Can We Forecast the Implied Volatility Surface Dynamics for CBOE Equity Options?

Predictability and Economic Value Tests

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Abstract

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Keywords: Implied volatility surface; Equity Options; Index Options, Predictability.

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Abstract

We investigate whether the dynamics in the volatility surface implicit in the prices of individual equity options traded on the CBOE contains any exploitable predictable patterns. In particular, we examine the possibility that the dynamics in the volatility surface implicit in S&P 500 index options may be associated and forecast subsequent movements in the implied volatility surface characterizing individual equity options. We find a strong relationship between equity and S&P 500 index option implied volatility surfaces. In addition, we discover a remarkable amount of predictability in the movements over time of both equity and stock index implied volatilities. We show that the predictability for equity options is increased by the incorporation in the model of recent dynamics in S&P 500 implied volatilities. Similarly, when we examine the economic value of these predictability patterns (by proposing and simulating trading strategies that exploit our 1-day head forecast of implied volatilities), we report that delta-hedged and straddle portfolios that take trade on the entire implied volatility surface and across all contracts examined produce high risk-adjusted profits which are maximum for the model that takes into account the feedback from past market implied volatility changes to subsequent dynamics in individual equity options implicit volatilities.

1. Introduction

It is well known that the volatilities implicit in observed option prices are not constant across strikes and time to maturity, as the Black–Scholes model would predict. Instead, they exhibit a smile/skew pattern across strikes for a given time to maturity, which extends to an entire volatility surface when different maturities are examined. These implied volatility (henceforth, IV) curves and surfaces also change through time, raising the need for an accurate modeling of their dynamics. While there is now abundant evidence on the existence and instability of an implied volatility surface (henceforth, IVS) in the case of stock index options (such as S&P 500 and S&P 100 options), much less is known with reference to (allegedly) similar phenomena involving the IVS of individual equity options, such as those traded on the Chicago Board Options Exchange (CBOE) market. In this paper we ask two related questions, which – to the best of our knowledge – have not been addressed by earlier research.[1] First, are CBOE equity IV surfaces as unstable and predictable over time as it has been shown to be case (see Dumas, Fleming and Whaley, 1998, Skiadopoulos *et al.* 1999, Cont and da Fonseca, 2002, Gonçalves and Guidolin, 2006) for the IVS that characterizes stock index options? Second, can we find any reliable, exploitable predictive links between the dynamics in the market index IVS (such as the IVS of S&P 500 index options) and the IV surfaces of individual equity options? Our paper provided a positive answer to both questions. Importantly, our analysis is not confined to the prediction aspects of the dynamic relationships and linkages that we document in our work, but systematically extends to the assessment of whether such relationships contain any economic value, i.e., whether appropriate trading strategies can be set up to bet on the predictability patterns.

Although the celebrated Black-Scholes (1973, BS henceforth) formula is very popular among market practitioners, when applied to (vanilla) call and put options it is very often reduced to a means of quoting option prices in terms of another parameter, the IV. Contrary to the constant volatility assumption of BS model, empirical research has revealed that implied volatilities usually depend on two parameters that define option contracts: their strike price (moneyness) and time-to-maturity. The function $\sigma_t^i : (K_i, \tau_i) \to \sigma(K_i, \tau_i)$ which represents this dependence is the IVS for options on underlying stock i at date t. Modelling the IVS at a given date is therefore equivalent to specifying prices of all (vanilla) calls and puts at that date. Two features of this surface have captured the attention of researchers. First, the non-flat instantaneous profile of the surface, whether it be a smile or skew, and the existence of a term structure, points out the insufficiency of the Black–Scholes model for pricing options. Second, the level of IVs changes with time, deforming the shape of the IVS. The evolution in time of this surface captures the evolution of prices in the options market (see, e.g., Canina and Figlewski 1989; Rubinstein 1994; and Campa and Chang 1995).

The shortcomings of the BS option pricing model when compared to empirical data from the options

[1] Rubinstein (1985), Dennis and Mayhew (2002), and more recently Goyal and Saretto (2009) have used individual equity options data to deal with issues related to implied volatility dynamics, although they ask rather different questions.

market have led to the development of a considerable literature on alternative option pricing models, in which the dynamics of the underlying asset is considered to be a nonlinear diffusion (e.g. Merton 1976), a jump-diffusion process (e.g. Ball and Torous 1985; Bates 1988; and Amin 1993) or a latent stochastic volatility model (e.g. Hull and White 1987; Scott 1987; Wiggins 1987; Johnson and Shanno 1987; and Heston 1993). A related class of models has captured latent stochastic volatility within ARCH-type frameworks (e.g., Duan 1995; and Heston and Nandi 2000). These models attempt to explain the various empirical deviations from BS by introducing additional degrees of freedom such as a local volatility function, a stochastic diffusion coefficient, jump intensities, jump amplitudes etc. Another strand of literature has insisted that the existence of an IVS is the result of precise economic risk factors and/or frictions being incorporated in the way in which state contingent claims are priced. For instance, Geske (1979) and Toft and Prucyk (1997) propose that leverage effects may be the reason for changes in implied volatilities. Franke, Stapleton, and Subrahmanyam (1999) investigate the influence in option pricing models of utility functions that introduce an undiversifiable background risk. Figlewski (1989) highlights the difficulty of implementing dynamic arbitrage strategies in option markets, due to transaction costs. In an extension of his seminal idea, Bollen and Whaley (2004) study the role of supply and demand (i.e. buying pressures) on specific option contracts and the limited ability of arbitrageurs in bringing option prices back to their fundamental value due to trading costs. David and Veronesi (2002) and Guidolin and Timmermann (2003) propose dynamic equilibrium pricing models in which incomplete learning and the need to filter information in the presence of structural instability would cause departures from BS baseline model. Although this literature has contributed significantly to our understanding of option prices behavior, there seems to be no consensus regarding a final explanation or on a optimal model for option pricing.

What the "structural" (pricing-type) and the economic explanations of IVS have in common is that these amount to adding additional "dimensions" – often, parameters – to BS model. However, these additional parameters usually enter the description of the infinitesimal, *local* stochastic evolution of the underlying asset while the market usually quotes options directly in terms of their IVs which are *global* quantities. In order to see whether a model reproduces empirical observations, one has to relate these two representations: the infinitesimal description via a stochastic differential equation on one hand, and the market description via IVs on the other hand. However, in the majority of these models it is impossible to compute directly the shape of the IVS in terms of the model parameters. Although it is possible to compute the IVS numerically, most of the papers in the literature have ended up concluding that simple jump processes, one-factor stochastic volatility models, and other economic (friction-based) explanations of departures from BS fail to reproduce correctly the profiles of empirically observed IV surfaces.[2] In fact,

[2]This problem, already present at the static level, becomes more acute if one examines the consistency of model dynamics with those observed in the options market. While a model with a large number of parameters – such as a non-parametric local

Conte and da Fonseca (2002) arrive to conjecture that the inability of models based on the underlying asset to describe the dynamic behavior of IV surfaces may be not simply due to their misspecification and that there may be a deeper reason: since the creation of organized option markets in 1973, these markets have become increasingly autonomous and option prices are driven, in addition to movements in the underlying asset, also by internal supply and demand in the options market. This observation can be accounted for by introducing sources of randomness which are specific to the options market and which are not present in the underlying asset dynamics. Hence the distinct need to model the structure as well as (or more importantly) the dynamics of the IVS, which is the objective pursued here.

Finally, it is easy to justify the role of producing economically valuable models that predict the IVS. From the standpoint of a vanilla option market maker, producing reliable forecasts for the evolution of the IVS on individual equity options is essential for two reasons. First, it provides an up-to-date indication of where the market stands with reference to each specific underlying asset, which is essential for trading. Second, it allows an efficient risk management of option portfolios that often include also (if not mostly) individual equity options. Moreover the interplay between individual equity IV surfaces and the market index IVS is potentially of great importance: in practice we do not observe simultaneous updates of the entire market IVS, as new quotes for individual trades become available at different points in time. New information is incorporated into the new trades and it becomes crucial for market makers to be able to learn about the IVS from the quotes of the available trades, which are typically few and sometimes concern individual equity options. This motivates us to also investigate the existence of predictive linkages between CBOE IV surfaces and a general market index (such as the S&P 500) IVS. In practice, although since Markowitz (1952, 1959) fathered finance into its modern era it is has been well understood that strong relationships ought to exist between the volatility of individual equity returns and the volatility of returns on the market portfolio, research remains scarce when it comes to investigate a similar relationship between the IVs of individual equity options and the IVs on index options. We therefore ask whether it is reasonable to think that equity and market index IVs may be related in any significant and exploitable way.

Our modelling strategy is inspired after the two-stage approach pursued by GG (2006): In the first stage, we fit daily deterministic IVS models that describe implied volatilities as a function of moneyness and time-to-maturity and of a few interpretable functions of these inputs. In the second stage, we fit time-series models of a VAR type (also augmented with information from the market index options segment) to capture the presence of time variation in the first stage estimated coefficients. Interestingly, as tighter and tighter restrictions are imposed on the second-state VAR model, we recover frameworks that have also

volatility function or implied tree – may calibrate well the strike profile and term structure of options on a given day, the same model parameters might give a poor fit at the next date, creating the need for constant re-calibration of the model. This time instability of model parameters leads to large variations in sensitivities and hedge parameters, which is problematic for risk management applications.

been used in the previous literature, such as GG's model applied to each individual underlying stock, and DFW's random walk framework.[3]

Our main results consists of showing the existence of a strong linear association – both contemporaneous and, more significantly, lagged, which implies the existence of forecasting power from the former to the latter – between the S&P 500 IVS and the coefficients characterizing the shape of the IVS for CBOE options. More generally – i.e., with or without incorporating these effects from the dynamics of the S&P 500 IVS to individual equity options IV surfaces – we report evidence of massive predictability of CBOE IV surfaces, thus extending the econometric results in GG (2006) for index options. In particular, when we evaluate the predictability performance of our baseline VAR model for IVS coefficients, we find that evidence of quite accurate forecasting performance (for future, 1-day ahead IVs) according to standard criteria such as root mean-squared forecast error and mean absolute percentage error. Our SPX-augmented version of GG (2006) two-step VAR framework out-performs a number of competing benchmarks, such as a DFW-style random walk model for the cross-sectional IVS coefficients and Duan and Simonato's (2001) option-GARCH model for American-style contracts.

We also investigate the realized, recursive out-of-sample performance of trading strategies built to exploit our preliminary evidence of statistical predictability. We simulate fixed investments of $1,000 per day using one-day-ahead delta-hedge and straddle portfolios, both (approximately) free of risk deriving from changes in the price of the underlying stocks and only exposed to changes in value driven by the dynamics in the IVS. Basically, we form these portfolios on a simple basis: when one of the competing models anticipates that for a given option contract, IV will increase (decrease) between t and $t+1$, the contract is purchased (sold). These strategies generate on average positive out-of-sample returns when transaction cost are not imposed. Similarly to GG (2006), however most of this profitability disappears when we increase the level of transaction costs, which is a finding consistent with the efficiency of option markets. All in all, it depends on the level of transaction costs faced by an investor or trading desk whether our evidence of cross-sectional CBOE IVS predictability may be put to service to generate abnormal profits.

A relatively small but growing literature on IVS modelling now exists. This is in part caused by the fact that modeling the IVS poses two challenges. First, IV data have a degenerated design: by "degenerated design" we refer to the institutional convention implying that option price data exist only for a small number of maturities such as 1, 3, 6, and 12 months to expiry. Hence, IV observations appear in "strings". As time passes, these strings move along the maturity axis toward expiry while changing their levels and shapes in a random fashion. A second challenge is that the observations do not always cover the desired estimation grid

[3] Competing models of the IVS are the dynamic Kalman filter approach in Bedendo and Hodges (2009), Fengler, Hardle, and Mammen's (2007) dynamic semiparametric factor models, or Daglish, Hull, and Suo's (2007) factor model in which the factors may be correlated with stock price movements.

completely and observations can be missing in certain sub-regions in the moneyness dimension. However, despite this peculiar structure, IVs are usually thought as being the observed structure of a smooth surface. However, a few papers (besides DFW, 1998, and GG, 2006) can be considered as closely related to our research. Empirical studies on the volatility dynamics normally consist in identifying the number and shapes of the shocks in the IV via principal component analysis (PCA) (see, for example, Skiadopoulos et al., 1999; Alexander, 2001; Cont and Da Fonseca, 2002). More recent contributions involve the specification of a deterministic or stochastic model for the IVS, which fully describes its evolution through time. The IVS models introduced by Derman (1999) assume that either the per-delta or the per-strike IVS has a deterministic evolution. Rosenberg (2000) proposed a stochastic process for the ATM IV, while keeping the shape of the curve fixed.

Dennis and Mayhew (2000, 2002) analyze different factors that may explain the volatility smile and risk-neutral skewness for nearest-to-maturity CBOE equity option contracts. They report that the volatility smile slope and the risk-neutral density on individual equity options are negative for equity and S&P 500 options, but that equity options smiles are much flatter than the smile implicit in the index options. They also find that equity betas, size and trading volume are related to the cross-sectional volatility smile slope and the risk-neutral density. Finally, they report a negative relationship between the level of market volatility and the volatility smile and identical association with the risk-neutral skew. However, they do not explore the term-structure dimension of the IVS and fail to assess the economic value of the empirical regularities they have uncovered. Goyal and Saretto (2009) document the existence of economic predictability in the cross-section of equity options IVs using delta-hedged and straddle positions. However, their strategies are based on the differences between historical realized volatilities and at-the-money one-month IVs and not on the predictable patterns affecting the entire IVS. They find abnormal profits using trading strategies that are long (short) in the positions with large positive (negative) differences between these two volatility proxies.

The paper is organized as follows. Section 2 provides an introduction to deterministic volatility surface models and reviews our two-step approach. Section 3 describes the data and reports our main in-sample estimation results. In section 4, we document the existence of cross-serial correlation structure in the relationship between CBOE IVs and index options IVs and therefore introduce issues of modelling and exploiting predictability. Sections 5 examines the out-of-sample statistical and economic performances. Finally, the conclusions are presented in section 6.

2. The Implied Volatility Surface

2.1. *A Deterministic IVS Model*

A convenient way to capture and quantify the shape characteristics of an IVS consists of fitting a linear model linking implied volatilities on a given underlying stock to (functions of) time-to-maturity and moneyness of a set of traded option contracts. Because these features are fully observable, the resulting models are often called deterministic IVS models (henceforth, DIVSMs). Dumas *et al.* (1998), Peña *et al.* (1999), and Gonçalves and Guidolin (2006) present competing specifications within the general class of polynomial/spline DIVSMs. In this paper we adopt the DIVSM functional form proposed and successfully applied by Gonçalves and Guidolin (2006), because in their empirical study they estimate a range of alternative functional specifications and find that other representations yield a worse fit to empirical option data.[4] Thus, the deterministic linear function used in our paper is:

$$\ln \sigma_i(M_{it}, \tau_{it}) = \beta^i_{0t} + \beta^i_{1t} M_{it} + \beta^i_2 M^2_{it} + \beta^i_3 \tau_{it} + \beta^i_4 [M_{it} \cdot \tau_{it}] + \varepsilon_{it}, \tag{1}$$

where $\sigma_i(M_{it}, \tau_{it})$ is the implied volatility for the option contract written on stock i, with time-to-maturity τ_{it} and moneyness M_{it} "estimated" (computed) at time $t = 1, ..., T$; N_t is the number of different types of options contracts available on each day t, i.e., $i = 1, ..., N_t$, each with a common underlying stock; ε_{it} is a random shock that may cause the estimated IV to deviate from what the DIVSM implies at time t, for given moneyness and time-to-expiry. We use log implied volatility as the dependent variable. This has the advantage of always producing non-negative fitted implied volatilities. In this paper, we follow a vast literature (see, e.g., Tompkins, 2001) and define moneyness in time- and dividend-adjusted terms:

$$M_{it} \equiv \frac{\ln\left(\frac{K_i}{\exp(r_{t\to\tau_{it}}\tau_{it})S_{it} - FVD_{it\to\tau}}\right)}{\sqrt{\tau_{it}}}, \tag{2}$$

where $r_{t\to\tau_{it}}$ is a default risk-free nominal interest rate applicable to the period $[t, \tau]$, S_{it} is the time t (closing) price of the underlying stock i, and $FVD_{it\to\tau}$ is the time t forward value of all future dividends to be paid by stock i over the period $[t, \tau]$,

$$FVD_{it\to\tau} \equiv \sum_{s=t+1}^{\tau} \exp[r_{t+s\to\tau}(\tau - s)] D^i_{t+s}, \tag{3}$$

and K_i is the option strike price (notice that strikes are constant over time). Clearly, M_{it} will be positive for out-of-the-money call (in-the-money puts) and negative for in-the-money calls (out-of-the-money puts).

[4] On the one hand, we have experimented with a few alternative (simpler) functional forms and found that degree and strenght of the IVS predictability patterns that can be captured in this way are generally weaker and yielding lower economic benefits (trading profits). On the other hand, we consider (1) a sort of "upper bound" to the degree of sophisitcation that we can afford in modelling cross-sectional IVS because (1) implies the need to estimate 5 parameters for each available time period, which already imposes strong data requirements on a significant portion of the CBOE option universe.

In (1), β^i_{0t} is an intercept/level coefficient: in the case in which $\beta^i_{jt} = 0$ ($j = 1, ..., 4$), i.e., under a simple constant volatility model, the level IV is simply given by $\sigma_i(M_{it}, \tau_{it}) = \exp(\beta^i_{0t})$ for all strikes and maturities.[5] β^i_{1t} characterizes the moneyness (smile/skew) slope of the DIVSM; β^i_{2t} captures the moneyness (smile/skew) curvature; β^i_{3t} is the maturity (term structure) slope coefficient. Finally, and β^i_{4t} will describe any interaction between moneyness and time-to-maturity effects in the IVS, as commonly observed in the empirical literature. The unknown but fixed (only at time t) coefficients in (1) are estimated on a daily basis using generalized least squares (GLS) as recommended by Hentschel (2003).[6] However, as a robustness check we have also estimated the parameters using simple OLS as well.

The linear spline specification proposed in equation (1) facilitates the reproduction of multiple IVS shapes with extraordinary flexibility on both moneyness and maturity dimensions. For instance, Figure 2 plots the IVS for Microsoft equity options on July 15, 2003 using market data and compares it with the empirical fit obtained from the DIVSM in (1).[7] The resemblance of the two functions is striking to say the least, which shows that even a relatively parsimonious model with very few estimable parameters may actually describe rather complex IVS characterized by asymmetric smiles, skews, and rich term-structure effects.

2.2. *Modelling the Joint Cross-Sectional Dynamics of Market vs. Cross-Sectional IV Surfaces*

The DIVSM in Section 2.2 is a simple extension of models known and used in the literature since and DFW (1998). Although its fit may be surprisingly good most of the time, it just captures the main features of the IVS for one or more underlying assets only at one point in time. However, as early as DFW (1998) (see also GG, 2006), it has been widely reported that such DIVSMs are extremely unstable, in the sense that the shape and structure of the IVS would continuously move over time, often undergoing rather abrupt and sudden changes. GG (2006) observe that what is actually challenging to empirical finance researchers is not really the task of fitting the IVS at one point in time, but to actually propose useful (i.e., displaying predictive power) econometric models able to capture the dynamics of the IVS over time.

Therefore in this sub-section we pursue the effort of proposing a model that is capable to test whether the equity option IVS movements over time may be statistically predictable and, subsequently, whether it

[5] However $\sigma_i(M_{it}, \tau_{it}) = \exp(\beta^i_{0t})$ fails to imply that option i volatility will be constant over time, i.e., it is not as simple as a Geometric Brownian motion, Black-Scholes type world. For instance, a Geometric Brownian motion stochastic volatility model with uncorrelated shocks to the underlying stock price and volatility will imply a particular process for β^i_{0t}.

[6] Hentschel (2003) shows that in application to IVS problems, the presence of pervasive measurement errors could introduce heteroskedasticity and autocorrelation in the standard OLS residuals, thus making simple OLS estimators inefficient.

[7] To plot the right-hand side, we fit on July 15, 2003 the estimated equation (1) to the Microsoft option contracts traded on that day. Subsequently, with the parameters estimated, we produce artificial data for the implied volatilities using different moneynesses and maturities. The left-hand side presented a continuous IVS only as a result of interpolation and to favor the comparison of the two plots.

may provide economic value to traders and derivative portfolio managers. While this question has been already researched by GG (2006) with reference to S&P 500 index (SPX) options only, here we extend their model to encompass the case in which the dynamics of individual CBOE IV surfaces is the object of interest.[8] Additionally, we also ask whether the dynamics of the "market IVS" – here surrogated by the IVS of SPX options – may influence the dynamics of individual equity options IV surfaces. Similarly to GG (2006), we propose a vector time series model of VARX(p,q) type to be fitted to the first-pass coefficients estimated from the DIVSM of Section 2.1:

$$\hat{\boldsymbol{\beta}}_t^i = \boldsymbol{\phi}_0^i + \sum_{j=1}^{p} \boldsymbol{\Phi}_j^i \hat{\boldsymbol{\beta}}_{t-j}^i + \sum_{j=1}^{q} \boldsymbol{\Psi}_j^i \hat{\boldsymbol{\beta}}_{t-j}^{SPX} + \mathbf{u}_t^i \quad \mathbf{u}_t \sim IID\ N(0, \boldsymbol{\Omega}), \tag{4}$$

where $\hat{\boldsymbol{\beta}}_t^i \equiv [\beta_{0t}^i\ \beta_{1t}^i\ \beta_{2t}^i\ \beta_{3t}^i\ \beta_{4t}^i]'$ is the 5×1 vector time series of the first-pass estimated DIVSM coefficients specific to each individual equity option $i = 1, ..., N$; $\hat{\boldsymbol{\beta}}_t^{SPX}$ is the similarly defined vector of 5×1 coefficients characterizing the SPX IVS at each point in time; $\mathbf{u}_t$ is a 5×1 vector of shocks that represent random influences affecting the variation in $\hat{\boldsymbol{\beta}}_t^i$ which cannot be explained by its own past and past values (more correctly, estimates) of the coefficients that characterize the SPX IVS. This is a simple vector time series model with a Markov structure, that implies that recent movements in the IVS of options written on stock i as well as recent move in the market portfolio IVS should forecast subsequent moves in the IVS of options written on i as well.[9] We select the number of lags to be used in forecasting (p and q) using standard information criterion, such as the Bayes Schwartz criterion (BIC), after setting an arbitrary maximum value of 3.[10] Notice that setting $q \geq 1$ is empirically justified by the casual observation (see Section 3.2 for additional details) that there exists a strong linear association between IVS coefficients for individual equity options and lagged coefficients describing the IVS for the S&P 500.

For comparative purposes, we compare our main model in (4) with three benchmarks that represent simplifications of our baseline framework. The first benchmark is identical to (4) but it imposes $q = 0$, i.e., it is a VARX$(p, 0)$ model very close in spirit to the empirical framework employed in GG (2006), in the sense that only lagged features of the IVS for stock i are allowed to affect the subsequent shape of the IVS

[8] Using the notation introduced in (4), GG (2006) actually concerns only the dynamic process followed by $\hat{\boldsymbol{\beta}}_t^{SPX}$ and disregards completely the estimation and economic value of the process followed by $\hat{\boldsymbol{\beta}}_t^i$, $i = 1, ..., N$.

[9] Our approach is a reduced form approach to modeling the time variation in the implied volatility surface that results from more structural models such as the investors' learning models of option prices. In particular, if the state variables that control the dynamics underlying the fundamentals in these models are persistent and follow a regime switching model (such as in David and Veronesi (2002) or Garcia, Luger and Renault (2003)), a VAR model appears to be a reasonable reduced form approach to model the predictability in the implied volatility surface.

[10] With reference to SPX options, GG find that usually more parsimonious models (here, with low values of p and q) tend to outperform richer model. Hence our choice of a relatively modest maximum value for p and q in our empirical analysis.

for the same set of options:[11]

$$\hat{\boldsymbol{\beta}}_t^i = \boldsymbol{\delta}_0^i + \sum_{j=1}^{p} \boldsymbol{\Delta}_j^i \hat{\boldsymbol{\beta}}_{t-j}^i + \mathbf{v}_t^i \quad \mathbf{v}_t^i \sim IID\ N(0, \boldsymbol{\Omega}_V). \tag{5}$$

Also in this case, we select p by minimizing the BIC, given a pre-selected maximum number of lags equal to 3.

A second benchmark is naturally suggested by the work by DFW and Christoffersen and Jacobs (2004): it is a simple random walk model for the CBOE IVS coefficients in which the best prediction for tomorrow's coefficients (hence, shape of the IVS) is simply given by today's coefficients. In practice, this corresponds to the case of a VARX(1,0) model in which $\boldsymbol{\Delta}_1^i = \mathbf{I}_5$ (the 5×5 identity matrix) for all options $i = 1, ..., N$:

$$\hat{\boldsymbol{\beta}}_t^i = \boldsymbol{\delta}_0^i + \hat{\boldsymbol{\beta}}_{t-1}^i + \mathbf{v}_t^i \quad \mathbf{v}_t^i \sim IID\ N(0, diag\{\boldsymbol{\xi}\}), \tag{6}$$

so that the process of each individual coefficients simply becomes $\beta_{jt}^i = \delta_j + \beta_{jt-1}^i + v_{jt}^i$. Although GG (2006) have found that for SPX options this model is severely out-performed by a VARX(p), whether or not this finding may extend to CBOE options remains an interesting empirical question.

A third benchmark consists of the IVS predicted by Duan and Simonato's (2001) American option pricing Markov Chain GARCH model (henceforth Opt-GARCH). The choice of an option-GARCH benchmark intends to compare the performance of our two-stage, sequential approach with at least one "representative" from the no-arbitrage, structural option pricing literature. In particular, recent years have proposed a number of discrete time, single-factor ARCH type models that when applied to option pricing have provided pricing performances often comparable to more complex, multi-factor structural models (e.g., including jumps in the underlying assets and/or in volatility). For instance, Heston and Nandi (2000) report the superior performance (in- and out-of-sample) of their ARCH option pricing model over DFW's "ad-hoc strawman" (our second benchmark) when estimated on weekly S&P 500 options data for the period 1992-1994. Notice that with American options, the early exercise decision depends on the level of volatility, because it determines the live value of option contracts. Therefore, with a GARCH-style approach it is important to consider all possible paths that could be taken by the future conditional volatility in the American contracts because they are relevant for the valuation function. Duan and Simonato (2001) develop a numerical method to enable the valuation of American option contract in the GARCH framework using Markov chains. However, in their study they do not estimate the parameters, they use an arbitrary set of parameters to simulate American option prices to evaluate their model. Instead, we use nonlinear least square (NLS) methods to estimate the parameters using trading options prices, which tries to match model

[11] Clearly, while in GG's (2006) "uni-dimensional" exercise (i.e., limited to SPX only) such a model is the most sensible one, in our research design it simply corresponds to a restriction of the general VARX(p, q) and it allows us to test whether there is a market model-type effect in the IVS space.

option values to observed option prices as closely as possible.[12] In contrast to the dynamic IVS models considered in our paper, notice that Duan and Simonato's (2001) American option pricing Markov Chain GARCH does not allow for time varying coefficients (although it implies time-varying risk neutral densities). Thus, it seems sensible to require that (4) be able to out-perform at least this no-arbitrage/structural benchmark.

3. **In-Sample Estimation Results**

3.1. *The Data*

We use a sample of daily data on equity and S&P 500 options (of American and European styles, respectively) traded on the U.S. CBOE option market (calls and puts). Data are extracted from the OptionMetrics IVY database covering the period January 4, 1996 - December 29, 2006 (i.e. a total of 2,764 trading days). The database includes information about individual equity as well as and stock index options for the entire U.S. option market, and contains daily closing bid and ask quotes, volume and open interest for each option contract, as defined by the "intersection" of its underlying stock (or index), maturity, and strike price. In addition, OptionMetrics provides a zero coupon, riskless yield curve, which we interpolate to match the time-to-maturity of each contract, to compute $r_{t\to\tau_{it}}$. This database also collects information on the price of the underlying stocks or indices (S_t^i, in our notation). However, Battalio and Schultz (2006) have recently questioned the quality of this portion of the OptionMetrics' contents and have reported that the database would often record the quotes for the options and the underlying asset prices at different times, which may represent be a potential source of biases when arbitrage conditions are studied.[13] Even though studying the presence of arbitrage violations is not one of our main goals and as such (similarly to the argument in Goyal and Saretto, 2009) these small mismatches between the time of recording of option and underlying prices may have at most second-order effects, we have carefully proceed to use the NYSE TAQ database to retrieve information on the posted bid and ask quotes for the underlying stocks with the reference to the same time stamp as the option quotes (i.e., at 4:02 p.m.).[14]

OptionMetrics does provide data for several hundreds sets of option contracts that over time have been

[12] We would like to thank Prof. Jin-Chuan Duan for making his Matlab codes available to us. GG (2006) adopt a related benchmark (Henston and Nandi's, 2000, NGARCH option pricing model) that would however be inappropriate to price American options. Hence our choice of Duan and Simonato's pricing framework.

[13] The main issue is that the OptionMetrics database matches underlying stock closing prices that occurred no later than 4:00 p.m. with closing option quotes that may have been posted as late as 4:02 p.m.

[14] Our claim on second order effects has one important caveats: non-synchroneous stock and option quotes that may show up as apparent (in fact, they are not) violations of the lower bound for option prices, will imply negative estimates of the underlying IV, which is clearly problematic (as we are taking logs of IVs) and makes little sense. After matching OptionMetrics options data with TAQ stock quotes, we had no such occurrences in our analysis. Notice that resorting to TAQ NYSE implicitly restricts our analysis only to options written on NYSE-listed companies.

traded on each given underlying stock. In our empirical analysis we have adopted two basic selection criteria. First, we focus only on options that have been continuously traded (with minor exceptions of a few days at most) over our entire sample period, 1996-2006. Second, we focus only on options on underlying stocks that are characterized by a high trading volume over our sample period, at least in average terms. Although volume and liquidity are not identical concepts in microstructure theory, the idea is to avoid that our conclusions on predictability and economic value may mostly depend on options that are infrequently traded and whose quotes tend to be systematically stale. In practice, we have applied this second selection criterion in the following way: after ranking all equity options on the basis of their daily average volumes (across all strikes and maturities) in our sample period, we have selected the top 70 options.

After selecting these 70 underlying stocks, we have proceed to apply five exclusionary criteria destined to filter out observations that contain obvious misrecordings and that may hardly be thought of as expressions of well-functioning (liquid) markets. First, we exclude all observations – as defined by the data K_i, τ_{it}, S_t^i, $\{D_s^i\}_{s=t+1}^{\tau_{it}}$, $\{r_s\}_{s=t+1}^{\tau_{it}}$, $CALL(K_i, \tau_{it})$, and $PUT(K_i, \tau_{it})$ – that violate at least one of the basic no-arbitrage conditions, because such violations are presumably due to misrecordings. In the case of the American-style equity options we require:

$$S_t^i - PVD_t^i(\tau_{it}) - PVK_t^i(\tau_{it}) \quad < \quad CALL(K_i, \tau_{it}) < S_t^i - PVD_t^i(\tau_{it}) \tag{7}$$

$$PVD_t^i(\tau_{it}) + PVK_t^i(\tau_{it}) - S_t^i \quad < \quad PUT(K_i, \tau_{it}) < K_i \tag{8}$$

where $CALL(K_i, \tau_{it})$ and $PUT(K_i, \tau_{it})$ are the call and put prices for strike K_i and expiry τ_{it}, respectively, and $PVD_t^i(\tau_{it})$ and $PVK_t^i(\tau_{it})$ are the present values of the series of future dividends on the underlying stock between t and τ_{it} and of the option strike, respectively.[15] In the case of the European-style SPX options, we impose again (7), plus:

$$PVD_t^i(\tau_{it}) + PVK_t^i(\tau_{it}) - S_t^i < PUT(K_i, \tau_{it}) < PVK_t^i(\tau_{it}). \tag{9}$$

Second, we exclude thinly traded option contracts with an arbitrary cut-off chosen at 10 transactions per day to avoid liquidity effects in option prices.[16] Third, we drop all contracts with less than six (trading) days and more than one year to expiration as their prices are usually noisy and, as argued by DFW, they usually contain little information on the IVS. Fourth, as in Bakshi *et al.* (1997) and GG (2006) we exclude contracts with prices lower than \$0.30 for equity options and \$3/8 for S&P 500 index options to mitigate the impact of price discreteness on the IVS structure.[17] Fifth, similarly to DFW (1998) and Heston and Nandi

[15]Their formal definitions are $PVD_t^i(\tau_{it}) \equiv \sum_{s=t+1}^{\tau} \exp[r_{t+s\to\tau}(\tau - s)]D_{t+s}^i$ and $PVK_t^i(\tau_{it}) \equiv \exp[r_{t\to\tau}\tau]K^i$, respectively. In the following, we assume perfect foresight on future dividends, as common in many empirical applications (see e.g., Bakshi, Cao, and Chen, 1996).

[16]For instance, Chan *et al.* (2002) exclude contracts with fewer than 20 transactions per day. We relax this constraint, because on each day we need enough contracts over different moneyness and maturities to estimate our DIVSM.

[17]This is due to the proximity of these prices to the minimum tick size: for equity options the minimum tick is \$0.05 for trading prices bellow \$3; for index options the smallest tick is \$1/16.

(2000), we exclude options contracts with moneyness either less than 0.9 or in excess of 1.1. Usually, deep in- and out-of-the money options suffer from liquidity issues that even our volume filters cannot precisely detect.

Finally, as in Bakshi *et al.* (1997) and DFW (1998), we assume dividend cash flows to be perfectly anticipated by market participants. In what follows, we calculate the implied volatilities for American options using a binomial tree model with the Cox, Ross, and Rubinstein (1979) approach, as in Goyal and Saretto (2009). In the case of European-style contracts, we numerically invert Black and Scholes' (1973) model to obtain estimates of implied volatilities.

3.2. *Fitting CBOE IV Surfaces and Their Dynamics*

Given our selection criteria for CBOE contracts, we have ended up estimating at a daily frequency 71 sets of DIVSM coefficients, i.e., 70 sets $\hat{\boldsymbol{\beta}}_t^i \equiv [\beta_{0t}^i \ \beta_{1t}^i \ \beta_{2t}^i \ \beta_{3t}^i \ \beta_{4t}^i]'$ for $i = 1, ..., 70$ and one additional set for SPX implied volatilities, $\hat{\boldsymbol{\beta}}_t^{SPX}$, for $t = 1, ...T$. To observe the in-sample fit, we present in the Tables 1 and 2 summary statistics of the coefficients estimated, for the equity and S&P 500 IV surfaces respectively. On average and similar to Dennis and Mayhew (2000) results, the IVS moneyness slopes are negative for equity options (-0.257), but not as negatively steeped as S&P 500 option (-0.882). In addition, the moneyness curvatures of the IV surfaces are on average higher for equity options (0.559) than index options (0.365). In relation to the maturity dimension, the maturity slopes of the IVS are on average negative for equity options (-0.039) and positive for S&P 500 options (0.082). The relations between maturity and moneyness are negative and stronger for index (-0.591) than the equity (-0.242) options IV surfaces. In addition, Table 3 presents on average the IVS coefficients for different industry groups. An important observation to note is that equity options in the technology group have IV surfaces with a particular behavior in relation to other industry groups. For instance, IV surfaces of technology equities are the most volatile among all industry groups, which is represented by the level coefficient (-0.839); the moneyness slopes and curvatures (-0.160 and 0.373) are smaller in absolute terms for technology IVS than other equity options IV surfaces; they have a steeper negatively sloped IVS term-structures (-0.071); and the weakest maturity and moneyness relationships (-0.175).

In relation to the equity-index IVS relationships, Table 4 presents correlations between equity and S&P 500 IVS coefficients estimated by equation (1). Practically, all coefficients are significant for the equity and index IV surfaces. It is particularly interesting to highlight the relationships between the level coefficient ($\beta_{0,Equity,t}$) for equity options IV surfaces and all coefficients of the S&P 500 options IVS. Similarly, a strong association is observed between the slopes of the equity IVS term-structures ($\beta_{3,Equity,t}$) and all coefficients of the S&P 500 options IVS. These relationships suggest a multivariate linear model for the set of estimated coefficients that should relate the equities and index IVS shape characteristics.

For instance, Table 5 presents linear correlations between equity and lagged S&P 500 options IV surfaces coefficients. Table 5 reveals a high linear relation between equities and one-day lagged index IVS coefficients, in which practically all values are significant. Similar to cross-sectional coefficients showed in Table 4, we find an exceptionally strong association of the level ($\beta_{0,Equity,t}$) and the term-structure slopes ($\beta_{3,Equity,t}$) of equity options IV surfaces in relation to all one-day lagged S&P 500 IVS coefficients. In addition, Table 6 and Table 7 exhibit linear correlations between equity options IVS coefficients and their lagged values, one-day and three-days respectively. Table 6 shows that all correlations are significant between equities options IVS coefficients and their one-day lagged coefficients. However, the highest correlation of each coefficient is in relation to itself but lagged one period ($\beta_{0,Equity,t}$ and $\beta_{0,Equity,t-1}$; $\beta_{1,Equity,t}$ and $\beta_{1,Equity,t-1}$;and so on). A similar effect is observed in Table 7 but with lower intensity due to the number lags in the coefficients. Our VARX approach seems to be a reasonable and intuitive since we use the linear relationships present in the data, which allow us to describe in a simple way the equity IVS dynamics.

4. **Predictability Performance**

4.1. *Statistical Measures of Predictability*

One of the main objectives of our research is to examine whether the movement over time of the IVS in equity options may be modeled using our dynamic approach. In this section, we present statistical measures to evaluate the IVS predictability for all models. Essentially, we analyze the level and the direction of change of one-day-ahead implied volatilities and their respective option prices. We estimate on a daily frequency all models for each equity options IVS over the period January 2, 1997 - December 28, 2006. The daily calibrations use six months rolling estimation windows (i.e. between day t and day t-(252/2)). Thus, we calculate daily one-day-ahead predictions of the equity options IVS coefficients $\widehat{\beta}^{E}_{t+1}$, which allow us to forecast implied volatilities for different moneyness and maturities on the equity IVS. Nevertheless, to calculate the option prices, we do not have predictions for the stock prices and interest rates. Following GG (2006), we suppose that good forecasts for those values on day t+1 are the prices on the day t. In addition, since we are predicting equity options, which are American style, we calculate the forecasted option prices using a binomial tree model with the Cox, Ross, and Rubinstein (1979) approach.

To evaluate the out-of-sample statistical performance of all models for each day and equity option IVS we calculate the following statistical measures of implied volatilities and option prices:

i) The root mean squared prediction error (RMSE) in implied volatilities (RMSE-V) and in option prices (RMSE-P).

ii) Since the RMSE statistic has scaling problems, and we are working with different equity options with different levels of implied volatilities and option prices, we use the Theil U statistic for implied volatilities (Utheil-V) and options prices (Utheil-P). The Theil U statistic is defined as:

$$Theil\ \ U_t = \sqrt{\frac{\frac{1}{N_t}\sum_{i=1}^{N_t}\left(y_{i,t} - f_{i,t|t-1}\right)^2}{\frac{1}{N_t}\sum_{i=1}^{N_t} y_{i,t}^2}} \tag{10}$$

where $i = 1, ..., N_t$, and N_t is the number of options contracts available on each day t and for each group of contracts with the same underlying. $y_{i,t}$ is the true value of the variable that we want to predict for the contract i, and $f_{i,t|t-1}$is the value of the forecasted variable at $t-1$.

iii) The mean absolute prediction error (MAE) in implied volatilities (MAE-V) and option prices (MAE-P).

iv) Similar to the RMSE, since the MAE statistic has scaling problems, we use the relative mean absolute prediction error (MAE-Rel) in implied volatilities (MAE-Rel-V) and in option prices (MAE-Rel-P). The MAE-Rel is defined as:

$$MAE\text{-}Rel_t = \frac{1}{N_t}\sum_{i=1}^{N_t}\left|\frac{y_{i,t} - f_{i,t|t-1}}{y_{i,t}}\right| \tag{11}$$

v) The mean correct prediction of direction of change (MCPDC) that is the average frequency (% of observations) for which the change in the value predicted by a model is of the same sign as the true value observed. We calculate this value for implied volatilities (MCPDC-V) and option prices (MCPDC-P).

Table 8 reports out-of-sample averages for the statistical measures of predictability for the equity IVS forecasts. We include additional to our Models (Model 1 and benchmarks models) a random walk for the implied volatilities (i.e. for a contract, the best prediction of tomorrow's implied volatility is the value of today). Table 4 shows that the random walk models (for the IVS coefficients and volatilities) have a better performance in relation to other models based on the statistics that evaluate the level of change in the implied volatilities and the respective option prices. However, the random walk models have a poor performance when it comes to evaluating the direction of change (i.e. MCPDC statistic) of one-day-ahead implied volatilities and option prices. The intuition behind these results is that implied volatilities have a high persistence, which can be observed in the correlation between the level coefficient ($\beta_{0,Equity,t}$) and its lagged values ($\beta_{0,Equity,t-1}$ and $\beta_{0,Equity,t-3}$) as reported in Table 6 and Table 7. This is not surprising since implied volatilities should have a similar behavior to equity returns volatilities, which are characterized by the grouping phenomenon and heteroscedasticity basis of GARCH models (a high persistence). However, the random walk models, which are static by definition, perform poorly in forecasting the direction of change of one-day-ahead predictions due to the fact that they do not take into account any dynamics of IVS movements over time. For this reason dynamic models are better to predict the direction of the movements over time, which is extremely relevant when it is necessary to implement trading strategies as we show in the following section. Among the dynamic models (i.e. Model 1; Model GG; and Option GARCH model), Model 1 has the best performance, and the best performance of all models when it is necessary to predict direction of movements of the implied volatilities (57.89%) and option prices (53.93%).

4.2. *The Economic Value of Predictability*

Additional to the statistical measures of predictability, it is important to examine whether our models may generate economic value to agents in the market. We evaluate whether trading strategies using different models may generate economic benefits to investors in the option markets. Similar to our statistical measures of predictability, we study out-of-sample forecast over the period January 2, 1997 - December 28, 2006, however we use trading strategies to evaluate the economic predictability of the models proposed. As in Day and Lewis (1992), Harvey and Whaley (1992) and Gonçalves and Guidolin (2006), the main idea of these strategies is: if a predictability model anticipates for the day t+1 an increase in the options price in relation to the day t (given for a increment in the respective forecasted implied volatility) this option contract is purchased. The contract is sold if the model anticipates that the implied volatility will decrease. For this reason, the MCPDC statistic in the statistical measures of predictability is crucial and highly related to our economic performance analysis. Correct prediction of directions of changes in the forecasted implied volatilities and option prices give us right trading decision about buying and selling. Our hypothetical trader invests $1,000 in delta-hedge and $1,000 in straddle portfolios, which are held for one trading day. Delta-hedge portfolios are less profitable than straddle portfolios, since delta hedge positions benefit from implied volatility predictabilities (through the IVS dynamics) of only one option (call or put). Instead, straddle portfolios take advantage of our implied volatilities forecasts using two options (call and put), which allow them to reach higher profits in each positions. However, delta-hedge portfolios have an inferior transaction cost in relation to straddle portfolios, because stock trading costs are lower than transactional costs in options. Similar to the procedure showed in the statistical measures of predictability, every day t we forecast for the day t+1 implied volatilities and option prices (using all models). For the delta-hedge positions, implied deltas are calculated with the binomial tree model with the Cox, Ross and Rubinstein (1979) approach.

Suppose that a trading rule suggests that Q contracts should be traded at time t. Therefore, if $Q = 0$ the trader invest the $1,000 in the riskless asset for one trading period. Let V_t^{D-H} be the total value of all delta-hedge positions on day t, and for a portfolio of equity options with the same underlying asset (i.e. to buy or sell, call and put option contacts):

$$
\begin{aligned}
V_t^{D-H} \quad = \quad & \sum_{i \in Q_+^{call}} \left(C_{i,t} - S_t \Delta_{i,t}^C\right) + \sum_{i \in Q_+^{put}} \left(P_{i,t} + S_t \Delta_{i,t}^P\right) - \\
& \sum_{i \in Q_-^{call}} \left(C_{i,t} - S_t \Delta_{i,t}^C\right) - \sum_{i \in Q_-^{put}} \left(P_{i,t} + S_t \Delta_{i,t}^P\right)
\end{aligned} \tag{12}
$$

where $C_{i,t}$ ($P_{i,t}$) denotes the price of a call (put) contract i and at time t. Q_+^{call}(Q_-^{call}) is the subset of call contracts that should be bought (sold), and the same definition for put contracts; and $\Delta_{i,t}^C$ ($\Delta_{i,t}^P$) is the call (put) delta ratio. If the net value of the delta-hedge portfolio is positive or zero (i.e. $V_t^{D-H} \geq 0$) the trader invests in her delta-hedge positions the quantity of $X_t^{D-H} = \$1000/V_t^{D-H}$ that has a total cost of \$1,000. Consequently, the one-day net gain (G_{t+1}^{D-H}) is:

$$
\begin{aligned}
G_{t+1}^{D-H} \quad = \quad & X_t^{D-H} \left[\sum_{i \in Q_+^{call}} \left(\left(C_{i,t+1} - S_{t+1} \Delta_{i,t}^C\right) - \left(C_{i,t} - S_t \Delta_{i,t}^C\right) \right) \right] + \\
& X_t^{D-H} \left[\sum_{i \in Q_+^{put}} \left(\left(P_{i,t+1} + S_{t+1} \Delta_{i,t}^P\right) - \left(P_{i,t} + S_t \Delta_{i,t}^P\right) \right) \right] + \\
& X_t^{D-H} \left[\sum_{i \in Q_-^{call}} \left(- \left(C_{i,t+1} - S_{t+1} \Delta_{i,t}^C\right) + \left(C_{i,t} - S_t \Delta_{i,t}^C\right) \right) \right] + \\
& X_t^{D-H} \left[\sum_{i \in Q_-^{put}} \left(- \left(P_{i,t+1} + S_{t+1} \Delta_{i,t}^P\right) + \left(P_{i,t} + S_t \Delta_{i,t}^P\right) \right) \right]
\end{aligned} \tag{13}
$$

If the net cost of the portfolio is negative (i.e. $V_t^{D-H} < 0$) she invests in a portfolio with the positions that have an active signal in the quantity $X_t^{D-H} = \$1000/|V_t^{D-H}|$, and she takes the \$1,000 generated by the inflows plus the \$1,000 initially on hand and invests this amount in a riskless asset that day. Therefore, in this scenario the net gain is $G_{t+1}^{D-H} + \$2,000 \cdot (\exp(r_t/252) - 1)$, where G_{t+1}^{D-H} is computed by the equation (13).

The same course of action is exactly applied in straddle portfolios. However, the total value of all straddle positions ($V_t^{Straddle}$) on day t and for the portfolio of equity options with underlying asset is defined as:

$$
\begin{aligned}
V_t^{Straddle} \quad = \quad & \sum_{i \in Q_+^{call}} \left(C_{i,t} + P_{i,t}^{C_{i,t}}\right) + \sum_{i \in Q_+^{put}} \left(P_{i,t} + C_{i,t}^{P_{i,t}}\right) - \\
& \sum_{i \in Q_-^{call}} \left(C_{i,t} + P_{i,t}^{C_{i,t}}\right) - \sum_{i \in Q_-^{put}} \left(P_{i,t} + C_{i,t}^{P_{i,t}}\right)
\end{aligned} \tag{14}
$$

where, $P_{i,t}^{C_{i,t}}$ is the put contract with the same features of the contract $C_{i,t}$ that causes the straddle position (i.e. same strike price, maturity and underlying asset); and in the same way, $C_{i,t}^{P_{i,t}}$ is the call contract with the same features of the contract $P_{i,t}$ that generates the straddle position. Similarly to delta-hedge portfolios, if the net cost of the portfolio is positive or zero (i.e. $V_t^{Straddle} \geq 0$) the trader invests in her straddle positions the quantity of $X_t^{Straddle} = \$1000/V_t^{Straddle}$ that has a total cost of \$1,000. Thus, the one-day net gain (Gt+1) is:

$$\begin{aligned} G_{t+1}^{Straddle} = & X_t^{Straddle}\left[\sum_{i\in Q_+^{call}}\left(\left(C_{i,t+1}+P_{i,t+1}^{C_{i,t}}\right)-\left(C_{i,t}+P_{i,t}^{C_{i,t}}\right)\right)\right]+ \\ & X_t^{Straddle}\left[\sum_{i\in Q_+^{put}}\left(\left(P_{i,t+1}+C_{i,t+1}^{P_{i,t}}\right)-\left(P_{i,t}+C_{i,t}^{P_{i,t}}\right)\right)\right]+ \\ & X_t^{Straddle}\left[\sum_{i\in Q_-^{call}}\left(-\left(C_{i,t+1}+P_{i,t+1}^{C_{i,t}}\right)+\left(C_{i,t}+P_{i,t}^{C_{i,t}}\right)\right)\right]+ \\ & X_t^{Straddle}\left[\sum_{i\in Q_-^{put}}\left(-\left(P_{i,t+1}+C_{i,t+1}^{P_{i,t}}\right)+\left(P_{i,t}+C_{i,t}^{P_{i,t}}\right)\right)\right] \end{aligned} \tag{15}$$

When the net cost of the portfolio is negative (i.e. $V_t^{Straddle} \leq 0$) she invests in a portfolio with the straddle positions that have an active signal in the quantity $X_t^{Straddle} = \$1000/|V_t^{Straddle}|$, and she takes the \$1,000 generated by the inflows plus the \$1,000 initially on hand and invests this amount in a riskless asset that day. Therefore, in this set-up the net gain is $G_{t+1}^{Straddle} + \$2,000 \cdot (\exp(r_t/252) - 1)$, where $G_{t+1}^{Straddle}$ is calculated using equation (15).

We consider different trading rules. First, a strategy in which our hypothetical trader invests in all contracts to exploit entirely the flexibility provided by our approach (henceforth Trading Rule A). Therefore, Trading Rule A reflects all economic benefits of our equity-market IVS dynamic models completely. However, due to the number of options and stock traded to use the complete equity IVS dynamics, the transaction costs are prominent in delta-hedge and straddle portfolios when our investor follows Trading Rule A. Second, in order to mitigate the effect of transaction costs, our investor selects one contract per option in the IVS, which has the highest expected trading profit after transaction costs to generate her delta-hedge and straddle portfolios (i.e. expected trading profit – expected transaction costs)(henceforth Trading Rule B). Third, following Harvey and Whaley (1992), we consider a trading rule in which trades only occur on closest-at-the-money and shortest-term contracts to produce a single position (henceforth Trading Rule C).

As we mentioned previously, we analyze the trading strategies before and after transaction costs. Dynamic transaction costs were incorporated using Bis-Ask spreads. In relation to equity options bid-ask spreads, De Fontnouvelle, Fisher, and Harris (2003) and Mayhew (2002) report that the ratio of effective to quoted spread is less than 0.5. Battalio, Hatch, and Jennings (2004) find that for a small group of equity options the ratio of effective spread to quoted spread is around 0.8. Therefore, similar to Goyal and Saretto (2009) we take two effective spread measures, 0.5 and 1.0 of the quoted spread, as our trading costs per option traded. Similar rule is applied to stock trading costs (i.e. 0.5 and 1.0 of the quoted stock spread is taken as trading costs per each transaction).

Table 9 and Table 10 present out-of-sample average returns before transaction costs for all trading strategies using delta-hedge and straddle portfolios respectively. Table 9 and Table 10 show that all IVS dynamic models (i.e. Model 1, Model GG and Opt-GARCH) are successful at generating profitable strategies when trading costs are not applied. However, the random walk model, which is static by construction, gives a poor economic performance, indeed negative profits for Trading Rule A in delta-hedge portfolios (-0.053%). Table 9 and Table 10 report that Model 1 has the highest returns when Trading Rule A is used in delta-hedge (0.476%) and straddle (4.513%) portfolios. These results are relevant for our research, because Trading Rule A exploits entirely all economic benefits of the equity IVS dynamic models. As expected, when our hypothetical trader follows trading rules that select a single option contract per equity IVS, Trading Rule B obtains highest profits in relation to Trading Rule C, since Trading Rule B chooses the equity option contract with highest expected benefits. Nevertheless, for all strategies, returns are statistical significant only when trading occurs using straddle portfolios.

Table 11 and Table 12 report out-of-sample average returns, after transaction costs, for our trading strategies with delta-hedge and straddle portfolios respectively. In Table 11 and Table 12, we use 0.5 of the quoted bid-ask spreads as proxies of trading costs (for the equity option contracts and underlying assets). Unsurprisingly, in Table 11 we can observe that Trading Rule A is affected enormously by transaction costs in delta-hedge portfolios, in which the Model 1 has the best performance (-7.705%). However, the highest effect of transaction costs in Trading Rule A is observed in Table 12 in straddle portfolios, since straddle positions involve 2 option contacts that have higher trading costs than stocks. In straddle portfolios using Trading Rule A, the best average returns are presented by Model 1 (-14.352). In addition, Table 11 and Table 12 shows that transaction costs have minor effects when trading strategies involve a single position (Trading Rule B and Trading Rule C). Nevertheless, all trading rules for delta-hedge and straddle portfolios are not profitable on average when this level of transaction cost is applied. Similar observable facts and effects are showed more intensely when a double level of transaction costs is used (Table 13). Table 13 reports out-of-sample average returns for the straddle-based trading strategies, when 1.0 of the quoted bid-ask spreads are used as proxies of transaction costs. Table 13 shows an incremented negative effect of transition costs, in relation to Tables 11 and 12, respectively, with all trading rules and portfolios, but specially in Trading Rule A with a the performance for model 1 in delta-hedge (-15.886%) and straddle (-33.217%) portfolios. The effects of transaction costs in our trading strategies and portfolios reveal that the equity IVS predictabilities found and modelled by our approach are not incompatible with market efficiency.

5. **Conclusions**

Empirical research has identified that implied volatilities tend to differ across strike prices and maturities. These observable facts are known as the IVS. In addition, recent studies confirm that the IVS changes over different periods as well. Even more, it has been suggested that there are some predictability patterns in the IVS which can be exploited in practice (e.g. Gonçalves and Guidolin, 2006). However, there is a lack in the literature in relations to research empirically IVS cross-sectional properties and predictable dynamics with groups of equity options. Researchers usually develop set-ups in which options on a single underlying asset are considered (principally index options). Furthermore, research remains scarce when it comes to investigate relationships between equity option IVS and the market IVS implicit in index options.

In this paper we study the relationships between the equity and the market IV surfaces, and we use these associations to model the equity IVS cross-sectional dynamics. First, we model 70 equity options IV surfaces and the S&P 500 options IVS daily along the moneyness and time-to-maturity dimension using a deterministic functional form to describe the IVS's shape characteristics. Second, with the estimated coefficients we calculate linear correlations between equity and index IVS shape characteristics to find relationships. We find a strong association, using the cross-section and one-period lagged coefficients, between equity options IV surfaces and S&P 500 IVS. In addition, we find a high association between equity options IVS coefficients and their lagged values (i.e. lagged equity options IVS coefficients). Third, with these relationships we forecast daily one-day-ahead values of the equity IVS shape characteristics using a VARX models that include the index options' IVS coefficients.

To empirically evaluate the statistical and economic performances of our approach we forecast daily one-day-ahead implied volatilities (and their respective prices) using the equity options IVS forecasted by our models. We show that our approach gives accurate forecast of the direction of change of one-day-ahead implied volatilities and option prices, both in absolute and compared with other benchmark models, which is extremely relevant when it is necessary to implement trading strategies to evaluate economically our models. Correct prediction of directions of changes in the forecasted implied volatilities and option prices give us right trading decisions about buying and selling in different trading strategies. In this fashion, to evaluate the economic predictability of our models, we simulate fixed investments of $1,000 per day using one-step ahead daily delta-hedge and straddle portfolios. For an option contract, if our models anticipate on the day t+1 an increment of the implied volatility in relation to the current day t, this option contract is purchased.

The contract is sold if the model anticipates that the implied volatility will decrease. These strategies generate on average positive out-of-sample returns when transaction cost are not imposed. However, most of this profitability disappears when we increase the level of transaction costs, which is in harmony with market efficiency.

Our approach is intuitive since we are using a functional form to characterize the IV surface shape characteristics, and it is easy-to-compute forecasts of implied volatilities and option prices using the equity IVS dynamics. We find the fit provided by our models is remarkable and describe the dynamics of different equity IVS. However, other interesting issues remain to be addressed. A complete explanation for the existence of the IVS is beyond the scope of this paper. In addition, the reason of the equity-market IVS relationships or proposes of possible option pricing model incorporating market information are left for future research.

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Table 1

Summary Statistics for Cross-Sectional CBOE IVS Coefficients: Averages Across Underlying Stocks and Over Time

The table reports summary statistics obtained across time as well as "names" (defined as the underlying stock on which option contracts are written) for the coefficients in the deterministic implied volatility function model:

$$\ln \sigma_{i,t} = \beta_0 + \beta_0 M_{i,t} + \beta_0 M_{i,t}^2 + \beta_3 \tau_{i,t} + \beta_0 (M_{i,t} \cdot \tau_{i,t}) + \varepsilon_{i,t} ,$$

estimated over time for each individual underlying stock. In the table, RMSE is the root mean squared error for implied volatilities, MAE the mean absolute error, and relative MAPE the mean absolute percentage error. The upper panel of the table reports GLS estimates of the coefficients that account for heterogeneous measurement error (as induced by heterogeneous liquidity and market frictions across moneyness and term structure dimensions) on the IVS (see Hentschel, 2003). The lower panel report simpler, raw OLS estimates.

Coefficients Statistics	Min.	Max.	Mean	Std. Dev.	Skew	Kurtosis
	Average GLS Estimates and Statistics (Across Equity Options)					
β_0	-2.384	2.392	-1.125	0.406	0.140	-0.190
β_1	-67.42	213.8	-0.257	1.177	73.36	11640.5
β_2	-2044.6	1384.9	0.559	11.838	-31.21	13237.6
β_3	-14.54	7.041	-0.039	0.189	-5.258	404.91
β_4	-458.0	176.4	-0.242	2.039	-99.00	22894.5
R^2	0.010	0.999	0.600	0.457	-7.546	171.80
R^2 adj.	0.009	0.999	0.452	0.657	-8.738	222.24
RMSE	0.001	0.790	0.038	0.027	3.266	29.290
Theil's U	0.002	1.181	0.038	0.034	5.762	69.236
MAE	0.001	0.600	0.026	0.017	4.408	58.668
MAPE	0.002	0.981	0.036	0.526	251.8	74738.3
	Average OLS Estimates and Statistics (Across Equity Options)					
β_0	-2.840	2.320	-1.125	0.410	0.140	-0.208
β_1	-143.2	104.4	-0.220	1.091	-1.202	4641.8
β_2	-2206.0	1561.6	0.551	11.362	-15.93	17358.0
β_3	-14.21	10.27	-0.039	0.215	-2.935	283.02
β_4	-444.8	195.8	-0.313	2.122	-76.30	17467.4
R^2	0.012	1.000	0.761	0.208	-1.266	1.009
R^2 adj.	0.008	0.999	0.675	0.287	-1.361	1.466
RMSE	0.002	0.659	0.029	0.019	4.280	53.370
Theil's U	0.002	0.918	0.030	0.028	6.393	80.334
MAE	0.002	0.549	0.023	0.016	5.044	72.04
MAPE	0.001	0.936	0.032	0.489	245.6	69948.1

Table 2

Summary Statistics for SPX IVS Coefficients: Averages Over Time

The table reports summary statistics obtained across time for the coefficients in the deterministic implied volatility function model:

$$\ln \sigma_{i,t} = \beta_0 + \beta_0 M_{i,t} + \beta_0 M_{i,t}^2 + \beta_3 \tau_{i,t} + \beta_0 (M_{i,t} \cdot \tau_{i,t}) + \varepsilon_{i,t}$$

applied to S&P 500 index options (SPX). In the table, RMSE is the root mean squared error for implied volatilities, MAE the mean absolute error, and relative MAPE the mean absolute percentage error. The upper panel of the table reports GLS estimates of the coefficients that account for heterogeneous measurement error (as induced by heterogeneous liquidity and market frictions across moneyness and term structure dimensions) on the IVS (see Hentschel, 2003). The lower panel report simpler, raw OLS estimates.

Coefficients Statistics	Min.	Max.	Mean	Std. Dev.	Skew	Kurtosis
	Average GLS Estimates and Statistics					
β_0	-2.426	-0.869	-1.729	0.323	-0.066	-0.574
β_1	-2.304	0.486	-0.882	0.367	-0.757	0.571
β_2	-2.115	3.301	0.365	0.664	0.388	0.952
β_3	-0.598	0.565	0.082	0.173	-0.294	-0.036
β_4	-6.946	1.301	-0.591	0.429	-1.584	18.864
R^2	0.128	0.995	0.802	0.200	-1.867	3.626
R^2 adj.	0.104	0.994	0.794	0.208	-1.862	3.606
RMSE	0.008	0.523	0.061	0.037	2.092	11.528
Theil's U	0.005	0.337	0.037	0.024	2.311	13.488
MAE	0.006	0.413	0.041	0.023	2.922	30.013
MAPE	0.004	0.395	0.026	0.017	5.435	86.746
	Average OLS Estimates and Statistics					
β_0	-2.441	-0.856	-1.747	0.325	0.010	-0.528
β_1	-2.167	0.478	-0.796	0.409	-0.389	0.236
β_2	-1.210	6.072	0.716	0.854	1.529	3.834
β_3	-0.584	0.814	0.105	0.202	-0.282	-0.097
β_4	-6.936	1.301	-0.889	0.828	-1.510	4.731
R^2	0.170	0.997	0.853	0.136	-1.731	3.214
R^2 adj.	0.146	0.997	0.847	0.142	-1.731	3.204
RMSE	0.008	0.512	0.053	0.031	2.423	20.379
Theil's U	0.005	0.329	0.032	0.020	2.656	22.012
MAE	0.006	0.421	0.041	0.024	2.711	25.851
MAPE	0.004	0.436	0.026	0.017	6.121	116.724

Table 3

Summary Statistics for Cross-Sectional CBOE IVS Coefficients: Averages and Standard Deviations by Industry Classifications and Over Time

The table reports summary statistics obtained across time as well as industries (as defined by the three-digit SIC code to which the underlying stock belongs to) for estimated GLS coefficients in the deterministic implied volatility function model:

$$\ln\sigma_{i,t} = \beta_0 + \beta_0 M_{i,t} + \beta_0 M_{i,t}^2 + \beta_3 \tau_{i,t} + \beta_0 (M_{i,t} \cdot \tau_{i,t}) + \varepsilon_{i,t} ,$$

estimated over time for each individual underlying stock.

Industry Groups	*β 0*		*β 1*		*β 2*		*β 3*		*β 4*	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**	**Mean**	**Std. Dev.**
Basic Materials	-1.231	0.327	-0.210	0.401	0.518	2.140	-0.040	0.149	-0.259	0.871
Conglomerates	-1.383	0.344	-0.369	0.321	0.706	1.902	0.004	0.179	-0.294	0.675
Consumer Goods	-1.298	0.350	-0.298	0.441	0.662	2.871	-0.001	0.177	-0.264	0.902
Financial	-1.254	0.404	-0.346	1.555	0.766	10.61	-0.011	0.222	-0.360	3.987
Healthcare	-1.178	0.314	-0.294	0.780	0.636	5.387	-0.050	0.186	-0.218	1.451
Industry Goods	-1.180	0.330	-0.239	0.412	0.437	2.213	-0.045	0.178	-0.300	0.790
Services	-1.216	0.332	-0.303	0.702	0.543	5.971	-0.023	0.164	-0.195	1.137
Technology	-0.839	0.396	-0.160	1.626	0.373	19.75	-0.071	0.194	-0.175	1.426

Table 4

Simultaneous Correlations for Cross-Sectional CBOE and SPX IVS Coefficients

The table reports pair-wise correlations statistics for average (across time and underlying stocks) CBOE cross-sectional IVS coefficients and for SPX IVS coefficients, all obtained by GLS estimation of the deterministic implied volatility function model

$$\ln \sigma_{i,t} = \beta_0 + \beta_0 M_{i,t} + \beta_0 M_{i,t}^2 + \beta_3 \tau_{i,t} + \beta_0 (M_{i,t} \cdot \tau_{i,t}) + \varepsilon_{i,t} \,,$$

estimated over time for each underlying stock or index.

	$\beta_{0,Equities,t}$	$\beta_{1,Equities,t}$	$\beta_{2,Equities,t}$	$\beta_{3,Equities,t}$	$\beta_{4,Equities,t}$	$\beta_{0,SPX,t}$	$\beta_{1,SPX,t}$	$\beta_{2,SPX,t}$	$\beta_{3,SPX,t}$	$\beta_{4,SPX,t}$
	Correlations									
$\beta_{0,Equities,t}$	1									
$\beta_{1,Equities,t}$	0.139**	1								
$\beta_{2,Equities,t}$	-0.029**	-0.381**	1							
$\beta_{3,Equities,t}$	-0.519**	-0.146**	-0.190**	1						
$\beta_{4,Equities,t}$	0.010**	-0.596**	-0.089**	0.288**	1					
$\beta_{0,SPX,t}$	0.614**	0.027**	-0.017**	-0.323**	0.046**	1				
$\beta_{1,SPX,t}$	0.342**	0.017**	-0.007*	-0.155**	0.019**	0.545**	1			
$\beta_{2,SPX,t}$	-0.288**	-0.019**	0.016**	0.123**	-0.028**	-0.410**	0.387**	1		
$\beta_{3,SPX,t}$	-0.402**	-0.009**	-0.004	0.329**	-0.028**	-0.752**	-0.515**	0.125**	1	
$\beta_{4,SPX,t}$	0.057**	0.000	-0.009**	-0.007*	0.017**	0.099**	-0.586**	-0.595**	0.011**	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Table 5

First Order Cross-Serial Correlations for CBOE and SPX IVS Coefficients

The table reports first order cross-serial correlations statistics for average (across time and underlying stocks) CBOE cross-sectional IVS coefficients and for SPX IVS coefficients, all obtained by GLS estimation of the deterministic implied volatility function model

$$\ln \sigma_{i,t} = \beta_0 + \beta_0 M_{i,t} + \beta_0 M_{i,t}^2 + \beta_3 \tau_{i,t} + \beta_0 (M_{i,t} \cdot \tau_{i,t}) + \varepsilon_{i,t} \,,$$

estimated over time for each underlying stock or index. The cross correlations intend to test whether the value of SPX IVS coefficients may forecast subsequent average values of IVS coefficients for individual equity options.

	$\beta_{0,Equities,t}$	$\beta_{1,Equities,t}$	$\beta_{2,Equities,t}$	$\beta_{3,Equities,t}$	$\beta_{4,Equities,t}$	$\beta_{0,SPX,t-1}$	$\beta_{1,SPX,t-1}$	$\beta_{2,SPX,t-1}$	$\beta_{3,SPX,t-1}$	$\beta_{4,SPX,t-1}$
	Correlations									
$\beta_{0,Equities,t}$	1									
$\beta_{1,Equities,t}$	0.139**	1								
$\beta_{2,Equities,t}$	-0.029**	-0.381**	1							
$\beta_{3,Equities,t}$	-0.519**	-0.146**	-0.190**	1						
$\beta_{4,Equities,t}$	0.010**	-0.596**	-0.089**	0.288**	1					
$\beta_{0,SPX,t-1}$	0.615**	0.029**	-0.018**	-0.327**	0.069**	1				
$\beta_{1,SPX,t-1}$	0.345**	0.017**	-0.011**	-0.164**	0.033**	0.545**	1			
$\beta_{2,SPX,t-1}$	-0.288**	-0.024**	0.012**	0.123**	-0.036**	-0.410**	0.387**	1		
$\beta_{3,SPX,t-1}$	-0.401**	-0.008**	-0.000	0.335**	-0.045**	-0.752**	-0.515**	0.125**	1	
$\beta_{4,SPX,t-1}$	0.054**	0.004	-0.005	0.000	0.019**	0.099**	-0.586**	-0.595**	0.011**	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Table 6

First Order Cross-Serial Correlations for CBOE and IVS Coefficients

The table reports first order cross-serial correlations statistics for average (across time and underlying stocks) CBOE cross-sectional IVS coefficients, all obtained by GLS estimation of the deterministic implied volatility function model in the main text.

	$\beta_{0,Equities,t}$	$\beta_{1,Equities,t}$	$\beta_{2,Equities,t}$	$\beta_{3,Equities,t}$	$\beta_{4,Equities,t}$	$\beta_{0,Equities,t-1}$	$\beta_{1,Equities,t-1}$	$\beta_{2,Equitiest-1}$	$\beta_{3,Equities,t-1}$	$\beta_{4,Equities,t-1}$
	Correlations									
$\beta_{0,Equities,t}$	1									
$\beta_{1,Equities,t}$	0.139**	1								
$\beta_{2,Equities,t}$	-0.029**	-0.381**	1							
$\beta_{3,Equities,t}$	-0.519**	-0.146**	-0.190**	1						
$\beta_{4,Equities,t}$	0.010**	-0.596**	-0.089**	0.288**	1					
$\beta_{0,Equities,t-1}$	0.990**	0.121**	-0.025**	-0.517**	0.079**	1				
$\beta_{1,Equities,t-1}$	0.171**	0.282**	0.190**	-0.168**	-0.280**	0.139**	1			
$\beta_{2,Equities,t-1}$	-0.041**	0.177**	0.192**	-0.054**	-0.222**	-0.029**	-0.381**	1		
$\beta_{3,Equities,t-1}$	-0.527**	-0.121**	-0.041**	0.825**	0.043**	-0.519**	-0.146**	-0.190**	1	
$\beta_{4,Equities,t-1}$	0.086**	-0.193**	-0.165**	0.037**	0.306**	0.010**	-0.596**	-0.089**	0.288**	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Table 7

Third Order Cross-Serial Correlations for CBOE and IVS Coefficients

The table reports third order cross-serial correlations statistics for average (across time and underlying stocks) CBOE cross-sectional IVS coefficients, all obtained by GLS estimation of the deterministic implied volatility function model in the main text.

	$\beta_{0,Equities,t}$	$\beta_{1,Equities,t}$	$\beta_{2,Equities,t}$	$\beta_{3,Equities,t}$	$\beta_{4,Equities,t}$	$\beta_{0,Equities,t-3}$	$\beta_{1,Equities,t-3}$	$\beta_{2,Equitiest-3}$	$\beta_{3,Equities,t-3}$	$\beta_{4,Equities,t-3}$
	Correlations									
$\beta_{0,Equities,t}$	1									
$\beta_{1,Equities,t}$	0.139**	1								
$\beta_{2,Equities,t}$	-0.029**	-0.381**	1							
$\beta_{3,Equities,t}$	-0.519**	-0.146**	-0.190**	1						
$\beta_{4,Equities,t}$	0.010**	-0.596**	-0.089**	0.288**	1					
$\beta_{0,Equities,t-3}$	0.980**	0.111**	-0.033**	-0.483**	0.090**	1				
$\beta_{1,Equities,t-3}$	0.153**	0.143**	0.056**	-0.119**	-0.106**	0.139**	1			
$\beta_{2,Equities,t-3}$	-0.037**	0.066**	0.064**	-0.020**	-0.084**	-0.029**	-0.381**	1		
$\beta_{3,Equities,t-3}$	-0.503**	-0.086**	-0.009**	0.729**	-0.005	-0.519**	-0.146**	-0.190**	1	
$\beta_{4,Equities,t-3}$	0.094**	-0.092**	-0.063**	0.003	0.161**	0.010**	-0.596**	-0.089**	0.288**	1

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Table 8

Predictive Performance Measures for Alternative Models of the IVS Dynamics

The table reports

	RMSE-V	Utheil-V (%)	MAE-V	MAE-Rel-V (%)	MCPDC-V (%)	RMSE-P	Utheil-P (%)	MAE-P	MAE-Rel-P (%)	MCPDC-P (%)
	OLS Estimates									
Deterministic IVS Model with SPX IVS	0.027	7.31%	0.018	4.73%	57.89%	0.539	12.67%	0.499	16.19%	53.93%
Goncalves-Guidolin Deterministic IVS Model	0.030	7.75%	0.020	4.90%	57.01%	0.545	13.44%	0.509	16.61%	53.90%
Random walk IVS (DWF "strawman")	0.017	4.98%	0.012	3.56%	51.35%	0.522	12.16%	0.490	15.61%	50.78%
Option GARCH(1,1)	0.032	7.94%	0.022	5.02%	56.89%	0.592	13.82%	0.544	17.38%	52.87%
Random walk IV	0.014	4.05%	0.010	2.99%	NA	0.514	11.96%	0.486	15.45%	NA

Table 9

Delta-Hedged Trading Performances for Alternative Models of the IVS Dynamics

The table reports average (across the cross-section of CBOE underlying stocks) mean trading strategy returns, their average standard deviation, t-ratio, and Sharpe ratios, when the trading strategy consists of

	Averg. Mean Profit (%)	Averg. Stdr. Dev (%)	Averg. t-ratio	Averg. Sharpe ratio
	Trading Rule A			
Deterministic IVS Model with SPX	0.476%	29.163%	0.734	1.578%
Goncalves-Guidolin Deterministic IVS	0.440%	28.578%	0.650	1.484%
Random walk IVS (DWF "strawman")	-0.053%	33.864%	-0.028	-0.202%
Option GARCH(1,1)	0.157%	40.367%	0.173	0.349%
	Trading Rule B			
Deterministic IVS Model with SPX	0.158%	4.033%	1.794	3.527%
Goncalves-Guidolin Deterministic IVS	0.162%	4.035%	1.872	3.613%
Random walk IVS (DWF "strawman")	0.135%	3.966%	1.566	3.001%
Option GARCH(1,1)	0.113%	4.688%	0.967	2.073%
	Trading Rule C			
Deterministic IVS Model with SPX	0.075%	2.488%	1.258	2.378%
Goncalves-Guidolin Deterministic IVS	0.071%	2.528%	1.222	2.198%
Random walk IVS (DWF "strawman")	0.062%	2.488%	1.056	1.864%
Option GARCH(1,1)	0.060%	2.763%	0.828	1.619%
	Benchmark			
S&P buy and hold	0.040%	1.112%	1.272	2.221%
T-Bill Portfolio	0.016%	0.007%	73.620	0.000%

Table 10

Trading Performances of Straddle Strategies Under Alternative Models of the IVS Dynamics

The table reports average (across the cross-section of CBOE underlying stocks) mean trading strategy returns, their average standard deviation, t-ratio, and Sharpe ratios, , when the trading strategy consists of

	Averg. Mean Profit (%)	Averg. Stdr. Dev (%)	Averg. t-ratio	Averg. Sharpe ratio
	Trading Rule A			
Deterministic IVS Model with SPX	4.513%	32.698%	5.958	13.754%
Goncalves-Guidolin Deterministic IVS	4.390%	33.085%	5.879	32.751%
Random walk IVS (DWF "strawman")	4.258%	33.000%	6.486	12.855%
Option GARCH(1,1)	3.889%	39.883%	4.123	9.712%
	Trading Rule B			
Deterministic IVS Model with SPX	1.793%	8.218%	9.015	21.628%
Goncalves-Guidolin Deterministic IVS	1.897%	8.302%	9.417	22.658%
Random walk IVS (DWF "strawman")	1.497%	8.423%	9.996	17.580%
Option GARCH(1,1)	1.479%	9.678%	6.874	15.118%
	Trading Rule C			
Deterministic IVS Model with SPX	1.119%	11.040%	4.151	9.990%
Goncalves-Guidolin Deterministic IVS	1.135%	11.038%	4.200	10.142%
Random walk IVS (DWF "strawman")	1.096%	11.040%	4.126	9.782%
Option GARCH(1,1)	1.038%	11.874%	3.176	8.609%
	Benchmark			
S&P buy and hold	0.040%	1.112%	1.272	2.221%
T-Bill Portfolio	0.016%	0.007%	73.620	0.000%

Table 11

Delta-Hedged Trading Performances for Alternative Models of the IVS Dynamics: Effects of Transaction Costs

The table reports average (across the cross-section of CBOE underlying stocks) mean trading strategy returns, their average standard deviation, t-ratio, and Sharpe ratios, when the trading strategy consists of

	Averg. Mean Profit (%)	Averg. Stdr. Dev (%)	Averg. t-ratio	Averg. Sharpe ratio
	Trading Rule A			
Deterministic IVS Model with SPX	-7.705%	39.838%	-8.593	-19.381%
Goncalves-Guidolin Deterministic IVS	-8.114%	39.339%	-8.994	-20.667%
Random walk IVS (DWF "strawman")	-8.688%	46.312%	-8.262	-18.793%
Option GARCH(1,1)	-9.584%	45.738%	-9.365	-20.989%
	Trading Rule B			
Deterministic IVS Model with SPX	-0.861%	4.067%	-9.535	-21.565%
Goncalves-Guidolin Deterministic IVS	-0.842%	4.070%	-9.271	-21.068%
Random walk IVS (DWF "strawman")	-0.794%	4.002%	-8.819	-20.232%
Option GARCH(1,1)	-1.477%	4.618%	-10.374	-32.319%
	Trading Rule C			
Deterministic IVS Model with SPX	-0.744%	2.503%	-13.654	-30.376%
Goncalves-Guidolin Deterministic IVS	-0.749%	2.544%	-13.423	-30.040%
Random walk IVS (DWF "strawman")	-0.757%	2.503%	-13.841	-30.879%
Option GARCH(1,1)	-0.780%	2.685%	-14.137	-29.628%
	Benchmark			
S&P buy and hold	0.040%	1.112%	1.272	2.221%
T-Bill Portfolio	0.016%	0.007%	73.620	0.000%

Table 12

Trading Performances of Straddle Strategies Under Alternative Models of the IVS Dynamics: Effects of Transaction Costs

The table reports average (across the cross-section of CBOE underlying stocks) mean trading strategy returns, their average standard deviation, t-ratio, and Sharpe ratios, , when the trading strategy consists of

	Averg. Mean Profit (%)	Averg. Stdr. Dev (%)	Averg. t-ratio	Averg. Sharpe ratio
	Trading Rule A			
Deterministic IVS Model with SPX	-14.352%	46.282%	-13.168	-31.044%
Goncalves-Guidolin Deterministic IVS	-15.314%	49.537%	-13.362	-30.946%
Random walk IVS (DWF "strawman")	-14.535%	45.484%	-13.988	-31.990%
Option GARCH(1,1)	-18.374%	52.288%	-14.238	-35.170%
	Trading Rule B			
Deterministic IVS Model with SPX	-1.077%	8.257%	-5.270	-13.238%
Goncalves-Guidolin Deterministic IVS	-1.027%	8.355%	-5.019	-12.480%
Random walk IVS (DWF "strawman")	-0.990%	8.467%	-4.800	-11.875%
Option GARCH(1,1)	-1.284%	9.617%	-6.837	-13.512%
	Trading Rule C			
Deterministic IVS Model with SPX	-3.151%	11.194%	-11.649	-28.290%
Goncalves-Guidolin Deterministic IVS	-3.143%	11.234%	-11.644	-28.115%
Random walk IVS (DWF "strawman")	-3.070%	11.199%	-11.351	-27.552%
Option GARCH(1,1)	-3.672%	12.454%	-12.374	-29.614%
	Benchmark			
S&P buy and hold	0.040%	1.112%	1.272	2.221%
T-Bill Portfolio	0.016%	0.007%	73.620	0.000%

Table 13

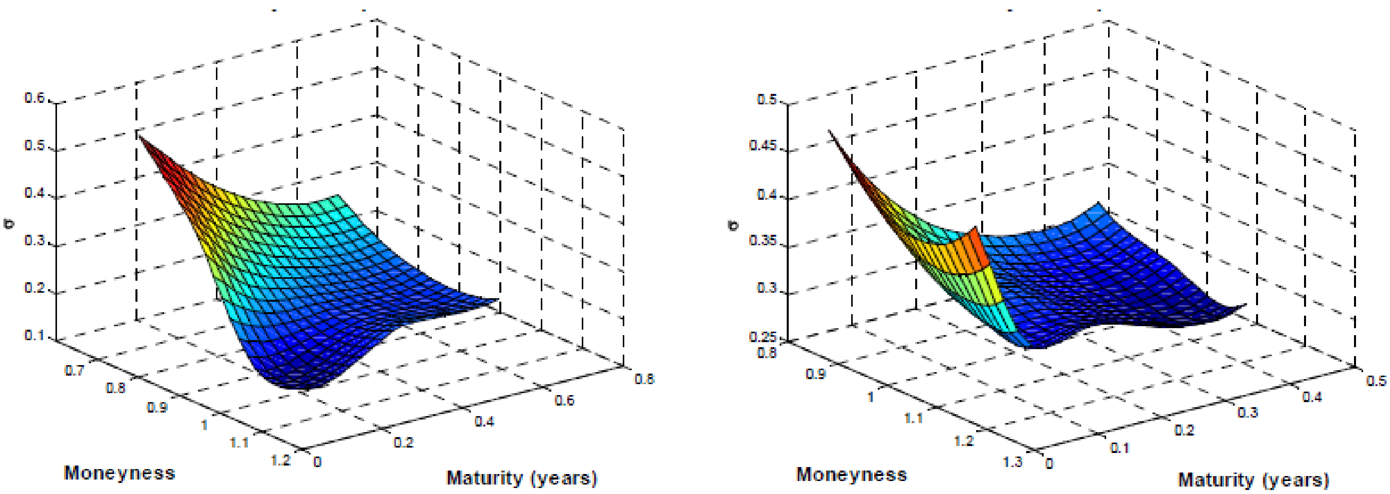
Trading Performances of Straddle Strategies Under Alternative Models of the IVS Dynamics: Profit-Destroying Transaction Costs

The table reports average (across the cross-section of CBOE underlying stocks) mean trading strategy returns, their average standard deviation, t-ratio, and Sharpe ratios, , when the trading strategy consists of

	Averg. Mean Profit (%)	Averg. Stdr. Dev (%)	Averg. t-ratio	Averg. Sharpe ratio
	Trading Rule A			
Deterministic IVS Model with SPX	-33.217%	83.134%	-16.780	-39.975%
Goncalves-Guidolin Deterministic IVS	-35.326%	89.221%	-16.871	-39.612%
Random walk IVS (DWF "strawman")	-35.143%	85.947%	-17.506	-40.908%
Option GARCH(1,1)	-39.738%	95.628%	-18.587	-41.571%
	Trading Rule B			
Deterministic IVS Model with SPX	-3.948%	8.574%	-18.767	-46.227%
Goncalves-Guidolin Deterministic IVS	-3.951%	8.684%	-18.677	-45.676%
Random walk IVS (DWF "strawman")	-4.100%	8.824%	-19.022	-46.641%
Option GARCH(1,1)	-3.318%	8.467%	-16.256	-39.369%
	Trading Rule C			
Deterministic IVS Model with SPX	-7.421%	11.707%	-26.074	-63.521%
Goncalves-Guidolin Deterministic IVS	-7.425%	11.748%	-26.131	-63.334%
Random walk IVS (DWF "strawman")	-7.340%	11.725%	-25.762	-62.730%
Option GARCH(1,1)	-7.937%	13.707%	-27.364	-58.021%
	Benchmark			
S&P buy and hold	0.040%	1.112%	1.272	2.221%
T-Bill Portfolio	0.016%	0.007%	73.620	0.000%

Figure 1

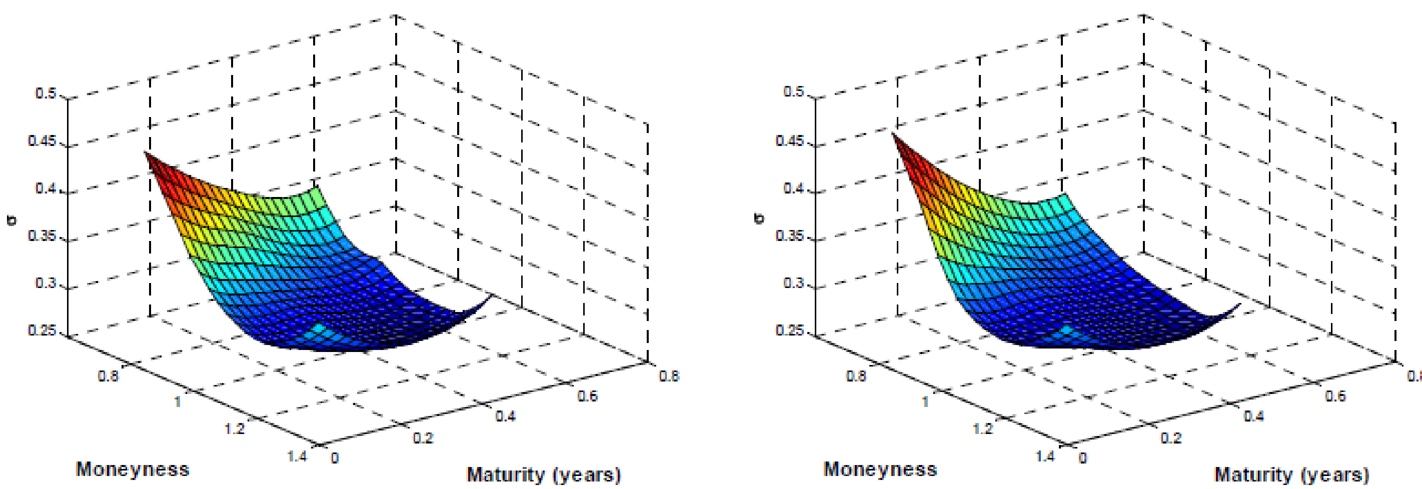
Dynamics of the IVS for Cisco System Options



June 30, 2006 July 31, 2006

Figure 2

Fit of the Deterministic IVS Model to Microsoft IVS on July 15, 2003



Market data Fitted Model