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OPTIMAL BENCHMARKING FOR ACTIVE PORTFOLIO MANAGERS UNDER LINEAR OR AFFINE COMPENSATION SCHEMES*

by

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Abstract

Within an agency theoretic framework adapted to the portfolio delegation issue, we show how to construct optimal benchmarks. In accordance with U.S. regulations, the benchmark-adjusted compensation scheme is taken to be symmetric. The investor's only control is to force the manager to adopt the appropriate benchmark. Solving simultaneously the manager's and the investor's dynamic optimization programs in a fairly general framework, we characterize the *optimal benchmark*. We then provide explicit solutions when the investor's and the manager's utility functions exhibit different CRRA parameters. We find that, even under optimal benchmarking, it is never optimal for the manager, and therefore for the investor, to follow *exactly* the benchmark, except in a very restrictive case. We finally assess by simulation the practical importance, in particular in terms of the investor's welfare, of selecting a sub-optimal benchmark.

Keywords: Benchmarking; Incentive Fees; Mutual Funds; Continuous Time Trading; Martingale Approach; Principal-Agent model.

J.E.L. Classification: G15, G23, G18.

I. INTRODUCTION

The compensation of active portfolio managers most often depends on their performance relative to a benchmark, a practice that is sensible to the extent that their skill (or lack of it) is best measured when the performance of the market(s) they trade on is taken into account. Earning 7% in a given period by investing in US stocks has evidently not the same meaning and the same implications according to whether the S&P 500 index has increased by 15% or decreased by 6% over the same period. The spectacular growth of the managed funds industry in recent years has elicited extensive research on the various aspects of the delegated portfolio management issue[1]. One particular strand of research has been concerned with the appropriate benchmark-adjusted compensation scheme that should be adopted. Within this literature, much attention has been devoted to the case where the part of the incentive fee that refers to the benchmark is symmetric, implying that a bonus is paid to the manager if the portfolio return exceeds that of the benchmark but a penalty is inflicted to him if the opposite occurs. This choice is essentially motivated by the regulation currently in force in the U.S. (Amendment to the Investment Advisors Act of 1940 passed by the Congress in 1970), as well as in many European countries, which prohibits mutual funds, pension funds and other publicly registered investment firms to use the asymmetric (bonus only) compensation scheme. Moreover, these institutions still represent the bulk of the delegated portfolio management industry, the recent success of hedge funds and other alternative management funds notwithstanding[2].

The literature relative to the optimal contract that should bind the principal (investor) and her agent (manager) addresses in general one of the following problems (detailed below): i) under what conditions is a linear or affine contract a first or a second best contract for the investor? ii) given that the contract is linear or affine, what parameters make it a first or a second best contract ? In the first issue, the benchmark is found as part of the solution to the principal's problem, while in the second issue it is exogenously given.

[1] Over $4 trillion is presently invested in U.S. equity funds alone.

[2] Note that some of these funds charge a linear symmetric incentive fee, although convex, asymmetric, compensation schemes have become popular.

In this paper, we adopt yet a different approach. We take the linear or affine structure of the contract, and its parameters, as given. This choice is motivated by: (i) the US legislation referred to above; (ii) the fact that the literature has largely focused on linear contracts, which will allow for useful comparisons with our results; and (iii) tractability and economic interpretation of the results, considering that they will be shown to be already rather involved. Under this assumption, we examine the issue of what constitutes the optimal benchmark for the *first best* contract. As is well known, there is a trade off between model generality (in particular regarding the assumed price dynamics for the traded assets and the relevant information structure) and tractable solutions. Adopting a setting that leads to a well defined second best contract, even not expressed in closed form, requires heroic assumptions that may blur the main message of the analysis. This is the reason why this paper is restricted to the study of first best contracts, the adopted setting being however fairly general.

Our approach extends or complements the extant literature in four directions.

1. While other studies did derive endogenous second best contracts under moral hazard and sometimes under adverse selection as well, they had to adopt very specific settings and assumptions (such as very specific processes for the traded asset prices, risk neutrality, control by the agent of only the drift of the value of the managed portfolio and not its volatility). By contrast, our analysis is fairly general as to the assumed utility functions and asset price dynamics. We allow for general Von Neuman-Morgenstern utility functions, as the CARA assumption is very restrictive and typically does not reflect actual investors' behavior. When we specialize the model to a sub-class of utility functions to obtain some closed form results, we select the CRRA (constant relative risk aversion) family, which is admittedly restrictive but nevertheless less problematic on empirical grounds than choosing the CARA utility. In addition, and more importantly, the stochastic processes generating asset returns are not mere geometric Brownian motions but are general diffusion processes. Furthermore, the riskless interest rate is not constant but obeys also a fairly general stochastic process. We thus are in a position to introduce state variables that will influence the investment opportunity set, instead of assuming the latter constant. This generalization is important in that it helps justifying why the principal should delegate her portfolio decisions to a manager in the first place. Even if the real world exhibited such stability that the assumption of i.i.d. returns that underlies geometric Brownian motions was reasonable, and consequently

that inferences by investors as to future return characteristics were easy, some arguments could still be advanced that justify the employment of professional managers. For instance, one could invoke the agent's lower transaction costs and the principal's lack of time or desire for active management. Yet, the argument that managers may have access to superior information is much more convincing. Suppose that, for reasons we review in section II below, asset returns are partially predictable, an assumption that is precisely ruled out by modeling returns as simple Brownian motions. Then it seems realistic to assume that typical investors have neither the technology nor the skill to make use of sophisticated Bayesian optimizing methods that can exploit this predictability. If they believe that professional managers do have these capabilities, then delegation makes more sense. In a way, we can think of markets being complete for managers but incomplete for investors, who then are willing to pay a fee to access truly optimal portfolios. Only is it necessary that the principal uses the appropriate benchmark so as she reaches her first best optimum.

2. In the continuous time literature, almost always the manager controls only the drift of his action's objective. This is particularly restrictive in the case of portfolio delegation, since there exists a risk-return trade off that the manager controls. By contrast, the agent in our setting controls both the drift and the volatility of the managed portfolio value.

3. Our solutions involve a well defined benchmark whose interpretation as a portfolio is straightforward and whose value is always non-negative. When the optimal contract structure is found endogenously, depending on the cost or effort function assumed for the manager, the benchmark may be difficult to interpret as a portfolio due to dimensionality problems[3]. Moreover, when the linear structure is taken as given, as here, nothing guarantees that the obtained parameters for the optimal incentive fee are reasonable from a practical viewpoint. Since our parameters are given, hence admissible by construction, our setting does not face this potential issue.

4. Contrary to what is done in related work where the investor's and the manager's programs are not solved simultaneously, there is no need here to introduce a constraint

[3] See for instance equation (9) in Ou-Yang (2003) defining the benchmark "portfolio".

relative to the agent's reservation utility in the investor's program. To the extent that the manager predetermines the fee parameters (and knows which optimal benchmark will be imposed by the principal), he will set them to levels such that the present value of his global fee is compatible with his reservation utility. By contrast, when the optimal contract form is to be found endogenously, the present value of the manager's fee is endogenous and therefore his reservation level must be added as a constraint. This is true also when the contract form is assumed to be linear, the benchmark is exogenously given and the fee parameters are found endogenously as part of the optimal solution to the principal's program. Since it is difficult to assess this reservation level with any degree of accuracy, not needing this parameter is a main advantage of our approach.

We analyze two alternative benchmark-adjusted compensation schemes. The first has a "linear symmetric" form, according to which the manager will receive at termination of the contract a fee proportional to the value of the managed portfolio plus a positive or negative fee based upon the difference between the value of the managed portfolio and that of the benchmark. The second scheme is "affine symmetric" in the sense that the agent receives a fixed dollar-amount plus a symmetric part as in the first scheme[4]. Our main results can be summarized as follows:

(i) the optimal portfolio managed by the agent always *differs* from the optimal benchmark chosen by the principal (except in one special case mentioned below in (v)). Since we consider first best contracts, the agent's behavior is nonetheless *optimal*.
(ii) under the "linear symmetric" scheme, the managed portfolio can be split in two components, a speculative part that depends on the manager's preferences but not on the principal's, and a hedging part against the adverse fluctuations of the value of the benchmark imposed by the principal;
(iii) under the "affine symmetric" scheme, the managed portfolio is more complicated as the two components above are intermingled and cannot be disentangled; in addition, due to the presence of a fixed fee, the value of this portfolio has an optional structure;

[4] It is of course possible to nest the two schemes into a single one comprising three components, and then to specialize it to the two schemes presented here. No economic insight, however, is lost with our simpler presentation.

(iv) under both compensation schemes, the optimal benchmark is generally more involved than the managed portfolio, even in the "linear" case, but can however be expressed as a combination of the principal's and the agent's optimal wealth; in addition, intuitive conditions are provided that greatly simplify its structure;

(v) when the investor's and the manager's utility functions are logarithmic, closed form solutions can be derived. Under both compensation schemes (with some minor restrictions on the fee parameters in the "affine symmetric" case), the managed portfolio is simply the optimum growth portfolio. The benchmark, however, depends on the compensation scheme. Only if the latter is "linear symmetric" does the benchmark perfectly match the managed portfolio. If it is "affine symmetric", the benchmark differs from the optimum growth portfolio, and thus the managed portfolio; this is due to the non-negativity constraint that binds the managed portfolio in presence of a fixed fee;

(vi) when the investor's and the manager's utility functions are iso-elastic, quasi-explicit solutions are provided which show that, in all cases, the benchmark and the managed portfolios differ, and also differ from the optimum growth portfolio, in a very complex way, which reinforces the view that commonly observed benchmarks are sub-optimal;

(vii) simulations show that differences between commonly adopted benchmarks and optimal ones are substantial in many situations, which implies tangible welfare losses for the principal.

The rest of the article is articulated as follows. Section II offers a brief review of the related literature on the delegated portfolio management and the asset return predictability issues. Section III presents the economic framework we adopt. Section IV analyzes the manager's optimization problem under the two alternative benchmark-adjusted compensation schemes ("linear symmetric" and "affine symmetric"). We then investigate in Section V the principal's problem, which consists in choosing the optimal benchmark that she imposes upon the agent and maximizes the expected utility of her terminal wealth, under both compensation schemes. Section VI derives explicit solutions when the principal's utility function and that of the agent both exhibit CRRA, in particular when they are logarithmic. Section VII assesses the practical importance of selecting an optimal benchmark for the principal by simulating the manager's and the agent's risk aversion coefficients and their optimal portfolios. Some concluding remarks are offered in the last Section. Most mathematical proofs are gathered in the Appendix for readability.

II. RELATED LITERATURE

Following the early lead by Ross (1973) on the principal-agent issue, Holmström and Milgrom (1987), whose work was generalized by Schättler and Sung (1993) and Sung (1995), proved in continuous time that if the principal's and agent's utility functions exhibit constant absolute risk aversion (CARA) and the principal cannot observe the agent's actions, linear contracts are optimal. Ou-Yang (2003)[5] proves that the symmetric compensation scheme is efficient in a continuous time principal-agent framework where all processes are geometric Brownian motions, the investor does not observe the value of the managed portfolio continuously, and the manager has CARA utility. He provides closed form results either when the investor has also CARA utility and a rather general class of cost functions for the manager is used, or the investor is endowed with a general utility function but the manager's cost function is a constant. Two recent contributions greatly extended this literature. Sung (2005) introduced in the standard Holmström and Milgrom (1987) adverse selection and also the possibility for the agent to control the volatility of the output of her actions. In a much more general stochastic environment, but assuming information is perfect, Cadenillas, Cvitanic and Zapatero (2006) prove that if the manager and the investor have the same CRRA coefficients, or possibly different CARA parameters, the optimal contract is (ex post) linear. If not, it is not linear and may be of a call type[6]. In this paper, as stated in the introduction, the principal takes the linear structure and the parameters of the compensation scheme as given but derives her optimal benchmark.

Some authors, however, questioned the optimality of such linear contracts. For example, Admati and Pfleiderer (1997) have shown in a static (one period) framework

[5] See Dybvig et al. (2001) for additional references to the literature before 2000.
[6] We do not discuss asymmetric compensation schemes, as they are forbidden by law for mutual funds. See Carpenter (2000) and Ross (2004) for insightful results on the way asymmetric fees impinge on managers' willingness to take more risk. See also Hodder and Jackwerth (2004) for a numerical evaluation of realistic incentive contracts on hedge-fund performance, and a comparison of managerial risk shifting that arises in the frameworks of Carpenter (2000), Goetzmann, Ingersoll and Ross (2003) and Basak, Pavlova and Shapiro (2003).

that the use of a risky benchmark portfolio of the type commonly adopted in practice, such as an index fund, cannot be in general rationalized when a linear and symmetric contract binds the investor and the manager. Li and Zhou (2005) showed that, in general, an optimal contract is an increasing, nonlinear function of final wealth, the shape of which depends on the principal's and the agent's risk aversions, the state price density function and the agent's reservation utility level.

Another strand of literature assumes the contract structure (linear symmetric or asymmetric) given, and derives the optimal parameters of the contract. In Golec (1992) for instance, while the principal takes the linear structure of the compensation scheme and the benchmark as given (which then faces the Admati and Pfleiderer (1997) criticism), she optimizes the fee parameters. An explicit solution for the latter parameters are obtained when the optimization is performed in a purely static framework. A similar approach has been adopted by Starks (1987) where symmetric and asymmetric contracts are compared. The former contract is shown to dominate the latter in aligning the manager's interest with that of the principal. In Kapur and Timmermann (2005), the agent has superior information and the investor chooses the parameters of the optimal contract subject to the manager's participation constraint.

All the previously quoted papers take the manager's reservation level as given. In particular, Kapur and Timmermann (2005) showed that whether the participation constraint is binding or not is crucial for the qualitative impact of performance fees on the capital market equilibrium. Here, we take the fee parameters as given. Consequently, the present value of the manager's compensation is decided by the manager and known (revealed) in advance. Note in addition that the positivity constraint on the parameters (necessary for the results to be plausible) is usually not taken into account in the extant literature, which may make the optimal contracts impossible to implement.

Another, related, strand of literature examines incentive conflicts between agents and principals in money management. In particular, Chevalier and Ellison (1997) investigate flows-induced risk taking by fund managers. Most agents are rewarded for increasing the value of managed assets, and there is a well documented positive relationship between relative performance and fund-flows. This conjunction creates an incentive for managers to exploit this relationship by manipulating their risk exposures near the end of the management period. Basak, Pavlova and Shapiro (2003), however, show that imposing a maximal shortfall of the managed fund return over that of an appropriate benchmark can reduce the adverse effects (on the investor's welfare) of such managerial incentives. Here, we ignore this issue for tractability and assume away free entry in and exit from the fund once it is started.

An (implicit) assumption of this paper is that skillful managers can exploit the predictability of asset returns. To obtain tractable or explicit solutions for the optimal contract, most previously quoted papers make restrictive assumptions on the dynamics of asset prices. In a static setting, asset returns are assumed to be Gaussian and in a dynamic framework, asset prices follow geometric Brownian motions. Although no consensus has yet emerged in the empirical literature as to which variables have predictive power and to what extent predictability is economically significant, it is now admitted that some predictability exists. Campbell (1987), Campbell and Shiller (1988a, b) and Fama and French (1989) reported that long term equity returns can be explained either by a short term interest rate, some measure of the term premium and the average dividend yield or by the dividend/price and the earnings/price ratios. Jegadesh and Titman (1993) showed that strategies exploiting some form of momentum can exhibit superior performance. Vila-Wetherilt and Wells (2004) confirmed the high predictability of U.K. equity returns in the long run using the approach Campbell and Shiller (1988a, b) had applied to U.S. stocks. Ferson, Heuson and Su (2004) recently reported that the time variation in expected returns remains economically important even after transaction costs. In works more closely related to this paper, Pastor and Stambaugh (2002), Busse and Irvine (2003), Geczy, Levin and Stambaugh (2003) and Jones and Shanken (2005) showed that the predictability embedded in observed managerial skills can be exploited. Avramov (2004) and Avramov and Chordia (2005) claimed that investment strategies involving individual stocks or benchmarks are more profitable when they incorporate macroeconomic variables as predictors. Avramov and Wermers (2004) using managers' skills, mutual fund risk-loadings and benchmark returns as predictors, provided convincing evidence that the predictability reported for single assets carries over to actively managed mutual funds and that portfolio strategies that exploit such a predictability significantly outperform those which do not.

These strands of research strongly suggest that active management may be more valuable than previously acknowledged under the standard market efficiency hypothesis. Therefore, solving for the optimal contract in the context of more general dynamics for asset prices provides a useful generalization. Our setting will allow for asset return predictability to the extent that the parameters of the diffusion processes characterizing our asset returns are driven by predictable (but unspecified) state variables.

III. THE ECONOMIC FRAMEWORK

The manager can trade continuously in a frictionless, arbitrage-free and complete financial market until time T, the horizon of the economy. There are N+1 financial assets available for trade in this market, namely a locally riskless asset and N risky assets. The former is a money market account yielding at each time t an instantaneous interest rate r(t). The investor does not intervene directly in the market but delegates her portfolio decisions to the manager. We give the model some structure by making the following additional assumptions.

- The drifts and diffusion parameters of all stochastic processes defined below depend on a number M of (unspecified) state variables Y(t). The latter evolve through time according to the following stochastic differential equation (SDE):

$$dY(t) = \mu_Y(t, Y(t))dt + \Sigma_Y(t, Y(t))dZ(t) \tag{1}$$

where $\mu_Y(.)$ is a bounded (M x 1) vector valued function of t and Y, $\Sigma_Y(.)$ is a bounded (M x N) matrix valued function of t and Y, with $M \leq N$, and Z denotes a standard Brownian Motion in R^N, $Z' = (Z_1, \ldots, Z_N)$, with the prime ' indicating a transpose. Hence the uncertainty is formalized by the complete filtered space $\left\{\Omega, F, \{F_t\}_{t\in[0,\tau_E]}, P\right\}$, where Ω is the state space, *F* is the σ-algebra representing measurable events, P is the actual (historical) probability and the filtration is the augmented filtration generated by the Brownian Motion assumed to satisfy the usual conditions[7]. Note that some of the

[7] The σ-algebra contains the events whose probability with respect to P is null. See Karatzas and Shreve (1991) p 89.

asset prices defined below may belong to the set of the M state variables. We assume M $\leq$ N so that the financial market is complete.

- The N risky asset prices obey the following SDE:

$$dS(t) = I_S(t)\mu_S(t, Y(t))dt + I_S(t)\Sigma_S(t, Y(t))dZ(t) \tag{2}$$

where $I_S(t)$ denotes the diagonal (N x N) matrix with the elements $S_i(t)$ (i = 1, ..., N), $\mu_S(.)$ is a bounded (N x 1) vector valued function, and $\Sigma_S(.)$ is a bounded (N x N) matrix valued function, assumed to be full rank when markets are assumed to be complete, and less than full rank when markets are assumed to be incomplete. To ease the exposition, we will denote $\mu_S(t, Y(t))$ by $\mu_S(t)$ and $\Sigma_S(t, Y(t))$ by $\Sigma_S(t)$.

Also, these assets are assumed to pay no dividends between 0 and T, which makes all admissible portfolios self-financing and thereby will allow us to solve our dynamic portfolio programs as if they were static.

- The instantaneous riskless interest rate at time t solves the following SDE:

$$dr(t) = \mu_r(t, Y(t))dt + \Sigma_r(t, Y(t))dZ_i(t) \tag{3}$$

where the drift and diffusion parameters, $\mu_r(t)$ and $\Sigma_r(t)$ for brevity, are general functions which can be specialized to preclude the spot rate to take on negative values. As was the case for $\mu_S(t)$, the drift $\mu_r(t)$ is assumed to satisfy the usual necessary conditions so that (3) has a unique solution. The diffusion process followed by r(t) is completely general and need not be made explicit. It determines the evolution of the whole term structure of interest rates in an endogenous manner.

- A particular portfolio will play an important role in the analysis to follow, namely the so-called “optimum growth”, or “log-optimal” or “numéraire” portfolio. Its value at time t ($\leq$ T) is denoted by h(t) and is normalized so that h(0) = 1. It is the optimal portfolio chosen by a logarithmic investor and has the convenient property to make the h-denominated value process of any admissible and self-financing portfolio a martingale under the true probability measure P[8]. This portfolio obeys the following dynamics:

$$\frac{dh(t)}{h(t)} = r(t, Y(t))dt + \kappa(t, Y(t))' dZ(t) \tag{4}$$

where $\kappa(t, Y(t))$ denotes the vector of the market prices of risk associated with the different sources of risk present in the economy. Given the dynamics of the primitive assets assumed in (2) and (3), this vector can be written explicitly, when markets are complete, as follows:

$$\kappa(t, Y(t)) = \Sigma_S(t, Y(t))^{-1}[\mu_S(t, Y(t)) - r(t)\mathbf{1}_N] \tag{5}$$

We now study the manager's program and the investor's problem in succession.

IV. THE MANAGER'S OPTIMIZATION PROBLEM

To act as an agent for his principal, the manager is assumed to impose on her either one of the following compensation schemes:

$$F_1(T) = \phi V_1^m(T) + \nu_1\left(V_1^m(T) - V_1^b(T)\right) \tag{6}$$

$$F_2(T) = \Phi + \nu_2\left(V_2^m(T) - V_2^b(T)\right) \tag{7}$$

where F(T) is the global fee received by the manager at terminal date T, ϕ, ν_1 and ν_2 are predetermined constants, Φ is a predetermined dollar amount, and V^m and V^b stand for the value of the managed fund and that of the benchmark, respectively. While both schemes are symmetric around the chosen benchmark, the first scheme involves a fee component proportional to the terminal value of the fund, whereas the second scheme involves a fixed part. This difference in the way the global fee is computed at date T will prove crucial as to the optimal strategy followed by the agent and the optimal benchmark imposed by the principal. To keep things tractable, we ignore the possibility of early withdrawal(s) from the fund on the part of the investor.

[8] See Long (1990) or Bajeux-Besnainou and Portait (1997).

We now make two commonly encountered assumptions regarding the optimization program the manager faces. First, the global fee he receives at the end of the period is considered his only source of wealth. Therefore, he will maximize the expected utility of this global fee. His utility function, $U^m(F(T))$, is an increasing and strictly concave Von Neuman-Morgenstern function that satisfies the usual Inada conditions, in particular that the marginal utility of zero wealth is plus infinity. Second, rather than using a cost function, we specify an effort function, which may seem more appropriate in a principal-agent type problem. Effort naturally involves disutility, and is fairly realistically assumed to be a positive function of the level of wealth (gross of compensation fees) restituted to the principal.

Since the financial market is complete, rather than solving the problem by way of the stochastic dynamic programming technique, it is easier to follow Duffie (1996) and the seminal contributions of Karatzas, Lehoczky and Shreve (1987) and Cox and Huang (1989, 1991) and adopt the martingale approach. As is well known, the latter substitutes a simpler, static, problem for a dynamic one. Also, to keep using the true (historical) probability measure P, and thus simplify the computation of the manager's optimal strategy, we make use of the optimum growth portfolio as the numéraire in this program.

Consequently, the manager's optimization program writes:

$$\begin{cases} \max\limits_{V_i^m} E^P\left[U^m\left(F_i(T)\right) - e_i V_i^m(T)\right] \\ \\ \text{s.t.} \qquad E^P\left[\dfrac{V_i^m(T)}{h(T)}\right] = V(0) \\ \qquad\qquad V_i^m(T) \geq 0 \\ \qquad\qquad F_i(T) \geq 0 \end{cases} \tag{8}$$

In this program, i = {1, 2} denotes the type of scheme proposed by the manager. Notice first that the effort coefficient e_i logically depends on the adopted scheme. Also, the manager monitoring the level of the principal's managed portfolio $\left[V_i^m(T)\right]$ through his decisions, he thereby *controls* his effort level $\left[e_i V_i^m(T)\right]$. For simplicity, this effort is assumed to be a linear function of the principal's portfolio. The first constraint, in which V(0) is the principal's initial wealth to be invested by the manager, is the *budget constraint* expressed in the usual martingale form (under P), and should be binding at the optimum. The second constraint forces the manager to follow strategies such that the terminal value of the managed portfolio is non-negative, and thus is a *solvency constraint*. It is well known that taking into account this constraint may have a decisive impact on the optimal decisions of risk averse agents[9]. The third constraint states that the overall fee must be non-negative. Due to the symmetric component of the fee, the manager who performs badly will have to pay the principal the difference between the managed portfolio value and the benchmark value. So adding this constraint is *a priori* necessary. It turns out that it is always satisfied under the linear symmetric compensation scheme since we have imposed Inada's conditions on the manager's utility function. It will be shown to be satisfied also at the optimum in the affine symmetric scheme.

A technicality worth mentioning is that the objective function could, in theory, be an increasing or a decreasing function of the manager's portfolio value because the manager's effort has a cost that increases with this value. Therefore, we will implicitly restrict the analysis to the region of the utility function where the objective function is increasing in the manager's portfolio value, i.e. where $\frac{\partial U^m}{\partial V_i^m} - e_i \geq 0$. In other words, the manager chooses only amongst those portfolios for which his marginal utility is larger than the cost of the effort necessary to construct them.

Using Cox and Huang (1991) one can easily verify that program (8) has a unique solution. Deriving the first-order condition for an optimum leads to the following solution for the portfolio chosen by the manager under the first compensation scheme:

Proposition 1:

The terminal value of the portfolio chosen by the manager under compensation scheme (6) is equal to:

[9] See Lioui and Poncet (1996) and Gollier et al. (1997).

$$V_1^m(T)=\frac{1}{\phi+\nu_1}Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right)+\frac{\nu_1}{\phi+\nu_1}V_1^b(T) \tag{9}$$

where $Y_m(.)$ *is the inverse of the marginal utility function U'*$_m$*, i.e.* $Y_m(y)$ *is x such that U'*$_m$*(x)=y, and the Lagrange multiplier* λ_1 *solves:*

$$\phi V(0)=E^P\left[h(T)^{-1}Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right)\right] \tag{10}$$

To ease the interpretation of this result, we note first that, by substituting in (6) the value of $V_1^m(T)$ given by (9), the agent's optimal compensation (and thus his optimal final wealth) under the first scheme is equal to:

$$F_1(T)=Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right) \tag{11}$$

so that the first term on the RHS of (9) is nothing but $F_1(T)/(\phi+\nu_1)$. If the compensation fee did not include a symmetric component ($\nu_1 = 0$), the optimal portfolio chosen by the agent would simply be the intuitive $V_1^m(T) = F_1(T)/\phi$, with obviously no role left for the benchmark. Here, however, because of the presence of a symmetric fee component, the value of the managed portfolio is the sum of two terms, the second one being a fraction $\nu_1/(\phi+\nu_1)$ of the value of the benchmark. Since neither the optimal wealth (11) nor the shadow price (10) of the budget constraint depend on the benchmark portfolio, they are also independent of the principal's risk aversion parameters. Therefore, a kind of "separation" property obtains in the case of the linear symmetric compensation scheme. Absent the effort cost, everything would be as if the manager received at the contract inception the present value of his fee and invested it optimally in the market. Only the presence of the effort cost prevents this to occur.

At each time t, the value of the manager's portfolio obeys:

$$\frac{V_1^m(t)}{h(t)}=\frac{1}{\phi+\nu_1}E^P\left[h(T)^{-1}Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right)\middle|F_t\right]+\frac{\nu_1}{\phi+\nu_1}\frac{V_1^b(t)}{h(t)} \tag{12}$$

where we have used both the martingale property of $\frac{V_1^m(t)}{h(t)}$ and the optimal solution.

The agent's optimal investment strategy consists of two parts. The first one can be interpreted as a "speculative" component. It depends on the manager's utility function, and on the value of the optimum growth portfolio h(t), but is completely independent of the principal's preferences and demand. The second one is reminiscent of a "hedging" term. It stems from the portfolio delegation granted to the agent and allows the latter to hedge against the unfavorable fluctuations of the benchmark portfolio that is imposed upon the agent by the principal. Unlike the first part, it does not depend directly on the optimum growth portfolio, and is free of the manager's preferences so that it can be considered as a minimum variance term. The crucial result here is that it is *not* optimal for the agent (except if his utility function is logarithmic and the compensation scheme is (6); see section VI below) to select the benchmark imposed by the principal as the delegated portfolio.

Turning to the second compensation scheme, the solution for the portfolio chosen by the manager is given in the following proposition:

Proposition 2:

The terminal value of the portfolio chosen by the manager under compensation scheme (7) is equal to:

$$V_2^m(T) = \frac{1}{\nu_2}\left[Y_m\left(\frac{\lambda_2}{\nu_2} h(T)^{-1} + \frac{e_2}{\nu_2}\right) + \nu_2 V_2^b(T) - \Phi \right]^+ \tag{13}$$

where the Lagrange multiplier λ_2 *is such that:*

$$V(0) = E^P\left[h(T)^{-1} \frac{1}{\nu_2}\left[Y_m\left(\frac{\lambda_2}{\nu_2} h(T)^{-1} + \frac{e_2}{\nu_2}\right) + \nu_2 V_2^b(T) - \Phi \right]^+ \right] \tag{14}$$

Under this compensation scheme, the solution is more involved because the presence of the fixed amount Φ could make $V_2^m(T)$ negative (see the appendix for the proof and a brief discussion). Embedded implicitly in (13) are long positions in a risky portfolio and in a long put, the latter being exercised if the portfolio value falls below the strike, which is of course zero. To see this, note that the agent's optimal final wealth is equal to:

$$F_2(T) = \max\left[Y_m\left(\frac{\lambda_2}{\nu_2} h(T)^{-1} + \frac{e_2}{\nu_2}\right); \left(\Phi + \nu_2\left[0 - V_2^b(T)\right]\right)\right] \tag{15}$$

where we have use the fact that max(x – y, 0) + y = max(x, y). Note that, due to our assumption regarding the manager's utility function, both equations (11) and (15) imply a strictly positive value for the final fee F(T).

An important difference with the linear compensation fee is that the "separation" property does not hold any more in the affine case. Indeed, the manager's optimal wealth now depends directly on the benchmark portfolio and thus indirectly on the principal's preference parameters. As a consequence, the manager is concerned in this case by the particular benchmark that the principal imposes on him. One practical implication is that managers could to a certain extent try to favor those investors whose preferences and thus desired benchmarks match best their management skills. Another implication is that investors, in turn, have under this compensation scheme an incentive to select those agents whose skills are reputed to fit best their investment objectives.

The value of the manager's portfolio at each time t then obeys:

$$\frac{V_2^m(t)}{h(t)} = E^P\left[h(T)^{-1} \frac{1}{\nu_2}\left[Y_m\left(\frac{\lambda_2}{\nu_2} h(T)^{-1} + \frac{e_2}{\nu_2}\right) + \nu_2 V_2^b(T) - \Phi \right]^+ \middle| F_t \right] \tag{16}$$

Here, it is *never* optimal for the manager to adopt the benchmark as the investor's portfolio, even in the log utility case (see section V below). In addition, the previous formal separation in two "funds" breaks down, because of the "positive only" feature of the solution. In particular, the implicit hedge against the benchmark's fluctuations is preference dependent and could now be interpreted as a kind of information-based Merton-Breeden hedging component. Also, due to the possibly binding solvency constraint of program (8), we recover here, as in Cox and Huang (1989), the optional feature of the strategy.

V. THE INVESTOR'S OPTIMAL BENCHMARK

Although the compensation scheme is imposed on the investor, the latter can nevertheless control the manager by selecting the appropriate benchmark. This is the reason why the principal's program and the agent's must be solved simultaneously. Formally, the investor chooses a benchmark portfolio that maximizes the expected utility of her terminal wealth. Recall her initial wealth is denoted by V(0). Her final wealth, net of all fees, is denoted by W(T). Depending on the compensation scheme selected by the manager, W(T) either writes:

$$\begin{aligned} W_1(T) &= V_1^m(T) - F_1(T) \\ &= (1-\phi-\nu_1)V_1^m(T) + \nu_1 V_1^b(T) \\ &= \frac{(1-\phi-\nu_1)}{\phi+\nu_1} Y_m\left(\frac{\lambda_1}{\phi+\nu_1} h(T)^{-1} + \frac{e_1}{\phi+\nu_1}\right) + \frac{\nu_1}{\phi+\nu_1} V_1^b(T) \end{aligned} \tag{17}$$

or:

$$\begin{aligned} W_2(T) &= V_2^m(T) - F_2(T) \\ &= (1-\nu_2)V_2^m(T) + \nu_2 V_2^b(T) - \Phi \\ &= \frac{1-\nu_2}{\nu_2}\left[Y_m\left(\frac{\lambda_2}{\nu_2} h(T)^{-1} + \frac{e_2}{\nu_2}\right) - \Phi + \nu_2 V_2^b(T)\right]^+ - \Phi + \nu_2 V_2^b(T) \end{aligned} \tag{18}$$

The principal's utility function, $U^p(W(T))$, is an increasing and strictly concave Von Neuman-Morgenstern function that, like the agent's utility, is assumed to satisfy the Inada conditions. She thus solves the following optimization program, for i = 1 or 2 according to the compensation scheme dictated by the manager:

$$\begin{cases} \max_{V_i^b} E^P\left[U^p(W_i(T))\right] \\ \\ \text{s.t.} \quad E^P\left[\dfrac{V_i^b(T)}{h(T)}\right] = V(0) \\ \qquad V_i^b(T) \geq 0 \\ \qquad W_i(T) \geq 0 \end{cases} \tag{19}$$

In this program, the first constraint is the budget constraint expressed in the martingale form and should be binding at the optimum. The second requires that the benchmark portfolio respects the solvency constraint. The third one will always be satisfied since we have imposed Inada's conditions on the principal's utility function.

Contrary to what is done in related research where the investor's and the manager's programs are not solved simultaneously, there is no need here to specify a constraint relative to the agent's reservation utility. To the extent that the manager predetermines the fee parameters (and knows what benchmark will be imposed by his principal), he can set them to a level such that the present value of his global fee is compatible with his reservation utility. That feature simplifies the investor's problem. The optimal benchmarks designed by the principal under compensation schemes (6) and (7) are described in the following two propositions.

Proposition 3:

Under compensation scheme (6), the benchmark portfolio chosen by the investor is such that:

$$V_1^b(T) = \frac{\phi+\nu_1}{\nu_1}\left[Y_p\left(\frac{\phi+\nu_1}{\nu_1}\delta_1 h(T)^{-1}\right) + \frac{\phi+\nu_1-1}{\phi+\nu_1}Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1} + \frac{e_1}{\phi+\nu_1}\right)\right]^+ \quad (20)$$

where $Y_p(.)$ is the inverse of the marginal utility function U'_p, and the Lagrange multiplier δ_1 associated with the investor's budget constraint solves:

$$V(0) = E^P\left[h(T)^{-1}\left[\frac{\phi+\nu_1}{\nu_1}Y_p\left(\frac{\phi+\nu_1}{\nu_1}\delta_1 h(T)^{-1}\right) - \frac{1-(\phi+\nu_1)}{\nu_1}Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1} + \frac{e_1}{\phi+\nu_1}\right)\right]^+\right] \quad (21)$$

It is noteworthy that the benchmark can be expressed by construction as a combination of the investor's and the manager's optimal wealth. The structure of the solution however is rather involved without further restrictions on the principal's and agent's utility functions. Note that we have to take the positive part of the solution only, not to make sure that the investor's terminal wealth is positive (since this is guaranteed by the Inada conditions), but to guarantee that the value of the benchmark is non-negative. Mere inspection of equation (18) reveals that, in general, a positive investor's wealth does not imply a positive value for the benchmark [the latter value is negative if: $\Phi <$ (1- ν_2) $V_2^m(T)$]. Similarly in equation (17), $W_1(T)$ may be positive while $V_1^b(T)$ is negative. However, the solution somewhat simplifies and becomes linear if the non-negativity feature (+) drops out of both equations. This is clearly the case when the sum (ϕ + ν_1) of the compensation parameters is (equal to or) larger than 1, an admittedly restrictive but plausible assumption.

To shed some light on the intricacy of general result (20), suppose for a moment that the parameters are such that the non-negativity constraint is not binding. Then, substituting in equation (17) for the value of $V_1^b(T)$ given by equation (20), the principal's optimal final wealth, net of all fees, writes:

$$W_1(T) = Y_p\left(\frac{\phi+\nu_1}{\nu_1}\delta_1 h(T)^{-1}\right) \tag{22}$$

Result (22) looks fairly simple as it involves the usual numéraire portfolio and the principal's utility function. This is deceitful however, as it does not exhibit the "separation" property: the principal's optimal net wealth still depends on the manager's preferences through the presence of the Lagrange multiplier δ_1 (see equation (21)).

Assuming now that the solvency constraint is binding, the principal's optimal wealth writes:

$$W_1(T) = \left[Y_p\left(\frac{\phi+\nu_1}{\nu_1}\delta_1 h(T)^{-1}\right); \frac{1-\phi-\nu_1}{\phi+\nu_1} Y_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1} + \frac{e_1}{\phi+\nu_1}\right)\right]$$

where the second term between parentheses can be interpreted as the optimal wealth achieved for a benchmark portfolio value set equal to zero (see equation (17) above).

The situation in which the compensation scheme includes a fixed amount is dealt with in the next proposition.

Proposition 4:

Under compensation scheme (7), the benchmark portfolio chosen by the investor is such that:

$$V_2^b(T)=\left[\Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi\right]^+ \tag{23}$$

where the Lagrange multiplier δ_2 associated with the investor's budget constraint solves:

$$V(0)=E^P\left[h(T)^{-1}\left[\Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi\right]^+\right] \tag{24}$$

The same comments as above essentially apply here, although the proof is slightly more involved (see the appendix). Suppose again the non-negativity constraint is not binding. Then, substituting in equation (18) for the value of $V_2^b(T)$ given by equation (23), the principal's optimal net wealth writes:

$$W_2(T)=\Upsilon_p\left(\delta_2 h(T)^{-1}\right) \tag{25}$$

As in (22), it does depend on the manager's preferences through the multiplier δ_2. A more detailed analysis and closed form solutions are provided in the next section where utility functions are restricted to the CRRA family. It is important to note that any member of this family satisfies the usual Inada conditions, in particular that the marginal utility of zero wealth is plus infinity. The previous general results then can be relied on.

VI. EXPLICIT SOLUTIONS WITH CONSTANT RELATIVE RISK AVERSION

A well known result in portfolio theory due to Merton (1971) is that for a well behaved utility function, the risky component of an investor's optimal portfolio can be decomposed into (M+1) parts. The first part is the optimum growth portfolio and others are the M Merton-Breeden terms hedging for the random shifts in the investment opportunity set brought about by the M state variables. In the special case of the log utility function, which uniquely possesses the myopic feature, the M hedging terms vanish. We then consider the log case first and then generalize to the case of the iso-elastic function with different risk aversion parameters for the principal and her agent.

VI.1 Logarithmic utility

To further simplify the results and focus on the main workings of the model, we now set the effort parameter e to zero. Although this is obviously restrictive, it is not damaging to the spirit of the model since only the first best contract is considered.

Under the linear compensation scheme (6), formula (9) for the managed portfolio reduces to:

$$V_1^m(T) = \frac{\phi}{\phi + \nu_1} V(0)h(T) + \frac{\nu_1}{\phi + \nu_1} V_1^b(T) \tag{26}$$

because the $\Upsilon_m(.)$ function is such that $\Upsilon_m(x) = 1/x$ in (9) and (10), which allows for an explicit solution for λ_1, and then for $V_1^m(T)$.

From (20) and (21), and using also $\Upsilon_m(x) = \Upsilon_p(x) = 1/x$, the benchmark portfolio simplifies to:

$$V_1^b(T) = \left[\frac{1}{\delta_1} - \frac{1-(\phi + \nu_1)}{\nu_1}\phi V(0)\right]^+ h(T) \tag{27}$$

where the Lagrange multiplier δ_1 solves:

$$V(0) = \left[\frac{1}{\delta_1} - \frac{1-(\phi + \nu_1)}{\nu_1}\phi V(0)\right]^+ \tag{28}$$

Combining the three previous equations then yields:

$$V_1^b(T) = V(0)h(T) = V_1^m(T) \tag{29}$$

This result, a primer to the best of our knowledge although very intuitive, is theoretically important as it is the only case where the manager will adopt exactly the benchmark as the investor's portfolio. A corollary to this result is that the predetermined coefficients (ϕ and ν_1) of the linear compensation scheme (6) bear no effect at all on the managed portfolio characteristics, since the latter portfolio is nothing but the optimum growth portfolio. Thus, in this case, Merton's result is recovered because both participants to the contract are myopic. Also, the manager will receive no benchmark-adjusted compensation in any state of nature (this part of the fee thus is riskless since equal to zero), his optimal fee being $\phi V_1^m(T)$.

Under the affine compensation scheme (7), results are strikingly different. First, the formula for the managed portfolio, obtained from equation (13), is more involved due to the presence of the fixed fee, which prevents us from deriving the Lagrange multiplier λ_2 explicitly:

$$V_2^m(T)=\left[\frac{1}{\nu_2}\left[\frac{\nu_2}{\lambda_2}h(T)-\Phi\right]+V_2^b(T)\right]^+ \tag{30}$$

where, from (19), λ_2 solves:

$$V(0)=E^P\left[h(T)^{-1}\left[\frac{1}{\nu_2}\left[\frac{\nu_2}{\lambda_2}h(T)-\Phi\right]+V_2^b(T)\right]^+\right] \tag{31}$$

From (23) and (24), the benchmark portfolio is given by:

$$V_2^b(T)=\left[\left(\frac{1}{\delta_2}-\frac{1-\nu_2}{\lambda_2}\right)h(T)+\frac{1}{\nu_2}\Phi\right]^+ \tag{32}$$

where δ_2 solves:

$$V(0)=E^P\left[h(T)^{-1}\left[\left(\frac{1}{\delta_2}-\frac{1-\nu_2}{\lambda_2}\right)h(T)+\frac{1}{\nu_2}\Phi\right]^+\right] \tag{33}$$

Combining (30) and (32) also yields:

$$V_2^m(T)=\mathrm{Max}\left[\frac{h(T)}{\lambda_2}-\frac{\Phi}{\nu_2};\left(\frac{1}{\delta_2}+\frac{\nu_2}{\lambda_2}\right)h(T)\right] \tag{34}$$

which clearly indicates the optional feature of the managed portfolio due to the fixed fee. In all cases, because utility functions were assumed logarithmic, the risky part of that portfolio is (a proportion of) the optimum growth portfolio. Notice that, in the case where $V_2^m(T)$ is equal to the first argument in the Max function above, the Lagrangian δ_2 plays no role and the fixed fee amount Φ appears explicitly, in addition to its indirect influence through λ_2. Also, the managed portfolio is a linear combination of the numéraire portfolio and the riskless asset that delivers $-\Phi/\nu_2$ at date T. In the case where $V_2^m(T)$ is equal to the second argument in the Max function, the managed portfolio is proportional to the numéraire portfolio, the proportion depending on the fee parameters, the chosen benchmark and both utility functions.

It is intuitive that for reasonable fee parameters, the second situation will prevail almost surely, and both $V_2^b(T)$ and $V_2^m(T)$ will have strictly positive values. In this case, the former rewrites:

$$V_2^b(T)=\left(\frac{1}{\delta_2}-\frac{1-\nu_2}{\lambda_2}\right)h(T)+\frac{1}{\nu_2}\Phi \tag{35}$$

where δ_2 solves:

$$V(0)=E^P\left[\left(\frac{1}{\delta_2}-\frac{1-\nu_2}{\lambda_2}\right)+\frac{1}{\nu_2}h(T)^{-1}\Phi\right] \tag{36}$$

Thus, we have:

$$\frac{1}{\delta_2}-\frac{1-\nu_2}{\lambda_2}=V(0)-\frac{1}{\nu_2}P(0,T)\Phi \tag{37}$$

and

$$V_2^b(T)=V(0)h(T)+\frac{1}{\nu_2}\Phi(1-P(0,T)h(T)) \tag{38}$$

where P(0,T) denotes the current value of a zero-coupon bond delivering \$1 at maturity T. Therefore, (30) becomes:

$$V_2^m(T)=\frac{1}{\nu_2}\left[\frac{\nu_2}{\lambda_2}h(T)-\Phi\right]+V_2^b(T) \tag{39}$$

and, substituting for (38) into (39) one also has:

$$V_2^m(T)=\frac{1}{\nu_2}\left[\frac{\nu_2}{\lambda_2}h(T)-\Phi\right]+V(0)h(T)+\frac{1}{\nu_2}\Phi(1-P(0,T)h(T)) \tag{40}$$

From (31), it follows that:

$$V(0)=E^P\left[\frac{1}{\nu_2}\left[\frac{\nu_2}{\lambda_2}-h(T)^{-1}\Phi\right]+h(T)^{-1}V_2^b(T)\right]$$

and therefore:

$$\frac{1}{\lambda_2}=\frac{1}{\nu_2}\Phi P(0,T) \tag{41}$$

Finally, the managed portfolio rewrites:

$$V_2^m(T)=V(0)h(T) \tag{42}$$

which is the numéraire portfolio. It does differ from the benchmark portfolio (38).

Whether the compensation fee is linear or affine (but in that case for fee parameters such that the probability of a non-positive value for the benchmark portfolio is zero), the portfolio managed by the agent turns out to be the same, namely the numéraire portfolio. However, in the affine case, the benchmark portfolio differs: as evidenced by equation (38), it is not equal to the optimum growth portfolio unless the fixed amount Φ is (uninterestingly) set to zero.

VI.2 Power utility

Let us assume now, more generally, that the principal and the agent both have iso-elastic utility functions, with possibly different risk aversion coefficients:

$$U^m(W)=\frac{W^{1-\gamma_m}}{1-\gamma_m}$$

$$U^p(W)=\frac{W^{1-\gamma_p}}{1-\gamma_p}$$

Under the first compensation scheme (6), formula (9) for the managed portfolio becomes:

$$V_1^m(T)=\frac{1}{\phi+\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-\frac{1}{\gamma_m}}h(T)^{\frac{1}{\gamma_m}}+\frac{\nu_1}{\phi+\nu_1}V_1^b(T) \tag{43}$$

where λ_1 is such that:

$$V(0)=\frac{1}{\phi+\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-\frac{1}{\gamma_m}}E^P\left[h(T)^{\frac{1-\gamma_m}{\gamma_m}}\right]+\frac{\nu_1}{\phi+\nu_1}V(0) \tag{44}$$

From (20) and (21), the benchmark portfolio writes:

$$V_1^b(T)=\left[\frac{\phi+\nu_1}{\nu_1}\left(\frac{\phi+\nu_1}{\nu_1}\delta_1 h(T)^{-1}\right)^{-\frac{1}{\gamma_p}}-\frac{1-(\phi+\nu_1)}{\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}\right]^+ \tag{45}$$

or else:

$$V_1^b(T)=\Lambda_{11}h(T)^{\frac{1}{\gamma_p}}\left[\Lambda_{12}-h(T)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}\right]^+ \tag{46}$$

with

$$\Lambda_{11} = \frac{1-(\phi+\nu_1)}{\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-\frac{1}{\gamma_m}}$$

$$\Lambda_{12} = \frac{(\phi+\nu_1)\left(\frac{\phi+\nu_1}{\nu_1}\delta_1\right)^{-\frac{1}{\gamma_p}}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{\frac{1}{\gamma_m}}}{1-(\phi+\nu_1)} \qquad (47)$$

where δ_1 solves:

$$V(0) = E^P\left[\Lambda_{11} h(T)^{\frac{1}{\gamma_p}-1}\left[\Lambda_{12} - h(T)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}\right]^+\right] \qquad (48)$$

It follows that the value of the manager's portfolio is equal to:

$$V_1^m(T) = \frac{1}{\phi+\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-\frac{1}{\gamma_m}}\left(1+(1-(\phi+\nu_1))\left[\Lambda_{12} h(T)^{\frac{1}{\gamma_p}-\frac{1}{\gamma_m}} - 1\right]^+\right) h(T)^{\frac{1}{\gamma_m}} \qquad (49)$$

Although the principal's and the agent's utility functions exhibit CRRA, the solution remains quite involved. In particular, both risk aversion coefficients are present (directly and through Λ_{12}) as a direct consequence of solving simultaneously the manager's and the investor's programs. Also, while the optimum growth portfolio still plays a pivotal role, the solution is highly non-linear in h(T).

One interesting particular case, sometimes dealt with in the literature, occurs when the principal and her agent have the same risk aversion parameter. Then one has:

$$V_1^m(T) = \frac{1}{\phi+\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-\frac{1}{\gamma_m}}\left(1+(1-(\phi+\nu_1))[\Lambda_{12}-1]^+\right) h(T)^{\frac{1}{\gamma_m}}$$

$$V_1^b(T) = \Lambda_{11}[\Lambda_{12}-1]^+ h(T)^{\frac{1}{\gamma_m}}$$

and the two portfolios are proportional to each other but not equal. In particular, they have identical volatility.

Under the second compensation scheme (7), formula (13) for the managed portfolio becomes:

$$V_2^m(T)=\left[\frac{1}{\nu_2}\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}-\frac{1}{\nu_2}\Phi+V_2^b(T)\right]^+ \tag{50}$$

where the Lagrange coefficient λ_2 solves:

$$V(0)=E^P\left[h(T)^{-1}\left[\frac{1}{\nu_2}\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}-\frac{1}{\nu_2}\Phi+V_2^b(T)\right]^+\right] \tag{51}$$

From (23) and (24), the benchmark portfolio writes:

$$V_2^b(T)=\left[\left(\delta_2 h(T)^{-1}\right)^{-\frac{1}{\gamma_p}}-\frac{1-\nu_2}{\nu_2}\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}+\frac{1}{\nu_2}\Phi\right]^+ \tag{52}$$

where the Lagrange multiplier δ_2 is such that:

$$V(0)=E^P\left[h(T)^{-1}\left[\left(\delta_2 h(T)^{-1}\right)^{-\frac{1}{\gamma_b}}-\frac{1-\nu_2}{\nu_2}\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}+\frac{1}{\nu_2}\Phi\right]^+\right] \tag{53}$$

Substituting for the benchmark portfolio into (50) yields:

$$V_2^m(T)=\max\left[\left(\delta_2 h(T)^{-1}\right)^{-\frac{1}{\gamma_p}}+\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}};\frac{1}{\nu_2}\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}-\frac{1}{\nu_2}\Phi\right] \tag{54}$$

where λ_2 solves:

$$V(0)=E^P\left[h(T)^{-1}\max\left[\left(\delta_2 h(T)^{-1}\right)^{-\frac{1}{\gamma_p}}+\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}};\frac{1}{\nu_2}\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right)^{-\frac{1}{\gamma_m}}-\frac{1}{\nu_2}\Phi\right]\right] \tag{55}$$

If the non-negativity constraint does not bind the value (52), the managed portfolio rewrites more simply:

$$V_2^m(T)=\delta_2^{-\frac{1}{\gamma_p}}h(T)^{\frac{1}{\gamma_p}}+\left(\frac{\lambda_2}{\nu_2}\right)^{-\frac{1}{\gamma_m}}h(T)^{\frac{1}{\gamma_m}} \tag{56}$$

Note that in all cases, because utility functions are not logarithmic, the managed and the benchmark portfolios are not proportions of the optimum growth portfolio, but are much more complex due to the non-myopic nature of the principal's and agent's strategies. With these preferences, the fact that asset returns are partially predictable becomes more important and justifies that the principal delegates her portfolio decisions to an agent against a fee. Even though the utility functions have been restricted to exhibit the convenient CRRA feature, the optimal benchmark is very intricate, which reinforces Admati and Pfleiderer's (1997) case against the use of commonly observed benchmarks.

VII. SIMULATIONS

The following simulations are intended to shed some light on the implications of compensation schemes on the manager's risk taking behavior and specifically on the risk/return tradeoff of his actual portfolio relative to that of the imposed benchmark. To this end, we adopt the CRRA utility of the previous section and assume that only two assets are traded, a stock index (the market portfolio) and a riskless asset (money market account). Because of the presence of the solvency constraint, this simplified setting is the only one for which an explicit solution is recovered. In the continuous time literature, this is a standard assumption, directly inspired by the CAPM.. Furthermore, the investment opportunity set is assumed to be constant for tractability and the dynamics of the two assets thus writes as follows:

$$\frac{dS(t)}{S(t)} = \mu_S \, dt + \sigma_S \, dZ(t) \tag{57}$$

and

$$\frac{dB(t)}{B(t)} = r \, dt \tag{58}$$

This assumption obviously *minimizes* the benefit of portfolio delegation from the investor's viewpoint, if the agent has managerial skills, since it ignores the asset return predictability discussed in section II. Unfortunately, closed form solutions for intertemporal portfolio choice are known to exist only in a few cases. The constant investment opportunity set considered here is one of them, but explicit solutions exist for more general specifications. For example, Kim and Omberg (1996) solved the optimal portfolio problem for a CRRA investor maximizing his expected utility of terminal wealth when the market price of risk is mean reverting and Wachter (2002) extended their setting to account for intermediate consumption. Bajeux-Besnainou et al. (2001), Lioui and Poncet (2001) and Brennan and Xia (2002) solved the problem under stochastic interest rates. Here, due to the binding solvency constraint on portfolio values, we cannot derive a closed form solution even with a constant investment opportunity set. Our results thus are best viewed as a worst case scenario for the principal, the actual incentive to delegate being *stronger* that what will be shown in the simulations.

The complete solutions to the investor's and the manager's problems, which still remain rather involved, are provided in Appendix B. The optimal benchmark is computed and compared to a classical benchmark, here the risky asset (the market portfolio). We performed the simulations for the linear compensation scheme (6) only, this being sufficient to illustrate our main point, namely, that imposing an optimal benchmark to the manager dramatically modifies the composition of the managed portfolio and increases substantially the principal's welfare. The parameters of our base case simulation are given in Table 1. Note that these are standard parameters, except that we have assumed that the principal is more risk averse than her agent (twice as much).

Insert Table 1 about here

Let us denote by α_1 the proportion of the relevant portfolio invested in the risky asset, $(1-\alpha_1)$ being invested in the riskless asset. Table 2 presents the initial compositions of the optimally managed $\left(\alpha_1^m\right)$ and benchmark $\left(\alpha_1^b\right)$ portfolios computed according to the results of Appendix B, and the initial composition of the sub-optimally managed portfolio $\left(\hat{\alpha}_1^m\right)$ when the adopted benchmark is the market portfolio, for different values of the compensation scheme parameters. Note that our calibration is such that $\hat{\alpha}_1^m$ is equal to 100% (up to the third decimal) in the baseline case, for any pair (ϕ, ν_1) of the fee parameters, that is the managed portfolio is (almost) identical to the (sub-optimal) benchmark.

Several striking features emerge from Table 2. First, the managed portfolio matches rather closely the optimal benchmark, the discrepancy ranging from (roughly) 1% to 6% only (compare the values of α_1^m and α_1^b). This highlights the fact that imposing the proper benchmark to the manager is of paramount importance as to the principal's welfare. Second, this result is all the more crucial because varying the parameters of the linear compensation scheme has very little impact on the composition of these portfolios, and on the (slight) difference between them. This is because the part ν_1 of the compensation that depends on the discrepancy between the portfolio and its benchmark is symmetric. Third, the optimal benchmark differs dramatically from the often adopted market portfolio (or any subset of it), a result that comforts Admati and Pfleiderer's (1997) analysis. The proportion of risky assets it includes is slightly less than one half (45 to 49%), as opposed to 100% (by definition) for the market portfolio. Since the manager closely follows his benchmark under the compensation schemes under scrutiny, whether the latter is optimal or not is of particular relevance to the investor.

Insert Table 2 about here

Table 3 reports the impact, on the managed and the optimal benchmark portfolios, of various assumptions regarding the volatility of the risky asset and the market price of risk. As expected, the proportion of both portfolios invested in the risky asset decreases (proportionately) with the asset volatility, while that invested in the market portfolio hardly decreases. For instance, for a baseline market price of risk κ equal to 0.3, the benchmark comprises 144.86% of the risky asset if the latter has a volatility of 5% and 36.22% only if the asset volatility is 20%. In addition, this proportion increases (proportionately) with κ, the market excess return per unit of risk, while that invested in the sub-optimal portfolio hardly increases. For example, for a baseline volatility of 15%, the proportion α_1^b is 32.19% if κ is equal to 0.2 and 64.38% if κ is equal to 0.4.

Insert Table 3 about here

Behavior towards risk also affects crucially the composition of the optimal benchmark and that of the managed portfolio. Table 4 shows the impact of both the principal's and the agent's risk aversion. As expected, this influence is huge. The proportion of the benchmark (and the managed portfolio) invested in the risky asset declines rapidly with the CRRA coefficients, and, for reasonable values of the latter, ranges from 100.00% to

19.54%. For a given manager's risk aversion, the more risk averse is the principal, the less is invested in the risky asset, both for the benchmark and the managed portfolios. The more risk averse are the agent and the principal, the more the managed and the benchmark portfolios diverge from the market portfolio and are invested in the riskless asset.

Insert Table 4 about here

Finally, we provide in Table 5 a measure of the *welfare loss* incurred by the investor when the manager follows the standard (i.e. market portfolio) benchmark. Table 5 presents the loss in absolute percentage return for different values of the principal's risk aversion coefficient γ_p, of the compensation scheme parameters ϕ and ν_1, and of the market price of risk κ. For tractability, we have assumed that the manager's utility is logarithmic ($\gamma_m = 1$), while the principal's is iso-elastic with various values of the risk aversion coefficient ($\gamma_p > 1$). If the manager's utility was not logarithmic, a change in the benchmark would imply a change in the manager's optimal wealth. Therefore, the welfare loss suffered by the principal would be an intricate combination of a *direct* loss due to the benchmark being sub-optimal and an *indirect* loss through the impact on the manager's optimal wealth. If the latter has log utility, however, his optimal wealth is completely independent of the investor's utility parameter since it is independent of the benchmark that she selects. Therefore, the welfare loss suffered by the principal is purely a direct one. As Table 4 has shown that the main qualitative influence is the *discrepancy* between the agent's and the investor's risk aversions, it is not too damaging to norm the former to one and let the latter be larger than one. The mathematical derivations and approximations are reported in Appendix C. The (approximate) welfare loss from the manager adopting a sub-optimal benchmark ranges roughly from 0.1% to 8%, which is quite sizeable if one recalls that the ϕ parameter, which represents the average fee for one dollar under management, ranges from 0.5% to 2.5%.

Insert Table 5 about here

To summarize, in spite of their limited scope (essentially due to the assumption of a constant opportunity set), our simulations offer a striking evidence as to the necessity for investors who face the compensation schemes imposed by the fund managers to design truly optimal, as opposed to routinely adopted, benchmarks. Introducing state variables and, thus, some predictability in asset returns would obviously amplify the reported differences between using an optimal or a sub-optimal benchmark.

VIII. CONCLUDING REMARKS

Admati and Pfleiderer (1997) have forcefully argued that benchmark-adjusted compensation based upon the unconditional, passive, benchmarks which are used in practice leads to a portfolio allocation that is sub-optimal for the investor and is useless to assess actual managerial skills. If, however, investors impose on managers appropriate active benchmarks in a multi-period context so that partial asset return predictability is properly taken into account, then delegation to talented managers makes sense and screening managers is both possible and useful. To show this, we have adopted general Von Neuman-Morgenstern utility functions for both the agent and the principal, and postulated fairly general diffusion processes for asset returns, the parameters of which are functions of (unspecified) state variables. Selecting two alternative, symmetric, benchmark-adjusted compensation schemes, we have solved the manager's and the investor's dynamic optimization programs simultaneously. Given that the manager imposes the compensation scheme on the investor, the latter's only control over her agent is to force him to adopt the benchmark that maximizes her welfare. The optimal portfolio managed by the agent has indeed been shown to always differ (with one minor exception) from the optimal benchmark chosen by the principal. Under the "affine symmetric" scheme, the value of the managed portfolio always has an optional structure. Under both compensation schemes, the optimal benchmark can be expressed as a combination of the principal's and the agent's optimal wealth.

We have also provided explicit solutions when both the investor's and the manager's utility functions exhibit constant relative risk aversion, in particular when they are logarithmic. In all cases but the logarithmic, the benchmark and the managed portfolios differ, and also differ from the optimum growth portfolio, in a very complex way, which reinforces the view that commonly observed benchmarks are sub-optimal. With logarithmic utility, under both compensation schemes the managed portfolio is simply the optimum growth portfolio. As a unique exception, the benchmark perfectly matches the managed portfolio under the "linear symmetric" scheme. It does differ from it, however, under the "affine symmetric" scheme.

In the more specialized context of a constant investment opportunity set, we have also shown by simulation that selecting a sub-optimal benchmark actually has material consequences, in particular in terms of the investor's welfare. Simulation in the more realistic but (much) more complicated environment assumed for the most part in this paper would lead to an even more dramatic discrepancy.

Appendix

Appendix A (sections IV and V)

Proof of Propositions 1 and 2.

- In the case of incentive fee (6), from program (8) the manager chooses his portfolio such that the latter is the positive solution to the following equation:

$$(\phi+\nu_1)U_m^{'}\left(\phi V_1^m(T)+\nu_1\left(V_1^m(T)-V_1^b(T)\right)\right)-e_1=\lambda_1 h(T)^{-1} \tag{A1}$$

where λ_1 is the Lagrange multiplier associated with the budget constraint. Therefore, the manager's optimal portfolio is given by:

$$V_1^m(T)=\frac{1}{\phi+\nu_1}\mathit{Y}_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right)+\frac{\nu_1}{\phi+\nu_1}V_1^b(T) \tag{A2}$$

where $\mathit{Y}_m(y)$ is x such that $U_m^{'}(x)=y$. Since U_m was assumed to be a well-behaved utility function (in particular is continuous and monotonic), its inverse always exists. This equation is equation (9) in the text. Using the budget constraint present in (8) yields:

$$V(0)=\frac{1}{\phi+\nu_1}E^P\left[h(T)^{-1}\mathit{Y}_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right)\right]+\frac{\nu_1}{\phi+\nu_1}V(0) \tag{A3}$$

Gathering the terms involving V(0) on the LHS yields (10).

- In the case of incentive fee (7), from program (8) the manager chooses his portfolio such that it is the positive solution to the following equation:

$$\nu_2 U_m^{'}\left(\Phi+\nu_2\left(V_2^m(T)-V_2^b(T)\right)\right)-e_2=\lambda_2 h(T)^{-1} \tag{A4}$$

where λ_2 is the Lagrange multiplier associated with the budget constraint.

A technical problem arises here due to the fixed amount Φ, which could make $V_2^m(T)$ negative. We thus make use of Cox and Huang's (1989) powerful equivalence result (Theorem 2.4 on p. 64) according to which the solution to the constrained program ($V_2^m(T)$ has to be non-negative) is the positive part of the solution to the unconstrained one (no restriction on the value of $V_2^m(T)$). Therefore, the optimal portfolio is equal to:

$$V_2^m(T)=\left[\frac{1}{\nu_2}\left[\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)-\Phi\right]+V_2^b(T)\right]^+ \tag{A5}$$

A slight rearranging of some terms yields (13). Then using as above the budget constraint in (8) yields (14).

Proof of Propositions 3 and 4:

- Given program (19) in the text, the optimal benchmark portfolio under compensation fee (6) is the positive solution to the following equation:

$$\frac{\nu_1}{\phi+\nu_1}U_p'\left(\frac{(1-\phi-\nu_1)}{\phi+\nu_1}\Upsilon_m\left(\frac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\frac{e_1}{\phi+\nu_1}\right)+\frac{\nu_1}{\phi+\nu_1}V_1^b(T)\right)=\delta_1 h(T)^{-1} \tag{A6}$$

where δ_1 is the Lagrange multiplier associated with the investor's budget constraint. Therefore, the optimal benchmark portfolio writes after some rearranging:

$$V_1^b(T)=\left[\begin{array}{l}\dfrac{\phi+\nu_1}{\nu_1}\Upsilon_p\left(\dfrac{\phi+\nu_1}{\nu_1}\delta_1 h(T)^{-1}\right)\\ -\dfrac{1-(\phi+\nu_1)}{\nu_1}\Upsilon_m\left(\dfrac{\lambda_1}{\phi+\nu_1}h(T)^{-1}+\dfrac{e_1}{\phi+\nu_1}\right)\end{array}\right]^+ \tag{A7}$$

which is result (20). Equation (21) for the Lagrange multiplier is obtained using the budget constraint present in program (19).

- Under compensation fee (7), the investor's terminal wealth writes according to equation (18):

$$W_2(T)=(1-\nu_2)\left[V_2^b(T)-\frac{1}{\nu_2}\left[\Phi-\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)\right]\right]^++\nu_2 V_2^b(T)-\Phi \tag{A8}$$

We solve as if the condition

$$V_2^b(T)\geq\frac{1}{\nu_2}\left[\Phi-\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)\right] \tag{A9}$$

holds, and then we show that the solution in fact satisfies this condition. When (A9) holds, the optimal wealth (18) writes:

$$W_2(T)=V_2^b(T)+\frac{(1-\nu_2)}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)-\frac{1}{\nu_2}\Phi \tag{A10}$$

Using the Lagrangian $E^P\left[U^p\left(W_2(T)\right)\right]-\delta_2\left[E^P\left[\frac{V_2^b(T)}{h(T)}\right]-V(0)\right]$, the optimal benchmark is equal to:

$$V_2^b(T)=\left[\Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi\right]^+ \quad \text{(A11)}$$

which is equation (23) and where δ_2 solves:

$$V(0)=E^P\left[h(T)^{-1}\left[\Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi\right]^+\right] \quad \text{(A12)}$$

which is equation (24).

To see why (A9) holds, use $[x]^+ = x + [-x]^+$ and write (A11) as:

$$\begin{aligned} V_2^b(T) &= \Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi \\ &+\left[-\left(\Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi\right)\right]^+ \end{aligned} \quad \text{(A13)}$$

Therefore:

$$\begin{aligned} & V_2^b(T)-\frac{1}{\nu_2}\left[\Phi-\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)\right] \\ &=\Upsilon_p\left(\delta_2 h(T)^{-1}\right)+\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}\right) \\ &+\left[-\left(\Upsilon_p\left(\delta_2 h(T)^{-1}\right)-\frac{1-\nu_2}{\nu_2}\Upsilon_m\left(\frac{\lambda_2}{\nu_2}h(T)^{-1}+\frac{e_2}{\nu_2}\right)+\frac{1}{\nu_2}\Phi\right)\right]^+ > 0 \end{aligned} \quad \text{(A14)}$$

and result (A9) follows since the first two terms are non-negative.

Appendix B (section VII)

We now provide an explicit solution to the manager's and the investor's problems in the case of a constant opportunity set. For readability, we rewrite here the equations whose explicit solutions are needed to derive the portfolio strategies.

A *portfolio strategy* consists in the dynamic allocation of the investor's wealth between the risky and the riskless assets. Therefore, the value V(t) of the strategy evolves over time as:

$$\frac{dV(t)}{V(t)} = [\alpha\mu_S + (1-\alpha)r]dt + \alpha\sigma_S\, dZ(t) \tag{B1}$$

where α is the proportion of wealth invested in the risky asset. Given the dynamics of the manager's portfolio and that of the benchmark, we can derive the exact composition of both portfolios.

With only one risky asset, the dynamics of the optimum growth portfolio reduces to:

$$\frac{dh(t)}{h(t)} = r\,dt + \kappa\, dZ(t) \tag{B2}$$

where:

$$\kappa = \frac{\mu_S - r}{\sigma_S} \tag{B3}$$

To proceed, we need the following useful

lemma:

Assume h has a log normal distribution according to (B14). Therefore:

$$E^P\left[\left(\frac{h(T)}{h(t)}\right)^x 1_{\left\{K-h(T)^y \geq 0\right\}}\right] = \exp\left\{x\left(r-\frac{1}{2}\kappa^2\right)(T-t)+\frac{1}{2}x^2\kappa^2(T-t)\right\}N(d) \tag{B4}$$

where

$$d = \frac{1}{y\kappa\sqrt{T-t}}\left\{\ln\frac{K}{h(t)^y} - y\left(r+\left(x-\frac{1}{2}\right)\kappa^2\right)(T-t)\right\} \tag{B5}$$

Proof:

We start from:

$$E^P\left[\left(\frac{h(T)}{h(t)}\right)^x 1_{\left\{K-h(T)^y\geq 0\right\}}\right]=E^P\left[\left(\frac{h(T)}{h(t)}\right)^x 1_{\left\{\left(\frac{h(T)}{h(t)}\right)^y\leq\frac{K}{h(t)^y}\right\}}\right] \tag{B6}$$

Since

$$\frac{h(T)}{h(t)}=\exp\left\{\left(r-\frac{1}{2}\kappa^2\right)(T-t)+\kappa(Z(T)-Z(t))\right\} \tag{B7}$$

it follows that:

$$\frac{h(T)}{h(t)}=\exp\left\{\left(r-\frac{1}{2}\kappa^2\right)(T-t)+\kappa\sqrt{T-t}u\right\} \tag{B8}$$

where u is a standard normal variate. Then we have:

$$\begin{aligned}E^P\left[\left(\frac{h(T)}{h(t)}\right)^x 1_{\left\{K-h(T)^y\geq 0\right\}}\right]&=\exp\left\{x\left(r-\frac{1}{2}\kappa^2\right)(T-t)\right\}\\&\quad E^P\left[\exp\left\{x\,\kappa\sqrt{T-t}u\right\}1_{\left\{u\leq\frac{1}{y\kappa\sqrt{T-t}}\left\{\ln\frac{K}{h(t)^y}-y\left(r-\frac{1}{2}\kappa^2\right)(T-t)\right\}\right\}}\right]\end{aligned} \tag{B9}$$

Explicit calculation using the standard Laplace transform yields the desired result.

Under the *linear compensation scheme*, the benchmark portfolio at each time t writes:

$$\frac{V_1^b(t)}{h(t)}=E^P\left[\Lambda_{11}h(T)^{\frac{1}{\gamma_p}-1}\left[\Lambda_{12}-h(T)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}\right]^+\right] \tag{B10}$$

and therefore:

$$\begin{aligned}V_1^b(t)&=\Lambda_{11}\Lambda_{12}h(t)^{\frac{1}{\gamma_p}}E^P\left[\left(\frac{h(T)}{h(t)}\right)^{\frac{1}{\gamma_p}-1}1_{\left\{\Lambda_{12}-h(T)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}\geq 0\right\}}\right]\\&\quad-\Lambda_{11}h(t)^{\frac{1}{\gamma_m}}E^P\left[\left(\frac{h(T)}{h(t)}\right)^{\frac{1}{\gamma_m}-1}1_{\left\{\Lambda_{12}-h(T)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}\geq 0\right\}}\right]\end{aligned} \tag{B11}$$

Using the lemma above yields:

$$V_1^b(t)=\Lambda_{11}\Lambda_{12}h(t)^{\frac{1}{\gamma_p}}\exp\left\{\frac{1-\gamma_p}{\gamma_p}\left(r-\frac{1}{2}\kappa^2\right)(T-t)+\frac{1}{2}\left(\frac{1-\gamma_p}{\gamma_p}\right)^2\kappa^2(T-t)\right\}N(d_1)$$
$$-\Lambda_{11}h(t)^{\frac{1}{\gamma_m}}\exp\left\{\frac{1-\gamma_m}{\gamma_m}\left(r-\frac{1}{2}\kappa^2\right)(T-t)+\frac{1}{2}\left(\frac{1-\gamma_m}{\gamma_m}\right)^2\kappa^2(T-t)\right\}N(d_2) \quad \text{(B12)}$$

where

$$d_1=\frac{1}{\left(\frac{1}{\gamma_m}-\frac{1}{\gamma_p}\right)\kappa\sqrt{T-t}}\left\{\ln\frac{\Lambda_{12}}{h(t)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}}-\left(\frac{1}{\gamma_m}-\frac{1}{\gamma_p}\right)\left(r+\left(\frac{1-\gamma_p}{\gamma_p}-\frac{1}{2}\right)\kappa^2\right)(T-t)\right\}$$

$$d_2=\frac{1}{\left(\frac{1}{\gamma_m}-\frac{1}{\gamma_p}\right)\kappa\sqrt{T-t}}\left\{\ln\frac{\Lambda_{12}}{h(t)^{\frac{1}{\gamma_m}-\frac{1}{\gamma_p}}}-\left(\frac{1}{\gamma_m}-\frac{1}{\gamma_p}\right)\left(r+\left(\frac{1-\gamma_m}{\gamma_m}-\frac{1}{2}\right)\kappa^2\right)(T-t)\right\}$$

Applying Ito's lemma and the Leibnitz rule then yields:

$$\frac{dV_1^b(t)}{V_1^b(t)}=(.)dt+\sigma_{V_1^b}(t)dZ(t) \quad \text{(B13)}$$

where

$$\sigma_{V_1^b}(t)=\frac{\Lambda_{11}\Lambda_{12}h(t)^{\frac{1}{\gamma_p}}\exp\left\{\frac{1-\gamma_p}{\gamma_p}\left(r-\frac{1}{2}\kappa^2\right)(T-t)+\frac{1}{2}\left(\frac{1-\gamma_p}{\gamma_p}\right)^2\kappa^2(T-t)\right\}N(d_1)}{V_1^b(t)}$$
$$\times\left[\frac{1}{\gamma_p}-\frac{1}{\gamma_m}-\frac{1}{\kappa\sqrt{T-t}}\frac{n(d_1)}{N(d_1)}+\frac{1}{\kappa\sqrt{T-t}}\frac{n(d_2)}{N(d_2)}\right]\kappa \quad \text{(B14)}$$
$$+\left[\frac{1}{\gamma_m}-\frac{1}{\kappa\sqrt{T-t}}\frac{n(d_2)}{N(d_2)}\right]\kappa$$

Using (B1), the optimal investment in the risky asset is:

$$\alpha_1^b(t)=\frac{\sigma_{V_1^b}(t)}{\sigma_S} \quad \text{(B15)}$$

The managed portfolio value at each time t writes:

$$\frac{V_1^m(t)}{h(t)}=E^P\left[\frac{1}{h(T)}\frac{1}{\phi+\nu_1}\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-\frac{1}{\gamma_m}}h(T)^{\frac{1}{\gamma_m}}+\frac{1}{h(T)}\frac{\nu_1}{\phi+\nu_1}V_1^b(T)\right] \quad \text{(B16)}$$

or else:

$$V_1^m(t) = \frac{1}{\phi + \nu_1}\left(\frac{\lambda_1}{\phi + \nu_1}\right)^{-\frac{1}{\gamma_m}} h(t)^{\frac{1}{\gamma_m}} E^P\left[\left(\frac{h(T)}{h(t)}\right)^{\frac{1}{\gamma_m}-1}\right] + \frac{\nu_1}{\phi + \nu_1} V_1^b(t) \tag{B17}$$

and explicit calculations lead to:

$$V_1^m(t) = \frac{\left(\frac{\lambda_1}{\phi + \nu_1}\right)^{-\frac{1}{\gamma_m}}}{\phi + \nu_1} h(t)^{\frac{1}{\gamma_m}} \exp\left\{\left(\frac{1}{\gamma_m} - 1\right)\left(r - \frac{1}{2}\kappa^2\right)(T - t) + \frac{1}{2}\left(\frac{1}{\gamma_m} - 1\right)^2 \kappa^2 (T - t)\right\} \tag{B18}$$

$$+ \frac{\nu_1}{\phi + \nu_1} V_1^b(t)$$

Applying Ito's lemma yields:

$$\sigma_{V_1^m}(t) = \left(1 - \frac{\nu_1}{\phi + \nu_1}\frac{V_1^b(t)}{V_1^m(t)}\right)\frac{1}{\gamma_m}\kappa + \frac{\nu_1}{\phi + \nu_1}\frac{V_1^b(t)}{V_1^m(t)}\sigma_{V_1^b}(t) \tag{B19}$$

so that the manager's optimal strategy reads:

$$\alpha_1^m(t) = \frac{\sigma_{V_1^m}(t)}{\sigma_S} \tag{B20}$$

It is instructive to compare this result with the manager's optimal strategy, had he been evaluated relative to the risky asset (the market portfolio) as the benchmark:

$$\hat{\sigma}_{V_1^m}(t) = \left(1 - \frac{\nu_1}{\phi + \nu_1}\frac{V(0)}{S(0)}\frac{S(t)}{V_1^m(t)}\right)\frac{1}{\gamma_m}\kappa + \frac{\nu_1}{\phi + \nu_1}\frac{V(0)}{S(0)}\frac{S(t)}{V_1^m(t)}\sigma_S \tag{B21}$$

and therefore:

$$\hat{\alpha}_1^m(t) = \frac{\hat{\sigma}_{V_1^m}(t)}{\sigma_S} \tag{B22}$$

Appendix C (section VII, welfare loss)

We now provide an explicit calculation of the welfare loss suffered by the investor when the manager adopts a sub-optimal benchmark. We assume that the manager has a logarithmic utility and the principal has a power utility. For brevity, only the first (linear) compensation scheme is considered.

The manager's portfolio under this scheme and logarithmic preferences is given by:

$$V_1^m(t) = \frac{\left(\frac{\lambda_1}{\phi+\nu_1}\right)^{-1}}{\phi+\nu_1} h(t) + \frac{\nu_1}{\phi+\nu_1} V_1^b(t) \tag{C1}$$

To compute explicitly the RHS of (C1), we need the Lagrange multiplier λ_1, which is given by equation (10). Its value is here equal to:

$$\lambda_1 = \frac{\phi+\nu_1}{\phi V(0)} \tag{C2}$$

Therefore:

$$V_1^m(t) = \frac{\phi}{\phi+\nu_1} V(0) h(t) + \frac{\nu_1}{\phi+\nu_1} V_1^b(t) \tag{C3}$$

The investor's optimal wealth then is:

$$W_1(T) = (1-\phi-\nu_1) V_1^m(T) + \nu_1 V_1^b(T) \tag{C4}$$

and consequently:

$$W_1(T) = \frac{(1-\phi-\nu_1)\phi}{\phi+\nu_1} V(0) h(T) + \frac{\nu_1}{\phi+\nu_1} V_1^b(T) \tag{C5}$$

If the standard (sub-optimal) benchmark was used, this wealth would be:

$$\hat{\hat{W}}_1(T) = \frac{(1-\phi-\nu_1)\phi}{\phi+\nu_1} V(0) h(T) + \frac{\nu_1}{\phi+\nu_1} \frac{V(0)}{S(0)} S(T) \tag{C6}$$

The optimal wealth using the optimal benchmark is such that:

$$\hat{W}_1(T) = \left(\frac{\phi+\nu_1}{\nu_1}\delta_1\right)^{-\frac{1}{\gamma_p}} h(T)^{\frac{1}{\gamma_p}} \tag{C7}$$

where δ_1 is the shadow price of the budget constraint.

The welfare loss θ associated with using the sub-optimal benchmark then is such that:

$$E^P\left[\frac{\hat{W}_1(T)^{1-\gamma_p}}{1-\gamma_p};V(0)\right]=E^P\left[\frac{\hat{\hat{W}}_1(T)^{1-\gamma_p}}{1-\gamma_p};(1+\theta)V(0)\right] \tag{C8}$$

The RHS cannot be computed explicitly, but we can use a first order approximation. We have:

$$\frac{\hat{\hat{W}}_1(T)^{1-\gamma_p}}{1-\gamma_p}\approx\frac{1}{1-\gamma_p}+\ln\hat{\hat{W}}_1(T) \tag{C9}$$

Note that:

$$\begin{aligned}\ln\hat{\hat{W}}_1(T)&=\ln\left[\frac{(1-\phi-\nu_1)\phi}{\phi+\nu_1}(1+\theta)V(0)h(T)+\frac{\nu_1}{\phi+\nu_1}\frac{(1+\theta)V(0)}{S(0)}S(T)\right]\\&=\ln(1+\theta)V(0)h(T)\left[\frac{(1-\phi-\nu_1)\phi}{\phi+\nu_1}+\frac{\nu_1}{\phi+\nu_1}\frac{1}{S(0)}\frac{S(T)}{h(T)}\right]\\&=\ln(1+\theta)V(0)h(T)+\ln\left[\frac{(1-\phi-\nu_1)\phi}{\phi+\nu_1}+\frac{\nu_1}{\phi+\nu_1}\frac{1}{S(0)}\frac{S(T)}{h(T)}\right]\\&=\ln(1+\theta)h(T)+\ln\left[\frac{(1-\phi-\nu_1)\phi}{\phi+\nu_1}+\frac{\nu_1}{\phi+\nu_1}\frac{S(T)}{h(T)}\right]\end{aligned} \tag{C10}$$

Expanding this expression around $\phi = 0$ yields:

$$\begin{aligned}\ln\hat{\hat{W}}_1(T)&\approx\ln(1+\theta)h(T)+\ln\frac{S(T)}{h(T)}+\phi\left[\frac{1-\nu_1}{\nu_1}\frac{h(T)}{S(T)}-\frac{1}{\nu_1}\right]\\&\approx\ln(1+\theta)+\ln S(T)+\frac{1-\nu_1}{\nu_1}\phi\frac{h(T)}{S(T)}-\frac{\phi}{\nu_1}\end{aligned} \tag{C11}$$

Plugging (C11) into (C9) then yields:

$$\frac{\hat{\hat{W}}_1(T)^{1-\gamma_p}}{1-\gamma_p}\approx\frac{1}{1-\gamma_p}+\ln(1+\theta)+\ln S(T)+\frac{\phi}{\nu_1}\frac{h(T)}{S(T)}-\phi\frac{h(T)}{S(T)}-\frac{\phi}{\nu_1} \tag{C12}$$

Therefore we have:

$$E^P\left[\frac{\hat{\hat{W}}_1(T)^{1-\gamma_p}}{1-\gamma_p}\right]\approx\frac{1}{1-\gamma_p}+\ln(1+\theta)+E^P[\ln S(T)]+\left[\frac{\phi}{\nu_1}-\phi\right]E^P\left[\frac{h(T)}{S(T)}\right]-\frac{\phi}{\nu_1} \tag{C13}$$

In addition, we know that:

$$\begin{aligned}\frac{S(T)}{S(0)}&=\exp\left\{\left(\mu_S-\frac{\sigma_S^2}{2}\right)T+\sigma_S\sqrt{T}u\right\}\\h(T)&=\exp\left\{\left(r-\frac{\kappa^2}{2}\right)T+\kappa\sqrt{T}u\right\}\end{aligned} \tag{C14}$$

where u is a standard normal variate. Using the fact that:

$$\mu_S = r + \kappa\sigma_S , \tag{C15}$$

(C14) becomes:

$$\begin{aligned} \frac{S(T)}{S(0)} &= \exp\left\{\left(\mu_S - \frac{\sigma_S^2}{2}\right)T + \sigma_S\sqrt{T}u\right\} \\ h(T) &= \exp\left\{\left(r - \frac{\kappa^2}{2}\right)T + \kappa\sqrt{T}u\right\} \end{aligned} \tag{C16}$$

Using the normalization V(0) = S(0) = 1 and T = 1 in the simulations, explicit computation of (C13) gives:

$$E^P\left[\frac{\hat{\hat{W}}_1(T)^{1-\gamma_p}}{1-\gamma_p}\right] \approx \frac{1}{1-\gamma_p} + \ln(1+\theta) + r + \kappa\sigma_S - \frac{\sigma_S^2}{2} + \left[\frac{\phi}{\nu_1} - \phi\right]e^{\sigma_S^2 - 2\kappa\sigma_S} - \frac{\phi}{\nu_1} \tag{C17}$$

On the other hand, using the optimal benchmark leads to:

$$\frac{\hat{W}_1(T)^{1-\gamma_p}}{1-\gamma_p} = \frac{1}{1-\gamma_p}\left(\frac{\phi+\nu_1}{\nu_1}\delta_1\right)^{-\frac{1-\gamma_p}{\gamma_p}} h(T)^{\frac{1-\gamma_p}{\gamma_p}} \tag{C18}$$

where δ_1 solves:

$$1 = \Lambda_{11}E^P\left[h(T)^{\frac{1}{\gamma_b}-1}\left[\Lambda_{12} - h(T)^{1-\frac{1}{\gamma_b}}\right]^+\right] \tag{C19}$$

with

$$\begin{aligned} \Lambda_{11} &= \frac{1-(\phi+\nu_1)}{\nu_1}\phi \\ \Lambda_{12} &= \frac{(\phi+\nu_1)\left(\frac{\phi+\nu_1}{\nu_1}\delta_1\right)^{-\frac{1}{\gamma_b}}\frac{1}{\phi}}{1-(\phi+\nu_1)} \end{aligned} \tag{C20}$$

Expanding (C18) around $\phi = 0$ yields:

$$\begin{aligned} \frac{\hat{W}_1(T)^{1-\gamma_p}}{1-\gamma_p} &= \frac{1}{1-\gamma_p}\left(\frac{\phi+\nu_1}{\nu_1}\delta_1\right)^{-\frac{1-\gamma_p}{\gamma_p}} h(T)^{\frac{1-\gamma_p}{\gamma_p}} \\ &= \frac{1}{1-\gamma_p}\left(\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}h(T)\right)^{\frac{1-\gamma_p}{\gamma_p}} \\ &\approx \frac{1}{1-\gamma_p} + \frac{1}{\gamma_p}\ln\left[\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}h(T)\right] \end{aligned} \tag{C21}$$

where we have neglected the dependence of the Lagrange multiplier δ_1 upon the risk aversion parameter γ_p. Therefore:

$$\begin{aligned} E^P\left[\frac{\hat{W}_1(T)^{1-\gamma_p}}{1-\gamma_p}\right] &\approx \frac{1}{1-\gamma_p}+\frac{1}{\gamma_p}E^P\left[\ln\left[\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}h(T)\right]\right] \\ &\approx \frac{1}{1-\gamma_p}+\frac{1}{\gamma_p}\ln\left[\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}\right]+\frac{1}{\gamma_p}E^P\left[\ln h(T)\right] \\ &= \frac{1}{1-\gamma_p}+\frac{1}{\gamma_p}\ln\left[\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}\right]+\frac{1}{\gamma_p}\left(r-\frac{\kappa^2}{2}\right) \end{aligned} \qquad \text{(C22)}$$

The welfare loss θ due to sub-optimal benchmarking then is such that:

$$\begin{aligned} &\frac{1}{1-\gamma_p}+\frac{1}{\gamma_p}\ln\left[\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}\right]+\frac{1}{\gamma_p}\left(r-\frac{\kappa^2}{2}\right) \\ &\quad = \frac{1}{1-\gamma_p}+\ln(1+\theta)+r+\kappa\sigma_S-\frac{\sigma_S^2}{2}+\left[\frac{\phi}{\nu_1}-\phi\right]e^{\sigma_S^2-2\kappa\sigma_S}-\frac{\phi}{\nu_1} \end{aligned} \qquad \text{(C23)}$$

which leads to the explicit value:

$$\theta=\left(\frac{\nu_1}{\phi+\nu_1}\frac{1}{\delta_1}\right)^{\frac{1}{\gamma_p}}\exp\left\{\frac{1}{\gamma_p}\left(r-\frac{\kappa^2}{2}\right)-r-\kappa\sigma_S+\frac{\sigma_S^2}{2}-\left[\frac{\phi}{\nu_1}-\phi\right]e^{\sigma_S^2-2\kappa\sigma_S}+\frac{\phi}{\nu_1}\right\}-1 \qquad \text{(C24)}$$

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Table 1 : Base case simulation parameters

ϕ	ν_1	γ_m	γ_p	R	κ	σ_s	V(0)	T	S(0)
1.5%	30%	2	4	2%	0.3	15%	1	1	1

ϕ and ν_1 are the constant parameters of the linear compensation scheme (6), γ_m and γ_p are the manager's and the principal's coefficients of risk aversion, respectively, r is the riskless rate of interest, κ is the market price of risk, σ_s is the volatility of the risky asset, T is the investment horizon (in years), and V(0) and S(0) stand for the (normalized) initial values of the managed fund and the risky asset (the market portfolio), respectively.

<u>Table 2 : Impact of the compensation scheme parameters</u>

The Table presents the initial compositions of the optimally managed $\left(\alpha_1^m\right)$ and benchmark $\left(\alpha_1^b\right)$ portfolios and the initial composition of the sub-optimally managed portfolio $\left(\hat{\alpha}_1^m\right)$ when the adopted benchmark is the market portfolio, for different values of the compensation scheme parameters. α_1 is the proportion of the risky asset in the relevant portfolio, $(1-\alpha_1)$ being invested in the riskless asset.

		$\nu_1 = 20\%$	$\nu_1 = 25\%$	$\nu_1 = 30\%$	$\nu_1 = 35\%$	$\nu_1 = 40\%$
	α_1^m	50.25%	50.25%	50.25%	50.25%	50.25%
ϕ = 0.5%	α_1^b	49.01%	49.26%	49.42%	49.54%	49.63%
	$\hat{\alpha}_1^m$	100.00%	100.00%	100.00%	100.00%	100.00%
	α_1^m	50.50%	50.50%	50.50%	50.50%	50.50%
ϕ = 1%	α_1^b	48.03%	48.52%	48.85%	49.09%	49.26%
	$\hat{\alpha}_1^m$	100.00%	100.00%	100.00%	100.00%	100.00%
	α_1^m	50.75%	50.75%	50.75%	50.75%	50.75%
ϕ = 1.5%	α_1^b	47.06%	47.80%	48.29%	48.64%	48.90%
	$\hat{\alpha}_1^m$	100.00%	100.00%	100.00%	100.00%	100.00%
	α_1^m	51.00%	51.00%	51.00%	51.00%	51.00%
ϕ = 2%	α_1^b	46.10%	47.08%	47.73%	48.20%	48.55%
	$\hat{\alpha}_1^m$	100.00%	100.00%	100.00%	100.00%	100.00%
	α_1^m	51.25%	51.25%	51.25%	51.25%	51.25%
ϕ = 2.5%	α_1^b	45.15%	46.38%	47.19%	47.77%	48.20%
	$\hat{\alpha}_1^m$	100.00%	100.00%	100.00%	100.00%	100.00%

Table 3 : Impact of the market parameters

The Table presents the initial compositions of the optimally managed $\left(\alpha_1^m\right)$ and benchmark $\left(\alpha_1^b\right)$ portfolios and the initial composition of the sub-optimally managed portfolio $\left(\hat{\alpha}_1^m\right)$ when the adopted benchmark is the market portfolio, for different values of the market parameters (market price of risk κ, and volatility of the risky asset, σ_S). α_1 is the proportion of the risky asset in the relevant portfolio, (1- α_1) being invested in the riskless asset.

		σ_S = 5%	σ_S = 10%	σ_S = 15%	σ_S = 20%	σ_S = 25%
	α_1^m	101.50%	50.75%	33.83%	25.38%	20.30%
κ = 0.2	α_1^b	96.57%	48.29%	32.19%	24.14%	19.31%
	$\hat{\alpha}_1^m$	104.76%	100.00%	98.41%	97.62%	97.14%
	α_1^m	126.87%	63.44%	42.29%	31.72%	25.37%
κ = 0.25	α_1^b	120.72%	60.36%	40.24%	30.18%	24.14%
	$\hat{\alpha}_1^m$	107.14%	101.19%	99.21%	98.21%	97.62%
	α_1^m	152.25%	76.12%	50.75%	38.06%	30.45%
κ = 0.3	α_1^b	144.86%	72.43%	48.29%	36.22%	28.97%
	$\hat{\alpha}_1^m$	109.52%	102.38%	100.00%	98.81%	98.10%
	α_1^m	177.63%	88.81%	59.21%	44.41%	35.53%
κ = 0.35	α_1^b	169.01%	84.50%	56.34%	42.25%	33.80%
	$\hat{\alpha}_1^m$	111.90%	103.57%	100.79%	99.40%	98.57%
	α_1^m	203.00%	101.50%	67.67%	50.75%	40.60%
κ = 0.4	α_1^b	193.15%	96.58%	64.38%	48.29%	38.63%
	$\hat{\alpha}_1^m$	114.29%	104.76%	101.59%	100.00%	99.05%

Table 4 : Impact of risk aversion

The Table presents the initial compositions of the optimally managed $\left(\alpha_1^m\right)$ and benchmark $\left(\alpha_1^b\right)$ portfolios and the initial composition of the sub-optimally managed portfolio $\left(\hat{\alpha}_1^m\right)$ when the adopted benchmark is the market portfolio, for different values of the risk aversion coefficients (γ_m and γ_p). α_1 is the proportion of the risky asset in the relevant portfolio, (1- α_1) being invested in the riskless asset.

		$\gamma_p = \gamma_m$	$\gamma_p = \gamma_m+1$	$\gamma_p = \gamma_m+2$	$\gamma_p = \gamma_m+3$	$\gamma_p = \gamma_m+4$
	α_1^m	100.00%	67.17%	50.75%	40.90%	34.33%
$\gamma_m = 2$	α_1^b	100.00%	65.53%	48.29%	37.94%	31.05%
	$\hat{\alpha}_1^m$	100.00%	100.00%	100.00%	100.00%	100.00%
	α_1^m	66.67%	50.25%	40.40%	33.83%	29.14%
$\gamma_m = 3$	α_1^b	66.67%	49.43%	39.09%	32.19%	27.27%
	$\hat{\alpha}_1^m$	98.41%	98.41%	98.41%	98.41%	98.41%
	α_1^m	50.00%	40.15%	33.58%	28.89%	25.38%
$\gamma_m = 4$	α_1^b	50.00%	39.66%	32.76%	27.84%	24.14%
	$\hat{\alpha}_1^m$	97.62%	97.62%	97.62%	97.62%	97.62%
	α_1^m	40.00%	33.43%	28.74%	25.23%	22.49%
$\gamma_m = 5$	α_1^b	40.00%	33.11%	28.18%	24.49%	21.61%
	$\hat{\alpha}_1^m$	97.14%	97.14%	97.14%	97.14%	97.14%
	α_1^m	33.33%	28.64%	25.13%	22.39%	20.20%
$\gamma_m = 6$	α_1^b	33.33%	28.41%	24.71%	21.84%	19.54%
	$\hat{\alpha}_1^m$	96.83%	96.83%	96.83%	96.83%	96.83%

Table 5 : Welfare loss associated with a sub-optimal benchmark

The Table presents the welfare loss in absolute percentage return when the adopted benchmark is the market portfolio, for different values of the principal's risk aversion coefficient γ_p, of the compensation scheme parameters ϕ and ν_1, and of the market price of risk κ.

γ_p	Welfare loss
2.500	0.08%
2.750	0.37%
3.000	0.69%
3.250	0.98%
3.500	1.22%
3.750	1.44%
4.000	1.63%
4.250	1.79%
4.500	1.94%
4.750	2.09%
5.000	2.33%
5.250	2.43%
5.500	2.60%

ϕ	Welfare loss
1.00%	0.14%
1.25%	0.68%
1.50%	1.22%
1.75%	1.77%
2.00%	2.31%
2.25%	2.86%
2.50%	3.41%
2.75%	3.96%
3.00%	4.53%
3.25%	5.14%
3.50%	5.77%

ν_1	Welfare loss
5.00%	2.89%
10.00%	1.83%
15.00%	1.53%
20.00%	1.38%
25.00%	1.29%
30.00%	1.22%
35.00%	1.18%
40.00%	1.14%
45.00%	1.12%
50.00%	1.10%

κ	Welfare loss
0.15	0.16%
0.20	0.31%
0.25	0.64%
0.30	1.22%
0.35	2.02%
0.40	3.24%
0.45	4.48%
0.50	6.11%
0.55	7.98%