Liquidity Supply in multiple markets?[1]

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Abstract

Using an inventory model based on Ho and Stoll (1983), this paper examines how two competing risk-averse dealers supply liquidity in two different market systems. We find that price formation and market spreads are directly impacted by the way order flows are correlated in systems. If order flows are negatively correlated, dealers expect to better manage their inventory position and markets spreads reduce. When order flows are positively correlated, dealers are more likely to be touched on the same side which increases their inventory risk and market spreads increase. Further, this model sheds new light on some empirical results (Gresse [2001], Hansh [2001] or Werner and Kleidon [1996]) and it has some new direct empirical predictions.

Keywords: Hybrid Market, Market competition, Interdealer trading, Market fragmentation.

EFM Classification code: 360.

1 Introduction

For a few decades, the proliferation of electronic trading systems, as a principal or an alternative trading venue, has stirred up the whole security industry. Financial markets seem to converge towards so-called "hybrid" organizations. In these structures, investors have the choice of routing their orders either towards a quote-driven market, where they negociate their trades with market markers, or towards an order-driven market, without intermediation (represented by one or several limit order books).

Most of order-driven markets, like Euronext, have developed an upstairs market, on which large size orders (*i.e.* "block trades") can be negociated with dealers. In parallel, markets which were traditionally dealership markets, like the Nasdaq or the London Stock Exchange, or with a Specialist like the New York Stock Exchange, now compete with order-driven "Electronic Communication Networks" (ECNs). Besides, most of quote-driven markets have recently adopted several reforms aimed at offering to their customers an hybrid organization in their own trading system. For intance, the London Stock Exchange has introduced in October 1997 an electronic limit order book , named SETS, for its most liquid securities. The Nasdaq has established an hybrid electronic system ("SuperMontage"), that organizes the competition between prices quoted by the dealers and a limit order book, while the New York Stock Exchange is currently re-thinking its operation mode to integrate the latest technological innovations. Finally, two merger's projects between a dominant stock exchange and an ECN are being launched in 2005, the NYSE and Archipelago on the one hand, and the Nasdaq and Instinet on the other hand. This convergence toward hybrid organizations, which are simulataneously an order- and quote-driven market, has spurred considerable interest and raises several questions.

The emergence of multiple markets regrouping different trading systems, which operate independantly and simultaneously, reopens the debate initiated by Hamilton (1979), on the benefits of competition between market centers and the drawbacks of the order flow fragmentation (*cf.* Pagano (1989), Madhavan (1995)). Recently, researchers have extended this controversy to the complex environment of hybrid markets, by modeling the impact of the differences in market structure on the performance of competing trading systems (*cf.* Henderschott and Mendelson (2000), Viswanathan and Wang (2002), Parlour and Seppi (2003) or Sabourin (2004)). These models all rely on the assumption that there exists, in each trading venue, a pool of agents who supply liquidity *independantly.*

Some recent empirical studies however suggest that in multiple markets, liquidity suppliers submit orders, or quote prices *inter-dependantly* in different systems. Dufour and Noel (2005) give evidence of the phenomenon for the London Stock Exchange, in which dealers in the quote-driven system compete with the limit order book SETS. Their results "suggest the existence of a pool of unexpressed liquidity outside SETS and that dealers are continuously monitoring the state of the order-book for profitable trading opportunities". The coexistence of two or many trading systems would thus enable market makers to supply liquidity in different trading venues. A case in point is how regulators, who aim at improving market performance, should react to this phenomenon. Our objective in this paper is to study the impact of different regulation's projects on the quality of hybrid markets, when the liquidity supply in several trading venues, which are operating simultaneously, is not independant.

Many researchers have studied the impact of the existence of an interdealer market on the spreads quoted in a dealership market (*cf.* Werner (1997), Saporta (1997) or Viswanathan and Wang (2004)). Werner (1997) for instance analyzes the introduction of an interdealer market organized as a limit order book, in which the market maker who has executed the order flow in the quote-driven market has the opportunity to submit an order, thus to share his risk with the other dealers. She predicts that the public quoted spread is lower than in the absence of the interdealer market. This later possibility has been empirically studied by Reiss and Werner (2003), who analyze the interactions between four anonymous interdealer limit order books operating on the London Stock Exchange, and the dealers' spreads. They note that "*although these anonymous systems seemingly favored dealers over other brockers and the public, the U.K. regulators permitted them on the grounds that they would reduce dealer inventory risk and thereby improve liquidity. The empirical evidence here and in Reiss and Werner (1998) lends to support to this logic*". However, these models of interdealer trading are, quite naturally, characterized by a sequential quote submission: dealers submit orders in an alternative trading venue only after the execution of a large order flow in the quote-driven market.

But, as Dufour and Noel (2005) highlight, "*dealers are afforded the opportunity to strategically enter into profitable trades and manage their inventory risks through either pre- or post-positioning of orders [on SETS]*". Our paper builds on this idea that in multiple markets, the dealers of the main trading system do not simply use an alternative trading venue to share risks that would be due to an inventory shock, but more generally act as liquidity suppliers in many trading systems.

In reality, when several markets are open simultaneously, the arrival of order flows in each market is extremely fast. Trading venues indeed adopt new technologies. Besides, they have to adapt to a seeminlingly growing demand from investors. Robert Greifeld, CEO and President of the Nasdaq Stock Market, testifies in 2003 that "*Speed is the critical consideration for many market participants [...]. At NASDAQ, the speed of execution is faster than ever and the spreads are tighter than ever*".[1] Furthermore, the *Securities and Exchanges Commission* has adopted in November 2000 the rule SEC 11 Ac1-5. This rule, named "*Dash 5*", imposes on execution centers the publication of a monthly report on their execution quality. This report includes traditional measures of the execution quality, like the quoted spread or the effective spread, but also information on the execution speed. For instance, the agregated "*Dash 5*" statistics published by the SEC in December 2004 show that the mean execution speed for a market order on a S&P 100 stock, which size is inferior to 499 shares (resp. between 500 and 1999 shares), is 9.9 seconds (resp. 12.9 seconds) in the NYSE and 1.3 seconds (resp. 2.2 seconds) in the Nasdaq. The execution speed has thus become one of the criteria used to evaluate market quality. In practice, when dealers trade in one trading venue, they may not have the opportunity to *cancel* an order submitted in an alternative trading system before its execution.

[1]Testimony of Robert Greifeld Before the Subcommittee on Capital Markets, Insurance and Government Sponsored Enterprises Of the House Financial Services Committee Hearing on Reviewing US Capital Market Structure: The New York Stock Exchange and Related Issues (October 16, 2003).

In this paper, we analyze the behavior of liquidity suppliers when, because of execution speeds, they do not have the time to cancel and revise theirs quotes before the arrival of the order flow in the alternative trading system. To this end, we consider two competing risk-adverse dealers, who simultaneously submit quotes in two quote-driven markets, named market A and market B, without knowing the direction of order flows in any of both trading venues. Unlike the existing literature, we thus study the *joint* liquidity supply of the market makers in two markets, when the arrival of order flows is simultaneous and correlated in two markets.

Our main results are as follows.

- Two effects impact the quotes of market makers in multiple markets: a “balancing” effect and a “dual-liability” effect. Dealers thus propose, in each market, a price which is different from the price that they would quote in a single market, for the same transaction size.
- Unlike the traditional paradigm, the dealer who initially has the longest (resp. shortest) inventory may be in position to be the first buyer (resp. seller) in one of the two trading systems.
- Dealers submit differentiated quotes in the two markets.
- The average best quoted spreads increase with the probability that the order flows in the two markets have the same direction.
- Obliging dealers to quote identical prices in the two markets increase the average best quoted spreads in the dominant market.

We thus show that the coexistence of two trading venues influence the price formation process in each market. We find that ask (resp. bid) prices quoted by market makers in each system depend on the transaction size, and on the initial inventory of the dealer who is in the shortest (resp. longest) position, as in Ho and Stoll (1983). Besides, in a multiple markets' environment, the quotes submitted by dealers in market A are impacted by the order flow that they expect to execute in market B, *i.e.* by the trade size and the direction of the order flow in this alternative trading system (and vice-versa in market B). Supplying liquidity in two trading venues indeed generates two opposite effects on the dealers' position in the risky asset.

On the one hand, dealers' quotes may be simultaneously touched on opposite sides, at the ask in one system and at the bid in the other, so that shocks in both systems tend to balance each other. Market makers thus expect to be able to better manage their inventory risk thanks to the existence of an alternative trading venue, which plays the role of an interdealer market. We name this effect a "balancing effect". Conversely, dealers' quotes may be simultaneously touched on the same side in the two systems. In this case, the dealers' inventory risk increases. We name this effect "*dual-liability*", since dealers are obliged to simultaneously execute two order flows which have the same direction. This second effect has not been studied in the literature.

The simultaneous set-up of our model thus enables us to shed light on some situations in which dealers' inventory risk increases in multiple markets. When, given execution speeds, dealers who have executed the order flow in one trading venue do not have the opportunity to cancel and resubmit their quotes before the arrival of the order flow in the alternative trading venue, they are exposed to a dual-liability risk. We show that dealers take this risk into account while choosing their quotes.

To illustrate this dual-liability phenomenon, we first study a benchmark model in which dealers have to quote identical prices in the two markets. In this case indeed, the dealer who submits the best ask price in one market also submit the best ask price in the second market. Consequently, this dealer cannot beneficiate from the balancing effect since his quotes will never be touched on opposite sides in the two markets, whatever the direction of the order flows. Conversely, if order flows in markets A and B have the same direction, the dealer is directly exposed to the dual-liability risk, since he must execute the whole order flow.

We then show that the minimal ask price at which this dealer is ready to trade an order flow of size Q_A in market A, when he expects a buy order flow Q_B in market B, corresponds to the minimal price at which he would be ready to trade a total order flow of size $(Q_A + Q_B)$

in a single market. Thus, when dealers are obliged to quote identical prices in multiple markets, they are characterized by higher reservation prices at the ask than on a single market, for an identical transaction size, which is reflected in their quotes.

Traditional inventory models show that the dealer who initially has the longest (resp. shortest) inventory submits the best ask (resp. bid) price (*cf.* Ho and Stoll (1983) or Biais (1993)). Similarly, we show that when the difference in dealers' initial inventories is sufficiently large, the dealer who initially has the longest position (for instance) submits the best ask price simultaneously in the two trading systems. Although the dual-liability risk deeply impacts his cost of liquidity supply, since his inital inventory is extremely long with respect to his competitor's inventory, he has the opportunity to submit sufficiently high ask prices to receive a positive profit from trading.

However, in some circumstances, this dealer with the longest position is better off letting his competitor undercutting his ask price in one of the two markets, although he could submit the best ask price in the two markets. We here introduce an asymmetry in markets A and B, by assuming that the volume traded in market A is strictly superior to the volume exchanged in market B. We then name market A the "dominant" market, and market B the "satellite" market.

In this later case, we show that in equilibrium, the dealer with the longest position submits the best ask price in the dominant market, but the best *bid* price in the satellite market. Assume for instance that the order flow in market A is a buy order flow. Then even if this dealer may submit the best ask price in both markets, his profit is strictly superior when he gives his opponent the opportunity to undercut his ask price in market B. Such a strategy enables a better risk-sharing among the two dealers. Indeed, they not only avoid a dual-liability risk, but they conversely beneficiate from a balancing effect. The combination of these two effects enables the market maker with the longest position to be the first buyer in market B, by undercutting the bid price of his competitor.

This strategy is also adopted by dealers when the difference in their initial inventories is sufficiently small. In this case, the dealer with the longest position cannot simultaneously submit the best ask price in the two systems. On the one hand, this dealer must take the dual-liability effect into account which increases his reservation price. Conversely, his opponent beneficiates from a balancing effect, which decreases his ask reservation price. Consequently, his reservation price becomes higher than the ask reservation price of his competitor in at least one of the two markets. He is thus forced not to post the best ask prices in the two systems. This leads to the following result.

Dealers submit differentiated quotes in the two trading venues.

Under the asymmetry's assumption described above, we find that the coexistence of an alternative trading system beneficiates to the dominant market. It indeed enables dealers to submit more aggressive quotes, because they take into account the opportunity to partially undo their position in the satellite market. Our analysis thus contributes to the controversy on the impact of alternative trading systems on the behavior of market makers and on the liquidity of the dominant market. It applies to stocks quoted in hybrid markets (multiplicity of execution centers). Many empirical studies focus on the impact of the competition of an alternative trading venue on the liquidity of the dominant market, in hybrid markets' environments. Gresse (2002) in particular studies the influence of "POSIT", a public and passive crossing network in the London Stock Exchange, on the quoted spreads in the dealership market. According to her, "*the crossing network activity appears to strenghten the competition between market makers and to give them a risk-sharing opportunity that leads them to improve quotes*". The positive impact of the balancing effect on dealers' aggressiveness in the dominant market has thus a coverage in the empirical literature.

Our theoretical study also applies to stocks for which there exist one or many substitutes, that may either be a deeply correlated stock, a derivative (an option for instance), or a cross-listed stock when markets are simultaneously open (multiplicity of markets). Thus, our model enables us to shed a new light on some of the results of Gresse (2001) for cross-listed stocks in Paris and in London, or of Hansch (2001) and Werner and Kleidon (1996) for cross-listed stocks in London and in the United States (as "American Depositary Receipts").

Besides, our model leads to new empirical predictions on the impact of the joint probability of order flows on market liquidity. In multiple markets, the spreads in two trading venues depend on the relative strenght of the balancing and the dual-liability effects. We find that the average best quoted spreads increase with the probability that the order flows in the two markets have the same direction. When this probability decreases, if dealers submit the best quotes on the same side of the market in both trading systems, the probability to bear a dual-liability risk decreases. In parallel, if dealers submits the best ask price in one market and the best bid price in the other, the probability to balance both shocks increases. This prediction suggests that it could be interesting to study the interactions between liquidity supply and demand in multiple markets, which opens a way for future research.

Finally, we study the implications of different reform projects on the liquidity of multiple markets. In particular, the *Securities and Exchange Commission* has adopted in 1997 a series of reform on the Nasdaq, called the "*Order Handling Rules*", and aimed at improving the access to dealership markets and at decreasing transaction costs. One of the main measures that has been implemented was the obligation on Nasdaq dealers to publicly report their most competitive quotes ("*Quote Rule*"). Our model enables us to bring new insights in this debate. To this end, we compare quoted spreads when dealers are obliged to quote identical prices in the two markets, and when they are allowed to display differentiated quotes.

We show that imposing identical quotes in the two markets increases the best quoted spreads in the dominant market. This result relies on two intuitions. First, when he submits identical quotes, the dealer is exposed to the dual-liability risk, whereas he can conversely beneficiate from the balancing effet by submitting the best ask price in one system and the best bid price in the other, when this constraint is relaxed. Price differenciation thus enables a better risk sharing among dealers. Second, when dealers must display identical quotes, the dealer with the shortest position posts unaggressive ask prices, which enables his competitor to easily undercut him while submitting relatively high ask prices. This cannot be the case when dealers have the opportunity to differentiate their prices. In this case indeed, the dealer with the shortest position may post an aggressive ask price in one market only, which induces his competitor to submit more aggressive prices in each market.

This paper is organized as follows. Section 2 describes the theoretical model. In Section

3, we determine the equilibrium pricing strategies of dealers in markets A and B. In Section 4, we suggest new empiriacal predictions. Section 5 discusses the implications of some reforms on market performance. Section 6 concludes. The proofs that are not presented are collected in the Appendix.

2 The Model

Assets and Market Structure

One risky security is traded on the market. Its final value $\tilde{v}$ is distributed as a random variable with mean v_0 and variance σ_v^2. We assume that two trading systems coexist in this market, named market A and market B.

Market Participants

In both trading venues, there exist liquidity suppliers and liquidity demanders.

1) Liquidity demanders

Let Q_i be the order flow routed towards market i, as the realization of a random variable $\tilde{Q}_i$. To simplify the analysis, we assume that the size of the order flow is exogenous and common knowledge, *i.e.* that it does not depend on the prices displayed in each system. This size is assumed to be pre-determined by the expected transactions costs (measured for instance by the expected effective spread), or by the services offered in each trading venue (represented for instance by the expected execution speed). However, the order flows routed towards each market do not necessarily have the same direction. By convention, we write $Q_i > 0$ when the realized order flow corresponds to a buy market order, and $Q_i < 0$ otherwise.

The directions of the order flows routed towards markets A and B are not independant. We assume a perfect symmetry between both markets, and we thus retain the marginal probabilities reported in Table 1 (before the Appendix) and summarized as follows.

Axiom 1 *The probability to observe a buy (resp. sell) order flow in market i, conditional* *on the direction of the order flow observed in market $-i$, is such that, for $i = A, B$:*

$$\begin{aligned}
\Pr\left(\tilde{Q}_i > 0 | \tilde{Q}_{-i} > 0\right) &= \Pr\left(\tilde{Q}_i < 0 | \tilde{Q}_{-i} < 0\right) = \gamma \\
\Pr\left(\tilde{Q}_i > 0 | \tilde{Q}_{-i} < 0\right) &= \Pr\left(\tilde{Q}_i > 0 | \tilde{Q}_{-i} < 0\right) = \mu
\end{aligned}$$

with $\gamma > \mu$.

Menkveld (2004) notes that various and opposite effects determine the correlation of order flows between two markets.[2] On the one hand, arbitrage between markets and the liquidity suppliers' inventory management have a negative impact on this correlation, but on the other hand, new information, program trading and the strategies of order flow fragmentation used by sophisticated traders have a positive impact on it. Finally, after controlling for the arrival of new information and for microstructure effects (which, as it turns out, do not modify the correlation), Menkveld (2004) reports an average positive correlation between the order flows routed towards the domestic market (London or Amsterdam) and the New York Stock Exchange for Dutch and English stocks that are cross-listed in the United States. This seems to show that, in practice, the probability γ would be superior to $\frac{1}{2}$. The author reports that the time-intervals which exhibit a negative correlation are a exception, and only occur at a very low frequency (less than 15%).

2) Liquidity suppliers

We assume that there exist two dealers, denoted D_1 and D_2, holding each an initial inventory I_j. It is common knowledge that the dealers' initial inventories are independant realizations drawn from a uniform distribution $[I_d, I_u]$, with $I_u > I_d + 2\gamma |Q_A|$.

These dealers are risk-adverse. Their expected utility is given by a mean-variance function, with an identical risk-aversion parameter ρ.[3] Dealer D_j' expected utility if he does

[2]Notice that whatever the joint and marginal distributions of the direction of order flows, the covariance between $\tilde{Q}_A$ and $\tilde{Q}_B$ equals zero. By abuse of language, we will however name this probability γ the "correlation" of the order flows.

[3]It is possible to show that the mean-variance function can be obtained by a Pratt-Arrow (1963)'s approximation of an exponential utility function (*cf.* for instance Biais (1993)).

not trade is thus as follows:

$$EU_j^0 = v_0 I_j - \frac{\rho\sigma_v^2}{2} I_j^2.$$

We name trading surplus the difference between the dealer's expected utility if he trades, and his expected utility if he does not trade:

$$S_j = EU_j - EU_j^0 \left(I_j\right).$$

Dealers post bid and ask quotes in markets A and B. Let a_j^i be the ask price displayed by dealer D_j in market i, and b_j^i his bid price, for $i \in \{A, B\}$. We define a pricing strategy in multiple markets as follows:

Definition 1 *A pricing strategy for dealer D_j in multiple markets is a price vector $\mathbf{v}_j = \left(a_j^A, b_j^A, a_j^B, b_j^B\right)$, where a_j^i (resp. b_j^i) represents the ask (resp. bid) price quoted by dealer D_j in market i, for $i = A, B$.*

For clarity, since the expected utility functions of both dealers are identical, we assume that dealer D_1 has the longest position, *i.e.* $I_1 > I_2$.

Besides, to simplify the analysis, we assume that each dealer observes his competitor's position.

Axiom 2 *Dealers observe their competitor's initial inventory.*

According to the terminology used in Biais (1993), this assumption, on which Ho and Stoll (1983) also base their analysis, corresponds to a "centralized" market organization. In contrast, Biais (1993) studies the dealers' pricing strategies in a "fragmented" market, where dealers do not observe their competitors' position. The author shows that, in the case of a single market, spreads are *on average* equal, whatever the market transparency. We thus assume in our model that dealers observe their competitor's position, which enables us to better shed light on the effects of a multiple markets' environment.

Timing and market environment

We consider the following timing. At date 0, dealers are endowed with their own inventory I_j, and observe the position of their competitor. A date 1, dealers simultaneously post bid and ask quotes in the two trading venues A and B, which are contingent on the transaction size, but without knowing the direction of order flows. At date 2, order flows simultaneously arrive in markets A and B, and in each market, the order flow is routed towards the best quoted price. At date 3, trading ends and the realized value of the security becomes common knowledge. The timing of trading is depicted in Figure 1.

In our model, dealers submit either quotes or limit orders in a trading system, without knowing the direction of the order flow arriving in each market. The Nasdaq represents an illustration of such an environment. Since the introduction of the quoting system "SuperMontage", Nasdaq's dealers have currently the possibility to post quotes in the system that are so-called "Auto-Executable". They thus quote prices that are valid for a fixed quantity, named the "quotity" (*i.e.* they know the transaction's maximal size), on the ask and the bid side (*i.e.* they do not know the direction of the order flow). In parallel, these dealers have the opportunity to submit buy and sell limit orders in the order book "INET", for which they not only choose a price, but also a quantity. On this order-driven market, the liquidity supply (produced by the limit orders) precede the liquidity demand (which takes the form of market or marketable limit orders). Thus, when they submit limit orders in INET, it seems reasonable to assume that dealers do not know the direction of the order flow in this market either.

This timing enables us to take into account the fact that dealers do not always have the opportunity to cancel and resubmit their quotes or limit orders in one of the trading venues, after the execution of a trade in the alternative trading system.

Trading mechanism and reservation prices

When inventories are common knowledge, the trading mechanism in each market corresponds to a second price auction, with independant private values. In equilibrium, the bidder who has the highest "reservation price" submits the best bid.

Definition 2 *Reservation prices are the minimum ask (resp. maximum bid) prices at which* *the dealer could execute the quantity Q, without incuring a decrease in the expected utility of his final wealth if he had to trade at those prices.*

In the mean-variance setup, in a single market, dealer D_j' s expected trading surplus writes as follows.

Result 1 *Dealer D_j' s expected trading surplus in a single market is:*

$$\begin{aligned} E\left(\bar{S}_j\left(Q\right)\right) &= \left(a_j - \left(v_0 + \frac{\rho\sigma_v^2}{2}\left(Q - 2I_j\right)\right)\right) \times Q \times \Pr\left(\tilde{Q} > 0\right) \times \Pr\left(a_j < a_{-j}\right) \\ &+ \left(\left(v_0 - \frac{\rho\sigma_v^2}{2}\left(\left(-Q\right) + 2I_j\right)\right) - b_j\right) \times \left(-Q\right) \times \Pr\left(\tilde{Q} < 0\right) \times \Pr\left(b_j > b_{-j}\right) \end{aligned} \quad (1)$$

Since the ask (resp. bid) reservation price of a dealer is conditional on the arrival of a buy (resp. sell) order flow, it is possible, with this expected surplus, to derive both dealers' ask and bid reservation prices. In the case of a one-period model in a single market, it is easily shown that dealer D_j's ask and bid reservation prices for a trade size Q when his inventory is I_j, respectively denoted $a_{r,j}\left(Q, I_j\right)$ and $b_{r,j}\left(-Q, I_j\right)$ are as follows.

Result 2 (Ho et Stoll (1983))

$$\begin{aligned} a_{r,j}\left(Q, I_j\right) &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q - 2I_j\right) \;\; \text{if } Q > 0 \\ b_{r,j}\left(-Q, I_j\right) &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left(-Q\right) + 2I_j\right) \;\; \text{if } Q < 0, \end{aligned}$$

where ρ is the dealer's risk-aversion parameter.

To lighten notations, in the absence of any ambiguity, we simply write these reservation prices $a_{r,j}\left(Q\right)$ and $b_{r,j}\left(Q\right)$.

Dominant market, satellite market

Markets A and B are perfectly symmetric. We now introduce an asymmetry between both markets.

Axiom 3 *The market A is "dominant", and the market B is "satellite", i.e. $|Q_A| > |Q_B|$.*

This assumption relies on the empirical observation according to which when many markets are simultaneously open for the same security, there always exists a dominant market, and one or more satellite markets. For instance, the domestic market of a cross-listed stock is a dominant market. Werner and Kleidon (1996) study the English stocks listed as ADRs on American stock markets (the AMEX and the NYSE). They note that the trading volume recorded on these American markets only represent 25% of the trading volume in London, in 1991. According to Hansch (2001), this proportion is even overestimated. Thus, the domestic market seems to capture the highest fraction of the order flow for the stocks cross-listed in London and in New York. Menkveld (2004) compares more specifically trading volumes in a domestic market (London or Amsterdam) and in the New York Stock Exchange when both markets are simultaneously open. He finds that the NYSE only captures one third of the total trading volume during this "overlap" period.

This notion of dominant market can also be found in hybrid markets. Gresse (2002) for instance finds that the crossing network "POSIT" only accounts for 1.28% to 2.11% of the trading volumes, depending on the sample period, in 2000-2001. Biais, Bisière and Spatt (2003) similarly find that in 2000-2001, the quote-driven Nasdaq market dominates the limit order book Instinet in terms of trading volume (Instinet represents only 13% of the total number of trades in the Nasdaq). Benhami and Bisière (2005) show that this difference decreases in December 2002, after the final establishment of the trading platform SuperMontage.

In parallel, Gresse (2002) and Werner and Kleidon (1996) show that the average trade size is higher, respectively in POSIT than in the LSE, and in the NYSE than in the domestic market. Conversely, Biais, Bisière and Spatt (2003) and Benhami and Bisière (2005) find that the average trade size in Instinet accounts for half of the average trade size in the Nasdaq. Thus, our assumption should be interpreted in terms of trading volumes rather than in trade sizes.

3 Dealers' equilibrium pricing strategies

In this section, we derive the dealers' equilibrium pricing strategies in the model described above. Dealers are not supposed to receive any information on the direction of the order flow, neither in market A, nor in market B, before posting their quotes. In contrast to traditional models, we cannot avoid a joint analysis of the ask and bid quotes in a given market. To illustrate this point, we first study the dealers' expected utility in multiple markets.

3.1 Dealers' expected utility

In a multiple markets' environment, where two trading systems coexist, dealer D_j's expected utility depends on his liquidity supply in both systems. When liquidity demand (*i.e.* the order flow's direction) is not known in advance by the dealer, he must evaluate his probability to execute the incoming order flows in each market. The dealer can indeed execute the order flow in market A, or in market B, or simultaneously in both markets. His expected utility thus take all the possible trading opportunities into account.

There exist three different cases in which the dealer executes a trade : (i) he executes the order flow in market A (resp. in market B) only, (ii) he executes a buy (resp. sell) order flow in both markets simultaneously, or (iii) he executes a buy order flow in one market, and a sell order flow in the alternative trading venue. The realization of these different situations not only depends on the joint probability of the order flows in markets A and B, but also on the dealers' relative positions.

Given the Table 1 reporting the conditional probabilities, there exist nine possible outcomes for the couple $\left(\tilde{Q}_A, \tilde{Q}_B\right)$. For each outcome, we compute the dealer's *conditional* expected utility, by evaluating the dealer's expected final inventory, and its variance. Then we weight each partial expected utility by its probability to find the dealer's expected utility from trading in multiple markets. To lighten the presentation, the developped form of the expected utility is given in Lemma 2 in the Appendix, and we only present here a simplified form of the dealers' expected trading surplus.

Lemma 1 *Dealer D_j's expected trading surplus can be written:*

$$E(S_j) = E\left(\bar{S}_j(Q_A)\right) + E\left(\bar{S}_j(Q_B)\right) + \Delta_j(\mathbf{v}_1, \mathbf{v}_2, \phi),$$

where Δ_j is defined in the Appendix after Lemma 2, and $E\left(\bar{S}_j(Q)\right)$ is described in Result 1.

Dealer D_j's expected surplus depends on his expected trading surplus from buying and for selling the asset, in each market. It is possible to decompose this total surplus such as to identify the trading surplus $E\left(\bar{S}_j(Q_A)\right)$ and $E\left(\bar{S}_j(Q_B)\right)$ given by Equation (1). The dealer takes into account the impact on his inventory risk of a potential execution of a quantity $|Q_A|$ in market A and of a quantity $|Q_B|$ in market B.

In multiple markets, the dealer's inventory risk, which can be measured by the variance of his final inventory, is also impacted by the covariance of potential trades in markets A and B. His total expected surplus is thus not only generated by the sum of the expected trading surplus in markets A and B, taken independantly, but also by a premium or a discount, that is due to the dependancy of the order flows in markets A and B (which creates the risk of a potential simultaneous execution in the two trading systems), and that we denote Δ_j.

In a single market's environment (a special case of our model when we impose $Q_B = 0$), Lemma 1 leads to the ask and bid reservation prices given by Result 2. In multiple markets however, as we show in the Appendix, it is not possible to determine unique reservation prices without any additional assumption. For instance, conditional on the arrival of a buy order flow in market A (*i.e.* $Q_A > 0$), the minimal price at which dealer D_j would be ready to sell a quantity Q_A in market A depends on the characteristics of the expected transaction in market B. These characteristics themselves rely on i) the direction of the order flow in market B (conditional on $Q_A > 0$), and ii) the ask and the bid quoted by both dealers in market B.

The reasoning is identical when we search for the reservation prices in market B, which has two main consequences. On the one hand, in multiple markets, it is not possible to determine the dealers' reservation prices independantly from the equilibrium prices in the alternative trading system. On the other hand, there exists a degree of freedom on reservation prices, which leads to a multiplicity of equilibria. For instance, even if he post the best ask prices simultaneously in markets A and B, dealer D_1 may be indifferent between various pricing strategies, as long as he is ensured that, at those prices, he stays in the first seller position in both markets (*i.e.* if his competitor cannot undercut those prices without incuring a loss). In the two cases studied below, an additional assumption however enables us to loose this degree of freedom and to find unique reservation prices.

3.2 A benchmark model: a market with identical quotes

In this section, we assume that dealers are obliged to submit identical quotes in the two trading systems. What prices do they post?

In this benchmark case, by definition, the dealer who displays the best ask price in market A will also attract a buy order flow in market B. This constraint therefore enables us to uniquely determine the dealers' ask and bid reservation prices.

Given the expected trading surplus defined in Lemma 1, we find the following reservation prices.

Result 3 *Dealer D_j's ask and bid reservation prices, if he is obliged to post identical quotes in both markets A and B, can be written*[4]*:*

$$\begin{aligned} a_{r,j}^{id} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2\times(1-\gamma)\times\frac{|Q_A\times Q_B|}{|Q_A|+|Q_B|} - 2I_j\right) \\ b_{r,j}^{id} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2\times(1-\gamma)\times\frac{|Q_A\times Q_B|}{|Q_A|+|Q_B|} + 2I_j\right) \end{aligned}$$

In the case where the directions of the order flows in both markets are always identical (*i.e.* $\gamma = 1$), dealers' reservation prices to trade a quantity $|Q_A|$ (or $|Q_B|$) corresponds to their reservation price to trade a quantity $|Q_A| + |Q_B|$ in a single market. In each market, dealers indeed take into account the execution in the alternative trading venue.

[4] The exponent "id" is used to distinguish dealers' reservation prices when they are obliged to submit identical quotes.

When quotes must be identical in both markets, the dealer who posts the best ask in one market cannot, by definition, submit the best bid in the other market. Consequently, if he trades in this alternative trading venue, this trade necessarily impacts his inventory in the same direction as a trade in the other market. For instance , the dealer who has the longest position may execute a buy order flow in market A and/or a buy order flow in market B, but cannot execute a sell order flow in any of the two markets. We name this effect "dual-liability".

In the opposite case where the directions of the order flows in both markets are never identical (*i.e.* $\gamma = 0$), then if the size of the transactions in both markets is equal (*i.e.* $|Q_A| = |Q_B|$), the coexistence of an alternative trading system has no impact on the dealers' reservation prices. In this case indeed, dealers never face a dual-liability risk, since their quotes cannot be simultaneously touched on the same side in both markets. Note however that if the trading size in markets A and B is different (*i.e.* $|Q_A| \neq |Q_B|$), since the dealers' reservation price is contingent on the order size, obliging dealers to quote identical prices in both markets induces them to condition their reservation prices on an *average* order size.

With these (unique) reservation prices, we find the dealers' equilibrium pricing strategies. In equilibrium, the dealer with the longest (resp. shortest) position has the lowest (resp. highest) reservation price at the ask (resp. at the bid), which enables him to post the best ask (resp. bid) price in both markets. Since D_2, who has by assumption the shortest position, cannot execute any buy order flow, his quoted ask price corresponds to his reservation price. Dealer D_1 can thus simply post an ask price which is slightly lower than his competitor's to be ensured to execute the order flow, and to earn a strictly positive trading surplus. This leads to the following Proposition.

Proposition 1 *When dealers must post identical prices in both markets (i.e. $a_j^A = a_j^B$ and $b_j^A = b_j^B$, $j = 1, 2$), the dealers' pricing strategies are such that:*

$$\begin{aligned} a_2^{id} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} - 2I_2\right) \\ a_1^{id} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} - 2I_2\right) - \varepsilon \end{aligned}$$

$$
\begin{aligned}
b_2^{id} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} + 2I_1\right) + \varepsilon \\
b_1^{id} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} + 2I_1\right)
\end{aligned}
$$

where ε represents the tick size.

Dealers' pricing strategies take into account the dual-liability effect, since liquidity suppliers's quotes can only be touched on the same side in both markets, when they must post identical quotes. Finally, given these pricing strategies, we find the best quoted spread.

Corollary 1 *When dealers must post identical prices in both markets, the best quoted prices are such that:*

$$
\begin{aligned}
a^{id*} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} - 2I_2\right) - \varepsilon \\
b^{id*} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} + 2I_1\right) + \varepsilon
\end{aligned}
$$

and the best quoted spread is equal to:

$$
s^{id*} = \rho\sigma_v^2\left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} + I_1 - I_2\right) - 2\varepsilon
$$

We show that when dealers are obliged to post identical quotes in both markets, the existence of an alternative trading system impacts dealers' quotes in the dominant market. For $|Q_A| = |Q_B|$, the best quoted spread is higher in multiple markets. We study the nature of this impact more precisely in Corollary 4. What happens when this constraint is relaxed?

3.3 Equilibrium pricing strategies

We assume that market A is dominant, which accounts for the asymmetry in trading volumes that is empirically observed in multiple markets. Given that assumption, under the competitive pressure generated by the existence of an alternative trading system, the objective of liquidity suppliers in the dominant market is to execute the order flow in the dominant system. This assumption enables us to uniquely determine the dealers' equilibrium pricing strategies.

In the general case where dealers can post differentiated prices in both markets, the following Proposition shows that when dealer D_1 has a very long position with respect to his competitor, he submits the best ask prices in both markets A and B.

Proposition 2 *When $I_1 - \gamma |Q_A| > I_2$, dealers' quotes are as follows:*

1) In market A:

$$\begin{aligned} a_2^A &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| - 2\left(I_2 + \mu |Q_B|\right)\right) \\ a_1^A &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| - 2\left(I_2 + \mu |Q_B|\right)\right) - \varepsilon \\ b_2^A &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + 2\left(I_1 - \mu |Q_B|\right)\right) + \varepsilon \\ b_1^A &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + 2\left(I_1 - \mu |Q_B|\right)\right) \end{aligned}$$

2) In market B:

$$\begin{aligned} a_2^B &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_B| - 2I_2\right) \\ a_1^B &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_B| - 2I_2\right) - \varepsilon \\ b_2^B &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_B| + 2I_1\right) + \varepsilon \\ b_1^B &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_B| + 2I_1\right) \end{aligned}$$

Since dealers' quotes are symmetric at the ask and at the bid, we focus on the ask side of the market. When $I_1 - \gamma |Q_A| > I_2$, dealer D_1 has a longer position than his competitor, even after accounting for the execution of a buy order flow in market A. He has thus the opportunity to submit the best ask prices simultaneously in markets A and B.

The ask price posted by each dealer in market A is composed of two elements. First, in the absence of an alternative trading venue (*i.e.* $|Q_B| = 0$), Proposition 2 is a simple restatement of Proposition 1 in Ho and Stoll (1983), for one period, and a unique security. Due to the dealer's inventory risk, his reservation price depends on his initial position, and increases with the size of the order flow, $|Q_A|$. Precisely, the dealer in the longest position posts an ask price which is slightly lower than his competitor's reservation price at the ask, *i.e.* $a_1(Q_A) = a_{r,2}(Q_A) - \varepsilon$. This reasoning holds for market B when $|Q_A| = 0$.

The coexistence of an alternative trading system, B, however impacts dealers' quotes in market A. Since dealer D_2's position is sufficiently short with respect to D_1's, dealer D_2 expects to be touched on the bid side in at least one of both markets. Consequently, this dealer do not bear any dual-liability risk: he cannot post the best ask price simultaneously in market A and in market B.

Conversely, when he determines his ask price in one of the two markets, he anticipates that since he has posted the best bid price in the alternative trading system, then if his ask quote leads to an execution (*i.e.* if it is touched), his potential trades in the two markets may compensate each other. This in turn decreases his inventory risk. We thus shed light on an effect which direction is opposite to the dual-liability effect, and that we name "balancing"effect. Since D_2 expects to execute a sell order flow in market B, his ask reservation price decreases in market A. The existence of an alternative trading venue thus enables D_2 to submit more aggressive prices in market A, not only at the bid (since he has the shortest position), but also at the ask.

However, since D_1 has the longest position, he expects to be touched at the ask in market B. He must take into account his dual-liability risk when he simultaneously determines his ask prices in markets A and B. But given dealer D_2's behavior, dealer D_1 must post a very aggressive ask quote in market A to execute the order flow. If he wants to avoid being undercut at the ask in the dominant market, dealer D_1 thus sets an aggressive ask price in market A, which he compensates by setting a less aggressive price in market B. In market B, he will thus be ready to execute $Q_B > 0$ but only at a price that accounts for the impact of a double execution on his inventory risk. Finally, everything is *as if* dealer D_1 takes into account a potential double execution (with a probability γ) by reasoning in market B on the basis of an *expected* inventory, *i.e.* $I_1 - \gamma |Q_A|$.

To sum up, each dealer takes advantage of market B to "adjust" his quotes and submit better prices in market A, in order to execute the order flow routed towards market A rather than towards market B, since $|Q_A| > |Q_B|$. On the one hand, the dealer with the shortest position beneficiates from the balancing effect, since he is the first buyer in market B, which enables him to submit an aggressive ask price in the dominant market. On the other hand, although the dealer in the longest position is exposed to a dual-liability risk, he is obliged to submit an aggressive ask price in the dominant market, if he does not want to be undercut. He is thus ready to trade in market B only at a high ask price.

Finally, we notice that in a multiple markets environment, (equilibrium) reservation prices at the ask and at the bid in a given market are not symmetric any more, as it is the case in a single market environment. Dealer D_2 for instance uses market B to try to undercut his competitor on the side where this competitor is (supposively) the best quoting dealer (*i.e.* the first seller), while posting an aggressive price on the side where he is the best quoting dealer (*i.e.* the bid side).

What happens when the difference in dealers' inventories is smaller?

Proposition 3 *When $I_1 - \gamma |Q_A| < I_2$, the dealers' equilibrium pricing strategies are as follows:*

1) In market A:

$$\begin{aligned} a_2^A &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| - 2I_2\right) \\ a_1^A &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_A| - 2I_2\right) - \varepsilon \\ b_2^A &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + 2I_1\right) + \varepsilon \\ b_1^A &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + 2I_1\right). \end{aligned}$$

2) In market B:

$$\begin{aligned} a_1^B &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_B| - 2\left(I_1 - \gamma |Q_A|\right)\right) \\ a_2^B &= v_0 + \frac{\rho\sigma_v^2}{2}\left(|Q_B| - 2\left(I_1 - \gamma |Q_A|\right)\right) - \varepsilon \\ b_1^B &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_B| + 2\left(I_2 + \gamma |Q_A|\right)\right) + \varepsilon \\ b_2^B &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_B| + 2\left(I_2 + \gamma |Q_A|\right)\right). \end{aligned}$$

The equilibrium described in this proposition arises when $I_2 > I_1 - \gamma |Q_A|$, but for slightly different reasons when $I_2 > I_1 - \mu |Q_B|$, or not.

If $I_2 > I_1 - \mu |Q_B|$, there exists no equilibrium such that dealer D_1 posts the best ask price simultaneously in both markets. In this case indeed, the dual-liability risk increases the dealers' reservation prices in both markets. At the same time, his opponent beneficiates from a balancing effect, which decreases his ask reservation price in at least one of the two markets. Thus, dealer D_2 has the opportunity to undercut dealer D_1 at the ask in at least one of the trading venues.

Since D_1 cannot submit the best ask prices in both markets, he lets his competitor undercut his ask in the satellite market (B). Since D_2 is the first seller in market B, he cannot be as aggressive at the ask in market A as in the previous case (because of the dual-liability effect). This in turn enables dealer D_1 to submit a less aggressive ask price in market A as in the case studied in Proposition 2, thus to earn a higher profit in this market. However, dealer D_1's ask price in market A must be sufficiently low to induce his competitor undercutting him in market A rather than in market B. Besides, such a strategy enables him to beneficiate from the balancing effect, since shocks in markets A and B could now compensate each other.

Thus, this equilibrium is characterized by a risk-sharing among dealers. Each dealer submits the best ask price in one market, and the best bid price in the other. The dealer with the longest position posts the best ask price in the dominant market.

If $I_1 - \mu |Q_B| > I_2 > I_1 - \gamma |Q_A|$, dealer D_1 could submit the best ask prices in both markets, but his expected trading surplus is strictly superior when he submits the best bid in market B. His competitor D_2 is in the same situation: he is better off letting D_1 undercut his bid price in the satellite market. Since the direction of the order flows in both markets is unknown, and since buy and sell orders are by assumption equally likely (*cf.* Table 1), dealer D_2's behavior is symmetrical, and he beneficiates from a better risk-sharing.

3.4 Best quoted spread

From Propositions 2 and 3, we determine the best quoted spreads.

When we observe equilibrium quotes in market A, we observe that the best quoted prices in this market correspond to the best prices that would be quoted for a quantity $|Q_A|$ in a single market, plus, in some cases, a premium δ^A. We thus easily compute the spread quoted in market A.

Corollary 2 *The average spread of best quoted prices in market A is as follows:*

$$\bar{s}^A(|Q_A|) = \rho\sigma_v^2\left(|Q_A| + \frac{(I_u - I_d)}{3}\right) - 2 \times \frac{\rho\sigma_v^2}{(I_u - I_d)^2} \times \mu|Q_B| \times (I_u - I_d - \gamma|Q_A|)^2 - 2\varepsilon$$

The average spread of best quoted prices in market A can be decomposed into two elements. First, for $|Q_B| = 0$, we find in this Corollary the average spread in a single market for a transaction size $|Q_A|$, *i.e.* $\bar{s}^u(|Q_A|) = \rho\sigma_v^2\left(|Q_A| + \frac{(I_u - I_d)}{3}\right)$. This result is traditional in the literature. When $|Q_B| > 0$, this spread is composed of a second element, which is strictly negative:

$$E\left(\delta^A\right) = -2 \times \rho\sigma_v^2 \times \mu|Q_B| \times \frac{(I_u - I_d - \gamma|Q_A|)^2}{(I_u - I_d)^2}$$

This last element depends on the dealers' relative positions, and is linked to trading opportunities in market B.

In market B, the quoted spread is not straightforward. The dealer in the longest position (resp. the shortest position) does indeed not necessarily submit the lowest ask price (resp. the highest bid price). We however find similar results for the best quoted spread.

Corollary 3 *The average spread of best quoted prices in market B is as follows:*

$$\bar{s}^B(|Q_B|) = \rho\sigma_v^2\left(|Q_B| + \frac{(I_u - I_d)}{3}\right) - 2 \times \frac{\rho\sigma_v^2}{(I_u - I_d)^2} \times (\gamma|Q_A|)^2 \times \left(-I_u + I_d + \frac{4}{3}\gamma|Q_A|\right) - 2\varepsilon$$

The average spread of best quoted prices is also composed of a second element, which is now strictly positive:

$$E\left(\delta^B\right) = -2 \times \frac{\rho\sigma_v^2}{(I_u - I_d)^2} \times (\gamma|Q_A|)^2 \times \left(-I_u + I_d + \frac{4}{3}\gamma|Q_A|\right)$$

In the next section, we study the characteristics of equilibrium quotes in multiple markets.

4 Empirical predictions

We now study the consequences of liquidity supply in multiple markets. The equilibrium pricing strategies defined in the previous section are characterized by two main particularities. First, dealers submit differentiated quotes in markets A and B. Second, dealers' quotes depend on the joint probability of order flows in both markets. These two results suggest new empirical predictions.

4.1 Differentiated quotes

One of the main predictions of our model is the following.

Prediction 1 *In multiple markets, the dealer's quotes in both markets are not identical.*

Quoted prices depend on the size of the order flow routed towards the alternative trading system. Thus in each market, if the dealers expect the size of the order flow in market A to be strictly superior than the size of the order flow in market B, then the premium (or the discount) that is proposed in multiple markets is larger (in absolute values) in market B than in market A. For instance, when $I_1 - \gamma \left| Q_A \right| > I_2$,

$$a_1^B(Q) - a_1^A(Q) = \rho \sigma_v^2 \mu \left| Q_B \right|.$$

In this case, quotes are *uniform* but not identical. Besides, when $I_1 - \gamma \left| Q_A \right| < I_2$, dealer D_1 lets his competitor undercut his ask price in market B. Thus, the dealer quotes the following ask prices:

$$a_1^B(Q) - a_1^A(Q) = \rho \sigma_v^2 \left(I_2 - I_1 + \gamma \left| Q_A \right| \right).$$

The prediction 1 is uniquely linked to the coexistence of many trading systems.[5] Under our asymmetry assumption (Assumption 3), we find that whatever dealers D_1 and D_2's

[5]We have checked the robustness of this claim by analyzing the symmetric case where markets are perfectly identical. However, since this case does not add much to the analysis with respect to the asymmetric case, we do not report it here.

respective initial positions, their quotes are more aggressive in the dominant market. Since $|Q_A| > |Q_B|$, dealers seek the execution of the largest volume, and are thus induced to reduce their quoted spreads in market A rather than in market B. This differentiation also applies to the average spreads.

Prediction 2 *On average, the spread of the best quoted prices in market A is inferior to the spread in market B.*

This prediction is corroborated by many empirical studies on multiple markets. Biais, Bisière and Spatt (2003) and Benhami and Bisière (2005) report smaller spreads in the Nasdaq than in Instinet (recall that on the sample period analyzed in these two papers, the Nasdaq attracts the largest part of the order flow).

For English stock traded as ADRs on the New York Stock Exchange, Werner and Kleidon (1996) find smaller spreads in the domestic market This phenomenon could be explained by a better liquidity in the London Stock Exchange, since the authors account for larger trading volumes and a larger number of trades in this market. However, they show in parallel that the volatility in the London market increases significantly with the opening of the NYSE. This effect should conversely increase the spreads quoted in the LSE. Hansch (2001)'s results are also in line with Prediction 2, since he finds that for the stocks traded as ADRs in New York, the spreads in London are significantly lower during the overlap period, when both markets are open.

Besides, although the empirical studies quoted above do specifically analyze the determinants of differences in the spreads quoted in the dominant and in the satellite markets, they underline these differences. It is indeed difficult to study the causes of such differences, that could be due to many factors. Quoted spreads are indeed known to be sensitive to trading volumes, to the asset volatility, or to adverse selection. Besides, Werner and Kleidon (1996) note that variations in the exchange rates could not only complexify any arbitrage strategy among markets, but also make the results of an empirical investigation noisy.

Our model enables us to account for one of the components of this difference, which would (partly) be due to the dealers' inventory management. In order to be able to empirically test this result, we propose a prediction on the variations of this difference, rather than on the difference itself.

Prediction 3 *For a given transaction size, the difference in the spreads of best quoted prices in markets A and B increases with σ_v^2 and with γ, and decreases with μ.*

To our knowledge however, there exists no empirical study on the determinants of these variations in the best quoted spreads in multiple markets, which could be done in future research.

4.2 Quoted spread and inventory

Besides, one of the main consequences of our model is the following.

Prediction 4 *The dealer with the longest initial position does not always post the best ask price in the satellite market.*

Proposition 3 shows that dealer D_1 does not submit the best ask price in market B, altough he initially has the longest position. We therefore find a result here that is similar to Lescourret and Robert (2002). The authors analyze the impact on price formation of an alternative form of order flow fragmentation, created by the practices of preferencing and of internalization.[6]

This prediction enables us to indirectly interpret a surprisng result of Hansch (2001). The author compares two classes of securities quoted in the LSE: some securities that are traded as ADRs in New York, and some that are not. He shows that the existence of an alternative trading venue for ADRs stocks has an economocially significant impact on the pricing behavior of London's market makers. In particular, he finds that this coexistence significantly decreases the mean reversion property of dealers' inventories. But this phenomenon could also be explained, in our model, by the observation that dealers who are in

[6] See for instance Biais and Davydoff (2002) for a review of the problems created by these practices in Europe and in the United States.

the most extreme positions do not always submit the best prices on the same side in both markets.

4.3 Liquidity supply in multiple markets

We now focus on the third main consequence of our model, *i.e.* the impact of the order flow correlation in both markets on the best quoted spreads.

Prediction 5 *The best spreads quoted in the dominant market (A) increase with the probability that the order flows in both markets have the same direction. Conversely, the best spreads quoted in the satellite market (B) decrease with this probability.*

Let us recall that γ represents the probability that the order flows routed towards the two markets have the same direction, and μ the probability that order flows have opposite directions.[7] Dealers in the dominant market use the alternative trading venue B to balance a potential shock in market A. Thus, when the probability μ increases, dealers' quotes are more likely to be touched on the opposite side of the market in market B. Dealers exploit the balancing effect in market A to post more aggressive quotes in the dominant market. Consequently, the best quoted spreads in market A decrease with μ.

Conversely, since dealers' quotes may be simultaneously touched on the same side of the market, dealers also account for the dual-liability effect. In order to set aggressive prices in the dominant market, the impact of that effect is completely taken into account in the quotes posted in the satellite market. The best spreads quoted in market B thus increase with γ. To our knowledge, this prediction has never been suggested nor tested in any empirical study.

[7] Although $\gamma + \mu + \theta = 1$, we keep the notations γ and μ in order to better illustrate the nature of the effects.

5 Implications

In this section, we aim at studying the impact of some regulation changes in a multiple markets environment.

5.1 Interdiction to supply liquidity in different systems

We have shown that although dealers' quotes are contingent to the arrival of the order flow in the market, they are impacted by the existence of an alternative trading system. What would happen if dealers were obliged to post quotes in one trading venue only?

Corollary 4 *If dealers only post quotes in a single market, then the best quoted spread increases on average in market A and decreases on average in market B.*

The proof is straightforward by observing the average best quoted spreads in Corollaries 2 and 3.■ Thus, the existence of an alternative trading venue beneficiates mainly to the investors of the dominant market, to the detriment of the investors of the satellite market. This idea is not new: it can also be found in traditional papers on order flow fragmentation, including Hamilton (1979), Pagano (1989), Chowdhry and Nanda (1991) or Menkveld (2005).[8] In these articles, the impact of the coexistence of multiple markets on welfare is due to a rupture in the order flow concentration, which increase transaction costs. In our model however, this effect arises because of the behavior of dealers in the dominant market.

There are two different ways of interpreting this Corollary. First, it enables to analyze the consequences of a regulation obliging dealers to post quotes (or submit orders) in a single trading venue. Second, although we assume that the fragmentation of the order flow is exogenous, this Corollary shows that whatever the nature of this fragmentation, the coexistent of an alternative trading system favors agents trading in the dominant system, which leads to the following empirical prediction.

[8] According to Menkveld (2005): "In equilibrium, wealth is transferred from local to sophisticated investors, since local investors are shown to be better off in a single, centralized market".

Prediction 6 *On average, for a given transaction size Q, the best quoted spread in market A is inferior to the best spread that would be quoted in a single market, and conversely for the best spread in market B.*

This result may look surprising. The articles quoted above highlight the existence of two opposite effects on liquidity, due to the introduction of an alternative trading system: competition versus fragmentation. In our model, we do not account for the positive effects that usually get along with the increase in competition since we assume that the size of the transactions in each market is exogenous. Thus, we should find a purely negative impact of the presence of multiple market on transaction costs, since it has been shown that fragmentation increases spreads (*cf.* Pagano and Röell (1996)). Werner and Kleidon (1996) for instance report that during the overlap period where both the London Stock Exchange and the New York Stock Exchange are simultaneously open, the spreads of cross-listed stocks quoted in New York are significantly lower than the spreads observed for the stocks of the same market belonging to a control sample, while the spreads quoted in London are significantly lower than those of their control sample. The authors interpret this result, that they find surprising given the economic rationale of fragmentation described above, by suggesting three possible explanations: i) even if assets quoted in New York and in London are *theoretically* susbtitutes, the market is highly segmented, ii) this overlap period corresponds to the opening of the NYSE, thus information flows during this period would be more important in New York than in London, and iii) the Specialist in New York earns a monopoly rent.

Our model enables us to shed light on an alternative explanation to this phenomenon. When the New York Stock Exchange opens, dealers from the LSE would indeed have the opportunity to submit orders in an alternative trading venue so as to submit more aggressive quotes in the domestic market.

5.2 Identical quotes

In 1997, the SEC has adopted various regulation reforms of the Nasdaq. These "*Order Handling Rules*" is composed of two main measures. First, dealers are now obliged to display their curstomers' limit orders, when they propose a best price than the dealer ("*Limit Order Display Rule*"). This rule betters market transparency and enables investors to execute a trade as counterpart of a limit order which was previously not exposed to the market. Second, dealers are obliged to publicly display their most competitive orders ("*Quote Rule*"). They must quote identical prices in all the systems where they supply liquidity.

Empirical studies illustrate the positive impact of this set of reforms on Nasdaq's liquidity (*cf.* McInish, Van Ness and Van Ness (1998) or Simaan, Weaver and Whitcomb (2003)). However, these studies do not enable us to distinguish the impact of each reform, taken separately. Besides, the authors suggest that the Quote Rule would only have had an advertising impact. For instance, according to McInish, Van Ness and Van Ness (1998), since the quotes to which the change in regulation apply were already posted outside of the Nasdaq system, its main effect would be to increase market transparency. According to Simaan, Weaver and Whitcomb (2003), this increase in transparency has enabled ECNs to attract a higher proportion of th order flow, thus allowing Nasdaq's dealers to avoid the collusion equilibrium by submitting anonymous orders in the ECNs.

Our model brings new insights to this debate. Our setup indeed enables us to compare quoted spreads when dealers are obliged to display identical quotes, or when they may submit differentiated prices.

Prediction 1 shows that dealers do not submit identical quotes in multiple markets. Consequently, obliging them to display such quotes has necessarily an impact on the liquidity supplied in these markets. Is this impact positive or negative for market liquidity?

To provide an answer to this question, we now compare the results of Propositions 2 and 3 to the equilibrium quotes of the benchmark model(Lemma 1). This leads to the following Corollary.

Corollary 5 *Let us consider a regulation obliging dealers to display identical quotes in the two markets. In this case:*

1) The best spread quoted in market A increases on average.

2) The best spread quoted in market A increases on average if and only if $\gamma < \gamma^*$.

When quotes in the two markets are identical, the dealer who has the longest position necessarily posts the best ask price in each market. Consequently, he may incur a dual-liability risk, but he never beneficiates from a balancing effect. This risk therefore is priced in the average spreads.

When the dealer has the opportunity to submit differentiated prices in the two markets, Proposition 2 shows that for $I_1 - \gamma |Q_A| > I_2$, he is exposed to this dual-liability risk. However, in this last case, the fragmentation of order flows induces him to submit aggressive prices in both markets. We show that he submits a better price in market A than in market B. However, price differentiation also decrease the best spreads quoted in market B, since his competitor has the possibility to undercut him in one market and to execute only a fraction of the total order flow. When he posts an ask price in the satellite market, the dealer who has the shortest position for instance is ensured not to be exposed to a dual-liability risk, since he submits the best bid in the dominant market. Consequently, the ask price of the dealer who has the longest position is also lower in market B when quotes may be differentiated.

When $I_1 - \gamma |Q_A| < I_2$, Proposition 3 shows that the dealer who initially has the longest position submits the best bid price in market. Price differentiation, which enabled risk-sharing among dealers when dealers' prices may differ in both markets, is no longer possible when they must display identical quotes. This is detrimental to the dominant market, in which dealers mainly beneficiate from the balancing effect when they may submit differentiated prices. However, unlike the previous case, the spread quoted in the satellite market may be lower in the identical quotes regime. Price differentiation may indeed in some circumstances induce dealers to submit prices in the satellite market that are not very competitive.

Finally, we show that the impact of such a regulation on the average spread in market B is ambiguous. It better liquidity in the satellite market if the probability that the order flows in markets A and B have the same direction is sufficiently high. We thus notice that the obligation to post identical quotes in multiple markets is more detrimental to the dominant market than to the satellite market. We here find an intuition that has been developped in industrial organization.

6 Conclusion

In this paper, we study the dealers' pricing strategies in multiple markets. Such an environment may characterize either an hybrid market (like the Nasdaq or the London Stock Exchange), or a dealership market with an inter-dealer trading device, or a domestic market and a foreign market where some securities are cross-listed. We find that the opportunity to supply liquidity in an alternative trading venue impacts risk-averse dealers' pricing strategies. On the one hand, th fragmentation of the order flow modifies the nature of competition among dealers. By analogy, this competition may be modeled by a single-unit auction in a single market, whereas it can only be modeled by a multi-unit auction in multiple markets. The auction theory shows that in the setup of multi-unit auctions, the nature of goods determines the bidders' strategies. In our model, goods may either be substitutes (when order flows have the same direction), or complementary (when order flows have opposite directions), which complefixies the determination of pricing strategies. Precisely, even if he executes the order flow in one market, a dealer may submit a quote so as to execute the order flow in the other market. Price formation in both markets is impacted by a "balancing effect" when order flows have opposite directions and a "dual-liability" effect when they have the same direction.

Financial markets thus have the particularity to be organized as "two side-markets": sellers and buyers may either supply or demand liquidity. This characteristic has long been neglected in quote-driven markets. Since liquidity supply is contingent on liquidity demand, it is possible to distinguish between the ask and bid sides of liquidity supply, and even to analyze one side of the market by symmetry with the other. Our results however show that this reasoning no longer holds in multiple markets, *i.e.* the ask price quoted in one market depends on the ask **and** bid prices quoted in the alternative trading venue. This suggests that in order to study hybrid markets, it becomes necessary to account for this characteristic if we want to fully understand liquidity supply and demand in each market, which are the determinants of investors' transaction costs.

Finally, our model has various empirical implications. Some of these predictions shed light on some existing empirical results (for instance, the inventories' mean reversion prorperty or quoted spread). Some other desserve attention, since they have not been explored yet. We for intance suggest new empirical predictions on the interactions of both markets on spreads, as a function of the order flow correlation.

A limitation of our model is the timing that we consider. We indeed focus on the special case where i) dealers submit quotes simultaneously in both markets, ii) without knowing the direction of the order flows routed towards the dominant market, nor towards the satellite market, and iii) when the arrival of order flows is simultaneous in both markets. One of the possible extensions of our model would be to check the robustness of our results i) when dealers possess further information on the direction of order flows and ii) when dealers have the opportunity to revise and resubmit their quotes in the alternative trading venue after an execution in one of the trading systems, with a probability that (in practice) depends on the execution speed. We currently work on these extensions, and our initial results seem to confirm the predictions and implications that are presented in this paper.

Table 1

Figure 1

7 References

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8 Appendix: proofs

Proof of Lemmas 1 and 2

Let us denote by $\mathbb{k}_x$ a dummy variable which takes the value 1 if condition $\{x\}$ holds and zero otherwise. The dealer's expected utility from trading is as follows:

$$\begin{aligned}
& EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B\right) \\
= \; & \Pr\left(Q_A > 0 \cap Q_B > 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A > 0 \cap Q_B > 0\right) \\
& + \Pr\left(Q_A > 0 \cap Q_B = 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A > 0 \cap Q_B = 0\right) \\
& + \Pr\left(Q_A > 0 \cap Q_B < 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A > 0 \cap Q_B < 0\right) \\
& + \Pr\left(Q_A = 0 \cap Q_B > 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A = 0 \cap Q_B > 0\right) \\
& + \Pr\left(Q_A = 0 \cap Q_B = 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A = 0 \cap Q_B = 0\right) \\
& + \Pr\left(Q_A = 0 \cap Q_B < 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A = 0 \cap Q_B < 0\right) \\
& + \Pr\left(Q_A < 0 \cap Q_B > 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A < 0 \cap Q_B > 0\right) \\
& + \Pr\left(Q_A < 0 \cap Q_B = 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A < 0 \cap Q_B = 0\right) \\
& + \Pr\left(Q_A < 0 \cap Q_B < 0\right) \times EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A < 0 \cap Q_B < 0\right)
\end{aligned}$$

We first study the 9 partial utilities.

1)

$$\begin{aligned}
& EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A > 0 \cap Q_B > 0\right) \\
= & \begin{pmatrix}
\left(a_1^A Q_A + a_1^B Q_B + v_0\left(I_1 - Q_A - Q_B\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_A - Q_B\right)^2\right) \\
\times I\left(a_1^A < a_2^A\right) \times I\left(a_1^B < a_2^B\right) \\
+ \left(a_1^B Q_B + v_0\left(I_1 - Q_B\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_B\right)^2\right) \\
\times I\left(a_1^A > a_2^A\right) \times I\left(a_1^B < a_2^B\right) \\
+ \left(a_1^A Q_A + v_0\left(I_1 - Q_A\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_A\right)^2\right) \\
\times I\left(a_1^A < a_2^A\right) \times I\left(a_1^B > a_2^B\right) \\
+ \left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(a_1^A > a_2^A\right) \times I\left(a_1^B > a_r^B\right)
\end{pmatrix}
\end{aligned}$$

2)

$$
\begin{aligned}
& EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A > 0 \cap Q_B = 0\right) \\
= & \left(\begin{array}{c}
\left(a_1^A Q_A + v_0\left(I_1 - Q_A\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_A\right)^2\right) \times I\left(a_1^A < a_2^A\right) \\
+\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(a_1^A > a_2^A\right)
\end{array}\right)
\end{aligned}
$$

3)

$$
\begin{aligned}
& EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A > 0 \cap Q_B < 0\right) \\
= & \left(\begin{array}{c}
\left(a_1^A Q_A - b_1^B\left(-Q_B\right) + v_0\left(I_1 - Q_A + \left(-Q_B\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_A + \left(-Q_B\right)\right)^2\right) \\
\times I\left(a_1^A < a_2^A\right) \times I\left(b_1^B > b_2^B\right) \\
\left(-b_1^B\left(-Q_B\right) + v_0\left(I_1 + \left(-Q_B\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_B\right)\right)^2\right) \\
\times I\left(a_1^A > a_2^A\right) \times I\left(b_1^B > b_2^B\right) \\
\left(a_1^A Q_A + v_0\left(I_1 - Q_A\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_A\right)^2\right) \\
\times I\left(a_1^A < a_2^A\right) \times I\left(b_1^B < b_2^B\right) \\
\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(a_r^A > a_2^A\right) \times I\left(b_1^B < b_2^B\right)
\end{array}\right)
\end{aligned}
$$

4)

$$
\begin{aligned}
& EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A = 0 \cap Q_B > 0\right) \\
= & \left(\begin{array}{c}
\left(a_1^B Q_B + v_0\left(I_1 - Q_B\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_B\right)^2\right) \\
\times I\left(a_1^B < a_2^B\right) \\
+\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(a_1^B > a_2^B\right)
\end{array}\right)
\end{aligned}
$$

5)

$$
EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A = 0 \cap Q_B = 0\right) = \left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right)
$$

6)

$$
\begin{aligned}
& EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A = 0 \cap Q_B < 0\right) \\
= & \left(\begin{array}{c}
\left(-b_1^B\left(-Q_B\right) + v_0\left(I_1 + \left(-Q_B\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_B\right)\right)^2\right) \\
\times I\left(b_{r,1}^B > b_{r,2}^B\right) \\
+\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(b_{r,1}^B < b_{r,2}^B\right)
\end{array}\right)
\end{aligned}
$$

7)

$$
\begin{aligned}
&EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A < 0 \cap Q_B > 0\right) \\
&= \left(\begin{array}{c}
\left(-b_1^A\left(-Q_A\right) + a_1^B Q_B + v_0\left(I_1 + \left(-Q_A\right) - Q_B\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_A\right) - Q_B\right)^2\right) \\
\times I\left(b_1^A > b_2^A\right) \times I\left(a_1^B < a_2^B\right) \\
+\left(a_1^B Q_B + v_0\left(I_1 - Q_B\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 - Q_B\right)^2\right) \\
\times I\left(b_1^A < b_2^A\right) \times I\left(a_1^B < a_2^B\right) \\
+\left(-b_1^A\left(-Q_A\right) + v_0\left(I_1 + \left(-Q_A\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_A\right)\right)^2\right) \\
\times I\left(b_1^A > b_2^A\right) \times I\left(a_1^B > a_2^B\right) \\
+\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(b_1^A < b_2^A\right) \times I\left(a_1^B > a_2^B\right)
\end{array}\right)
\end{aligned}
$$

8)

$$
\begin{aligned}
&EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A < 0 \cap Q_B = 0\right) \\
&= \left(\begin{array}{c}
\left(-b_1^A\left(-Q_A\right) + v_0\left(I_1 + \left(-Q_A\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_A\right)\right)^2\right) \times I\left(b_1^A > b_2^A\right) \\
+\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(b_1^A < b_2^A\right)
\end{array}\right)
\end{aligned}
$$

9)

$$
\begin{aligned}
&EU_1\left(a_1^A, b_1^B, a_1^B, b_1^B | Q_A < 0 \cap Q_B < 0\right) \\
&= \left(\begin{array}{c}
\left(\begin{array}{c}
-b_1^A\left(-Q_A\right) - b_1^B\left(-Q_B\right) + v_0\left(I_1 + \left(-Q_A\right) + \left(-Q_B\right)\right) \\
-\frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_A\right) + \left(-Q_B\right)\right)^2
\end{array}\right) \\
\times I\left(b_1^A > b_2^A\right) \times I\left(b_1^B > b_2^B\right) \\
+\left(-b_1^A\left(-Q_A\right) + v_0\left(I_1 + \left(-Q_A\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_A\right)\right)^2\right) \\
\times I\left(b_1^A > b_2^A\right) \times I\left(b_1^B < b_2^B\right) \\
+\left(-b_1^B\left(-Q_B\right) + v_0\left(I_1 + \left(-Q_B\right)\right) - \frac{\rho\sigma_v^2}{2}\left(I_1 + \left(-Q_B\right)\right)^2\right) \\
\times I\left(b_1^A < b_2^A\right) \times I\left(b_1^B > b_2^B\right) \\
+\left(v_0 I_1 - \frac{\rho\sigma_v^2}{2} I_1^2\right) \times I\left(b_1^A < b_2^A\right) \times I\left(b_1^B < b_2^B\right)
\end{array}\right)
\end{aligned}
$$

After re-arranging terms, we find:

Lemma 2 *Dealer D_1's expected utility can be writen:*

$$EU_1\left(a_1^A, b_1^A, a_1^B, b_1^B\right)$$

$$\begin{aligned}
&= EU_1^0(I_1) \\
&+\left(a_1^A - v_0 - \frac{\rho\sigma_v^2}{2}\times(Q_A - 2I_1)\right)\times Q_A \times \Bbbk\left(a_1^A < a_2^A\right)\times \Pr\left(\tilde{Q}_A > 0\right) \\
&+\left(v_0 - \frac{\rho\sigma_v^2}{2}\times((-Q_A) + 2I_1) - b_1^A\right)\times(-Q_A)\times \Bbbk\left(b_1^A > b_2^A\right)\times \Pr\left(\tilde{Q}_A < 0\right) \\
&+\left(a_1^B - v_0 - \frac{\rho\sigma_v^2}{2}\times(Q_B - 2I_1)\right)\times Q_B \times \Bbbk\left(a_1^B < a_2^B\right)\times \Pr\left(\tilde{Q}_B > 0\right) \\
&+\left(v_0 - \frac{\rho\sigma_v^2}{2}\times((-Q_B) + 2I_1) - b_1^B\right)\times(-Q_B)\times \Bbbk\left(b_1^B > b_2^B\right)\times \Pr\left(\tilde{Q}_B < 0\right) \\
&-\rho\sigma_v^2 \times Q_A \times Q_B \times \Bbbk\left(a_1^B < a_2^B\right)\times \Bbbk\left(a_1^A < a_2^A\right)\times \Pr\left(\tilde{Q}_B > 0 \cap \tilde{Q}_A > 0\right) \\
&+\rho\sigma_v^2 \times Q_A \times (-Q_B) \times \Bbbk\left(b_1^B > b_2^B\right)\times \Bbbk\left(a_1^A < a_2^A\right)\times \Pr\left(\tilde{Q}_B < 0 \cap \tilde{Q}_A > 0\right) \\
&+\rho\sigma_v^2 \times (-Q_A) \times Q_B \times \Bbbk\left(a_1^B < a_2^B\right)\times \Bbbk\left(b_1^A > b_2^A\right)\times \Pr\left(\tilde{Q}_B > 0 \cap \tilde{Q}_A < 0\right) \\
&-\rho\sigma_v^2 \times (-Q_A) \times (-Q_B) \times \Bbbk\left(b_1^B > b_2^B\right)\times \Bbbk\left(b_1^A > b_2^A\right)\times \Pr\left(\tilde{Q}_B < 0 \cap \tilde{Q}_A < 0\right)
\end{aligned}$$

If we define:

$$\begin{aligned}
&\Delta_j(\mathbf{v}_1, \mathbf{v}_2, \phi) = -\rho\sigma_v^2 \times Q_A \times Q_B \times \Bbbk\left(a_1^B < a_2^B\right)\times \Bbbk\left(a_1^A < a_2^A\right)\times \Pr\left(\tilde{Q}_B > 0 \cap \tilde{Q}_A > 0\right) \\
&+\rho\sigma_v^2 \times Q_A \times (-Q_B) \times \Bbbk\left(b_1^B > b_2^B\right)\times \Bbbk\left(a_1^A < a_2^A\right)\times \Pr\left(\tilde{Q}_B < 0 \cap \tilde{Q}_A > 0\right) \\
&+\rho\sigma_v^2 \times (-Q_A) \times Q_B \times \Bbbk\left(a_1^B < a_2^B\right)\times \Bbbk\left(b_1^A > b_2^A\right)\times \Pr\left(\tilde{Q}_B > 0 \cap \tilde{Q}_A < 0\right) \\
&-\rho\sigma_v^2 \times (-Q_A) \times (-Q_B) \times \Bbbk\left(b_1^B > b_2^B\right)\times \Bbbk\left(b_1^A > b_2^A\right)\times \Pr\left(\tilde{Q}_B < 0 \cap \tilde{Q}_A < 0\right)
\end{aligned}$$

This leads to Lemma 1.■

The indeterminacy of reservation prices: an illustration

In a single market (special case for $Q_B = 0$), we find the ask and bid reservation prices given by Result 2. For instance, dealer D_1's ask reservation price $a_{r,1}$ is such that his expected utility if he trades $\tilde{Q}_A > 0$, is equal to his expected utility if he does not trade, *i.e.* such that:

$$\left(a_{r,1} - v_0 - \frac{\rho\sigma_v^2}{2}\times(Q_A - 2I_1)\right)\times Q_A = 0$$

In multiple markets though, these reservation prices cannot be uniquely identified.

We illustrate this idea according to which, in multiple markets, reservation prices cannot be evaluated independantly in each market when quotes are simultaneous. Assume for instance that dealer D_1 submits the best ask price in market B, whatever the (partial) equlibrium quotes in market A. What is his ask reservation price in market A? If he trades $Q_A > 0$, his trading surplus is thus:

$$\begin{aligned} & E\left(S_1^E\right) \\ = & \left(a_1^A - v_0 - \frac{\rho\sigma_v^2}{2} \times (Q_A - 2I_1)\right) \times Q_A \\ & + \left(a^B_{1|D_1\text{trades } Q_A>0} - v_0 - \frac{\rho\sigma_v^2}{2} \times (Q_B - 2I_1)\right) \times Q_B \times \Pr\left(\tilde{Q}_B > 0|\tilde{Q}_A > 0\right) \\ & -\rho\sigma_v^2 \times Q_A \times Q_B \times \Pr\left(\tilde{Q}_B > 0|\tilde{Q}_A > 0\right) \end{aligned}$$

If he does not trade $\tilde{Q}_A > 0$, his trading surplus can only be due to a potential sale in market B. However, if the dealer does not trade $\tilde{Q}_A > 0$, he may submit an ask price in market B different from the price that he would post if he trades $\tilde{Q}_A > 0$. His expected surplus if he does not submit the best ask price in market A is thus:

$$E\left(S_1^{NE}\right) = \left(a^B_{1|D_1 \text{ do not trade} Q_A>0} - v_0 - \frac{\rho\sigma_v^2}{2} \times (Q_B - 2I_j)\right) \times Q_B \times \Pr\left(\tilde{Q}_B > 0|\tilde{Q}_A > 0\right)$$

Conditional on $Q_A > 0$, dealer D_1's ask reservation price in market A depends on i) the expected direction of the order flow in market B, conditional on $Q_A > 0$, and ii) ask and bid equilibrium prices in market B. As this example clearly illustrates, it is not possible to determine dealers' reservation prices independantly from the equilibrium prices in the alternative trading system.

Instead of defining a (unique) reservation price at the ask (resp. at the bid) in market i, such that the dealer is indifferent between trading $Q_i > 0$ or not (resp. $Q_i < 0$), in multiple markets we must define a *vector* of reservation prices, $\mathbf{v}_{r,j} = \left(a^A_{r,j}, b^A_{r,j}, a^B_{r,j}, b^B_{r,j}\right)$ such that, conditional on the realization of the couple $\left(\tilde{Q}_A, \tilde{Q}_B\right)$, the dealer is indifferent between trading simultaneously $\tilde{Q}_A$ in market A at the ask price $a^A_{r,j}$ (or the bid price $b^A_{r,j}$), **and** $\tilde{Q}_B$ at the ask price $a^B_{r,j}$ (or at the bid price $b^B_{r,j}$) in market B, or not trading. Vector $\mathbf{v}_r$ of reservation prices is thus such that:

$$EU_j\left(a^A_{r,j}, b^A_{r,j}, a^B_{r,j}, b^B_{r,j}\right) = EU_j^0\left(I_j\right) \tag{2}$$

This definition does not enable us to uniquely determine the vector $\mathbf{v}_r$. Indeed, we still have two degrees of freedom, which are the result of Equation 2.

The multiplicity in the vectors of reservation prices leads to a multiplicity of equilibria. Each equilibrium can be sustained by a given vector, which would satisfy condition 2. However, it is possible, under some conditions, to bypass this multiplicity.■

Proof : identical quotes, Proposition 1

If the price in both markets must be identical, even for different quantities, then dealer D_1's ask reservation price is such that:

$$
\begin{aligned}
0 \;=\; & \left(a_{r,1} - v_0 - \frac{\rho\sigma_v^2}{2} \times (Q_A - 2I_1)\right) \times Q_A \times \Pr\left(\tilde{Q}_A > 0\right) \\
& + \left(a_{r,1} - v_0 - \frac{\rho\sigma_v^2}{2} \times (Q_B - 2I_1)\right) \times Q_B \times \Pr\left(\tilde{Q}_B > 0\right) \\
& - \rho\sigma_v^2 \times |Q_A| \times |Q_B| \times \Pr\left(\tilde{Q}_B > 0 \cap \tilde{Q}_A > 0\right)
\end{aligned}
$$

We finally find:

$$a_{r,1}^{id} = v_0 + \frac{\rho\sigma_v^2}{2}\left(\frac{(Q_A^2 + Q_B^2 + 2 \times \gamma \times Q_A \times Q_B)}{Q_A + Q_B} - 2I_1\right)$$

Which writes:

$$a_{r,1}^{id} = v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A + Q_B - 2 \times (1-\gamma) \times \frac{Q_A \times Q_B}{Q_A + Q_B} - 2I_1\right)$$

■

Proof of Propositions 2 and 3.

We first come back on Assumption 3, according to which A is the dominant market. We conjecture that under this assumption, dealers deterine theirs quotes so as to execute the order flow Q_A.

- We first look for equilibria such that D_1 submits the best ask prices in both markets.

Since his competitor may execute a sell order flow in market B, his minimal reservation price in market A is:

$$a_{r,2}^{A} = v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2\left(I_2 + \mu\left|Q_B\right|\right)\right)$$

while D_1's minimal reservation price is such that he only accounts for his dual-liability risk in market B:

$$a_{r,1}^{A} = v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2I_1\right)$$

Thus,

$$a_{r,1}^{A} < a_{r,2}^{A} \Leftrightarrow I_1 - \mu\left|Q_B\right| > I_2$$

Reservation prices in market B are:

$$\begin{aligned} a_{r,2}^{B} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_B - 2I_2\right) \\ a_{r,1}^{B} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_B - 2\left(I_1 - \gamma\left|Q_A\right|\right)\right) \end{aligned}$$

We check that in this case,

$$a_{r,1}^{B} < a_{r,2}^{B} \Leftrightarrow I_1 - \gamma\left|Q_A\right| > I_2$$

The necessary condition for D_1 to submit the best ask prices in both markets is finally:

$$I_1 - \max\left(\gamma\left|Q_A\right|, \mu\left|Q_B\right|\right) > I_2$$

Similarly, we show that if D_1 is first seller in both markets, his bid reservation prices write:

$$\begin{aligned} b_{r,1}^{A} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left|Q_A\right| + 2\left(I_1 - \mu\left|Q_A\right|\right)\right) \\ b_{r,1}^{B} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left|Q_B\right| + 2I_1\right) \end{aligned}$$

Indeed, for the following reservation prices' vector:

$$\begin{aligned}
a_{r,1}^{A} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2I_1\right) \\
b_{r,1}^{A} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + 2\left(I_1 - \mu\,|Q_B|\right)\right) \\
a_{r,1}^{B} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_B - 2\left(I_1 - \gamma\,|Q_A|\right)\right) \\
b_{r,1}^{B} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_B| + 2I_1\right)
\end{aligned}$$

we have:

$$EU_1\left(a_{r,1}^{A}, b_{r,1}^{A}, a_{r,1}^{B}, b_{r,1}^{B}\right) = EU_1^{\mathbf{0}}\left(I_1\right)$$

Symmetrically for D_2 :

$$\begin{aligned}
a_{r,2}^{A} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2\left(I_2 + \mu\,|Q_B|\right)\right) \\
b_{r,2}^{A} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_A| + 2I_2\right) \\
a_{r,2}^{B} &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_B - 2I_2\right) \\
b_{r,2}^{B} &= v_0 - \frac{\rho\sigma_v^2}{2}\left(|Q_B| + 2\left(I_2 + \gamma\,|Q_B|\right)\right)
\end{aligned}$$

Remark: In this case, ask and bid quotes in a given market are not symmetric! Both dealers seek the execution of Q_A whatever its direction, and use market B to couter-balance their aggressivity in market A.

We check that under condition $I_1 - \max\left(\gamma\,|Q_A|\,, \mu\,|Q_B|\right) > I_2$, D_2 is indeed first buyer in each market.

$$b_{r,2}^{A} > b_{r,1}^{A} \Leftrightarrow I_2 < I_1 - \mu\,|Q_B|$$

Since by assumption, $\gamma > \mu$ and $|Q_A| > |Q_B|$, this leads us to the equilibrium described in Proposition 1.■

When $I_1 - \max\left(\gamma\,|Q_A|\,, \mu\,|Q_B|\right) < I_2$, such an equilibrium cannot be reached, since D_2 expects to be touched at the bid in market B, which enables him to submit aggressive ask prices in market A.

- We now look for equilibria such that D_1 submits the best ask price in market A and the best bid price in market B. Dealer D_1's reservation prices such that his ask reservation price in market A is minimal and his bid reservation price is maximal are:

$$\begin{aligned}
a_{r,1}^A &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2\left(I_1 + \mu\left|Q_B\right|\right)\right) \\
b_{r,1}^A &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left|Q_A\right| + 2I_1\right) \\
a_{r,1}^B &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_B - 2\left(I_1 - \gamma\left|Q_A\right|\right)\right) \\
b_{r,1}^B &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left|Q_B\right| + 2I_1\right)
\end{aligned}$$

Dealer D_2's reservation prices are symmetric:

$$\begin{aligned}
a_{r,2}^A &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2I_2\right) \\
b_{r,2}^A &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left|Q_A\right| + 2\left(I_2 - \mu\left|Q_B\right|\right)\right) \\
a_{r,2}^B &= v_0 + \frac{\rho\sigma_v^2}{2}\left(Q_A - 2I_2\right) \\
b_{r,2}^B &= v_0 - \frac{\rho\sigma_v^2}{2}\left(\left|Q_B\right| + 2\left(I_2 + \gamma\left|Q_A\right|\right)\right)
\end{aligned}$$

The necessary condition to reach such an equilibrium is that both following conditions simultaneously hold:

$$\begin{aligned}
a_{r,1}^A &< a_{r,2}^A \Leftrightarrow \left(I_1 + \mu\left|Q_B\right|\right) > I_2 \text{ true} \\
b_{r,1}^B &> b_{r,2}^B \Leftrightarrow I_1 - \gamma\left|Q_A\right| < I_2
\end{aligned}$$

Which leads to Proposition 3.■

- Conjecture according to which dealers determine their quotes so as to execute the order flow in market A when $|Q_A| > |Q_B|$.

When $I_1 - (\gamma + \mu)\left|Q_A\right| > I_2$, dealer D_1 has a sufficiently long position so as to post the best ask price in both markets. In this case indeed, event if D_1 executes $Q_A > 0$ and

$Q_B > 0$, his maximal reservation price in each market is strictly inferior to his opponent's reservation price, even if he expected to execute a sell order flow in one market.

$$\begin{aligned} a_{r,1}(Q_A, I_1 + \gamma Q_B) &< a_{r,2}(Q_A, I_2 - \mu |Q_B|) \\ a_{r,1}(Q_B, I_1 + \gamma Q_A) &< a_{r,2}(Q_B, I_2 - \mu |Q_A|) \end{aligned}$$

When $I_1 - \gamma |Q_A| > I_2 > I_1 - (\gamma + \mu)|Q_A|$, D_2 could deviate from the equilibrium defined in Proposition 2, for instance in submitting an ask price in market B that would be inferior to D_1's, *i.e.* $a_1^B = v_0 + \frac{\rho\sigma_v^2}{2}(|Q_B| - 2I_2) - \varepsilon$ (and a higher ask price in market A). Such a deviation is not sustainable. If he deviates, then D_2 submits an ask price in market A such that his expected surplus from trading, in the case where a buy order flow is routed towards market A, is weak (due to the long initial position of D_1). Besides, this deviation induces D_1 to act symmetrically and undercut him at the id. Since the direction of order flows is unknown, and since the probability to observe a sell order flow is equal to the probability to observe a buy order flow, D_2 would earn a higher expected profit if he does not deviate: he would not only decrease his inventory risk by executing sell order flow, and earn a large profit since his opponent quotes at the bid are less aggressive.

When $I_1 - \gamma |Q_A| < I_2$, dealer D_1 is no longer in a position to post the best quotes in both markets. For $|Q_A| > |Q_B|$, both dealers are better off giving their competitor the opportunity to undercut in market B rather than in market A, so as to use the largest transaction to decrease their inventory risk (again, this is due to the uncertainty on the direction of the order flows).■

Proof of Corollary 2: Average spread in market A

In a single market, the average spread for a transaction size Q when two dealers with inventories (I_1, I_2) compete for the order flow, the quoted spread can be deduced from Propositions 2 and 3 by imposing $Q_B = 0$:

$$s^u(Q, I_1, I_2) = \rho\sigma_v^2 (Q + \max(I_1, I_2) - \min(I_1, I_2))$$

The average spread is thus:

$$\bar{s}^u(q) \equiv \int_{I_d}^{I_u}\int_{I_d}^{I_u} s^u(q, I_1, I_2) f(I_1) f(I_2) dI_1 dI_2 = \rho\sigma_v^2 \left(q + \frac{(I_u - I_d)}{3}\right)$$

In market A, we show that:

$$s^A(q, I_1, I_2) = \rho\sigma_v^2 (q + \max(I_1, I_2) - \min(I_1, I_2)) + \delta^A_{|I_1,I_2}(\mu, \gamma, |Q_B|)$$

where $\delta^A_{|I_1,I_2}(\mu, \gamma, |Q_B|)$ is such that:

- If $I_1 - \gamma|Q_A| > I_2$ or if $I_2 - \gamma|Q_A| > I_1$ then:

$$\delta^A_{|I_1,I_2} = -2 \times \rho\sigma_v^2 \times \mu|Q_B| \equiv \delta^A_1$$

- If $I_1 > I_2 > I_1 - \gamma|Q_A|$ or if $I_2 > I_1 > I_2 - \gamma|Q_A|$, then:

$$\delta^A_{|I_1,I_2} = 0 \equiv \delta^A_0$$

Since:

$$E(\delta^A) = -2 \times \rho\sigma_v^2 \times \mu|Q_B| \int_{I_d}^{I_u}\int_{I_d}^{I_u} \delta^A_{|I_1,I_2} f(I_1) f(I_2) dI_1 dI_2$$

and since

- for $I_1 < I_d + \gamma|Q_A|$, then $I_1 - \gamma|Q_A| < I_d$ so $I_1 - \gamma|Q_A| < I_2$ even if $I_1 > I_2$.

- for $I_1 > I_u - \gamma|Q_A|$, then $I_1 > I_2 - \gamma|Q_A|$ even if $I_1 < I_2$.

Finally:

$$\begin{aligned}
&(I_u - I_d)^2 \times E(\delta^A) \\
= \; & \int_{I_d}^{I_d+\gamma|Q_A|}\left(\int_{I_d}^{I_1}\delta^A_0 dI_2 + \int_{I_1}^{I_1+\gamma|Q_A|}\delta^A_0 dI_2 + \int_{I_1+\gamma|Q_A|}^{I_u}\delta^A_1 dI_2\right) dI_1 \\
& + \int_{I_d+\gamma|Q_A|}^{I_u-\gamma|Q_A|}\left(\begin{array}{c}\int_{I_d}^{I_1-\gamma|Q_A|}\delta^A_1 dI_2 + \int_{I_1-\gamma|Q_A|}^{I_1}\delta^A_0 dI_2 \\ + \int_{I_1}^{I_1+\gamma|Q_A|}\delta^A_0 dI_2 + \int_{I_1+\gamma|Q_A|}^{I_u}\delta^A_1 dI_2\end{array}\right) dI_1 \\
& + \int_{I_u-\gamma|Q_A|}^{I_u}\left(\int_{I_d}^{I_1-\gamma|Q_A|}\delta^A_1 dI_2 + \int_{I_1-\gamma|Q_A|}^{I_1}\delta^A_0 dI_2 + \int_{I_1}^{I_u}\delta^A_0 dI_2\right) dI_1
\end{aligned}$$

Which writes

$$\frac{(I_u-I_d)^2}{-2\times\rho\sigma_v^2\times\mu\,|Q_B|}\times E\left(\delta^A\right)=\int_{I_d}^{I_d+\gamma|Q_A|}\left(\int_{I_1+\gamma|Q_A|}^{I_u}dI_2\right)dI_1$$
$$+\int_{I_d+\gamma|Q_A|}^{I_u-\gamma|Q_A|}\left(\int_{I_d}^{I_1-\gamma|Q_A|}dI_2+\int_{I_1+\gamma|Q_A|}^{I_u}dI_2\right)dI_1+\int_{I_u-\gamma|Q_A|}^{I_u}\left(\int_{I_d}^{I_1-\gamma|Q_A|}dI_2\right)dI_1$$

And leads to:

$$\bar{s}^A\left(|Q_A|\right)=\rho\sigma_v^2\left(|Q_A|+\frac{(I_u-I_d)}{3}\right)-2\times\rho\sigma_v^2\times\mu\,|Q_B|\times\frac{(I_u-I_d-\gamma\,|Q_A|)^2}{(I_u-I_d)^2}$$

Proof of Corollary 3: Average spread in market B

The average spread in market B is such that:

$$\begin{aligned}
&(I_u-I_d)^2\times\bar{s}^B\left(|Q_B|\right)\\
=\;&\int_{I_d}^{I_d+\gamma|Q_A|}\left(\int_{I_d}^{I_1}s^B_{0,I_1>I_2}dI_2+\int_{I_1}^{I_1+\gamma|Q_A|}s^B_{0,I_1<I_2}dI_2+\int_{I_1+\gamma|Q_A|}^{I_u}s^B_{1,I_1<I_2}dI_2\right)dI_1\\
&+\int_{I_d+\gamma|Q_A|}^{I_u-\gamma|Q_A|}\left(\begin{array}{c}\int_{I_d}^{I_1-\gamma|Q_A|}s^B_{1,I_1>I_2}dI_2+\int_{I_1-\gamma|Q_A|}^{I_1}s^B_{0,I_1>I_2}dI_2\\+\int_{I_1}^{I_1+\gamma|Q_A|}s^B_{0,I_1<I_2}dI_2+\int_{I_1+\gamma|Q_A|}^{I_u}s^B_{1,I_1<I_2}dI_2\end{array}\right)dI_1\\
&+\int_{I_u-\gamma|Q_A|}^{I_u}\left(\int_{I_d}^{I_1-\gamma|Q_A|}s^B_{1,I_1>I_2}dI_2+\int_{I_1-\gamma|Q_A|}^{I_1}s^B_{0,I_1>I_2}dI_2+\int_{I_1}^{I_u}s^B_{0,I_1<I_2}dI_2\right)dI_1
\end{aligned}$$

with:

$$\begin{aligned}
s^B_{1,I_1>I_2}&=\rho\sigma_v^2\left(|Q_B|+I_1-I_2\right)\\
s^B_{1,I_1<I_2}&=\rho\sigma_v^2\left(|Q_B|+I_2-I_1\right)\\
s^B_{0,I_1>I_2}&=\rho\sigma_v^2\left(|Q_B|+I_2-I_1+2\gamma\,|Q_A|\right)\\
s^B_{0,I_1<I_2}&=\rho\sigma_v^2\left(|Q_B|+I_1-I_2+2\gamma\,|Q_A|\right)
\end{aligned}$$

- For $|Q_B|=0$

$$\frac{(I_u - I_d)^2}{\rho\sigma_v^2} \times \bar{s}_{\mathbf{0}}^B(|Q_B|) = \int_{I_d}^{I_u}\int_{I_d}^{I_u} |Q_B|\, dI_2 dI_1$$
$$+ \int_{I_d}^{I_d+\gamma|Q_A|} \left(\begin{array}{c} \int_{I_d}^{I_1} (I_2 - I_1)\, dI_2 + \int_{I_1}^{I_1+\gamma|Q_A|} (I_1 - I_2)\, dI_2 \\ + \int_{I_1+\gamma|Q_A|}^{I_u} (I_2 - I_1)\, dI_2 \end{array} \right) dI_1$$
$$+ \int_{I_d+\gamma|Q_A|}^{I_u-\gamma|Q_A|} \left(\begin{array}{c} \int_{I_d}^{I_1-\gamma|Q_A|} (I_1 - I_2)\, dI_2 + \int_{I_1-\gamma|Q_A|}^{I_1} (I_2 - I_1)\, dI_2 \\ + \int_{I_1}^{I_1+\gamma|Q_A|} (I_1 - I_2)\, dI_2 + \int_{I_1+\gamma|Q_A|}^{I_u} (I_2 - I_1)\, dI_2 \end{array} \right) dI_1$$
$$+ \int_{I_u-\gamma|Q_A|}^{I_u} \left(\begin{array}{c} \int_{I_d}^{I_1-\gamma|Q_A|} (I_1 - I_2)\, dI_2 + \int_{I_1-\gamma|Q_A|}^{I_1} (I_2 - I_1)\, dI_2 \\ + \int_{I_1}^{I_u} (I_1 - I_2)\, dI_2 \end{array} \right) dI_1$$

So:

$$\bar{s}_{\mathbf{0}}^B(|Q_B|) = \rho\sigma_v^2 \left(|Q_B| + \frac{1}{3}(I_u - I_d) - 2 \times \frac{(\gamma|Q_A|)^2}{(I_u - I_d)^2} \times \left(I_u - I_d + \frac{1}{3}\gamma|Q_A| \right) \right)$$

- Plus, for $|Q_B| > 0$

$$\begin{aligned} & \frac{(I_u - I_d)^2}{\rho\sigma_v^2 \times 2\gamma|Q_A|} \times E\left(\delta^B\right) \\ = & \int_{I_d}^{I_d+\gamma|Q_A|} \left(\int_{I_d}^{I_1+\gamma|Q_A|} dI_2 \right) dI_1 + \int_{I_d+\gamma|Q_A|}^{I_u-\gamma|Q_A|} \left(\int_{I_1-\gamma|Q_A|}^{I_1+\gamma|Q_A|} dI_2 \right) dI_1 \\ & + \int_{I_u-\gamma|Q_A|}^{I_u} \left(\int_{I_1-\gamma|Q_A|}^{I_u} dI_2 \right) dI_1 \end{aligned}$$

So:

$$E\left(\delta^B\right) = 4\rho\sigma_v^2 \frac{(\gamma|Q_A|)^2}{(I_u - I_d)^2} \times \left(I_u - I_d - \frac{1}{2}\gamma|Q_A| \right)$$

- Finally

$$\bar{s}^B(|Q_B|) = \rho\sigma_v^2 \left(|Q_B| + \frac{1}{3}(I_u - I_d) - 2 \times \frac{(\gamma|Q_A|)^2}{(I_u - I_d)^2} \times \left(-I_u + I_d + \frac{4}{3}\gamma|Q_A| \right) \right)$$

Identical quotes

- According to Corollary 1, the quoted spread in this case is, if $I_1 > I_2$:

$$s^{id} = \rho\sigma_v^2 \left(|Q_A| + |Q_B| - 2 \times (1-\gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} + I_1 - I_2 \right) - 2\varepsilon$$

When $I_1 - \gamma |Q_A| > I_2$, Proposition 2 leads to:

$$\begin{aligned} s_1^A &= \rho\sigma_v^2 \left(|Q_A| + I_1 - I_2 - 2\mu |Q_B| \right) \\ s_1^B &= \rho\sigma_v^2 \left(|Q_B| + I_1 - I_2 \right) \end{aligned}$$

and when $I_1 > I_2 > I_1 - \gamma |Q_A| > I_2$, Proposition 3 leads to:

$$\begin{aligned} s_2^A &= \rho\sigma_v^2 \left(|Q_A| + I_1 - I_2 \right) \\ s_2^B &= \rho\sigma_v^2 \left(|Q_B| + I_2 + 2\gamma |Q_A| - I_1 \right) \end{aligned}$$

- Let us compare the spreads quoted in market A.

$$s_1^A < s^{id} \Leftrightarrow \left(-\mu \times \frac{|Q_B|}{|Q_A| + |Q_B|} + \theta \times \frac{|Q_A|}{|Q_A| + |Q_B|} \right) < \frac{1}{2}$$

which always holds for $\gamma > \frac{1}{2}$.

$$s_2^A < s^{id} \Leftrightarrow (1-\gamma) \times \frac{|Q_A|}{|Q_A| + |Q_B|} < \frac{1}{2}$$

which always holds for $\gamma > \frac{1}{2}$.

Consequently:

$$\bar{s}^A < \bar{s}^{id}$$

- Let us compare the spreads quoted in market B.

$$s_1^B < s^{id} \Leftrightarrow (1-\gamma) \times \frac{|Q_B|}{|Q_A| + |Q_B|} < \frac{1}{2}$$

which always holds for $\gamma > \frac{1}{2}$.

However, the following inequality does not necessarily hold.

$$\begin{aligned} s_2^B &< s^{id} \Leftrightarrow (I_2 - (I_1 - \gamma |Q_A|)) < \frac{1}{2}\left(1 - 2 \times (\mu + \theta) \times \frac{|Q_B|}{|Q_A| + |Q_B|}\right) \times |Q_A| \\ &\Leftrightarrow -(I_1 - I_2) < \left(\frac{1}{2}\left(1 - 2 \times (\mu + \theta) \times \frac{|Q_B|}{|Q_A| + |Q_B|}\right) - \gamma\right) \times |Q_A| \end{aligned}$$

But $\left(\frac{1}{2}\left(1 - 2 \times (\mu + \theta) \times \frac{|Q_B|}{|Q_A| + |Q_B|}\right) - \gamma\right)$ can be either positive or negative.

- Let us finally compute the difference in average spreads.

$$\bar{s}^{id} = \rho\sigma_v^2 \left(|Q_A| + |Q_B| - 2 \times (1 - \gamma) \times \frac{|Q_A \times Q_B|}{|Q_A| + |Q_B|} + \frac{I_u - I_d}{3}\right)$$

Let us compare with th average spread quoted in market B:

$$\bar{s}^{id} - \bar{s}^B(|Q_B|) = \frac{1}{2} - \frac{|Q_B|}{|Q_A| + |Q_B|} - \gamma \times \left(\frac{\gamma |Q_A|}{(I_u - I_d)^2} \times \left(I_u - I_d - \frac{4}{3}\gamma |Q_A|\right) + \frac{|Q_B|}{|Q_A| + |Q_B|}\right)$$

Since $\left(\frac{\gamma|Q_A|}{(I_u - I_d)^2} \times \left(I_u - I_d - \frac{4}{3}\gamma |Q_A|\right)\right) < 1$, the inequality $\bar{s}^{id} - \bar{s}^B(|Q_B|) > 0$ only holds for some parameter values.

But:

$$\frac{\partial\left(\bar{s}^{id} - \bar{s}^B(|Q_B|)\right)}{\partial\gamma} = -\frac{2\gamma |Q_A|}{(I_u - I_d)^2} \times (I_u - I_d - 2\gamma |Q_A|) - \frac{|Q_B|}{|Q_A| + |Q_B|} < 0$$

Since $\left(\bar{s}^{id} - \bar{s}^B(|Q_B|)\right)_{|\gamma=\frac{1}{2}} < 0$, and $\left(\bar{s}^{id} - \bar{s}^B(|Q_B|)\right)_{|\gamma=1} > 0$, there exists a unique γ^* such that $\left(\bar{s}^{id} - \bar{s}^B(|Q_B|)\right)_{|\gamma=\gamma^*} = 0$.

■