 + \sigma_{a,r,t+1}^2\right)} \right]^{\frac{1}{\gamma-1}} \qquad \text{(A10e)}$$

A4: Derivation for the Momentum Factor

After substituting budget constraint (2b) into (17), we have the following expressions:

$$\overline{R}_{t+1} = \left[\frac{\left[\left(\frac{A_t (A_{t-1} W_{t-1} - C_{t-1}) R_t^A}{S_t^A + \sum_{j=1}^{K} \Delta S_{j,t}} \right)^{-\gamma} \beta^{-1} - COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma} \right) \right]}{E_t\left(A_{t+1}^{1-\gamma} \right) e^{\frac{1}{2}(1-\gamma)^2 \sigma_{r,t+1}^2}} \right]^{\frac{1}{1-\gamma}} \qquad \text{(A11a)}$$

$$\overline{R}_{t+1} = \left[\frac{\left[\left(\frac{A_t (\beta A_{t-1} W_{t-1} \left[E_{t-1}\left(\beta A_t^{1-\gamma} R_t^{1-\gamma} \right) \right]^{\frac{1}{\gamma}}) R_t^A}{S_t^A + \sum_{j=1}^{K} \Delta S_{j,t}} \right)^{-\gamma} \beta^{-1} - COV_t\left(A_{t+1}^{1-\gamma}, R_{t+1}^{1-\gamma} \right) \right]}{E_t\left(A_{t+1}^{1-\gamma} \right) e^{\frac{1}{2}(1-\gamma)^2 \sigma_{r,t+1}^2}} \right]^{\frac{1}{1-\gamma}}. \qquad \text{(A11b)}$$

Appendix B: Derivations and Proofs under Heterogeneity

The economy

There are N representative heterogeneous investors and M representative risky assets in one economy in which the number of investors is strictly greater than the number of risky assets (N > M). We naturally assume that each investor can have a different risk aversion level and wealth shock, and the same power utility function $U(C_{i,t}) = \frac{C_{i,t}^{1-\gamma i} - 1}{1-\gamma i}$ where $\gamma i > 1$. Therefore, each investor will face the following optimization problem:

$$\underset{C_{i,t+\tau}}{MAX}\ E_t \sum_{\tau=0}^{\infty} (\beta^{\tau} U(C_{i,t+\tau})) , \tag{B1}$$

subject to: $(A_{i,t+\tau} W_{i,t+\tau} - C_{i,t+\tau}) R_{i,t+\tau+1}^{p} = W_{i,t+\tau+1}$, i=1,..,N, $\tau = 0,..,\infty$,

(B1a)

where the $R_{i,t+1}^{p} = \sum_{j=1}^{M} w_{i,t}^{j} R_{t+1}^{j}$ is the expected next return of the investor's investment portfolio in which the weights of each risky asset j are determined by the traditional mean-variance portfolio optimization or another optimization solution[11] based on the portfolio constraint $\sum_{j=1}^{M} w_{i,t}^{j} = 1$ (B1b). The portfolio weights are exogenous in this optimization problem because investors will first decide on their optimal consumption level as a proportion of their wealth, and then make decisions on how to allocate the absolute value of consumption and investment in assets and goods. These second level or separated portfolio and consumption allocation decisions belong to portfolio theory and consumer-producer theory, and can be exogenous.

[11] A new way of optimizing the weights of each risky asset as the portfolio optimization problem is beyond the scope of this paper. We will undertake a study of this issue under demand shocks in the future.

Without a loss of generality, we assume that $R_{t+1}^{i} \geq R_{t+1}^{j}$ while $\sigma_t^i \geq \sigma_t^j$. The special case of this economy is that there is one representative investor and multiple heterogeneous risky assets. The risky asset in the previous section can be viewed as the market portfolio $R_{p,t} = R_{m,t} = \sum_{i=1}^{M} w_{i,t-1} R_{i,t}$ ($w_{i,t-1} = \frac{S_{i,t-1}}{S_{m,t-1}}$ >0) where there is one representative investor. The Lagrangian of each investor and their optimal consumption, demand and demand shock for risky assets will be the following:

$$L_i = E_t \sum_{\tau=0}^{\infty} (\beta^{\tau} U(C_{i,t+\tau}) + \mu_{i,t+\tau}((A_{i,t+\tau} W_{i,t+\tau} - C_{i,t+\tau}) R_{i,t+\tau+1}^{p} - W_{i,t+\tau+1})), \qquad \text{i=1,..,N.} \quad \text{(B1c)}$$

Each Investor's Consumption

Without Shocks: $$C_{i,t}^{1} = W_{i,t}(1 - \beta^{\frac{1}{\gamma_i}} E_t (R_{i,t+1}^{p^{1-\gamma_i}})^{\frac{1}{\gamma_i}}) \quad \text{(B2a)}$$

With Shocks: $$C_{i,t}^{A} = A_{i,t} W_{i,t} (1 - \left[E_t \left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p^{1-\gamma_i}} \right) \right]^{\frac{1}{\gamma_i}}) \quad \text{(B2b)}$$

Each Investor's Demand for Risky Assets

Without Shocks: $$W_{i,t} - C_{i,t}^{1} = W_{i,t} \beta^{\frac{1}{\gamma_i}} E_t (R_{i,t+1}^{p^{1-\gamma_i}})^{\frac{1}{\gamma_i}} = D_{i,t}^{1} \quad \text{(B3a)}$$

With Shocks: $$A_{i,t} W_{i,t} - C_{i,t}^{A} = A_{i,t} W_{i,t} \left[E_t \left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p^{1-\gamma_i}} \right) \right]^{\frac{1}{\gamma_i}} = D_{i,t}^{A} \quad \text{(B3b)}$$

The Demand Shocks generated by each investor

$$\Delta D_{i,t}^{d} = D_{i,t}^{A} - D_{i,t}^{1} = A_{i,t} W_{i,t} \left[E_t \left(\beta A_{i,t+1}^{1-\gamma} R_{i,t+1}^{p^{1-\gamma_i}} \right) \right]^{\frac{1}{\gamma_i}} - W_{i,t} \beta^{\frac{1}{\gamma_i}} E_t (R_{i,t+1}^{p^{1-\gamma_i}})^{\frac{1}{\gamma_i}} \quad \text{(B4)}$$

The demand shock equilibrium condition for each risky asset will be the following.

Risky Asset j, $$S_{t-1}^{j} R_t^{j,d} = S_{t-1}^{j} \left(R_t^{j,A} - R_t^{j,1} \right) = \sum_{i=1}^{N} w_{i,t}^{j} \Delta D_{i,t} \quad \text{(B5)}$$

The Demand Shock Realized Premium:

Risky Asset j: $$R_t^{j,d} = \sum_{i=1}^{N} w_{i,t}^{j} \Delta D_{i,t} / S_{t-1}^{j} \tag{B6}$$

The Cross Sectional Market Equilibrium

$$S_t^{j,A} = \sum_{i=1}^{N} w_{i,t}^{j} D_{i,t}^{A} = \sum_{i=1}^{N} \left(w_{i,t}^{j} A_{i,t} W_{i,t} \left[E_t \left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p\,1-\gamma_i} \right) \right]^{\frac{1}{\gamma_i}} \right) \tag{B7}$$

The Cross Sectional Market Equilibrium Price of Risky Assets

$$P_t^{j,A} = \left(\sum_{i=1}^{N} w_{i,t}^{j} D_{i,t}^{A} \right) / SH_t^{j} = \left(\sum_{i=1}^{N} \left(w_{i,t}^{j} A_{i,t} W_{i,t} \left[E_t \left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p\,1-\gamma_i} \right) \right]^{\frac{1}{\gamma_i}} \right) \right) / SH_t^{j}$$

(B8a)

or $$P_t^{j,A} = \left(\sum_{i=1}^{N} D_{i,t}^{A} - \sum_{\substack{k=1, \\ k \neq j}}^{M} S_t^{k,A} \right) / SH_t^{j} = \left(\sum_{i=1}^{N} \left(A_{i,t} W_{i,t} \left[E_t \left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p\,1-\gamma_i} \right) \right]^{\frac{1}{\gamma_i}} \right) - \sum_{\substack{k=1, \\ k \neq j}}^{M} S_t^{k,A} \right) / SH_t^{j}$$

(B8b)

, where SH_t^{j} is the number of shares outstanding of stock j at time t.

Appendix C: Derivations and Proofs under Global Economy

Solving the investor's optimization problem, we obtain the optimal consumption and demand for risky assets together with the market equilibrium conditions as follows.

Each Investor's Consumption in Local Currency

Without Shocks: $$C_{i,t}^{1} = W_{i,t}(1-\beta^{\frac{1}{\gamma_i}} E_t (R_{i,t+1}^{p^{1-\gamma_i}})^{\frac{1}{\gamma_i}}) \quad \text{(C1a)}$$

With Shocks: $$C_{i,t}^{A} = A_{i,t} W_{i,t}(1-\left[E_t\left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p^{1-\gamma_i}}\right)\right]^{\frac{1}{\gamma_i}}) \quad \text{(C1b)}$$

Each Investor's Demand for Risky Assets in Local Currency

Without Shocks: $$W_{i,t} - C_{i,t}^{1} = W_{i,t}\beta^{\frac{1}{\gamma_i}} E_t (R_{i,t+1}^{p^{1-\gamma_i}})^{\frac{1}{\gamma_i}} = D_{i,t}^{1}$$

(C2a)

With Shocks: $$A_{i,t}W_{i,t} - C_{i,t}^{A} = A_{i,t}W_{i,t}\left[E_t\left(\beta A_{i,t+1}^{1-\gamma_i} R_{i,t+1}^{p^{1-\gamma_i}}\right)\right]^{\frac{1}{\gamma_i}} = D_{i,t}^{A} \quad \text{(C2b)}$$

Demand and Supply Equilibrium Condition

Home: $$R_t^{1,A} S_{t-1}^{1,A} = S_t^{1,A} = w_{1,t}^{1} D_{1,t}^{A} + \sum_{i=2}^{N} w_{i,t}^{1} D_{i,t}^{A} / X_{i,t} \quad \text{(C3a)}$$

Foreign: $$R_t^{j,A} S_{t-1}^{j,A} = S_t^{2,A} = w_{1,t}^{1} D_{1,t}^{A} X_{j,t} + w_{j,t}^{j} D_{j,t}^{A} + \sum_{\substack{k=2 \\ k\neq j}}^{N}\left(w_{i,t}^{j} D_{i,t}^{A}\frac{X_{j,t}}{X_{k,t}}\right) \quad \text{(C3b)}$$

Demand Shock Equilibrium Condition

Home: $$R_t^{1,d} S_{t-1}^{1,A} = \left(R_t^{1,A} - R_t^{1,1}\right) S_{t-1}^{1,A} = w_{1,t}^{1}\Delta D_{1,t}^{A} + \sum_{i=2}^{N}\left(w_{i,t}^{1}\Delta D_{i,t}^{A} / X_{i,t}\right) \quad \text{(C4a)}$$

Foreign: $$\begin{aligned} R_t^{j,d} S_{t-1}^{j} &= \left(R_t^{j,A} - R_t^{j,1}\right) S_{t-1}^{j} \\ &= w_{1,t}^{1}\Delta D_{1,t}^{A} X_{2,t} + w_{j,t}^{j}\Delta D_{j,t}^{A} + \sum_{\substack{k=2 \\ k\neq j}}^{N}\left(w_{i,t}^{j}\Delta D_{i,t}^{A}\frac{X_{j,t}}{X_{k,t}}\right) \end{aligned} \quad \text{(C4b)}$$

Table 1: The wealth risk premium and implied risk aversion reduction of Fama-French 5x5 size and value portfolios for the first wealth shock proxy.

This table reports the wealth risk premium and implied risk aversion reduction of Fama-French size and value portfolios. The wealth shock proxy is the one plus the ratio of national income deducted from capital gain from equity market to the aggregate market capitalization.

Size	Book-to-Market Ratio	Compound Risk Premium	Risk Premium Plus Variance	Wealth Risk Premium	Implied Risk Aversion Reduction
Small	Low	2.16%	7.87%	-2.40%	-30.50%
	2	8.71%	13.01%	-2.38%	-18.30%
	3	10.34%	13.46%	-2.35%	-17.50%
	4	12.47%	15.33%	-2.37%	-15.40%
	High	15.24%	18.46%	-2.39%	-12.90%
2	Low	2.04%	5.65%	-1.66%	-29.40%
	2	7.34%	9.62%	-1.35%	-14.00%
	3	10.15%	12.19%	-1.71%	-14.00%
	4	10.48%	12.57%	-1.48%	-11.80%
	High	11.56%	14.04%	-1.53%	-10.90%
3	Low	4.11%	6.92%	-1.37%	-19.80%
	2	8.06%	9.93%	-1.48%	-14.90%
	3	8.54%	10.18%	-1.25%	-12.20%
	4	10.43%	12.37%	-1.51%	-12.20%
	High	11.33%	13.58%	-1.62%	-11.90%
4	Low	5.53%	7.58%	-1.53%	-20.20%
	2	6.74%	8.26%	-1.32%	-15.90%
	3	9.52%	10.95%	-1.24%	-11.30%
	4	9.50%	11.20%	-1.78%	-15.90%
	High	10.32%	12.62%	-1.07%	-8.50%
Big	Low	5.33%	6.79%	-1.33%	-19.70%
	2	6.82%	7.86%	-1.06%	-13.40%
	3	7.86%	9.07%	-1.00%	-11.10%
	4	7.99%	9.42%	-1.09%	-11.60%
	High	8.55%	10.48%	-1.00%	-9.60%

Table 2: The wealth risk premium and implied risk aversion reduction of Fama-French 5x5 size and value portfolios for the second wealth shock proxy.

This table reports the wealth risk premium and implied risk aversion reduction of Fama-French size and value portfolios. The wealth shock proxy is the one plus the ratio of the sum of national income deducted from capital gain from equity market and the property appreciation adjusted for interest payment to the aggregate market capitalization.

Size	Book-to-Market Ratio	Compound Risk Premium	Premium Plus Variance	Wealth Risk Premium	Implied Risk Aversion Reduction
Small	Low	2.16%	7.68%	-0.09%	-1.12%
	2	8.71%	12.86%	-0.76%	-5.89%
	3	10.34%	13.35%	-0.96%	-7.22%
	4	12.47%	15.23%	-0.44%	-2.91%
	High	15.24%	18.34%	-0.53%	-2.87%
2	Low	2.04%	5.53%	-0.33%	-5.93%
	2	7.34%	9.54%	-0.85%	-8.87%
	3	10.15%	12.12%	-0.67%	-5.55%
	4	10.48%	12.49%	-0.40%	-3.21%
	High	11.56%	13.96%	-0.70%	-4.99%
3	Low	4.11%	6.83%	-0.56%	-8.26%
	2	8.06%	9.86%	-0.87%	-8.82%
	3	8.54%	10.12%	-0.65%	-6.45%
	4	10.43%	12.30%	-0.41%	-3.34%
	High	11.33%	13.50%	-0.95%	-7.00%
4	Low	5.53%	7.51%	-1.08%	-14.42%
	2	6.74%	8.21%	-0.64%	-7.77%
	3	9.52%	10.90%	-0.99%	-9.09%
	4	9.50%	11.14%	-0.72%	-6.42%
	High	10.32%	12.54%	-0.93%	-7.38%
Big	Low	5.33%	6.74%	-1.07%	-15.93%
	2	6.82%	7.82%	-0.89%	-11.43%
	3	7.86%	9.03%	-0.80%	-8.91%
	4	7.99%	9.37%	-1.10%	-11.76%
	High	8.55%	10.41%	-1.17%	-11.21%

Table 3 : The wealth risk premium of Fama-French size portfolios

This table reports the monotonic decreasing of wealth risk premium for the three Fama-French size portfolios. Stocks are sorted into bottom 30%, middle 40% and top 30% on their market capitalization at the end of last year. The small portfolio contains stocks in the bottom 30%. The medium portfolio contains stocks in the middle 40%. Finally, the large portfolio contains stocks in the top 30%.

Characteristics	Small	Medium	Large
Risk Premium	7.40%	6.46%	3.85%
Premium Plus Variance	13.24%	10.19%	5.87%
Wealth Risk Premium	-2.04%	-1.40%	-0.61%
Implied Risk Aversion Reduction	-15.41%	-13.78%	-10.44%