The information content of implied volatilities and model-free volatility expectations: Evidence from options written on individual stocks

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Abstract

The volatility information content of stock options for individual firms is measured using option prices for 149 U.S. firms during the period from January 1996 to December 1999. ARCH models and OLS regressions are used to compare volatility forecasts defined by historical stock returns, at-the-money implied volatilities and model-free volatility expectations for every firm. For one-day-ahead estimation, a historical ARCH model outperforms both of the volatility estimates extracted from option prices for 36% of the firms, but the option forecasts are nearly always more informative for those firms that have the more actively traded options. When the prediction horizon extends until the expiry date of the options, the option forecasts are more informative than the historical volatility for 86% of the firms. The results also show that, overall, there is less volatility information contained in the model-free volatility expectations than in the at-the-money implied volatilities.

JEL classifications: C22; C25; G13; G14
Keywords: Volatility; Stock options; Information content; Implied volatility; Model-free volatility expectations; ARCH models

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1. Introduction

The volatility implicit in an option price can be interpreted as an estimate of the average volatility of the underlying asset over the life of the option. If markets are efficient and option pricing models are correctly specified, then option implied volatilities are expected to subsume all information contained in historical volatility. There is already a fruitful literature that investigates the information content of stock index options and exchange rate options. However, a good understanding of the information content of options written on individual stocks is also of great importance for risk management, volatility forecasting and option pricing.

1.1 Prior literature

The ability of option implied volatility to provide good estimates of stock index volatility has now been established. However, some early empirical studies challenge the usefulness of implied volatility as a guide to the future variability of index returns. Day and Lewis (1992) do not find conclusive evidence that option implied volatilities contain incremental information relative to the conditional volatility from GARCH and EGARCH models, by using weekly data on the S & P 100 index. Canina and Figlewski (1993), who relied on regression tests, find that implied volatility does not have a statistically significant correlation with realized volatility for most of their subsamples and it is less informative than a simple historical measure of volatility.

These negative conclusions might be caused by a lack of data, mis-measurement of implied volatilities, or inappropriate statistical inference. After correcting various methodological errors, later studies of the S & P 100 index provide a consensus that the at-the-money (henceforth ATM) option implied volatility is a more efficient estimate of the subsequent realized volatility than estimates based solely on historical information. Fleming (1998), in agreement with Christensen and Prabhala (1998), shows that the implied volatility, although biased, subsumes all information contained in the historical volatility. In more depth, Blair, Poon and Taylor (2001) use the Chicago Board Options Exchange (CBOE) volatility index (VIX) as the measure of option implied volatility and daily and intra-day returns sources to measure historical volatility. They find no evidence for incremental information contained in daily index returns beyond that provided by VIX. The implied volatilities in their sample outperform intraday returns for both in-sample estimation and out-of-sample forecasting. Similar conclusions are obtained for S & P 500 index options by Ederington and Guan (2002a). Studies that focus on exchange rate options also favor the conclusion that the option implied volatility is an efficient estimate of the future realized volatility (Jorion (1995), Xu and Taylor (1995), Pong et al (2004)).

However, nearly all of the previous studies investigate the information content of ATM or nearest-to-the-money option implied volatilities, and thus fail to incorporate the information contained in out-of-the-money (henceforth OTM) options, which are also actively traded for indices. Ederington and Guan (2002b) investigate the information content contained in Black-Scholes implied volatilities, corresponding to different strike price intervals, using S & P 500 data. They find that the implied volatilities in several strike prices intervals are upward biased compared with the realized volatility and that the nearest-to-the-money implied volatilities do not provide the most information. The use of either a single option or only a few options may not be sufficient to extract all the relevant information, so that the forecasting ability of option prices may be underestimated. Furthermore, the studies that have investigated the information content of Black-Scholes implied volatilities could be affected by model misspecification errors.

Constructive theoretical relationships between volatility and option prices have been developed by Carr and Madan (1998) and Demeterfi, Derman, Kamal and Zou (1999). They show that the fair value of a variance swap rate, which is a risk-neutral forecast of subsequent realized variance, can be replicated by taking a static position in options of all strike prices. Likewise, Britten-Jones and Neuberger (2000) build on the pioneering work of Breeden and Lizenberger (1978) to show that a complete set of call options can be used to infer the risk-neutral expectation of the integrated variance until the options' maturity. This risk-neutral expectation of future variance is independent of any option pricing model and it incorporates the information across all strike prices. In September 2003, the model-free volatility expectation was adopted by the CBOE as a new method for calculating the components of its volatility index (VIX). The new VIX provides a 30-day volatility expectation for the S & P 500 index. Carr and Wu (2004) synthesize the variance swap rates of five stock indices and 35 individual stocks using option prices. They find their estimates of the variance swap rates are significant variables when explaining the movements of realized variance for all the indices and the majority of the stocks.

The model-free volatility expectation is theoretically more appealing than alternative volatility estimates, including Black-Scholes implied volatility, because it contains information from a complete set of option prices and it does not rely on restrictive model assumptions. Jiang and Tian (2005) generalize the model-free volatility expectation to processes with prices jumps and they develop a method for implementing its calculation using prices for options written on the S & P 500 index. They also investigate the information content and the forecasting ability of the model-free volatility expectation. Their results show that the model-free volatility expectation subsumes all information contained in both the ATM implied volatility and the past realized volatility, calculated from intraday index returns. Lynch and Panigirtzoglou (2005) also examine the information content of the model-free volatility expectation in comparison with historical volatility measured by intraday returns. Their results for S & P 500, FTSE 100, Eurodollar and short sterling futures show that the model-free volatility expectation is an efficient but biased estimate of future volatility.

However, nearly all of the previous equity studies, whether they use the Black-Scholes implied volatility or the model-free volatility expectation, only investigate the information content of option prices that are written on stock indices. There are very few studies that test the information content of individual stock options. We may anticipate that the volatility information contained in the prices of stock options is less efficient when estimating and predicting volatility, compared with index option prices, since stock options are traded far less frequently. Lamoureux and Lastrapes (1993) study two years of daily data for ten U.S. firms and their results indicate that the simple GARCH(1,1) model is more informative than a model that uses implied volatility alone. Their results also show that implied volatilities have predictive power, although they are biased forecasts of future volatilities. It is inevitably difficult to draw firm conclusions from their small quantity of data.

1.2 Scope

This paper is the first to examine the volatility information content of individual stock options based on a large sample of U.S. stocks. In addition to the ATM option implied volatility, the model-free volatility expectation is also adopted as a predictor of realized volatility. We develop a method to calculate the model-free volatility expectation for individual stock options that are less liquid than stock index options. For each firm out of the 149 in our sample, we use both ARCH models and OLS regressions to compare the historical information from daily stock returns, the information contained in the ATM implied volatility and the information provided by the model-free volatility expectation.

In contrast to previous studies about stock index options, our empirical research shows that for one-day-ahead estimation neither the ATM implied volatility nor the model-free volatility expectation is consistently superior to an ARCH model when estimating the volatility of individual stock returns. Especially for firms with few traded options, it is better to use an asymmetric ARCH model to estimate the conditional volatility of future stock returns. When the estimation horizon extends until the end of the option lives, it is found that both of the volatility estimates extracted from option prices outperform the historical volatility for a substantial majority of our firms. For the firms in our sample, the ATM implied volatility outperforms the model-free volatility expectation overall when predicting the volatilities of individual stock returns.

The paper proceeds as follows. Section 2 describes the newly developed model-free volatility expectation and our method for calculating this expectation. The descriptions of our data and empirical methodology are provided in Sections 3 and 4 respectively. Section 5 presents the empirical results. Our conclusions are stated in the final section.

2. Model-free volatility expectation and implementation issues

This section describes our methods for calculating the model-free volatility expectation from a small number of option prices.

2.1 Theory

At time 0 it is supposed that there is a complete set of European option prices for an expiry time *T*. For a general strike price *K*, these option prices are denoted by $c(K,T)$ and $p(K,T)$ respectively. For a risk-neutral measure *Q*, the price of the underlying asset S_t is assumed to satisfy the equation $dS = (r-q)Sdt + \sigma SdW$, where r is the risk-free rate, q is the dividend yield, W_t is a Wiener process and σ_t is the risk-neutral stochastic volatility. The integrated squared volatility of the asset over the horizon T is defined as:

$$V_{0,T} = \int_0^T \sigma_t^2 dt.$$

Britten-Jones and Neuberger (2000) show that the risk-neutral expectation of the integrated squared volatility is given by the following function of the continuum of European OTM option prices:

$$E^Q\left[V_{0,T}\right] = 2e^{rT}\left[\int_0^{F_{0,T}} \frac{p(K,T)}{K^2}dK + \int_{F_{0,T}}^{\infty} \frac{c(K,T)}{K^2}dK\right]. \tag{1}$$

Here $F_{0,T}$ is the forward price at time 0 for a transaction at the expiry time T. Following previous literature, the quantity defined by equation (1) will be referred to as the *model-free variance expectation* and its square root as the *model-free volatility expectatio*n. Dividing the variance expectation by T defines its annualized value.

The key assumption required to derive (1) is that the stochastic process for the underlying asset price is continuous. It is then possible to produce a volatility expectation that does not rely on a specific option-pricing formula. The expectation is then 'model-free', in contrast to the Black-Scholes implied volatility. Both Carr and Wu (2004) and Jiang and Tian (2005) relax the assumption of continuity. They show that (1) is an excellent approximation when there are jumps in prices.

2.2 The discrete formula

The CBOE has calculated the model-free volatility expectation of the S & P 500 index over the next 30 calendar days since September 2003. They calculate the volatility index, VIX, using *M* strike prices as[1]:

$$\sigma_{VE} = \sqrt{\frac{2}{T}e^{rT}\sum_{i=1}^{M}\frac{\Delta K_i}{K_i^2}Q(K_i,T) - \frac{1}{T}\left[\frac{F_{0,T}}{K_0} - 1\right]^2} \tag{2}$$

[1] This equation is included in http://www.cboe.com/micro/vix/vixwhite.pdf.

where K_0 is the strike price used to select call or put options, $Q(K_i, T)$ is the call price with strike price K_i when $K_i \geq K_0$ and otherwise it is the put price, and ΔK_i is set equal to $\frac{K_{i+1} - K_{i-1}}{2}$ [2]. The quantity σ_{VE} is the annualized value of the model-free volatility expectation from time 0 until time T.

The CBOE sets the strike K_0 just below the forward price $F_{0,T}$. We also employ equation (2). However, we use a small number of strikes to estimate a risk-neutral density and hence we can infer option prices for as many strikes as necessary. Consequently we can set $K_0 = F_{0,T}$ and the final term in equation (2) then disappears. Thus $Q(K_i, T)$ always represents an OTM option price in our calculations, as K_0 is the ATM strike price.

2.3 Construction of implied volatility curves

Equation (1) shows that the model-free volatility expectation is obtained from the integral of a function of option prices at all strikes. However, stock option prices are usually only available for a small number of strike prices. In order to obtain consistent option prices for a large number of strikes, we must estimate implied volatility curves from small sets of observed option prices.

[2] ΔK for the lowest strike is the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.

We implement a variation of the practical strategy described by Malz (1997), who proposed estimating the implied volatility curve as a quadratic function of the option's delta; previously a quadratic function of the strike price had been suggested by Shimko (1993). As stated by Malz (1997), making implied volatility a function of delta, rather than of the strike price, has the advantage that the away-from-the-money implied volatilities are grouped more closely together than the near-the-money implied volatilities. Also, extrapolating a function of delta provides sensible limits for the magnitudes of the implied volatilities.

The quadratic specification is chosen because it is the simplest function that captures the basic properties of the volatility smile. Furthermore, there are insufficient stock option prices to estimate higher-order polynomials. Only three strike prices are required to estimate the parameters of a quadratic implied volatility function.

Delta is defined here as the first derivative of the Black-Scholes call option price with respect to the underlying forward price:

$$\Delta_i = \frac{\partial c(F_{0,T}, K_i, T)}{\partial F_{0,T}} = e^{-rT} N(d_1(K_i)) \tag{3}$$

with

$$d_1(K_i) = \frac{\ln(F_{0,T} / K_i) + 0.5\sigma^* T^2}{\sigma^* \sqrt{T}} .$$

Following Bliss and Panigirtzoglou (2002, 2004), σ^* is a constant that permits a convenient one-to-one mapping between delta and the strike price. In this study, σ^* is the implied volatility for the option price whose strike price is nearest to the forward price $F_{0,T}$. The value of the call delta Δ increases from zero for deep out-of-the-money call options to e^{-rT} for deep in-the-money call options.

The parameters of the quadratic function have been estimated by minimizing the following function:

$$\sum_{j=1}^{N} w_j (IV_j - I\hat{V}_j(\Delta_j, \Phi))^2 , \tag{4}$$

where N is the number of observed strike prices, IV_j is the observed implied volatility for a strike price K_j, Δ_j is given by (3), w_j equals $\Delta_j(1-\Delta_j)$, Φ is the vector of the three parameters of the quadratic function, and $I\hat{V}_j(\Delta_j, \Phi)$ is the fitted implied volatility. The minimization is subject to the constraint that the fitted implied volatility curve is always positive when Δ is between 0 and e^{-rT}.

The squared errors of the fitted implied volatilities are weighted by $\Delta_j(1-\Delta_j)$, to ensure that the most weight is given to near-the-money options. Far-from-the-money options are given low weights because their contracts are less liquid and hence their prices are the most susceptible to non-synchronicity errors. Introducing weights when fitting the quadratic function reduces the impact of any outliers obtained from far-from-the-money options.

After the implied volatility function is fitted, we use 1000 equally spaced values of delta (that cover the range from 0 to e^{-rT}) to calculate OTM option prices for the corresponding strike prices. If either the least call price or the least put price exceeds

0.001 cents then we extend the range of strike prices[3], to eliminate any error caused by truncating the integral shown in (1). The OTM prices are then used to evaluate (2).

3. Data

This section introduces the data used in our study and the methods we used to select our sample of firms.

3.1 Sources

The option data used in this study are from the Ivy DB database of OptionMetrics and the stock return data are from CRSP. The Ivy DB contains the prices for all US listed equities and market indices and all US listed index and equity options, based on closing quotes at the CBOE. The dataset also includes interest rate curves, dividend projections and option implied volatilities. Our sample starts on 4 January 1996 and ends on 31 December 1999. There are 1009 trading days during this period.

Both Carr and Wu (2004) and our study use the implied volatilities provided by Ivy DB, rather than option prices. Each implied volatility provided by Ivy DB is based on the midpoint of the highest closing bid price and the lowest closing offer price across all exchanges on which the option trades. For the European options, implied volatilities are inferred from the Black-Scholes option pricing formula adjusted for projected dividends. For the American options, the implied volatilities are calculated

[3] The extrapolation in either tail occurs with an equal spacing of 0.01 in moneyness, defined as the ratio $K/F_{0,T}$. It continues until the OTM prices are less than 0.001 cents. The implied volatility equals the appropriate end-point of the quadratic function.

from a binomial tree model, which takes into account the early exercise premium and dividends. Whenever call and put implieds are both available, for the same firm, trading day, expiry date and strike price, the average of the two implied volatilities is used. Options with less than eight days to maturity are excluded in order to avoid any liquidity and market microstructure effects around expiry.

The interest rate that corresponds to each option's expiration is obtained by linearly interpolating between the two closest zero-coupon rates, that are provided by the zero-curve file included in the Ivy DB. We also calculate the corresponding forward stock price $F_{0,T}$ for each option, that has the same time to maturity, T. It is defined as the future value of the difference between the current spot price and the present value of all future dividend distributions between times 0 and T inclusive. The dividend distribution information is also included in the Ivy DB.

Daily stock returns for each firm have been obtained from CRSP, for the period from January 1988 to March 2000. These returns incorporate adjustments for both dividends and changes in the capital structure of the firm. They are transformed into continuously compounded returns, such that $r_t = \log(1 + r_t^*)$, where r_t^* is the CRSP stock return.

3.2 Selection of firms

All firms with sufficient option trading activity are included in our study. Two criteria are used to select firms from the database. The first criterion selects firms based on the number of option trading days during the sample period and then the second selects firms depending on the number of market option observations per day, which is the daily number of available strike prices.

Firstly, only firms that have options written on them throughout the whole sample period are included. As a result, every firm that is selected has option observations for 1009 trading days from January 1996 to December 1999. Consequently, the comparison of different volatility measures using GARCH specifications and OLS regressions, introduced in the following section, will not be influenced by either the sample period or the sample size.

Secondly, a firm must have sufficient option trading activity, where sufficient is defined by us as enough to construct implied volatility curves for at least 989 (i.e. 98%) of the 1009 trading days. If the firm has too many days of missing data, the firm’s options are considered to be illiquid and then the information content of option prices is expected to be reduced.

Our method for constructing the implied volatility curve requires at least three strike prices and their corresponding implied volatilities to estimate the quadratic curve. Whenever there are less than three available strike prices on a trading day, we are unable to construct the implied volatility curve and thus unable to calculate the model-free volatility expectation. The options with the nearest time-to-maturity are usually chosen. When there are less than three available strike prices for the nearest time-to-maturity, we switch to the second nearest time-to-maturity, which is usually in the month after the trading day. However, when it is impossible to estimate the implied volatility curve for the two nearest-to-maturity sets of option contracts, both the model-free and the ATM volatility estimates are treated as missing data for that trading day and instead we assume both values remain unchanged from the previous trading day.

A total of 149 firms pass both filters. The number of option observations during the sample period varies from firm to firm and could be viewed as a measure of liquidity. We anticipate that more option observations for a firm during the sample period will be associated with a higher efficiency of the firm's options when estimating the underlying asset's future variability. There are less option observations in 1996, compared with later periods. The maximum number of daily observations occurs in 1999 for most firms.

Figure 1 shows the distribution of the average number of daily available strike prices for the 149 selected firms. The average number for firm i, $\overline{N}_i$, equals the total number of available strike prices for firm i during the sample period divided by the number of trading days, which is 1009 for all firms; for those trading days when it is impossible to construct an implied volatility curve because of a lack of observations, the number of available strike prices is set to zero. The minimum, median and maximum values of $\overline{N}_i$ are 3.7, 5.1 and 12.9 respectively. More than a half of the averages $\overline{N}_i$ are between 4 and 6. As shown by Figure 1, the stock options in our sample have far less observations than the stock index options studied in previous literature.

4. Empirical methodology

4.1 Econometric specifications

Both ARCH and regression models have been estimated in many previous comparisons of the information content of different volatility estimates. ARCH models can be estimated from daily returns, while regression models are estimated for a data-frequency that is determined by the expiration dates of the option contracts. The primary advantages of ARCH models are the availability firstly of more observations and secondly of maximum likelihood estimates of the model parameters. A disadvantage of ARCH models, however, is that the data-frequency is usually very different to the forecasting horizon that is implicit in option prices, namely the remaining time until expiry. This fact may weaken the relative performance in an ARCH context of volatility estimates extracted from option prices. To learn as much as we can about volatility from the option prices, our study therefore uses both ARCH specifications for one-day returns and regressions that employ a forecast horizon equal to the options' time to maturity.

4.1.1 ARCH specifications

To compare the performance of historical daily returns, ATM implied volatilities and model-free volatility expectations, when estimating future volatility, three different ARCH specifications that use different daily information sets are estimated for daily stock returns r_t, from 4 January 1996 to 31 December 1999. The specifications include an MA(1) term in the conditional mean equation to capture any first order autocorrelation.

The general specification is similar to that of Blair et al (2001). It is as follows:

$$r_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1},$$
$$\varepsilon_t = h_t^{1/2} z_t, \quad z_t \sim i.i.d.(0,1), \tag{5}$$
$$h_t = \frac{\omega + \alpha\varepsilon_{t-1}^2 + \alpha^- s_{t-1}\varepsilon_{t-1}^2}{1-\beta L} + \frac{\gamma\sigma_{VE,t-1}^2}{1-\beta_\gamma L} + \frac{\delta\sigma_{ATM,t-1}^2}{1-\beta_\delta L}.$$

Here L is the lag operator, h_t is the conditional variance of the return in period t and s_{t-1} is 1 if $\varepsilon_{t-1} < 0$ and it is 0 otherwise. The terms $\sigma_{VE,t-1}$ and $\sigma_{ATM,t-1}$ are respectively the daily estimates of model-free volatility expectation and the ATM implied volatility, computed at time $t-1$ by dividing the annualized values by $\sqrt{252}$.

By placing restrictions on selected parameters in the conditional variance equation, three different volatility models based upon different information sets are obtained:

(1) The GJR(1,1)-MA(1) model, as developed by Glosten et al (1993): $\gamma = \beta_\gamma = \delta = \beta_\delta = 0$.

(2) The model that uses the information provided by model-free volatility expectations alone: $\alpha = \alpha^- = \beta = \delta = \beta_\delta = 0$.

(3) The model that uses information provided by ATM implied volatilities alone: $\alpha = \alpha^- = \beta = \gamma = \beta_\gamma = 0$.

The parameters are estimated by maximising the quasi-log-likelihood function, defined by assuming that the standardized returns z_t have a normal distribution. To ensure that the conditional variances of all models remain positive, constraints such as $\omega > 0$, $\alpha \ge 0$, $\alpha + \alpha^- \ge 0$, $\beta \ge 0$, $\beta_\gamma \ge 0$ and $\beta_\delta \ge 0$ are placed on the parameters. Inferences are made through $t-$ratios, constructed from the robust standard errors of

Bollerslev and Wooldridge (1992). The three special cases listed above are ranked by comparing their log-likelihood values; a higher value indicates that the information provides a better description of the conditional distributions of daily stock returns.

4.1.2 OLS regressions

Univariate and encompassing regressions are estimated for each firm, as in the index studies by Canina and Figlewski (1993), Christensen and Prabhala (1998) and Jiang and Tian (2005). While a univariate regression can assess the information content of one volatility estimation method, the encompassing regression addresses the relative importance of competing volatility estimates.

The most general regression equation is specified as follows:

$$\sigma_{RE,t,T} = \beta_0 + \beta_{HV}\sigma_{HV,t,T} + \beta_{VE}\sigma_{VE,t,T} + \beta_{ATM}\sigma_{ATM,t,T} + \varepsilon_{t,T}, \qquad (6)$$

where $\sigma_{RE,t,T}$ is some measure of the realized volatility from time t to time T, and $\sigma_{HV,t,T}$ is a historical volatility forecast calculated from the GJR(1,1)–MA(1) model using the information up to time t. The terms $\sigma_{VE,t,T}$ and $\sigma_{ATM,t,T}$ are non-overlapping measures of the model-free volatility expectation and the ATM implied volatility. Inferences are made using the robust standard errors of White (1980), that take account of heteroscedasticity in the residual terms $\varepsilon_{t,T}$.

4.2 Volatility calculations

Option measures

We use daily estimates of the ATM implied volatility and the model-free volatility expectation in the estimation of the ARCH models. The ATM implied volatility is the implied volatility for the available strike price that is closest to the forward price. The model-free volatility expectation is calculated as described in Section 2.3. Firstly, the implied volatility curve is constructed on each trading day for each firm’s stock. From each implied volatility curve, a large number of fitted implied volatilities are converted into Black-Scholes call and put option prices. Finally, the annualized value of the model-free volatility expectation is calculated from equation (2).

Figure 2 shows the two time series of option measures of volatility, for General Electric and IBM, during our sample period from January 1996 to December 1999. The dark line represents the model-free volatility expectation and the dotted line the ATM implied volatility. It is seen that these two volatility measures move closely with each other and that the ATM implied volatility tends to be slightly lower than the model-free volatility expectation.

For the OLS regressions, both the model-free volatility expectation and the ATM implied volatility on the trading date that follows the previous maturity date are selected, so that non-overlapping samples of volatility expectations are obtained. We are able to use sets of 49 monthly observations, with maturity days from January 1996 to January 2000, for 129 of the 149 firms. For each of the remaining 20 firms, the number of observations is 46, 47 or 48 because of the occasional illiquidity of option trading for some firms. To match the horizon of all the variables in the OLS regressions with the one-month horizon of the options information, realized volatility measures and historical volatility forecasts are required for the remaining lives of the options.

Realized volatility

Two measures of the realized volatility from a day t until the options' maturity date T are predicted. The first measure applies the well-known formula of Parkinson (1980) to daily high and low stock prices, to give:

$$\sigma^{(1)}_{RE,t,T} = \sqrt{\frac{252}{H} \sum_{i=1}^{H} \frac{[\log(high_i) - \log(low_i)]^2}{4\log(2)}} \tag{7}$$

where $high_i$ and low_i are, respectively, the highest and lowest stock price for day i, and H is the number of days until the options expire. The second measure is the annualized variance of the daily returns:

$$\sigma^{(2)}_{RE,t,T} = \sqrt{\frac{252}{H} \sum_{i=1}^{H} (r_i - \bar{r}_{t,T})^2} \tag{8}$$

where r_i is the stock return for day i and $\bar{r}_{t,T}$ is the average stock return from time t until T.

The first estimator is expected to be a more accurate measure of realized volatility than the second, because intraday prices theoretically contain more volatility information than daily prices.

Historical volatility

Historical forecasts of volatility are evaluated using the GJR(1,1)-MA(1) model. The historical information up to the observation day *t* provides the conditional variance h_{t+1} for the next day. The forecast of the aggregate variance until the expiry time *T*, whose square root represents the historical volatility forecast σ_{HV} in the regressions, is given by:

$$\sigma^2_{HV,t,T} = H\sigma^2 + \frac{1-\phi^H}{1-\phi}(h_{t+1} - \sigma^2) \quad (9)$$

where $\phi = \alpha + \frac{1}{2}\alpha^{-} + \beta$ and $\sigma^2 = \omega/(1-\phi)$ are respectively equal to the persistence parameter and the unconditional variance of the returns.

The parameters of the ARCH models used to define the historical forecasts are estimated by maximizing the log-likelihood of a set of *n* returns that do not go beyond time t. Ninety of the 149 firms have continuous price histories from January 1988 until January 2000. For these firms, we initially use $n = 2024$ returns for the trading days between 4 January 1988 and 4 January 1996, as our first forecasts are made on 4 January 1996; the subsequent forecasts use parameters estimated from the 2024 most recent returns. For each of the other firms, whose histories commence after January 1988, we use all the daily returns until the observation day *t* (although we stop adding to the historical sample if *n* reaches 2024).

4.3 Descriptive Statistics

Table 1 presents summary statistics for all the volatility estimates used in either the ARCH or regression models. Statistics are first obtained for each firm from time series of volatility estimates. Then the cross-sectional mean, median, lower quartile and upper quartile values of each statistic, across the 149 firms, are calculated and displayed in Table 1. Panel A provides summary statistics for daily estimates of the model-free volatility expectation, the ATM implied volatility, and the difference between them; while Panel B shows statistics for the non-overlapping volatility estimates that are used in the OLS regressions, including the realized volatility measured by high and low stock prices, the realized volatility measured by the standard deviation of returns, the model-free volatility expectation, the ATM implied volatility and the historical volatility forecasts.

From Panel A, on average the model-free volatility expectation is higher than the ATM implied volatility although occasionally the latter is higher than the former. This also occurs in the study by Jiang and Tian (2005) of S & P 500 index options. We have tested and rejected the null hypothesis that the ATM implied volatility is an unbiased estimate of the model-free volatility expectation, at the 1% significance level, for each of the 149 firms. We therefore conclude that the ATM implied volatility tends to be a downward biased measure of the risk-neutral expected variance.

Panel B shows the cross-sectional statistics of the time-series means of the ATM implied volatility are very close to those of the realized volatility measured by the standard deviation of returns. The estimates of the realized volatility measured by high and low prices are lower, as these estimates are biased when the intraday price process is not geometric Brownian motion.

Table 2 presents the cross-sectional mean and median values of the correlation matrix, which is formed using time-series of non-overlapping, monthly, volatility estimates for each firm. Comparing the correlations of each volatility estimate with realized volatility, the ATM implied volatility provides the highest correlation values and historical volatility the lowest, for both measurements of realized volatility. The correlations of the model-free volatility expectation with the two measures of realized volatility are slightly lower than, but very close to, those of ATM implied volatility. Realized volatility measured by high and low prices is more highly correlated with the three volatility estimates, than the realized volatility measured by the standard deviation of returns. The highest correlation statistics are between the model-free volatility expectation and the ATM implied volatility, with the mean and median respectively equal to 0.932 and 0.952. These high values reflect the similar information that is used to price ATM and OTM options.

5. Results

In this section, the results from fitting both the GARCH specifications and the OLS regressions, defined by equations (5) and (6), to the data from 149 firms during the period from 4 January 1996 to 31 December 1999 are discussed. The results for models estimated across all firms are first presented. We then consider results for groups of firms, with the groups defined in two different ways by using measures of the liquidity of options trading.

5.1 ARCH specifications

5.1.1 Estimates of parameters

Table 3 provides the summary statistics of the sets of 149 point estimates (their median, lower quartile L_q, and upper quartile U_q) from the three ARCH specifications defined by equation (5). Summary statistics are shown in Panel A for the GJR(1,1)-MA(1) model, in Panel B for the model that only uses the information provided by the model-free volatility expectation, and in Panel C for the model that uses the information provided by the ATM implied volatility alone. The last two rows in each panel are the percentages of the estimates that are significantly different from zero at the 5% and the 10% levels.

The first model is the standard GJR(1,1)-MA(1) model, which uses previous stock returns to calculate the conditional variance. The value of α measures the symmetric impact of new information (defined by ε_t) on volatility while the value of α^- measures the additional impact of negative information (when $\varepsilon_t < 0$) on volatility. Approximately 75% of all firms have a value of $\alpha + \alpha^-$ that is more than twice the estimate of α, indicating a substantial asymmetric effect for individual stocks. For the majority of firms, the estimates of α and α^- are not significantly different from zero at the 5% level. This is probably a consequence of the relatively short sample period. The volatility persistence parameter, assuming returns are symmetrically distributed, is $\alpha + 0.5\alpha^- + \beta$. The median estimate of persistence equals 0.94.

The second model uses only the information contained in the time series of model-free volatility expectations, $\sigma_{VE,t-1}$, to calculate conditional variances. The series

$\sigma_{VE,t-1}$ is filtered by the function $\gamma/(1-\beta_\gamma L)$ of the lag operator *L*. For half of the firms, the estimates of γ are between 0.48 and 0.85; also, half of the estimates are significantly different from zero at the 5% level. In contrast, most of the estimates of β_γ are near zero. This suggests that a conditional variance calculated from the model-free volatility expectation given by the latest option prices can not be improved much by using older option prices.

The third model uses only the information contained in the ATM implied volatility series, $\sigma_{ATM,t-1}$, to calculate the conditional variances. The interquartile range for δ is from 0.62 to 0.88 and almost one-half of the estimates are significantly different from zero at the 5% level. More than half of the estimates of the lag coefficient, β_δ, are zero and few of them are far from zero. On average, δ exceeds γ and β_δ is less than β_γ.

The total weight in the conditional variance equation given to the model-free volatility expectations and the ATM implied volatilities are respectively defined by the quantities $\gamma/(1-\beta_\gamma)$ and $\delta/(1-\beta_\delta)$. A higher value of these quantities may imply the information provided is more relevant to the conditional variance movements, or it may also indicate a lower level of the volatility estimates. The summary statistics for $\gamma/(1-\beta_\gamma)$ and $\delta/(1-\beta_\delta)$ are shown in the last two columns of Table 3. Figure 4 is a scatter diagram of these two variables for the 149 firms. It is seen that there is a strong positive correlation between these two variables. Most points are closer to the x-axis than to the y-axis because $\delta/(1-\beta_\delta)$ is usually higher than $\gamma/(1-\beta_\gamma)$.

5.1.2 Comparisons of log-likelihoods

A higher log-likelihood value indicates a more accurate description of the conditional distributions of daily stock returns. Panel A of Table 5 provides frequency counts that show how often each of the three ARCH specifications has the highest log-likelihood for the observed returns. The first column shows the percentage frequencies across all firms. We use L_{HV}, L_{VE} and L_{ATM} to represent the log-likelihoods of the three models defined after equation (5). The percentage numbers in the table show how many firms satisfy each of the six possible orderings of L_{HV}, L_{VE} and L_{ATM}.

More than a third of the firms (35.6% or 53 firms) have a log-likelihood, L_{HV}, for the GJR(1,1)–MA(1) model that is higher than both of the values, L_{VE} and L_{ATM}, obtained from the options information. For the 64.4% (96 firms) whose log-likelihoods are maximized using option specifications, the ATM specification (37%) is best more often than the model-free volatility expectation (27%). This is evidence for the superior efficiency of ATM option implied volatilities when estimating individual stock volatility.

This high frequency for the historical specification is contrary to the studies on options written on stock indices, which reach a consensus that option prices perform much better than ARCH models estimated from daily returns. However, our results are consistent with the in-sample conclusions of Lamoureux and Lastrapes (1993), who show that the GARCH(1,1) model has a slightly higher log-likelihood than the model that uses ATM implied volatilities, for all the 10 U.S. firms in their sample.

There are two obvious reasons why the GJR model performs the best for so many firms. Firstly, the key difference between our data for individual stock options and the stock index option data of previous studies is that the latter options are much more liquid than the former. As individual stock options are far less liquid, they may be unable to provide informative volatility expectations. Secondly, our ARCH specifications and the model in Lamoureux and Lastrapes (1993) are estimated with a horizon of one day, while volatility estimates from option prices represent the expected average daily variation until the end of the options' lives. The mismatch between the estimation horizon and the options' time to maturity may enhance the performance of the GJR (1,1)–MA(1) model relative to the other two models.

5.2 OLS regressions

The regression results are for non-overlapping observations, defined so that the estimation horizon is matched with the options' time to maturity. Table 4 reports the results from both univariate and encompassing regressions. The results from the two measures of realized volatility are presented in separate panels. As before, Med, L_q and U_q are the median, lower quartile, and upper quartile of the point estimates across 149 firms. The two numbers in parentheses for each parameter estimate are the numbers of firms whose estimates are significantly different from zero at the 5% and 10% levels. The last two sets of columns show summary statistics for the regression mean squared errors (MSE) and the Durbin-Watson statistics.

We begin our discussion with the results from the univariate regressions in Panel A, when realized volatility is measured by high and low stock prices. The null hypothesis

$\beta = 0$ is rejected for more than 80% of the tests at the 5% level. The values of the adjusted R^2 are highest for the ATM implied volatility (median 0.282), but the values for the model-free volatility expectation are similar (median 0.260); the values for historical volatility, however, are much lower (median 0.119). This evidence suggests that volatility estimates extracted from option prices are much more informative than historical daily stock returns when the estimation horizons match the lives of the options.

We next consider the encompassing regressions with two independent variables in Panel A. The bivariate regression models that include the historical volatility variable increase the median adjusted R^2 values slightly from the univariate levels for option specifications; from 0.282 to 0.284 for ATM implied volatility and from 0.260 to 0.273 for the model-free expectation. For these bivariate regressions, only a small number of firms reject the null hypothesis $\beta_{HV} = 0$ at the 5% level (37 for historical volatility and model-free volatility, and 33 for historical volatility and ATM implied volatility). Therefore most firms can not reject the hypothesis that the historical volatility of the underlying asset is redundant when forecasting future volatility, which may be a consequence of the informative option prices and the small number of forecasts that are evaluated.

The bivariate regressions involving the model-free volatility expectation and the ATM implied volatility have a median value of adjusted R^2 equal to 0.282, which is fractionally less than for the bivariate regressions involving the historical and the ATM volatilities. This can be explained by the very high correlation between the model-free volatility expectation and the ATM implied volatility. For most firms, both the null hypotheses $\beta_{VE} = 0$ and $\beta_{ATM} = 0$ can not be rejected, showing that we can not decide that one option measure subsumes all the information contained in the other.

The regression involving all three volatility estimates has a median adjusted R^2 equal to 0.302. The median values of β_{HV}, β_{VE} and β_{ATM} are respectively 0.09, 0.13 and 0.46 which suggests the ATM forecasts are the most informative.

Panel B in Table 4 presents the regression results when the realized volatility is measured by the standard deviation of daily returns. The comparisons between specifications provide the same conclusions as those deduced from Panel A, but now the regressions have lower values of adjusted R^2 and, therefore, less firms with significant coefficient estimates. This lower values of R^2 are expected from the information in the correlation matrix shown in Table 2, which shows that the realized volatility measured by high and low stock prices has higher correlations with the three volatility forecasts under consideration, than does the realized volatility measured by the standard deviation of daily returns.

Although the values of the Durbin-Watson statistic are often low, most of the test values are not significantly different from two so that the null hypothesis that the regression residuals are not correlated is accepted.

Regressions results have also been obtained when the volatility variables are replaced either by variances or by their logarithms. The results are similar to those presented in Table 4.

Panel B of Table 5 provides frequency counts that show how often each of the three univariate forecasts has the highest value of R^2, when realized volatility is measured by daily high and low stock prices. The first column of percentage frequencies shows how many firms satisfy each of the six possible orderings of adjusted R^2 values from the three univariate regressions, which are denoted by R^2_{HV}, R^2_{VE} and R^2_{ATM}.

There are important differences between the frequencies in Panels A and B of Table 5. Only 14% (i.e. 21) of the firms have historical volatility ranking highest in Panel B, compared with 36% in Panel A. Both the model-free volatility expectation and the ATM implied volatility rank highest more often in Panel B than in Panel A. The ATM implied volatility has the best regression results for 48% (i.e. 72) of the firms, while the model-free volatility expectation performs the best for 38% (i.e. 56 firms). Thus, only when the estimation horizon is matched do we find that the option prices are clearly more informative than the historical daily returns.

5.3 Comparisons for groups defined by available strike prices

We now allocate the firms to groups according to the average number of daily available strike prices, $\overline{N}_i$, defined in Section 4.2. We do this to try and discover if the liquidity of options trading can explain why some firms appear to have informative option prices (relative to historical forecasts) and others do not. A successful liquidity relationship would also help to explain why our results for stock options are different from those reported elsewhere for stock index options.

When a firm has more available strike prices, we may conjecture that the firm's option prices are more efficient when estimating volatility. Therefore, for a higher value of $\overline{N}_i$, the ATM implied volatility and the model-free volatility expectation should tend to perform better than historical volatility, especially for the model-free expectation which is estimated across all available strike prices.

The 149 firms have been divided into three groups. Group 1 $(n = 63)$ contains the firms that have $\overline{N}_i$ between 3 and 4 and they are considered to be the firms with the least liquidly traded options in our sample. The firms in Group 2 $(n = 50)$ are those that have $\overline{N}_i$ between 4 and 5. Group 3 $(n = 36)$, which is the most liquid group, contains the firms with $\overline{N}_i$ higher than 5.

The second, third and fourth columns of Table 5 provide frequency counts about the best source of information for the three groups, that can be compared with the counts in the first column for all the firms.

From Panel A of Table 5, that compares ARCH likelihoods, we conclude that the historical volatility is outperformed by options when the stock options are most actively traded. The percentage of firms for which the GJR(1,1)–MA(1) model performs the best decreases from 51% for Group 1, to 40% for Group 2 and then to only 3% for Group 3. The percentages are similar, across groups, for the model-free volatility expectation. They increase for the ATM implied volatility from 24% for Group 1, to 32% in Group 2 and then to 67% in Group 3. The null hypothesis that there is no relationship between our groups and the highest log-likelihood values is rejected by the 3×3 contingency table (chi-squared) test at the 0.5% level, with the chi-square test statistic equal to 27.38.

Panel B of Table 5, that compares adjusted R^2 values for the regresions, displays similar patterns to Panel A, but the changes in frequencies between groups are now less pronounced. Again we see that the historical forecasts are clearly inferior to the ATM implied volatility forecasts for the firms that have the most recorded option prices. Although the general pattern is the same, the chi-squared test statistic, equal to 4.39, does not reject the null hypothesis of no association between the group and the best method at the 5% level. The different chi-squared test conclusions may simply reflect the small sample sizes used in the regressions (ranging from 46 to 49) that make it difficult to estimate the true level of R^2 for each forecasting method.

The median values of the ARCH parameter estimates for the three specific ARCH models, across the firms in each group, are provided in Table 6. The numbers in parentheses are the percentages of the estimates that are significantly different from zero at the 10% level. Panel A shows the median values for the parameters in the GJR(1,1)-MA(1) model. Panel B shows the median values for the ARCH specification that uses only the information provided by the model-free volatility expectation. As the options of the firms become more liquid, moving from Group 1 to

Group 3, the estimates of γ and the percentages of significant estimates increase monotonically, from 0.57 to 0.81 and from 47% to 89%, coupled with a decrease in the median estimate of β_γ from 0.17 to 0.03. Meanwhile, the median value of the weight put on the σ_{VE} series, $\gamma/(1-\beta_\gamma)$, rises from 0.77 for Group 1 to 0.86 for Group 3, reflecting an overall increase in information content. The same pattern is seen in Panel C, which provides the median values for the ARCH specification that only uses the information provided by the ATM implied volatility.

5.4 Comparisons for groups defined by comparing ATM and OTM liquidity

From Table 5, it is seen that the ATM implied volatility outperforms the model-free volatility expectation for a majority of the firms; this is true even for the firms that have the more actively traded options. Firms are again allocated to groups, but now using the time-series average of the trading volume of ATM options as a fraction of the total volume across all strikes. We might conjecture that when trading is concentrated in the firm’s ATM options then the information provided by OTM options will become redundant; consequently, the model-free volatility expectation will then be unable to provide efficient forecasts of the future variability.

The 149 firms are separated into three groups according to their average level of ATM trading activity relative to the total trading activity. Group 1 contains the firms that have an average ratio from 24% to 40%. The firms in Group 2 are those that have average ratios from 40% to 45% and for Group 3 the range is higher than 45%. We might expect more firms in Group 3 to have ATM implied volatility predictions that outperform the model-free volatility expectation than in Groups 1 and 2.

The middle group of three columns in Table 5 present the frequency counts, defined in the same way as for all the other columns in this table. In Panel A, the ATM predictions outperform the model-free expectations for 60% of the firms in Group 1, 56% in Group 2 and 59% in Group 3. The corresponding frequencies in Panel B are 50%, 67% and 54%. There is therefore no evidence to support the conjecture that the ATM predictions are more likely to be best when OTM trading volume is relatively low. Furthermore, for Group 1 when the OTM volume is relatively high, the model-free volatility expectation remains less efficient than the ATM predictions for a majority of the firms.

The chi-square test statistics are, 1.51 and 3.64, respectively for the orderings defined by ARCH specifications, and for those defined by the R^2 values. The null hypothesis that there is no association between the second grouping method and the best of the three forecasting variables can not be rejected for either Panel A or Panel B.

5.5 Comparisons for groups defined by relative trading activity of intermediate delta options

We also group all firms according to the time-series average of trading volume of intermediate delta options as a fraction of the total volume across all strike prices. The options with delta values between 0.25 and 0.75 are defined to be intermediate delta options. When the relative trading activity of these options is high, there is relatively less trading in very high and very low strike prices. Therefore, the model-free volatility expectation, which includes all strike prices, might then lose its advantage and thus the ATM implied volatility concentrated on only one central strike price might tend to perform better.

The 149 firms are again separated into three groups according to the average level of intermediate delta trading activity relative to the total trading activity. Group 1 (n=49) contains the firms where average ratio is lower than 62%. Group 2 (n=44) contains those that have the ratio between 62% and 65%. Group 3 (n=56) contains the firms that have an average ratio higher than 65%. We might expect more firms in Group 3 to have ATM implied volatility predictions that outperform the model-free volatility expectation than in Groups 1 and 2.

The last three columns in Table 5 present the frequency counts when the firms are grouped by near-the-money trading activity. In Panel A, that compares ARCH likelihoods, the ATM predictions outperform the model-free volatility expectation for 63% of the firms in Group 1, 60% in Group 2, and 54% in Group 3. The corresponding frequencies in Panel B, when comparing the adjusted R^2, are 65%, 59%, and 50%. There is no evidence showing that the ATM implied volatility is more often superior to the model-free volatility expectation when relative intermediate delta trading activity is high.

The chi-square test statistic is 9.00 for Panel A, and 3.03 for Panel B. Thus, we cannot reject the null hypothesis that there is no association between the third grouping method and the best method, at the 5% level.

5.6 Cross-sectional regression analysis

When we try to discover when one prediction method is better than the others, the allocation of firms to different groups loses some information. We now use the same three variables, average number of available strike prices, relative liquidity of ATM options, and relative liquidity of intermediate delta options, to try and explain the numerical differences between the likelihood values from ARCH models and the numerical differences between R^2 values from univariate OLS regressions.

The general cross-sectional regression equation is specified as follows:

$$y_i = \alpha + \beta_1 \times \overline{N}_i + \beta_2 \times TV_{ATM,i} + \beta_3 \times TV_{DELTA,i} + \varepsilon_i \qquad (10)$$

where y_i, for firm i, represents the difference between any two loglikelihood values from ARCH models or any two adjusted R^2 values from univariate regressions. The variables $\overline{N}_i$, $TV_{ATM,i}$ and $TV_{DELTA,i}$ are respectively the average number of available strike prices, the relative trading volume of ATM options and the relative trading volume of intermediate delta options for firm i, defined in Sections 5.3, 5.4 and 5.5. Regressions involving one, two or three explanatory variables are estimated for all 149 firms.

Tables 7, 8 and 9 show the regression results. Table 7 reports the results when explaining the difference in the performance of the ATM implied volatility and that of the model-free volatility expectation. In Panel A, y_i equals the difference between the loglikelihood value of the ARCH model using ATM implied volatility alone and that of the ARCH model using model-free volatility expectation alone. In Panel B, y_i equals adjusted R^2 from the univariate regression using ATM implied volatility minus the adjusted R^2 from the univariate regression using model-free volatility expectation. Similarly, Table 8 shows the regression results that explain the differences between the performance of ATM implied volatility and that of historical volatility; Table 9 reports the results for differences between model-free volatility expectation and historical volatility. In all three tables, Panel A shows the results when the dependent variable is obtained from the loglikelihood values of ARCH models, while Panel B shows the results when the dependent variable is obtained from the adjusted R^2 values of univariate regressions.

We begin our discussion with the results in Table 7. As we expect the ATM implied volatility to perform better, compared with the model-free volatility expectation, when there is relatively more trading in ATM and intermediate delta options, here the coefficients of $TV_{ATM,i}$ and $TV_{DELTA,i}$ are expected to be positive.

In Panel A of Table 7, we find that the adjusted R^2 of the three univariate regressions are low and two are negative. The most significant variable in the univariate regressions is the average number of available strike prices, with a coefficient of 0.599 and a t-value of 1.80. We can conclude that none of the three variables can explain the difference in the performance between ATM implied volatility and model-free volatility expectation, on its own.

A high relative trading volume for ATM options might occur when there is illiquid trading for all options. Under this circumstance, both the ATM implied volatility and the model-free volatility expectation become inefficient and thus we cannot explain the difference between them using the relative trading volume of ATM options alone.

However, after combining it with the average number of available strike prices in the fourth regression in Panel A, both of the explanatory variables become more significant and the adjusted R^2 value of 0.026 is higher than those of the univariate regressions. We might conclude that when the firm's option trading is liquid overall but most trading is of the ATM options, then the ATM implied volatility tends to perform better than the model-free volatility expectation.

The highest adjusted R^2 value is 0.034, from the encompassing regression including all three explanatory variables. We cannot find any evidence supporting the hypothesis that the model-free volatility expectation is relatively inefficient when the relative trading volume of the intermediate delta options is high. On the contrary, the coefficient of the relative trading volume of the intermediate delta options is negative, with a t-value equal to –1.50.

None of the coefficients are significant in Panel B of Table 7. The low number of observations in our OLS regressions (46 to 49), defined in section 4.1.2, might explain the low explanatory power of these estimates.

We next consider the results shown in Table 8, which measure the difference in the performance between ATM implied volatility and historical volatility. When any one of $\overline{N}_i$, $TV_{ATM,i}$ or $TV_{DELTA,i}$ is higher, we conjecture that the predictions from option prices become more efficient than historical forecasts. Therefore we expect the coefficients of the explanatory variables to be positive.

In both panels of Table 8, we find that all the coefficients of $\overline{N}_i$ are positive and significantly different from zero at the 5% level. It is seen that the average number of available strike price is the most important variable to explain the difference between the performances of ATM implied volatility and historical volatility. Especially in Panel B, there is no additional explanatory power when combining $\overline{N}_i$ with the other two variables. The positive coefficients imply that for firms with more recorded strike prices, the ATM implied volatility is more accurate in estimating future volatility, compared to historical volatility.

Similar to results in Table 7, we find that the coefficient of relative trading volume of ATM options becomes more significant when it is combined with the number of available strike prices. The highest adjusted R^2 values in Panel A and Panel B are both from the bivariate regressions that use the relative trading volume of ATM options and the number of available strike prices. This shows again that when the firm's option trading is liquid overall, but the trading is more concentrated on ATM options than the others, then ATM implied volatility tends to be the most informative estimate of future volatility.

The coefficient of $TV_{DELTA,i}$ is positive and significant for two of the regressions in Panel A. When the relative trading of intermediate delta options is high, volatility estimates from options tend to be better than historical estimates. However, the relative trading activity of intermediate delta options loses its significance when combined with the average number of strike prices.

Finally, we consider the results in Table 9, which compare the performance of model-free volatility expectation and historical volatility. The significant results in Table 9 are similar to those in Table 8, except that the coefficient of relative trading volume of ATM options, which measures the liquidity of ATM options, is always insignificant in Table 9. When there are more available strike prices, or when trading is more concentrated on intermediate delta options, the model-free volatility expectation tends to perform better than historical volatility. The significant coefficients in Panel A of Table 9 are slightly lower than those in Panel A of Table 8. This implies that when the values of $\overline{N}_i$ and $TV_{DELTA,i}$ increase, the enhancement in the performance of ATM implied volatility is slightly more than that of model-free volatility expectation.

6. Conclusions

There is a consensus from previous studies about the informational efficiency of options written on stock indices that option prices are more informative than daily stock returns when estimating and predicting the volatility of indices. Our analysis of 149 firms shows, however, that a different estimation conclusion applies to options for individual firms. For one-day-ahead estimation, more than a third of our firms do not have volatility estimates, extracted from option prices, that are more accurate than those provided by a simple ARCH model estimated from daily stock returns. When the prediction horizon extends until the expiry date of the options, the historical volatility becomes much less informative than either the ATM implied volatility or the newly developed model-free volatility expectation. Our results also show that both volatility estimates from options are more likely to be more informative than historical returns when the number of different strike prices traded is higher.

Although the model-free volatility expectation has been demonstrated to be the most accurate predictor of realized volatility by Jiang and Tian (2005) for the S & P 500 index, for our firms it only outperforms both the ATM implied volatility and the historical volatility for about one-third of the firms. In contrast, the ATM implied volatility is the method that most often performs the best. The relatively poor performance of the model-free volatility expectation might be explained by the relative levels of ATM and OTM trading activity, when the firm's option trading overall is liquid. Another possible explanation is that the OTM options are mispriced and, therefore, the model-free volatility expectation, which is a combination of option prices across all strike prices, is outperformed by the information provided by ATM options alone. A third explanation may be that option prices for all strikes can not be inferred reliably from a handful of traded strikes.

References

Blair, B.J., Poon, S.-H., Taylor, S.J., 2001, Forecasting S&P 100 volatility: The incremental information content of implied volatilities and high frequency index returns, *Journal of Econometrics*, 105, 5-16.

Bliss, R. and Panigirtzoglou, N., 2002, Testing the stability of implied probability density functions, *Journal of Banking and Finance*, 26, 381-422.

Bliss, R. and Panigirtzoglou, N., 2004, Option-implied risk aversion estimates, *The Journal of Finance,* 6, 406-446.

Bollerslev, T., and Wooldridge, J.M., 1992, Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances, *Econometric Reviews*, 11, 143-179.

Breeden, D.T., and Litzenberger, R.H., 1978, Prices of state-contingent claims implicit in option prices, *Journal of Business*, 51, 621-651.

Britten-Jones, M., and Neuberger, A., 2000, Option prices, implied price processes, and stochastic volatility, *Journal of Finance*, 55, 839-866.

Canina, L., and Figlewski, S., 1993, The information content of implied volatility, *Review of Financial Studies*, 6, 659-681.

Carr, P., and Madan, D., 1998, Towards a theory of volatility trading, in: R.A. Jarrow (ed.), *Volatility* (Risk Books, London), 417-427.

Carr, P., and Wu, L., 2004, Variance risk premia, (May 2004), AFA 2005 Philadelphia Meeting Paper.

Christensen, B.J., and Prabhala, N.R., 1998, The relation between implied and realized volatility, *Journal of Financial Economics*, 59, 125-150.

Day, T.E., and Lewis, C.M., 1992, Stock market volatility and the informational content of stock index option, *Journal of Econometrics*, 52, 267-287.

Dermeterfi, K., Derman, E., Kamal, M., and Zou, J., 1999, A guide to volatility and variance swaps, *Journal of Derivatives*, 6, 9-32.

Ederington, L., and Guan, W., 2002a, Is implied volatility an informationally efficient and effective predictor of future volatility?, *Journal of Risk*, 4, 1-18.

Ederington, L., and Guan, W., 2002b, Measuring implied volatility: Is an average better? Which average?, *Journal of Futures Markets*, 9, 811-837.

Fleming, J., 1998, The quality of market volatility forecast implied by S&P100 index option price, *Journal of Empirical Finance* 5, 317-345.

Fleming, J., Ostdiek, B., and Whaley, R.E., 1995, Predicting stock market volatility: A new measure, *Journal of Futures Markets*, 15, 265-302.

Jiang, G., and Tian, Y., 2005, The model-free implied volatility and its information content, *Review of Financial Studies*, 18, 1305-1342.

Jorion, P., 1995, Predicting volatility in the foreign exchange market, *Journal of Finance*, 50, 507-528.

Lamoureux, C.G., and Lastrapes, W.D., 1993, Forecasting stock-return variance: Toward an understanding of stochastic implied volatilities, *Review of Financial Studies*, 6, 293-326.

Lynch, D., and Panigertzoglou, N., 2005, Option implied and realised measures of variance, (March 15, 2004), EFA 2004 Maastricht Meeting Paper.

Malz, A.M., 1997, Option-based estimates of the probability distribution of exchange rates and currency excess returns, Federal Reserve Bank of New York.

Parkingson, M., 1980, The extreme value method for estimating the variance of the rate of return, *Journal of Business*, 53, 61-65.

Pong, S., Shackleton, M.B., Taylor, S.J., and Xu, X., 2004, Forecasting currency volatility: A comparison of implied volatilities and AR(FI)MA models, *Journal of Banking and Finance,* 28, 2541-2563.

Taylor, S.J., 2005, Asset Price Dynamics, Volatility, and Prediction, Princeton University Press.

White, H, 1980, A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity, *Econometrica,* 4, 817-838.

Xu, X., and Taylor, S.J., 1995, Conditional volatility and the informational efficiency of the PHLX currency options markets, *Journal of Banking and Finance*, 19, 803-821.

Figure 1 Distribution of the average number of daily available strike prices for 149 firms

Sample period is from January 4, 1996 to December 31, 1999. The average number of daily available strike prices for firm i, $\overline{N}_i$, is defined as the total number of option observations for firm i during the sample period divided by the number of trading days, which is 1009 for all firms.

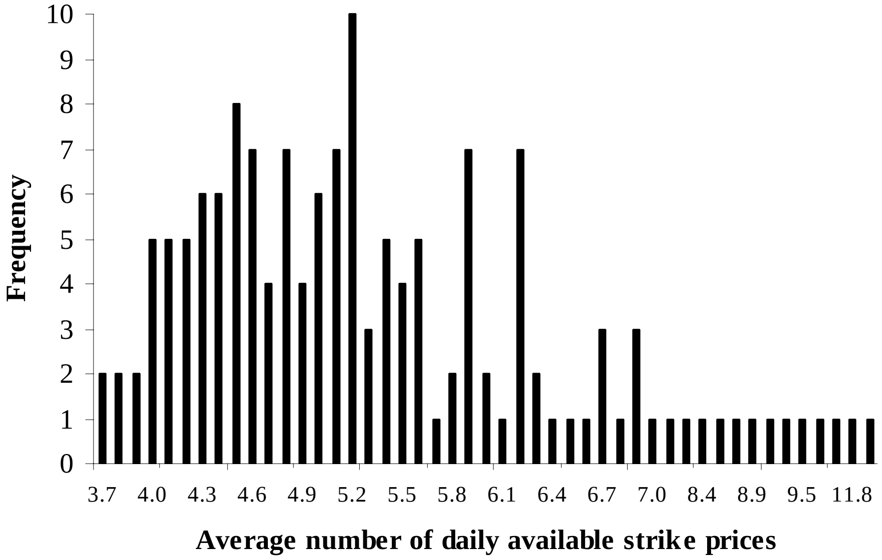
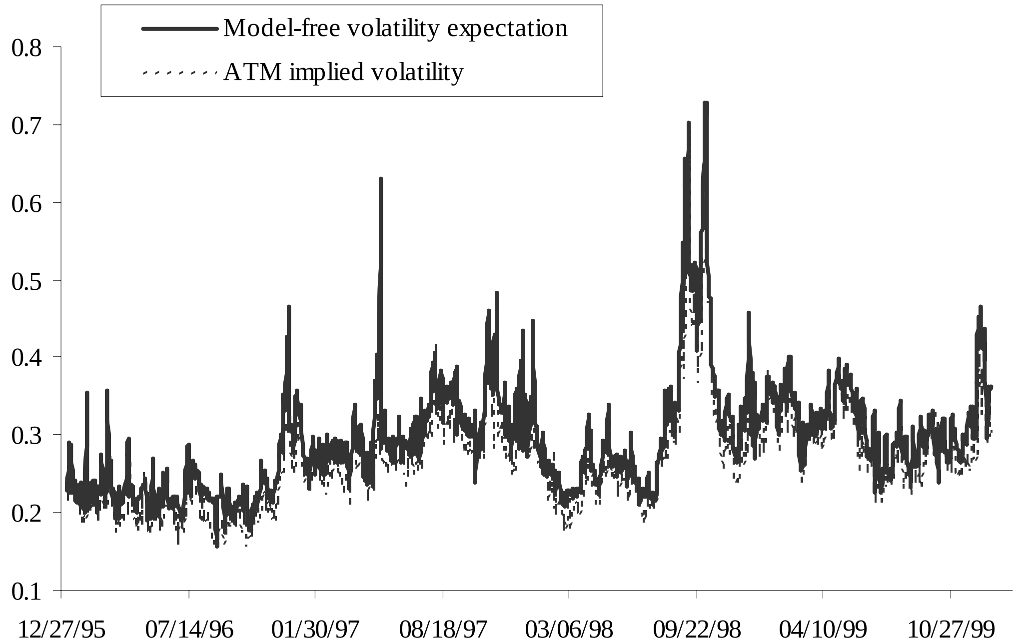


Figure 2 Examples of the model-free volatility expectation and the ATM implied volatility

The figure plots the time series of daily estimates of model-free volatility expectation and ATM implied volatilities over the sample period from January 1996 to December 1999.

Panel A: Time series plot for General Electronic



Panel B: Time series plot for IBM

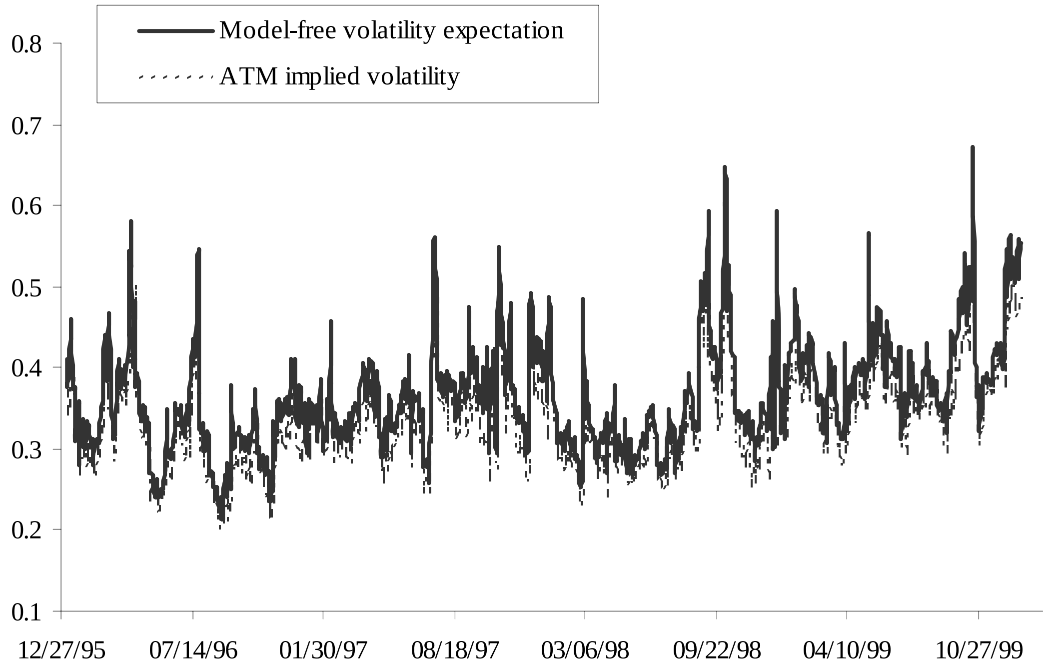


Figure 3 Comparison of the estimated values of $\frac{\gamma}{1-\beta_\gamma}$ and $\frac{\delta}{1-\beta_\delta}$ for 149 firms

For each firm, $\frac{\gamma}{1-\beta_\gamma}$ and $\frac{\delta}{1-\beta_\delta}$ are, respectively, obtained from the estimates of the model using information provided by the model-free volatility expectation only and the estimates of the model using information provided by the ATM implied volatility only. The straight line is the 45-degree line.

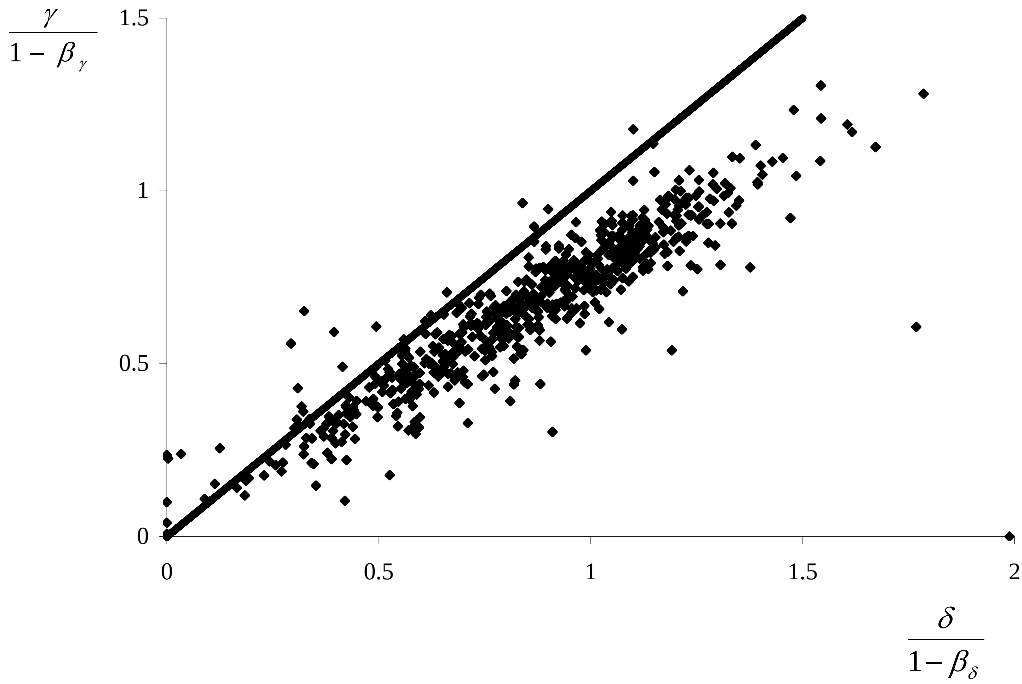


Table 1 Summary statistics of volatility estimates

Sample period is from January 1996 to December 1999. Numbers are cross-sectional statistics calculated from time series statistics, for a cross-section of 149 firms. Mean, Med, L_q and U_q are respectively the mean, median, lower and upper quartile values of each estimate across 149 firms. Panel A provides the summary statistics of the daily estimates used in ARCH models and Panel B the non-overlapping volatility estimates used in OLS regressions. σ_{VE} represents the model-free volatility expectation, σ_{ATM} the ATM implied volatility, $\sigma_{RE}^{(1)}$ the realized volatility measured by daily high and low prices, $\sigma_{RE}^{(2)}$ the realized volatility measured by daily returns, and σ_{HV} the historical volatility forecasts. All volatility numbers are annualized.

Time series stat.	Mean				Std Dev				Max				Min			
Cross-sec. stat.	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q
Panel A: summary statistics for daily measures of model-free volatility expectation and ATM implied volatility																
σ_{VE}	0.523	0.522	0.371	0.646	0.124	0.106	0.078	0.131	1.539	1.176	0.826	1.527	0.316	0.311	0.222	0.404
σ_{ATM}	0.487	0.484	0.351	0.610	0.099	0.094	0.072	0.114	1.093	1.023	0.755	1.278	0.296	0.285	0.205	0.381
$\sigma_{VE} - \sigma_{ATM}$	0.036	0.032	0.024	0.043	0.051	0.035	0.026	0.048	0.634	0.301	0.218	0.415	-0.105	-0.067	-0.132	-0.042
Panel B: summary statistics for non-overlapping volatility estimates																
$\sigma_{RE}^{(1)}$	0.370	0.371	0.254	0.474	0.105	0.098	0.077	0.130	0.713	0.703	0.504	0.894	0.210	0.195	0.140	0.275
$\sigma_{RE}^{(2)}$	0.484	0.490	0.341	0.607	0.173	0.162	0.122	0.210	1.096	1.021	0.722	1.314	0.230	0.222	0.156	0.293
σ_{VE}	0.517	0.518	0.367	0.639	0.112	0.092	0.072	0.116	0.909	0.807	0.595	0.987	0.361	0.364	0.245	0.454
σ_{ATM}	0.484	0.489	0.351	0.605	0.093	0.087	0.063	0.107	0.771	0.722	0.558	0.901	0.340	0.335	0.233	0.428
σ_{HV}	0.493	0.507	0.323	0.600	0.109	0.068	0.046	0.101	0.924	0.708	0.510	0.925	0.353	0.342	0.229	0.473

Table 2 Summary statistics of the correlation matrices

Sample period is from January 1996 to December 1999. Mean and Med are respectively the cross-sectional mean and median values of correlations across 149 firms. Correlations for each firm are calculated using the non-overlapping observations of volatility estimates. $\sigma_{RE}^{(1)}$ is realized volatility measured by high and low prices. $\sigma_{RE}^{(2)}$ is realized volatility measured by daily returns. σ_{VE} is model-free volatility expectation. σ_{ATM} is ATM implied volatility. σ_{HV} is historical volatility .

	$\sigma_{RE}^{(1)}$		$\sigma_{RE}^{(2)}$		σ_{VE}		σ_{ATM}		σ_{HV}	
Cross sectional stat.	Mean	Med	Mean	Med	Mean	Med	Mean	Med	Mean	Med
$\sigma_{RE}^{(1)}$	1	1								
$\sigma_{RE}^{(2)}$	0.852	0.869	1	1						
σ_{VE}	0.510	0.522	0.445	0.433	1	1				
σ_{ATM}	0.521	0.528	0.454	0.465	0.937	0.952	1	1		
σ_{HV}	0.368	0.370	0.314	0.330	0.542	0.572	0.558	0.578	1	1

Table 3 Summary statistics of ARCH parameter estimates across 149 firms

Daily stock returns r_t are modelled by the ARCH specification: $r_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t$, $\varepsilon_t = h_t^{1/2} z_t$, $z_t \sim i.i.d.(0,1)$, $h_t = \dfrac{\omega + \alpha\varepsilon_{t-1}^2 + \alpha^- s_{t-1}\varepsilon_{t-1}^2}{1-\beta L} + \dfrac{\gamma\sigma_{VE,t-1}^2}{1-\beta_\gamma L} + \dfrac{\delta\sigma_{ATM,t-1}^2}{1-\beta_\delta L}$,

s_t is 1 if ε_t is negative, otherwise s_t is zero. σ_{VE} is a measure of model-free volatility expectation. σ_{ATM} is a measure of ATM implied volatility. Parameters are estimated by maximizing the quasi-log-likelihood function. Panel A contains the estimation results for the GJR (1,1)-MA (1) model; Panel B and Panel C for models that uses information provided by model-free volatility expectation and ATM implied volatility respectively. Inferences are made through t–ratios, constructed from robust standard errors. Numbers in the parentheses are the percentage of estimates which are significantly different from zero at the 5% and the 10% significance level. The persistence estimate is $\alpha + 0.5\alpha^- + \beta$. Med, L_q and U_q are respectively the median, lower and upper quartile values of each estimate across 149 firms.

Parameters	$\mu \times 10^3$	θ	$\omega \times 10^5$	α	α^-	β	γ	β_γ	δ	β_δ	Persistence	$\frac{\gamma}{1-\beta_\gamma}$	$\frac{\delta}{1-\beta_\delta}$
Panel A: estimates of GJR (1,1)-MA (1) model													
Med	0.85	0.00	5.91	0.03	0.08	0.86					0.94		
L_q	0.40	-0.04	1.57	0.00	0.04	0.66					0.81		
U_q	1.42	0.04	17.89	0.06	0.13	0.93					0.98		
At 5%	(11.4%)	(10.7%)	(63.8%)	(20.1%)	(43.0%)	(93.3%)							
At 10%	(22.8%)	(14.8%)	(74.5%)	(32.2%)	(54.4%)	(93.3%)							
Panel B: estimates of ARCH specification that uses model-free volatility expectation only													
Med	0.73	0.00	0.49				0.71	0.03				0.83	
L_q	0.33	-0.03	0.00				0.48	0.00				0.72	
U_q	1.22	0.05	10.15				0.85	0.34				0.90	
At 5%	(11.4%)	(10.7%)	(0.7%)				(50.3%)	(7.4%)					
At 10%	(22.8%)	(16.1%)	(2.0%)				(61.1%)	(8.7%)					
Panel C: estimates of ARCH specification that uses ATM implied volatility only													
Med	0.71	0.01	0.00						0.88	0.00			0.96
L_q	0.35	-0.03	0.00						0.62	0.00			0.84
U_q	1.23	0.05	7.27						1.01	0.20			1.04
At 5%	(10.7%)	(9.4%)	(0.0%)						(42.3%)	(4.0%)			
At 10%	(20.1%)	(16.8%)	(0.7%)						(57.0%)	(6.0%)			

Table 4 Summary statistics of estimates for univariate and encompassing regressions across 149 firms

Panel A contains the results of OLS regressions when realized volatility is measured using high and low stock prices; Panel B for results when realized volatility is measured by the standard deviation of returns. Inferences are made through standard errors, computed following a robust procedure taking account of heteroscedasticity [White (1980)]. Numbers in parentheses are the number of firms whose estimates are significantly different from zero at the 5% and the 10% levels. Med, L_q and U_q are respectively the median, lower and upper quartile values of each estimate across 149 firms. MSE is the mean squared regression error.

α			β_{HV}			β_{VE}			β_{ATM}			Ad. R square			MSE			Durbin-Watson		
Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q
Panel A: Realized volatility is calculated using high and low stock prices																				
0.096	0.019	0.224	0.535	0.284	0.737							0.119	0.031	0.261	0.008	0.004	0.015	1.50	1.28	1.68
	(58/66)			*(102/114)*																
0.071	0.024	0.152				0.559	0.415	0.678				0.260	0.126	0.392	0.007	0.003	0.013	1.59	1.43	1.81
	(39/51)						*(131/139)*													
0.066	0.005	0.143							0.612	0.473	0.767	0.282	0.141	0.401	0.007	0.003	0.013	1.61	1.42	1.81
	(35/47)									*(137/141)*										
0.043	-0.011	0.125	0.134	-0.048	0.361	0.502	0.304	0.624				0.273	0.150	0.422	0.007	0.003	0.013	1.73	1.54	1.90
	(19/32)			*(37/47)*			*(100/115)*													
0.028	-0.032	0.122	0.117	-0.06	0.347				0.544	0.387	0.735	0.284	0.166	0.415	0.007	0.003	0.013	1.71	1.53	1.90
	(19/31)			*(33/43)*						*(108/116)*										
0.066	0.003	0.147				0.143	-0.112	0.523	0.464	0.000	0.816	0.282	0.154	0.410	0.007	0.003	0.013	1.63	1.45	1.86
	(36/50)						*(24/34)*			*(34/48)*										
0.036	-0.024	0.127	0.093	-0.086	0.329	0.132	-0.183	0.487	0.457	-0.009	0.727	0.302	0.164	0.432	0.007	0.003	0.013	1.75	1.53	1.92
	(15/31)			*(31/45)*			*(22/29)*			*(27/40)*										
Panel B: realized volatility is calculated using returns																				
0.126	0.010	0.330	0.670	0.344	1.000							0.090	0.008	0.190	0.023	0.012	0.042	1.79	1.60	1.96
	(54/69)			*(89/104)*																
0.072	0.004	0.186				0.764	0.581	0.934				0.172	0.082	0.309	0.022	0.010	0.035	1.78	1.60	2.06
	(19/34))						*(114/129)*													
0.052	-0.019	0.151							0.882	0.667	1.059	0.202	0.107	0.287	0.021	0.010	0.036	1.78	1.62	2.03
	(16/25)									*(123/132)*										
0.045	-0.046	0.161	0.187	-0.116	0.493	0.672	0.356	0.881				0.192	0.103	0.318	0.021	0.010	0.035	1.90	1.71	2.09
	(19/27)			*(31/42)*			*(89/103)*													
0.034	-0.066	0.141	0.113	-0.147	0.409				0.744	0.538	1.052	0.210	0.109	0.294	0.021	0.010	0.036	1.88	1.71	2.06
	(17/26)			*(24/33)*						*(94/109)*										
0.056	-0.018	0.155				0.201	-0.249	0.756	0.563	-0.021	1.152	0.199	0.106	0.326	0.021	0.009	0.035	1.82	1.63	2.05
	(15/28)						*(16/27)*			*(25/34)*										
0.044	-0.052	0.152	0.095	-0.149	0.400	0.190	-0.275	0.737	0.563	-0.053	1.078	0.213	0.115	0.324	0.021	0.009	0.035	1.90	1.71	2.08
	(17/27)			*(28/33)*			*(14/24)*			*(20/27)*										

Table 5 Frequency counts for the variables that best describe the volatility of stock returns

L_{HV}, L_{VE} and L_{ATM} represent the log-likelihood values of the ARCH models that only use, respectively, historical volatility, the model-free volatility expectation and ATM implied volatility. R^2_{HV}, R^2_{VE} and R^2_{ATM} represent, respectively, the Adjusted R-squared of the three univariate regressions when realized volatility is measured by high and low prices. The figures are the percentages of firms that satisfy the orderings in the first column. Groups are firstly separated according to average values of traded strike prices for each firm. Group 1 has the lowest average values for the number of traded strike prices: $3 \le \overline{N}_i < 4$, group 2 has $4 \le \overline{N}_i < 5$, and group 3 has $\overline{N}_i \ge 5$. Groups are secondly separated according to the relative trading activity of at-the-money options for each firm. Groups 1, 2 and 3 are respectively the firms whose at-the-money trading volume as a fraction of total trading volumes are below 40%, between 40% and 45% and higher than 45%. Groups are thirdly separated according to the relative trading activity of intermediate delta options for each firm, Groups 1, 2 and 3 are respectively the firms whose intermediate delta trading volume as a fraction of the total trading volume are lower than 62%, between 62% and 65%, and above or equal to 65%, where a intermediate delta is defined as a delta value between 0.25 and 0.75. n represents the number of firms in each group.

	All firms	Grouped by $\overline{N}_i$			Grouped by ATM trading activity			Grouped by Intermediate delta trading activity		
		Group 1	*Group 2*	*Group 3*	*Group 1*	*Group 2*	*Group 3*	*Group 1*	*Group 2*	*Group 3*
	(n=149)	(n=63)	(n=50)	(n=36)	(n=48)	(n=55)	(n=46)	(n=49)	(n=44)	(n=56)
Panel A: Frequency counts for the ARCH specifications that maximize the likelihoods of observed stock returns										
GJR (1,1) – MA (1) model performs the best	**35.6%**	**50.8%**	**40.0%**	**2.8%**	**37.5%**	**30.9%**	**39.1%**	**46.9%**	**20.5%**	**37.5%**
$L_{HV} > L_{VE} > L_{ATM}$	14.1%	19.1%	18.0%	0.0%	16.7%	12.7%	13.0%	18.4%	6.8%	16.1%
$L_{HV} > L_{ATM} > L_{HV}$	21.5%	31.8%	22.0%	2.8%	20.8%	18.2%	26.1%	28.6%	13.6%	21.4%
σ_{VE} **performs the best**	**27.5%**	**25.4%**	**28.0%**	**30.6%**	**22.9%**	**30.9%**	**28.3%**	**18.4%**	**36.4%**	**28.6%**
$L_{VE} > L_{ATM} > L_{HV}$	25.5%	22.2%	26.0%	30.6%	20.8%	29.1%	26.1%	14.3%	34.1%	28.6%
$L_{VE} > L_{HV} > L_{ATM}$	2.0%	3.2%	2.0%	0.0%	2.1%	1.8%	2.2%	4.1%	2.3%	0.0%
σ_{ATM} **performs the best**	**36.9%**	**23.8%**	**32.0%**	**66.7%**	**39.6%**	**38.2%**	**32.6%**	**34.7%**	**43.2%**	**33.9%**
$L_{ATM} > L_{VE} > L_{HV}$	32.9%	20.6%	28.0%	61.1%	33.3%	36.4%	28.3%	30.6%	38.6%	30.4%
$L_{ATM} > L_{HV} > L_{VE}$	4.0%	3.2%	4.0%	5.6%	6.3%	1.8%	4.3%	4.1%	4.5%	3.6%
Panel B: Frequency counts for the univariate regression model that has the highest adjusted R squared										
σ_{HV} **performs the best**	**14.1%**	**19.1%**	**12.0%**	**8.3%**	**16.7%**	**10.9%**	**13.0%**	**16.3%**	**13.6%**	**10.7%**
$R^2_{HV} > R^2_{VE} > R^2_{ATM}$	4.7%	7.9%	0.0%	5.6%	6.3%	1.8%	6.5%	6.1%	2.3%	5.4%
$R^2_{HV} > R^2_{ATM} > R^2_{VE}$	9.4%	11.1%	12.0%	2.8%	10.4%	9.1%	6.5%	10.2%	11.4%	5.4%
σ_{VE} **performs the best**	**37.6%**	**41.3%**	**36.0%**	**33.3%**	**43.8%**	**30.9%**	**39.1%**	**28.6%**	**38.6%**	**44.6%**
$R^2_{VE} > R^2_{ATM} > R^2_{HV}$	33.6%	34.9%	34.0%	30.6%	39.6%	27.3%	34.8%	26.5%	31.8%	41.1%
$R^2_{VE} > R^2_{HV} > R^2_{ATM}$	4.0%	6.4%	2.0%	2.8%	4.2%	3.6%	4.3%	2.0%	6.8	3.6%
σ_{ATM} **performs the best**	**48.3%**	**39.7%**	**52.0%**	**58.3%**	**39.6%**	**58.2%**	**47.8%**	**55.1%**	**47.7%**	**44.6%**
$R^2_{ATM} > R^2_{VE} > R^2_{HV}$	41.6%	30.2%	46.0%	55.6%	33.3%	54.6%	37.0%	49.0%	36.4%	41.1%
$R^2_{ATM} > R^2_{HV} > R^2_{VE}$	6.7%	9.5%	6.0%	2.8%	6.3%	3.6%	10.9%	6.1%	11.4%	3.6%

Table 6 Median values of ARCH parameter estimates across firms in different groups

Daily stock returns r_t are modelled by the ARCH specification: $r_t = \mu + \theta\varepsilon_{t-1} + \varepsilon_t$, $\varepsilon_t = h_t^{1/2} z_t$, $z_t \sim i.i.d.(0,1)$, $h_t = \dfrac{\omega + \alpha\varepsilon_{t-1}^2 + \alpha^- s_{t-1}\varepsilon_{t-1}^2}{1-\beta L} + \dfrac{\gamma\sigma_{VE,t-1}^2}{1-\beta_\gamma L} + \dfrac{\delta\sigma_{ATM,t-1}^2}{1-\beta_\delta L}$,

s_t is 1 if ε_t is negative, otherwise s_t is zero. σ_{VE} and σ_{ATM} are measures of model-free volatility expectation and ATM implied volatility. Parameters are estimated by maximizing the quasi-log-likelihood function. Panel A, Panel B and Panel C are the estimation results for the GJR(1,1)-MA(1) model and models using information provided by the model-free volatility expectation and ATM implied volatility, respectively. Inferences are made through t–ratios, constructed from robust standard errors. Numbers in parentheses are the percentage of estimates that are significantly different from zero at the 10% significance level. The persistence estimate is $\alpha + 0.5\alpha^- + \beta$. Groups 1, 2 and 3 are the firms that have $3 \le \overline{N}_i < 4$, $4 \le \overline{N}_i < 5$ and $\overline{N}_i \ge 5$ respectively, where $\overline{N}_i$ is the average number of daily available strike prices for firm i.

Parameters	$\mu \times 10^3$	θ	$\omega \times 10^5$	α	α^-	β	γ	β_γ	δ	β_δ	Persistence	$\frac{\gamma}{1-\beta_\gamma}$	$\frac{\delta}{1-\beta_\delta}$
Panel A: estimates of GJR (1,1)-MA (1) model													
Group 1	0.61	0.00	5.91	0.03	0.06	0.87					0.96		
	(10%)	*(20%)*	*(62%)*	*(41%)*	*(48%)*	*(92%)*							
Group 2	0.83	0.00	5.81	0.02	0.10	0.86					0.94		
	(16%)	*(16%)*	*(82%)*	*(20%)*	*(54%)*	*(94%)*							
Group 3	1.40	0.00	8.01	0.03	0.08	0.83					0.92		
	(56%)	*(3%)*	*(86%)*	*(33%)*	*(67%)*	*(94%)*							
Panel B: estimates of ARCH specification that uses model-free volatility expectation only													
Group 1	0.55	0.00	2.23				0.57	0.17				0.77	
	(10%)	*(19%)*	*(3%)*				*(48%)*	*(14%)*					
Group 2	0.78	0.00	0.00				0.73	0.00				0.85	
	(20%)	*(14%)*	*(2%)*				*(58%)*	*(2%)*					
Group 3	1.22	0.01	0.05				0.81	0.03				0.86	
	(50%)	*(14%)*	*(0%)*				*(89%)*	*(8%)*					
Panel C: estimates of ARCH specification that uses ATM implied volatility only													
Group 1	0.56	0.01	1.58						0.75	0.00			0.89
	(6%)	*(21%)*	*(2%)*						*(44%)*	*(8%)*			
Group 2	0.77	0.01	0.00						0.90	0.00			1.00
	(14%)	*(14%)*	*(0%)*						*(52%)*	*(6%)*			
Group 3	1.21	0.01	0.00						0.97	0.00			1.03
	(53%)	*(14%)*	*(0%)*						*(86%)*	*(3%)*			

Table 7 Cross-sectional analysis of the relative performance of ATM implied volatility and model-free volatility expectation

The most general OLS regression results are for the equation $y_i = \alpha_0 + \beta_1 \times \overline{N}_i + \beta_2 \times TV_{ATM,i} + \beta_3 \times TV_{DELTA,i} + \varepsilon_i$, where $y_i = L_{ATM,i} - L_{VE,i}$ in Panel A and $y_i = R^2_{ATM,i} - R^2_{VE,i}$ in panel B. $L_{ATM,i}$ and $L_{VE,i}$ represent the log-likelihood values of the ARCH models that only use, respectively, the ATM implied volatility and the model-free volatility expectation; $R^2_{ATM,i}$ and $R^2_{VE,i}$ represent, respectively, the adjusted R^2 values of the univariate regressions using ATM implied volatility and model-free volatility expectation for firm i, when realized volatility is measured by high and low prices. $\overline{N}_i$ denotes the average value of the traded strike prices for firm i. $TV_{ATM,i}$ denotes the relative trading activity of at-the-money options for firm i, calculated as the time-series mean of the daily trading volume of the at-the-money option divided by the total trading volume. $TV_{DELTA,i}$ denotes the trading activity of intermediate delta options, calculated as the time-series mean of the daily trading volume of intermediate delta options divided by the total trading volume, where intermediate delta is defined as between 0.25 and 0.75. Estimation is across 149 firms. Numbers in parentheses are t-values, and the asterisk * indicates a significant estimate at the 5% level using a two tailed t-test.

Intercept	Number of strikes	ATM volume	Intermediate delta volume	R-Square	Adj. R-square
Panel A: when the dependent variable is $y_i = L_{ATM,i} - L_{VE,i}$.					
-2.239	0.599			0.021	0.015
(-1.19)	*(1.80)*				
-0.902		4.572		0.002	-0.005
(-0.26)		*(0.56)*			
-0.727			2.810	0.001	-0.006
(-0.16)			*(0.38)*		
-10.109	0.892*	14.892		0.039	0.026
(-1.95)	*(2.36)*	*(1.63)*			
-2.129	0.601		-0.192	0.021	0.008
(-0.45)	*(1.75)*		*(-0.03)*		
-1.362		3.994	1.128	0.002	-0.011
(-0.28)		*(0.43)*	*(0.13)*		
-8.063	1.291*	27.234*	-15.102	0.054	0.034
(-1.51)	*(2.81)*	*(2.22)*	*(-1.50)*		
Panel B: when the dependent variable is $y_i = R^2_{ATM,i} - R^2_{VE,i}$.					
0.014	-0.001			0.000	-0.006
(0.90)	*(-0.24)*				
-0.002		0.029		0.001	-0.006
(-0.05)		*(0.42)*			
0.054			-0.071	0.009	0.002
(1.43)			*(-1.16)*		
-0.000	-0.000	0.027		0.001	-0.012
(-0.00)	*(-0.04)*	*(0.35)*			
0.054	0.000		-0.071	0.009	-0.005
(1.40)	*(0.02)*		*(-1.13)*		
0.041		0.083	-0.105	0.017	0.004
(1.04)		*(1.09)*	*(-1.53)*		
0.021	0.004	0.154	-0.155	0.024	0.004
(0.47)	*(1.03)*	*(1.50)*	*(-1.85)*		

Table 8 Cross-sectional analysis of the relative performance of ATM implied volatility and historical volatility

The most general OLS regression results are for the equation $y_i = \alpha_0 + \beta_1 \times \overline{N}_i + \beta_2 \times TV_{ATM,i} + \beta_3 \times TV_{DELTA,i} + \varepsilon_i$, where $y_i = L_{ATM,i} - L_{HV,i}$ in Panel A and $y_i = R^2_{ATM,i} - R^2_{HV,i}$ in panel B. $L_{ATM,i}$ and $L_{HV,i}$, respectively, represent the log-likelihood values of the ARCH models that only use ATM implied volatility and the loglilkelihood value of the GJR(1,1)-MA(1) model; $R^2_{ATM,i}$ and $R^2_{HV,i}$ represent, respectively, the adjusted R^2 values of the univariate regressions using ATM implied volatility and historical volatility for firm i, when realized volatility is measured by high and low prices. $\overline{N}_i$ denotes the average value of the traded strike prices for firm i. $TV_{ATM,i}$ denotes the relative trading activity of at-the-money options for firm i, calculated as the time-series mean of the daily trading volume of the at-the-money option divided by the total trading volume. $TV_{DELTA,i}$ denotes the trading activity of intermediate delta options, calculated as the time-series mean of the daily trading volume of intermediate delta options divided by the total trading volume, where intermediate delta is defined as between 0.25 and 0.75. Estimation is across 149 firms. Numbers in parentheses are t-values, and the asterisk * indicates a significant estimate at the 5% level using a two tailed t-test.

Intercept	Number of strikes	ATM volume	Intermediate delta volume	R-Square	Adj. R-square
Panel A: when the dependent variable is $y_i = L_{ATM,i} - L_{HV,i}$.					
-17.389	3.735*			0.090	0.084
(-3.14)	*(3.81)*				
1.344		3.803		0.000	-0.007
(0.13)		*(0.15)*			
-32.105			56.212*	0.042	0.036
(-2.31)			*(2.54)*		
-49.548	4.931*	60.848*		0.121	0.109
(3.28)	*(4.49)*	*(2.28)*			
-39.862	3.326*		39.606	0.110	0.098
(-2.93)	*(3.33)*		*(1.80)*		
-27.046		-31.847	69.624*	0.051	0.038
(-1.86)		*(-1.16)*	*(2.79)*		
-51.091	4.631*	51.54	11.390	0.122	0.104
(-3.26)	*(3.43)*	*(1.43)*	*(0.39)*		
Panel B: when the dependent variable is $y_i = R^2_{ATM,i} - R^2_{HV,i}$.					
0.039	0.016*			0.033	0.026
(0.99)	*(2.24)*				
0.126		-0.005		0.000	-0.007
(1.72)		*(-0.03)*			
0.020			0.167	0.008	0.001
(0.20)			*(1.08)*		
-0.081	0.020*	0.226		0.042	0.029
(-0.74)	*(2.53)*	*(1.18)*			
-0.014	0.015*		0.094	0.035	0.022
(-0.15)	*(2.04)*		*(0.60)*		
0.038		-0.115	0.215	0.010	-0.003
(0.37)		*(-0.60)*	*(1.23)*		
-0.073	0.021*	0.271	-0.054	0.043	0.023
(-0.65)	*(2.21)*	*(1.05)*	*(-0.26)*		

Table 9 Cross-sectional analysis of the relative performance of model-free volatility expectation and historical volatility

The most general OLS regression results are for the equation $y_i = \alpha_0 + \beta_1 \times \overline{N}_i + \beta_2 \times TV_{ATM,i} + \beta_3 \times TV_{DELTA,i} + \varepsilon_i$, where $y_i = L_{VE,i} - L_{HV,i}$ in Panel A and $y_i = R^2_{VE,i} - R^2_{HV,i}$ in panel B. $L_{VE,ii}$ and $L_{HV,i}$, respectively, represent the log-likelihood values of the ARCH models that only use ATM implied volatility and the loglilkelihood value of the GJR(1,1)-MA(1) model; $R^2_{VE,i}$ and $R^2_{HV,i}$ represent, respectively, the Adjusted R^2 values of the univariate regressions using model-free volatility expectation and historical volatility for firm i, when realized volatility is measured by high and low prices. $\overline{N}_i$ denotes the average value of the traded strike prices for firm i. $TV_{ATM,i}$ denotes the relative trading activity of at-the-money options for firm i, calculated as the time-series mean of the daily trading volume of the at-the-money option divided by the total trading volume. $TV_{DELTA,i}$ denotes the trading activity of intermediate delta options, calculated as the time-series mean of the daily trading volume of intermediate delta options divided by the total trading volume, where intermediate delta is defined as between 0.25 and 0.75. Estimation is across 149 firms. Numbers in parentheses are t-values, and the asterisk * indicates a significant estimate at the 5% level using a two tailed t-test.

Intercept	Number of strikes	ATM volume	Intermediate delta volume	R-Square	Adj. R-square
Panel A: when the dependent variable is $y_i = L_{VE,i} - L_{HV,i}$.					
-15.150	3.136*			0.067	0.061
(-2.78)	*(3.25)*				
2.246		-0.769		0.000	-0.007
(0.22)		*(-0.03)*			
-31.378			53.403*	0.040	0.034
(-2.33)			*(2.48)*		
-39.439	4.039*	45.956		0.086	0.074
(-2.63)	*(3.71)*	*(1.74)*			
-37.732	2.725*		39.799	0.088	0.076
(-2.82)	*(2.78)*		*(1.84)*		
-25.684		-35.842	68.496*	0.052	0.039
(-1.82)		*(-1.34)*	*(2.83)*		
-43.028	3.340*	24.307	26.492	0.091	0.073
(-2.78)	*(2.51)*	*(0.68)*	*(0.91)*		
Panel B: when the dependent variable is $y_i = R^2_{VE,i} - R^2_{HV,i}$.					
0.025	0.016*			0.033	0.027
(0.61)	*(2.25)*				
0.127		-0.033		0.000	-0.007
(1.68)		*(-0.19)*			
-0.035			0.237	0.015	0.008
(-0.35)			*(1.48)*		
-0.081	0.020*	0.200		0.040	0.027
(-0.72)	*(2.46)*	*(1.00)*			
-0.069	0.015*		0.165	0.040	0.027
(-0.68)	*(1.96)*		*(1.01)*		
-0.003		-0.198	0.321	0.021	0.008
(-0.03)		*(-0.99)*	*(1.78)*		
-0.094	0.017	0.117	0.101	0.041	0.022
(-0.81)	*(1.74)*	*(0.44)*	*(0.46)*		