Option Valuation with Long-run and Short-run Volatility Components

Peter Christoffersen
McGill University and CIRANO

Kris Jacobs
McGill University and CIRANO

Yintian Wang*
McGill University

October 24, 2005

Abstract

This paper presents a new model for the valuation of European options. In our model, the volatility of returns consists of two components. One of these components is a long-run component, and it can be modeled as fully persistent. The other component is short-run and has a zero mean. Our model can be viewed as an affine version of Engle and Lee (1999), allowing for easy valuation of European options. We investigate the model through an integrated analysis of returns and options data. The performance of the model is spectacular when compared to a benchmark single-component volatility model that is well-established in the literature. The improvement in the model's performance is due to its richer dynamics which enable it to jointly model long-maturity and short-maturity options.

JEL Classification: G12

Keywords: option valuation; long-run component; short-run component; unobserved components; persistence; GARCH; out-of-sample.

*The first two authors are grateful for financial support from FQRSC, IFM^2 and SSHRC. We would like to thank David Bates, Tim Bollerslev, Peter Boswijk, Frank De Jong, Frank Diebold, Joost Driessen, Rob Engle, Bjorn Eraker, Steve Heston, Robert Hauswald, Yongmiao Hong, Chris Jones, Mark Kamstra, Eric Jacquier, Nour Meddahi, Michel Robe, Gordon Sick, Jean-Guy Simonato, Daniel Smith and seminar participants at American University, HEC Montreal, ISCTE, McGill University, the St. Louis Fed, the Swiss Banking Institute, Queen's University, University of Amsterdam, University of Porto, York University, the UBC Summer Finance Conference, and the Winter Meetings of the Econometric Society for helpful remarks and discussions. Any remaining inadequacies are ours alone. Correspondence to: Peter Christoffersen, Faculty of Management, McGill University, 1001 Sherbrooke Street West, Montreal, Canada H3A 1G5; Tel: (514) 398-2869; Fax: (514) 398-3876; E-mail: peter.christoffersen@mcgill.ca.

1 Introduction

There is a consensus in the equity options literature that combining time-variation in the conditional variance of asset returns (Engle (1982), Bollerslev (1986)) with a leverage effect (Black (1976)) constitutes a potential solution to well-known biases associated with the Black-Scholes (1973) model, such as the implied volatility smirk. These asymmetric dynamic volatility models generate negative skewness in the distribution of asset returns which in turn generates higher prices for out-of-the-money put options as compared to the Black-Scholes formula. In the continuous-time option valuation literature, the Heston (1993) model addresses some of these biases. This model contains a leverage effect as well as stochastic volatility.[1] In the discrete-time literature, the NGARCH(1,1) option valuation model proposed by Duan (1995) contains time-variation in the conditional variance as well as a leverage effect. The model by Heston and Nandi (2000) is closely related to Duan's model.

Many existing empirical studies have confirmed the importance of time-varying volatility, the leverage effect and negative skewness in continuous-time and discrete-time setups, using parametric as well as non-parametric techniques.[2] However, it has become clear that while these models help explain the biases of the Black-Scholes model in a qualitative sense, they come up short in a quantitative sense. Using parameters estimated from returns or options data, these models reduce the biases of the Black-Scholes model, but the magnitude of the effects is insufficient to completely resolve the biases. The resulting pricing errors have the same sign as the Black-Scholes pricing errors, but are smaller in magnitude. We therefore need models that possess the same qualitative features as the models in Heston (1993) and Duan (1995), but that contain stronger quantitative effects. These models need to generate more flexible skewness and volatility of volatility dynamics in order to fit observed option prices.

One interesting approach in this respect is the inclusion of jump processes. In most existing studies, jumps are added to models that already contain time-variation in the conditional variance as well as a leverage effect. The empirical findings in this literature have been mixed. In general, for Poisson processes, jumps in returns and volatility improve option valuation when parameters are estimated using historical time series of returns, but not always when parameters are estimated using the cross-section of option prices.[3] Huang and

[1] The importance of stochastic volatility is also studied in Hull and White (1987), Melino and Turnbull (1990), Scott (1987) and Wiggins (1987).

[2] See for example Ait-Sahalia and Lo (1998), Amin and Ng (1993), Bakshi, Cao and Chen (1997), Bates (2000), Benzoni (1998), Bollerslev and Mikkelsen (1999), Chernov and Ghysels (2000), Duan, Ritchken and Sun (2004), Engle and Mustafa (1992), Eraker (2004), Heston and Nandi (2000), Jones (2003), Nandi (1998) and Pan (2002).

[3] See for example Bakshi, Cao and Chen (1997), Bates (2000), Broadie, Chernov and Johannes (2004), Eraker, Johannes and Polson (2003), Eraker (2004) and Pan (2002).

Wu (2004) find that other types of jump processes may provide a better fit.

This paper takes a different approach. We attempt to remedy the remaining option biases by modeling richer volatility dynamics.[4] It has been observed using a variety of diagnostics that it is difficult to fit the dynamics of return volatility using a benchmark model such as a GARCH(1,1). Similar remarks apply to stochastic volatility models such as Heston (1993). The main problem is that volatility autocorrelations are too high at longer lags to be explained by a GARCH(1,1), unless the process is extremely persistent. This extreme persistence may impact negatively on other aspects of option valuation, such as the valuation of short-maturity options.

In fact, it has been observed in the literature that volatility may be better modeled using a fractionally integrated process, rather than a stationary GARCH process.[5] Andersen, Bollerslev, Diebold and Labys (2003) confirm this finding using realized volatility. Bollerslev and Mikkelsen (1996, 1999) and Comte, Coutin and Renault (2001) investigate and discuss some of the implications of long memory for option valuation. Using fractional integration models for option valuation is somewhat cumbersome. Optimization is time-intensive and certain ad-hoc choices have to be made regarding implementation. This paper addresses the same issues using a different type of model that is easier to implement and captures the stylized facts addressed by long-memory models at horizons relevant for option valuation. The model builds on Heston and Nandi (2000) and Engle and Lee (1999). In our model, the volatility of returns consists of two components. One of these components is a long-run component, and it can be modeled as (fully) persistent. The other component is short-run and mean zero. The model is able to generate autocorrelations that are richer than those of a GARCH(1,1) model while using just a few additional parameters. We illustrate how this impacts on option valuation by studying the term structure of volatility.

Unobserved component or factor models are very popular in the finance literature. See Fama and French (1988), Poterba and Summers (1988) and Summers (1986) for applications to stock prices. In the option pricing literature, Bates (2000) and Taylor and Xu (1994) investigate two-factor stochastic volatility models. Duffie, Pan and Singleton (2000) provide a general continuous-time framework for the valuation of contingent claims using multifactor affine models. Eraker (2004) suggests the usefulness of a multifactor approach based on his empirical results. Alizadeh, Brandt and Diebold (2002) uncover two factors in stochastic volatility models of exchange rates using range-based estimation. Bollerslev and Zhou (2002), Brandt and Jones (2005), Chacko and Viceira (2003), Chernov, Gallant, Ghysels and Tauchen (2003), and Maheu (2002) also find that two-factor stochastic volatility models outperform single factor models when modeling daily asset return volatility. Adrian and Rosenberg (2005) investigate the relevance of a two-component volatility model for pricing the cross-section of stock returns. Unobserved component models are also very popular in the term structure

[4] Adding jumps to the new volatility specification may of course improve the model further.

[5] See Baillie, Bollerslev and Mikkelsen (1996).

literature, although in this literature the models are more commonly referred to as multifactor models.[6] There are very interesting parallels between our approach and results and stylized facts in the term structure literature. In the term structure literature it is customary to model short-run fluctuations around a time-varying long-run mean of the short rate. In our framework we model short-run fluctuations around a time-varying long-run volatility.

Dynamic factor and component models can be implemented in continuous or discrete time.[7] We choose a discrete-time approach because of the ease of implementation. In particular, our model is related to the GARCH class of processes and volatility filtering and forecasting are relatively straightforward, which is critically important for option valuation. An additional advantage of our model is parsimony: the most general model we investigate has seven parameters. We speculate that parsimony may help our model's out-of-sample performance.

We investigate the model using options data, while imposing consistency with the time series of underlying returns.[8] We study two models: one where the long-run component is constrained to be fully persistent and one where it is not. We refer to these models as the persistent component model and the component model respectively. When persistence of the long-run component is freely estimated, it is very close to one. The performance of the component model is spectacular when compared with a benchmark GARCH(1,1) model. When using all available option data, the dollar RMSE of the component model is on average 16.5% lower than that of the benchmark GARCH model in-sample and 22.1% out-of-sample. When using long-maturity options only, the dollar RMSE improvement is on average 17.8% in-sample and 33.7% out-of-sample. The improvement in the model's performance is due to its richer dynamics, which enable it to jointly model long-maturity and short-maturity options. Our out-of-sample results strongly suggest that these richer dynamics are not simply due to spurious in-sample overfitting. The persistent component model performs better than the benchmark GARCH(1,1) model, but it is inferior to the component model both in- and out-of-sample. We also provide a detailed study of the term structure of volatilities for our proposed models and the benchmark model.

We use the GARCH(1,1) as a benchmark model mainly for the following three reasons: First, Christoffersen, Jacobs and Mimouni (2005) find that the GARCH(1,1) model outperforms the benchmark stochastic volatility model in Heston (1993), which is the most commonly used benchmark in the literature. Second, the component model is a natural

[6] See for example Dai and Singleton (2000), Duffee (1999), Duffie and Singleton (1999) and Pearson and Sun (1994).

[7] Duffie, Pan and Singleton (2000) suggest a multifactor continuous-time model that captures the spirit of our approach, but do not investigate the model empirically.

[8] The literature does not contain a large number of studies that provide this type of analysis. See Bates (2000, 2004), Chernov and Ghysels (2000), Eraker (2004), Heston and Nandi (2000), Jones (2003) and Pan (2002) for empirical results and discussions of different techniques that can be used to perform this type of analysis.

generalization of the GARCH(1,1) model, and its implementation uses similar techniques. A third motivation for adopting the GARCH(1,1) model as a benchmark is that Heston and Nandi (2000) find that the GARCH(1,1) slightly outperforms the ad-hoc implied volatility benchmark model in Dumas, Fleming and Whaley (1998).

The paper proceeds as follows. Section 2 introduces the model. Section 3 discusses the volatility term structure and Section 4 discusses option valuation. Section 5 discusses the empirical results, and Section 6 concludes.

2 Return Dynamics with Volatility Components

In this section we first present the Heston-Nandi GARCH(1,1) model which will serve as the benchmark model throughout the paper. We then construct the component model as a natural extension of a rearranged version of the GARCH(1,1) model. Finally the persistent component model is presented as a special case of the component model.

2.1 The Heston and Nandi GARCH(1,1) Model

Heston and Nandi (2000) propose a class of GARCH models that allow for a closed-form solution for the price of a European call option. They present an empirical analysis of the GARCH(1,1) version of this model, which is given by

$$\begin{aligned} R_{t+1} &\equiv \ln(S_{t+1}/S_t) = r + \lambda h_{t+1} + \sqrt{h_{t+1}} z_{t+1} \qquad (1) \\ h_{t+1} &= w + b h_t + a(z_t - c\sqrt{h_t})^2 \end{aligned}$$

where S_{t+1} denotes the underlying asset price, r the risk free rate, λ the price of risk and h_{t+1} the daily variance on day $t+1$ which is known at the end of day t. The z_{t+1} shock is assumed to be i.i.d. $N(0,1)$. The Heston-Nandi model captures time variation in the conditional variance as in Engle (1982) and Bollerslev (1986),[9] and the parameter c captures the leverage effect. The leverage effect captures the negative relationship between shocks to returns and volatility (Black (1976)), which results in a negatively skewed distribution of returns.[10] Note that the GARCH(1,1) dynamic in (1) is slightly different from the more conventional NGARCH model used by Engle and Ng (1993) and Hentschel (1995), which is used for option valuation in Duan (1995). The reason is that the dynamic in (1) is engineered to yield a closed-form solution for option valuation, whereas a closed-form solution does not obtain for the more conventional GARCH dynamic. Hsieh and Ritchken (2000) provide

[9] For an early application of GARCH to stock returns, see French, Schwert and Stambaugh (1987).

[10] Its importance for option valuation has been emphasized among others by Benzoni (1998), Chernov and Ghysels (2000), Christoffersen and Jacobs (2004), Eraker (2004), Eraker, Johannes and Polson (2003), Heston (1993), Heston and Nandi (2000) and Nandi (1998).

evidence that the more traditional GARCH model may actually slightly dominate the fit of (1). Our main point can be demonstrated using either dynamic. Because of the convenience of the closed-form solution provided by dynamics such as (1), we use this as a benchmark in our empirical analysis and we model the richer component structure within the Heston-Nandi framework.[11]

To better appreciate the workings of the component models presented below, note that by using the expression for the unconditional variance

$$E\left[h_{t+1}\right] \equiv \sigma^2 = \frac{w+a}{1-b-ac^2}$$

the variance process can now be rewritten as

$$h_{t+1} = \sigma^2 + b\left(h_t - \sigma^2\right) + a\left((z_t - c\sqrt{h_t})^2 - (1 + c^2\sigma^2)\right) \tag{2}$$

2.2 Building a Component Volatility Model

The expression for the GARCH(1,1) variance process in (2) highlights the role of the parameter σ^2 as the constant unconditional mean of the conditional variance process. A natural generalization is then to specify σ^2 as time-varying. Denoting this time-varying component by q_{t+1}, the expression for the variance in (2) can be generalized to

$$h_{t+1} = q_{t+1} + \beta\left(h_t - q_t\right) + \alpha\left((z_t - \gamma_1\sqrt{h_t})^2 - (1 + \gamma_1^2 q_t)\right) \tag{3}$$

This model is similar in spirit to the component model of Engle and Lee (1999). The difference between our model and Engle and Lee (1999) is that the functional form of the GARCH dynamic (3) allows for a closed-form solution for European option prices. This is similar to the difference between the Heston-Nandi (2000) GARCH(1,1) dynamic and the more traditional NGARCH(1,1) dynamic discussed in the previous subsection. In specification (3), the conditional volatility h_{t+1} can most usefully be thought of as having two components. Following Engle and Lee (1999), we refer to the component q_{t+1} as the long-run component, and to $h_{t+1} - q_{t+1}$ as the short-run component. We will discuss this terminology in some more detail below. Note that by construction the unconditional mean of the short-run component $h_{t+1} - q_{t+1}$ is zero.

The model can also be written as

$$\begin{aligned} h_{t+1} &= q_{t+1} + (\alpha\gamma_1^2 + \beta)\left(h_t - q_t\right) + \alpha\left((z_t - \gamma_1\sqrt{h_t})^2 - (1 + \gamma_1^2 h_t)\right) \\ &= q_{t+1} + \tilde{\beta}\left(h_t - q_t\right) + \alpha\left((z_t - \gamma_1\sqrt{h_t})^2 - (1 + \gamma_1^2 h_t)\right) \end{aligned} \tag{4}$$

[11] See Bollerslev and Mikkelsen (1996), Engle and Mustafa (1992), Christoffersen and Jacobs (2004), and Hsieh and Ritchken (2000) for other empirical studies of European option valuation using GARCH dynamics. Ritchken and Trevor (1999) discusses the pricing of American options with GARCH processes.

where $\tilde{\beta} = \alpha\gamma_1^2 + \beta$. This representation is useful because we can think of

$$\begin{aligned} v_{1,t} &\equiv \left(z_t - \gamma_1\sqrt{h_t}\right)^2 - (1 + \gamma_1^2 h_t) \\ &= \left(z_t^2 - 1\right) - 2\gamma_1\sqrt{h_t}z_t \end{aligned} \tag{5}$$

as a mean-zero innovation.

The model is completed by specifying the functional form of the long-run volatility component. In a first step, we assume that q_{t+1} follows the process

$$q_{t+1} = \omega + \rho q_t + \varphi\left(\left(z_t^2 - 1\right) - 2\gamma_2\sqrt{h_t}z_t\right) \tag{6}$$

Note that $E\left[q_{t+1}\right] = E\left[h_{t+1}\right] = \sigma^2 = \dfrac{\omega}{1-\rho}$ as long as $\rho < 1$. We can therefore write the component volatility model as

$$\begin{aligned} h_{t+1} &= q_{t+1} + \tilde{\beta}\left(h_t - q_t\right) + \alpha v_{1,t} \\ q_{t+1} &= \omega + \rho q_t + \varphi v_{2,t} \\ &= \sigma^2 + \rho(q_t - \sigma^2) + \varphi v_{2,t} \end{aligned} \tag{7}$$

with

$$v_{i,t} = \left(z_t^2 - 1\right) - 2\gamma_i\sqrt{h_t}z_t, \text{ for } i = 1, 2. \tag{8}$$

and $E_{t-1}\left[v_{i,t}\right] = 0$, $i = 1, 2$. Also note that in addition to the price of risk, λ, the model contains seven parameters: α, $\tilde{\beta}$, γ_1, γ_2, ω, ρ and φ.

2.3 A Fully Persistent Special Case

In our empirical work, we also investigate a special case of the model in (7). Notice that in (7) the long-run component of volatility will be a mean reverting process for $\rho < 1$. We also estimate a version of the model which imposes $\rho = 1$. The resulting process is

$$\begin{aligned} h_{t+1} &= q_{t+1} + \tilde{\beta}\left(h_t - q_t\right) + \alpha v_{1,t} \\ q_{t+1} &= \omega + q_t + \varphi v_{2,t} \end{aligned} \tag{9}$$

and $v_{i,t}$, $i = 1, 2$ are as in (8). In addition to the price of risk, λ, the model now contains six parameters: $\alpha, \tilde{\beta}, \gamma_1, \gamma_2, \omega$ and φ .

In this case the process for long-run volatility contains a unit root and shocks to the long-run volatility never die out: they have a "permanent" effect. Recall that following Engle and Lee (1999) in (7) we refer to q_{t+1} as the long-run component and to $h_{t+1} - q_{t+1}$ as the short-run component. In the special case (9) we can also refer to q_{t+1} as the "permanent" component, because innovations to q_{t+1} are truly "permanent" and do not die out. It is then customary to refer to $h_{t+1} - q_{t+1}$ as the "transitory" component, which reverts to zero. It is in fact this permanent-effects version of the model that is most closely related to models which have been studied more extensively in the finance and economics literature, rather than the more general model in (7).[12] We will refer to this model as the persistent component model.

It is clear that (9) is nested by (7). It is therefore to be expected that the in-sample fit of (7) is superior. However, out-of-sample this may not necessarily be the case. It is often the case that more parsimonious models perform better out-of-sample if the restriction imposed by the model is a sufficiently adequate representation of reality. The persistent component model may also be better able to capture structural breaks in volatility out-of-sample, because a unit root in the process allows it to adjust to a structural break, which not possible for a mean-reverting process. It will therefore be of interest to verify how close ρ is to one when estimating the more general model (7).

3 Variance Term Structures

To intuitively understand the shortcomings of existing models such as the GARCH(1,1) model in (1) and the improvements provided by our model (7), it is instructive to graphically illustrate the workings of both models in a dimension that critically affects their performance. In this section we illustrate some properties of the models that are key for option valuation, namely variance term structures and their impulse response functions.

3.1 The Variance Term Structure for the GARCH(1,1) Model

Following the logic used for the component model in (7), we can rewrite the GARCH(1,1) variance dynamic in (2). We have

$$h_{t+1} = \sigma^2 + \tilde{b}\left(h_t - \sigma^2\right) + a\left((z_t^2 - 1) - 2c\sqrt{h_t}z_t\right) \tag{10}$$

where $\tilde{b} = b + ac^2$ and where the innovation term has a zero conditional mean. From (10) the multi-step forecast of the conditional variance is

$$E_t\left[h_{t+k}\right] = \sigma^2 + \tilde{b}^{k-1}(h_{t+1} - \sigma^2)$$

where the conditional expectation is taken at the end of day t. Notice that $\tilde{b}$ is directly interpretable as the variance persistence in this representation of the model.

[12] See Fama and French (1988), Poterba and Summers (1988) and Summers (1986) for applications to stock prices. See Beveridge and Nelson (1981) for an application to macroeconomics.

We can now define a convenient measure of the variance term structure for maturity K as

$$h_{t+1:t+K} \equiv \frac{1}{K}\sum_{k=1}^{K} E_t\left[h_{t+k}\right] = \frac{1}{K}\sum_{k=1}^{K}\sigma^2 + \tilde{b}^{k-1}(h_{t+1}-\sigma^2) = \sigma^2 + \frac{1-\tilde{b}^K}{1-\tilde{b}}\frac{(h_{t+1}-\sigma^2)}{K}$$

This variance term structure measure succinctly captures important information about the model's potential for explaining the variation of option values across maturities.[13] To compare different models, it is convenient to set the current variance, h_{t+1}, to a simple m multiple of the long run variance. In this case the variance term structure relative to the unconditional variance is given by

$$h_{t+1:t+K}/\sigma^2 \equiv 1 + \frac{1-\tilde{b}^K}{1-\tilde{b}}\frac{(m-1)}{K}$$

The dash-dot lines in the top panels of Figures 1 and 2 show the term structure of variance for the GARCH(1,1) model for a low and high initial conditional variance respectively. We use parameter values estimated via MLE on daily S&P500 returns (the estimation details are in Table 2 and will be discussed further below). We set $m = \frac{1}{2}$ in Figure 1 and $m = 2$ in Figure 2. The figures present the variance term structure for up to 250 days, which corresponds approximately to the number of trading days in a year and therefore captures the empirically relevant term structure for option valuation. It can be clearly seen from Figures 1 and 2 that for the GARCH(1,1) model, the conditional variance converges to the long-run variance rather fast.

We can also learn about the dynamics of the variance term structure though impulse response functions. For the GARCH(1,1) model, the effect of a shock at time t, z_t, on the expected k-day ahead variance is

$$\partial(E_t\left[h_{t+k}\right])/\partial z_t^2 = \tilde{b}^{k-1}a\left(1 - c\sqrt{h_t}/z_t\right)$$

and thus the effect on the variance term structure is

$$\partial E_t\left[h_{t:t+K}\right]/\partial z_t^2 = \frac{1-\tilde{b}^K}{1-\tilde{b}}\frac{a}{K}\left(1 - c\sqrt{h_t}/z_t\right)$$

The bottom-left panels of Figures 3 and 4 plot the impulse responses to the term structure of variance for $h_t = \sigma^2$ and $z_t = 2$ and $z_t = -2$ respectively, again using the parameter estimates from Table 2. The impulse responses are normalized by the unconditional variance. Notice that the effect of a shock dies out rather quickly for the GARCH(1,1) model. Comparing

[13] Notice that due to the price of risk term in the conditional mean of returns, the term structure of variance as defined here is not exactly equal to the conditional variance of cumulative returns over K days.

across Figures 3 and 4 we see the asymmetric response of the variance term structure from a positive versus negative shock to returns. This can be thought of as the term structure of the leverage effect. Due to the presence of a positive c, a positive shock has less impact than a negative shock along the entire term structure of variance.

3.2 The Variance Term Structure for the Component Model

In the component model we have

$$\begin{aligned} h_{t+1} &= q_{t+1} + \tilde{\beta}\,(h_t - q_t) + \alpha v_{1,t} \\ q_{t+1} &= \sigma^2 + \rho(q_t - \sigma^2) + \varphi v_{2,t} \end{aligned}$$

The multi-day forecast of the two components are

$$\begin{aligned} E_t\left[h_{t+k} - q_{t+k}\right] &= \tilde{\beta}^{k-1}(h_{t+1} - q_{t+1}) \\ E_t\left[q_{t+k}\right] &= \sigma^2 + \rho^{k-1}\left(q_{t+1} - \sigma^2\right) \end{aligned}$$

The simplicity of these multi-day forecasts is a key advantage of the component model. The multi-day variance forecast is a simple sum of two exponential components. Notice that $\tilde{\beta}$ and ρ correspond directly to the persistence of the short-run and long-run components respectively.

We can now calculate the variance term structure in the component model for maturity K as

$$\begin{aligned} h_{t+1:t+K} &\equiv \frac{1}{K}\sum_{k=1}^{K} E_t\left[q_{t+k}\right] + E_t\left[h_{t+k} - q_{t+k}\right] \\ &= \frac{1}{K}\sum_{k=1}^{K} \sigma^2 + \rho^{k-1}\left(q_{t+1} - \sigma^2\right) + \tilde{\beta}^{k-1}(h_{t+1} - q_{t+1}) \\ &= \sigma^2 + \frac{1-\rho^K}{1-\rho}\frac{q_{t+1} - \sigma^2}{K} + \frac{1-\tilde{\beta}^K}{1-\tilde{\beta}}\frac{h_{t+1} - q_{t+1}}{K} \end{aligned}$$

If we set q_{t+1} and h_{t+1} equal to m_1 and m_2 multiples of the long run variance respectively, then we get the variance term structure relative to the unconditional variance simply as

$$h_{t+1:t+K}/\sigma^2 = 1 + \frac{1-\rho^K}{1-\rho}\frac{m_1 - 1}{K} + \frac{1-\tilde{\beta}^K}{1-\tilde{\beta}}\frac{m_2 - m_1}{K} \tag{11}$$

The solid lines in the top panels in Figures 1 and 2 show the term structure of variance for the component model using parameters estimated via MLE on daily S&P500 returns from Table 2. We set $m_1 = \frac{3}{4}$, $m_2 = \frac{1}{2}$ in Figure 1 and $m_1 = \frac{7}{4}$, $m_2 = 2$ in Figure 2. By picking m_2 equal to the m used for the GARCH(1,1) model, we ensure comparability across models within each figure because the spot variances relative to their long-run variances are identical.[14] The main conclusion from Figures 1 and 2 is that compared to the dash-dot GARCH(1,1), the conditional variance converges more slowly to the unconditional variance in the component model. This is particularly so on days with a high spot variance. The middle and bottom panels show the contribution to the total variance from each component. Notice the strong persistence in the long-run component.

We can also calculate impulse response functions in the component model. The effects of a shock at time t, z_t on the expected k-day ahead variance components are

$$\begin{aligned} \partial E_t\left[q_{t+k}\right]/\partial z_t^2 &= \rho^{k-1}\varphi\left(1-\gamma_2\sqrt{h_t}/z_t\right) \\ \partial E_t\left[h_{t+k}-q_{t+k}\right]/\partial z_t^2 &= \tilde{\beta}^{k-1}\alpha\left(1-\gamma_1\sqrt{h_t}/z_t\right) \\ \partial E_t\left[h_{t+k}\right]/\partial z_t^2 &= \tilde{\beta}^{k-1}\alpha\left(1-\gamma_1\sqrt{h_t}/z_t\right)+\rho^{k-1}\varphi\left(1-\gamma_2\sqrt{h_t}/z_t\right) \end{aligned}$$

Notice again the simplicity due to the component structure. The impulse response on the term structure of variance is then

$$\partial E_t\left[h_{t:t+K}\right]/\partial z_t^2 = \frac{1-\tilde{\beta}^K}{1-\tilde{\beta}}\frac{\alpha}{K}\left(1-\gamma_1\sqrt{h_t}/z_t\right)+\frac{1-\rho^K}{1-\rho}\frac{\varphi}{K}\left(1-\gamma_2\sqrt{h_t}/z_t\right)$$

The top-left panels of Figures 3 and 4 plot the impulse responses to the term structure of variance for $h_t = \sigma^2$ and $z_t = 2$ and $z_t = -2$ respectively. The figures reinforce the message from Figures 1 and 2 that using parameterizations estimated from the data, the component model is quite different from the GARCH(1,1) model. The effects of shocks are much longer lasting in the component model using estimated parameter values because of the parameterization of the long-run component. Comparing across Figures 3 and 4 it is also clear that the term structure of the leverage effect is more flexible. As a result current shocks and the current state of the economy potentially have a much more profound impact on the pricing of options across maturities in the component model.

It has been argued in the literature that the hyperbolic rate of decay displayed by long memory processes may be a more adequate representation for the conditional variance of returns.[15] We do not disagree with these findings. Instead, we argue that Figures 1 through 4 demonstrate that in the component model the combination of two variance components with exponential decay gives rise to a slower decay pattern that sufficiently adequately

[14] Note that we need $m_1 \neq m_2$ in this numerical experiment to generate a "short-term" effect in (11). Changing m_1 will change the picture but the main conclusions stay the same.

[15] See Bollerslev and Mikkelsen (1996,1999), Baillie, Bollerslev and Mikkelsen (1996) and Ding, Granger and Engle (1993).

captures the hyperbolic decay pattern of long memory processes for the horizons relevant for option valuation. This is of interest because although the long memory model may be a more adequate representation of the data, it is harder to implement.

4 Option Valuation

We now turn to the ultimate purpose of this paper, namely the valuation of derivatives on an underlying asset with dynamic variance components. For the purpose of option valuation we need the risk-neutral return dynamics rather than the physical dynamics in (1), (7) and (9).

4.1 The Risk-Neutral GARCH(1,1) Dynamic

The risk-neutral dynamics for the GARCH(1,1) model are given in Heston and Nandi (2000)[16] as

$$\begin{aligned} R_{t+1} &= r - \tfrac{1}{2}h_{t+1} + \sqrt{h_{t+1}}z^*_{t+1} \\ h_{t+1} &= w + bh_t + a(z^*_t - c^*\sqrt{h_t})^2 \end{aligned} \tag{12}$$

with $c^* = c + \lambda + 0.5$ and $z^*_{t+1} \sim N(0,1)$.

4.2 The Risk-Neutral Component GARCH Dynamic

Appendix A demonstrates that the risk-neutral Component GARCH dynamic is given by

$$\begin{aligned} h_{t+1} &= q_{t+1} + \tilde{\beta}^* (h_t - q_t) + \alpha\left(\left(z^*_t - \gamma^*_1\sqrt{h_t}\right)^2 - \left(1 + \gamma^{*2}_1 h_t\right)\right) \\ q_{t+1} &= \omega + \rho^* q_t + \varphi\left(\left(z^*_t - \gamma^*_2\sqrt{h_t}\right)^2 - \left(1 + \gamma^{*2}_2 h_t\right)\right) \end{aligned} \tag{13}$$

where the risk neutral parameters are defined as follows

$$\begin{aligned} \tilde{\beta}^* &= \tilde{\beta} + \alpha\left(\gamma^{*2}_1 - \gamma^2_1\right) + \varphi\left(\gamma^{*2}_2 - \gamma^2_2\right) \\ \rho^* &= \rho + \alpha\left(\gamma^{*2}_1 - \gamma^2_1\right) + \varphi\left(\gamma^{*2}_2 - \gamma^2_2\right) \\ \gamma^*_i &= \gamma_i + \lambda + 0.5,\ i = 1,2. \end{aligned}$$

The moment generating function for the risk-neutral Component GARCH process is therefore equal to the one for the physical Component GARCH process, setting $\lambda = -0.5$ and using the risk neutral parameters $\gamma^*_1, \gamma^*_2, \rho^*, \tilde{\beta}^*$ as well as ω, α and φ.

[16] For the underlying theory on risk neutral distributions in discrete time option valuation see Rubinstein (1976), Brennan (1979), Amin and Ng (1993), Duan (1995), Camara (2003), and Schroder (2004).

4.3 The Option Valuation Formula

Given the risk-neutral dynamics, option valuation is relatively straightforward. We use the result of Heston and Nandi (2000) that at time t, a European call option with strike price K that expires at time T is worth

$$\begin{aligned}\text{Call Price} &= e^{-r(T-t)}E_t^*[Max(S_T-K,0)] \quad (14)\\ &= S_t\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \text{Re}\left[\frac{K^{-i\phi}f^*(t,T;i\phi+1)}{i\phi S_t e^{r(T-t)}}\right]d\phi\right)\\ &\quad -Ke^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \text{Re}\left[\frac{K^{-i\phi}f^*(t,T;i\phi)}{i\phi}\right]d\phi\right)\end{aligned}$$

where $f^*(t,T;i\phi)$ is the conditional characteristic function of the logarithm of the spot price under the risk neutral measure. For the return dynamics in this paper, we can characterize the generating function of the stock price with a set of difference equations, using the techniques in Heston and Nandi (2000). Appendix B demonstrates that for the Component GARCH model we have

$$f(t,T;\phi) = S_t^{\phi}\exp[A_t + B_{1,t}(h_{t+1}-q_{t+1}) + B_{2,t}q_{t+1}]$$

with coefficients

$$\begin{aligned}A_t &= A_{t+1} - (\alpha B_{1,t+1} + \varphi B_{2,t+1}) - 1/2\ln(1-2\alpha B_{1,t+1} - 2\varphi B_{2,t+1}) + B_{2,t+1}\omega\\ B_{1,t} &= B_{1,t+1}\tilde{\beta} - 1/2\phi + 2\frac{\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi}{1-\alpha B_{1,t+1} - \varphi B_{2,t+1}}\\ B_{2,t} &= B_{2,t+1}\rho - 1/2\phi + 2\frac{\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi}{1-\alpha B_{1,t+1} - \varphi B_{2,t+1}}\end{aligned}$$

and terminal conditions

$$A_T = B_{1,T} = B_{2,T} = 0.$$

For the moment generating function in the GARCH(1,1) case we refer to Heston and Nandi (2000).

5 Empirical Results

This section presents the empirical results. We first discuss the data, followed by an empirical evaluation of the model estimated under the physical measure using a historical series of stock returns. Subsequently we present estimation results obtained by estimating the risk-neutral version of the model using options data.

5.1 Data

We conduct our empirical analysis using six years of data on S&P 500 call options, for the period 1990-1995. We apply standard filters to the data following Bakshi, Cao and Chen (1997). We only use Wednesday options data. Wednesday is the day of the week least likely to be a holiday. It is also less likely than other days such as Monday and Friday to be affected by day-of-the-week effects. For those weeks where Wednesday is a holiday, we use the next trading day. The decision to pick one day every week is to some extent motivated by computational constraints. The optimization problems are fairly time-intensive, and limiting the number of options reduces the computational burden. Using only Wednesday data allows us to study a fairly long time-series, which is useful considering the highly persistent volatility processes. An additional motivation for only using Wednesday data is that following the work of Dumas, Fleming and Whaley (1998), several studies have used this setup.[17]

We perform a number of in-sample and out-of-sample experiments using the options data. We first estimate the model parameters using the 1990-1992 data and subsequently test the model out-of-sample using the 1993 data. We also estimate the model parameters using the 1992-1994 data and subsequently test the model out-of-sample using the 1995 data. For both estimation exercises we use a volatility updating rule for the 500 days predating the Wednesday used in the estimation exercise. This volatility updating rule is initialized at the model's unconditional variance. We also perform an extensive empirical analysis using return data. Ideally we would like to use the same sample periods for these estimation exercises, but it is well-known that it is difficult to estimate GARCH parameters precisely using relatively short samples on returns. We therefore use a long sample of returns 1963-1995 on the S&P 500.

Table 1 presents descriptive statistics for the options data for 1990-1995 by moneyness and maturity. Panels A and B indicate that the data are standard. We can clearly observe the volatility smirk from Panel C and it is clear that the slope of the smirk differs across maturities. Descriptive statistics for different sub-periods (not reported here) demonstrate that the slope also changes across time, but that the smirk is present throughout the sample. The top panel of Figure 5 gives some indication of the pattern of implied volatility over time. For the 312 days of options data used in the empirical analysis, we present the average implied volatility of the options on that day. It is evident from Figure 5 that there is substantial clustering in implied volatilities. It can also be seen that volatility is higher in the early part of the sample. The bottom panel of Figure 5 presents a time series for the 30-day at-the-money volatility (VIX) index from the CBOE for our sample period. A comparison with the top panel clearly indicates that the options data in our sample are representative of market conditions, although the time series based on our sample is of course a bit more noisy due to the presence of options with different moneyness and maturities.

[17] See for instance Heston and Nandi (2000).

5.2 Empirical Results using Returns Data

Table 2 presents estimation results obtained using returns data for 1963-1995 for the physical model dynamics. We present results for three models: the GARCH(1,1) model (1), the component model (7) and the persistent component model (9). Almost all parameters are estimated significantly different from zero at conventional significance levels.[18] In terms of fit, the log likelihood values indicate that the fit of the component model is much better than that of the persistent component model, which in turn fits much better than the GARCH(1,1) model.

The improvement in fit for the component GARCH model over the persistent component GARCH model is perhaps somewhat surprising when inspecting the persistence of the component GARCH model. The persistence is equal to 0.996. It therefore would appear that equating this persistence to 1, as is done in the persistent component model, is an interesting hypothesis, but apparently modeling these small differences from one is important. It must of course be noted that the picture is more complex: while the persistence of the long-run component (ρ) is 0.990 for the component model as opposed to 1 for the persistent component model, the persistence of the short-run component ($\tilde{\beta}$) is 0.644 versus 0.764 and this may account for the differences in performance. Note that the persistence of the GARCH(1,1) model is estimated at 0.955, which is consistent with earlier literature. It is slightly lower than the estimate in Christoffersen, Heston and Jacobs (2004) and a bit higher than the average of the estimates in Heston and Nandi (2000).

The ability of the models to generate richer patterns for the conditional versions of leverage and volatility of volatility is critical. For option valuation, the conditional versions of these quantities and their variation through time are just as important as the unconditional versions. The conditional versions of leverage and volatility of volatility are computed as follows. For the GARCH(1,1) model the conditional variance of variance is

$$\begin{aligned} Var_t(h_{t+2}) &= E_t\left[h_{t+2} - E_t\left[h_{t+2}\right]\right]^2 \\ &= 2a^2 + 4a^2c^2h_{t+1} \end{aligned} \tag{15}$$

and the leverage effect can be defined as

$$\begin{aligned} Cov_t(R_{t+1}, h_{t+2}) &= E_t\left[\left(R_{t+1} - E_t\left[R_{t+1}\right]\right)\left(h_{t+2} - E_t\left[h_{t+2}\right]\right)\right] \\ &= E_t\left[\sqrt{h_{t+1}}z_{t+1}\left(az_{t+1}^2 - 2acz_{t+1}\sqrt{h_{t+1}} - a\right)\right] \\ &= -2ach_{t+1} \end{aligned} \tag{16}$$

The conditional variance of variance in the component model is

$$Var_t(h_{t+2}) = 2\left(\alpha + \varphi\right)^2 + 4\left(\gamma_1\alpha + \gamma_2\varphi\right)^2 h_{t+1} \tag{17}$$

[18] The standard errors are computed using the outer product of the gradient at the optimal parameter values.

and the leverage effect in the component model is

$$Cov_t(R_{t+1}, h_{t+2}) = -2\left(\gamma_1\alpha + \gamma_2\varphi\right)h_{t+1} \tag{18}$$

Figures 6 and 7 present the conditional leverage and conditional variance of variance for the GARCH(1,1) model and the component model over the option sample 1990-1995 using the MLE parameter values in Table 2. It can be clearly seen that the level as well as the time-series variation in these critical model features are fundamentally different in the two models. In Figure 6 the leverage effect is much more volatile in the component model and it takes on much more extreme values on certain days. In Figure 7 the variance of variance in the component model is in general much higher than in the GARCH(1,1) model and it also more volatile. Thus the more flexible component model is capable of generating not only more flexible term structures of variance, it is also able to generate more skewness and kurtosis dynamics which are key for explaining the variation in index options prices.

Table 2 also presents some unconditional summary statistics for the different models. The computation of these statistics deserves some comment. For the GARCH(1,1) model and the component model, the unconditional versions of the volatility of volatility are computed using the estimate for the unconditional variance in the expressions for the conditional moments (15) and (17). For the persistent component model, the unconditional volatility and the unconditional variance of variance are not defined. To allow a comparison of the unconditional leverage for all three models, we report the moments in (15) and (17) divided by h_{t+1}. While the unconditional volatility of the GARCH(1,1) model (0.137) is very similar to that of the component GARCH model (0.141), the leverage and the variance of variance of the component GARCH model are larger in absolute value than those of the GARCH(1,1) model. The leverage for the persistent component model is of the same order of magnitude as that of the component model.

We previously discussed Figures 1-4, which emphasize other critical differences between the models. These figures are generated using the parameter estimates in Table 2. Figures 1 and 2 indicate that for the GARCH(1,1) model, forecasted model volatility reverts much more quickly towards the unconditional volatility over long-maturity options' lifetimes than is the case for the component model. Figures 3 and 4 demonstrate that the effects of shocks are much longer lasting in the component model because of the parameterization of the long-run component. As a result current shocks and the current state of the economy have a much more profound impact on the pricing of maturity options across maturities.

Figures 8 and 9 give another perspective on the component models' improvement in performance over the benchmark GARCH(1,1) model. These figures present the sample path for volatility in all three models, as well as the sample path for volatility components for the component model and persistent component model. In each figure, the sample path is obtained by iterating on the variance dynamic starting from the unconditional volatility 500 days before the first volatility included in the figure, as is done in estimation. Initial conditions are therefore unlikely to affect comparisons between the models in these figures.

Figure 8 contains the results for the component model. The overall conclusion seems to be that the mean zero short run component in the top-right panel adds short-horizon noise around the long-run component in the bottom-right panel. This results in a volatility dynamic for the component model in the top-left panel that is more noisy than the volatility dynamic for the GARCH(1,1) model in the bottom-left panel. The more noisy sample path in the top-left panel is of course confirmed by the higher value for the variance of variance in Table 2. This increased flexibility results in a better fit. The results for the permanent component model in Figure 9 confirm this conclusion, even though the sample paths for the components in Figure 9 look different from those in Figure 8.[19]

5.3 Empirical Results using Option and Return Data

Tables 3-10 present the empirical results for the option-based estimates of the risk-neutral parameters. We present four sets of results. Table 3 presents results for parameters estimated using options data for 1990-1992 using all option contracts in the sample. Note that the shortest maturity is seven days because options with very short maturities were filtered out. Table 4 contains results for 1990-1992 obtained using options with more than 80 days to maturity, because we expect the component models to be particularly useful to model options with long maturities. Tables 5 and 6 present results obtained using options data for 1992-1994, using all contracts and contracts with more than 80 days to maturity respectively. When using the 1990-1992 sample in estimation, we test the model out-of-sample using data for 1993. When using the 1992-1994 sample in estimation, we test the model out-of-sample using 1995 data. Tables 7-10 present results for the two in-sample and two out-of-sample periods by moneyness and maturity. In all cases we obtain parameters by minimizing the dollar mean squared error

$$\$MSE = \frac{1}{N^T} \sum_{t,i} \left(C_{i,t}^D - C_{i,t}^M \right)^2 \tag{19}$$

where $C_{i,t}^D$ is the market price of option i at time t, $C_{i,t}^M$ is the model price, and $N^T = \sum_{t=1}^{T} N_t$. T is the total number of days included in the sample and N_t the number of options included in the sample at date t. The parameters in Tables 3-6 are found by applying nonlinear least squares (NLS) estimation techniques on the $\$MSE$ expression in (19). The variance dynamic is used to update the variance from one Wednesday to the next using daily returns

[19] The figures presented so far have been constructed from the return-based MLE estimates in Table 2. Below we will present four new sets of (risk-neutral) estimates derived from observed option prices. In order to preserve space we will not present new versions of the above figures from these estimates. The option-based estimates imply figures which are qualitatively similar to the return-based figures presented above.

and the option valuation formula in Section 4.2 is used to compute the model prices on each Wednesday.[20]

In Table 3 we present results for the 1990-1992 period (in-sample) and the 1993 period (out-of-sample). The standard errors indicate that almost all parameters are estimated significantly different from zero.[21] There are some interesting differences with the parameters estimated from returns in Table 2, but the parameters are mostly of the same order of magnitude. This is also true for critical determinants of the models' performance, such as unconditional volatility, leverage and volatility of volatility. It is interesting to note that in both tables the persistence of the GARCH(1,1) model and the component GARCH model is close to one. This of course motivates the use of the persistent component model, where the persistence is restricted to be one. Note also that the persistence of the short-run components and the long-run components is not dramatically different from Table 2. In the in-sample period, the RMSE of the component model is 89.7% of that of the benchmark GARCH(1,1) model. For the out-of-sample period, it is 76.5%. For the persistent component model, this is 95.5% and 97.1% respectively. Table 4 confirms that the same results obtain when estimating the models using only long-maturity options.

Tables 5 and 6 present the results for the 1992-1994 period (in-sample) and the 1995 period (out-of-sample). The results largely confirm those obtained in Tables 3 and 4. The most important difference is that the in-sample and out-of-sample performance of the component model is even better relative to the benchmark, as compared with the results in Tables 3 and 4. For the 1992-1994 in-sample period, the component model's RMSE is 77.3% of that of the GARCH(1,1) model in Table 5 and 74.8% in Table 6. For the 1995 out-of-sample period, this is 79.2% and 60.4% respectively. The performance of the persistent component model in some cases does not improve much over the performance of the GARCH(1,1) model, and in other cases its performance is actually worse than that of the benchmark. Another interesting difference with Tables 3 and 4 is that in Tables 5 and 6, the persistence of the short-run component is much higher. Finally note that the persistence of the GARCH(1,1) process in Table 5 is lower than in Table 3 but in line with the MLE estimate in Table 2.

In order to provide more detail on the overall model performance, Tables 7-10 present MSE results by moneyness and maturity. To save space we only report for the samples that include all options. Note that the tables contain information on MSEs, not RMSEs. In each table, Panel A contains the MSE for the GARCH(1,1) model. To facilitate the interpretation of the table, panels B and C contain MSEs that are normalized by the corresponding MSE for the GARCH(1,1) model. It is clear that an overall MSE which is not too different across the three models as in Table 3 can mask large differences in the models' performance for a given

[20] Notice from the risk-neutral dynamics (12) and (13) that the parameter λ is not separately identified using option prices. We therefore simply set λ equal to the MLE estimate from Table 2 for the respective models, which identifies the other parameters, and we do not report λ in Tables 3-6.

[21] The standard errors are again computed using the outer product of the gradient at the optimum.

moneyness/maturity cell. Inspection of the out-of-sample results in Tables 8 and 10 is very instructive. The overwhelming conclusion is that the improved out-of-sample performance of the component models is due to the improved valuation of long-maturity options. This is perhaps not surprising given the differences in the impulse response functions discussed above.

Figure 10 graphically represents some related information. For different moneyness bins, we first compute the average Black-Scholes implied volatility for all the options in our sample. Subsequently we compute implied volatilities based on model prices and also average this for all options in the sample. Note that while the implied volatility fit is not perfect, the component model matches the volatility smirk better than the two other models.

Figures 11 and 12 evaluate the performance of the three models along a different dimension, by presenting average weekly bias (average observed market price less average observed model price) over the 1990-1993 and 1992-1995 sample periods respectively. The bias seems to be more highly correlated across models in the 1990-1993 sample. In the 1993-1995 sample, the persistent component model in particular has a markedly different fit from the two other models. The most important conclusion is that the improved performance of the component model does not derive from any particular sample sub-period: the bias of the component GARCH model is smaller than that of the GARCH(1,1) model in most weeks.

Our overall conclusion is that the performance of the component GARCH model is very impressive. Its RMSE is between 60.4% and 89.7% of the RMSE of the benchmark GARCH(1,1) model. The performance of the persistent component model is less impressive, both in-sample and out-of-sample.

5.4 Discussion

It must be emphasized that this improvement in performance is remarkable and to some extent surprising. The GARCH(1,1) model is a good benchmark which itself has a very solid empirical performance (see Heston and Nandi (2000)). The model captures important stylized facts about option prices such as volatility clustering and the leverage effect (or equivalently negative skewness). When estimating models from option prices, Christoffersen and Jacobs (2004) find that GARCH models with richer news impact parameterizations do not improve the model fit out-of-sample. Christoffersen, Heston and Jacobs (2004) find that a GARCH model with non-normal innovations improves the model's fit in-sample and for short out-of-sample horizons, but not for long out-of-sample horizons. Although we do not report the results in the paper, we have also compared the performance of the GARCH(1,1) model with the implied Black-Scholes model in Dumas, Fleming and Whaley (1998). We confirm the finding of Heston and Nandi (2000) that the GARCH(1,1) model outperforms the implied Black-Scholes model out-of-sample.

One may wonder how the GARCH(1,1) performs compared with the popular continuous-time stochastic volatility model in Heston (1993), which is routinely used as a benchmark model in the continuous-time literature. Christoffersen, Mimouni and Jacobs (2005) find that the GARCH(1,1) model outperforms the Heston (1993) model in-sample as well as out-of-sample. This is somewhat surprising because it is often believed that the Heston and Nandi model converges to the Heston (1993) model. However, this convergence result only applies to a special case.[22] Christoffersen, Mimouni and Jacobs (2005) suggest that the better performance of the GARCH(1,1) model may be due to its ability to model time-varying conditional correlation between returns and volatility, while the Heston (1993) model is characterized by constant conditional correlation.

To better appreciate the richness of the component model and its relationship to continuous-time processes, consider a finite time interval $[0,1]$ and divide this time interval into n subintervals of length $s = 1/n$. Following Duan (1997), the approximating process corresponding to the GARCH component model can be defined as

$$\begin{aligned} \Delta \ln(S_k) &= (r + \lambda h_k)\, s + \varepsilon_k \sqrt{s} \\ \Delta (h_k - q_k) &= \left(1 - \widetilde{\beta}\right)(q_k - h_k)\, s + \alpha v_{1,k}\sqrt{s} \\ \Delta q_k &= (1-\rho)\,(\sigma^2 - q_k)s + \varphi v_{2,k}\sqrt{s} \end{aligned} \tag{20}$$

for $k = 1, 2, ...n$, and where $\varepsilon_k \equiv \sqrt{h_k} z_k$. This representation suggests a continuous time limit with two stochastic volatility components and three shocks: ε, v_1, v_2. We now derive and plot the covariance dynamics of these shocks.[23]

First note that the conditional variances of the innovations to returns and volatility components are

$$\begin{aligned} Var_{k-1}[\varepsilon_k] &= h_k \\ Var_{k-1}[v_{i,k}] &= 2 + 4\gamma_i^2 h_k, \text{ for } i = 1, 2. \end{aligned}$$

The conditional covariance between the component shocks is

$$Cov_{k-1}[v_{1,k}, v_{2,k}] = 2 + 4\gamma_1\gamma_2 h_k$$

while the conditional covariances between the return and component shocks are

$$Cov_{k-1}[\varepsilon_k, v_{i,k}] = -2\gamma_i h_k, \text{ for } i = 1, 2.$$

[22] Heston and Nandi's limit argument assumes that the GARCH asymmetry parameter c goes to plus or minus infinity as the sampling frequency approaches continuous time. The limit of this special case of the GARCH model is the Heston (1993) model with a correlation parameter of +1 or -1. However, for a finite c a different limit may exist with a potentially different conditional correlation.

[23] In the Heston and Nandi model, $v_{i,t}$ cannot be written as $h_t f_i(z_t)$. Thus, this model does not belong to the class of GARCH models studied by Duan (1997). Therefore, to the best of our knowledge standard techniques cannot be used to explicitly derive the continuous time limit in this case.

We can now compute the conditional correlations between the component shocks as

$$Corr_{k-1}\left[v_{1,k}, v_{2,k}\right] = \frac{2 + 4\gamma_1\gamma_2 h_k}{\sqrt{\left(2 + 4\gamma_1^2 h_k\right)\left(2 + 4\gamma_2^2 h_k\right)}}$$

The conditional correlations between the return and the volatility components are

$$Corr_{k-1}\left[\varepsilon_k, v_{i,k}\right] = \frac{-2\gamma_i h_k}{\sqrt{h_k\left(2 + 4\gamma_i^2 h_k\right)}}, \text{ for } i = 1, 2.$$

Figure 13 presents these conditional correlations at the daily frequency. The results indicate that the model is able to capture substantial time variation in the correlation between the return innovation and the volatility components, as well as time variation in the correlation between the volatility components. Figure 13 also presents the conditional correlation between the return and volatility innovations for the Heston-Nandi GARCH(1,1) model. The conditional correlation of the innovation in the short-run volatility component with the return innovation is on average more negative than the conditional correlation between the return and the volatility innovation in the GARCH(1,1) model, and it is also less variable. In contrast, the conditional correlation of the innovation in the long-run volatility component with the return innovation is on average less negative than the conditional correlation between the return and the volatility innovation in the GARCH(1,1) model, and it is more variable.

Interestingly, the structure of the approximating model in (20) is similar to that of the continuous time model proposed in Duffie, Pan and Singleton (2000), with one important exception. In (20), the conditional correlations between the return innovation and the volatility innovations are time-varying. It would be interesting to compare the component model suggested here with the continuous time model proposed in Duffie, Pan and Singleton (2000), but this investigation is beyond the scope of this paper.

Much of the continuous-time literature has attempted to improve the performance of the Heston (1993) model by adding to it (potentially correlated) jumps in returns and volatility. The empirical findings in this literature have been mixed. In general, Poisson jumps in returns and volatility improve option valuation when parameters are estimated using historical time series of returns, but not always when parameters are estimated using the cross-section of option prices.[24] In a recent paper, Broadie, Chernov and Johannes (2004) use a long data set on options and find evidence of the importance of jumps for option pricing. Carr and Wu (2004) and Huang and Wu (2004) model a different type of jump process and find that they are better able to fit options out-of-sample. Finally, Duan, Ritchken and Sun (2002) find that adding jumps to discrete-time models leads to a significant improvement in fit. We

[24] See for example Andersen, Benzoni and Lund (2002), Bakshi, Cao and Chen (1997), Bates (1996, 2000), Chernov, Gallant, Ghysels and Tauchen (2003), Eraker, Johannes and Polson (2003), Eraker (2004) and Pan (2002).

therefore speculate that adding jumps or fat-tailed shocks to our model may further improve the fit, but we leave such investigations for future research.

6 Conclusion and Directions for Future Work

This paper presents a new option valuation model based on the work by Engle and Lee (1999) and Heston and Nandi (2000). The empirical performance of the new variance component model is significantly better than that of the benchmark GARCH (1,1) model, in-sample as well as out-of-sample. This is an important finding because the literature has demonstrated that it is difficult to find empirical models that improve on the GARCH(1,1) model or the Heston (1993) model. The improved performance of the model is due to a richer parameterization which allows for improved joint modeling of long-maturity and short-maturity options. This parameterization can capture the stylized fact that shocks to current conditional volatility impact on the forecast of the conditional variance up to a year in the future. Given that the estimated persistence of the model is close to one, we also investigate a special case of our model in which shocks to the variance never die out. The performance of the persistent component model is satisfactory in some dimensions, but it is strictly dominated by the component model. Note that this is not a trivial finding: even though the persistent component model is nested by the component model, a more parsimonious model can easily outperform a more general one out-of-sample. This is not the case here.

Given the success of the proposed model, a number of further extensions to this work are warranted. First, the empirical performance of the model should of course be validated using other datasets. In particular, it would be interesting to test the model using LEAPS data, because the model may excel at modeling long-maturity LEAPS options. In this regard a direct comparison between component and fractionally integrated volatility models may be interesting. It could also be useful to combine the stylized features of the model with other modeling components that improve option valuation. One interesting experiment could be to replace the Gaussian innovations in this paper by a non-Gaussian distribution in order to create more negative skewness in the distribution of equity returns. Combining the model in this paper with the inverse Gaussian shock model in Christoffersen, Heston and Jacobs (2004) may be a viable approach. Finally, in this paper we have proposed a component model that gives a closed form solution using results from Heston and Nandi (2000) who rely on an affine GARCH model. We believe that this is a logical first step, but the affine structure of the model may be restrictive in ways that are not immediately apparent. It may therefore prove worthwhile to investigate non-affine variance component models.

7 Appendix A

The physical Component GARCH dynamic is given by

$$\begin{aligned}
\ln(S_{t+1}) &= \ln(S_t) + r + \lambda h_{t+1} + \sqrt{h_{t+1}} z_{t+1} \\
h_{t+1} &= q_{t+1} + \tilde{\beta}\left(h_t - q_t\right) + \alpha\left(\left(z_t - \gamma_1\sqrt{h_t}\right)^2 - \left(1 + \gamma_1^2 h_t\right)\right) && (21) \\
q_{t+1} &= \omega + \rho q_t + \varphi\left(\left(z_t - \gamma_2\sqrt{h_t}\right)^2 - \left(1 + \gamma_2^2 h_t\right)\right) && (22)
\end{aligned}$$

Under the risk neutral measure, we need $E^*\left[S_{t+1}/S_t\right] = \exp(r)$, which requires that

$$\ln(S_{t+1}) = \ln(S_t) + r - 0.5h_{t+1} + \sqrt{h_{t+1}} z^*_{t+1}$$

This implies in turn that

$$z^*_{t+1} = z_{t+1} + (\lambda + 0.5)\sqrt{h_{t+1}}. \tag{23}$$

We also need to ensure that

$$Var_t\left(R_{t+1}\right) = Var^*_t\left(R_{t+1}\right)$$

In order to have the same conditional variances under the two measures, we need to have the same variance innovations under the two measures. Thus we need

$$\left(z_t - \gamma_i\sqrt{h_t}\right)^2 = \left(z^*_t - \gamma^*_i\sqrt{h_t}\right)^2 \qquad i = 1, 2$$

which can be achieved by defining a new risk neutral parameter

$$\gamma^*_i = \gamma_i + \lambda + 0.5, i = 1, 2.$$

Consider the following candidate for the risk-neutral Component GARCH dynamic

$$\begin{aligned}
h_{t+1} &= q_{t+1} + \tilde{\beta}^*\left(h_t - q_t\right) + \alpha\left(\left(z^*_t - \gamma^*_1\sqrt{h_t}\right)^2 - \left(1 + \gamma^{*2}_1 h_t\right)\right) && (24) \\
q_{t+1} &= \omega + \rho^* q_t + \varphi\left(\left(z^*_t - \gamma^*_2\sqrt{h_t}\right)^2 - \left(1 + \gamma^{*2}_2 h_t\right)\right) && (25)
\end{aligned}$$

where $z^*_t \sim N(0, 1)$ and the risk neutral parameters are defined as follows

$$\begin{aligned}
\tilde{\beta}^* &= \tilde{\beta} + \alpha\left(\gamma^{*2}_1 - \gamma^2_1\right) + \varphi\left(\gamma^{*2}_2 - \gamma^2_2\right) && (26) \\
\rho^* &= \rho + \alpha\left(\gamma^{*2}_1 - \gamma^2_1\right) + \varphi\left(\gamma^{*2}_2 - \gamma^2_2\right)
\end{aligned}$$

For this candidate risk-neutral dynamic to be valid, we have to verify that it is consistent with (21) and (22). Using (23), (26) and (25) in (24) we get

$$h_{t+1} = \omega + \rho q_t + \varphi\left(\left(z_t - \gamma_2\sqrt{h_t}\right)^2 - \left(1 + \gamma_2^2 h_t\right)\right) + \tilde{\beta}\left(h_t - q_t\right) + \alpha\left(\left(z_t - \gamma_1\sqrt{h_t}\right)^2 - \left(1 + \gamma_1^2 h_t\right)\right)$$

which is identical from what we get using the physical Component GARCH dynamic (21) and (22).

8 Appendix B

This Appendix derives the moment generating function for the component GARCH process. The component GARCH process is given by

$$\begin{aligned} h_{t+1} &= q_{t+1} + \tilde{\beta}\left(h_t - q_t\right) + \alpha\left((z_t - \gamma_1\sqrt{h_t})^2 - (1+\gamma_1^2 h_t)\right) \\ q_{t+1} &= \omega + \rho q_t + \varphi\left((z_t - \gamma_2\sqrt{h_t})^2 - (1+\gamma_2^2 h_t)\right) \end{aligned}$$

Let $x_t = \ln(S_t)$. For convenience we will denote the time t conditional generating function of S_T (or equivalently the conditional moment generating function (MGF) of x_T) by f_t instead of the more cumbersome $f(t;T,\phi)$. By definition

$$f_t = E_t[\exp(\phi x_T)]$$

We shall guess that the moment generating function has the log-linear form. We again use the more parsimonious notation A_t to indicate $A(t;T,\phi)$.

$$f_t = \exp\left(\phi x_t + A_t + B_{1,t}(h_{t+1} - q_{t+1}) + B_{2,t} q_{t+1}\right) \tag{28}$$

We have the terminal condition $A_T = B_{i,T} = 0$. Applying the law of iterated expectations to f_t we get

$$f_t = E_t\left[f_{t+1}\right] = E_t \exp\left(\phi x_{t+1} + A_{t+1} + B_{1,t+1}(h_{t+2} - q_{t+2}) + B_{2,t+1} q_{t+2}\right)$$

Substituting the dynamics of x_t gives

$$\begin{aligned} f_t &= E_t \exp\begin{pmatrix} \phi(x_t + r) + \phi\lambda h_{t+1} + \phi\sqrt{h_{t+1}} z_{t+1} + A_{t+1} + B_{1,t+1}(h_{t+2} - q_{t+2}) + \\ B_{2,t+1} q_{t+2} \end{pmatrix} \\ &= E_t \exp\begin{pmatrix} \phi(x_t + r) + \phi\lambda h_{t+1} + \phi\sqrt{h_{t+1}} z_{t+1} + A_{t+1} + \\ B_{1,t+1}\left(\tilde{\beta}\left(h_{t+1} - q_{t+1}\right) + \alpha\left((z_{t+1} - \gamma_1\sqrt{h_{t+1}})^2 - (1+\gamma_1^2 h_{t+1})\right)\right) + \\ B_{2,t+1}\left(\omega + \rho q_{t+1} + \varphi\left((z_{t+1} - \gamma_2\sqrt{h_{t+1}})^2 - (1+\gamma_2^2 h_{t+1})\right)\right) \end{pmatrix} \\ &= E_t \exp\begin{pmatrix} \phi(x_t + r) + \phi\lambda h_{t+1} + A_{t+1} + B_{1,t+1}\tilde{\beta}\left(h_{t+1} - q_{t+1}\right) + \\ B_{2,t+1}(\omega + \rho q_{t+1}) - (\alpha B_{1,t+1} + \varphi B_{2,t+1}) + \\ (\alpha B_{1,t+1} + \varphi B_{2,t+1}) + \left(z_{t+1} - \frac{\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi}{(\alpha B_{1,t+1} + \varphi B_{2,t+1})}\sqrt{h_{t+1}}\right)^2 - \\ \frac{(\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi)^2}{(\alpha B_{1,t+1} + \varphi B_{2,t+1})} h_{t+1} \end{pmatrix} \end{aligned}$$

Using the result

$$E\left[\exp(x(z+y)^2)\right] = \exp(-\frac{1}{2}\ln(1-2x) + xy^2/(1-2x))$$

we get

$$f_t = E_t \exp \begin{pmatrix} \phi(x_t + r) + A_{t+1} - (\alpha B_{1,t+1} + \varphi B_{2,t+1}) \\ -1/2 \ln(1 - 2\alpha B_{1,t+1} - 2\varphi B_{2,t+1}) + B_{2,t+1}\omega + \\ B_{1,t+1}\tilde{\beta}\,(h_{t+1} - q_{t+1}) + B_{2,t+1}\rho q_{t+1}) + \\ \left(\lambda\phi + 2\frac{\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi}{1 - \alpha B_{1,t+1} - \varphi B_{2,t+1}}\right) h_{t+1} \end{pmatrix} \tag{29}$$

Matching terms in (29) and (28) gives

$$\begin{aligned} A_t &= A_{t+1} - (\alpha B_{1,t+1} + \varphi B_{2,t+1}) - 1/2 \ln(1 - 2\alpha B_{1,t+1} - 2\varphi B_{2,t+1}) + B_{2,t+1}\omega \\ B_{1,t} &= B_{1,t+1}\tilde{\beta} - 1/2\phi + 2\frac{\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi}{1 - \alpha B_{1,t+1} - \varphi B_{2,t+1}} \\ B_{2,t} &= B_{2,t+1}\rho - 1/2\phi + 2\frac{\alpha\gamma_1 B_{1,t+1} + \varphi\gamma_2 B_{2,t+1} - 0.5\phi}{1 - \alpha B_{1,t+1} - \varphi B_{2,t+1}} \end{aligned}$$

References

[1] Adrian, T. and J. Rosenberg (2005) "Stock Returns and Volatility: Pricing the Long-Run and Short-Run Components of Market Risk," Working Paper, the Federal Reserve Bank of New York.

[2] Ait-Sahalia, Y, and A. Lo (1998) "Nonparametric Estimation of State-Price Densities Implicit in Financial Asset Prices," Journal of Finance, 53, 499-547.

[3] Alizadeh, S., Brandt, M. and F. Diebold (2002) "Range-Based Estimation of Stochastic Volatility Models," Journal of Finance, 57, 1047-1091.

[4] Andersen, T., Bollerslev, T., Diebold, F. and P. Labys (2003) "Modeling and Forecasting Realized Volatility," Econometrica, 71, 529-626.

[5] Amin, K. and V. Ng (1993), "ARCH Processes and Option Valuation," Manuscript, University of Michigan.

[6] Andersen, T., Benzoni, L. and J. Lund (2002), "An Empirical Investigation of Continuous-Time Equity Return Models," Journal of Finance, 57, 1239-1284.

[7] Baillie, R.T, Bollerslev, T. and H.O. Mikkelsen (1996), "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, 74, 3-30.

[8] Bakshi, C., Cao, C. and Z. Chen (1997), "Empirical Performance of Alternative Option Pricing Models," Journal of Finance, 52, 2003-2049.

[9] Bates, D. (1996), "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options," Review of Financial Studies, 9, 69-107.

[10] Bates, D. (2000), "Post-87 Crash Fears in S&P500 Futures Options," Journal of Econometrics, 94, 181-238.

[11] Bates, D. (2004), "Maximum Likelihood Estimation of Latent Affine Processes," Working Paper, University of Iowa.

[12] Benzoni, L. (1998), "Pricing Options Under Stochastic Volatility: An Econometric Analysis," Manuscript, University of Minnesota.

[13] Beveridge, S. and C. Nelson (1981), "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle," Journal of Monetary Economics, 7, 151-174.

[14] Black, F. (1976), "Studies of Stock Price Volatility Changes," in: Proceedings of the 1976 Meetings of the Business and Economic Statistics Section, American Statistical Association, 177-181.

[15] Black, F. and M. Scholes (1973), "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81, 637-659.

[16] Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics, 31, 307-327.

[17] Bollerslev, T. and H.O. Mikkelsen (1996), "Modeling and Pricing Long Memory in Stock Market Volatility," Journal of Econometrics, 73, 151-184.

[18] Bollerslev, T. and H.O. Mikkelsen (1999), "Long-Term Equity AnticiPation Securities and Stock Market Volatility Dynamics," Journal of Econometrics, 92, 75-99.

[19] Bollerslev, T. and H. Zhou (2002), "Estimating Stochastic Volatility using Conditional Moments of Integrated Volatility," Journal of Econometrics, 109, 33-65.

[20] Brandt, M. and C. Jones (2005), "Forecasting Volatility with Range-Based EGARCH Models," Journal of Business and Economic Statistics. Forthcoming.

[21] Brennan, M. (1979), "The Pricing of Contingent Claims in Discrete-Time Models," Journal of Finance, 34, 53-68.

[22] Broadie, M., Chernov, M. and M. Johannes (2004), "Model Specification and Risk Premiums: The Evidence from the Futures Options," manuscript, Graduate School of Business, Columbia University.

[23] Camara, A. (2003), "A Generalization of the Brennan-Rubenstein Approach for the Pricing of Derivatives," Journal of Finance, 58, 805-819.

[24] Carr, P. and L. Wu (2004), "Time-Changed Levy Processes and Option Pricing," Journal of Financial Economics, 17, 113–141.

[25] Chacko, G. and L. Viceira, (2003), "Spectral GMM estimation of Continuous-Time Processes," Journal of Econometrics, 116, 259-292.

[26] Chernov, M. and E. Ghysels, (2000) "A Study Towards a Unified Approach to the Joint Estimation of Objective and Risk Neutral Measures for the Purpose of Option Valuation," Journal of Financial Economics, 56, 407-458.

[27] Chernov, M., R. Gallant, E. Ghysels and G. Tauchen (2003) "Alternative Models for Stock Price Dynamics," Journal of Econometrics, 116, 225-257.

[28] Christoffersen, P., S. Heston, and K. Jacobs (2004), "Option Valuation with Conditional Skewness," Journal of Econometrics, forthcoming.

[29] Christoffersen, P. and K. Jacobs (2004), "Which GARCH Model for Option Valuation?" Management Science, 50, 1204-1221.

[30] Christoffersen, P., K. Mimouni and K. Jacobs (2005), "An Empirical Comparison of Affine and Non-affine Models for Equity Index Options" working paper, McGill University.

[31] Comte, F., Coutin, L. and E. Renault (2001), "Affine Fractional Stochastic Volatility Models", working paper, University of Montreal.

[32] Dai, Q. and K.J. Singleton (2000), "Specification Analysis of Affine Term Structure Models," Journal of Finance, 55, 1943-1978.

[33] Ding, Z., C.W.J. Granger, and R.F. Engle (1993), "A Long Memory Property of Stock Market Returns and a New Model," Journal of Empirical Finance, 83–106.

[34] Duan, J.-C. (1995), "The GARCH Option Pricing Model," Mathematical Finance, 5, 13-32.

[35] Duan, J.-C. (1997), "Augmented GARCH(p,q) Process and its Diffusion Limit," Journal of Econometrics, 79, 97-127.

[36] Duan, J.-C., P. Ritchken and Z. Sun (2002), "Option Valuation with Jumps in Returns and Volatility," manuscript, Rotman School, University of Toronto.

[37] Duffee, G.R. (1999), "Estimating the Price of Default Risk," Review of Financial Studies, 12, 197-226.

[38] Duffie, D. and K.J. Singleton (1999), "Modeling Term Structures of Defaultable Bonds," Review of Financial Studies, 12, 687-720.

[39] Duffie, D., J. Pan and K. Singleton (2000), "Transform Analysis and Asset Pricing for Affine Jump-Diffusions," Econometrica, 68, 1343-1376.

[40] Dumas, B., F. Fleming and R. Whaley (1998), "Implied Volatility Functions: Empirical Tests," Journal of Finance, 53, 2059-2106.

[41] Engle, R. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation," Econometrica, 50, 987-1008.

[42] Engle, R. and C. Mustafa (1992), "Implied ARCH Models from Options Prices," Journal of Econometrics, 52, 289-311.

[43] Engle, R. and G. Lee (1999), "A Long-Run and Short-Run Component Model of Stock Return Volatility," in Cointegration, Causality and Forecasting, edited by R. Engle and H. White, Oxford University Press.

[44] Engle, R. and V. Ng (1993), "Measuring and Testing the Impact of News on Volatility," Journal of Finance, 48, 1749-1778.

[45] Eraker, B. (2004), "Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices," Journal of Finance, 59, 1367-1403.

[46] Eraker, B., M. Johannes and N. Polson (2003), "The Impact of Jumps in Volatility and Returns," Journal of Finance, 58, 1269-1300.

[47] Fama, E. and K. French (1988), "Permanent and Temporary Components of Stock Prices," Journal of Political Economy, 96, 246-273.

[48] French, K., G. W. Schwert, and R. Stambaugh (1987), "Expected Stock Returns and Volatility," Journal of Financial Economics, 19, 3-30.

[49] Gallant, R., Chernov, M., Ghysels, E. and G. Tauchen (2003), "Alternative Models for Stock Price Dynamics," Journal of Econometrics, 116, 225-257.

[50] Hentschel, L. (1995), "All in the Family: Nesting Symmetric and Asymmetric GARCH Models," Journal of Financial Economics, 39, 71-104.

[51] Heston, S. (1993), "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options," Review of Financial Studies, 6, 327-343.

[52] Heston, S. and S. Nandi (2000), "A Closed-Form GARCH Option Pricing Model," Review of Financial Studies, 13, 585-626.

[53] Hsieh, K. and P. Ritchken (2000), "An Empirical Comparison of GARCH Option Pricing Models," manuscript, Case Western Reserve University

[54] Huang, J.-Z. and L. Wu (2004), "Specification Analysis of Option Pricing Models Based on Time-Changed Levy Processes," Journal of Finance, 59, 1405–1439.

[55] Hull, J. and A. White (1987), "The Pricing of Options with Stochastic Volatilities," Journal of Finance, 42, 281-300.

[56] Jones, C. (2003), "The Dynamics of Stochastic Volatility: Evidence from Underlying and Options Markets," Journal of Econometrics, 116, 181-224.

[57] Maheu, J. (2002), "Can GARCH Models Capture the Long-Range Dependence in Financial Market Volatility?," manuscript, University of Toronto.

[58] Melino, A. and S. Turnbull (1990), "Pricing Foreign Currency Options with Stochastic Volatility," Journal of Econometrics, 45, 239-265.

[59] Nandi, S. (1998), "How Important is the Correlation Between returns and Volatility in a Stochastic Volatility Model? Empirical Evidence from Pricing and Hedging in the S&P 500 Index Options Market," Journal of Banking and Finance 22, 589-610.

[60] Pan, J. (2002), "The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study," Journal of Financial Economics, 63, 3–50.

[61] Pearson, N.D. and T.-S. Sun (1994), "Exploiting the Conditional Density in Estimating the Term Structure: An Application to the Cox, Ingersoll, and Ross Model," Journal of Finance, 49 (4), 1279-1304.

[62] Poterba, J. and L. Summers (1988), "Mean Reversion in Stock Returns: Evidence and Implications",. Journal of Financial Economics, 22, 27-60.

[63] Ritchken, P. and R. Trevor (1999), "Pricing Options under Generalized GARCH and Stochastic Volatility Processes," Journal of Finance, 54, 377-402.

[64] Rubinstein, M. (1976), "The Valuation of Uncertain Income Streams and The Pricing of Options," Bell Journal of Economics, 7, 407-425.

[65] Schroder, M. (2004), "Risk-Neutral Parameter Shifts and Derivative Pricing in Discrete Time," Journal of Finance, 59, 2375-2401.

[66] Scott, L. (1987), "Option Pricing when the Variance Changes Randomly: Theory, Estimators and Applications," Journal of Financial and Quantitative Analysis, 22, 419-438.

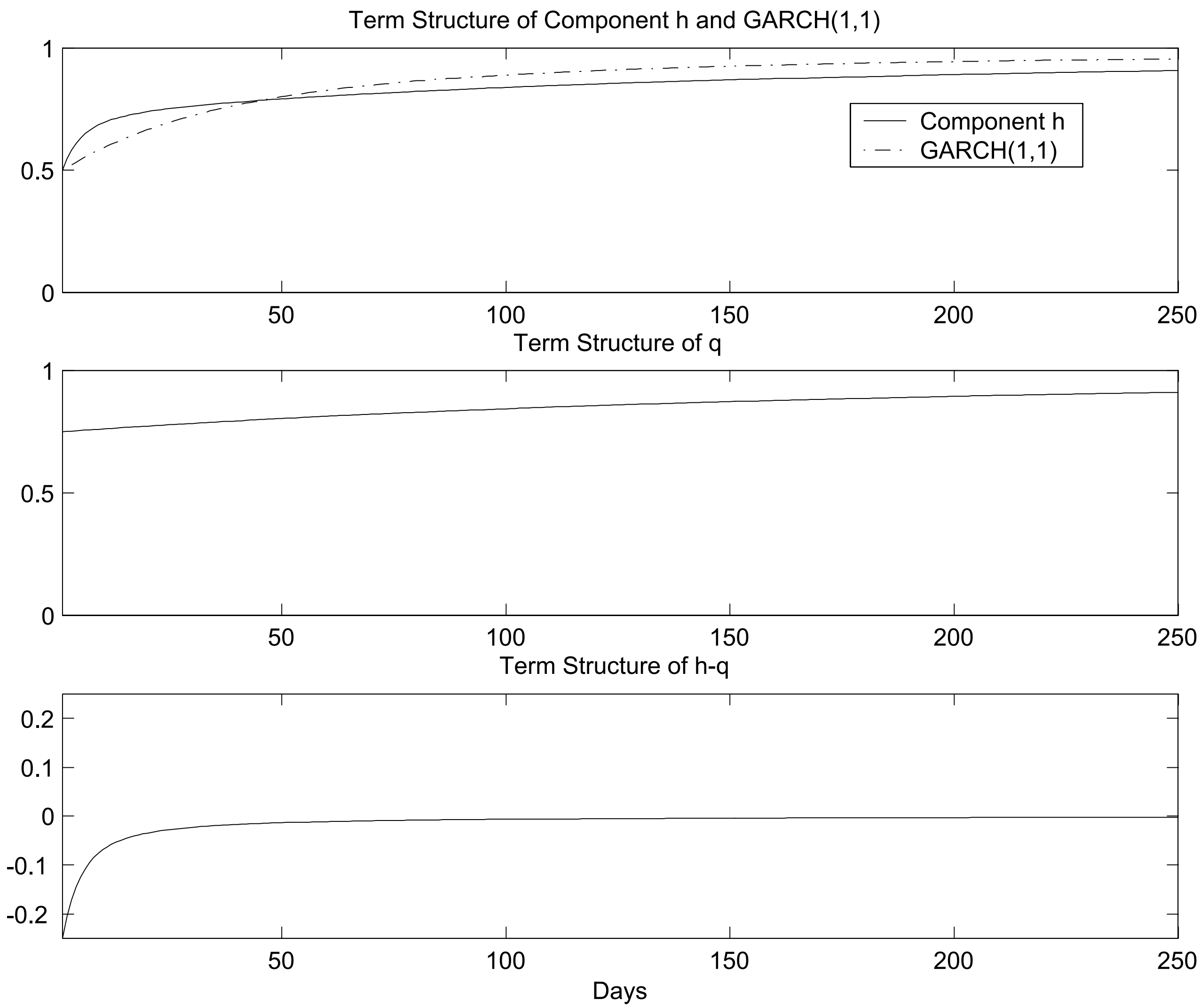
[67] Summers, L. (1986), "Does the Stock Market Rationally Reflect Fundamental Values," Journal of Finance, 41, 591-600.

[68] Taylor, S. and X. Xu (1994), "The Term Structure of Volatility Implied by Foreign Exchange Options," Journal of Financial and Quantitative Analysis, 29, 57-74.

[69] Wiggins, J.B. (1987), "Option Values under Stochastic Volatility: Theory and Empirical Evidence," Journal of Financial Economics, 19, 351-372.

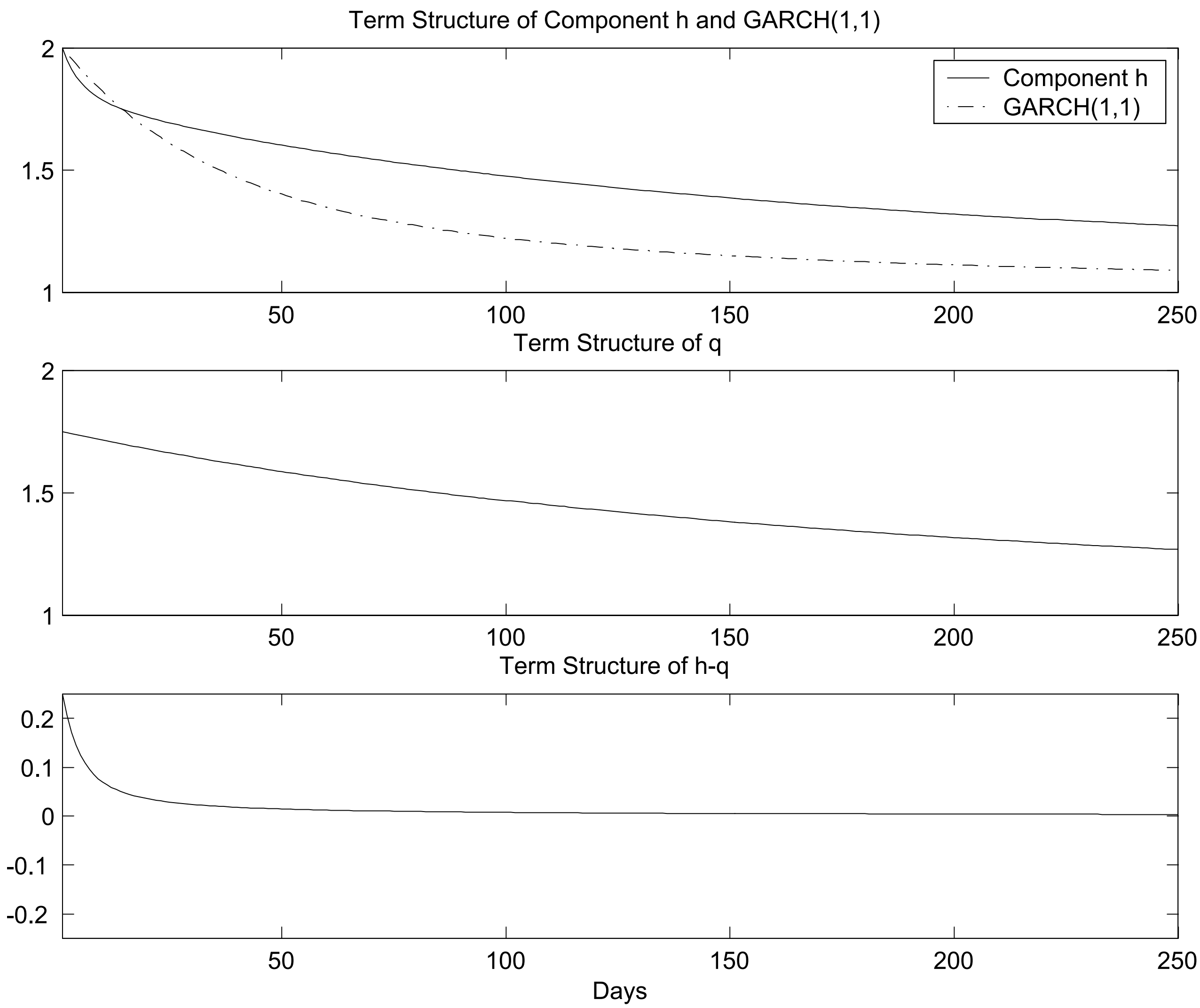
[70] White, H. (1982), "Maximum Likelihood Specification of Misspecified Models," Econometrica, 50, 1-25.

Figure 1. Term Structure of Variance with Low Initial Variance.
Component Model Versus GARCH(1,1).
Normalized by Unconditional Variance. Estimates Obtained from MLE.



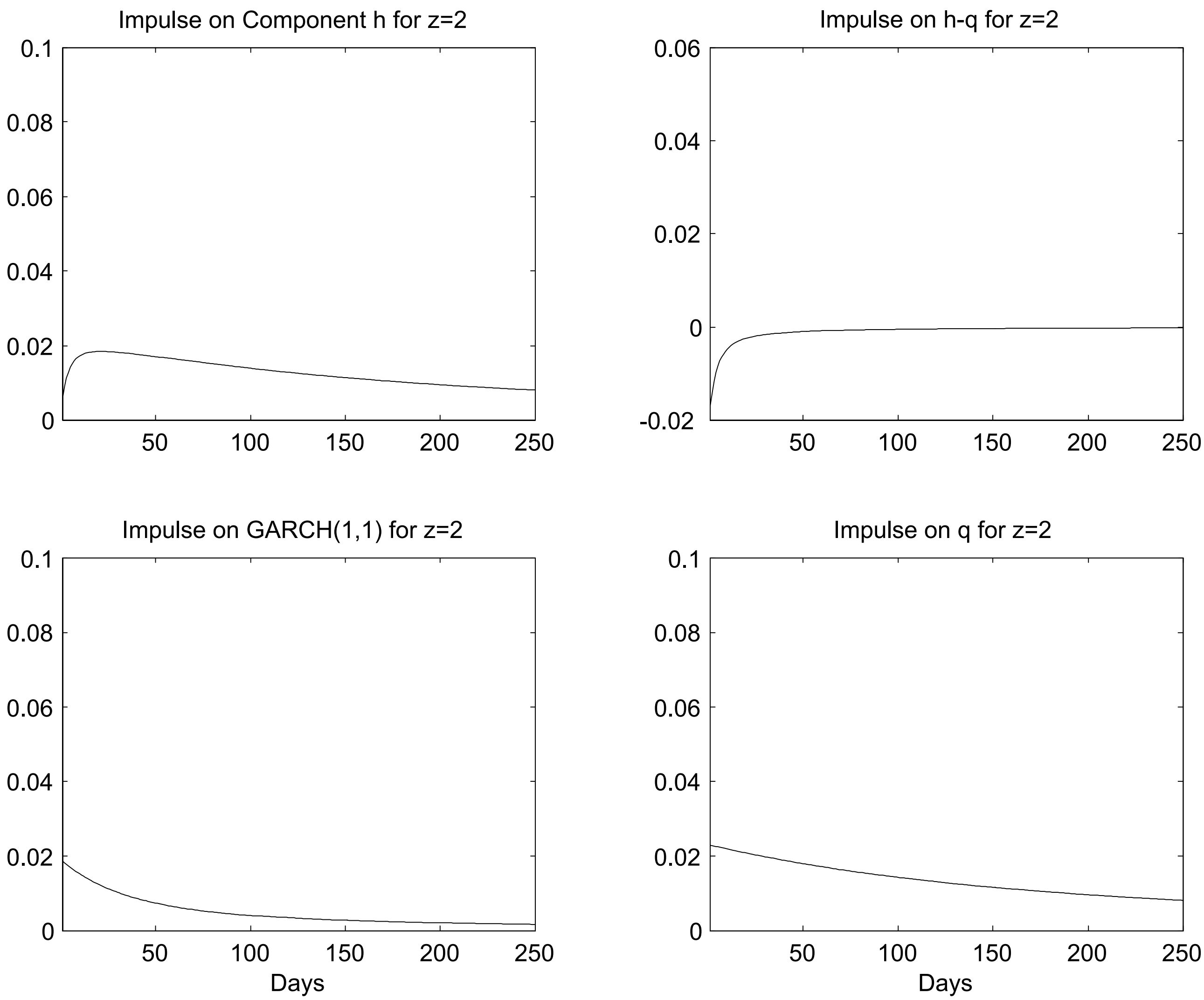
Notes to Figure: In the top panel we plot the variance term structure implied by the component GARCH and GARCH(1,1) models for 1 through 250 days. In the second and third panel we plot the term structure of the individual components for the component model. The parameter values are obtained from MLE estimation on returns in Table 2. The initial value of q_{t+1} is set to $0.75\sigma^2$ and the initial value of h_{t+1} is set to $0.5\sigma^2$. The initial value for h_{t+1} in the GARCH(1,1) is set to $0.5\sigma^2$ as well. All values are normalized by the unconditional variance σ^2.

Figure 2. Term Structure of Variance with High Initial Variance.
Component Model Versus GARCH(1,1).
Normalized by Unconditional Variance. Estimates Obtained from MLE.



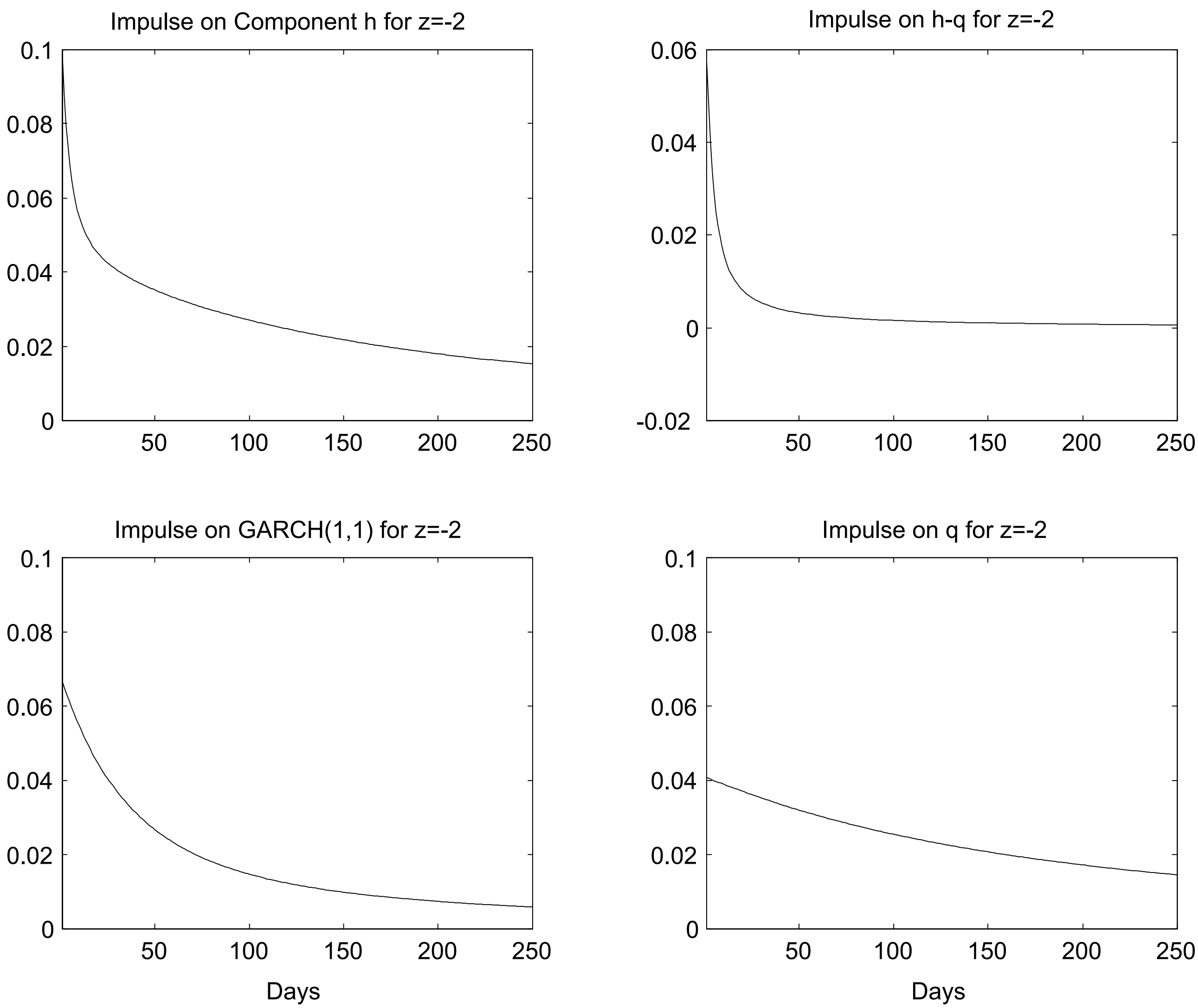
Notes to Figure: In the top panel we plot the variance term structure implied by the component GARCH and GARCH(1,1) models for 1 through 250 days. In the second and third panel we plot the term structure of the individual components for the component model. The parameter values are obtained from MLE estimation on returns in Table 2. The initial value of q_{t+1} is set to $1.75\sigma^2$ and the initial value of h_{t+1} is set to $2\sigma^2$. The initial value for h_{t+1} in the GARCH(1,1) is set to $2\sigma^2$ as well. All values are normalized by the unconditional variance σ^2.

Figure 3. Term Structure Impulse Response to Positive Return Shock ($z_t = 2$). Component Model Versus GARCH(1,1). Normalized by Unconditional Variance. Estimates Obtained from MLE.



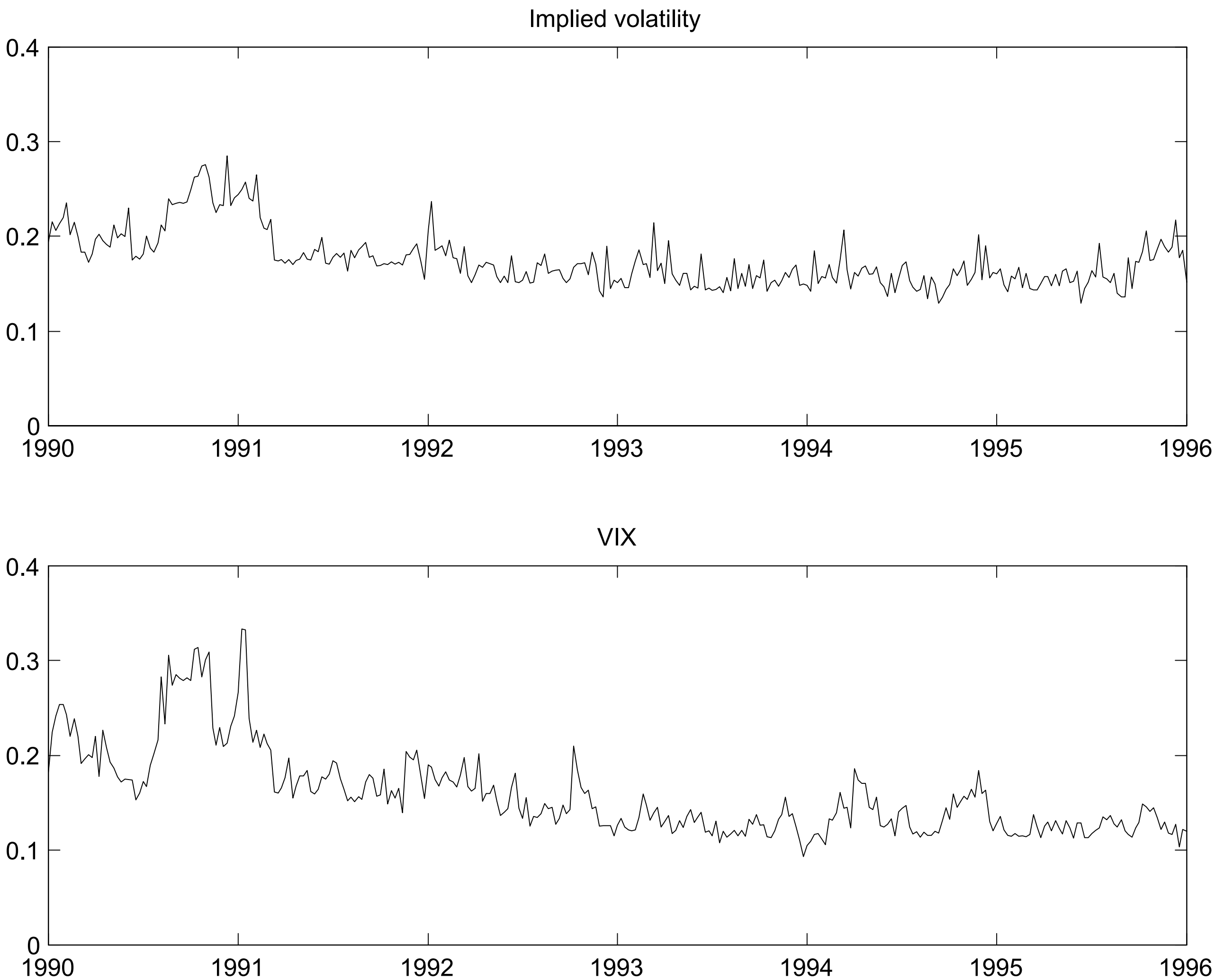
Notes to Figure: In the left-hand panels we plot the variance term structure response to a $z_t = 2$ shock to the return in the component and GARCH(1,1) models. For the component model, the right-hand panels show the response of the individual components. The parameter values are obtained from the MLE estimation on returns in Table 2. The current variance is set equal to the unconditional value. All values are normalized by the unconditional variance.

Figure 4. Term Structure Impulse Response to Negative Return Shock, ($z_t = -2$).
Component Model Versus GARCH(1,1).
Normalized by Unconditional Variance. Estimates Obtained from MLE.



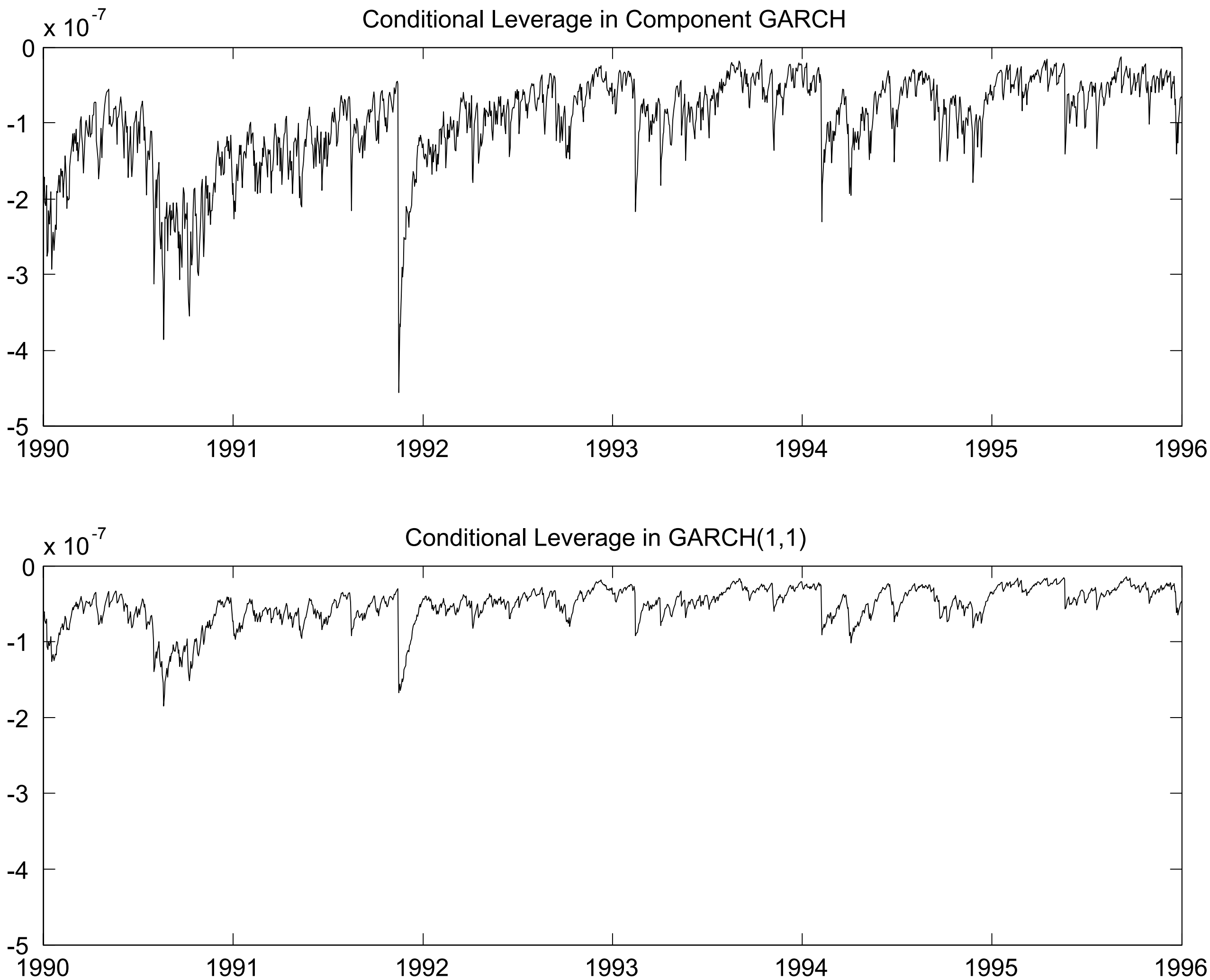
Notes to Figure: In the left-hand panels we plot the variance term structure response to a $z_t = -2$ shock to the return in the component and GARCH(1,1) models. For the component model, the right-hand panels show the response of the individual components. The parameter values are obtained from the MLE estimation on returns in Table 2. The current variance is set equal to the unconditional value. All values are normalized by the unconditional variance.

Figure 5. Sample Average Weekly Implied Volatility and VIX.



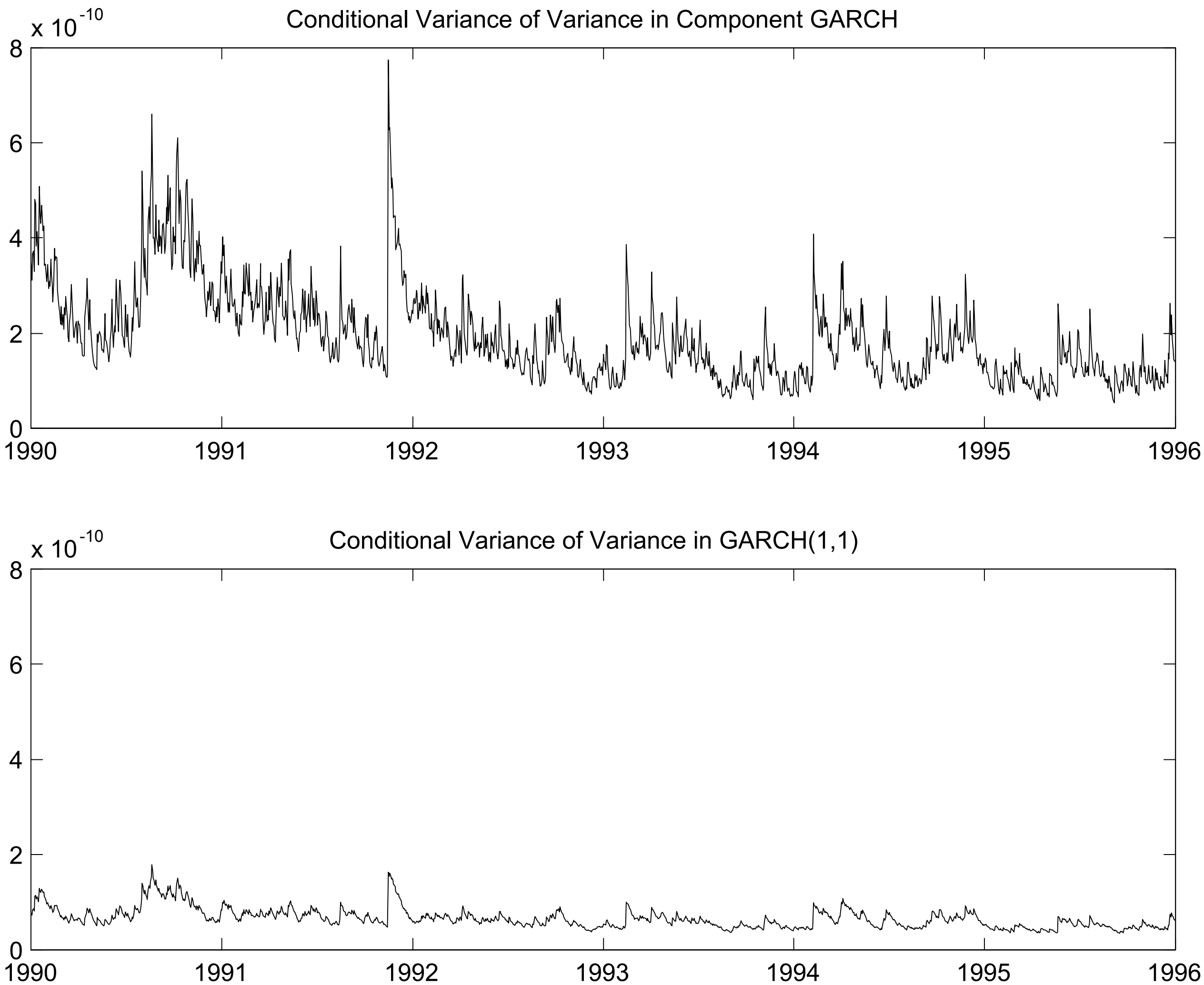
Notes to Figure: The top panel plots the average weekly implied Black-Scholes volatility for the S&P500 call options in our sample. The bottom panel plots the VIX index from the CBOE for comparison.

Figure 6. Conditional Leverage Paths.
Estimates Obtained from MLE.



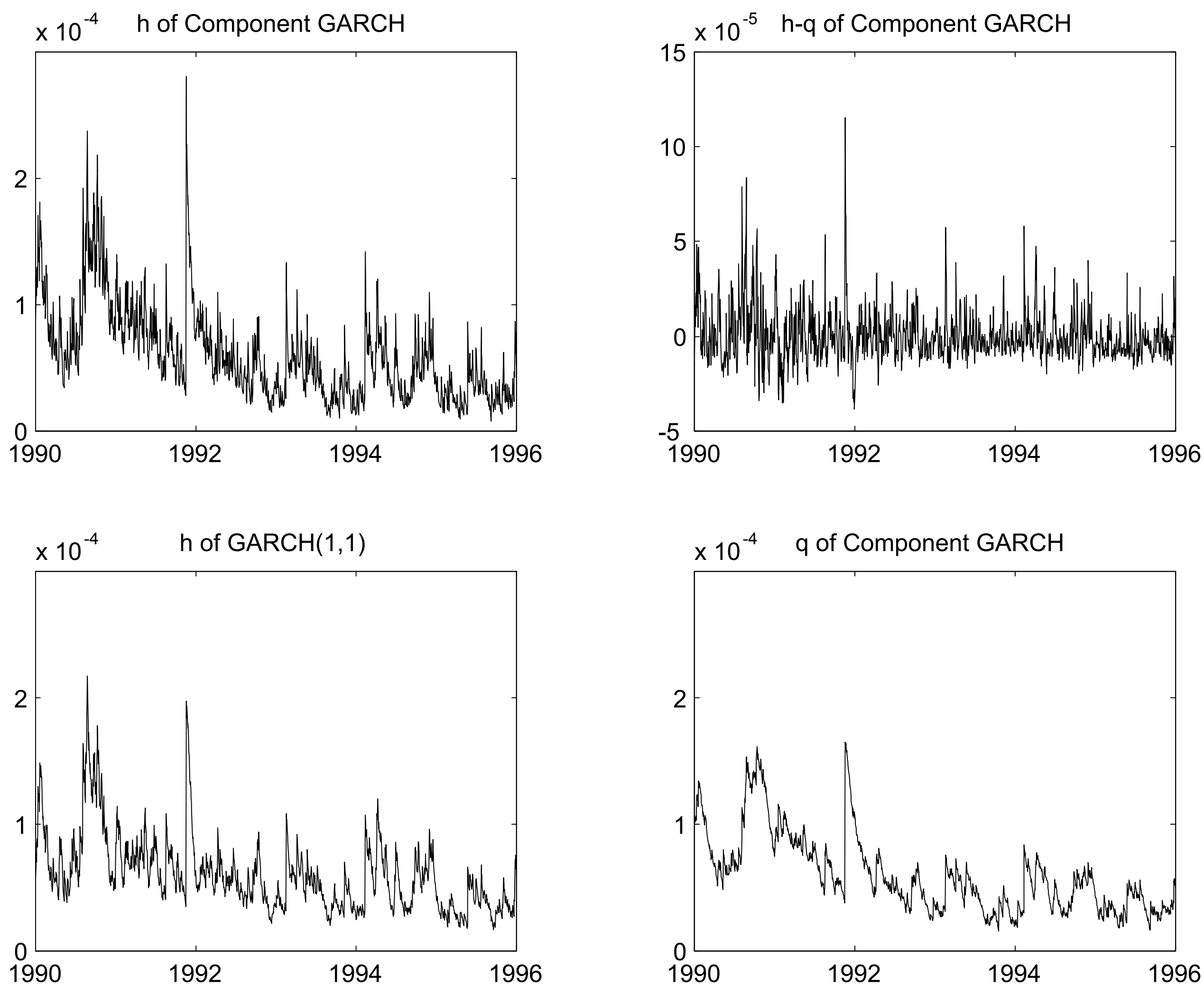
Notes to Figure: We plot the conditional covariance between return and next-day variance as implied by the GARCH models and refer to it as conditional leverage. The top panel shows the component model and the bottom panel shows the GARCH(1,1) model. The scales are identical across panels to facilitate comparison across models. The parameter values are obtained from the MLE estimates on returns in Table 2.

Figure 7: Conditional Variance of Variance Paths.
Estimates Obtained from MLE.



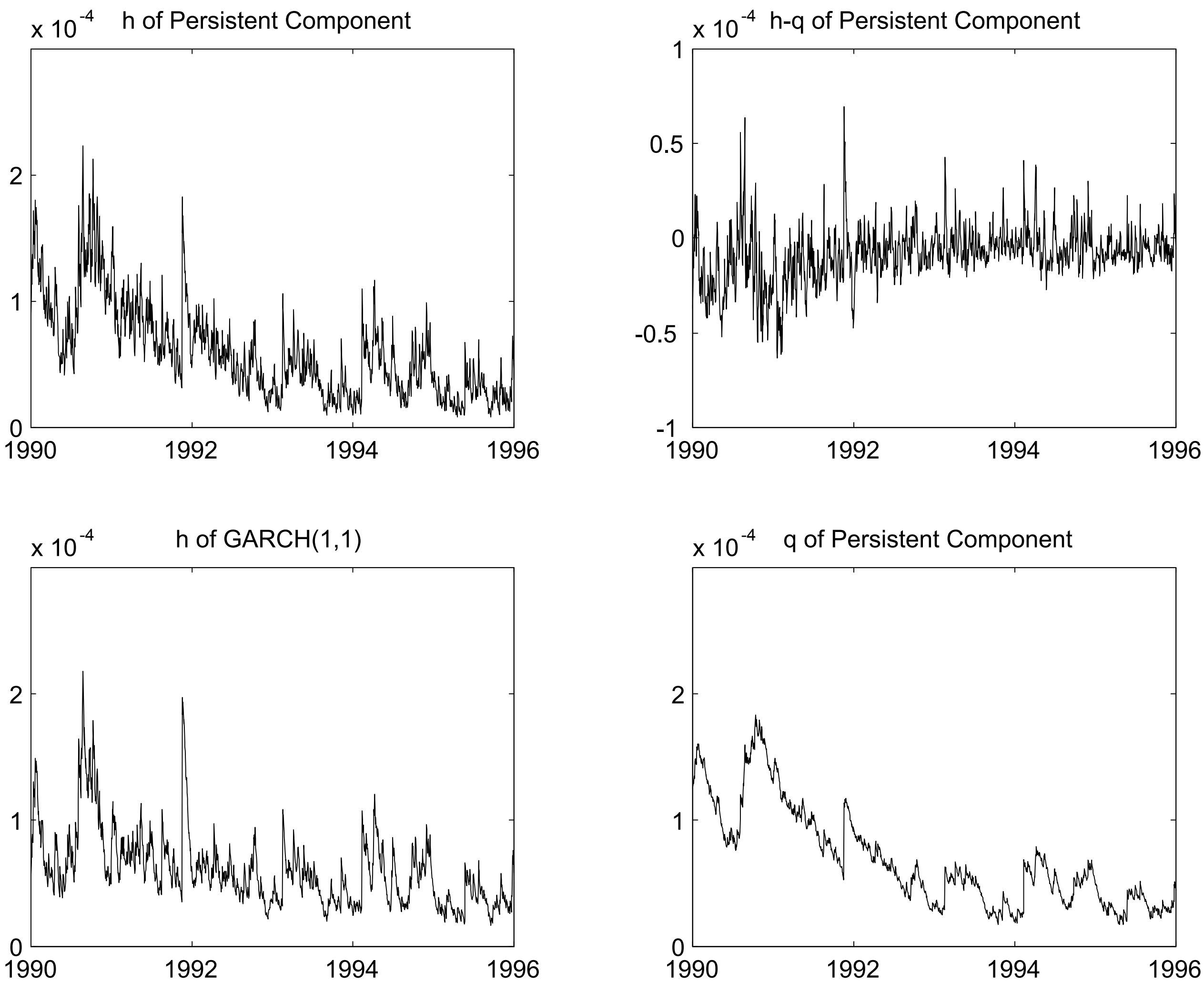
Notes to Figure: We plot the conditional variance of next day's variance as implied by the GARCH models. The top panel shows the component model and the bottom panel shows the GARCH(1,1) model. The scales are identical across panels to facilitate comparison across models. The parameter values are obtained from the MLE estimates on returns in Table 2.

Figure 8. Spot Variance of Component GARCH versus GARCH(1,1).
Estimates Obtained from MLE.



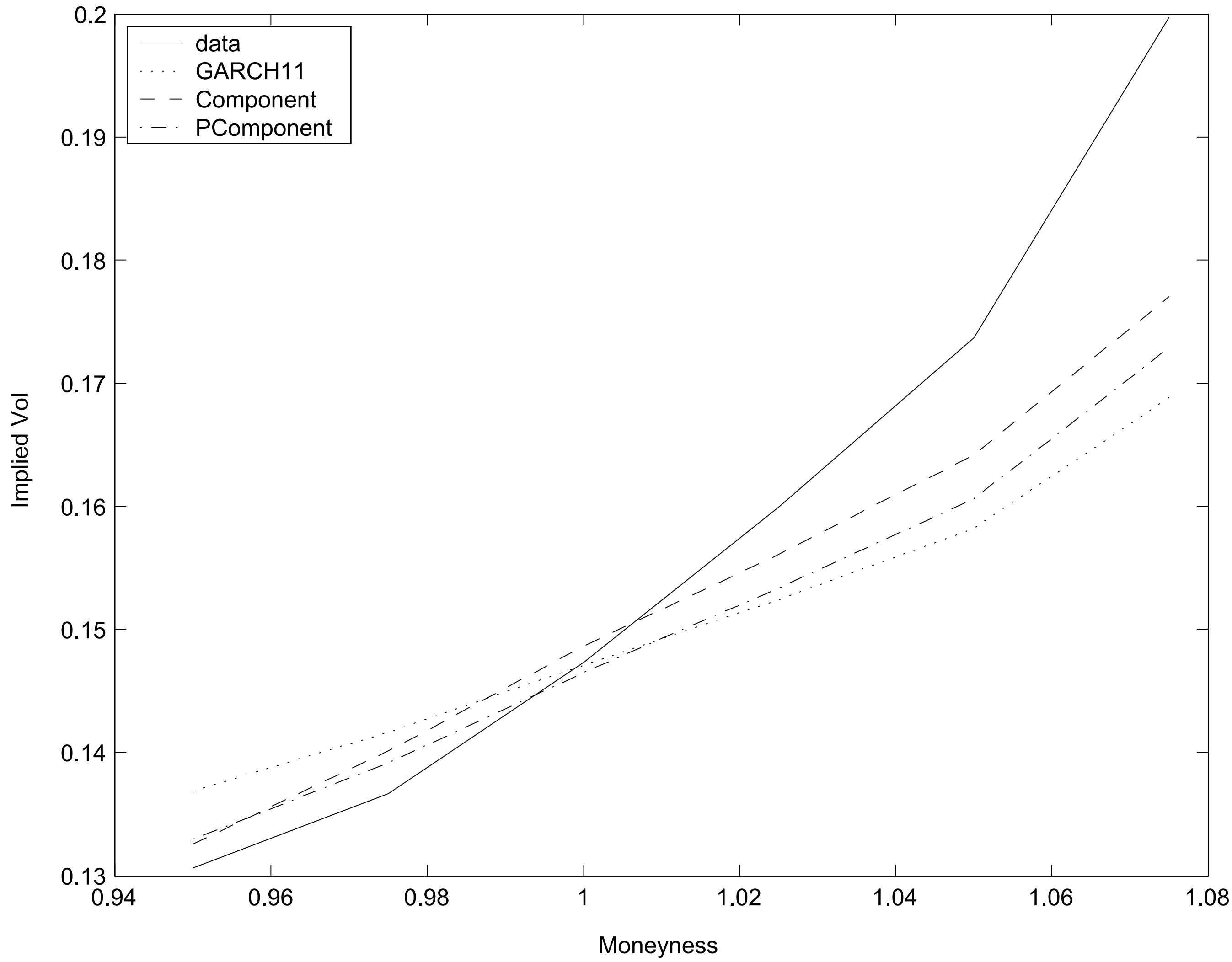
Notes to Figure: The left-hand panels plot the variance paths from the component and GARCH(1,1) models. The right-hand panels plot the individual components. The parameter values are obtained from MLE estimation on returns in Table 2.

Figure 9. Spot Variance of Persistent Component Model versus GARCH(1,1).
Estimates Obtained from MLE.



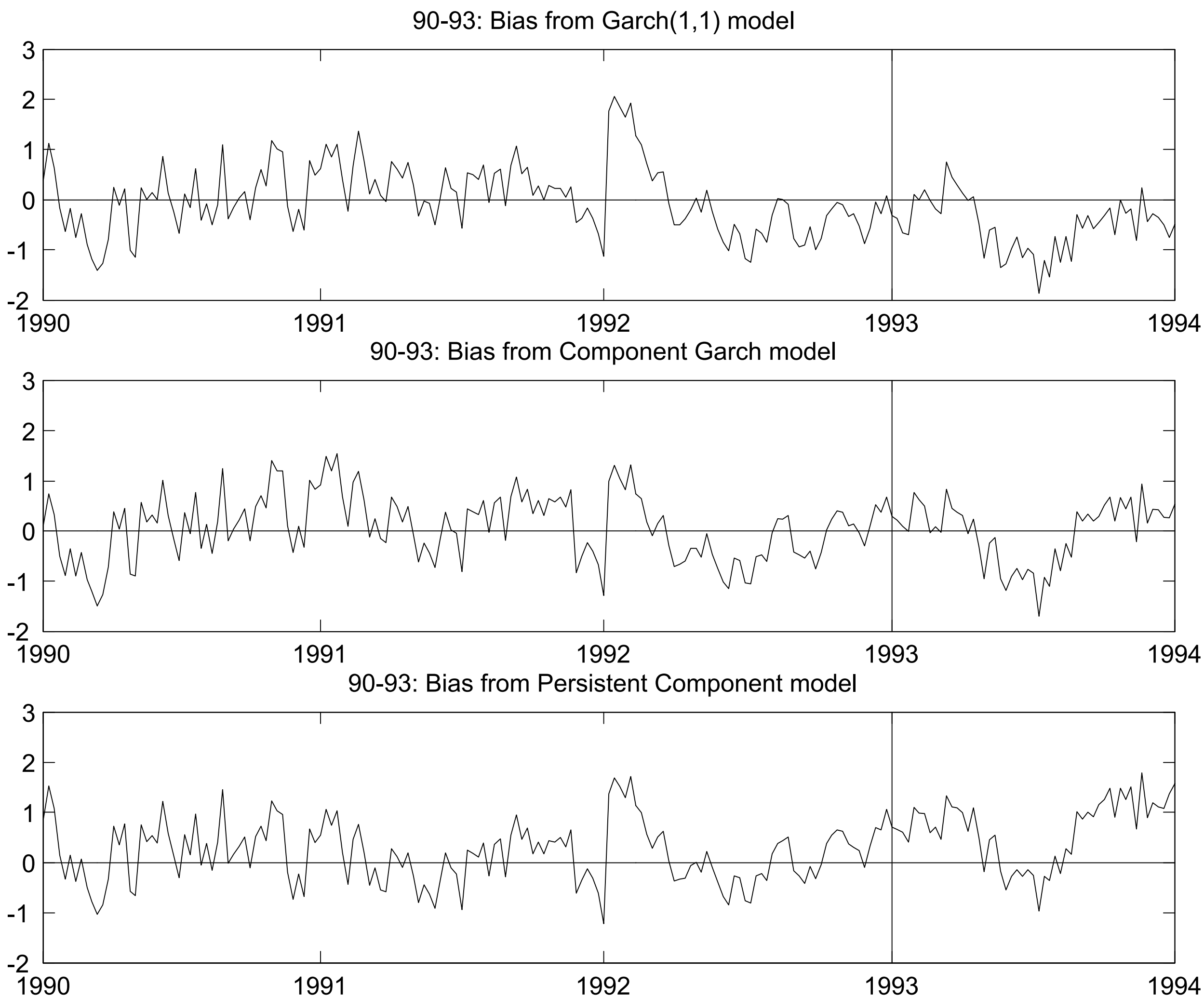
Notes to Figure: The left-hand panels plot the variance paths from the persistent component ($\rho = 1$) and GARCH(1,1) models. The right-hand panels plot the individual components. The parameter values are obtained from MLE estimation on returns in Table 2.

Figure 10. Average Implied Volatility Smiles: Data and Models.
Estimates Obtained from NLS.



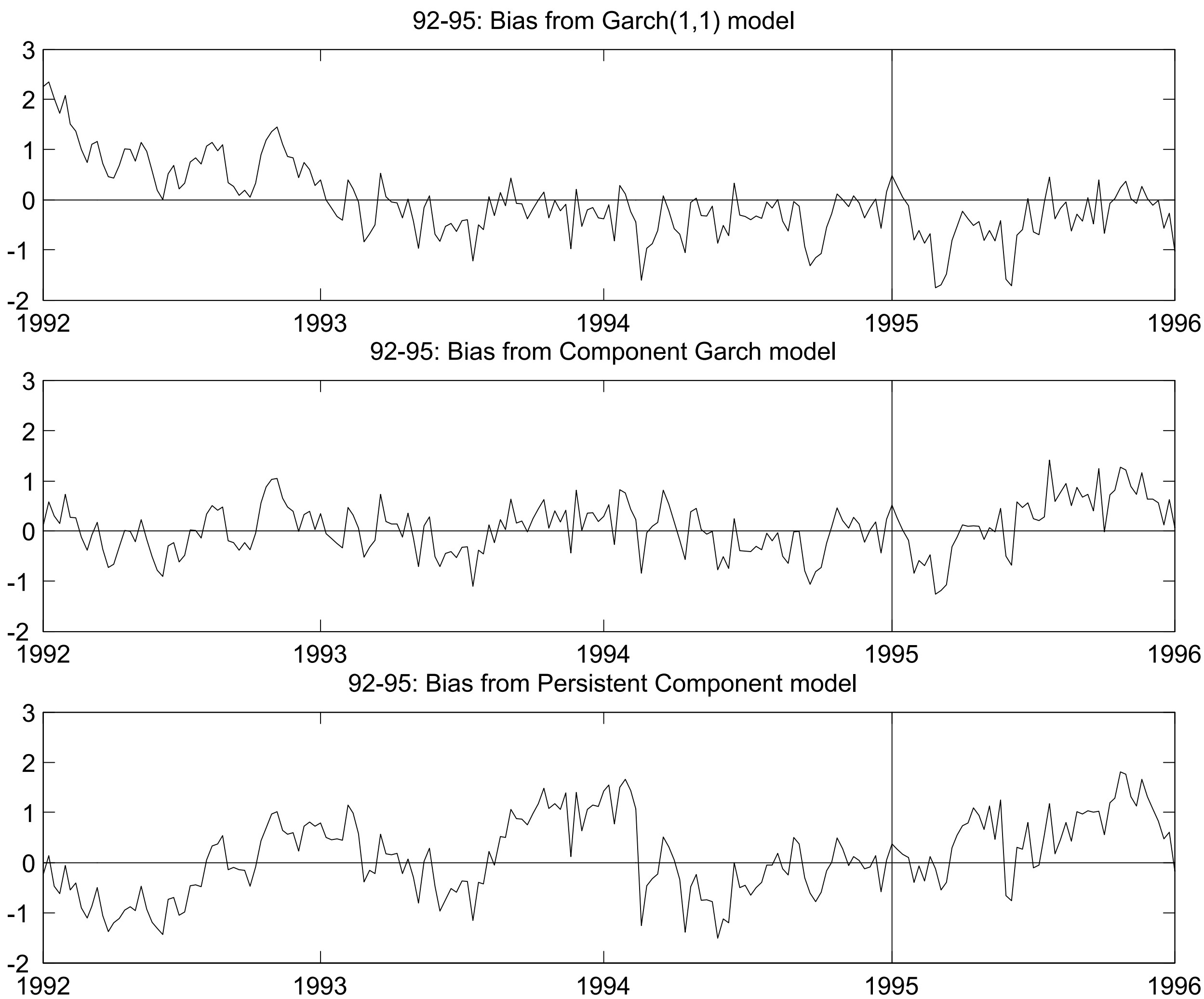
Notes to Figure: We plot the average Black-Scholes implied volatility from observed price data (solid line) and from our three sets of model prices against moneyness. The options are from the 1990-1992 sample, and the option valuation model parameter values are taken from the NLS estimation in Table 3.

Figure 11. Weekly Average Dollar Bias from Sample 90-93.
Estimates Obtained from NLS.



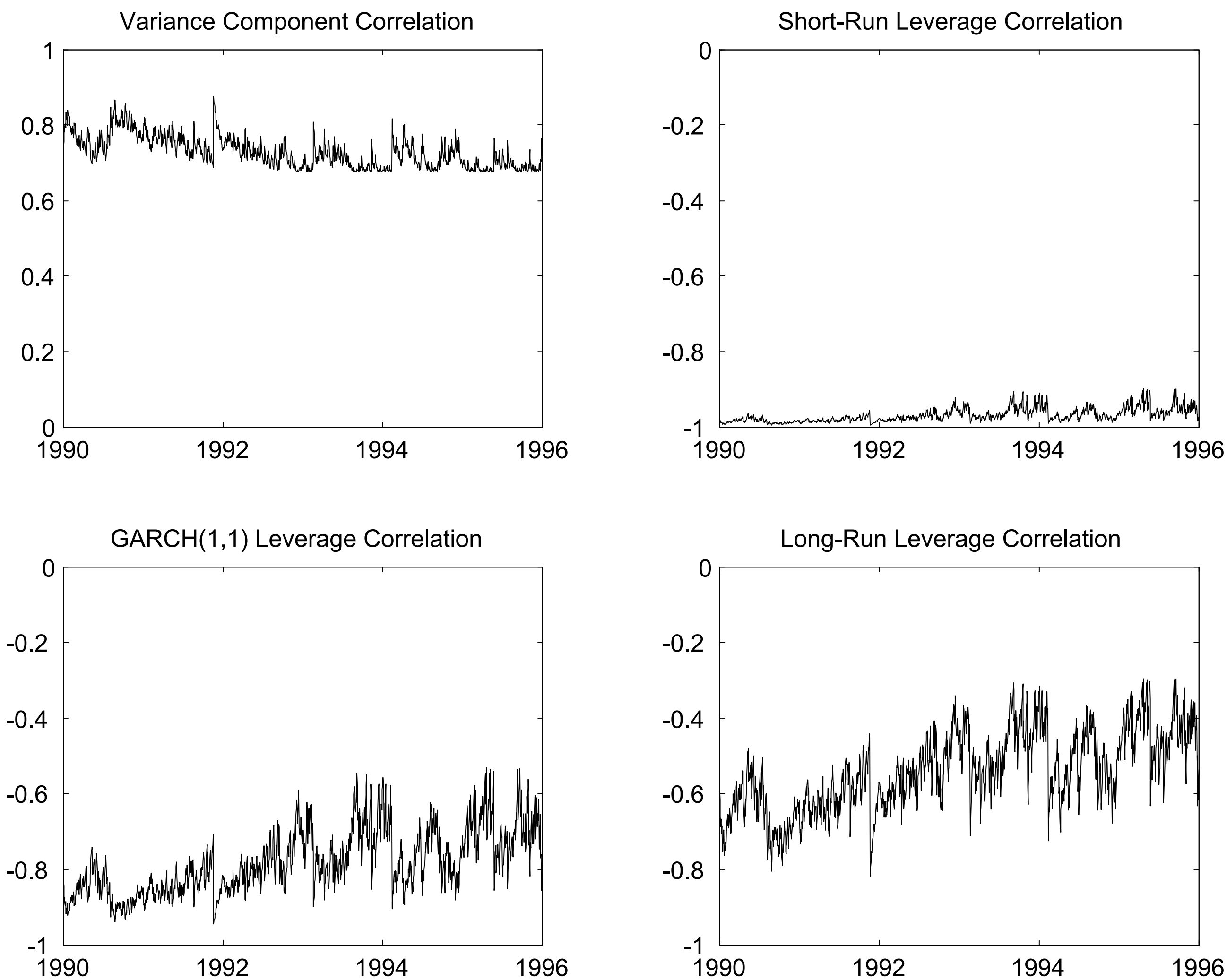
Notes to Figure: We plot the average weekly bias (market price less model price) for the three GARCH models during the 1990-1993 sample. The parameter values are obtained from NLS estimation in Table 3 on options quoted during the 1990-1992 period. The vertical lines denote the end of the estimation sample period. The horizontal lines are at zero and the scales are identical across panels to facilitate comparison across models.

Figure 12. Weekly Average Dollar Bias from Sample 92-95.
Estimates Obtained from NLS.



Notes to Figure: We plot the average weekly bias (market price less model price) for the three GARCH models during the 1992-1995 sample. The parameter values are obtained from NLS estimation in Table 5 on options quoted during the 1992-1994 period. The vertical lines denote the end of the estimation sample period. The horizontal lines are at zero and the scales are identical across panels to facilitate comparison across models.

Figure 13. Correlation Between Shocks to Returns and Volatility.
Estimates Obtained from MLE.



Notes to Figure: The top left panel plots the conditional correlation between the innovations to the two variance components in the component model. The top right picture plots the conditional correlation between the innovation to the short-run volatility component and the return innovation, and the bottom right panel plots the conditional correlation between the innovation to the long-run volatility component and the return innovation. The bottom left panel plots the conditional correlation between the return innovation and the volatility innovation in the Heston-Nandi GARCH(1,1) model. The parameter values are taken from the MLE estimation on returns in Table 2.

Table 1: S&P 500 Index Call Option Data (1990-1995)

Panel A. Number of Call Option Contracts

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	147	2,503	2,322	1,119	6,091
0.975<S/X<1.00	365	1,604	871	312	3,152
1.00<S/X<1.025	378	1,524	890	382	3,174
1.025<S/X<1.05	335	1,462	797	311	2,905
1.05<S/X<1.075	307	1,315	713	297	2,632
1.075<S/X	736	3,096	2,112	982	6,926
All	2,268	11,504	7,705	3,403	24,880

Panel B. Average Call Price

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.91	2.67	6.86	11.94	5.93
0.975<S/X<1.00	2.64	7.95	16.99	27.50	11.77
1.00<S/X<1.025	9.37	15.37	24.90	34.41	19.62
1.025<S/X<1.05	19.64	24.53	33.13	42.14	28.21
1.05<S/X<1.075	30.06	34.33	41.98	48.83	37.54
1.075<S/X	57.42	59.05	65.29	68.34	62.10
All	27.65	26.66	32.07	36.07	29.71

Panel C. Average Implied Volatility from Call Options

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.1553	0.1284	0.1348	0.1394	0.1335
0.975<S/X<1.00	0.1331	0.1329	0.1461	0.1562	0.1389
1.00<S/X<1.025	0.1555	0.1489	0.1572	0.1605	0.1534
1.025<S/X<1.05	0.1940	0.1676	0.1679	0.1656	0.1705
1.05<S/X<1.075	0.2445	0.1855	0.1792	0.1739	0.1894
1.075<S/X	0.3877	0.2371	0.1996	0.1869	0.2345
All	0.2484	0.1738	0.1642	0.1607	0.1758

Notes to Table: We use European call options on the S&P500 index. The prices are taken from quotes within 30 minutes from closing on each Wednesday during the January 1, 1990 to December 31, 1995 period. The moneyness and maturity filters used by Bakshi, Cao and Chen (1997) are applied here as well. The implied volatilities are calculated using the Black-Scholes formula.

Table 2: MLE Estimates and Properties

Sample: Daily Returns, 1963-1995

GARCH(1,1)			**Component GARCH**			**Persistent Component**		
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
w	2.101E-17	1.120E-07	$\tilde{\beta}$	6.437E-01	1.892E-01	$\tilde{\beta}$	7.643E-01	5.963E-03
b	9.013E-01	4.678E-03	α	1.580E-06	1.200E-07	α	7.639E-07	1.230E-07
a	3.313E-06	1.380E-07	γ_1	4.151E+02	3.156E+02	γ_1	7.645E+02	1.121E+01
c	1.276E+02	8.347E+00	γ_2	6.324E+01	7.279E+00	γ_2	1.137E+02	8.202E+00
λ	2.231E+00	1.123E+00	ω	8.208E-07	1.860E-07	ω	2.448E-07	7.280E-09
			φ	2.480E-06	1.200E-07	φ	1.482E-06	3.500E-08
			ρ	9.896E-01	1.950E-03	ρ	1.000E+00	
			λ	2.092E+00	7.729E-01	λ	-6.659E+00	5.410E+00

Total Persist	0.9552	Total Persist	0.9963	Total Persist	1.0000
Annual Vol	0.1366	Annual Vol	0.1413		
Var of Var	8.652E-06	Var of Var	1.557E-05		
Leverage	-8.455E-04	Leverage	-1.626E-03	Leverage	-1.505E-03
Ln Likelihood	33,955	Ln Likelihood	34,102	Ln Likelihood	34,055

Notes to Table: We use daily total returns from January 1, 1963 to December 31, 1995 on the S&P500 index to estimate the three GARCH models using Maximum Likelihood. Robust standard errors are calculated from the outer product of the gradient at the optimum parameter values. Total Persist refers to the persistence of the conditional variance in each model. Annual Vol refers to the annualized unconditional standard deviation as implied by the parameters in each model. Var of Var refers to the unconditional variance of the conditional variance in each model. Leverage refers to the unconditional covariance between the return and the conditional variance. Ln Likelihood refers to the logarithm of the likelihood at the optimal parameter values.

Table 3: NLS Estimates and Properties
Sample: 1990-1992 (in-sample) 1993 (out-of-sample).
7 - 365 Days to Maturity

GARCH(1,1)			Component GARCH			Persistent Component		
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
w	3.891E-14	3.560E-12	$\tilde{\beta}$	6.998E-01	1.345E-01	$\tilde{\beta}$	6.870E-01	2.049E-01
b	6.801E-01	3.211E-03	α	1.788E-06	7.121E-09	α	1.127E-06	4.409E-09
a	2.666E-07	6.110E-09	γ_1	5.592E+02	4.524E+01	γ_1	7.386E+02	6.904E+01
c	1.092E+03	5.432E+01	γ_2	5.612E+02	2.136E+02	γ_2	4.651E+02	1.204E+02
			ω	2.382E-07	1.093E-07	ω	4.466E-07	1.909E-08
			φ	5.068E-07	2.305E-10	φ	7.474E-07	8.344E-09
			ρ	9.966E-01	9.970E-04	ρ	1.000E+00	
Total Persist	0.9981		Total Persist	0.9990		Total Persist	1.0000	
Annual Vol	0.1857		Annual Vol	0.1320				
Var of Var	6.820E-06		Var of Var	2.161E-05				
Leverage	-5.822E-04		Leverage	-2.569E-03		Leverage	-2.360E-03	
RMSE (in)	1.038		RMSE (in)	0.931		RMSE (in)	0.991	
Normalized	1.000		Normalized	0.897		Normalized	0.955	
RMSE (out)	1.284		RMSE (out)	0.983		RMSE (out)	1.247	
Normalized	1.000		Normalized	0.765		Normalized	0.971	

Notes to Table: We use Wednesday option prices from from January 1, 1990 to December 31, 1992 on the S&P500 index to estimate the three GARCH models using Nonlinear Least Squares on the valuation errors. Robust standard errors are calculated from the outer product of the gradient at the optimum parameter values. RMSE refers to the square root of the mean-squared valuation errors. RMSE(in) refers to 1990-1992 and RMSE(out) to 1993. Normalized values are divided by the RMSE from GARCH(1,1).

Table 4: NLS Estimates and Properties

Sample: 1990-1992 (in-sample) 1993 (out-of-sample)

80 - 365 Days to Maturity

GARCH(1,1)			**Component GARCH**			**Persistent Component**		
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
w	1.023E-14	1.346E-14	$\tilde{\beta}$	6.031E-01	2.599E-01	$\tilde{\beta}$	7.964E-01	3.935E-01
b	6.842E-01	4.673E-02	α	4.707E-06	7.896E-10	α	2.757E-07	7.099E-08
a	2.679E-07	7.836E-09	γ_1	3.378E+02	1.838E+01	γ_1	1.891E+03	6.921E+02
c	1.082E+03	5.024E+01	γ_2	6.922E+02	2.683E+02	γ_2	5.369E+02	2.300E+02
			ω	1.596E-07	9.790E-08	ω	4.612E-07	2.964E-07
			φ	3.793E-07	7.540E-10	φ	6.643E-07	8.733E-09
			ρ	9.975E-01	2.900E-03	ρ	1.000E+00	
Total Persist	0.9980		Total Persist	0.9990		Total Persist	1.0000	
Annual Vol	0.1833		Annual Vol	0.1277				
Var of Var	6.705E-06		Var of Var	3.065E-05				
Leverage	-5.798E-04		Leverage	-3.705E-03		Leverage	-1.756E-03	
RMSE (in)	1.133		RMSE (in)	1.013		RMSE (in)	1.112	
Normalized	1.000		Normalized	0.895		Normalized	0.982	
RMSE (out)	1.452		RMSE (out)	1.048		RMSE (out)	1.553	
Normalized	1.000		Normalized	0.722		Normalized	1.070	

Notes to Table: See notes to Table 3. Only options with at least 80 days to maturity are used here.

Table 5: Model Estimates and Properties

Sample: 1992-1994 (in-sample) 1995 (out-of-sample)

7 to 365 Days to Maturity

GARCH(1,1)			Component GARCH			Persistent Component		
Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
w	7.521E-16	3.498E-09	$\tilde{\beta}$	9.241E-01	3.780E-01	$\tilde{\beta}$	9.763E-01	3.435E-01
b	4.694E-01	1.251E-01	α	1.849E-06	1.103E-09	α	1.678E-06	3.430E-08
a	1.936E-06	3.986E-07	γ_1	5.827E+02	1.505E+02	γ_1	2.552E+02	1.123E+02
c	5.078E+02	1.041E+02	γ_2	5.714E+02	2.110E+02	γ_2	1.924E+02	3.425E+01
			ω	2.043E-07	1.301E-07	ω	1.246E-07	2.054E-08
			φ	2.420E-07	1.035E-08	φ	7.191E-07	2.453E-08
			ρ	9.958E-01	1.039E-03	ρ	1.000E+00	
Total Persist	0.9687		Total Persist	0.9997		Total Persist	1.0000	
Annual Vol	0.1250		Annual Vol	0.1113				
Var of Var	1.572E-05		Var of Var	1.731E-05				
Leverage	-1.967E-03		Leverage	-2.432E-03		Leverage	-1.133E-03	
RMSE (in)	1.107		RMSE (in)	0.855		RMSE (in)	0.994	
Normalized	1.000		Normalized	0.773		Normalized	0.898	
RMSE (out)	1.227		RMSE (out)	0.972		RMSE (out)	1.076	
Normalized	1.000		Normalized	0.792		Normalized	0.877	

Notes to Table: See notes to Table 3. RMSE(in) now refers to 1992-1994 and RMSE(out) to 1995.

Table 6: Model Estimates and Properties

Sample: 1992-1994 (in-sample) 1995 (out-of-sample)

80 to 365 Days to Maturity

GARCH(1,1)		
Parameter	Estimate	Std. Error
w	3.238E-07	1.099E-08
b	1.341E-01	9.100E-02
a	1.894E-06	5.232E-07
c	6.624E+02	8.010E+01
Total Persist	0.9650	
Annual Vol	0.1264	
Var of Var	2.016E-05	
Leverage	-2.509E-03	
RMSE (in)	1.190	
Normalized	1.000	
RMSE (out)	1.743	
Normalized	1.000	

Component GARCH		
Parameter	Estimate	Std. Error
$\tilde{\beta}$	9.448E-01	2.935E-01
α	1.125E-06	2.542E-09
γ_1	7.337E+02	2.400E+02
γ_2	5.318E+02	1.769E+02
ω	2.260E-07	1.390E-07
φ	2.864E-07	1.109E-08
ρ	9.957E-01	1.009E-02
Total Persist	0.9998	
Annual Vol	0.1147	
Var of Var	1.427E-05	
Leverage	-1.956E-03	
RMSE (in)	0.890	
Normalized	0.748	
RMSE (out)	1.052	
Normalized	0.604	

Persistent Component		
Parameter	Estimate	Std. Error
$\tilde{\beta}$	9.756E-01	2.104E-01
α	4.569E-07	9.353E-10
γ_1	9.492E+02	3.091E+02
γ_2	7.317E+02	6.055E+01
ω	2.127E-07	8.785E-08
φ	3.319E-07	4.936E-10
ρ	1.000E+00	
Total Persist	1.0000	
Leverage	-1.353E-03	
RMSE (in)	0.926	
Normalized	0.778	
RMSE (out)	1.201	
Normalized	0.689	

Notes to Table: See notes to Tables 3. RMSE(in) refers to 1992-1994 and RMSE(out) to 1995. Only options with at least 80 days to maturity are used here.

Table 7: 1990-1992 (in-sample) MSE and Ratio MSE by moneyness and maturity
Contracts with 7-365 days to maturity

Panel A. GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.191	0.791	1.205	1.628	1.162
0.975<S/X<1.00	0.441	1.112	1.260	1.297	1.110
1.00<S/X<1.025	0.331	0.914	1.100	0.987	0.913
1.025<S/X<1.05	0.309	0.822	1.060	0.901	0.844
1.05<S/X<1.075	0.472	0.979	1.360	1.236	1.064
1.075<S/X	0.412	1.156	1.511	1.044	1.164
All	0.372	0.953	1.262	1.324	1.079

Panel B. Ratio of Component GARCH to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.882	0.666	0.718	0.855	0.762
0.975<S/X<1.00	0.802	0.761	0.852	1.101	0.860
1.00<S/X<1.025	0.851	0.839	0.912	0.999	0.896
1.025<S/X<1.05	0.760	0.777	0.914	1.130	0.876
1.05<S/X<1.075	0.813	0.721	0.821	1.005	0.813
1.075<S/X	0.942	0.769	0.718	0.900	0.780
All	0.851	0.749	0.780	0.922	0.808

Panel C. Ratio of Persistent Component to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.720	0.702	0.819	1.263	0.972
0.975<S/X<1.00	0.707	0.749	0.777	1.206	0.848
1.00<S/X<1.025	0.824	0.805	0.807	1.205	0.886
1.025<S/X<1.05	0.789	0.805	0.794	1.112	0.843
1.05<S/X<1.075	0.840	0.807	0.775	1.090	0.849
1.075<S/X	0.950	0.834	0.785	0.894	0.830
All	0.830	0.780	0.799	1.173	0.891

Notes to Table: We use the NLS estimates from Table 3 to compute the mean squared option valuation error (MSE) for various moneyness and maturity bins during 1990-1992. Panel A shows the MSEs for the GARCH(1,1) model. Panel B shows the ratio of the component GARCH MSEs to the GARCH(1,1) MSEs from Panel A. Panel C shows the ratio of the persistence component GARCH MSEs to the GARCH(1,1) MSEs.

Table 8: 1993 (out-of-sample) MSE and Ratio MSE by moneyness and maturity
Contracts with 7-365 days to maturity

Panel A. GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.083	1.339	1.764	3.780	2.133
0.975<S/X<1.00	0.335	2.282	3.241	5.924	2.659
1.00<S/X<1.025	0.248	1.316	2.132	5.243	1.839
1.025<S/X<1.05	0.351	0.524	1.309	4.056	1.015
1.05<S/X<1.075	0.422	0.427	0.696	2.496	0.740
1.075<S/X	1.316	1.345	0.983	1.965	1.359
All	0.661	1.263	1.582	3.318	1.648

Panel B. Ratio of Component GARCH to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.513	0.371	0.340	0.259	0.309
0.975<S/X<1.00	0.638	0.453	0.434	0.284	0.415
1.00<S/X<1.025	1.181	0.712	0.569	0.347	0.560
1.025<S/X<1.05	0.933	1.158	0.758	0.475	0.780
1.05<S/X<1.075	1.256	1.076	1.080	0.386	0.813
1.075<S/X	0.981	0.943	1.091	0.862	0.954
All	0.989	0.685	0.606	0.425	0.586

Panel C. Ratio of Persistent Component to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.440	0.306	0.714	1.664	1.065
0.975<S/X<1.00	0.629	0.297	0.495	0.454	0.394
1.00<S/X<1.025	1.207	0.557	0.770	0.710	0.680
1.025<S/X<1.05	1.007	1.499	1.124	0.413	0.985
1.05<S/X<1.075	1.390	1.511	2.135	0.920	1.417
1.075<S/X	0.984	1.094	1.889	1.119	1.259
All	1.010	0.690	0.990	1.139	0.934

Notes to Table: See Table 7. We use the NLS estimates from Table 3 to compute the out-of-sample mean squared option valuation error (MSE) for various moneyness and maturity bins during 1993.

Table 9: 1992-1994 (in-sample) MSE and Ratio MSE by moneyness and maturity
Contracts with 7-365 days to maturity

Panel A. GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.233	0.862	1.199	1.861	1.259
0.975<S/X<1.00	0.977	1.646	1.673	1.953	1.625
1.00<S/X<1.025	0.818	1.469	1.401	2.248	1.507
1.025<S/X<1.05	0.347	0.908	0.997	1.818	1.004
1.05<S/X<1.075	0.617	0.646	0.785	2.398	0.982
1.075<S/X	0.850	0.750	0.735	2.094	1.065
All	0.734	1.018	1.092	2.023	1.225

Panel B. Ratio of Component GARCH to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.661	0.410	0.371	0.332	0.365
0.975<S/X<1.00	0.597	0.592	0.530	0.479	0.558
1.00<S/X<1.025	0.750	0.679	0.529	0.395	0.578
1.025<S/X<1.05	0.922	0.669	0.511	0.619	0.624
1.05<S/X<1.075	0.845	0.706	0.513	0.564	0.616
1.075<S/X	0.992	0.906	0.738	0.830	0.844
All	0.826	0.654	0.514	0.553	0.591

Panel C. Ratio of Persistent Component to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.652	0.770	1.086	1.335	1.114
0.975<S/X<1.00	0.488	0.746	1.083	1.290	0.905
1.00<S/X<1.025	0.582	0.658	0.984	1.213	0.867
1.025<S/X<1.05	0.917	0.698	0.988	0.993	0.858
1.05<S/X<1.075	0.971	0.795	1.012	0.727	0.822
1.075<S/X	0.989	1.162	1.180	0.958	1.061
All	0.780	0.808	1.072	1.110	0.978

Notes to Table: See Table 7. We use the NLS estimates from Table 5 to compute the mean squared option valuation error (MSE) for various moneyness and maturity bins during 1992-1994.

Table 10: 1995 (out-of-sample) MSE and Ratio MSE by moneyness and maturity
Contracts with 7-365 days to maturity

Panel A. GARCH(1,1)

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.150	0.745	2.120	6.033	3.135
0.975<S/X<1.00	0.991	1.381	2.954	4.381	2.391
1.00<S/X<1.025	0.566	1.134	2.292	3.505	1.930
1.025<S/X<1.05	0.289	0.827	1.599	2.103	1.233
1.05<S/X<1.075	0.816	0.380	0.751	1.963	0.803
1.075<S/X	0.415	0.380	0.326	0.929	0.463
All	0.553	0.716	1.408	3.415	1.507

Panel B. Ratio of Component GARCH to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	1.700	1.404	0.947	0.378	0.572
0.975<S/X<1.00	0.988	0.815	0.571	0.481	0.613
1.00<S/X<1.025	0.703	0.562	0.543	0.424	0.501
1.025<S/X<1.05	1.074	0.559	0.432	0.465	0.497
1.05<S/X<1.075	0.959	0.691	0.588	0.550	0.634
1.075<S/X	1.016	1.160	1.190	0.870	1.052
All	0.957	0.863	0.717	0.449	0.613

Panel C. Ratio of Persistent Component to GARCH(1,1) MSE

	DTM<20	20<DTM<80	80<DTM<180	DTM>180	All
S/X<0.975	0.749	0.987	0.647	0.372	0.475
0.975<S/X<1.00	0.824	0.780	0.564	0.660	0.670
1.00<S/X<1.025	0.748	0.678	0.675	0.605	0.648
1.025<S/X<1.05	1.216	0.899	0.671	0.712	0.767
1.05<S/X<1.075	1.059	1.492	1.364	0.641	1.041
1.075<S/X	0.927	1.753	2.457	1.099	1.589
All	0.922	1.034	0.822	0.536	0.718

Notes to Table: See Table 7. We use the NLS estimates from Table 5 to compute the out-of-sample mean squared option valuation error (MSE) for various moneyness and maturity bins during 1995.