Analyzing the Interest Rate Risk of Banks Using Time Series of Accounting-Based Data: Evidence From Germany*

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Abstract

Recently, the Basel Committee on Banking Supervision (2004b) has stressed the significance of interest rate risk for the stability of the banking system and suggested ‘Principles for the Management and Supervision of Interest Rate Risk’ in the banking book that go far beyond current practice. However, there is little data available concerning the interest rate risk of banks. In this paper we develop a new method relying on accounting-based data to quantify the interest risk of banks and apply it to analyze German banks. In detail, the contribution to the literature is threefold: first, common models proposed in the literature use accounting-based data at a single point in time, whereas our approach considers time series of accounting-based data. Second, using unique bank internal data exclusively available to the Deutsche Bundesbank allows us to evaluate the model results on a subsample of German banks. Third, we examine for the first time the interest rate risk of the whole German universal banking system on an individual bank level. We find evidence that i) our model yields a significantly better fit of banks’ internally quantified interest rate risk than a standard approach that relies on one-point-in-time data, and that the interest rate risk differs between banks of different ii) size and iii) banking group. Additionally, we find structural differences between trading book and non-trading book institutions.

Keywords: German financial institutions; interest rate risk; accounting-based approach; maturity transformation; banking supervision; model evaluation

JEL classification: G18, G21
1 Introduction

While over the past few years both banking supervisors and researchers have nearly exclusively focused their attention on banks’ credit and operational risk, the spotlight is now being turned again on interest rate risk. One reason for this is its threat to the stability of the financial system as a kind of systematic risk. In 2004, the Basel Committee on Banking Supervision (2004b) suggested ‘Principles for the Management and Supervision of Interest Rate Risk’ that go far beyond current practice. Within these principles the Committee states that “this [interest rate] risk is a normal part of banking and can be an important source of profitability” but stresses that it is “essential to the safety and soundness of banks” that interest rate risk is maintained within prudent levels. A historical example of a banking crisis where interest rate risk played an integral role is the ‘Savings and Loan Crisis’ which occurred in the US during the 1980s. Between 1980 and 1988, 563 of the approximately 4,000 then existing savings and loan institutions failed, while further failures were prevented by 333 supervisory mergers. The total costs of the crisis are estimated at USD 160 bn.\(^1\)

The German banking system has some features that make an investigation of interest rate risk in this market particularly interesting. First, German banks still typically act as qualitative asset transformers which makes the German banking system quite particular in comparison to other banking systems like in the US or UK (Schmidt et al., 1999; Allen and Santomero, 2001). Interest rate risk hence still arises from the basic banking business. So far, there is no reliable analysis of the interest rate risk of most German banks and its determinants, but there are indications that the level of interest rate risk might be comparatively high: in 2006 the Deutsche Bundesbank conducted a stress test among a sample of 25 banks. For 10 medium-sized and smaller banks a 150 basis point interest rate shock caused an average loss of 15% of regulatory capital (Deutsche Bundesbank, 2006a). Via scaling, this corresponds to the Basel Committee’s definition of ‘outlier banks’;\(^2\) Second, the German universal banking system has a special structure: in ad-

\(^1\)See Federal Deposit Insurance Corporation (1997) for a detailed analysis.

\(^2\)The Basel Committee on Banking Supervision (2004a,b) recommends the supervisors to be particularly attentive to those banks (= outlier banks) whose regulatory capital declines by more than 20% when a 200 basis point interest rate shock occurs.
dition to private commercial banks (‘Kreditbanken’), there are the state-owned savings banks (‘Sparkassen’) and the member-owned cooperative banks (‘Genossenschaftsbanken’). The banking groups differ in their business model (Schmidt and Tyrell, 2004) and the relevance of their net interest income (Deutsche Bundesbank, 2006b). Third, there is a large number of German universal banks, approximately 2,000 in 2005. This fact facilitates empirical statements.

Because there is still no standardized access to banks’ internally quantified interest rate risk, most models proposed in the literature and applied by banking supervisors rely on accounting-based data. These include Bennett et al. (1986), Planta (1989), Patnaik and Shah (2004), and the Federal Reserve’s Economic Value Model (EVM) presented by Houpt and Embersit (1991) and analyzed by Wright and Houpt (1996), Sierra (2004), and Sierra and Yeager (2004), as well as the ‘standardized framework’ suggested by the Basel Committee on Banking Supervision (2004b). The Net Portfolio Value Model applied by the Office of Thrift Supervision (OTS) is similar to these models but requires far more detailed information on the assets and liabilities of banks that is exclusively available to the OTS (Office of Thrift Supervision, 2000).

Accounting-based data typically shows the amount of positions within certain time bands: the total outstanding amount of a given position is distributed among a number of time bands according to remaining time to maturity (e.g. in the US) and/or initial maturity (e.g. in Germany). Given this information, the cash flow structure of a bank’s on-balance positions and its interest rate sensitivity can be estimated, but the results depend on the assumed distribution of maturities within these time bands. For example, the Federal Reserve’s EVM assumes a concentration in the middle of the time bands to estimate the interest rate risk of US commercial banks, which is also suggested by the Basel Committee on Banking Supervision (2004b). Bennett et al. (1986) assume a uniform distribution.

In this context, our contribution to the literature is threefold: first, we present a new method to derive a bank’s interest rate risk using time series of accounting-based data. This allows us to estimate the distribution of maturities within the time bands. By doing so, we get greater

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3 There is an extensive literature on the interest rate sensitivity of stock returns of exchange-traded banks. See Staikouras (2003, 2006) for recent surveys. However, as most German banks are not listed, this approach for analyzing the interest rate risk is not applicable for the vast majority of German banks.
precision with our estimates of the interest rate risk. The framework is a generalization of the aforementioned models such as the EVM and the proposal of the Basel Committee on Banking Supervision (2004b) and is flexible enough to capture different actual reporting practices, for example in the US and Germany, as well as the ‘reporting framework’ suggested by the Basel Committee on Banking Supervision (2004b). Additionally, it allows the integration of several data sources that are available to regulators and/or external analysts.


Third, we examine for the first time the interest rate risk of the German universal banking system on an individual bank level and analyze its determinants with respect to banks’ attributes. So far, the interest rate risk has only been analyzed for small samples of German banks using market values of equity (e.g. Oertmann et al., 2000) or bank internal data (e.g. Deutsche Bundesbank, 2006a).

The empirical evidence in this paper can be summarized as follows: first, our model is able to explain the cross-sectional variation of the interest rate risk of banks more accurately than a standard approach that relies on one-point-in-time data. Second, bigger banks have a higher level of interest rate risk than smaller banks and third, the interest rate risk of German banks differs between the banking groups. Savings banks and cooperative banks have a higher level of interest rate risk than private commercial banks. Additionally, there are structural differences in the interest rate risk of trading book and non-trading book institutions.

The remainder of the paper is organized as follows. Section 2 outlines our hypotheses. The method is presented in Section 3. Section 4 contains the description of the data sources, the model evaluation and the analysis of the interest rate risk of German banks. Section 5 concludes.
2 Hypotheses

The aforementioned models that aim to quantify the interest rate risk of banks using accounting-based information rely on one-point-in-time data. To derive the cash flow structure of a bank and its interest rate risk, these models use information on time bands of the maturities of the bank’s assets and liabilities and assume a certain distribution of maturities within the time bands. Wright and Houpt (1996) argue that the Federal Reserve’s EVM, acting as a representative of these simple models, yields similar results to the more complex Net Portfolio Value Model of the OTS. We in turn expect that the integration of time series information and different data sources – such as the breakdown by initial maturity – improves the ability to explain the cross-sectional variation of banks’ interest rate risk significantly even though our model relies on data that basically has the same simple structure as the input data of the EVM.

Hypothesis 1 (H1) Our model is able to explain the cross-sectional variation of banks’ interest rate risk better than a standard approach that relies on one-point-in-time data.

Maturity transformation is often seen as a specific function of banks (e.g. Niehans, 1978): customers tend to borrow long-term capital and to lend short-term capital. Additionally, banks may have an incentive to lend out long-term and refinance short-term since the slope of the term structure is usually positive (e.g. Bhattacharya and Thakor, 1993): on average, long-term interest rates (for assets) exceed the short-term interest rates (for liabilities). Thus, on average, the bank achieves a premium from maturity transformation. On the other hand, the resulting maturity mismatch between the assets and liabilities causes interest rate risk. While there is the incentive to bear interest rate risk due to the expected yield, there are several economic reasons why firms should manage and limit their risks (e.g. Allen and Santomero, 1998). These include managerial self-interest (Stulz, 1984), non-linearity of taxes (Smith and Stulz, 1985), costs of financial distress (Warner, 1977) and capital market imperfections (Froot et al., 1993; Froot and Stein, 1998). We expect that smaller banks have a higher incentive to keep their exposure to interest rate risk low than bigger banks for several reasons: first, bigger banks may be assumed to be “too big to fail” by their investors and other stakeholders and hence face lower financial distress costs (e.g Saunders et al., 1990). Second, bigger banks are more diversified than smaller banks
and hence have a lower level of idiosyncratic risk. To obtain a certain level of total risk, these banks may bear more systematic (interest rate) risk (e.g. Demsetz and Strahan, 1997). Third, the information risk may be lower for investors of bigger banks and hence may be substituted by interest rate risk (e.g. Banz, 1981). Fourth, bigger banks may have more opportunities to trade their risk on the capital market and hence to alter their exposure to interest rate risk via off-balance activities quickly, once a stress situation occurs. Hence, we hypothesize for the German banking system:

**Hypothesis 2 (H2)** Bigger banks have a higher level of interest rate risk than smaller banks.

We expect that the costs of bearing (interest rate) risk does not only depend on a bank’s size but also on its banking group. While German savings banks and cooperative banks rather merge within their respective banking group than default (Koetter et al., 2005; Porath, 2006), German private commercial banks are faced with possible financial distress costs more directly. Hence, we expect that the latter have higher incentives to keep their exposure to interest rate risk low, and we hypothesize for the German banking system:

**Hypothesis 3 (H3)** The interest rate risk differs between banking groups. Savings banks and cooperative banks have a higher level of interest rate risk than private commercial banks.

3 Model

3.1 Definition of Interest Rate Risk

In order to apply a comparable and widely accepted measure for the interest rate risk of banks, we follow the ‘standardized interest rate shock’ approach also proposed within the new Basel Capital Accord (Basel II) and the ‘Principles for the Management and Supervision of Interest Rate Risk’ that are published by the Basel Committee on Banking Supervision (2004a,b):

**Definition 1 (Interest Rate Risk)** The interest rate risk (IRR) of a bank is given by the maximum absolute change of its economic value caused by an upward and downward 200 basis point parallel interest rate shock in relation to its regulatory capital.
Approximating the interest rate sensitivity by the duration, the interest rate risk of a bank in \( t_{\text{ref}} \) (‘reference date’) is measured by:

\[
IRR(t_{\text{ref}}) = 0.02 \left| \frac{\sum \limits_{t_{\text{CF}} > t_{\text{ref}}} (t_{\text{CF}} - t_{\text{ref}}) \sum \limits_{\text{pos} \in \text{POS}^A} CF(\text{pos}, t_{\text{ref}}, t_{\text{CF}}) - \sum \limits_{\text{pos} \in \text{POS}^L} CF(\text{pos}, t_{\text{ref}}, t_{\text{CF}})}{(1 + R_{\text{ac}}(t_{\text{ref}}, t_{\text{CF}}))(t_{\text{CF}} - t_{\text{ref}} + 1)} \right| \frac{RC(t_{\text{ref}})}{RC(t_{\text{ref}})}. \tag{1}
\]

\( RC(t_{\text{ref}}) \) denotes the regulatory capital in \( t_{\text{ref}} \). \( \text{POS}^A \) is the set of all interest rate-sensitive asset positions, \( \text{POS}^L \) the set of all interest rate-sensitive liability positions and \( \{t_{\text{CF}}\} \) the set of all points in time when cash flows are due. \( CF(\text{pos}, t_{\text{ref}}, t_{\text{CF}}) \) denotes the cash flow of position \( \text{pos} \) in \( t_{\text{CF}} > t_{\text{ref}} \) from the perspective of \( t_{\text{ref}} \).\(^5\) The set of all \( CF(\text{pos}, t_{\text{ref}}, t_{\text{CF}}) \) will be referred to as ‘cash flow structure’. Finally, \( R_{\text{ac}}(t_{\text{ref}}, t_{\text{CF}}) \) represents the annually compounded spot rate in \( t_{\text{ref}} \) for the date \( t_{\text{CF}} \). In line with the earlier accounting-based models (See Section 1) we only capture here the interest rate risk of the net portfolio value, excluding other components such as the exposure of the going concern value (See Samuelson, 1945).

The key to the analysis is determining the detailed cash flow structure in (1) that is usually unknown to regulators, external analysts and a bank’s stake holders. In the following, we present a new method to derive this cash flow structure using accounting-based data. Since the distinctive feature of our model is the ability to integrate time series information, we refer to the model as ‘Time Series Accounting-Based Model’ (TAM). To highlight the underlying idea, we start with a simplified example before we present the general framework.

### 3.2 Introductory Example

The bank is assumed to contract only one type of interest rate-sensitive, default-free business in each year with initial maturities of 1, 2, 3, 4, and/or 5 years. To keep the example as simple as possible, we omit here payments other than those due to the face value at maturity. Thus, the

\(^4\)It is well known that the duration of a defaultable cash flow usually differs from the duration of a corresponding default-free cash flow. Jacoby and Roberts (2003) provide a recent literature overview. In line with the approaches proposed in the literature (See Section 1) we assume default-free cash flows and apply the effective duration (See Ho, 1990).

\(^5\)The cash flow due in \( t_{\text{CF}} = t_{\text{ref}} \) does not influence the bank’s interest rate risk and is hence omitted.
This figure shows the bank structure in $t = 0$. The rows refer to the time of contraction ($t_{beg}$), whereas the columns refer to the time of maturity ($t_{end}$). $X_{t_{beg}, t_{end}}$ denotes the face value of business that is contracted in $t_{beg}$ and that matures in $t_{end}$.

Figure 1: Bank structure in $t = 0$.

cash flow structure is completely determined by the face values of contracted business and the corresponding maturity dates. For the reference date $t_{ref} = 0$, the bank structure\(^6\) is shown in Figure 1, where

$$X_{t_{beg}, t_{end}} \geq 0 \quad \forall \ t_{beg}, t_{end}$$ \hspace{1cm} (2)

denotes the face value of business that has been contracted in $t_{beg}$ (‘time of contraction’) and that matures in $t_{end}$ (‘time of maturity’). The cash flow in $t_{CF} > t_{ref}$ is the sum of all business with maturity date $t_{CF}$ that has been contracted until $t_{ref}$:

$$CF(t_{ref}, t_{CF}) = \sum_{t \leq t_{ref}} X^{t, t_{CF}}.$$

(3)

If each $X_{t_{beg}, t_{end}}$ in (3) or each cash flow $CF(t_{ref}, t_{CF})$ were known to the regulators or external analysts, they could easily calculate the interest rate risk in $t_{ref}$ via (1). However, banks do not usually report the $X_{t_{beg}, t_{end}}$ nor the detailed cash flows. But they regularly report the outstanding amount of the business within certain time bands according to the remaining time to maturity or/and the initial maturity.

\(^6\)‘Bank structure’ refers to the composition of the portfolio that is held by the bank. The formal definition is given in Section 3.3.
Assume that the bank’s annual report in $t$ gives the outstanding amount of the business with a remaining time to maturity of up to 1 year, denoted by $RTM_{0,1}^t$, and of more than 1 year and up to 3 years, denoted by $RTM_{1,3}^t$. If regulators only take $RTM_{0,1}^0$ and $RTM_{0,1}^{1,3}$ into account, they know in $t = 0$ the cash flow that is due in one year but cannot distinguish between the cash flows in 2 and 3 years, because only its sum $RTM_{1,3}^0$ is reported. However, they can gain further information if they incorporate past reports: since the remaining time to maturity of a certain business decreases from date to date, the business migrates between the time bands over time. This effect can be used to restrict the estimates of future cash flows.

For illustration, we restrict our example to two reporting dates: $t = 0$ and $t = -1$. As $RTM_{0,1}^0$ denotes the business in $t = 0$ that has a remaining time to maturity of 1 year it consists of business that had a remaining time to maturity of 2 years in the previous period $t = -1$ and new business that is contracted in $t = 0$ and that matures in 1 year. Hence, we know that $CF(-1, 1) \leq RTM_{0,1}^0$. This relation also provides information on $CF(-1, 2)$, as the sum of both cash flows equals the report item $RTM_{-1}^{1,3} = CF(-1, 1) + CF(-1, 2)$. Some minor rearrangements yield $CF(-1, 2) \geq RTM_{-1}^{1,3} - RTM_{0,1}^0$. Additionally, $CF(-1, 2)$ cannot exceed $CF(0, 2)$, since the latter consists of the former plus the business contracted in $t = 0$ that matures in $t = 2$. Hence, we can conclude that

$$CF(0, 2) \geq RTM_{-1}^{1,3} - RTM_{0,1}^0.$$ (4)

This relationship restricts possible values of $CF(0, 3)$, too, as the sum of both equals the amount reported in the time band $RTM_{0,1}^{1,3}$:

$$CF(0, 3) \leq RTM_{0,1}^{1,3} - (RTM_{-1}^{1,3} - RTM_{0,1}^0).$$ (5)

This simple example shows that former reports can indeed add information on today’s cash flow structure by restricting the possible estimates and, hence, can improve the estimation of a bank’s interest rate risk. When we model a realistic bank with a lot of different positions, time bands and reporting dates, the system of equations becomes rather complex and unhandy. However, the representation can be conceptually simplified by using the above-defined variables $X_{t_{beg}, t_{end}}$ to express each single report item as a function of these variables.
This figure shows the bank structure in $t = 0$ (top) and $t = -1$ (bottom), respectively. The left-hand side shows the business items with a remaining time to maturity of up to 1 year (light shading) and those of more than 1 year and up to 3 years (dark shading). The right-hand side shows the items with an initial maturity within these ranges, respectively. The rows refer to the time of contraction ($t_{\text{beg}}$), whereas the columns refer to the time of maturity ($t_{\text{end}}$). $X^{t_{\text{beg}}-t_{\text{end}}}$ denotes the amount of business that is contracted in $t_{\text{beg}}$ and that matures in $t_{\text{end}}$.

Figure 2: Bank structure and bank reports in $t = 0$ and $t = -1$. 

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For instance, Figure 2 shows the bank structure in $t = 0$ (top) and $t = -1$ (bottom). The outstanding amounts with a remaining time to maturity of up to 1 year and those of more than 1 year and up to 3 years, respectively, are marked on the left-hand side. Since the reported amounts equal the sum of the business within the respective time bands, the following equations must hold:

\[
X^{-4,1} + X^{-3,1} + X^{-2,1} + X^{-1,1} + X^{0,1} = RTM_0^{0,1},
\]

\[
X^{-5,0} + X^{-4,0} + X^{-3,0} + X^{-2,0} + X^{-1,0} = RTM_{-1}^{0,1},
\]

\[
X^{-3,2} + X^{-2,2} + X^{-1,2} + X^{0,2} + X^{-2,3} + X^{-1,3} + X^{0,3} = RTM_0^{1,3},
\]

\[
X^{-4,1} + X^{-3,1} + X^{-2,1} + X^{-1,1} + X^{-3,2} + X^{-2,2} + X^{-1,2} = RTM_{-1}^{1,3}.
\]

This formulation additionally allows us to easily integrate further sources of relevant information. In Germany for instance, also time bands according to the initial maturity are available to the regulators. Assume that the bank reports in $t$ the amount of business with an initial maturity of up to 1 year and of more than 1 year and up to 3 years, denoted as $ITM_t^{0,1}$ and $ITM_t^{1,3}$, respectively. The corresponding variables $X_t^{beg,end}$ are marked on the right-hand side of Figure 2 and we have:

\[
X^{0,1} = ITM_0^{0,1},
\]

\[
X^{-1,0} = ITM_{-1}^{0,1},
\]

\[
X^{-2,1} + X^{-1,1} + X^{-1,2} + X^{0,2} + X^{0,3} = ITM_0^{1,3},
\]

\[
X^{-3,0} + X^{-2,0} + X^{-2,1} + X^{-1,1} + X^{-1,2} = ITM_{-1}^{1,3}.
\]

The approaches proposed in the literature only use the information of (6) and (8) and omit the remaining information. Equations (6) to (13) and the non-negativity restriction (2) form a system of linear equations that restricts the possible bank structure, and hence via (3) and (1) the bank’s interest rate risk.

However, in general the system of equations is not uniquely determined. Instead, there exists a space of solutions of bank structures that are all consistent with the reported data.\footnote{Note that an ambiguous bank structure does not necessarily imply an ambiguous level of interest rate risk.}

\[\text{10}\]
To identify an economically sensible solution, further assumptions are necessary. For instance, one could assume that the bank has a constant level of interest rate risk over time or that it always contracts business with the same maturities. For illustration, we here assume that the cash flow structure of the bank is as constant as possible over the last two periods $t = 0$ and $t = -1$. This can be expressed by optimizing a function $F$ on the space of solutions: we select that solution that minimizes the sum of the quadratic deviations between $t = 0$ and $t = -1$ of the cash flows due in 1, 2, ..., 5 years in relation to the sum of all cash flows from the perspective of the respective period. Formalizing this procedure, we obtain:

$$\text{minimize } F(\mathbf{X}) = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2$$

subject to

(1) Goals

$$d_t = \frac{CF(0, t)}{\sum_{t_{CF}} CF(0, t_{CF})} - \frac{CF(-1, t - 1)}{\sum_{t_{CF}} CF(-1, t_{CF})} \quad \forall t \in \{1, ..., 5\}$$

(2) Restrictions

(6), (7), (8), (9), (10), (11), (12), (13), (2),

where $\mathbf{X}$ denotes the set of all variables $X_{t_{beg}t_{end}}$. We go on formalizing the ideas presented above in a general framework.

### 3.3 General Framework

Analogous to the previous example, we start by describing the bank structure. We choose a discrete setting that allows us to capture the bank structure by a finite number of variables. Due to the discrete granularity of the available data, no information is omitted.

**Model Component 1 (Bank Structure)** *The bank contracts interest rate-sensitive default-free business in certain time points with a fixed time to maturity and a fixed, periodically paid coupon. Once contracted, the business is on-balance with a constant amount until its maturity. The cash flow implied by each business consists of the repayment of the face value and coupon payments.*
The assumption of a fixed, periodically paid coupon as well as the omission of amortization payments and default of assets can be relaxed easily. However, calculations on a subsample of banks have shown that these assumptions do not affect our later results but yield a reduction of calculation burden. A possible bias implied by these or the remaining assumptions is compensated as shown in Section 4. We go on formalizing model component 1.

The set of points in time when business is contracted and/or cash flows are due is given by \( T = \{ t_i \}_{i \in \mathbb{Z}} \) where we set \( t_0 = 0 \) for simplicity. In each \( t_{beg} \in T \) the bank contracts certain amounts of business of a position \( pos \in POS \) (such as loans on the asset side or interbank liabilities) that mature in \( t_{end} > t_{beg} \).\(^8\) The amount is denoted \( X^{pos,t_{beg},t_{end}} \) and assumed to be non-negative:

\[
X^{pos,t_{beg},t_{end}} \geq 0 \quad \forall \ pos, t_{beg}, t_{end}. \tag{14}
\]

We will refer to these variables as ‘business items’. The set of all business items is referred to as ‘bank structure’. The corresponding coupons are denoted \( c^{pos,t_{beg},t_{end}} \).

Under these assumptions, the cash flow in \( t_{CF} > t_{ref} \) related to position \( pos \) is given by:

\[
CF(pos,t_{ref},t_{CF}) = \sum_{t_i \leq t_{ref}} X^{pos,t_i,t_{CF}} + \sum_{t_i < t_{ref}, t_j \geq t_{CF}} X^{pos,t_i,t_j} c^{pos,t_i,t_j}. \tag{15}
\]

The first part of the right-hand side of (15) denotes the payments due to the repayment of the face values (‘maturity structure’) of all business items that are on-balance in \( t_{ref} \) and that mature \( t_{CF} \). The second part represents the corresponding coupon payments.

**Model Component 2 (Reports on the Bank Structure)** *At certain points in time the bank reveals information on its structure by reporting time bands of the outstanding amount for each position broken down by remaining time to maturity and/or initial maturity, respectively.*

This assumption captures current reporting practice and allows us to derive the bank’s maturity structure, which is the core of the model. In each reporting date \( t_{obs} \in T_{obs} \subseteq T \) the bank reports the sum of all business (classified by position) within specified ranges of remaining time to maturity and/or initial maturity, respectively. We assume that for each \( pos \in POS \) in each

\(^8\)Daily maturing business is not included, since – applied for the German banking system – the model cannot distinguish between daily and monthly maturing business due to the monthly granularity of the reports.
\( t \in T_{\text{obs}}^{rtm} \subseteq T_{\text{obs}} \) there are \( |N^{pos}| \) report items characterized by the business’ remaining time to maturity: \( RTM_{t}^{pos,n} \) with \( n \in N^{pos} \) denotes the amount of \( pos \) in \( t \) with a remaining time to maturity \( t_{\text{end}} - t \) within the time band \( (h^{pos,n}_{\text{lower}}, h^{pos,n}_{\text{upper}}) \). Analogously, in each \( t \in T_{\text{obs}}^{itm} \subseteq T_{\text{obs}} \) there are \( |M^{pos}| \) items characterized by the business’ initial maturity: \( ITM_{t}^{pos,m} \) with \( m \in M^{pos} \) denotes the amount of \( pos \) in \( t \) with an initial maturity \( t_{\text{end}} - t_{\text{beg}} \) within the time band \( (h^{pos,m}_{\text{lower}}, h^{pos,m}_{\text{upper}}) \).

Since the amount within a certain time band equals the sum of the relevant business items, the former can be expressed as a linear function of the latter. Thus, we obtain one function (restriction) for each reported time band and each reporting date, leading to a system of linear equations:

\[
\sum_{h^{pos,n}_{\text{lower}} < t_{j} - t \leq h^{pos,n}_{\text{upper}}} X_{pos,t_{i},t_{j}} = RTM_{t}^{pos,n} \quad \forall t \in T_{\text{obs}}^{rtm}, n \in N^{pos}, \quad (16)
\]

\[
\sum_{h^{pos,m}_{\text{lower}} < t_{j} - t \leq h^{pos,m}_{\text{upper}}} X_{pos,t_{i},t_{j}} = ITM_{t}^{pos,m} \quad \forall t \in T_{\text{obs}}^{itm}, m \in M^{pos}, \quad (17)
\]

where (16) captures the restrictions due to reports on the remaining time to maturity and (17) those on the initial maturity.

**Model Component 3 (Objective Function)** An economically sensible solution is obtained by the optimization of a function on the space of solutions.

The system of equations given by (16), (17) and (14) restricts the values of the business items \( X_{pos,t_{\text{beg}},t_{\text{end}}} \) and hence via (15) and (1) the bank’s interest rate risk. However, it is clear that the system is in general not uniquely determined, i.e. it is not possible to infer the bank structure and the bank’s interest rate risk unambiguously using accounting-based data. Thus, analogously to the example, further assumptions on the bank structure are necessary in order to identify an economically sensible solution. These can again be formulated in terms of the optimization of a function \( F \) (‘objective function’\(^{10}\)) on the space of solutions, and the optimization problem can

\(^{9}\) According to German reporting practice, we model left open intervals.

\(^{10}\) Note that we present a description model here, thus ‘objective function’ does not refer to the bank’s objective.
Table 1: German universal banks

<table>
<thead>
<tr>
<th>Sample</th>
<th>All Banks</th>
<th>Complete Sample</th>
<th>Trading Book</th>
<th>Merged</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Number</td>
<td>Number</td>
<td>Number</td>
</tr>
<tr>
<td></td>
<td>Fraction</td>
<td>Fraction</td>
<td>Fraction</td>
<td>Fraction</td>
</tr>
<tr>
<td>Total Number</td>
<td>1,934</td>
<td>1,785</td>
<td>179</td>
<td>1,167</td>
</tr>
<tr>
<td>Commercial Banks</td>
<td>175</td>
<td>159</td>
<td>49</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>9.05%</td>
<td>8.91%</td>
<td>27.37%</td>
<td>9.17%</td>
</tr>
<tr>
<td>Savings Banks</td>
<td>463</td>
<td>424</td>
<td>98</td>
<td>146</td>
</tr>
<tr>
<td></td>
<td>23.94%</td>
<td>23.75%</td>
<td>54.75%</td>
<td>12.51%</td>
</tr>
<tr>
<td>Cooperative Banks</td>
<td>1,296</td>
<td>1,202</td>
<td>32</td>
<td>914</td>
</tr>
<tr>
<td></td>
<td>67.01%</td>
<td>67.34%</td>
<td>17.88%</td>
<td>78.32%</td>
</tr>
</tbody>
</table>

This table shows the breakdown of banks according to their banking group as of 31 December 2005, both for all German universal banks and for the sample analyzed in this paper. ‘Trading Book’ refers to banks that have a trading book and ‘Merged’ refers to banks that took part in a merger during the sample period (1999 to 2005). ‘Number’ refers to the absolute number of banks, whereas ‘Fraction’ refers to the relative share. Branches of foreign banks are not included in commercial banks. Commercial banks include ‘Landesbanken’ and cooperative central banks as they show more similarities to commercial banks than to savings banks or cooperative banks. Deutsche Bundesbank (2006a) uses a similar classification.

be expressed as follows:

\[
\text{optimize } F(X) \\
\text{subject to } (16), (17), (14),
\]

where \( X \) denotes the set of all business items \( X^{pos, beg, end} \).

4 Empirical Analysis

4.1 Data

The analysis is based on German universal banks as of 31 December 2005. Table 1 shows the composition of our sample. Banks for which data is incomplete are excluded from our analysis, which results in a total of 1,785 banks, accounting for 92.3% of all German universal banks. We make use of the following major data sources: the data schedule pursuant to the auditor’s report (‘Sonderdatenkatalog’) and the monthly balance sheet statistics (‘Monatliche Bilanzstatistik’) for estimating the bank structure and an interest rate risk survey for evaluating our model. Due to structural breaks within the data sources at the end of 1998, we restrict our sample to the period
from January 1999 to December 2005. All data is provided by the Deutsche Bundesbank. 

The data schedule pursuant to the auditor’s report contains asset and liability positions broken down by remaining time to maturity for all banks on a yearly basis. In contrast, the monthly balance sheet statistics contain on a monthly basis a respective breakdown by initial maturity. We consider 3 interest rate-sensitive asset and 4 liability positions. The former include ‘interbank loans’, ‘customer loans’, and ‘debt securities held’, the latter consist of ‘interbank liabilities’, ‘customer liabilities’, ‘debt securities issued’, and ‘non-maturing deposits’. The positions and the reported time bands are shown in Table 2.

Additionally, we use the Bundesbank’s survey of lending and deposit rates (‘Erhebung über Soll- und Habenzinsen’) and the MFI interest rate statistics, which replaced the former in 2003. Both statistics contain on a monthly basis information on the coupons charged and paid by banks on new business classified by time bands. For mid- and long-term default-free interest rates, we use the term structure estimated by the Deutsche Bundesbank from German governmental bonds according to Svensson (1994), and for the short-term rates we use the money market rates reported by Frankfurt banks (‘Geldmarktsätze am Frankfurter Bankplatz’). Regulatory capital (own funds) is taken from the reports of Principle I.

To evaluate our model results for the German banking system we use a non-published interest rate risk survey (IRRS) that was carried out by the Deutsche Bundesbank and the German Federal Financial Supervisory Authority (BaFin). All German banks were asked to report the losses in their banking book as a consequence of a, among others, 200 basis point shift in the term structure according to their internal risk-management system. The participation rate was nearly 60%, so we have bank internal interest rate risk reports of some 1,200 banks for the reference period.

---

11Note that the TAM is able to capture structural breaks. However, before 1998 German banks did not report time bands according to remaining time to maturity. Hence, the integration of the structural break would only have resulted in a one year extension of the sample period.

12See Section 68 of the German Auditor’s Report Regulation (‘Prüfberichtsverordnung’) for detailed information.

13See Deutsche Bundesbank (2003) for detailed information.

14See European Central Bank (2003) and the respective report forms for detailed information.

15According to Sections 10 and 53 of the German Banking Act (‘Kreditwesengesetz’).
<table>
<thead>
<tr>
<th>Asset positions ($POS_A$)</th>
<th>[0]</th>
<th>(0; 3/12]</th>
<th>(3/12; 1]</th>
<th>(1; 2]</th>
<th>(2; 5]</th>
<th>(5; ∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>interbank loans</td>
<td>RTM</td>
<td>SON01.354</td>
<td>SON01.355</td>
<td>SON01.356</td>
<td></td>
<td>SON01.357</td>
</tr>
<tr>
<td></td>
<td>ITM</td>
<td>A1.100/01</td>
<td>A1.100/02</td>
<td>A1.100/03</td>
<td>A1.100/04</td>
<td></td>
</tr>
<tr>
<td>customer loans</td>
<td>RTM</td>
<td>SON01.358</td>
<td>SON01.359</td>
<td>SON01.360</td>
<td></td>
<td>SON01.361</td>
</tr>
<tr>
<td></td>
<td>ITM</td>
<td>B1.500/01</td>
<td>B1.500/02</td>
<td>B1.500/03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>debt securities held</td>
<td>RTM</td>
<td>SON01.379</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ITM</td>
<td>E1.100/02</td>
<td>E1.100/03</td>
<td></td>
<td></td>
<td>E1.100/04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Liability positions ($POS_L$)</th>
<th>[0]</th>
<th>(0; 3/12]</th>
<th>(3/12; 1]</th>
<th>(1; 2]</th>
<th>(2; 5]</th>
<th>(5; ∞)</th>
</tr>
</thead>
<tbody>
<tr>
<td>interbank liabilities</td>
<td>RTM</td>
<td>–</td>
<td>SON01.362</td>
<td>SON01.363</td>
<td>SON01.364</td>
<td>SON01.365</td>
</tr>
<tr>
<td></td>
<td>ITM</td>
<td>A2.100/01</td>
<td>A2.100/02</td>
<td>A2.100/03</td>
<td>A2.100/04</td>
<td></td>
</tr>
<tr>
<td>customer liabilities</td>
<td>RTM</td>
<td>–</td>
<td>SON01.370</td>
<td>SON01.371</td>
<td>SON01.372</td>
<td>SON01.373</td>
</tr>
<tr>
<td></td>
<td>ITM</td>
<td>C1.100/01</td>
<td>C1.100/02</td>
<td>C1.100/03</td>
<td></td>
<td>C1.100/04</td>
</tr>
<tr>
<td>debt securities issued</td>
<td>RTM</td>
<td>–</td>
<td>SON01.174$^2$</td>
<td>SON01.175$^2$</td>
<td>SON01.176$^2$</td>
<td>SON01.177$^2$</td>
</tr>
<tr>
<td>non-maturing deposits</td>
<td>ITM</td>
<td>F1.100/01+F2.400/01$^1$</td>
<td>F1.100/02+F2.400/02$^1$</td>
<td>F1.100/03+F2.400/03$^1$</td>
<td></td>
<td>HV21.231</td>
</tr>
</tbody>
</table>

This table shows the positions we use in our analysis and the time bands that are available for the German banking system. The rows refer to the balance sheet positions $POS$ and whether the time band is according to remaining time to maturity (RTM) included in the data schedule pursuant to the auditor's report or initial maturity (ITM) included in the monthly balance sheet statistics. The columns refer to the range of the respective bands (in years). The cells refer to the report items, that are denoted in line with Deutsche Bundesbank, i.e. \text{form}.\text{item} and \text{form}.\text{row}/\text{column} for the data schedule and the monthly balance sheet statistics, respectively. The report items from the data schedule according to the auditor's report are available on a yearly basis, those from the monthly balance sheet statistics on a monthly basis. The amounts in the data schedule according to the auditor's report do not always fit the corresponding data in the monthly balance sheet statistics for a small number of banks. To obtain consistent data, we scale the former to fit the latter for these banks.

$^1$: The amount of business within the time band is bigger than the report item.

$^2$: The complete outstanding amount of the position ‘debt securities issued’ is given on a monthly basis by $F_{1.100/01} + F_{2.400/01} + F_{1.100/02} + F_{2.400/02} + F_{1.100/03} + F_{2.400/03} + HV_{21.234}$.

$^3$: On a yearly basis, the outstanding amount of the position ‘debt securities issued’ with a remaining time to maturity of up to 1 year (i.e. $RTM^0_{1.38}$) is reported by $SON_{01.380} + SON_{01.374} + SON_{01.375} - HV_{21.233}$. 

| Table 2: Positions and time bands |
date of 30 September 2005 that are consistent with our definition of interest rate risk.

4.2 Model Specification

In order to test our hypotheses we estimate the interest rate risk of the German banking system for the reference date of 31 December 2005 on the basis of the TAM using the data from January 1999 to December 2005. We have to specify the system of equations (16) and (17) and the coupons of the respective business items to calculate the cashflow structure via (15). In this context, we have to decide how to deal with non-maturing deposits as there is no reliable information available on the economic maturity structure or time-bands of non-maturing deposits. Additionally, we have to define an objective function to select a sensible solution from the space of solutions.

The system of equations is based on the positions and the time bands that are reported within the data schedule pursuant to the auditor’s report and the monthly balance sheet statistics (See Table 2). Within our sample period 7 yearly observations of the former and 84 monthly observations of the latter report are available. Identifying the reference date December 2005 with \( t_{ref} = t_0 = 0 \) we obtain \( T^\text{rtn}_{obs} = \{-6, -5, ..., 0\} \) and \( T^\text{rtm}_{obs} = \{-83/12, -82/12, ..., 0\} \). To reduce calculation burden the initial maturity of business \( X_{t_{beg},t_{end}} \) is restricted to \( t_{end} - t_{beg} \in \{0, 1/12, ..., 6/12, 9/12, ..., 24/12, 30/12, ..., 60/12, 72/12, ..., 120/12\} \). This results in 22,368 variables and 2,254 linear equations plus the non-negativity restriction for each variable.\(^{16}\)

We assume that coupons of positions that are contracted with banks or the capital market equal the default-free market interest rate. For the remaining positions we take the coupons from the statistics presented in Section 4.1 and assume that they refer to the middle of the reported time bands. The coupons for the remaining maturities are obtained by interpolating the spreads to the default-free capital market rates linearly between the coupons that are given.

To identify an economically sensible solution we assume a stable bank structure over the time period 1999 to 2005, i.e. we assume that the fraction of the outstanding amount with a certain initial and/or remaining time to maturity in relation to the complete outstanding amount of the

\(^{16}\)The system of equations turned out to be not solvable for a small number of smaller banks. For these banks our assumptions were not consistent with the reported data. To obtain a solution we selected that bank structure that best fitted the system of equations.
respective period remains as constant as possible during the sample period.\footnote{Deshmukh et al. (1983) show in a simple framework that banks may have an incentive to alter their exposure to interest rate risk depending on the magnitude and volatility of interest rates. Since the term structure of interest rates was quite stable during the sample period, we maintain the assumption of a stable bank structure. However, the model can be adapted easily.} We implement this by minimizing the sum of the quadratic deviations of the relative outstanding amount with a certain remaining time to maturity and initial maturity, respectively, over all possible maturities and all observation dates. For the actual specification, see Appendix A. It was selected in a simulation-based analysis. We modeled several synthetic banks with reasonable business strategies such as constant or varying maturity transformation strategies. We then created reports that the respective banks would have had submitted to the Deutsche Bundesbank and applied several specifications of the objective function to compare the respective model results with the theoretical ones.\footnote{The specifications differed in three aspects: first, the degree of the deviations (absolute versus quadratic), second, the definition of maturity (initial maturity versus remaining time to maturity versus initial maturity and remaining time to maturity), third, the definition of the reference maturity structure (average maturity structure versus maturity structure of the last observation date).} Based on several measures of fit like the sum of the absolute or quadratic deviations between the model implied and the real cash flow structure at the reference date, we identified the above-mentioned specification as superior.

Non-maturing deposits can be interpreted as short-term liabilities. However, it is well known that they can exhibit a high level of interest rate risk (e.g. Hutchison and Pennacchi, 1996, Jarrow and van Deventer, 1998, and O’Brien, 2000). This can to a large extent be explained by the fact that the development of their volume and deposit rates turns out to be sticky, yielding a duration or, equivalently, an economic maturity that is higher than that of other short-term products. For estimating the most ‘reasonable’ economic maturity of non-maturing deposits, we make use of the IRRS above. For different assumptions about the maturity, we calculate the TAM and estimate the following linear regression on the IRRS-subsample of banks:

\[
RM^\text{IRRS}_i = \alpha_{\text{TAM}} + \beta_{\text{TAM}} \cdot RM^\text{TAM}_i + \varepsilon_i, \tag{18}
\]

where \(RM^\text{IRRS}_i\) and \(RM^\text{TAM}_i\) denote the interest rate risk for bank \(i\) derived from the IRRS and the TAM, respectively. We choose the maturity which yields the highest \(R^2\), i.e. that can
explain the largest part of the cross-sectional variation of the IRRS-risk measure. The best fit is obtained when we assume that the maturity of non-maturing deposits equals the legal maturity of 3 months.

Interestingly, when we apply the bank-individual assumption to the maturity of non-maturing deposits that is also reported in the IRRS, we obtain a rather poor fit. This suggests that a number of banks did not include their internally assumed exposure of non-maturing deposits when reporting the interest rate risk, which implies that the evaluation of our model in the following is quite conservative (i.e. we underestimate the model quality using the IRRS), since we cannot distinguish between the respective banks.

4.3 Model Evaluation

To benchmark our model to standard approaches and hence to test hypothesis H1, we additionally estimate a ‘Benchmark Model’ (BM) that relies on one-point-in-time data.\textsuperscript{19} We consider the same asset and liability positions as for the TAM, assume also a maturity of 3 months for the deposits and use the breakdown by initial maturity on 31 December 2005 for the remaining positions, assuming that the business is concentrated in the middle of the reported time bands.\textsuperscript{20} Based on this, we assume that the business has continuously been contracted over time and has a coupon of 5%. This allows us to derive a (continuous) cash flow structure for each position and to estimate the interest rate risk analogously to (1).

For the evaluation we run regression (18) for the TAM and the analogue regression

$$RM_{i}^{IRS} = \alpha_{BM} + \beta_{BM} RM_{i}^{BM} + \varepsilon_{i}$$

for the BM on the IRRS-subsample of banks (IRRS-banks). If the models produce unbiased estimates of the true interest rate risk – assuming that the results of the IRRS represent the banks’ true interest rate risk – then the respective constants $\alpha$ should be zero and the slopes

\textsuperscript{19}We first estimated a model based on the remaining time to maturity in line with the standardized framework proposed by the Basel Committee on Banking Supervision (2004b) which, however, yielded a rather poor fit of the IRRS. We therefore specify the BM based on the initial time to maturity, whereas all other assumptions are in line with the Basel Committee on Banking Supervision (2004b).

\textsuperscript{20}For the time bands (2; $\infty$) and (5; $\infty$), we assume a concentration at 4 and 6, respectively.
Table 3: Evaluation of the model quality

<table>
<thead>
<tr>
<th></th>
<th>TAM</th>
<th>BM</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>Slope</td>
<td>Intercept</td>
</tr>
<tr>
<td>IRRS-banks</td>
<td>0.0365 0.26 0.2733</td>
<td>0.0283 0.49 0.1915</td>
<td>0.0020 1,047</td>
</tr>
<tr>
<td></td>
<td>(3.88)***</td>
<td>(19.82)***</td>
<td>(2.32)**</td>
</tr>
</tbody>
</table>

This table shows the results of the regression \( R_{IRRS}^i = \alpha + \beta R_{Model}^i + \epsilon_i \) for the subsample of banks that took part in the IRRS (IRRS-banks). \( R_{IRRS}^i \) denotes the interest rate risk of bank \( i \) reported in the IRRS and \( R_{Model}^i \) represents the risk measure of the TAM and the BM, respectively. \( t \)-ratios are in parentheses. The p-value is based on \( t \)-statistics and correspond to the null hypothesis \( H_0: R^2_{TAM} = R^2_{BM} \). \( N \) denotes the number of observations. Significance at the 10%/5%/1% level is marked */**/***.

\( \beta \) should be one. A positive (negative) \( \alpha \) and a \( \beta \) bigger (smaller) than one means that the respective model tends to underestimate (overestimate) the interest rate risk. The results of the regressions are reported in Table 3. Obviously, both models produce biased estimates of the level of interest rate risk. The slope for both models is positive but clearly smaller than one. As the constants are comparatively small this implies that both the TAM and the BM overestimate the interest rate risk of banks. This was to be expected as it can be explained by our assumptions: we exclude provisions and premature redemptions on the asset side and prolongation options on the liability side and assume a very small economic maturity of non-maturing deposits, i.e. we tend to overestimate the interest rate sensitivity of the asset side and underestimate the interest rate sensitivity of the liability side. As banks usually have a positive duration gap, these assumptions hence cause higher estimates of the level of interest rate risk. Additionally, we do not consider off-balance positions, such as interest derivatives, that on average can be expected to be used to reduce interest rate risk by the banks (e.g. Schrand, 1997). Similar biases are also reported by Wright and Houpt (1996), Sierra (2004) and Sierra and Yeager (2004) when evaluating the EVM.

How can we test hypothesis H\(_1\)? The part of the cross-sectional variation of interest rate risk explained by the respective models is measured by the coefficients of determination \( R^2 \), 27.33% for the TAM and 19.15% for the BM, respectively (See Table 3). However, as we do not know the joint distribution of the two \( R^2 \), we cannot decide whether the possible differences in the \( R^2 \) are significant. To be able to make statistical statements, we develop a procedure that can tell which of two unbiased measures is more accurate.
First, the above-mentioned bias can be compensated by a simple linear transformation (e.g. Greene, 2003). Define

\[
\tilde{RM}_{Model}^i = \hat{\alpha}_{Model} + \hat{\beta}_{Model} RM_{Model}^i, \quad Model = TAM, BM,
\]  

(20)

where \(\hat{\alpha}_{Model}\) and \(\hat{\beta}_{Model}\) denote the results of the respective regressions (18) and (19). Then, \(\tilde{RM}_{TAM}^i\) and \(\tilde{RM}_{BM}^i\) are unbiased estimates of the level of interest rate risk of bank \(i\).

Second, note that the \(R^2\) remain unchanged if we replace \(RM_{TAM}^i\) and \(RM_{BM}^i\) by \(\tilde{RM}_{TAM}^i\) and \(\tilde{RM}_{BM}^i\) in the regressions (18) and (19), respectively, as the coefficient of determination is invariant to any linear transformations.

Third, let \(err_{TAM}^i = \tilde{RM}_{TAM}^i - RM_{IRRS}^i\) and \(err_{BM}^i = \tilde{RM}_{BM}^i - RM_{IRRS}^i\) be the error of the unbiased TAM and BM measure, respectively. Then the coefficient \(w\) in the following regression provides information on the relative quality of the two measures as proved in Appendix B:\textsuperscript{21}

\[
err_{TAM}^i = w (err_{TAM}^i - err_{BM}^i) + \eta_i.
\]  

(21)

If both measures are of equal accuracy, the coefficient \(w\) equals 0.5. In case the TAM measure (BM measure) is more accurate, then the coefficient \(w\) will be greater (smaller) than 0.5. That is why testing the hypothesis of equal accuracy is equivalent to testing whether the coefficient in regression (21) is equal to 0.5. Table 3 shows the result for the hypothesis \(H_0: R^2_{TAM} = R^2_{BM}\). We find strong evidence that the \(R^2\) of the TAM exceeds that of the BM and conclude:

**Result 1** There is strong evidence that the Time Series Accounting-Based Model is able to explain the cross-sectional variation of banks’ interest rate risk better than the Benchmark Model.

However, the model quality differs depending on the characteristics of the banks. Table 4 shows the \(R^2\) for some subsamples of IRRS-banks. We stress two distinctive features here: first, the TAM fits better the internal data of those banks that have not merged during the sample period. Mergers are dealt with by creating an artificial bank before the date of the merger, aggregating the balance sheet positions. As the assumption of a stable structure over time is quite strong for merged banks, our model provides better quality for banks that did not merge

\textsuperscript{21}Morgan (1939) uses a similar approach.
Table 4: Evaluation of the model quality on subsamples of IRRS-banks

<table>
<thead>
<tr>
<th></th>
<th>R²</th>
<th>p-Value</th>
<th>( H_0: R^2_{TAM} = R^2_{BM} )</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Sample (IRRS-banks)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TAM</td>
<td>0.2733</td>
<td>0.1915</td>
<td>0.0020</td>
<td>1,047</td>
</tr>
<tr>
<td>BM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>By Banks’ Assumption on Maturity of Non-Maturing Deposits</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Quintile (shortest maturity)</td>
<td>0.5018</td>
<td>0.2386</td>
<td>0.0000</td>
<td>198</td>
</tr>
<tr>
<td>2nd Quintile</td>
<td>0.2679</td>
<td>0.2315</td>
<td>0.2264</td>
<td>219</td>
</tr>
<tr>
<td>3rd Quintile</td>
<td>0.2300</td>
<td>0.1517</td>
<td>0.3375</td>
<td>211</td>
</tr>
<tr>
<td>4th Quintile</td>
<td>0.1900</td>
<td>0.1509</td>
<td>0.3599</td>
<td>207</td>
</tr>
<tr>
<td>5th Quintile (longest maturity)</td>
<td>0.2493</td>
<td>0.2389</td>
<td>0.7518</td>
<td>208</td>
</tr>
<tr>
<td><strong>Non-Merged Banks (IRRS-banks)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TAM</td>
<td>0.3089</td>
<td>0.1782</td>
<td>0.0001</td>
<td>585</td>
</tr>
<tr>
<td>BM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>By Banks’ Assumption on Maturity of Non-Maturing Deposits</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st Quintile (shortest maturity)</td>
<td>0.6680</td>
<td>0.2561</td>
<td>0.0000</td>
<td>103</td>
</tr>
<tr>
<td>2nd Quintile</td>
<td>0.2089</td>
<td>0.1489</td>
<td>0.0982</td>
<td>121</td>
</tr>
<tr>
<td>3rd Quintile</td>
<td>0.2462</td>
<td>0.1050</td>
<td>0.0957</td>
<td>121</td>
</tr>
<tr>
<td>4th Quintile</td>
<td>0.2254</td>
<td>0.1893</td>
<td>0.4545</td>
<td>116</td>
</tr>
<tr>
<td>5th Quintile (longest maturity)</td>
<td>0.2315</td>
<td>0.2400</td>
<td>0.8565</td>
<td>118</td>
</tr>
</tbody>
</table>

This table shows the results of the regression \( RM_{IRRS}^{IRRS} = \alpha + \beta \cdot RM_{Model} + \varepsilon_i \) for subsamples of banks that took part in the IRRS (IRRS-banks). \( RM_{IRRS}^{IRRS} \) denotes the interest rate risk of bank \( i \) reported in the IRRS and \( RM_{Model} \) the risk measure of the TAM and of the BM, respectively. The p-values are based on \( t \)-statistics and correspond to the null hypothesis \( H_0: R^2_{TAM} = R^2_{BM} \). \( N \) denotes the number of observations. The quintile analysis is based on the bank internal assumptions on the (economic) maturity of non-maturing deposits as reported in the IRRS. ‘Non-merged banks’ refers to those banks that did not merge during the sample period (1999 to 2005).

during the sample period. However, in both samples, the TAM fits the banks’ internal data on the interest rate risk better than the BM.

Second, as reported in Section 4.2, we find evidence that a number of banks did not include their internally assumed exposure of non-maturing deposits when reporting the interest rate risk in the IRRS. If some banks include the respective exposure, we underestimate the model quality using the IRRS as a benchmark. However, this problem should be less severe for banks that assume a low maturity for non-maturing deposits. In fact, Table 4 shows that we obtain much better results for these banks. In the first quintile of the reported bank internal assumption about the maturity of non-maturing deposits, the coefficient of determination yields more than 50%, and for non-merged banks even more than 66%.
Table 5: Analysis of the TAM’s bias

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Slope TAM</th>
<th>Slope Log(Assets)</th>
<th>Dummies SB</th>
<th>Dummies CB</th>
<th>Dummies TB</th>
<th>Adj. R²</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRRS-banks</td>
<td>0.1046</td>
<td>0.23</td>
<td>-0.0033</td>
<td>0.0376</td>
<td>0.0432</td>
<td>0.0270</td>
<td>0.2747</td>
<td>1,077</td>
</tr>
<tr>
<td></td>
<td>(1.55)</td>
<td>(16.79)***</td>
<td>(-1.13)</td>
<td>(2.62)***</td>
<td>(2.77)***</td>
<td>(2.59)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table shows the results of the multiple regression $RM_{i}^{IRRS} = \alpha + \beta_1 RM_{i}^{TAM} + \beta_2 \log(\text{Assets}_i) + \delta_{SB} D_{SB}^i + \delta_{CB} D_{CB}^i + \delta_{TB} D_{TB}^i + \varepsilon_i$ in the subsample of banks that took part in the IRRS (IRRS-banks). $RM_{i}^{IRRS}$ denotes the interest rate risk of bank $i$ reported in the IRRS and $RM_{i}^{TAM}$ the risk measure of the TAM. $D_{SB}^i$ ($D_{CB}^i, D_{TB}^i$) is a dummy for savings banks (cooperative banks, trading book institutions). The basis scenario is a bank that has no trading book and is not a savings bank or a cooperative bank. The $t$-ratios are in parentheses. Significance at the 10%/5%/1% level is marked */**/***.

4.4 The Interest Rate Risk of German Banks

The next step is testing the hypotheses H2 and H3. The previous Section 4.3 showed that our estimates of the interest rate risk of banks are biased. Since the bias could depend on the banks’ characteristics, we unbias our estimates for size and banking group. In addition, we include a control dummy variable for trading book institutions and estimate the following regression for the banks that took part in the IRRS:

$$RM_{i}^{IRRS} = \alpha + \beta_1 RM_{i}^{TAM} + \beta_2 \log(\text{Assets}_i) + \delta_{SB} D_{SB}^i + \delta_{CB} D_{CB}^i + \delta_{TB} D_{TB}^i + \varepsilon_i,$$

where $D_{SB}^i$ ($D_{CB}^i, D_{TB}^i$) is a dummy for savings banks (cooperative banks, trading book institutions). Table 5 shows the results.

The coefficients imply that the bias is independent of the bank size but depends on the banking group and whether the bank has a trading book. The positive sign of the dummies for the banking group suggests that – as we overestimate the interest rate risk for all investigated subgroups on average (not reported here) – the overestimation is less severe for savings banks and cooperative banks than for private commercial banks. This implies that the effect of the simplifying assumptions, discussed in Section 4.3, is more severe for the latter banks. One specific reason could be off-balance activities. The more a bank uses derivatives to decrease (increase) the interest rate risk, the more (less) we overestimate the risk. So our result could indicate that private commercial banks use interest derivatives for hedging purposes to a higher extent than the other banks. Analogue considerations hold for trading book institutions. However, we would
Table 6: Determinants of the interest rate risk

<table>
<thead>
<tr>
<th></th>
<th>Log(Assets)</th>
<th>Dummies</th>
<th>Adj. R²</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete Sample</td>
<td>0.0059</td>
<td>0.1163</td>
<td>0.1460</td>
<td>0.0426</td>
</tr>
<tr>
<td></td>
<td>(5.06)***</td>
<td>(22.07)***</td>
<td>(28.54)***</td>
<td>(8.12)***</td>
</tr>
</tbody>
</table>

This table shows the results of the regression \( \hat{RM}_{i}^{TAM} = \alpha + \beta \log(Assets_i) + \delta^{SB} D_{SB}^i + \delta^{CB} D_{CB}^i + \delta^{TB} D_{TB}^i + \epsilon_i \) on the whole data sample where \( \hat{RM}_{i}^{TAM} \) denotes the unbiased TAM-risk measure (23) of bank \( i \). \( D_{SB}^i \) (\( D_{CB}^i \), \( D_{TB}^i \)) is a dummy for savings banks (cooperative banks, trading book institutions). The basis scenario is a bank that has no trading book and is no savings bank nor a cooperative bank. We do not report the value of the intercept since the level of interest rate risk for the German banking system is handled confidentially by the Deutsche Bundesbank. The \( t \)-ratios are in parentheses. Significance at the 10%/5%/1% level is marked ∗/∗∗/∗∗∗.

have expected a negative sign of the dummy, as our approach includes the on-balance positions of the banking and the trading book, whereas the IRRS only captures the banking book. On the other hand, trading book institutions may assume a higher level of interest rate risk via derivatives, because once a stress situation occurs they can more quickly alter their exposure to interest rate risk via capital market transactions. When doing so, there are regulatory incentives to take the interest rate risk rather in the banking book than in the trading book (e.g. Jones, 2000). This could be done by ‘hedging’ the on-balance positions on the liability side of the banking book via derivatives that are included in the IRRS.

With the coefficients from regression (22) we calculate for each bank in the whole data sample the unbiased measure of interest rate risk

\[
\hat{RM}_{i}^{TAM} = \hat{\alpha} + \hat{\beta}_1 \log(Assets_i) + \hat{\delta}^{SB} D_{SB}^i + \hat{\delta}^{CB} D_{CB}^i + \hat{\delta}^{TB} D_{TB}^i \tag{23}
\]

and run the regression

\[
\hat{RM}_{i}^{TAM} = \alpha + \beta \log(Assets_i) + \delta^{SB} D_{SB}^i + \delta^{CB} D_{CB}^i + \delta^{TB} D_{TB}^i + \epsilon_i \tag{24}
\]

on the whole data sample to test hypotheses H2 and H3. Table 6 shows the results.

The coefficient of size turns out to be significantly positive, i.e. we find evidence that there is a positive correlation between size and interest rate risk which supports hypothesis H2.

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**Result 2** There is strong evidence that bigger banks have a higher level of interest rate risk than smaller banks.

The costs of bearing interest rate risk seem to be lower for bigger banks than for smaller banks or are compensated for by other effects. This could be related to the above-mentioned arguments: bigger banks are more diversified and can thus assume a higher level of systematic risk, or even assume a higher level of total risk, since they may be “too big to fail”. Additionally, bigger banks may have more opportunities to trade their risk on the capital market and hence to alter their exposure to interest rate risk via off-balance activities quickly. This argument is supported by the fact that banks with a trading book (i.e. that have a better access to the capital market) have a higher level of interest rate risk than non-trading book institutions.

Result 2 seems to contradict the results of the stress test among German banks, where – based on a sample of 25 banks – medium-sized and smaller banks turn out to have a higher level of interest rate risk than commercial banks and central institutions (See Deutsche Bundesbank, 2006a). However, we find evidence that the difference of interest rate risk quantified within the stress test is rather attributable to the banking group than to size: The analyzed subgroup ‘medium-sized and smaller banks’ mainly consists of savings banks and cooperative banks. The coefficients of the respective dummies in our empirical analysis are both highly significantly positive. More precisely, the interest rate risk of savings banks and cooperative banks is higher than that of the remaining banks. This supports the hypothesis that bearing risk might be more costly for private banks than for state-owned savings banks and member-owned cooperative banks.

**Result 3** There is strong evidence that the interest rate risk differs between banks of different banking groups and that the interest rate risk of savings banks and cooperative banks is higher than that of the remaining banks.

**5 Conclusion**

In this paper we developed a model to quantify the interest rate risk of banks using time series of accounting-based data. The framework is flexible enough to include different data sources and
to capture the standardized reporting framework suggested by the Basel Committee on Banking Supervision (2004b) and different actual reporting practices like in the US and in Germany. Further, we examined – for the first time – the interest rate risk of the German universal banking system on an individual bank level and analyzed its determinants.

Based on a subsample of German banks to whose internally quantified interest rate risk we have access via a unique interest rate risk survey conducted by the Deutsche Bundesbank and BaFin, we could evaluate our model results. In line with earlier models and studies we found that our accounting-based approach produces biased estimates of the interest rate risk of banks, which is attributed to some simplifying assumptions. However, we found convincing evidence that our model is able to explain the cross-sectional variation of banks’ interest rate risk better than a benchmark model that relies on assumptions similar to models proposed in the literature and applied by banking supervisors. We controlled for the bias and analyzed factors that affect the level of interest rate risk of German banks. We found strong evidence that bigger banks have a higher level of interest rate risk than smaller banks. Further, we found evidence that the interest rate risk differs among banks of different banking groups. Savings banks and cooperative banks have a higher level of interest rate risk than private commercial banks. Lastly, there seem to be structural differences for trading book institutions.

Banks may also have incentives to alter their exposure depending on the magnitude and volatility of interest rates. Within our sample period, the term structure of interest rates was quite stable in Germany. As a structural break in the data at the end of 1998 bars us from going back in time, an analysis of the time variability of interest rate risk in the German banking system must be left for future research.
Appendix A  Specification of the Objective Function

Due to the structure of the TAM, the asset and liability side can be modeled separately. Since both sides are modeled analogously, we only present the asset side here.

\[
\begin{align*}
\text{minimize} \quad & F(X) = \sum_{t \in T_{\text{obs}}} \sum_{rtm \in RTM} d^2_{\text{assets},t,rtm} + \sum_{t \in T_{\text{obs}}} \sum_{itm \in ITM} d^2_{\text{assets},t,itm} \\
\text{subject to} \quad & \\
& \quad d_{\text{assets},t,rtm} = \frac{\sum_{pos \in POS^A} \sum_{j \geq 0} X_{\text{pos},-j/12,rtm} \sum_{i=1}^{120} X_{\text{pos},-j/12,i/12}}{\sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i=1}^{120} X_{\text{pos},-j/12,i/12}} - \frac{\sum_{pos \in POS^A} \sum_{j \geq 0} X_{\text{pos},t-j/12,t+rtm} \sum_{i=1}^{120} X_{\text{pos},t-j/12,t+i/12}}{\sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i=1}^{120} X_{\text{pos},t-j/12,t+i/12}} \quad \forall \ t \in T_{\text{obs}}, \ t \in RTM, \\
& \quad d_{\text{assets},t,itm} = \frac{\sum_{pos \in POS^A} X_{\text{pos},0,itm} \sum_{j \geq 0} \sum_{i=1}^{120} X_{\text{pos},-j/12,i/12}}{\sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i=1}^{120} X_{\text{pos},-j/12,i/12}} - \frac{\sum_{pos \in POS^A} X_{\text{pos},t-itm} \sum_{j \geq 0} \sum_{i=1}^{120} X_{\text{pos},t-j/12,t+i/12}}{\sum_{pos \in POS^A} \sum_{j \geq 0} \sum_{i=1}^{120} X_{\text{pos},t-j/12,t+i/12}} \quad \forall \ t \in T_{\text{obs}}, \ t \in ITM, 
\end{align*}
\]

where \(d_{\text{assets},t,itm}\) and \(d_{\text{assets},t,rtm}\) denote the deviational variables and \(F()\) the objective function. The first part of the respective right-hand side denotes the outstanding amount in year 0 with a remaining time to maturity (initial maturity) of \(rtm\) (\(itm\)) months in relation to the complete outstanding amount of the respective period. The second part of the respective right hand side denotes the respective fraction \(t\) months before.\(^{22}\) To reduce the calculation burden, the included maturities are restricted to:

\(ITM = \{0, 1/12, \ldots, 6/12, 9/12, \ldots, 24/12, 30/12, \ldots, 60/12, 72/12, \ldots, 120/12\}\)

\(RTM = \{1/12, 2/12, \ldots, 6/12, 12/12, 18/12, \ldots, 120/12\}\).

\(^{22}\)To include information on business that is on-balance during the observation period but was contracted before, the relation for the initial maturity was also calculated for this business. The complete outstanding amount of these periods of contraction are assumed to correspond to the amount in the first observation time.
Appendix B  Comparison of the Models’ Accuracy

Let $err_i$ be the (in-sample) prediction error of the two models $i = 1, 2$. We assume that the measures are unbiased, i.e. $E(err_i) = 0 \forall i$. The quality of the prediction is measured by the standard deviation of the prediction error $\sigma_i$: the lower the standard deviation, the more accurate the forecast. The errors $err_1$ and $err_2$ can, but need not be, correlated. The covariance between the two errors is denoted by $\sigma_{12}$.

Note the following regression:

$$err_1 = w (err_1 - err_2) + \eta.$$  \hspace{1cm} (B.1)

As both of the measures are unbiased, there is no intercept in the regression. The regression coefficient $w$ is calculated as

$$w = \frac{cov(err_1, err_1 - err_2)}{\text{var}(err_1 - err_2)} = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}.$$  

Minor manipulation shows that the following relation holds:

$$w \leq \frac{1}{2} \iff \sigma_1 \leq \sigma_2.$$  

For instance, the inequality $w < \frac{1}{2}$ can be transformed in

$$\frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} < \frac{1}{2} \iff 2\sigma_1^2 - 2\sigma_{12} < \sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \iff \sigma_1^2 < \sigma_2^2.$$  

Therefore, testing the equality of the accuracy is equivalent to running the OLS regression (B.1) and then carrying out the $t$–test if $w$ equals $\frac{1}{2}$. Incidentally, the coefficient of determination $R_i^2$ which we apply as the evaluation tool in Section 4.3 is closely related to the error’s standard deviation ($y$ denotes the variable to be predicted):

$$R_i^2 = 1 - \frac{\sigma_i^2}{\text{var}(y)}.$$
References


