Volatility Components: Evidence of the Behaviour of the Portuguese Stock Market

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Abstract

This paper studies the time series volatility of the Portuguese stock market. We document the patterns of market-wide and firm-specific risks using daily stock data from 1986 to 2002. We further look into the relation between these measures and economic cycles. Unlike previous studies we find no evidence of a statistically significant rise in firm-level volatility meaning that the number of stocks to diversify fully has remained stable. In contrast with previous studies we find that volatility measures and proxies of economic activity are weakly positively correlated. We discuss these findings in the light of the important changes in the Portuguese stock market in the recent years.

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1. Introduction

Changes in the volatility of stock returns and its systematic and idiosyncratic components have important implications in terms of portfolio management, risk management or option pricing (see, for example, Campbell, Lettau, Malkiel and Xu (2001) or Bennett and Sias (2005)). It is well-established that aggregate stock market volatility is time-varying (e.g., Schwert (1989); Campbell *et al.* (2001)). Recent evidence in the literature has shown further that total volatility components are also not constant (e.g., Campbell et al (2001)). In particular, these studies decompose the total risk of individual stocks into market-wide, industry-specific and firm-specific components, and find strong evidence that the idiosyncratic volatility of individual stock returns has increased over time, both in absolute terms, and relative to systematic market and industry-specific variances. This effect is present for the U.S., developed and emerging markets stocks (e.g. Morck, Yeung and Yu (2000); Li, Morck, Yang and Yeung (2004)).[1] A recent study by Ferreira and Gama (2004) extends the analysis to an international setting: total volatility is now decomposed into four sources of risk: world, country, industry and individual assets components and their results also indicate evidence of rising idiosyncratic risk, over the 1990s, particularly between 1995 and 2000[2]. Kearney and Potì (2004) also examine the dynamics of market and idiosyncratic risks for the constituent stocks of the Eurostoxx50 and find that idiosyncratic volatility has also trended upwards in the euro-zone area.

A series of studies has suggested upon and looked at the forces that could drive this stock market behaviour. Return co-movement may be linked to fundamentals co-movement, and could therefore be impacted by trade or capital market openness, stricter

[1] Campbell et al. (2001) estimate market, industry and idiosyncratic variances for the US market from 1962 to 1997 and find that, while market and industry variances have been fairly stable, firm-level variance increased significantly during that period.

[2] Ferreira and Gama (2004) study 21 developed markets over the 1974-2001 period using local industry data (instead of individual stock data) and report a sharp increase of local and industry variances after 1995 and that these effects dominate world and country risks over the period of 1996-2001.

regulation and investment protection or efficiency in capital allocation (Morck, Yeung and Yu (2000); Li *et al.* (2004) and references therein).[3] Co-movement could also reflect the role of certain types of investors (informed traders) or portfolio management practices (institutional passive investing) (Xu and Malkiel (2003); Campbell *et al.* (2001); Morck, Yeung and Yu (2000)).[4] An alternative explanation is that the changes in co-movement merely reflect changes in the composition of securities/industries that are prevalent in stock markets for a particular period (Bennett and Sias (2004))[5].

The objective of this study is to document the historical behaviour of volatility in the Portuguese stock market from 1986 to 2002 and investigate if the pattern of rising idiosyncratic volatility extends to the Portuguese market. This case analysis can be informative in terms of the variables or factors that drive co-movement in stock returns. Over the last 20 years, there were important changes in the Portuguese stock market in terms of its industry composition and the role of large stocks; over that same period Portugal has witnessed greater capital market openness, improving fundamentals and regulation, and larger institutional presence. If changes in co-movement reflect changes in trade or capital openness or fundamental changes in the economy or in the stock market, one would expect to observe non-trivial changes in the volatility components of Portuguese stock returns.

Our sample consists of all Portuguese stocks listed during the period from 1986 to 2002. We use Campbell *et al.* (2001) decomposition to estimate the volatility components (at the market and firm-specific levels). As in previous studies, we examine co-variation and

[3] Morck, Yeung and Yu (2000) report that, over the 90's, the ratio of idiosyncratic variation to total variation in individual stock returns is greater in higher-income countries. Li et al. (2004) study the pattern and components of volatility in 17 emerging markets for the period of 1990-2001. They find that higher idiosyncratic risk is significantly correlated with greater capital openness in markets with "good government" (low corruption). Yet trade openness is generally associated with greater co-movement.

[4] For example, Malkiel and Xu (1999) relate the rising trend in firm-specific risk to the increasing presence of institutional investors in the stock market.

[5] Bennett and Sias (2004) re-examine the role of market, industry and firm-specific components in volatility for all CRSP securities over the period from 1962 to 1997. They show that the rising trend in firm-specific risk arises from three (technical) factors: growth of riskier industries; increased role of smaller firms in the market; and decrease in within-industry concentration.

lead-lag relationships in volatility components and document the relation between market volatility and economic activity. Our main results are the following:

1. Over the period of analysis, there is no evidence of a statistically significant increase in aggregate volatility.
2. Unlike previous results for developed and emerging markets, we find no increase in firm-specific volatility. Consequently, the number of stocks needed to obtain a desired level of diversification remains stable. Nonetheless, in the more recent years, the results suggest that one could use fewer stocks to obtain the same level of diversification.
3. Volatility measures and economic activity proxies are weakly correlated. Results are not supportive of previous empirical findings for mature markets that suggest that volatility measures are countercyclical and anticipate recessions.

The paper proceeds as follows. Section 2 provides a brief overview of the methodology and describes the data. In Section 3 we present our empirical results and discuss the main findings. Section 4 concludes.

2. Methodology and Data

In this section we present the return decomposition used in this study. Next we present the data and describe the sample.

2.1 Total Volatility Decomposition

We use the methodology proposed by Campbell *et al.* (2001). They decompose the return of a stock into three components (market, industry and firm components) without having to estimate co-variances or betas for individual stocks.

Given the small size of the Portuguese market, industry portfolios would reflect in many cases the behaviour of individual stocks, and would thus dry out firm-specific effects.

To overcome that, we rely on the same methodology Campbell *et al.* (2001) but we use only two sources of risk: market-wide and firm specific risks. This is similar to what has been done by Li *et al.* (2004) and Kearney and Potì (2004). The tables and discussion focus on the results for the decomposition of total risk into market and idiosyncratic components. In a separate section we briefly discuss the results of decomposing total variance into three components: market, industry and firm specific risks.[6]

The excess return of an individual firm *j* in period *t* is denoted by R_{jt}. Excess returns are measured over the risk-free rate. w_{jt} is the weight of firm *j* in the market.

In the analysis that follows we analyse the results for two weighting schemes: market-value weights (as of end of period *t-1*) and equal weights. Market returns are computed accordingly. For the value-weighted index we have: $R_{mt} = \sum_j w_{jt} R_{jt}$..

As in Campbell *et al.* (2001) we assume a simplified individual stock return decomposition. The return of a stock is decomposed into a market-wide return and a firm specific residual based on the market-adjusted return model:

$$R_{jt} = R_{mt} + \eta_{jt} \, . \tag{1}$$

where ε_{jt} is the difference between the individual stock return R_{jt} and the market return R_{mt}.

In a CAPM framework a firm's excess return is expressed as:

$$R_{jt} = \beta_{jm} R_{mt} + \tilde{\eta}_{jt} \, , \tag{2}$$

As such,

$$\eta_{jt} = \tilde{\eta}_{jt} + (\beta_{jm} - 1) R_{mt} \, . \tag{3}$$

and

$$Var(R_{jt}) = \beta_{jm}^{2} Var(R_{mt}) + Var(\tilde{\eta}_{jt}), \tag{4}$$

[6] Tables in appendix A show these results. Please refer to Appendix B for the decomposition proposed by Campbell *et al.* (2001).

where β_{jm} is the market beta for stock j and $\tilde{\eta}_{jt}$ is the firm specific residual. R_{mt} and $\tilde{\eta}_{jt}$ are orthogonal, by construction. The residual $\tilde{\eta}_{jt}$ equals η_{jt} only when beta is 1 or the market return is zero.

For the simplified version the variance of an individual stock is given by:

$$Var\,(R_{jt}) = Var\,(R_{mt}) + Var\,(\eta_{jt}) + 2Cov\,(R_{mt},\ \eta_{jt})$$

$$= Var\,(R_{mt}) + Var\,(\eta_{jt}) + 2(\beta_{jm} - 1)Var\,(R_{mt}), \tag{5}$$

This decomposition requires the estimation of stock betas but this is unproblematic given that we are concerned with average variances across stocks. Given that

$$\sum_j w_{jt}\beta_{jm} = 1 \tag{6}$$

there is no need to estimates betas and the volatility of a typical stock can be computed as:

$$\sum_j w_{jt}Var(R_{jt}) = Var(R_{mt}) + \sum_j w_{jt}Var(\eta_{jt})$$

$$= \sigma^2_{mt} + \sigma^2_{\eta t}, \tag{7}$$

where $\sigma^2_{mt} \equiv Var(R_{mt})$ e $\sigma^2_{\eta t} \equiv \sum_j w_{jt}Var(\eta_{jt})$.

The weighted average $\sum_j w_{jt}Var(R_{jt})$ can be interpreted as the volatility of a random selected firm (where the probability of drawing firm j is equal to its weight w_j). The methodology is easily extended to an equally-weighting scheme.

2.2 Estimation Procedures

We estimate the volatility components in equation (7) using daily data. The sample volatility estimate of the market return in month t (MKT_t), is computed as

$$MKT_t = \hat{\sigma}^2_{mt} = \sum_{s\in t}\left(R_{ms} - \mu_m\right)^2 \tag{8}$$

where μ_m is the daily mean of the market return (R_{ms}) over the sample period. *s* denotes the trading days in a particular month[7]. We construct value (and equally)-weighted estimates of market-wide effects using all firms in the sample.

The measure of individual stock or firm volatility is obtained as:

$$FIRM_t = \hat{\sigma}^2_{\eta t} = \sum_j w_{jt} \hat{\sigma}^2_{\eta jt} . \qquad (9)$$

This is the value (or equally)-weighted average across firms of the monthly estimates

$$\hat{\sigma}^2_{\eta jt} = \sum_{s \in t} \eta^2_{js} \qquad (10)$$

constructed from the squares of the residuals of daily returns obtained from equation (1).

2.3 Data

2.3.1 The Portuguese Stock Market: a Brief Overview

In this study we analyze the behaviour of total volatility and its systematic and idiosyncratic components for the Portuguese stock market from January 1986 to December 2002. Over this period there were major changes in the Portuguese stock market in terms of its size, industry composition, foreign and institutional presence and regulation. We briefly describe some of those changes below.

In 1986, the aggregate market capitalization of the Portuguese stock market was below 4 thousand million euros and increased more than 10 times over the sample period. Average daily turnover changed from less than 2 million euros a day to more than 87 million euros by 2002 (more than 40x). Market Capitalization rose in the late 80's and that growth was sustained during the 90's due to large privatization public offerings and new private IPOs in the late 90s. In 1993, the aggregate market capitalization was over 6 million euros and surged up in 1995 after the sale of two of the major Portuguese stated owned firms (Portugal Telecom and Portucel). By that time, market capitalization of privatized

[7] We also use weekly returns to obtain quarterly variance estimates.

firms was almost 42 percent of the Portuguese Stock Exchange but only a small part of this was free-floated. In 1998/1999 the market capitalization expanded further thanks again to the entry of the newly privatized firms (EDP and Brisa). Aggregate market capitalization peaked to over 68 thousand million euros in 2001 just before the internet bubble burst. In the 2 years that followed the capitalization decreased to around 50 000 million euros. By December 2004, aggregate market capitalization had recovered but it was still well below the level of 2001.

Over the sample period, the Portuguese economy observed two main business cycles. After the important growth phase brought by the entry of Portugal in EEC (European Economic Community) initiated in 1986, growth rates slowed down and a recession was hit in 1993. Then, between 1994 and 1998, economic growth surged up due both to capital investment and consumption. After 1999, the economy slowed down again and in 2003 the economy was again depressed.

2.3.2 Sample

We use data gathered from Dathis, which is a database compiled by the Portuguese stock exchange and that is the most comprehensive data set on Portuguese stocks. We collected firm-level data (total returns and market capitalization) for all stocks listed on the Portuguese stock exchange.[8, 9] Our sample period runs from January 1986 to December 2002. The total number of stocks increases from 5 in December 1986 to 57 in December

[8] Up to 1994, Portuguese stocks traded on two exchanges: BVP - Bolsa de Valores do Porto and BVL - Bolsa de Valores de Lisboa. After June 1994, the spot trades were concentrated on BVL while BVP kept the derivatives market. In 2000, the two exchanges merged into BVLP - Bolsa de Valores de Lisboa e Porto. In 2002, Euronext took over BVLP and Portuguese stocks trade now on Euronext Lisbon.

[9] In April 1991, the new Capital Markets law (*Lei Sapateiro*) set up three market segments in the Portuguese stock exchange. Regular firms, meeting all exchange requirements (in terms of capital dispersion, market capitalization, turnover and solvency), are listed on *Mercado de Cotações Oficiais* (Market with Official Quotations). Small and medium firms list on *Segundo Mercado* (Second Market). The firms that do not meet the exchange requirements are listed on *Mercado Sem Cotações* (Market Without Quotations). Our sample includes only the stocks listed on the main market (Market with Official Quotations).

2002[10]. In aggregate, the sample covers 93 different stocks some of which were in the meanwhile de-listed or dropped from the main market.

Table A1 in appendix shows the number of firms as well as industry weights in total market capitalization[11]. The number and of stocks has changed dramatically over time and so as the number and the relative market capitalization weight of the different industries. The coefficient of variation of market capitalization has monotonically increased from 0.79 in 1986 to 2.18 in 2002. This higher cross industry dispersion reflects the concentration of market capitalization in a few very large industries. If we look at the dispersion of market capitalization at the stock level, a similar pattern holds. The coefficient of variation monotonically increases from 1.02 to 4.36 over the sample period and this dispersion reflects merely that there a few very large stocks.

The industry with most firms, on average, is Banks (increasing from 0 to 9 stocks over the sample period) and the industry with the fewest is Transport (on average, 2 stocks). Over time, *Miscellaneous Manufacturing*, *Real Estate* and *Insurance* sectors lost ground to sectors like *Banks*, *Information Technology* and *Building and Related Activities*. By the end of 2002, the three largest industries were *Banks* (17%), *Miscellaneous Manufacturing* (15%) and *Equipment Manufacturing* (17%). Over time there were also important changes in the number and characteristics of firms in the market mainly because of Privatization new listings and re-structuring as a result of regulatory changes (as mentioned, the Capital Markets law of 1991 had a significant impact on the firms listed in the main market due to stricter requirements in terms of minimum size and turnover).

We use daily (and weekly) excess returns (in excess of the Interbank Monetary Market daily or weekly rates) to estimate monthly (and quarterly) variances. We kept only one class of shares per firm. We excluded preferred stocks because of lower liquidity.

[10] We include only the most liquid class of shares for a given stock. Preferred stocks were dropped.
[11] Stocks were classified into industries according to their CAE (*Classificação de Actividade Económica*) which is a fairly close equivalent in Portugal of SIC.

GDP data is on a quarterly basis and was obtained from INE (Bureau of Portuguese Statistics).

3. Findings

3.1 Trends in Volatility

3.1.1 Aggregate Volatility

Figure 1 plots the annualized (multiplied by $\sqrt{12}$) standard deviation of monthly raw returns of a value weighted portfolio of all stocks in sample over the period from January 1986 to December 2002. The figure confirms previous international evidence of time-varying volatility (Schwert, 1989; Campbell *et al.,* 2001). Over time, there have been episodes of increased volatility but there is no obvious upward trend. In particular, we observe that the highest levels of volatility occurred in 1987 and 1988 and later in 2002. The average annual standard deviation for the aggregate period was 24 percent. When we look at sub-periods, we observe that, for the period from 1986 to 1992, average standard deviation was 32 percent, against 13 percent between 1993 and 1997, and 24 percent in the latter period (1998-2002) suggesting that total market volatility is higher in periods just before economic recessions.

In the sections that follow we investigate if the patterns of the volatility components are the same as the described for total volatility.

3.1.2 Volatility Components

Graphical Analysis

We first present the graphs for the time behaviour of market-wide and firm-specific volatility components. Our base analysis relies on value-weighted estimates using daily return data within a month. For robustness purposes we also look at quarterly estimates obtained using weekly returns. We report annualized (multiplied by 12) variances for the sake of comparing the different estimates.

Figures 2 and 3 show thus market-wide (*MKT*) and firm-specific (*FIRM*) volatility measures obtained using, respectively, equation (8) and equations (9) and (10). Figures A1 and A2 in appendix show the corresponding 3-month moving average of *MKT* and *FIRM*. Similarly to what was observed for total volatility in the preceding section, the highest levels in volatility (variance measures) occur around the US stock market crash of October 1987.

The maximum value of *MKT* was 1.36 (which implies an annualized standard deviation of 117 percent) in February 1988. The second highest value was in January 1986 with 1.04 (annualized standard deviation of 102 percent) and the third in September 1986 with 0.55 (annualized standard deviation of 74 percent). After this initial period of volatility turmoil, from 1986 to the first quarter in 1988, the behaviour of *MKT* is fairly stable with the highest values observed well below 0.16 (which implies an annual standard deviation below 40 percent). The moving average plot confirms this analysis. From 1989 to 1992, the plot suggests a somewhat decreasing trend but, from that year on, volatility rises again. Yet the higher levels of volatility in recent years (1998 onwards) are well below the ones observed in the period of 1986-1988[12].

As for *FIRM*, the level is larger than *MKT* confirming the stylized fact that firm-specific volatility is the most important component of total volatility. The all-times high for *FIRM*, occurs in February 1988 (FIRM=2.97). Overall and unlike the evidence in previous studies (Campbell *et al.* (2001); Li *et al.* (2004)), there is no obvious upward trend in the specific volatility of Portuguese stocks. On the contrary, the graph suggests a slight downward trend. The 3-month moving average plot confirms these preliminary remarks.

[12] All these figures have a much higher magnitude than the ones observed in more mature markets. Campbell *et al.* (2001) reports average figures for *MKT* in the US that are less than one third of the average observed for the Portuguese market.

Figures 4 and 5 show the behaviour of annualized estimates (multiplied by 4) of *MKT* and *FIRM* at a lower frequency[13]: we compute variance measures using end-of the week returns. The plots show the same features highlighted above.

Figures 6 and 7 plot the annualized variances of *MKT* and *FIRM* using equally-weighted averages of all firms in the sample. We observe the same clusters of volatility found for the value-weighted estimates. Yet the levels of *MKT* and *FIRM* are now of lower magnitude. Given that the plots show variances in absolute terms, this could merely reflect that the total variance of an equally-weighted portfolio is lower than its value-weighted counterpart. In the next section, we explore whether equal-weighting reflects different roles of *MKT* and *FIRM* in total volatility. The *FIRM* plot also reveals another salient feature: while for the value-weighted estimates, the behaviour of *FIRM* over time seems fairly stable after 1988, for equally-weighted series we now observe several spikes of high volatility particularly, after 1995. This could reflect that, as expected, small firms do indeed observe higher firm-specific volatility that was previously concealed as a result of their tiny market weights. The counterparts of the plots 6 and 7, using weekly returns, are shown in appendix (figures A3 and A4) and are suggestive of the same behaviour.

In any of figures analysed above, *MKT* and *FIRM* seem to move together but there are some differences. For example, in 1993 and 1995, firm volatility increases (and this is more obvious in figure 7) while market volatility remains constant. As in 1989, market-wide volatility increases while firm-specific volatility remains constant. This evidence might reflect that not all shocks influence the different volatility components in the same way.

Descriptive Statistics and Linear Trends

To investigate further the trends in volatility components, we first test whether such trends are deterministic or stochastic. Table 1 reports the autocorrelation coefficients

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[13] The weekly *MKT* and *FIRM* estimates are based upon a maximum of 831 weeks against 4030 when using daily returns.

for the two volatility measures. We compare the two series of estimates – value and equally weighted. Overall the results are not affected by the weighting scheme and show serial correlation, particularly in the short run. Table 2 shows the Augmented Dickey-Fuller (1979) *t*-statistics to test for unit roots in the series (including a constant, and a constant and time trend). The number of lagged values included in the regression was chosen in order to account for the serial correlation observed in table 1. The hypothesis of a unit root is rejected for all series, reflecting that deviations from the long run mean are, for the most part, temporary.

Given these results, we proceed in analysing volatility in levels. Table 3 reports descriptive statistics for the annualized volatility estimates based on daily and weekly returns. For the whole sample, the mean of *MKT* is about 5.53 x 10^{-2} which implies an annual standard deviation of 23.5 percent. This is much lower than the mean of *FIRM* (17.16 x 10^{-2} which implies an annual standard deviation of 41.4 percent). Altogether, these figures confirm what we remarked on when looking at plots: the share of unconditional variance of an average stock that is due to market effects is small of only about 24 percent. This share is above what has been found in the US.[14] Li *et al.* (2004) report an average R^2 of 24% (ranging from a minimum of 9.9% to a maximum of 43%) for 17 emerging markets between 1990 and 2001[15].

To check the robustness of results based on daily returns, we looked at the statistics of volatility measures using weekly returns. The means of *MKT* increases while the mean of *FIRM* decreases[16]. The results suggest that market returns are positively serially correlated while firm-specific returns are negatively correlated. This is similar to what has been found by previous studies (see, for, example, Campbell *et al.* (2001)).

[14] Campbell *et al.* (2001) report that, over the period from 1962 to 1997, the share of systematic variation in total variance is about 17% based on averages of, respectively, 12 and 25 percent for *MKT* and *FIRM*.
[15] Morck, Yeung and Yu (2000) and Li *et al.* (2004) suggest using the average R^2 of individual stocks' market model regressions to gauge the importance of systematic return variation as a fraction of total return variation.
[16] In relative terms, *MKT* represents now 34% of total variance.

We contrast the value-weighted results with the results for the equally weighted (EW) series[17]. Our prior expectation was that we would find a stronger effect for *FIRM* both in absolute and relative terms (in relation to *MKT*) given that an EW index gives more weight to small firms. The last two columns of table 3 show that the average estimates are lower for *MKT* (reflecting that the variance of the EW portfolio is lower[18]) and *FIRM*. In absolute terms, the result for *FIRM* is surprising reflecting that the excess volatility for small firms was lower[19]. In relative terms, we confirm our prior: the non-systematic variation is now higher than observed for the value-weighted series. This result reflects the effect of firm size: small firms show higher *FIRM* effects. This is also similar to what has been found in previous studies (see, for, example, Campbell *et al.* (2001)).

In any case, all series exhibit substantial variation over time. The standard deviation of respectively the *MKT* and *FIRM* are 13.67 x 10^{-2} and 35.05 x 10^{-2} but most of this variation reflects the behaviour of the volatility series around the crash of October 1987[20]. To further explore the behaviour of the volatility measures over time, we compute descriptive statistics for three 4-year non-overlapping periods (1986-1992; 1993-1997; and 1998-2002)[21]. Table A2 in appendix show the results. The first sub-period is the highest volatility period for the two series. The second sub-period indicates a relatively low volatility for *MKT* and *FIRM*. In the last sub-period, *MKT* rises again while *FIRM* remains

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17 We use a difference t-test to compare VW and EW estimates. We reject the null of similar average estimates for *MKT* (*t*-statistic of 2.24) but we could not reject the null for the *FIRM* average estimates (*t*-statistic of 1.09).
18 This result reflects that the EW index shows better diversification.
19 This atypical behaviour could be associated with thin trading.
20 Related studies (Campbell et al. (2001), Ferreira and Gama (2005), for example) report "downweighted crash" results by replacing the more extreme observation with the second-largest observation in the respective series. We did not adopt that procedure given that, in our case, the second-largest observation (*MKT*: 1.36; *FIRM*: 2.97) is very close to the first (*MKT*: 1.04; *FIRM*: 1.31) and that is also the case for the third-largest (*MKT*: 0.55; *FIRM*: 2.25).
21 The definition of sub-periods is always controversial. Sub-periods were here defined according to the business cycles described above. The chosen break dates might also account for important changes observed in the Portuguese stock market over the period analysed. In particular, the first 4 years capture the re-emergence of the stock market after the 1974 revolution. The second sub-period reflects the enforcement of the new Capital Markets law and the liberalization to foreign participation. The latter sub-period coincides with the "upgrade" of Portugal to the developed markets group (Portugal was reclassified as a developed market by Morgan Stanley Capital International and IFC/Standard & Poor's in 1999).

fairly constant. In relative terms, and comparing the three sub-periods, we observe that the (value-weighted) average R^2 of a market model is about 24 percent in the first sub-period, 14 percent in the second and 35 percent toward the final years[22]. These results could suggest a decrease in firm-specific volatility; alternatively the rise in the latter period could merely reflect a short-term phenomenon. Weekly estimates are very similar to these.

As for the variability of *MKT* and *FIRM* measures over the three 4-year non-overlapping periods, we confirm that the first period is atypical: the standard deviation of respectively the *MKT* and *FIRM* are 20.56 x 10^{-2} and 52.22 x 10^{-2} against 1.7 x 10^{-2} and 8.05x 10^{-2} in the period 1993-1997, and 3.69 x 10^{-2} and 5.50 x 10^{-2} in the period 1998-2002.

To assess if such a deterministic trend exist in the time series, we run single OLS regressions of the volatility measures on time. We assess significance with a Vogelsang (1998) $t\text{-}PS_T$ test which is a more robust test in the presence of serial correlation[23]. Table 3 shows the trend coefficients and the *PS* statistics. For the whole sample, we observe a negative effect for *MKT* but the coefficient is very small and flips sign when we use weekly returns. As for *FIRM*, the coefficient is also negative but higher and could suggest thus a decrease in firm-specific volatility over time. Yet, in both cases, we are unable to reject the null of no deterministic time trend in the series.

The time pattern for *MKT* seems to be fairly robust to the weighting scheme. Yet for the *FIRM* series, that is when we give extra weight to small stocks, the slight apparent decrease observed above vanishes, implying that (as observed above) the decrease in firm-specific volatility was prevalent for large firms but not for all firms. Again the results are robust to the return horizon.

[22]Equally weighted average R^2 are 28%, 7% and 9%. Comparing these results with those for the VW estimates, it looks like the *MKT* effect is playing an increasing role for large firms but not for small firms. For small firms, this result suggest that there is no evidence of a decrease in firm-specific volatility.
[23] Please refer to Campbell *et al.* (2001) for more details about this test.

Overall, our results contrast with previous results (Campbell *et al.* (2001), Li *et al.* (2004)) that report a very positive and significant trend coefficient for *FIRM* that is magnified for EW estimates.

Relative importance of systematic and idiosyncratic volatility components

From the results above we know the importance of the different volatility components is not the same and changes over the sample period. To assess the relative importance of each volatility component to the total volatility of a typical stock, we perform mean and variance decompositions. Given that $\sigma^2_{jt} = \sigma^2_{mt} + \sigma^2_{\eta t}$, the decomposition of the mean of volatility, is such that:

$$E(\sigma^2_{mt}) / E(\sigma^2_{jt}) + E(\sigma^2_{\eta t}) / E(\sigma^2_{jt}) = 1. \quad (11)$$

For the variance of total volatility, the decomposition is such that:

$$Var(\sigma^2_{mt}) / Var(\sigma^2_{jt}) + Var(\sigma^2_{\eta t}) / Var(\sigma^2_{jt}) + 2\,Cov(\sigma^2_{mt}, \sigma^2_{\eta t}) / Var(\sigma^2_{jt}) = 1. \quad (12)$$

Table 4 reports the decomposition of the mean and the total volatility of a typical stock for the overall period and for the three sub-periods and confirms the preliminary remarks made above. First, *FIRM* represents the largest share of total risk (75.6%). Second, over time, the share of *FIRM* has changed, increasing from 1986-1992 to 1993-1997 but decreasing again in the latter period (1998-2002) to a level below the observed in the first sub-period[24]. The results are consistent with Campbell et al. (2001) in respect to *FIRM* being the dominant share but the dynamics are not alike in particular if we look at the VW measures. When we focus on the EW measures, we observe that the share of *FIRM* increased from the first to the second period and remained at that level in the latter period. That is, when we give more weight to small firms, we observe that *FIRM* effect is more significant and these results in particular are more alike US results. These results are

[24] This surely has something to do with the large weight of a few large firms in the index.

consistent with the analysis above in terms of the behaviour of the absolute firm-specific effects. Below we discuss the implication of these results in terms of portfolio diversification.

As for the variance of total volatility, again the highest contribution is given by the variance of FIRM (56.2%) and its covariance with MKT (35.2%). Yet when we look at the decomposition of the conditional series of *MKT* and *FIRM* to diminish the effect of random variation, the contribution of *MKT* is the largest but *FIRM* continues to play a very important role [25].

Dynamics in Volatility Components

To clarify the above apparently contradictory effects, we look at the dynamics of the two series: As noted in the graphical analysis, the two volatility components appear to be correlated. Panel A of table 5 reports the contemporaneous correlation structure of the two volatility measures (estimate of 0.807 and 0.638, respectively, for VW and EW estimates). To find out how causality works in this relation, we look at a bivariate vector autoregression (VAR). The VAR lasg length was chosen using the Akaike information criterion. Panel B shows the *p*-value of a standard *F*-test for the hypothesis that the three lags of a measure do not help to forecast the other measure. Results suggest that *FIRM* appears to Granger-cause *MKT* but not the other way round. Thus, our evidence suggest that firm-specific risk leads market-wide variations and is ultimately responsible for total variation in average stock returns. Campbell *et al.* (2001) report a similar result for the US market, but also find that market volatility appears to lead industry and firm-level measures.

3.1.3 Implications for Portfolio Diversification

[25] The conditional series were obtained by regressing each detrended volatility series on four lags of itself and of the other detrended volatility series.

Investors' ability to eliminate firm specific risk is obviously affected by the amount of firm specific risk. Yet to find out the number of securities needed to form a well diversified portfolio, it is necessary to identify the behaviour of correlation between stocks over time. Kearney and Potì (2004) show that it is incorrect to infer about the trend in average correlation only on the basis of the time pattern of idiosyncratic risk. Rising idiosyncratic risk only implies a decrease in average correlation of stocks, if market risk is constant. They show that the variation in average correlation, that has implications in terms of diversification, is a (negative) function of the variation in the ratio of *FIRM* to *MKT*[26]. As such the results we now present for the correlation analysis have to be consistent with our previous findings in terms of the relative importance of *FIRM* effects.

Campbell *et al.* (2001) showed that, in the US, the number of securities needed to form a well-diversified portfolio has increased. This was not only because of the observed rising trend in firm-specific risk but also because market-wide risk remained fairly constant. In other words, the average correlation between stocks decreased.

We compute quarterly pair-wise correlation coefficients using daily returns for the prior 12 months (except for 1986, when fewer observations were available). We compute correlations between all pairs of stocks for which return observation was available in respect to a particular quarter. The number of pair-wise correlations ranges between 10 and 2485 over the sample period. Figure 8 shows the equal-weighted average of those pair-wise correlation coefficients over the sample period. We observe that average correlation across stocks over the sample period is very low (0.087) and time varying but there seems to be no discernible trend (the trend coefficient is negative but very close to zero, -0.0002).[27] We test the null of constant average correlation for the three sub-periods. We reject the null of

[26] Kearney and Potì (2004) show that this correspondence holds exactly for EW estimates. In fact, and as we referred previously, the average R^2 of model market regressions across stocks is informative in respect of the importance of idiosyncratic risk in total volatility. The EW average of pair-wise correlation across stocks gives that same information.

[27] The average correlation for the three sub-periods is, respectively, 0.102 (1986-1992), 0.044 (1993-1997) and 0.109 (1998-2002).

constant correlation from 1986-1992 to 1993-1997 and from 1993-1997 to 1997-2002 but not from 1986-1992 to 1997-2002. The evidence is thus consistent with what we report for the relative importance of firm-specific risk over the sample period for the VW results. For the equal-weighting results, the results were slightly different. Recall that the share of the *FIRM* effect increased in the first sub-period and was more or less unchanged in the latter period, reflecting thus lower common effects. Yet those effects are not reflected in the dynamics observed for the average correlation across stocks[28]. Unlike previous studies we find no downward trend in average correlation. Consequently, the number of stocks to form a well diversified portfolio or to attain a certain level of risk is barely the same, over time. Panel A in figure 9 shows the excess standard deviation for different portfolios of 2, 5, 15 and 30 stocks. The excess standard deviation is the difference between each portfolio standard deviation and the standard deviation of an equally-weighted portfolio of all stocks used in the calculations[29]. For each year, equally-weighted portfolios were formed of randomly selected stocks (without replacement). Excess standard deviations are computed on the basis of portfolio daily returns and then averaged across portfolios. The number of the n-stock portfolios varies over time depending on the total number of stocks in sample.[30] Table 6 shows the average excess standard deviation for the aggregate period and for the three sub-periods[31]. Panel B in figure 9 plots these figures. The highest levels of excess standard deviation were observed in 1987 and in 2000. This is consistent with our analysis above: these were the periods for which the role of *MKT* was dominant.

The figures reveal that excess standard deviation is pretty small if we use 15 stocks or more. To reduce excess standard deviation in the aggregate period below 0.01, one

[28] To check the consistency stated in footnote 21 between the average pair-wise correlation coefficients and the average R^2 of market model regressions, we obtained the R^2 of the market model regressions on a quarterly basis using daily returns for the previous 12 months for the same stocks used in the pair-wise correlations. The plot of the average R^2 is very close to figure 8.

[29] We use the same stocks for which we compute pair-wise correlations, between 5 and 71 stocks as the stocks with complete records change over time.

[30] For example, in 2002 we formed 26, 11, 4 e 2 portfolios respectively for the 2-, 5-, 15- and 30-stock portfolios.

[31] Data for these plots are obtained by averaging quarterly estimates of excess standard deviations.

would need more than 30 stocks. This is also true for the sub-periods of 1993-1997 and 1998-2002. Yet, over the first sub-period one could have halved that figure with the same number of stocks reflecting lower correlation among stocks.

When we compare of 1993-1997 and 1998-2002 we observe that the former required a larger number of stocks to attain the same level of excess standard deviation. This occurs even if correlation was lower (0.04 in 1993-1997 against 0.11 in 1998-2002) – the role of *FIRM* in total volatility was higher – because, in absolute terms, idiosyncratic volatility in excess to market volatility was higher[32]. The results thus suggest that the foregoing diversification benefits of holding bad-diversified portfolios are larger when aggregate volatility is lower. Conversely, when aggregate volatility is higher, increases in the number of securities do not produce as significant additional risk reductions: the risk of a portfolio of 10 stocks will be only slight greater than the amount of market risk.

3.1.4 Controlling for Industry Effects and Individual Industries

We performed the same tests as above considering the *MKT+IND+FIRM* decomposition shown in appendix B.

The hypothesis of a unit root in the series is rejected for the three series (autocorrelation coefficients and unit root tests results are available upon request).

Table A.3 reports the descriptive statistics for the annualised (value-weighted) volatility measures based on daily returns. For the whole sample we observe that the mean of *IND* is 9.5 x 10^{-2} (which implies an annual standard deviation of 30.8%) while the mean of *FIRM* is now only 7.69 x 10^{-2} (which implies an annual standard deviation of 27.7% against the 41.4% observed for the *MKT+FIRM* decomposition). As we conjectured, when we include an industry risk factor, the firm-specific volatility effects are subsumed under

[32] Lower correlation results in lower *total* market volatility and therefore higher *excess* standard deviation.

the industry factor. Consequently, in terms of the relative importance of the different effects we observe that *FIRM* no longer represents the largest share of total risk (please refer to table A.4 in appendix). *IND* is now dominant with 41.9% against 33.8% and 24.3%for *FIRM* and *MKT* effects, respectively.

That goes also for the standard deviation of the total volatility of a typical stock: the standard deviation of *FIRM* declines from 35.05 x 10^{-2} (*MKT+FIRM* decomposition) to 10.29 x 10^{-2} (*MKT+IND+FIRM* decomposition) while the standard deviation of *IND* is 25.22 x 10^{-2}.[33] As for the variance of total volatility, the highest contributor is now the variance of *IND* (32.8%) and its covariance with *MKT* (24.9%). *FIRM* now contributes to only 13.5% of the total variation of a typical compared with over 90% for the *MKT+FIRM* decomposition.

To find out whether the effects are significant different across industries, we compute the *IND* and *FIRM* for each industry. Table A.5 shows these results. The standard deviation for the mean (variance) of *IND* and *FIRM* across industries is respectively, 0.8% and 1% (6% and 8.5%).[34] The *FIRM* effect is always dominant and the ratio *FIRM/IND* ranges from 104% to 233%.[35] When we exclude the more extreme sub-period, the magnitude of these figures declines dramatically but *FIRM* remains the dominant effect.

3.2 Cyclical Behaviour of Volatility Measures

[33] These large variations occur due to the first years in the sample. When we compute the statistics for the series excluding this period, the standard variation is only 8 x 10^{-2} and 2.5 x 10^{-2} for the *FIRM* and *IND* effects.

[34] These results are not directly comparable to the (average) statistics of *IND* and *FIRM* shown above. The decomposition at the industry-level requires the estimation of industry betas given that we no longer average across industries. Please see Campbell *et al.* (2001) for more details

[35] The heterogeneity (and the number of firms) within a particular industry may have a non trivial impact on these results. The maximum value for the *FIRM/IND* ratio occurs for *Miscellaneous Manufacturing* (#18 stocks) followed by *Tourism and Leisure* (#8) while the minimum occurs for *Retailers* (# 2 stocks) closely followed by *Telecoms* (#2) stocks). The fact that some industries include only a few constituents, on one hand, and the subjectivity in classifying stocks into industries, on the other hand, may thus impact the results. The base decomposition (*MKT+FIRM*) avoids these potential biases.

The preceding sections provide evidence of the patterns in total volatility and its components. We also analyzed the dynamics of the two volatility measures (autoregressive effects and correlation and causality relations between *MKT* and *FIRM*). In this section, we report the results for the relation between volatility measures and economic cycles. Earlier studies (see, for example, Schwert (1989) and the references therein) report a negative relation between market volatility and economic activity. The question that arises from previous literature to our study is whether market-wide and firm-level volatility measures are countercyclical.

We use an indicator variable to proxy recession/expansion and GDP data (measured on a quarterly basis) and compute contemporaneous and lead-lag correlation coefficients between these variables and the volatility measures. We also explore whether these volatility measures can forecast GDP growth.

In tables 7 and 8 we present these results (for the unconditional, the conditional and the innovation volatility measures). Overall the correlation coefficients are very low and do not seem supportive of previous empirical findings for mature markets that suggest that volatility measures are countercyclical. Panel A in table 7 shows that there are only a few negative (lead) coefficients associated with *FIRM*. The contemporaneous correlations are stronger but results are mixed: the coefficient is negative for the *MKT* innovation but positive for the conditional series. The pattern seems to be similar for the correlations with GDP growth in Panel B: we observe some evidence that market-wide volatility is higher when the economy is down. Yet it does not seem to anticipate the downturn. On the contrary, the results suggest that both volatility series lag the business cycle.

To find out whether these measures have some ability to forecast GDP growth, we run an OLS regression with GDP growth as a dependent variable. As regressors we included the lagged GDP growth, the lagged market index (computed as an equally weighted index of the constituent stocks in sample) and the two volatility measures. All variables are individually significant and the R^2 of the regression is about 20%.[36] There seems to be some forecasting power but it is difficult to interpret the signs of the estimates of these regressions (in particular the very negative sign for the lagged GDP growth parameter and the positive signs for the parameters associated with *MKT* and *FIRM*).

In sum, this evidence is inconclusive and thus, for the particular case of the Portuguese stock market, one cannot establish that, when there was a recession, portfolios were more exposed to volatility or that lack of diversification was more costly.

3.3 Discussion of main findings

What factors might explain the evidence found for the Portuguese stock market? Unlike previous studies, our results do not show an upward trend in firm specific volatility and even if we confirm that the role of *FIRM* is dominant, there is no evidence of an increasing important role over the sample period. Hence, we find no downward trend in average correlation and, as a result, the number of stocks to form a well-diversified portfolio or to attain a certain level of risk is more or less unchanged over the sample period. This is to some extent striking evidence, given the huge changes observed in the stock market, in terms of the number of listed stocks, its average capitalization, its industry representation, etc[37]. This could be indirect evidence that, in spite of a more regulated market and with more sophisticated issuers and investors that would, in theory, make room for more innovation and less stickiness (of poor shared practices, for example in terms of corporate governance), and even if there are more industries in the stock market - that would yield in theory, also reduce correlation -, the type of industries and firms that

[36] When *MKT* and *FIRM* are included together, none of them is statistically significant and this is not surprising given that they are very positively correlated.

[37] Take, for example, *Banks*. This is a sector that has increased its weight over the sample period and that observes large *FIRM* effects both in absolute and relative terms. Banking is a high regulated activity but that is free from government interference.

became listed in recent years are large firms, more "normal" and eventually with government damaging intervention – for example, partially-privatized firms –.

The results, in particular those of the VW estimates, could also reflect the effect of increasing market concentration: three stocks represented in 2002 over 60% of total market capitalization while the large 20 accounted for over 90% of total market capitalization. These stocks are the constituents of PSI20, the reference index for the Portuguese stock market[38]. Several authors claim that stocks that belong to a major stock index present a common pattern in returns and show a permanent increase in stock *price* volatility (see, for example, Coopers and Woglom (2002)). These effects could be information-driven given that stocks that belong to a major index have a wider analyst and media coverage. Alternatively these could be purely trading-effects as a result of increased trading by trackers or herding behaviour. Altogether the direction of the effects of greater concentration in idiosyncratic *return* volatility is ambiguous

4. Conclusions

This paper documents the historical behaviour of volatility in the Portuguese stock market from 1986 to 2002 and investigates if the pattern of rising idiosyncratic volatility extends to the Portuguese market. Unlike previous studies we find no evidence of a statistically significant rise in firm-level volatility meaning that the number of stocks to diversify fully has remained stable. Overall our results are difficult to reconcile with previous evidence. Capital market openness, increasing regulation and more sophisticated investors would dictate higher firm-specific variation. Yet this might have been counter-balanced by an increasing weight of large privatized (regulated) firms in industries, such as utilities, with lower firm-specific variation and/or by the increasing role of passive management

[38] PSI20 is a free-float value-weighted index that includes the 20 largest, most liquid stocks.

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Table 1
Autocorrelation Structure

This table reports the autocorrelation structure of monthly volatility series computed with squared daily deviations. ρ_k is the autocorrelation of order *k*. *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). We present statistics for variances weighted by the market value of stocks and equally weighted variances.

Autocorrelation order	Value-weighted		Equally-weighted	
	MKT	*FIRM*	*MKT*	*FIRM*
ρ_1	0.321	0.299	0.472	0.254
ρ_2	0.206	0.327	0.162	0.133
ρ_4	0.125	0.125	0.077	0.051
ρ_{12}	0.034	0.007	0.091	-0.072

Table 2
Unit Root Tests

This table reports Dickey-Fuller (1997) augmented unit-root *t*-tests for monthly volatilities series constructed from daily data. *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). We present statistics for equally weighted variances and for variances weighted by the market value of stocks. The unit-root tests are based on regressions that include a constant or a constant and a time trend. The critical values are for the Dickey-Fuller *t*-test. The number of lags was determined empirically in order to remove serial correlation.

	Value-weighted		Equally-weighted	
	MKT	*FIRM*	*MKT*	*FIRM*
Constant				
t -statistic	-4.381	-4.664	-4.741	-4.726
Critical value at 1%	-3.465	-3.465	-3.465	-3.465
Critical value at 5%	-2.876	-2.876	-2.876	-2.876
Lag order	5	4	4	4
Constant and trend				
t -statistic	-4.585	-5.340	-5.056	-4.723
Critical value at 1%	-4.007	-4.007	-4.007	-4.007
Critical value at 5%	-3.434	-3.434	-3.433	-3.433
Lag order	5	4	4	4

Table 3
Volatility Components: Descriptive Statistics and Linear Trends

This table reports summary statistics for annualized monthly (quarterly) volatility series computed with squared daily (weekly) deviations. *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). Means and standard deviations of the monthly variances are multiplied 100. *Linear Trend* is the slope coefficient of a linear trend regression multiplied by 10^2 or 10^4?. Significance is assessed with a Vogelsang test statistic (*PS*-statistic) for deterministic linear trends. * denotes significance at 10% level. We present statistics for value and equally weighted variances.

	MKT	*FIRM*	*MKT*	*FIRM*
	value-weighted		equally-weighted	
Daily				
Mean * 10^2	5.53	17.16	3.05	14.17
Standard Deviation * 10^2	13.67	35.05	7.91	17.70
Linear Trend * 10^2	-0.05	-0.19	-0.05	-0.02
PS-statistic	-1.125	-1.991	-1.177	-0.385
Weekly				
Mean * 10^2	7.33	14.41	3.97	12.51
Standard Deviation * 10^2	19.71	22.93	8.12	14.18
Linear Trend * 10^2	0.09	-0.14	-0.06	-0.05
PS-statistic	-1.251	-1.364	-1.431	-0.853

Table 4
Total Volatility Mean and Variance Decomposition

This table shows the shares of mean and variance decomposition of total volatility of a typical stock (σ^2_{jt}) for the entire sample period and for sub-periods. σ^2_{mt} refers to market-level volatility; $\sigma^2_{\eta t}$ refers to firm-level volatility. For the mean of volatility, the decomposition is such that $E(\sigma^2_{mt})/E(\sigma^2_{jt}) + E(\sigma^2_{\eta t})/E(\sigma^2_{jt}) = 1$. For the variance of volatility, the decomposition is such that $Var(\sigma^2_{mt})/Var(\sigma^2_{jt}) + Var(\sigma^2_{\eta t})/Var(\sigma^2_{jt}) + 2\,Cov(\sigma^2_{mt}, \sigma^2_{\eta t})/Var(\sigma^2_{jt}) = 1$. *MKT* is an estimate of σ^2_{mt} calculated using equation (8); *FIRM* is an estimate of $\sigma^2_{\eta t}$, calculated using equations (9) and (10). The conditional series of *MKT* and *FIRM* are obtained by regressing the detrended volatility series on four lags of the two detrended volatility series. All estimates are value-weighted monthly variances computed with squared daily deviations.

	MKT	*FIRM*	*MKT*	*FIRM*
	Value-weighted		Equally-weighted	
Mean				
1986 – 2002	0.244	0.756	0.177	0.823
1986 – 1992	0.238	0.762	0.281	0.719
1993 – 1997	0.139	0.861	0.068	0.932
1998 – 2002	0.349	0.651	0.095	0.905
Variance				
Raw series				
MKT	0.086	0.352	0.113	0.321
FIRM		0.562		0.566
Conditional series				
MKT	0.077	0.386	0.068	0.340
FIRM		0.537		0.592

Table 5
Correlation Structure and Granger-causality Tests

This table reports contemporaneous correlation structure (Panel A) of the monthly volatility measures constructed from daily data. Panel B shows the p-values of Granger-causality VAR tests. The *p*-values refer to the *F*-test of the null hypothesis that the lags of 1 to *k* (in parentheses) of each variable are jointly equal to zero. *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). The volatility measures are value-weighted variances.

Panel A. Correlation

value-weighted		equally-weighted	
MKT	*FIRM*	*MKT*	*FIRM*
1.000	0.807	1.000	0.638
	1.000		1.000

Panel B. *p*-values of Granger-causality tests

	Bivariate VAR	
	MKT_t	$FIRM_t$
$MKT_{t\text{-}k}$	–	0.1162 (3)
$FIRM_{t\text{-}k}$	0.0000 (3)	–

Table 6
Diversification Benefits against Number of Stocks

This table shows the excess standard deviation of a portfolio for randomly selected portfolios containing 2, 5, 15 and 30 stocks for the entire sample period and for three sub-periods (January 1986-December 1992; January 1993-December 1997 and January 1998-December 2002). The excess standard deviation of a portfolio is defined as the difference between the portfolio's standard deviation and the standard deviation of an equally weighted index containing all stocks used in the calculations as constituents. Excess standard deviation is calculated from daily data within the years and annualized.

	2 Stocks	5 Stocks	15 Stocks	30 Stocks
1987 – 2002	0.1965	0.0978	0.0331	0.0106
1987 – 1992	0.2114	0.0943	0.0289	0.0044
1993 – 1997	0.1974	0.1075	0.0390	0.0139
1998 – 2002	0.1778	0.0922	0.0313	0.0111

Table 7
Cyclical Behaviour

This table reports the correlation of *MKT* and *FIRM* with cyclical indicators. Panel A shows correlation with a dummy variable that is one during an expansion and zero during a recession. Panel B shows correlation with GDP growth. The volatility measures are value-weighted variances computed with squared daily deviations, aggregated to a quarterly frequency. The cyclical indicators are measured with j lags (up to 4 quarters) relative to the volatility series; thus the correlation with positive (+) js at the first column of each panel measure the extent to which the volatility series lead the business cycle and the negative (-) js at the bottom of each panel measure the extent to which the volatility series lag the business cycle. The largest correlation (in absolute value) for each column is indicated in bold. For each volatility measure ($v_t = MKT_t$ or $FIRM_t$), we calculate conditional expectations $E_{t-1}(v_t)$ by regressing the volatility series on four lags of the two volatility measures. The innovation to volatility ξ_t, is defined as $v_t - E_{t-1}(v_t)$. *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). The output data, obtained on a quarterly basis from INE (Bureau of Portuguese Statistics), range from 1988 to 2002.

Panel A. Correlation with Bivariate Cyclical Indicator (Dummy Variable)

Volatility Lead/Lag (Quarters)	*MKT*			*FIRM*		
	v_t	$E_{t-1}(v_t)$	ξ_t	v_t	$E_{t-1}(v_t)$	ξ_t
+4	0.081	0.063	0.025	-0.108	-0.087	-0.030
+2	0.069	0.063	0.011	-0.105	-0.030	**-0.076**
+1	0.085	0.025	0.066	0.045	0.090	-0.047
0	0.031	**0.185**	**-0.152**	**0.162**	**0.120**	0.027
-1	**0.097**	0.078	0.064	0.106	0.073	0.071
-2	**0.097**	0.071	0.095	0.059	0.062	-0.021
-4	0.046	0.030	0.058	0.032	0.022	0.022

Panel B. Correlation with GDP Growth

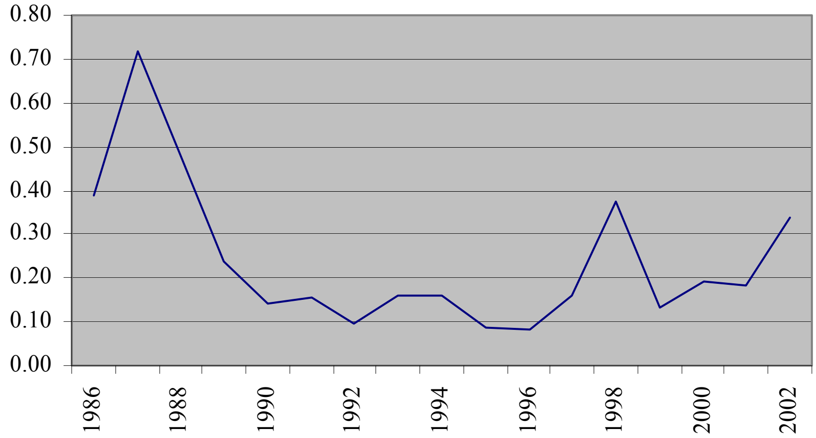
Volatility Lead/Lag (Quarters)	*MKT*			*FIRM*		
	v_t	$E_{t-1}(v_t)$	ξ_t	v_t	$E_{t-1}(v_t)$	ξ_t
+4	0.049	0.065	-0.012	-0.002	0.034	-0.034
+2	-0.002	-0.097	0.093	-0.085	-0.128	0.047
+1	0.042	-0.087	0.129	-0.060	0.072	**-0.135**
0	-0.092	0.026	-0.121	0.072	0.074	-0.013
-1	-0.033	-0.002	-0.103	-0.015	0.015	-0.079
-2	-0.015	-0.055	**0.146**	0.030	0.009	0.053
-4	**0.190**	**0.171**	0.070	**0.191**	**0.162**	0.043

Table 8
Forecasting GDP growth

This table reports OLS regressions with GDP growth as the dependent variable *(ΔGDP_t)*. *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). The two volatility measures are value-weighted variances computed with squared daily deviations, aggregated to a quarterly frequency. *RM* denotes the quarterly return on the all stocks value-weighted portfolio. The output data, obtained on a quarterly basis from INE (Bureau of Portuguese Statistics), range from 1988 to 2002. Coefficients are reported with Newey-West (1994) *t*-statistics in parentheses. The last column reports the R^2 and the *p*-value for a test of the joint significance of the two volatility measures.

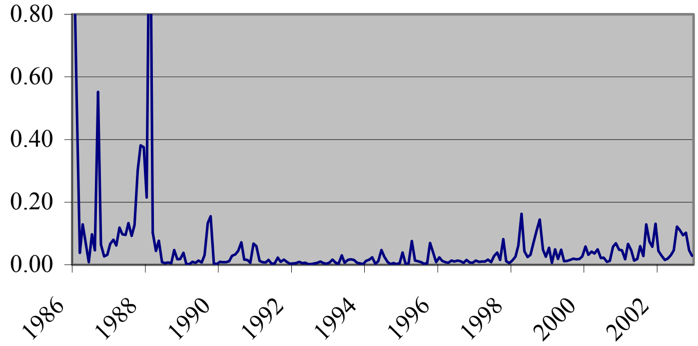
ΔGDP_{t-1}	RM_{t-1}	MKT_{t-1}	$FIRM_{t-1}$	R^2 (*p*-value)
-0.438 (-4.259)	0.023 (1.698)			21.2%
-0.451 (-4.476)	0.021 (2.097)	0.524 (2.701)		24.4%
-0.449 (-4.627)	0.024 (1.908)		0.176 (1.725)	23.0%
-0.454 (-4.596)	0.027 (2.137)	0.407 (1.685)	0.098 (0.755)	23.7% (0.001)

Figure 1
Volatility of the Portuguese Stock Market



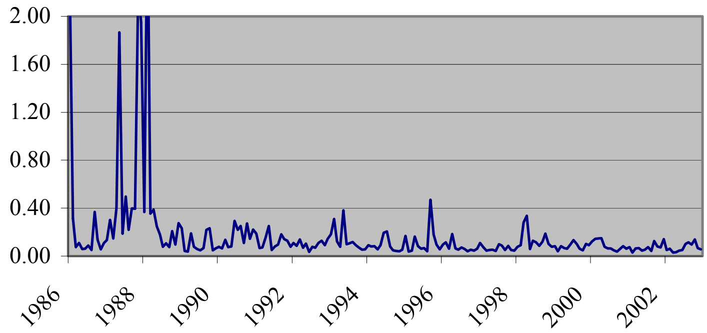
This figure shows the annualized standard deviation of monthly returns for the Portuguese stock market (value-weighted returns of the stocks in sample) for the period from 1986 to 2002.

Figure 2
Market Volatility
- Daily Data (value-weighted) -



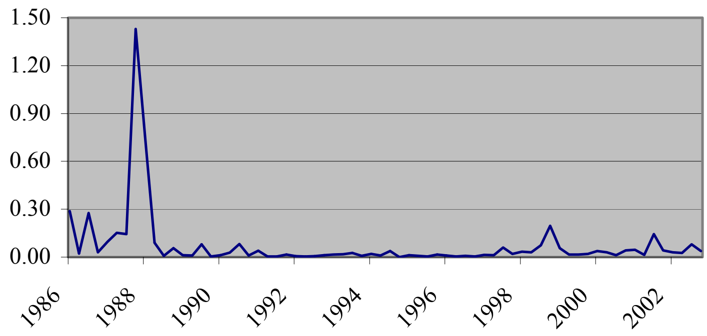
This figure shows annualized variance within each month of daily market (value-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

Figure 3
Firm Volatility
- Daily Data (value-weighted) -



This figure shows annualized variance within each month of daily firm (value-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

Figure 4
Market Volatility
- Weekly Data (value-weighted) -



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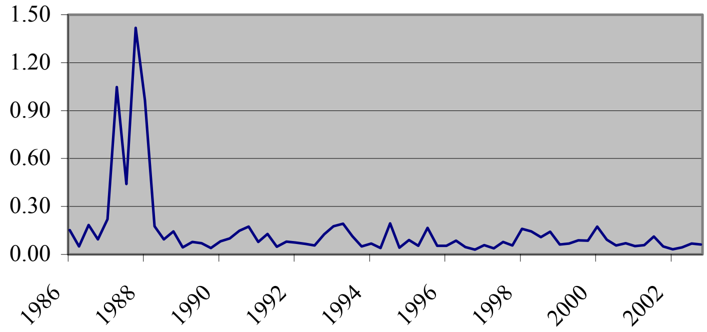
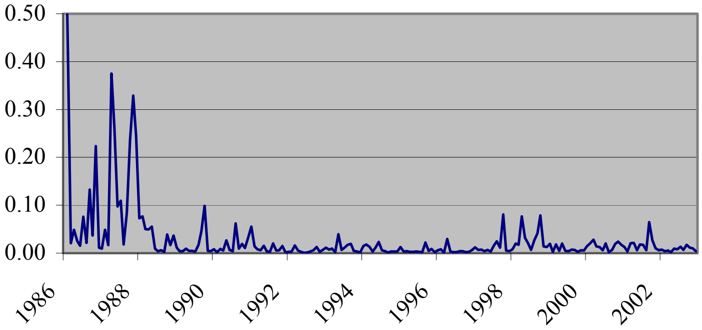
This figure shows annualized variance within each quarter of weekly market (value-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

Figure 5
Firm Volatility
- Weekly Data (value-weighted) -



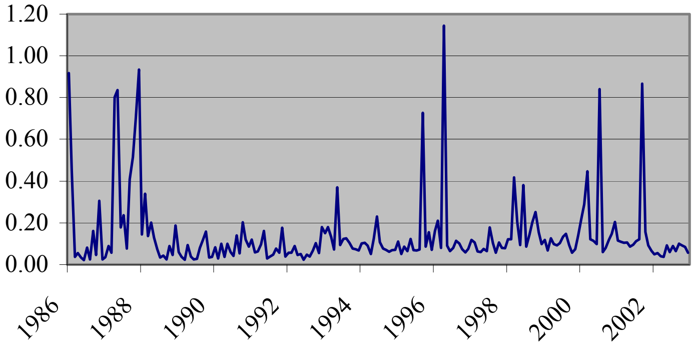
This figure shows annualized variance within each quarter of weekly firm (value-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

Figure 6
Market Volatility
- Daily Data (equally-weighted) -



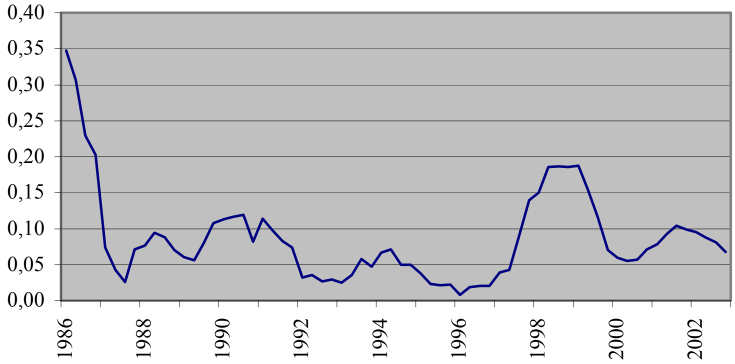
This figure shows annualized variance within each month of daily market (equally-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

Figure 7
Firm Volatility
- Daily Data (equally-weighted) -



This figure shows annualized variance within each month of daily firm (equally-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

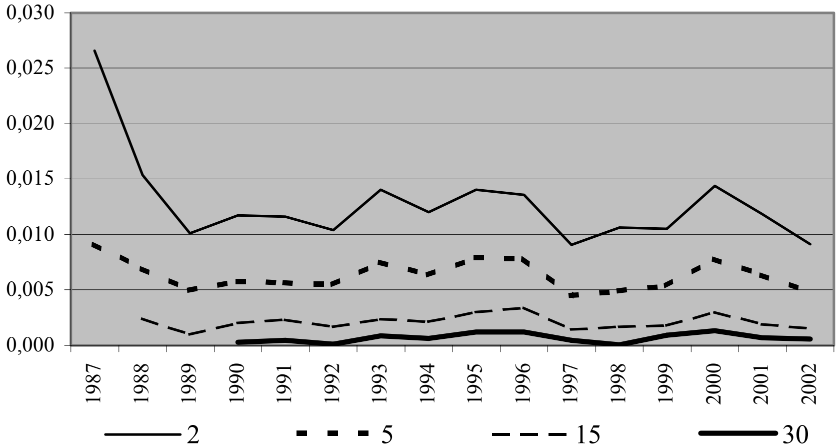
Figure 8
Average Correlation among Individual Stocks



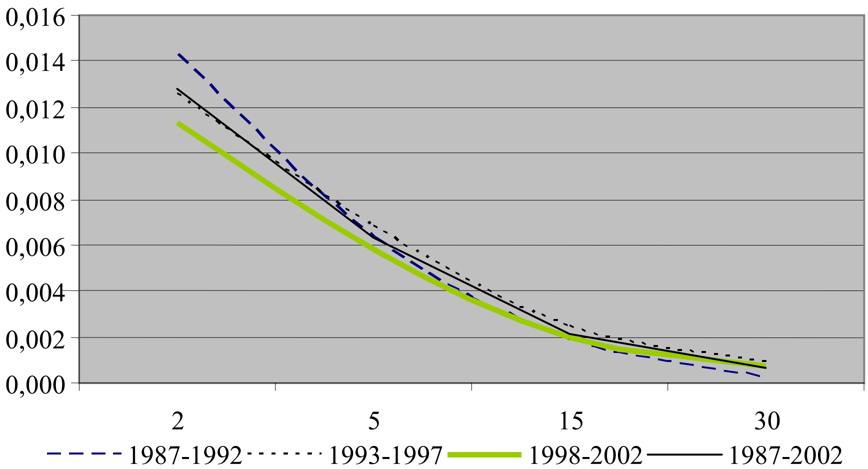
This figure shows equally weighted average pairwise correlation across individual stocks traded on the Portuguese Stock Exchange, for the period from 1986 to 2002. The estimates of the correlation coefficients were computed on a quarterly basis over the past 12 months of daily data.

Figure 9
Diversification Benefits against Time and Number of Stocks

Panel A. Excess Standard Deviation against Time



Panel B. Excess Standard Deviation against Number of Stocks



The figure plots the excess standard deviation of a portfolio against time and number of constituents stocks. The excess standard deviation of a portfolio is defined as the difference between the portfolio's standard deviation and the standard deviation of an equally weighted index containing all stocks used in the calculations as constituents. In Panel A the annualized excess standard deviation is calculated from daily data within the year, for randomly selected portfolios containing 2, 5, 15 and 30 stocks. In Panel B, the excess standard deviation of a portfolio is plotted against the number of stocks and compares the diversification risk reduction for the all sample period with three sub-periods (January 1986-December 1992; January 1993-December 1997 and January 1998-December 2002).

Appendix A - Additional Tables and Graphs

Table A1
Descriptive Statistics for the Market and Industry Portfolios

# indicates the number of constituent stocks in the market and in each industry. Weight indicates the weight of each industry in the market.

Industry	12/1985		12/1992		12/1997		12/2002		Average	
	#	Weight	#	weight	#	weight	#	weight	#	weight
Banks	0	0.0%	7	14.9%	13	17.3%	9	17.0%	18	19.4%
Telecom Services	1	20.0%	1	2.1%	2	2.7%	2	3.8%	3	3.2%
Equipment Manufacturing	2	40.0%	7	14.9%	11	14.7%	9	17.0%	13	14.0%
Miscellaneous Manufacturing	1	20.0%	13	27.7%	17	22.7%	8	15.1%	18	19.4%
Retailers	0	0.0%	2	4.3%	3	4.0%	3	5.7%	3	3.2%
Information Technology	0	0.0%	2	4.3%	2	2.7%	6	11.3%	7	7.5%
Building Contractors and Related	0	0.0%	2	4.3%	6	8.0%	5	9.4%	7	7.5%
Insurance	0	0.0%	3	6.4%	4	5.3%	0	0.0%	4	4.3%
Other Services	0	0.0%	2	4.3%	5	6.7%	5	9.4%	7	7.5%
Tourism and Leisure	0	0.0%	4	8.5%	7	9.3%	3	5.7%	8	8.6%
Real Estate	1	20.0%	2	4.3%	3	4.0%	1	1.9%	3	3.2%
Transportation	0	0.0%	2	4.3%	2	2.7%	2	3.8%	2	2.2%
TOTAL	5		47		75		53		93	

Table A2

Volatility Components: Descriptive Statistics and Linear Trends in Sub-periods

This table reports summary statistics for annualized monthly (quarterly) volatility series computed with squared daily (weekly) deviations for three sub-periods (January 1986-December 1992; January 1993-December 1997 and January 1998-December 2002). *MKT* refers to market-level volatility (calculated using equation (8)); *FIRM* refers to firm-level volatility (calculated using equations (9) and (10)). Means and standard deviations of the monthly variances are multiplied 100. We present statistics for value and equally weighted variances.

	MKT	*FIRM*	*MKT*	*FIRM*
	value-weighted		equally-weighted	
JANUARY 1986-DECEMBER 1992				
Daily				
Mean * 10^2	8.84	28.24	5.61	14.34
Standard Deviation * 10^2	20.56	52.22	11.77	20.03
Linear Trend * 10^2	-0.33	-0.64	-0.22	-0.29
Weekly				
Mean * 10^2	6.22	22.79	7.49	14.93
Standard Deviation * 10^2	29.72	33.89	11.78	20.03
Linear Trend * 10^2	-0.43	-0.50	-0.24	-0.33
JANUARY 1993-DECEMBER 1997				
Daily				
Mean * 10^2	1.59	9.83	0.94	12.93
Standard Deviation * 10^2	1.70	8.05	1.21	16.43
Linear Trend * 10^2	0.01	-0.14	0.00	-0.01
Weekly				
Mean * 10^2	1.54	8.45	1.28	10.68
Standard Deviation * 10^2	13.87	5.43	1.11	8.96
Linear Trend * 10^2	0.01	-0.16	-0.01	-0.01
JANUARY 1998-DECEMBER 2002				
Daily				
Mean * 10^2	4.82	9.00	1.59	15.19
Standard Deviation * 10^2	3.69	5.40	1.58	15.54
Linear Trend * 10^2	0.03	-0.12	-0.03	-0.14
Weekly				
Mean * 10^2	4.95	8.65	1.75	10.95
Standard Deviation * 10^2	4.60	4.09	1.79	6.66
Linear Trend * 10^2	-0.02	-0.15	-0.03	-0.12

Table A3
Volatility Components (MKT+IND+FIRM): Descriptive Statistics and Linear Trends

This table reports summary statistics for annualized monthly volatility series computed with squared daily deviations for the periods of January 1986-December 2002. *MKT* refers to market-level volatility (calculated using equation (B15)); *IND* refers to industry-level volatility (calculated using equation (B16) and (B17)); *FIRM* refers to firm-level volatility (calculated using equations (B19), (B19) and (B20)). Means and standard deviations of the monthly variances are multiplied 100. *Linear Trend* is the slope coefficient of a linear trend regression multiplied by 10^2 or 10^4?. Significance is assessed with a Vogelsang test statistic (*PS*-statistic) for deterministic linear trends. * denotes significance at 10% level. We present statistics for value and equally weighted variances.

	MKT	IND value-weighted	*FIRM*
Mean * 10^2	5,53	9,51	7,69
Standard Deviation * 10^2	13,67	25,22	10,29
Linear Trend * 10^2	-0,05	-0,14	-0,03
PS-statistic	-1.125	-1.892	-1.118

Table A4
Total Volatility Mean Decomposition
- Considering Industry Effects -

This table shows the shares of mean and variance decomposition of total volatility of a typical stock (σ^2_{rt}) for the entire sample period and for sub-periods. σ^2_{mt} refers to market-level volatility; $\sigma^2_{\varepsilon t}$ refers to industry-level volatility; $\sigma^2_{\eta t}$ refers to firm-level volatility. *MKT* is an estimate of σ^2_{mt} calculated using equation (B15). *IND* is an estimate of $\sigma^2_{\varepsilon t}$ calculated using equation (B16 and B17); *FIRM* is an estimate of $\sigma^2_{\eta t}$ calculated using equations (B18), (B19) and (B20). All estimates are value-weighted monthly variances computed with squared daily deviations. For the mean of volatility, the decomposition is such that:

$$E(\sigma^2_{mt}) / E(\sigma^2_{rt}) + E(\sigma^2_{\varepsilon t}) / E(\sigma^2_{rt}) + E(\sigma^2_{\eta t}) / E(\sigma^2_{rt}) = 1.$$

For the variance of volatility, the decomposition is such that:

$$Var(\sigma^2_{mt}) / Var(\sigma^2_{rt}) + Var(\sigma^2_{\varepsilon t}) / Var(\sigma^2_{rt}) + Var(\sigma^2_{\eta t}) / Var(\sigma^2_{rt}) + [2\,Cov(\sigma^2_{mt}, \sigma^2_{\eta t}) + 2\,Cov(\sigma^2_{mt}, \sigma^2_{\varepsilon t}) + 2\,Cov(\sigma^2_{\varepsilon t}, \sigma^2_{\eta t})] / Var(\sigma^2_{rt}) = 1.$$

	MKT	*IND*	*FIRM*
Mean			
1986 – 2002	0.243	0.419	0.338
1986 – 1992	0.245	0.510	0.245
1993 – 1997	0.128	0.223	0.649
1998 – 2002	0.339	0.264	0.397
Variance			
Raw Series			
MKT	0.096	0.249	0.080
IND		0.328	0.192
FIRM			0.055

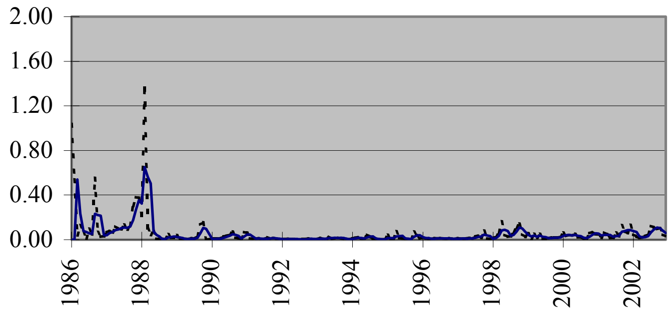
Table A5
Volatility Components: Descriptive Statistics in Individual Industries

This table reports summary statistics for monthly volatility series computed with squared daily deviations. *IND* refers to industry-level volatility (calculated using equation (B16 and B17)); *FIRM* refers to firm-level volatility (calculated using equations (B18), (B19) and (B20)). Statistics are for value-weighted variances. Time-series means and standard deviations of the monthly variances are annualized and multiplied by 100. # indicates the number of constituent stocks in each industry.

Industry	#	*IND*		*FIRM*	
		Mean	StDev	Mean	StDev
Banks	18	2.53	18.07	3.86	27.65
Telecom Services	3	0.58	1.89	0.63	1.89
Equipment Manufacturing	13	1.99	17.76	3.01	21.39
Miscellaneous Manufacturing	18	0.51	2.00	1.19	2.98
Retailers	3	2.29	13.43	2.38	13.43
Information Technology	7	1.14	4.08	1.46	4.28
Building Contractors and Related	7	0.98	3.62	1.36	4.23
Insurance	4	0.46	1.69	0.73	2.71
Other Services	7	0.41	0.53	0.61	0.64
Tourism and Leisure	8	0.60	1.35	1.30	2.77
Real Estate	3	1.05	4.26	1.63	7.99
Transportation	2	1.88	6.67	2.12	6.73

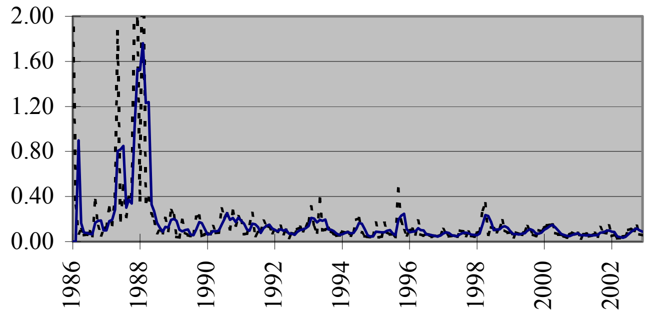
Appendix A - Additional Tables and Graphs (ctd.)

Figure A1
Market Volatility
- *Moving Average, daily data* (value-weighted) -



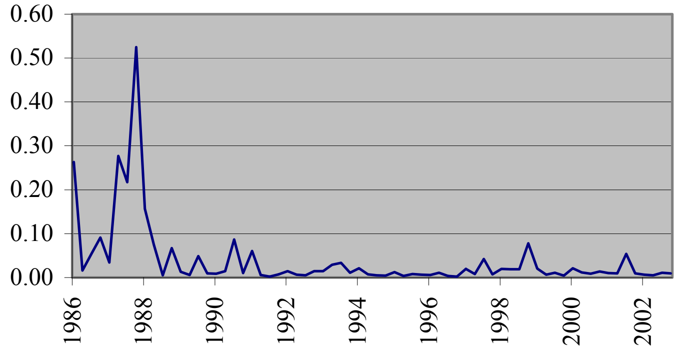
This figure shows annualized variance within each month of daily market (value-weighted) returns (dashed line), calculated using equation (8), for the period from 1986 to 2002. The solid line refers to the backwards 3-month moving average.

Figure A2
Firm Volatility
- *Moving Average, daily data* (value-weighted) -



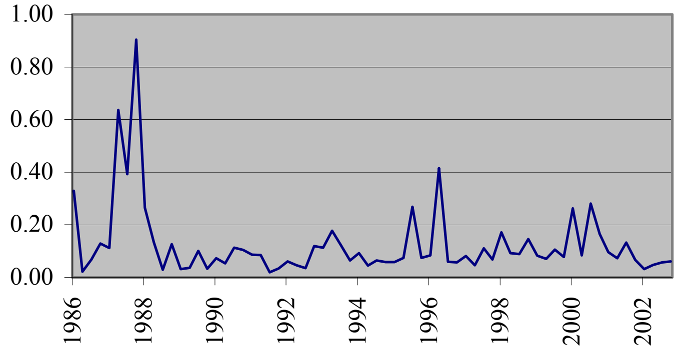
This figure shows annualized variance within each month of daily firm (value-weighted) returns (dashed line), calculated using equation (9) and (10)), for the period from 1986 to 2002. The solid line refers to the backwards 3-month moving average.

Figure A3
Market Volatility
- Weekly Data (equally-weighted) -



This figure shows annualized variance within each quarter of weekly market (equally-weighted) returns, calculated using equation (8), for the period from 1986 to 2002.

Figure A4
Firm Volatility
- Weekly Data (equally-weighted) -



This figure shows annualized variance within each quarter of weekly firm (equally-weighted) returns, calculated using equation (9) and (10)), for the period from 1986 to 2002.

Appendix B – Volatility Decomposition into Market, Industry and Firm Specific Components

As in Campbell *et al.* (2001) we first consider the return decomposition for a particular industry assuming a simplified return decomposition.

The return of an industry *i* is decomposed into a market-wide return and an industry specific residual based on the market-adjusted return model:

$$R_{it} = R_{mt} + \varepsilon_{it} . \tag{B1}$$

where ε_{it} is the difference between the industry return R_{it} and the market return R_{mt}.

The return of a stock *j* is decomposed into a industry-specific return and a firm specific residual based on the market-adjusted return model:

$$R_{jit} = R_{it} + \eta_{jit} , \tag{B2}$$

where η_{it} is the difference between the stock return R_{jt} and the industry return R_{it}.

In a CAPM framework industry and firm excess returns are expressed as:

$$R_{it} = \beta_{im} R_{mt} + \tilde{\varepsilon}_{it} , \tag{B3}$$

$$R_{jit} = \beta_{ji} R_{it} + \tilde{\eta}_{jit} = \beta_{ji} \beta_{im} R_{mt} + \beta_{ji} \tilde{\varepsilon}_{it} + \tilde{\eta}_{jit} , \tag{B4}$$

As such,

$$\varepsilon_{it} = \tilde{\varepsilon}_{it} + (\beta_{im} - 1) R_{mt} . \tag{B5}$$

$$\eta_{it} = \tilde{\eta}_{jit} + (\beta_{ji} - 1) R_{it} \tag{B6}$$

and

$$Var(R_{it}) = \beta_{im}^2 Var(R_{mt}) + Var(\tilde{\varepsilon}_{it}), \tag{B7}$$

$$Var(R_{jit}) = \beta_{jm}^2 Var(R_{mt}) + \beta_{ji}^2 Var(\tilde{\varepsilon}_{it}) + Var(\tilde{\eta}_{jit}) \tag{B8}$$

In equations B3, B5 and B7, β_{im} is the market beta for industry i and $\tilde{\varepsilon}_{it}$ is the industry specific residual. R_{mt} and $\tilde{\varepsilon}_{it}$ are orthogonal by construction. The residual $\tilde{\varepsilon}_{it}$ equals ε_{it} only when industry beta is 1 or the market return is zero.

In equations B4, B6 and B8, β_{ji} is the industry beta for stock j and $\tilde{\eta}_{jit}$ is the firm specific residual. $\tilde{\eta}_{jit}$ is orthogonal to R_{it}, R_{mt} and $\tilde{\varepsilon}_{it}$ by construction. Therefore the industry beta of a particular stock (β_{jm}) is given by $\beta_{ji}\beta_{im}$.

For the simplified version the variance of an individual industry is given by:

$$Var\,(R_{it}) = Var\,(R_{mt}) + Var\,(\varepsilon_{it}) + 2Cov\,(R_{mt}, \varepsilon_{it})$$

$$= Var\,(R_{mt}) + Var\,(\varepsilon_{it}) + 2(\beta_{im} - 1)Var\,(R_{mt}), \quad \text{(B9)}$$

Similarly, the variance of an individual stock is given by:

$$Var(R_{jit}) = Var(R_{it}) + Var(\eta_{jit}) + 2Cov(R_{it}, \eta_{jit})$$

$$= Var(R_{it}) + Var(\eta_{jit}) + 2(\beta_{ji} - 1)Var(R_{it}) \quad \text{(B10)}$$

These decompositions require the estimation of betas but this problem is bypassed given that we are concerned with average variances across industries or stocks. Given that

$$\sum_i w_{it}\beta_{im} = 1, \quad \sum_{j\in i} w_{jit}\beta_{ji} = 1 \quad \text{(B11)}$$

there is no need to estimates betas.

The volatility of a typical industry can be computed as:

$$\sum_i w_{it} Var(R_{it}) = Var(R_{mt}) + \sum_i w_{it} Var(\varepsilon_{it})$$

$$= \sigma_{mt}^2 + \sigma_{\varepsilon t}^2, \quad \text{(B12)}$$

where $\sigma_{mt}^2 \equiv Var(R_{mt})$ e $\sigma_{\varepsilon t}^2 \equiv \sum_i w_{it} Var(\varepsilon_{it})$. The weighted average $\sum_i w_{it} Var(R_{it})$ can be interpreted as the volatility of a random drawn industry (where the probability of drawing industry i is equal to its weight w_i).

The volatility of a typical stock in a particular industry can be computed as:

$$\sum_{j \in i} w_{jit} Var(R_{jit}) = Var(R_{it}) + \sigma^2_{\eta it} , \tag{B13}$$

where the weighted average $\sigma^2_{\eta it} \equiv \sum_{j \in i} w_{jit} Var(\eta_{jit})$ can be interpreted as the volatility of a random drawn of stock *j* out of industry *i*(where the probability of drawing stock *j* is equal to its weight w_{ij}). Averaging (B13) across industries we obtain:

$$\begin{aligned} \sum_i w_{it} \sum_{j \in i} w_{jit} Var(R_{jit}) &= \sum_i w_{it} Var(R_{it}) + \sum_i w_{it} \sum_{j \in i} w_{jit} Var(\eta_{jit}) \\ &= Var(R_{mt}) + \sum_i w_{it} Var(\varepsilon_{it}) + \sum_i w_{it} \sigma^2_{\eta it} \\ &= \sigma^2_{mt} + \sigma^2_{\varepsilon t} + \sigma^2_{\eta t} , \end{aligned} \tag{B14}$$

where $\sigma^2_{\eta t} \equiv \sum_i w_{it} \sigma^2_{\eta it} = \sum_i w_{it} \sum_{j \in i} w_{jit} Var(\eta_{jit})$ is the volatility of a random drawn of stock *j* out of the entire sample.

To estimate the volatility components with monthly estimates constructed with daily data. The sample volatility estimate of the market return in month *t* (MKT_t), is computed as

$$MKT_t = \hat{\sigma}^2_{mt} = \sum_{s \in t} \left(R_{ms} - \mu_m \right)^2 \tag{B15}$$

where μ_m is the daily mean of the market return (R_{ms}) over the sample period. *s* denotes the trading days in a particular month. We construct value (and equally)-weighted estimates of market-wide effects using all firms in the sample.

The measure for average industry volatility is given by

$$IND_t = \sum_i w_{it} \hat{\sigma}^2_{\varepsilon it} . \tag{B16}$$

This the weighted average across industries of the monthly estimates of each industry effect.

$$\hat{\sigma}^2_{\varepsilon it} = \sum_{s \in t} \varepsilon^2_{is} . \tag{B17}$$

The measure of individual stock or firm volatility is obtained as:

$$FIRM_t = \sum_i w_{it} \hat{\sigma}^2_{\eta it} . \tag{B18}$$

This is the weighted average across industries of the monthly estimates of firm effects in each industry given by:

$$\hat{\sigma}^2_{\eta it} = \sum_{j \in i} w_{jit} \hat{\sigma}^2_{\eta jit} \tag{B19}$$

Each of the monthly estimates of firm effects in a particular industry is the weighted average across firms of the monthly estimates of each firm effect

$$\hat{\sigma}^2_{\eta jit} = \sum_{s \in t} \eta^2_{jis} . \tag{B20}$$