

# Basle-2 revised standard approach and beyond: Credit risk valuation of short-term loan commitments

By

**J.-P. CHATEAU**

The 2005-2006 Reinhard Selden Chair Professor of Finance, Otto-von-Guericke  
Universitaet Magdeburg, FWW – Dekanat, P.O. Box 4120, D-39016 Magdeburg. Email  
address: [jean-pierre.chateau@ww.uni-magdeburg.de](mailto:jean-pierre.chateau@ww.uni-magdeburg.de)

## **ABSTRACT**

This research makes three contributions. The first one prices the credit risk of short-term bank commitments and determines their duration-dependent funding proportion. By combining these factors, the second one computes the ‘fair’ capital charge corresponding to the commitment ‘true’ credit risk; a charge that is then compared to the accounting-based ones computed with the Basel-1 and Basel-2 credit-conversion and principal-risk factors. The advantage of the fair-value procedure is that (i) the capital charges computed are quite moderate and internally consistent for all commitment types and (ii) the commitment put values impose some market discipline. The third one finally proposes a new two-dimensional risk-weighting system, which accounts for the borrower’s rating ranges of public credit agencies.

**Key words:** Gram-Charlier put option, duration-dependent commitment funding, option-versus accounting-based capital charge, and standard risk-weighting system.

**JEL classification:** G13 and G21

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## 1. INTRODUCTION

Basel-2 simplified standardized approach<sup>1</sup> (from now on Basel-2 SSA [2004]) proposes to change the way the capital charge for short-term irrevocable credit commitments was previously computed under the Bank for International Settlement (BIS) initial guidelines (Basle-1 [1988]). Regarding commitment duration, Basel-2 does maintain Basel-1 distinction between short-term irrevocable commitments, namely those with an initial term to maturity less than one year, and longer-term irrevocable ones, e.g., those with an initial term to maturity longer than one year. Regarding commitment risk, Basel-2 also leaves most of Basel-1 coefficients unchanged. The credit conversion factor (CCF) and principal risk factor (PRF) remain both 0% for all revocable commitments irrespective of their term-to-maturity, and the 50% CCF and 100% PRF also remain in force for longer-term commitments. Yet, Basel-2 introduces a new 20% CCF and a new 100% PRF<sup>2</sup> for short-term irrevocable credit commitments –both weights being previously nil. (For the sake of clarity we abstract here from any credit risk mitigation.) Since the end of 1992, a minimum total capital requirement of 8% applies to the commitment risk-weighted balances.

The new 20% CCF and 100% PRF for short irrevocable commitments are introduced to obviate a well-known Basel-1 induced arbitrage: banks were adjusting their commitment portfolio toward those in the low risk-weight class (the short irrevocable commitments) and away from those in the high risk-weight class (the long-term irrevocable commitments).<sup>3</sup> Their interest in using this regulatory arbitrage is both

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<sup>1</sup> This approach is to be implemented by the end of 2006, whereas the internal ratings-based approach would not apply before the end of 2007. The revised standard approach thus constitutes Basel-1 relevant extension for now.

<sup>2</sup> The 100% PRF may however be reduced to 75% for the bank's retail portfolio under regulatory and granularity conditions (see SSA, pp. 22 and 67, and Annex 9, pp. 230-233 in BIS-2 [2004]).

<sup>3</sup> This regulatory arbitrage is also recognized in André *et al.* (2001). Critiques of Basel-1 guidelines and analyses of Basel-2 ones can be found, among others, in Barrios and Blanco (2003), Decamp, Rochet, and Roger (2004), Ferguson (2003), Hall (2004), Himino (2004), and Rochet (2004).

financial and strategic: the absence of a capital charge for short-term irrevocable commitments and all revocable commitments allows the banks to increase the return on the regulatory capital committed to other instruments. That Basle-2 directly relates credit risk of short-term irrevocable commitment to their capital charge is a welcome step forward. But the adjustment will be more complete if the new accounting-based coefficients set forth in Basel-2 first pillar were replaced by market-based concepts, which would also be more consistent with the “market discipline” extolled in Basel-2 third pillar. The proposed adjustment is based on the following substitution. The BIS credit-conversion factor makes way for the forward funding proportion (namely the average amount of the credit line draw down when the line is exercised) and the principal-risk factor is replaced by the value of the put option embedded in the commitment contract. This substitution does raise a number of questions. 1) Does this embedded put value capture the commitment ‘true’ credit risk? 2) Is this put value affected by the nonnormal skewness and kurtosis of the mark-to-market value of the credit line? 3) Why is credit line funding dependent on the time left to commitment expiry? And 4) how is the ‘fair’ capital charge for short-commitment ‘true’ credit risk computed?

According to Thakor *et al.* (1981), when the interest rate on a commitment contract is lower than that on an equivalent spot loan, the borrower receives the credit line face value but is only indebted for its lower marked-to-market value --usually referred to as the **indebtedness value**. More concretely, the borrower's claim on the lending bank constitutes an embedded, yet valuable, commitment put option. Several researchers have derived alternative formulas for valuing credit-line commitments. Thakor *et al.* (1981), and Ho and Saunders (1983) derived option-like values for fixed-rate commitments, Thakor (1982) and Chateau (1990) obtained put formulas for variable-rate commitments, and Hawkins (1982) priced revolving credit lines. According to the existing literature, most commitment put values are estimated with the Black-Scholes (1973) or Barone-Adessi and Whaley (1987) formulas and so, are not adjusted for any potential skewness and kurtosis in the empirical distribution of the underlying indebtedness value. Fortunately, there have been advances in research on contingent claims with nonnormal skewness and kurtosis. Corrado and Su (1996 and 1997) derived the formula for a call

option adjusted for skewness and kurtosis. It was based on the Gram-Charlier series expansion (type A) of a normal density function and their skewness coefficient was corrected by Brown and Robinson (2002). By contrast Rubinstein (1998) uses an Edgeworth form of the type-A series for valuing European and American derivatives by way of binomial trees. Jarrow and Rudd (1982) had pioneered this type of expansion when valuing derivatives based on lognormal distributions. Since then the non-zero skewness and greater-than-three kurtosis of various returns have been found, beyond the above-mentioned references, in Andersen et al. (2002), Backus et al. (1997), Jondeau and Rockinger (2000 and 2001), Jurczenko et al. (2004), Ki et al. (2004), Lekkos (1999) and Li (2000), among others. Here we propose a value formula for the Gram-Charlier put option implicit in short-term loan commitments.

To the extent the put value captures the credit risk of short commitments, it seems sensible to determine the impact of this implicit liability on the bank's capital adequacy at any BIS audit date. The aggregate face value of unused short commitments is reported as an off-balance-sheet entry to the bank annual consolidated balance sheet. Yet, at the BIS capital-adequacy audit date, the time remaining to commitment expiry is less than the initial one-year term for many of the still-unused commitments. To account for this, the average time remaining to the commitment expiry date,  $T$ , is standardized at  $T - s$ , with  $s = 0$  denoting the date at which the BIS audit takes place. Within a fixed-audit framework, the put option is European and generated by the fixed markup of commitments with a floating prime-rate formula --namely those with "a fixed markup over a stochastic index cost of funds"<sup>4</sup>. And this value corresponds to the bank's notional liability for carrying the commitment at the audit date.

Granted the above, the research proceeds as follows. A risk-neutrality argument is used in Section 2 to value the Gram-Charlier put option comprised in short-term commitments; the latter is then compared to the Black-Scholes put option to detect any systematic over- or underestimation. An exercise-cum-takedown proportion is defined next: it combines an exercise-indicator function that captures the line exercise decision to

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<sup>4</sup> According to the Federal Reserve survey of year 2000 (Board [2000]), about eighty percent of U.S. commercial and industrial lending is done via loan commitments, with the vast majority being of the floating-rate type.

a takedown proportion that increases with the time left to the commitment expiry. As commitment puts are but the notional values of embedded credit-risk derivatives, simulations are used in Section 3 to quantify the credit-risk cost of short commitments. Not unexpectedly, these cost curves look very much like in-the-money put curves.

Based on these simulations, three regulatory implications are then considered in Section 4. To start with, the structural shifts in the commitment aggregates of a large international bank testify to the Basel-1 induced arbitrage between short and longer-term commitments. We then consider why the Basel-2 solution will likely induce a new arbitrage based on its new credit-risk coefficients. It is shown next how put value and funding proportion are combined to compute the ‘fair’ or option-based regulatory capital charge for short commitments. This computation highlights that (i) the capital charges computed with market-based risk weights are moderate and internally consistent for all commitment types and (ii) the embedded put values constitute a finer credit-risk grid than the BIS accounting-based values. Finally, the fair procedure is generalized to arrive at a new two-dimensional risk-weighting system applicable to the balances of all short commitments. According to this system, the new standard credit-risk weights become sensitive to three parameters: the borrowers’ risk ratings of public credit agencies, the commitment duration, and the line-funding proportion, respectively. Interestingly, the credit-risk weights, which represent the bank’s notional credit-risk costs per \$100 of line commitment, are more sensitive to the borrowers’ credit-rating ranges than to the forward-funding proportions. The computation of these weights is summarized in a stepwise procedure.

The layout of the paper is as follows. Section 2 provides the analytical value of the European put embedded in short commitments; it also determines the exercise-cum-takedown proportion. Simulation results are presented in Section 3 and used in Section 4 to quantify the link between commitment credit risk and bank’s capital charge. Short concluding remarks close the paper in Section 5.

## 2. VALUATION OF SHORT-TERM LOAN COMMITMENTS

### 2.1 Short-term credit commitments in the BIS framework

The salient features of a short-term commitment with a fixed forward markup are stylized in the decision chart below. In part (a) of the chart, the bank writes at date 0 an off-balance-sheet commitment contract for a short-term credit line (CL) with the following features: (i) the commitment period,  $[0, T]$ , is one year, (ii) loan duration,  $[T, T^*]$ , is one year from date T if the credit line is drawn down (it is explained later on in this subsection why an European exercise date T is selected here), (iii) the CL face value is standardized at a maximum of  $L = \$100$ , and (iv) the transaction rate is  $c_T + \bar{m}_0$ . The first component,  $c_T$ , of this floating prime-rate is the bank's **stochastic** cost of funds at exercise date T, with the rate on banker's acceptance (BAs) or certificates of deposit (CDs) being generally used as exogenous index<sup>5</sup>. The other component, the fixed forward markup  $\bar{m}_0$ , is determined at date  $t=0$  when the commitment contract is written. For instance, a commitment for a \$100-maximum CL has a time-0 (time-T) prime rate of 4.5% p.a. (5% p.a.) made up of a 3%-p.a. (3.5%-p.a.) stochastic cost of funds plus at both dates a fixed forward markup of 1.5% p.a. The fixed markup thus only hedges credit risk as the borrower bears the funding risk,  $c_T$ <sup>6</sup>.

Thakor and Udell (1987) provide the economic rationale for the bank's optimal deployment of up-front and rear-end fees in commitment pricing. In their competitive equilibrium model, the screening device resolves the bank-borrower asymmetries of information and the presence of adverse selection gives rise to split fees at the commitment end-dates<sup>7</sup>. The short-commitment sorting variables are shown in part (a) of the decision chart. The initial fee,  $f_0^U$ , is an upfront fee of 1/4 of 1% per annum of the

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<sup>5</sup> Libor is often used as the international index (see Athavale and Edminster [2004] or Greenbaum and Thakor [1995]).

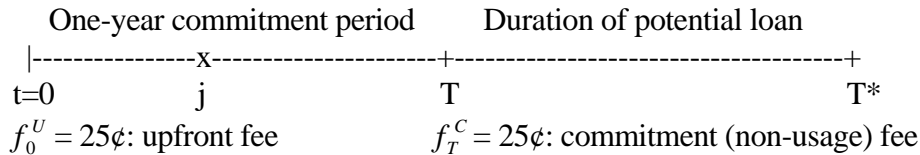
<sup>6</sup> It is also worth pointing out that the bank does not hedge fixed markup and commitment funding at the underwriting date. At the actual funding date, the bank uses its own available demand deposits and/or sells CDs or banker's acceptances in the spot (wholesale) market.

<sup>7</sup> Typically, commitments with an up-front fee only are sold to high-credit-quality firms. But commitments with up-front and rear-end fees are sold to medium size firms whose

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**DECISION CHART of a short-term credit commitment with a fixed forward markup in its floating prime-rate formula.**

a) Initial situation at  $t=0$ : The contractual terms of reference abstract from compensating balances, bank reserve requirement and annual fees, which are explained in the note at the bottom of the chart.



Characteristics: maximum of  $L = \$100$ , fixed markup  $\bar{m}_0 = 1.5\%$  p.a., and MAC clause in force from  $t = 0$  to  $T$ .

b) Regulatory time frame: BIS valuation takes place at the audit date  $s$ . Yet at  $s$ , the time left to commitment expiry can vary from  $j = 0, \dots, 12$  months. So  $T - j/s$  is the time left to the expiry date with  $s$  functioning as the option valuation date, usually 0.

- S1:  $f_T^C = 25\phi$  as  $L = 0$  when the line is left totally unexercised
- |                   |   |
|-------------------|---|
| x-----            | • S2: The fraction $0 < \pi_{T-j} < 1$ of $L = \$100$ becomes a loan, and   |
| At $s = j$ ,      | $f_T^C$ is paid on its un-funded portion $(1 - \pi_{T-j})$                  |
| Audit = valuation | • S3: Full funding, $L = \$100$ , results in a loan, with $\pi_{T-j} = 1$ . |
| date              |   |

Note: Commitments only rarely involve compensating deposit balances (Berger and Udell [1995]), and this disguised cost is usually treated as a scaling problem. U.S. reserve requirement for transaction deposits ranges from 3 to 10% depending on the size of the financial institution (Rose and Marquis [2005]). There also exist minor annual expense and annulation expense, in the order each of 3 to 5 basis points per annum (0.03 to 0.05 % p.a.).

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line maximum face value, namely here 25 cents per \$100, and the second one is an

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credit quality is poorer (Petersen and Rajan [1994]). This most prevalent type is examined here. Borrower self-selection as a screening and risk-sharing device with optimal fee mix is also examined in Ergungor (2001), Fery *et al.* (2003), Greenbaum and Thakor

exercise-contingent commitment fee,  $f_T^C$ , of the same magnitude as the initial commitment fee. This 25-cent payment really is a non-usage fee paid at T on the undrawn portion of the credit line<sup>8</sup>.

Most irrevocable commitments (ditto obviously for revocable ones) also comprise a material-adverse-change (MAC) clause. This escape covenant allows the bank to limit or even deny credit funding if the borrower's financial condition deteriorates over the commitment period. In practice, the MAC covenant renders the commitment default-free as the bank can deny funding<sup>9</sup>. In addition, retail commitments from certain countries are considered fully and unconditionally cancelable if the bank can cancel them under credit protection or related legislation (BIS [2004], p 232).

In its annual consolidated statement, the bank reports off-balance-sheet the aggregate contractual value of all short commitments. In order to set these commitments in the BIS regulatory time frame, we now introduce two additional dates, the age of the commitment and the BIS annual audit date. We first introduce in part (a) of the chart date  $j$ , with  $0 \leq j \leq T$ . It defines two periods:  $T - j$ , the time remaining in the initially one-year loan commitment and  $j - t$ , the age of the commitment. Depending on the commitment age,  $j$  varies from 0 to 12 months: for instance, if  $j = 3$  months, the initial one-year commitment is now three-month old ( $j - 0 = 3$ ) and has nine months left to commitment expiry (namely  $T - j = 9$ ). The second date, in part (b) of the chart, is the BIS audit date  $s$ , with  $j = s$ , since the  $j$ -month-old commitment is valued at the audit date. Indeed, for most of the commitments at the BIS annual reporting date, the time remaining to the expiry date is less than the initial one-year period. In what follows, date  $j=s$  functions as the date-0 valuation date of a European put option (see also Merton [1977] for a similar

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[1995], Shockley and Thakor (1997), and Thakor (1989).

<sup>8</sup> According to Shockley and Thakor (1997, Table 1) for the years 1989 and 1990, the mean upfront fee on short-term (liquidity, working capital, and trade and finance) commitments was 24.2 basis points while the mean annual commitment or usage fee was 22.8 basis points. Angbazo *et al.* (1998) note that both fees are declining since the mid-90s due to strong competition; the commitment fee also has a tendency to be somewhat lower than the upfront fee (Gottesman and Roberts [2004]).

<sup>9</sup> This MAC clause should not be confused with the borrower's potential default on principal and/or interests after the commitment has been exercised and the credit line drawn down. The bank then holds a vulnerable counterparty call; namely the bank has the ability to call the borrower's outstanding loan at any time.



argument<sup>10</sup>). This put is used later on when the risk-adjusted commitment balances are computed. The time subscript  $s$  is dropped from now on when there is no ambiguity: so  $T - s/j$  becomes  $T - j$ .

Depending on the amount of forward funding from the audit date, three outcomes, labeled scenarios S1, S2, and S3 respectively, are possible at date  $T$  in part (b) of the chart. In the first scenario, the commitment is never exercised, no funding takes place, and the borrower pays the non-usage fee,  $f_T^C$ , on the full unused \$100. In S2, the commitment is exercised and partial take down of the initial \$100 results in an on-balance-sheet corporate loan --it is explained in subsection 2.4 how the exercise-cum-takedown proportion,  $0 < \pi_{T-j} \leq 1$ , is arrived at. In addition, the borrower also pays  $f_T^C$  on the un-funded portion  $(1 - \pi_{T-j})$  of the exercised line, namely  $f_T^C [(1 - \pi_{T-j}) L]$ . Finally in S3, the commitment is exercised and full funding results in a \$100 on-balance-sheet loan, with  $f_T^C = 0$ . It is already worth pointing out that the markup assumption will be relaxed in the subsequent developments. For non-prime commitments, the forward markup that captures credit risk is to be adjusted with add-ons or discounts ( $\pm 25$  basis points,  $\pm 50$  basis points, and so on)<sup>11</sup>. And in subsection 4.3, these higher markups will be associated with the borrowers' lower risk ratings of public credit agencies.

## 2.2 Indebtedness value and its log-relatives

Thakor et al. (1981) were the first to define the marked-to-market value of a credit line, a forward debt value (with respect to the contract writing date) often referred to as the borrower's indebtedness value,  $X$ . The  $j$ -month-old indebtedness value is thus computed as

$$X_j = L \exp\{(\bar{m}_0 - m_j)(T - T^*)\}, \quad (1)$$

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<sup>10</sup> The exogenous audit date and European put option are extended in Merton (1978) to random audits and American options. See also Bhattacharya et al. (2002) for random audits.

<sup>11</sup> The magnitude of such spreads over the floating prime rate is examined in Angbazo *et al.* (1998), Athavale and Edminster (2004), Elsas and Krahen (1998), Gottesman and Roberts (2004), or Shockley and Thakor (1997).

where  $L$  is the constant par value,  $(T - T^*)$  is loan duration once the commitment has been exercised, and  $(\bar{m}_0 - m_j)$  is the difference between  $\bar{m}_0$ , the fixed forward markup set at date 0 when the commitment was written, and  $m_j = (l_j - c_j)$ , the date- $j$  stochastic spot markup defined as the difference between the spot prime credit rate,  $l_j$ , and the funding rate in the banker's acceptance market,  $c_j$ . For instance,  $X_3$  refers to a three-month-old indebtedness value of a one-year commitment with still nine months to go  $(T - j = 9)$ <sup>12</sup>. At date  $j$ , the commitment holder decides to draw on the line only if *ceteris paribus*<sup>13</sup>  $\bar{m}_0 < m_j$ , namely when the fixed forward markup is less than the stochastic spot markup. To wit, if the 1.5-% forward markup of the previous subsection is combined with, say, a 2.5-% spot markup, the markup differential in eq. (1) is negative at -1%. The inequality  $X_j < L$  then gives rise to an implicit put option as the borrower's debt value is less than the option strike price. For the above-mentioned three-month-old indebtedness value, a nine-month put option is embedded in the original one-year commitment. As in Thakor et al. (1981), the dynamics of indebtedness-value changes is given by

$$\ln[X(j)/X(j-1)] = \mu dj + \sigma dz_X(j), \quad (2)$$

where the constant terms  $\mu$  and  $\sigma$  are the drift and standard deviation of the distribution, and  $dz_X(j)$  the differential of the Wiener process  $z_X(j)$ . To save space, we list in the stub of Exhibit 1 only the log- $X$ -relatives from the third to the ninth month. From the statistical evidence presented in the second column, the mean of the indebtedness-value changes is practically 0% for indebtedness values computed from the end of the third month to that of the ninth month. In the next column, the volatility of the empirical

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<sup>12</sup> Although this value is not likely to trade directly, the difficulty is overcome (i) by appealing to Merton's (1973) intertemporal CAPM or (ii) by observing that the spot markup is a "quasi-price" as it results from the actual (equilibrium) prices in the continuous primary lending and funding markets.

<sup>13</sup> The alternative approach (not relevant however in the Basel-2 credit-risk context) is the all-in-cost basis, in which markup fees are computed and compared for credit commitments and spot loans.

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**EXHIBIT 1: Statistical analysis of  $X_j$ : {j: 3, 4, ..., 9m}, the indebtedness-value monthly time series computed from eq. (1) for the period 1966.01 to 2004.12.**

$\text{Ln}[X(j)/X(j-1)]$	Mean	Std. dev.	Skewness	Kurtosis	Min	Max
$X_{j=3m}$	-.000029	2.17	.442	8.80	-.025	.034
$X_{j=4m}$	-.000030	2.08	.044	9.92	-.035	.037
$X_{j=5m}$	-.000029	2.20	.030	9.96	-.037	.037
$X_{j=6m}$	-.000034	2.06	.256	12.82	-.037	.040
$X_{j=7m}$	-.000039	2.15	.099	9.63	-.037	.033
$X_{j=8m}$	-.000038	2.01	-.128	11.24	-.037	.034
$X_{j=9m}$	-.000034	2.14	-.563	9.74	-.038	.029

Note: For a sample size  $n = 468$  observations, the 95% confidence intervals for normal sample skewness and kurtosis coefficients are  $\pm 1.96(6/468)^{1/2} = \pm 0.222$  and  $3 \pm 1.96(24/468)^{1/2} = 3 \pm 0.437$ , respectively.

Source: Spot markups, markup differentials, and indebtedness values computed in eq. (1) are based on Statistics Canada monthly time series V122495 and V122504 of the prime credit rate and one-month banker's acceptance of chartered (commercial) banks, respectively.

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distribution fluctuates between 2.01% per annum and 2.20% per annum for log-X-relatives computed for the same months. The confidence intervals for the normal sample skewness and kurtosis coefficients are computed in the exhibit bottom note. For the statistics computed in the fourth and fifth columns, a few of both positive and negative skewness coefficients fall outside the confidence intervals and all the sample kurtosis coefficients fall outside the confidence intervals. This indicates statistically significant departures from normality: it indicates that the empirical distribution presents mostly weak positive asymmetry coupled with a strongly leptokurtic pattern. The historical values reported in Exhibit 1 are only approximations of the volatility, asymmetry and

kurtosis coefficients to be used when pricing the embedded put option in the next subsection<sup>14</sup>.

### 2.3 Valuing the skewness- and kurtosis-adjusted European put embedded in credit-line commitments

To incorporate value adjustments for nonnormal skewness and kurtosis in the Black-Scholes commitment put option, we use a truncated (for terms beyond the fourth moment) Gram-Charlier series expansion (type A) of the normal density function<sup>15</sup>. This approximation yields the following density function:

$$f(z) = n(z) \{ 1 + (\mu_3/6)(z^3 - 3z) + (1/24)(\mu_4 - 3)(z^4 - 6z^2 + 3) \}, \quad (3)$$

where

$$n(z) = (\sqrt{2\pi})^{-1} e^{-1/2z^2}$$

is the standard normal density, and

$$z = \{ \ln (X_T/X_j) - \mu T \} / (\sigma \sqrt{T}). \quad (4)$$

In eq. (3),  $\mu_3$  and  $\mu_4$  are the standardized coefficients of skewness and kurtosis respectively. In (4)  $X_j$  denotes the indebtedness value at date  $j$ ,  $T$  the final date,  $\mu T$  the conditional mean, and  $\sigma \sqrt{T}$  the standard deviation of the indebtedness-value relative changes.

Under risk neutrality, the European put value is the present value of the expected payoff at commitment expiry,  $e^{-rT} E[F(X_T)]$ , where  $E$  is the expectation operator under the

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<sup>14</sup> Since commitment puts are not traded options, one cannot extract the implied values of the volatility, asymmetry and kurtosis coefficients. McDonald [2003] also points out that using the B-S model (where for instance the volatility is assumed constant) to track changes in the implied volatility, asymmetry and kurtosis is internally inconsistent.

<sup>15</sup> Consult Johnson, Kotz and Balakrishman [1994] or Stuart and Ort [1994] for the Gram-Charlier expansion. Corrado and Su (1996 and 1997) first used the Gram-Charlier expansion for call options, and Jarrow and Rudd (1982) or Rubinstein (1998) initially

risk-neutral pricing measure  $Q$  and  $F$  the payoff. To compute the latter, the expected value of the indebtedness value under the risk-neutral pricing measure,  $E(X_T)$ , is needed. In addition, Longstaff's traditional martingale restriction,  $\mu T = rT - 1/2\sigma^2 T$  for  $E(X_T) = X_0 e^{rT}$ , has to be replaced by a moment restriction for the Gram-Charlier distribution (see for instance, Longstaff [1995], Kochard [1999], Jurczenko et al. [2004] and Ki et al. [2004]). The moment-generating function of the Gram-Charlier distribution is given by

$$M(\theta) = e^{m\theta} \int_{-\infty}^{\infty} \exp(\sigma \theta z) f(z) dz,$$

where  $\theta$  denotes the moment order and  $m$  refers to the mean. The restriction on the mean of the probability measure  $Q$  is analogous to the first-moment restriction for the Gram-Charlier distribution. Thus the latter mean should satisfy the moment restriction

$$m = \ln X_0 + (r - \frac{1}{2} \sigma^2)T - \ln(1 + \omega)$$

with  $\omega = (1/6) \mu_3 T^{3/2} + (1/24) (\mu_4 - 3) \sigma^4 T^2$ . As a result for  $\theta = 1$ , the expected value of the indebtedness value in the risk-neutral measure is

$$M(1) = E(X_T) = X_0 e^{rT} \{1 + \omega\}$$

Once  $E(X_T)$  and the moment restriction are defined, we can compute  $P_{GC}$ , the value of the embedded commitment put option based on the truncated Gram-Charlier density function in equation (3) above<sup>16</sup>:

$$P_{GC} = P_{BS} + \mu_3 Q_3 + (\mu_4 - 3)Q_4 \quad (5)$$

where

$$P_{BS} = L \exp(-rT) N(-d) - X N(-d) \quad (6)$$

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used an Edgeworth form of the Type A series.

<sup>16</sup> Equation (5) obtains by (i) combining Corrado and Su (1996) call formula corrected by Brown and Robinson (2002) with the Gram-Charlier moment restriction (as in Jurczenko et al. [2004] or Ki et al. [2005]) and (ii) introducing the resulting expression in the put-

is the Black-Scholes commitment put value in which  $d_{\pm} = \{\ln(X/L) + (r \pm \frac{1}{2}\sigma^2)T\}/(\sigma\sqrt{T})$ .  
In equation (5) also

$$Q_3 = [6(1 + \omega)]^{-1} X\sigma\sqrt{T} [2\sigma\sqrt{T} - d_*]n(d_*),$$

$$Q_4 = [24(1 + \omega)^{-1} X\sigma\sqrt{T} [d_*^2 - 1 - 3\sigma\sqrt{T}d_* + 3\sigma^2T] n(d_*),$$

and

$$d_* = \{\ln(X/L) + (r + \frac{1}{2}\sigma^2)T - \ln(1 + \omega)\}/(\sigma\sqrt{T}),$$

with  $\omega$  and the other terms having been defined previously. More concretely, the Gram-Charlier commitment put value in eq. (5) is the sum of a Black-Scholes commitment put and two nonzero adjustment terms for non-normal skewness and kurtosis. But, if the indebtedness-value relative changes are normally distributed, then  $\mu_3 = 0$ ,  $\mu_4 = 3$ , and equation (4) collapses to eq. (6), the Black-Scholes commitment put value. In brief, the Gram-Charlier put option captures the credit risk embedded in short-term loan commitments.

## 2.4 Modelling the forward-funding proportion

Once the embedded put value is computed, it remains to determine the proportion of the still unused credit line that can be drawn down forwards, from date  $j$  to  $T$ . Recall how this was formalized in part (b) of the decision chart in subsection 2.1: first commitment exercise occurs or not, and next for the exercised commitments, partial or total funding takes place. Modelling the exercise-cum-funding proportion of individual commitments in the BIS regulatory framework is exceedingly difficult<sup>17</sup>. So, we propose the following bank-level solution at the BIS audit date:

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call. parity.

<sup>17</sup> Some of the reasons of the difficulty are: (i) individual commitments are written continuously with varying initial (maximum) amounts, (ii) draw downs take place on different dates, (iii) some lines are completely drawn down, others are partially drawn down and in stages, and some are left unexercised altogether, and (iv) banks take advantage of the MAC clause or some credit-protection legislation to limit or even cancel

- (i) the \$100 commitment functions as a reference unit that is either completely drawn down or completely left un-drawn, with partial takedown reallocated to these two extreme proportions, and
- (ii) the forward-funding proportion applies at the bank level to the aggregate amount of still alive commitments whose time left to expiry,  $T - j$ , is variable.

The conditional average proportion at date  $j$  for the period  $T-j$ , written  $\pi_{T-j}$  is:

$$\pi_{T-j} = E [d_j \times 1_{X_j \leq L}] = d_j \times P(X_j \leq L), \quad (7)$$

where  $d_j$  is the funding proportion of lines of age  $j$ ,  $1_{\text{condition}}$  is equal to one if the condition is verified at date  $j$  and zero otherwise, and  $T-j: 1, \dots, 12$  refers to the duration of the embedded put option. The complementary proportion is thus

$$1 - \pi_{T-j} = (1 - d_j) \times P(X_j \leq L) + P(X_j > L). \quad (8)$$

More concretely, the total amount of potential funding of  $j$ -month old commitments is the average proportion  $\pi_{T-j}$  times the aggregate amount of still unused outstanding commitments at the end of month  $j$ . And  $(1 - \pi_{T-j})$ , applied to the same-month aggregate amount, determines its total un-funded amount --from the un-drawn portions of the exercised commitments as well as from the totally un-funded lines.

From the empirical evidence reported in Athanavale and Edminster (2004), Gottesman and Roberts (2004) and Morgan (1993)<sup>18</sup>, we select a proportion,  $\pi_{T-j}$ , that increases with the time remaining to the put option expiry. To wit, a proportion of  $\pi_{T-j} \equiv \pi_{3m} = 45\%$  means that, for initially one-year commitments with only three months left to expiry, 45% of the aggregate still unused dollar amount is taken down; this funding proportion increases to  $\pi_{9m} = 75\%$  for commitments with nine months left to expiry. The

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funding.

<sup>18</sup> Morgan (1993) indicates that between 1988 and 1990, the fraction of the loan limit actually borrowed by prime-rate borrowers is about 55%; unfortunately, he is not reporting the number of commitments left unexercised. In Athavale and Edminster (2004), the dollar amounts of loans are reported, but not the commitment initial sizes; conversely, the facility

percentage chosen for commitments with little time left to expiry is relatively low because (i) there is little time and opportunities left to draw down these lines and (ii) the borrower intentionally refrains from taking down full funding so as to avoid being charged higher commitment fees in the next period (Ergungor [2001]). The funding proportion is likely to increase however, as borrowers have more time and investment opportunities to draw down (even cumulatively) irrevocable credit lines.

The forward-funding proportion along with the embedded put value is all that we need to compute the bank's capital charge corresponding to the credit risk of short-term commitments. Equations (1) to (8) form the credit-risk valuation programme of short commitments, which is estimated in the next section.

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sizes are reported in Gottesman and Roberts (2004), but not the funding amounts.



## SIMULATION RESULTS

### 3.1 Simulation

As credit-risk derivatives, the Gram-Charlier put values implicit in short commitments are but notional values. We thus rely on simulations to compute their values, and our simulation parameters are based on the statistical evidence reported in Exhibit 1 in subsection 2.2. The reported min-max relative changes in indebtedness values imply that historically most indebtedness values (the  $X_j$ s) vary in the value range \$96.5 to \$103.5: it is thus sensible to set  $X$  at \$100, \$99.5, \$99, \$98.5, \$98, and 97.5 for a commitment put that moves progressively in the money<sup>19</sup>. For a line par value of \$100, the slightly in-the-money indebtedness values simulate small increases in the spot markup of the class of floating prime-rate borrowers over the twelve-month term. Granted these indebtedness values, the simulation experiments are performed for commitments with a remaining time to maturity,  $T - j$ , from 3 to 9 months. The values of the volatility, skewness, and kurtosis coefficients reported in Exhibit 1 are used in the commitment put simulations. Furthermore, two parameters are common to all simulations: the credit-line strike price,  $L$ , is \$100 and the risk-free interest rate,  $r$ , is 4% or 0.04.

Before reporting on the simulations, we first clarify the meaning of computed put values. Consider the time-risk scenario represented by the entries  $T - j = 6$  months and  $X = \$99$  in the first matrix of Table 1. This cell corresponds to an embedded put with six months remaining to expiry, and an indebtedness value slightly in the money at  $X = \$99$ . According to the (underlined) estimate  $P_{GC} = 0.096$ , the Gram-Charlier put has an equilibrium value of 0.096% of the \$100 par value if the floating prime-rate commitment with a 1.5%-p.a. fixed forward markup is priced when the stochastic markup on spot loans is 2.5% p.a. By comparison, the corresponding Black-Scholes put value in the second matrix of the table is larger at 0.211; this implies that the (underlined) magnitude (-54.7%) of the value adjustment induced by nonnormal skewness and kurtosis is negative and significant, as shown in the corresponding cell in the table third matrix.

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<sup>19</sup> To consider indebtedness values below \$97.5 is of limited interest since there are but a few values (outliers) lower than \$97.5 out of the 468 observations.

## Table 1 about here

### 3.2 Credit-risk assessment in terms of Gram-Charlier commitment put values

Two credit-risk patterns are emerging from the matrices of Table 1. The first tendency focuses on the magnitude of the cost incurred by the bank for carrying unused credit lines with varying time to maturity at the annual audit date. The  $P_{GC}$  matrix of Table 1 shows the sensitivity of commitment puts to risk variations (as one moves down each column) as well as time-to-maturity variations (when moving across each row). To wit, in the third row of the  $P_{GC}$  matrix for an indebtedness value of \$99, Gram-Charlier put values fluctuate up and down from \$0.113 for nine-month put options to \$0.303 for three-month ones (both shown in bold values). The other rows also depict mixed time patterns. On the other hand, the matrix columns capture the indebtedness-value risk. For instance, for the fourth column of the  $P_{GC}$  matrix in which  $T-j = 6$  months, put values are increasing continuously from \$0.121 for the par indebtedness value ( $X = \$100$ ) to \$0.688 for the in-the-money indebtedness value ( $X = \$97.5$ ). Similar put-like down-sloping patterns are observed for commitments with different maturity terms; a sample of these cost curves is shown in Figure 1. In brief, the  $P_{GC}$  matrix 1 of Table 1 clearly indicates that put values, and hence the bank's credit-risk costs, are more sensitive to risk changes in the indebtedness value than to maturity changes in the put options. The comment regarding the second matrix can be rather short: Black-Scholes put values duplicate the patterns observed for the Gram-Charlier put values.

## Figure 1 about here

The other pattern, revealed by the rows and columns of matrix 3, pertains to the over- or underestimation of the Gram-Charlier put values expressed as a percentage of the  $P_{BS}$  values. Black-Scholes values mostly overestimate (up to 56.6%) Gram-Charlier put values; yet this overestimation is neither systematic nor continuous since some Gram-Charlier put values are greater than the corresponding B-S ones for all par indebtedness

values and for some in-the-money ones. These simulation results are used in the next section to quantify the link between commitment credit risk and its risk-weighted capital charge.

## 4. LINKING COMMITMENT CREDIT RISK TO CAPITAL SUFFICIENCY

The BIS *credit-risk* guidelines of off-balance-sheet loan commitments (see BIS-1 [1988] and BIS-2 [2004])<sup>20</sup> can be summarized as follows. First convert by way of a **credit conversion factor (CCF)** the commitment contractual amount to a “credit equivalent [on-balance-sheet] amount”; and weight next the resultant amount by a **principal risk factor (PRF)** to arrive at the commitment risk-weighted balance. Since the end of 1992, a minimum total capital requirement of 8% applies to this risk-weighted balance. Regarding commitment credit risk, Basel-2 simplified standardised approach leaves most of Basel-1 risk weights unchanged. In Basel-2 as in Basel-1, the credit conversion factor (CCF) and principal risk factor (PRF) are nil or 0% for all revocable commitments irrespective of their term-to-maturity; and they are 50% and 100% respectively for irrevocable commitments with an initial term longer than one year. Basel-2 novelty is to introduce a 20% CCF and a 100% PRF for the short-term irrevocable credit commitments –both weights being previously nil<sup>21</sup>. In the sequel, we abstract from any credit risk mitigation for the sake of clarity.

Granted this brief review of commitment risk weights, three questions are now dealt with in turn. (1) Is Basel-1 regulatory arbitrage between short and long commitments likely to be displaced by a new one induced by Basel-2 new risk weights? (2) How is the ‘fair’ or option-based capital charge for short commitments computed? And (3) how can this fair procedure be generalized to arrive at new standard credit-risk weights for short loan commitments?

### 4.1 Displacing Basel-1 regulatory arbitrage

Basel-1 credit-risk weights have induced a well-known regulatory arbitrage among commitments. Banks prefer offering commitments with no risk weights (0% CCF and 0% PRF), namely 364-day irrevocable commitments or revocable commitments

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<sup>20</sup> We focus here on the simplified standard approach for credit risk, and do not dwell on the other risks, such as legal, market, operational, and so forth.

<sup>21</sup> The 100% PRF may however be reduced to 75% for the bank’s retail portfolio under regulatory and granularity conditions (see SSA, pp. 22, 67, & 230-233 in BIS-2 [2004]).

without distinction of maturity, rather than longer-term irrevocable commitments with substantial risk weights (50% CCF and 100% RPF). To obviate this arbitrage, Basel-2 introduces a 20% CCF and a 100% PRF for short-term irrevocable commitments: yet the 0% CCF and nil PRF remain in force for short-term revocable commitments. In other terms, the Basel-2 new risk weights may only displace the previous arbitrage between short- and longer-term commitments and replace it by a new one between short-term irrevocable commitments (now the lower credit-risk weight class) and short-term revocable ones (the no credit-risk weight class).

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**Exhibit 2: Structural changes in the balances of short- and long-term commitments of the Royal Bank of Canada for the period 1989 to 2005.**

Contractual amount of still unused commitments with a remaining term to maturity:

	1989	1995	2000	2003	2004	2005
≤ 1 year, C\$ in billions	40.9	44.5	98.0	100.1	106.7	95.8
of which - irrevocable ones				40.3	45.7	50.8
- revocable ones				59.8	61.0	44.9
> 1 year, C\$ in billions	28.8	23.7	41.6	28.2	28.9	30.5
of which - irrevocable ones				28.2	28.9	30.5
- revocable ones				0.0	0.0	0.0
Total of all commitments	69.7	68.2	139.6	128.3	135.6	126.3

---

This slippage seems to be borne out by the structural shifts observed in the main commitment aggregates reported in Exhibit 2 for a large international bank. For instance, the amount of less-than-one-year commitments has increased by 134.3% over the sixteen-year period, rising from \$40.9 billion in 1989 to \$95.8 billion in 2005, and their share of the commitment total has grown significantly from 58.7% to 73.6%. Simultaneously, the minimal growth (5.9%) exhibited by longer-term commitments over the same period has resulted in their share of the commitment total contracting from 41.3% to 24.2%. Since 2003 moreover, short commitments are split between irrevocable and revocable (also

called uncommitted) commitments. While both balances had nil risk weights under Basel-1, this is not the case under Basel-2: the credit-conversion and principal-risk factors are 20% and 100% respectively for short irrevocable commitments, but both coefficients remain nil for short revocable commitments. In 2005 44.9 billion, that is 46.9% of short commitments or 36.5% of all commitments, are in this unweighted risk class. For longer commitments on the other hand, all 30.5 billion are irrevocable commitments, with a nil balance for revocable ones. In short, this bank is already implementing the new arbitrage ahead of the Basel-2 risk weights coming into force in 2006. Basel-2 simplified standard approach may thus constitute but a partial remedy to the arbitrage, with the option-based approach proposed in the next subsection as a more complete solution.

#### **4.2 ‘Fair’ capital charge corresponding to short-commitment ‘true’ credit risk.**

The solution put forward is based on two premises. Firstly, the proportion of [off-balance-sheet] commitments that is likely to become [on-balance-sheet] outstanding loans is captured by the funding proportion that depends on commitment duration. Secondly, the commitment credit risk is determined by the Gram-Charlier put value implicit in short commitments. More concretely, the funding proportion and commitment put value play the role of the BIS credit-conversion factor and principal-risk factor, respectively. The procedure is illustrated in Table 2 where the benchmark scenario,  $X = \$99$ , from the first matrix in Table 1 is combined with the year-2005 data shown in Exhibit 2. The assumptions underlying this scenario thus are: (i) the commitment contractual amount,  $L$ , is \$95.8 billion, (ii) the time left to commitment expiry is six months,  $T-j = 6$ , and (iii) the forward funding proportion,  $\pi_{T-j}$ , is 60%. The computation is as follows:

$$\$95.8 \text{ billion} \times 0.6 = \$57.48 \text{ billion}$$

$$\$57.48 \text{ billion} \times 0.00096 (= \text{commitment put value per } \$ \text{ billion}) = \$55.18 \text{ million}$$

$$55.18 \times 0.08 = 4.41 \text{ million}$$

#### **Table 2 about here**

On the first line, the duration-dependent takedown proportion of 60% converts the off-balance-sheet contractual amount into an on-balance-sheet credit-equivalent amount --also

reported on line (3) in the last column of Table 2. The latter result is then multiplied by the Gram-Charlier put value to arrive on the second line at the risk-adjusted balance of short-term commitments --an amount also shown on line (5) in the last column of Table 2. On the third line finally, the capital charge obtains by applying the 8% capital requirement to the just-computed risk-weighted balance --this corresponds to line (6b) in the last column of Table 2. More concretely, the capital charge for short commitments is moderate, in the order of a couple of millions. This is to be contrasted with the Basel-1 dichotomy of no capital charge for both short irrevocable commitments and all revocable ones, and an extremely substantial one (1.252 billion on line (6a) in Table 2) for longer-term irrevocable commitments. Furthermore, under the Basel-2 scenario, the capital charge for less-than-one-year irrevocable commitments has increased from nil under Basel-1 to 812.8 million (that is roughly two thirds of the charge for over-one-year commitments) -- an amount also shown on line (6a) in Table 2. One can surmise that banks may prefer a 'fair' valuation procedure that requires a very minimal charge against both short irrevocable commitments and all revocable ones without distinction of maturity. Regulators, on the other hand, could be attracted to the solution by the fact that any regulatory arbitrage between these two commitment types vanishes as well as by the market discipline induced by the put liability value. Finally, the proposed procedure is developed one step further to arrive at a two-dimensional credit-risk weighting system.

### 4.3 New standard credit-risk weights for loan commitments

We now propose that standard risk weights applicable to short commitments be based on the Gram-Charlier put,  $P_{T-j}^{GC}$ , and the duration-dependent funding proportion,  $\pi_{T-j}$ . This amounts to determining the sensitivity of  $P_{T-j}^{GC}$  with respect to  $X$  and  $T-j$ , and that of  $\pi_{T-j}$  in terms of  $T-j$ . And the resultant risk-weighting system relies on two not-unreasonable assumptions: (i) the Gram-Charlier put value is mainly a function of the indebtedness value, the latter being itself a market proxy for the borrowers' risk ratings of public credit agencies, and (ii) the forward-funding proportion varies with the commitment expiry date.

Firstly, regarding the put sensitivity to the indebtedness value  $X$ , we make the following observation: the floating credit rate and hence forward markup of line

commitments are generally set below the credit rate and markup set in spot loans. It is moreover sensible to assume that the differential between spot markup and forward markup grows larger as the borrowers' risk rating by external credit agencies declines. In essence, we propose to associate the progressively in-the-money indebtedness values with the declining risk-rating ranges proposed for on-balance-sheet loans in the Second Consultative Document (BIS [2000] or Fischer [2001]). The argument runs as follows. For prime-rate borrowers (those in the credit-risk range [AAA to AA<sup>-</sup>]), the bank is likely to charge a spot markup that is equal to the forward markup of a credit commitment: in that case the indebtedness value, X, is equal to the line par value, L. But for spot loans and credit lines of borrowers with a risk rating in the range or risk bucket [A<sup>+</sup> to A<sup>-</sup>], the loan spot markup is slightly higher than the corresponding forward markup charged on credit lines. Hence, according to expression (1), the indebtedness value corresponding to this risk bucket is lower than the line \$100 par value, say \$99.5. With lower risk ranges correspond progressively deeper in-the-money indebtedness values. This holds true up to the lowest risk range, defined as less than B<sup>-</sup>, which corresponds to the indebtedness value \$98. For un-rated borrowers, the X value is \$97.5.

### **Table 3 about here**

Secondly, as pointed out in subsection 2.4, the proportion of line funding is likely to be somewhat greater the longer the time left to commitment expiry. Generally speaking, borrowers have more opportunities to draw cumulatively on the credit line the longer the time left to commitment maturity. The computation of the new risk weights is based on the following scale: the forward-funding proportion increases progressively from 45% of the initial \$100 maximum for commitments with a three-month remaining life of contract to 75% for commitments with nine months left to expiry. Given the above assumptions, the matrix of standard risk weights has the advantage of being simultaneously a function of the risk rating range of external credit agencies, the variable funding proportion, and the commitment duration. The granularity of this time-risk system is indeed richer (although it could be improved by increasing the number of risk grades when using any internal ratings-based approach) than the Basel-2 coefficients



characterized by the superficial time-to-maturity dichotomy for irrevocable commitments and their only two risk weights.

## Figure 2 about here

The new risk weights per \$100 of borrower's commitment are presented in Table 3: its columns refer to rating ranges from public credit agencies and its rows to the duration-dependent funding proportion<sup>22</sup>. Not unexpectedly, the table rows reveal that, for a commitment with a given forward-funding proportion, the credit-risk weights present a put-like down-sloping pattern. To wit, for a commitment with a 60% forward funding proportion over the coming six months (namely for  $\pi_{6m} = 0.60$  on the matrix fourth row), the risk weights decline from \$0.413 per \$100 of commitment for un-rated (and below investment-grade) borrowers to \$0.073 per \$100 of commitment for top investment-grade borrowers (3As to 2A's). Yet, when a given risk bucket is chosen, the reading down the column is surprising. For top credit borrowers in the matrix sixth column, the risk weights do not exhibit any systematic or unambiguous up or down pattern. But for unrated borrowers (in the first column), the risk weights vary inversely with the time left to commitment expiry. Visually, Figure 2 confirms what Table 3 highlights: that credit-risk weights, and hence the bank's notional costs per \$100 of line commitment, are more sensitive to the borrowers' credit-rating range (across a row) than to the forward-funding proportion (down a column). Finally, each weight is simply multiplied by 8%, the capital ratio, to determine the actual dollar amount of capital per \$100 of commitment funding. This stepwise procedure can be summarized as follows:

- 1. Compute the sensitivity of the Gram-Charlier put values, the  $P_{T-j}^{GC}$ , to the indebtedness value  $X$  and the time left to the put expiry  $T-j$ , respectively;**
- 2. Multiply the rows of the resultant matrix by the funding proportions, the  $\pi_{T-j}$ ;**  
**and**
- 3. Apply the 8% regulatory capital requirement to the new standard credit-risk weights.**

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<sup>22</sup> Gottesman and Roberts (2004) also explore the relationship between facility maturity

**Or analytically,  $\$100 \times \{ [P_{T-j}^{GC} = f(\mathbf{X}, T-j)] \times [\pi_{T-j} = g(T-j)] \} \times 0.08 =$  the credit-risk capital charge per \$100 of short-term commitments.**

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and credit spreads.

## 5. CONCLUSION

This research makes three contributions. The first one prices the credit risk embedded in short-term commitments and determines the funding proportion that depends on commitment duration. By combining these factors, the second one relates the commitment 'true' credit risk to its 'fair' capital charge; a charge that is then compared to the accounting-based ones computed with the Basel-1 and Basel-2 credit-conversion and principal-risk factors. The third one proposes a new two-dimensional risk-weighting system that accounts for the borrowers' rating ranges of public credit agencies. In this three-step process, the BIS credit-conversion and principal-risk factors are replaced with the duration-dependent takedown proportion and the Gram-Charlier put value embedded in the commitment contract, respectively. This fair-value procedure has the advantage that (i) the capital charges computed are quite moderate and internally consistent for all commitment types and (ii) the put values impose some market discipline. Further work will consider the Basel-2 transition from the simplified standardised approach to the internal ratings-based approach. It is already clear from the computation of the Gram-Charlier put and the assumptions underlying the funding proportion that any system internal to the bank will require an extensive data base, and be complicated and time consuming. Another point to elaborate further is the change of mark-up class by the bank borrower: a matrix of transition probabilities between mark-up states seems a promising start.

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**Table 1: Gram-Charlier put values embedded in short-term commitments**

Entries in matrix 1:  $P_{GC}$  from eq. (5), Gram-Charlier put values embedded in short-term credit commitments. Entries in matrix 2:  $P_{BS}$  from eq. (6), Black-Scholes put values embedded in same commitments. Entries in matrix 3: nonnormal skewness + kurtosis adjustments as a percentage of the Black-Scholes put values, namely  $(P_{GC} - P_{BS})/P_{BS}$ . Parameter definition:  $L$  = credit line exercise value in \$;  $r$  = short-term rate of interest, in % per annum;  $T$  = commitment maturity date;  $T - j$  = time remaining to commitment put expiry; and  $X$  = indebtedness value in \$ computed from eq. (1).

		T - j:	9 ms	8 ms	7 ms	6 ms	5ms	4 ms	3ms
$P_{GC}$	X = 100.0		.094	.119	.125	<b>.121</b>	.102	.101	.112
	99.5		.103	.127	.130	.103	.115	.104	.151
	99.0		<b>.113</b>	.133	.144	<u>.096</u>	.171	.176	<b>.303</b>
	98.5		.142	.158	.199	.157	.323	.397	.622
	98.0		.212	.238	.332	.348	.602	.782	1.071
	97.5		.351	.407	.571	<b>.688</b>	.999	1.267	1.567
$P_{BS}$	X = 100		.043	.042	.062	.056	.077	.072	.100
	99.5		.079	.080	.116	.113	.154	.160	.221
	99.0		.136	.145	.202	<u>.211</u>	.281	.314	.425
	98.5		.224	.246	.332	.363	.472	.550	.721
	98.0		.351	.394	.514	.580	.735	.871	1.101
	97.5		.524	.597	.756	.865	1.068	1.265	1.541
$(P_{GC} - P_{BS})/P_{BS}$									
	X = 100		116.2	184.7	100.8	115.2	32.4	41.4	12.1
	99.5		30.0	58.6	12.3	-9.3	-25.4	-35.0	-31.7
	99.0		-16.9	-8.2	-28.7	<u>-54.7</u>	-39.0	-43.8	-28.6
	98.5		-36.7	-35.6	-40.0	-56.6	-31.6	-27.7	-13.8
	98.0		-39.5	-39.5	-35.4	-40.0	-18.0	-10.1	-2.7
	97.5		-33.1	-31.7	-24.4	-20.5	-6.4	0.2	1.7

Common parameters:  $L = 100$ ;  $r = 0.04$ ;  $T = 12$  months;  $T - j = \{3, \dots, 9 \text{ months}\}$ .



TABLE 2: Computing the regulatory capital charge for short-term commitments: the BIS accounting-based capital charges versus the option-based one

	<b>Accounting-based</b> computation				<b>Option-based</b> computation
	Irrevocable <sup>a</sup>		Revocable <sup>a</sup>		Scenario: X = \$99 & T-j = 6 months
	< 1 yr B1 <sup>b</sup>	> 1 yr B2 <sup>b</sup>	> 1 yr B1≡B2 <sup>c</sup>	B1≡B2 <sup>c</sup>	all < 1-yr irrevocable commitments + all revocable commitments
(1) Contractual amount, C\$ in billions	50.8	50.8	34.4	44.9	95.8
(2) <b>Credit conversion</b> factor, CCF in %	0	20%	50%	0	60%
(3) Credit-equivalent amount, C\$ in billions	nil	10.16	17.2	nil	57.48
(4) <b>Principal risk</b> factor, PRP in %	0	100%	100%/91% <sup>d</sup>	0	0.00096
(5) Risk-weighted balance, C\$ in billions	nil	10.16	15.66	nil	0.05518
(6a) <b>Regulatory</b> capital charge, C\$ in <b>billions</b>	0	0.8128	1.252	0	
(6b) <b>Fair</b> capital charge, C\$ in <b>millions</b>					4.41

Notes:

a Irrevocable commitments are unused portions of firm authorizations to extend credit and revocable commitments are offers but no obligations to extend credit.

b B1 refers to the Basel-1 scenario and B2 to the Basel-2 standardized simplified approach.

c B1≡B2 indicates that the risk weights remained unchanged from Basel-1 to Basel-2

d The first figure refers to the BIS-set percentage and the second to the actual weighted average of counterparty risk within this class. The latter figure is used to compute line (5).

Source: Royal Bank of Canada, 2005 annual report, Table 32 p. 60, Table 39 p. 66, and note 25 p. 122.

**TABLE 3: Two-dimensional risk-weighting system <sup>a</sup>: credit-risk cost per \$100 of short commitments with three to nine months remaining to expiry.**

Borrowers' risk bucket or Indebtedness value, X in \$	Unrated	B <sup>-</sup>	[2B <sup>+</sup> -B <sup>-</sup> ]	[3B <sup>+</sup> -3B <sup>-</sup> ]	[A <sup>+</sup> -A <sup>-</sup> ]	[3A-2A <sup>-</sup> ]	
	97.5	98.0	98.5	99.0	99.5	100.0	
Line takedown <sup>b</sup> , $\pi_{T-j}$ in %:							
$\pi_{9m} = 0.75$	\$	.263	.159	.107	.085	.077	<b>.071</b>
$\pi_{8m} = 0.70$		.285	.167	.111	.093	.089	.083
$\pi_{7m} = 0.65$		.371	.216	.129	.094	.085	.081
$\pi_{6m} = 0.60$		<b>.413</b>	.209	.094	.058	.062	<b>.073</b>
$\pi_{5m} = 0.55$		.549	.331	.178	.094	.063	.056
$\pi_{4m} = 0.50$		.634	.391	.199	.088	.052	.051
$\pi_{3m} = 0.45$		.705	.482	.280	.136	.068	<b>.050</b>

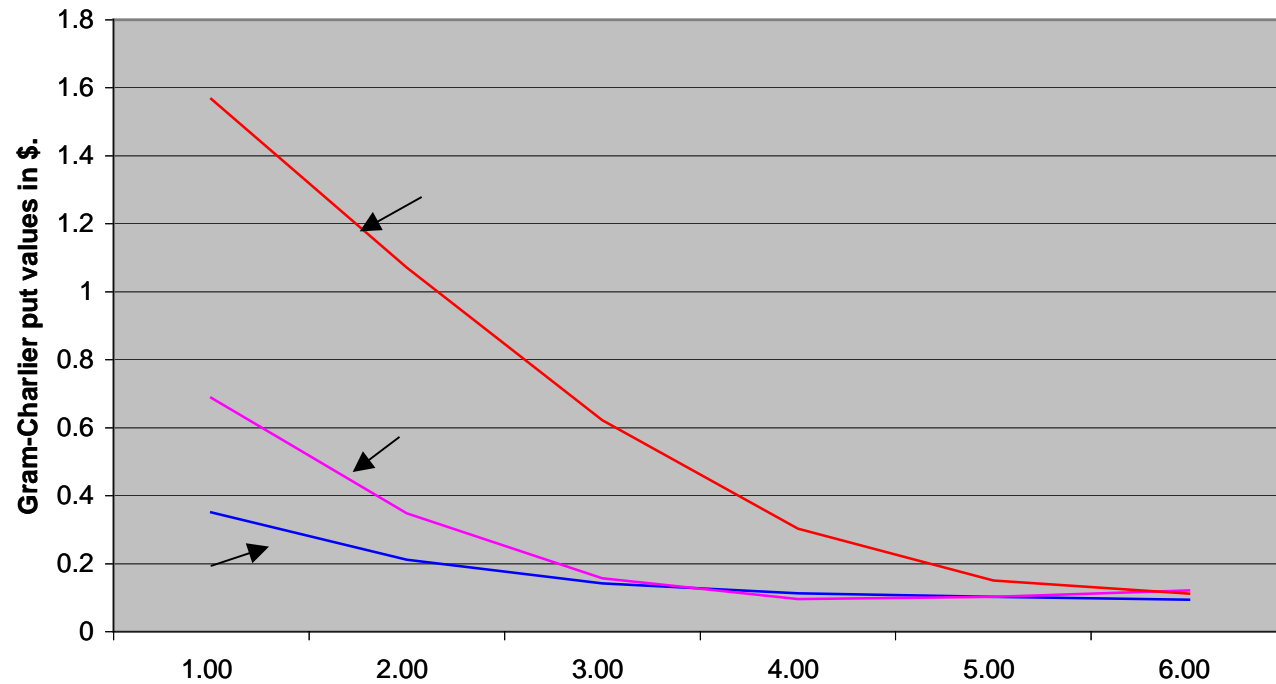
Common parameters: L = 100; r = 0.04; T = 12 months; T - j = {3, ..., 9 months}.

Note:

a The matrix captures the sensitivity of the Gram-Charlier put value to the indebtedness value and the time left to option expiry, namely  $\partial^2 P_{GC}^{T-j} / \partial X \partial (T - j)$ . Each row of the resultant matrix is next multiplied by a different funding proportion,  $\pi_{T-j}$ .

b The funding proportion varies with the time to expiry of the embedded put option, denoted by the  $\pi$ -subscript, T-j: {3, ..., 9}.

**Figure 1: Cost curves depicted by the Gram-Charlier 3-, 6- and 9-month commitment put values, in \$. Vertical differences between the curves measure the time effect: to wit, at #1 or  $X = \$97.5$ ,  $\$0.688 - \$0.351 = \$0.317$  between 3- and 6-month puts**



X= indebtedness values; from #1= \$97.5 (in-the-money value) to #6 = \$100 (par value).

**Figure 2: Sensitivity of credit-risk weights to the borrowers' risk ranges (on the X axis) and the funding proportion that varies with the commitment term (on the Y axis).**

