

Conglomeration with Bankruptcy Costs: Separate or Joint Financing?*

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ABSTRACT

When should projects be financed jointly rather than separately? We show that bankruptcy costs alone generate a non-trivial tradeoff between the benefit of *co-insurance* and the cost of *risk contamination* associated to joint financing. We characterize this tradeoff for projects with binary returns, depending on the mean, variability, skewness, and correlation of returns, the bankruptcy recovery rate, the tax rate advantage of debt relative to equity, the number of projects, and their heterogeneity. We discuss cases in which separate financing is more profitable even though joint financing is available at lower interest rate and results in lower probability of bankruptcy.

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This paper characterizes the determinants of the optimal corporate structure in the presence of bankruptcy costs. Consider a firm with access to two risky projects with positive net present value. The firm can either finance these projects jointly within a single company or finance them separately by setting up two independent companies. In either case, financing is obtained from a competitive credit market through standard non-recourse debt.¹ If a company's returns fall below the debt repayment obligation, a fraction of the returns are lost because of bankruptcy costs. When should the firm finance the projects jointly and when separately? By answering this question we can determine the profitability of:

- corporate spin-offs, whereby a firm's divisions are set up as independent companies;
- project finance, whereby corporate projects are financed through special purpose vehicles;
- conglomerate mergers, whereby the resources of originally separate companies are combined; and
- mortgage securitization, whereby separate mortgage claims are pooled.

The conventional wisdom in corporate finance has largely settled on the view that the purely financial synergies from savings in bankruptcy costs achieved through conglomeration are always positive. According to this view, conglomeration should bring about a reduction in the probability of bankruptcy by allowing the firm to use the proceeds of a successful project to save an unsuccessful one, which would have failed otherwise. By aggregating imperfectly correlated cash flows, the argument goes, the firm should be able to reduce expected bankruptcy costs and increase borrowing capacity—see Lewellen (1971). As aptly summarized by Brealey, Myers, and Allen's (2006, page 880) textbook, “merging decreases the probability of financial distress, other things equal. If it allows increased borrowing, and increased value from the interest tax shields, there can be a net gain to the merger.”

In this paper, we amend this conventional view by revisiting the *purely financial* effects of conglomeration in the presence of bankruptcy costs. We show that bankruptcy costs alone create a non-trivial tradeoff for conglomeration, even abstracting from tax considerations. Our analysis clarifies conditions for when the logic of the conventional argument is reversed—sometimes when projects are financed jointly, failure of one project can drag down another profitable project that would have stayed afloat otherwise. As we also show, this effect can

¹When a project is financed through non-recourse debt, if the firm does not meet the repayment obligation on one project, creditors do not have access to the returns of the other project.

be so strong to make it optimal in some cases for a firm to finance projects *separately*, even though joint financing would involve paying a *lower* interest rate or would result in a *lower* probability of bankruptcy.

While the literature has focused mostly on the *co-insurance* effect of conglomeration, our analysis uncovers the determinants of the *risk contamination* effect, whereby aggregating risky assets can generate incremental distress costs. As argued informally by Esty (2002), “this phenomenon [risk contamination] must be balanced against the benefits of co-insurance received from the project.” We characterize when pooling risks allows to better cushion the blow from a negative shock, and when, instead, it magnifies the shock’s effect, bringing down the entire house of cards. As we discuss in the paper, our characterization of the tradeoff between co-insurance and risk contamination delivers predictions broadly in line with empirical evidence.

To best uncover the tradeoff between co-insurance and risk contamination, we analyze the simplest model in which each project has binary returns, either low or high. We focus on the interesting case in which separate financing of a project results in default when the return is low. In that case, creditors are able to recover a fraction of the project’s return, while the remaining fraction is lost due to bankruptcy costs. Perfect competition among risk-neutral creditors drives down the required repayment obligation (consisting of principal plus interest) to a level at which creditors expect to exactly recoup the initial investment outlay, partly through the full repayment in case the project’s return is high and through the recovered fraction of the low return.

Next, suppose instead the two projects are financed jointly through a single loan. Clearly, default will result when both projects yield a low return. The interesting case to consider is when one project has a low return and the other has a high return. Suppose first that the sum of these two returns is sufficiently high to meet the required repayment obligation to creditors. If so, the high return project saves the low return project from bankruptcy. Given that the probability of bankruptcy is reduced by joint financing compared to separate financing, creditors are forced to further reduce the required repayment obligation. This is the logic of the “good” conglomeration stressed by Lewellen (1971).

If, instead, the sum of low and high returns is not sufficiently high to meet the required repayment obligation to creditors, conglomeration is “bad” for profits. We derive conditions on the exogenous parameters for good and bad diversification to arise, taking into account that the required repayment obligation is endogenously determined by the creditors’ zero

profit condition. We fully characterize when joint financing dominates separate financing, depending on the distribution of projects' returns, the recovery rate in case of bankruptcy, the correlation across projects, the number of projects, and the heterogeneous characteristics of projects.

We find that an increase in the bankruptcy recovery rate as well as an increase in the probability of a high return increase the profitability of joint financing relative to separate financing. On the other hand, an increase in the riskiness of (sufficiently negatively skewed) projects, an increase in the negative skewness of projects (with sufficiently high return), and an increase in the correlation of projects all make separate financing relatively more profitable than joint financing. Also, joint financing is always more profitable than separate financing of a sufficiently large number of independent projects.

Our results offer a simple explanation for the common use of project finance, which involves the transfer of a subset of a company's assets into a special purpose vehicle financed with non-recourse debt.² The few existing papers that analyze the motivations for using project finance concentrate on agency problems (see e.g. Subramanian et al. (2007)). With the notable exception of Leland (2007) only a few papers have been devoted to risk contamination (see Esty (2003) for a survey of the explanations). While the literature proceeds under the assumption that it is necessary to introduce agency conflicts within the firm to justify the practice of project finance, we show that project finance is an optimal firm response to minimize expected bankruptcy costs.

Many empirical studies of project finance find that in project finance a disproportionate proportion of the funding is in "debt". By also allowing for equity financing, we show that when it is strictly optimal to finance separately, only debt should be used. Instead, when it is strictly optimal to finance jointly, it might be optimal to use equity. One might be willing to use equity (and pay more taxes) if a rate that avoids intermediate bankruptcy can be obtained. And, of course, we do that only when it is optimal to finance jointly. In this sense, a higher proportion of debt in project finance arises endogenously in our model.

Our analysis has also implications for securitizations, one of the key elements of the crisis. Our framework should also provide hints as to when securitisation creates value. Prior to the crisis, banks had transferred to bankruptcy remote vehicles large amounts of (perceived) low-risk cash flows such as mortgages and credit card payments (Keys et al. (2008)). Improving

²See, for example, Gorton and Souleles (2006) for an introduction to project finance and a discussion of the importance of bankruptcy costs in that context.

risk sharing was one of the main perceived benefits behind this expansion (Pennacchi (1988)). But, as we have just witnessed, this risk-sharing and shifting mechanisms do not ensure that large-scale upheavals are contained and, in fact, they might have induced it.

We also show, as argued by Esty (2003), that project finance might increase investment. Allowing for heterogeneous projects, one project (the sponsor) might give but not receive co-insurance benefits from another (the investment opportunity). If this opportunity is financed with corporate finance, the firm's expected distress costs should increase. Assuming the project's NPV is marginally positive, a rational manager will forgo the investment. By investing through a separately incorporated project company financed with nonrecourse project debt, the sponsoring firm can dramatically reduce the potential for risk contamination and the need to supply co-insurance.

Our analysis delivers two additional managerial implications. At first it seems plausible that (i) the financing option with the lowest interest rate has the lowest likelihood of bankruptcy and (ii) the financing option with the lowest probability of bankruptcy is more profitable. We show that these two rules of thumb are false in general.

When the recovery rate in case of bankruptcy is sufficiently high, creditors are forced by competition to offer low repayment rates to firms that finance projects jointly. Given that creditors break even whether the projects are financed separately or jointly, the firm acts as residual claimant of the projects' expected returns net of the bankruptcy costs. Hence, the firm should optimally finance the projects separately at a higher interest rate, thereby reducing expected bankruptcy costs. Even though the interest rate is lower with joint financing, the firm bears the brunt for the inefficient increase in expected bankruptcy costs associated with joint financing.

In terms of the literature, we depart from Modigliani and Miller's (1958) world without financial synergies by introducing bankruptcy costs.³ By clarifying the conditions for the value of conglomeration, this paper contributes to a voluminous literature on the purely financial motives for mergers. In his discussion to Lewellen (1971), Higgins (1971) notes that project bundling affects also the riskiness of the lender's returns—we instead abstract from risk by assuming risk neutrality. Scott (1977) and Sarig (1985) show that if cash flows can be negative, a firm can exploit the shelter of limited liability by financing projects in

³In the absence of bankruptcy costs, diversification is not a valid argument for mergers. This is because investors can obtain the same diversification themselves by purchasing appropriate amounts of the unmerged firms. See, for example, Levy and Sarnat (1970).

separate companies. In our analysis we explicitly abstract from this limited liability effect, so that the mode of financing does *not* affect the payoff of third parties, but only that of the firm and its creditor.

Our results are most closely related to three recent contributions to the corporate finance literature. First, Winton (1999) in the third case of his Proposition 3.1 discusses a situation in which a bank prefers to specialize even though the repayment rate for pooled projects is lower. Despite the differences in the model, our Proposition 3 is similar to Winton's earlier result. Second, Inderst and Müller (2003) analyze the pros and cons of project bundling in a two-project version of Bolton and Scharfstein's (1990) dynamic model of debt. In their setting, financing two projects within the same company can reduce the firm's ability to borrow when the firm is able to make follow-up investments without having to return to the capital market.⁴ This a very different channel through which bad conglomeration arises, as we explain in Appendix B. Third, Leland (2007) shows that financial separation can be beneficial when it allows a firm to fine tune the capital structure (mix of debt and equity) to the specific characteristics of projects with heterogeneous returns.⁵ In our paper, instead, we explicitly rule out the possibility of re-optimizing the capital structure by requiring projects to be financed with debt only. In addition, we provide simple analytical results which clarify what drives the sign of the profitability of conglomeration.⁶

The paper proceeds as follows. Section I formulates the model. Focusing on the baseline version of the model with two identically and independently distributed projects financed with debt, Section II analyzes the conditions setting apart good from bad conglomeration and performs comparative statics with respect to the distribution (mean, variance, and skewness) of returns and the bankruptcy recovery rate. Section III analyzes the effect of correlation across projects and the results for a high number of projects. Section IV extends the analysis to the case when projects have heterogeneous returns. Section V allows the entrepreneur to

⁴See also Faure-Grimaud and Inderst (2005).

⁵A number of papers (e.g., Higgins and Schall, 1975, and Kim and McConnell, 1977) have analyzed the effect of the current capital structure on merger incentives. These papers noted that, while mergers may increase total firm value, bondholders may gain at the expense of shareholders. We abstract from such a distributional conflict among (cashless) stakeholders, by considering the ex-ante choice of corporate structure by shareholders and forcing bondholders to compete and therefore obtain no surplus.

⁶Our results are very different from those of Shaffer (1994), who studies the effect of joint financing on the probability of *joint* failure. Instead, we compare the firm's expected payoff when the interest rate is endogenously determined by competition among creditors.

use equity to finance part of the investment. Section VI concludes. Appendix A collects the proofs omitted from the text. Appendix B analyzes a dynamic version of the model with non-verifiable returns and optimal financial contracting.

I. Model

This section formulates the simplest possible model to analyze how firms should finance multiple assets in the presence of bankruptcy costs. In the rest of the paper we derive results for special cases of this model.

A risk-neutral entrepreneur is endowed with n projects. Each project i requires at $t = 1$ an investment outlay normalized to $I = 1$ and yields at $t = 2$ a random return r^i with a binary distribution: the return is either low, $r^i = r_L^i > 0$, with probability $1 - p_i$, or high, $r^i = r_H^i > r_L^i$, with probability p_i . Each project has positive net present value, $(1 - p_i)r_L^i + p_i r_H^i - 1 > 0$. The low return is insufficient to cover the initial investment outlay, $r_L^i < 1$. Returns are possibly correlated across projects.

Before raising external finance, the entrepreneur chooses how to group projects into separate non-recourse “companies”. That means that investors in each company have access to the returns of all projects in that company, but they do not have access to the returns of the projects in the other companies. Financing can be obtained in competitive credit and equity markets. For notational simplicity, we stipulate that the entrepreneur accepts to be financed only when expecting to obtain a *strictly* positive expected payoff.

Creditors are risk neutral and lend money by way of standard debt contracts. Without loss of generality we normalize their required interest rate to $r_D = 0$.⁷ Therefore, creditors expect to make zero expected profits. This is equivalent to assuming that each company j makes a take-it-or-leave-it repayment offer to a single creditor, promising to repay $r_j^*(D_j)$ for each D_j borrowed, where D_j is the per-project fraction of debt financing.⁸ In other words, the loan of company j , which consists of n_j projects, stipulates a payment of $n_j r_j^*$ at $t = 2$ in exchange of $n_j D_j$ (the debt value) at $t = 1$. According to our accounting convention,

⁷To see that $I = 1$ and $r_D = 0$ are innocuous normalizations, suppose that the investment outlay is equal to I and that the creditors’ required interest rate is $r_D > 0$. Denoting the random cash flow by R_i , the project’s return in the model can be reinterpreted in terms of percentage gross return for each unit of period-2 equivalent outlay: $r_i = R_i/[I(1 + r_D)]$. Thus, it is without loss of generality to set $I = 1$ and $r_D = 0$.

⁸Thus, for the case in which each loan (or company) is financed by multiple creditors, we implicitly assume that there are no coordination failures across the creditors who syndicate the same loan.

the entrepreneur’s repayment obligation comprises principal as well as net interest. The net interest rate i satisfies $1+i = r_j^*/D_j$ and therefore, in the case of full debt financing ($D_j = 1$), the repayment obligation can be interpreted as the gross interest rate.

On each loan, the borrower repays the creditor in full when the total realized return of the projects pledged is sufficient to cover the promised repayment, r_j^* . If instead the total realized return falls short of the repayment obligation, the borrower defaults and the ownership of the projects’ realized returns is transferred to the creditor. Due to bankruptcy costs, following default the creditor is able to recover only a fraction β of the returns. The remaining fraction $1 - \beta$ of the returns is lost. The bankruptcy recovery rate $\beta \in [0, 1]$ measures the efficiency of bankruptcy and is industry specific.⁹

The equity market is also assumed to be competitive. Equity investors expect a net return equal to the risk premium from holding equity, denoted by r_E . This is equivalent to assuming that company j makes a take-it-or-leave-it offer to a single investor, for a fraction α_j of the company in exchange of a fraction E_j of the necessary investment, where $E_j = 1 - D_j$. Provided that some shares are sold to outside investors ($\alpha_j > 0$), the total equity value, including the entrepreneur’s return, at $t = 1$ is given by $n_j E_j / \alpha_j$. Debt payments are tax deductible and therefore exempt from taxes but equity has a (net) corporate tax τ .

As will be shown in Section V, full debt financing ($D_j = 1$) is optimal if taxes and/or the required equity premium are high. A full debt contract would also be the optimal contractual arrangement if we assumed that returns were privately observed by the borrower and could be verified only at a cost (equal to the bankruptcy cost), as in the costly state verification model of Townsend (1978) and Gale and Hellwig (1985). See Appendix B for an alternative derivation of the optimality of the debt contract in a dynamic version of our model developed along the lines of Bolton and Scharfstein (1990) and Hart and Moore (1998). Thus, we start our analysis assuming that “companies” can only be financed with debt.

Notice that our simple framework can be used to model merges and spin-offs, to model the decision to use corporate finance versus project finance, and also capture some features of the decision to securitize. The entrepreneur’s problem can be interpreted as a problem of a firm that considers whether to spin out a division or to use project finance to finance specific projects instead of corporate finance.

⁹For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2006).

II. Two Identical and Independent Projects

This section analyzes the simplest possible specification of the model with $n = 2$ identically and independently distributed projects financed with debt only. Each project i yields a low return $r_L^i \equiv r_L$ with probability $1 - p_i \equiv 1 - p$ and a high return $r_H^i \equiv r_H > r_L$ with probability $p_i \equiv p$. We proceed by first examining the conditions for when the borrower is able to finance the two projects separately and jointly (Section A). Second, we compare the profitability of separate and joint financing, when they are both feasible (Section B). Third, we illustrate that separate financing can be easily optimal for empirically plausible parameter values (Section C). Finally, we derive a set of comparative statics predictions for the occurrence of joint and separate financing (Section D).

A. Financing Conditions

Consider first the possibility of financing the two projects through two separate non-recourse loans (or, equivalently, through two different limited liability companies). Given that the two projects are ex ante identical, when each of them is financed, financing will take place at the same equilibrium repayment rate (equal to the nominal repayment obligation). Such rate r_i^* must satisfy $r_L < 1 < r_i^* < r_H$, so that there is a positive probability that the loan is not repaid in full. Indeed, the firm would not accept to be financed at a rate above r_H , because this would result in zero payoff for the firm. Also, the rate must be above 1 because at rates at or below 1 the creditor would make negative expected profits (by obtaining a return never above the investment outlay of 1 and strictly below 1 with strictly positive probability) and therefore would not be willing to extend the loan.

In a competitive credit market, creditors make zero expected profits. Thus, the repayment requested by the creditor is r_i^* such that the gross profits, $pr_i^* + (1 - p)\beta r_L$, are equal to the initial investment outlay 1. As a result, each project can be financed through a separate loan if and only if

$$r_i^* := \frac{1 - (1 - p)\beta r_L}{p} < r_H. \tag{1}$$

Clearly, $r_i^* > 1$.

Next, consider joint financing of the two projects through a single loan (or, equivalently, within the same company). Denote by r_m^* the equilibrium repayment obligation *per unit of investment*, so that $2r_m^*$ is the total repayment promised to investors in return for the initial

financing of the two projects, $2I = 2$. Two cases need to be distinguished, depending on whether or not the required repayment rate induces default when one project yields a high return while the other project yields a low return—this is the case with intermediate returns.

Suppose first that the equilibrium repayment rate r_m^* is such that $r_L < r_m^* < \frac{r_H+r_L}{2}$, so that there is no default with intermediate returns. As a result, the probability of default of the loan is reduced to $(1-p)^2$. Substituting again in the expected creditor profits, the borrower would only be able to obtain this rate in a competitive market if and only if

$$r_m^* := \frac{1 - (1-p)^2 \beta r_L}{1 - (1-p)^2} < \frac{r_H + r_L}{2}. \quad (2)$$

Suppose now that the equilibrium repayment rate r_m^{**} is such that $\frac{r_H+r_L}{2} < r_m^{**} < r_H$ and therefore the borrower defaults in the event of a high and a low return. Hence, default occurs with probability $1 - p^2 = (1-p)^2 + 2p(1-p)$. In a competitive credit market, this case arises if and only if

$$r_m^{**} := \frac{1 - (1-p) \beta (pr_H + r_L)}{p^2} < r_H. \quad (3)$$

Since the borrower's expected profits for a given distribution are decreasing in the equilibrium rate, if both conditions (2) and (3) are satisfied, the borrower prefers rate r_m^* to rate r_m^{**} .¹⁰ Summarizing the results so far, we have the following proposition.

PROPOSITION 1: *Projects can be financed separately if and only if $r_i^* < r_H$ (Condition 1), in which case the equilibrium repayment rate is r_i^* . When the borrower seeks joint finance, if $r_m^* < (r_H + r_L)/2$ (Condition 2), then the equilibrium rate is r_m^* ; if $r_m^* > (r_H + r_L)/2$ and $r_m^{**} < r_H$ (Condition 3), then the equilibrium rate is r_m^{**} ; otherwise, the projects cannot be financed.*

Figure 1 depicts how *per-project* expected returns are divided between borrower and creditor in the three scenarios described by Proposition 1. The area above the distribution function in all the panels is equal to the project's expected return. When the two projects are financed separately, the return of each project is a binary random variable with a cumulative distribution represented in Panel (a). When the two projects are bundled and financed jointly, there are three possible realized returns. Panels (b) and (c) display the cumulative distributions of the (per-project) returns resulting with joint financing for two different

¹⁰It is straightforward to show that if $r_m^* > (r_H + r_L)/2$, then $r_m^{**} > (r_H + r_L)/2$. Therefore, if it is not possible to obtain r_m^* , then we can disregard the $r_m^{**} > (r_H + r_L)/2$ constraint.

examples—in each graph the dashed distribution corresponds to the returns resulting with separate financing. Note that the distribution of (per-project) returns with separate financing is a mean-preserving spread of the distribution with joint financing. Intuitively, joint financing steepens the return distribution around the center by inducing an anti-clockwise rotation.

For any given repayment rate r , the net expected return for the borrower corresponds to the area above the cumulative distribution of (per-project) returns at r , $F(r)$, and to the right of r (in blue). The gross expected return of the creditor is the sum of (i) the area above $F(r)$ and to the left of r (in yellow) and (ii) the fraction β of the area below $F(r)$ and above the distribution function (in pink). The first area is equal to pr , which is the full repayment of the outstanding obligation multiplied by the probability that the project stays afloat, p . The second area is equal to $(1-p)\beta r_L$, capturing the expected returns obtained in case of bankruptcy. The remaining fraction $1-\beta$ of the pink area is equal to the expected bankruptcy costs. This is also equal to the difference between the net present value of the company, the area above the distribution function and below 1, and the sum of creditor's and borrower's profits.

The equilibrium rate r^* in the three panels is such that the gross expected return of the creditor is equal to 1. Projects can be financed separately as long as the creditor's gross returns at a rate r_H are greater than 1, as in Panel (a). Projects can be financed jointly at a rate below the crossing point as long as the per-project creditor returns at $(r_H + r_L)/2$ are greater than 1, as in Panel (b). Projects can only be financed jointly at a rate above the crossing point if the per-project creditor returns at $(r_H + r_L)/2$ are lower than 1 and at r_H are greater than 1, as in Panel (c).

B. *Good and Bad Conglomeration*

When both separate and joint financing are feasible, which one should the borrower choose? Obviously, in the absence of bankruptcy costs (i.e., when $\beta = 1$) the borrower is indifferent between financing the projects separately or jointly. The next proposition states the gains and losses when $\beta < 1$.

PROPOSITION 2: *When the borrower can finance both projects separately and jointly:*

(a) *If the joint rate is r_m^* , then the borrower should finance the projects jointly because of the co-insurance effect. The per-project incremental surplus for the borrower of joint rather*

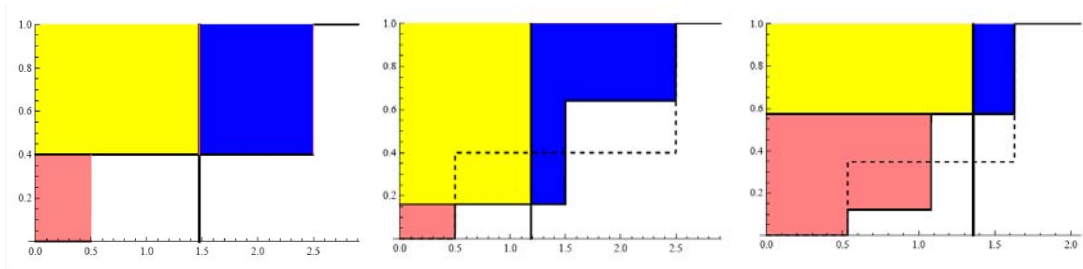


Figure 1. Distribution of Returns. The area above the distribution function represents the project's expected return. For a given repayment rate r , the net expected return for the borrower corresponds to the area above the distribution function and to the right of r (in blue). The gross expected return for the creditor is the sum of (i) the area above $F(r)$ and to the left of r (in yellow) and (ii) the fraction β of the area below $F(r)$ and above the distribution function (in pink). The equilibrium rate r^* is such that the gross expected return for the creditor is equal to 1. Projects can be financed separately if the creditor's gross expected return at the rate r_H are greater than 1, as in Panel (a). Projects can be financed jointly at a rate below the crossing point if the per-project creditor returns at $(r_H + r_L)/2$ are greater than 1, as in Panel (b). Projects can only be financed jointly at a rate above the crossing point if the creditor's per-project returns at $(r_H + r_L)/2$ are smaller than 1 and at r_H are greater than 1, as in Panel (c).

than separate financing is $p(1-p)(1-\beta)r_L$.

(b) If the joint rate is r_m^{**} , then the borrower should finance the projects separately because of the risk contamination effect. The per-project incremental surplus for the borrower of separate rather than joint financing is $p(1-p)(1-\beta)r_H$.

Intuitively, when the borrower obtains a rate that avoids intermediate bankruptcy, the probability of default under joint financing is lower than under separate financing. The low-return project is saved from default when the other project yields a high return, thereby reducing the inefficiency associated with bankruptcy. Per-project expected savings when the projects are financed jointly rather than separately—the “co-insurance effect”—are equal to the probability that the first project yields a low return while the second project yields a high return, $p(1-p)$, multiplied by the avoided losses due to bankruptcy costs, $(1-\beta)r_L$. Graphically, per-project savings due to the co-insurance effect associated with joint financing are represented by a fraction $(1-\beta)$ of the red area in Panel (a) of Figure 2.

If, instead, the borrower obtains a joint rate that does not avoid intermediate bankruptcy, a low-performing project drags down the other, increasing the probability of default. Per-project expected losses when projects are financed jointly rather than separately—the “risk contamination effect”—are equal to the probability that the first project yields a high return while the second project yields a low return, $p(1-p)$, multiplied by the additional losses in bankruptcy costs incurred, $(1-\beta)r_H$. Graphically, the per-project costs due to the risk contamination effect associated with joint financing are represented by a fraction $(1-\beta)$ of the red area in Panel (b) of Figure 2.

The key question is whether the equilibrium repayment rate for joint financing is below or above the crossing point, $(r_H + r_L)/2$. In conclusion, joint financing is optimal in case (a) when condition (2) is satisfied—otherwise separate financing is optimal.

Notice that the crossing point is not necessarily at the mean. In particular, if $p > 1/2$, so that the distribution is skewed to the left (negatively skewed), the crossing point is below the mean. As a result, equilibrium rates above the crossing point are consistent with a probability of default below 50%. The resulting default probabilities would be $1-p$ for separate financing and $1-p^2$ for joint financing, which for p high enough may be very low, as illustrated in the following numerical example.

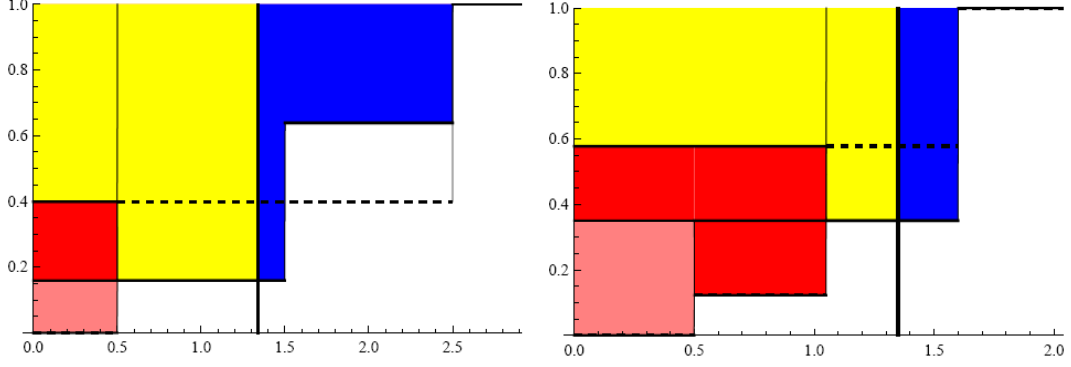


Figure 2. Good and Bad Conglomeration. Panel (a): Projects can be financed jointly at a rate below the crossing point. The reduction in expected bankruptcy costs obtained with joint rather than separate financing (co-insurance effect) is equal to the red area. Panel (b): Projects cannot be financed jointly at a rate below the crossing point. The increase in expected bankruptcy costs obtained with joint rather than separate financing (risk contamination effect) is equal to the red area.

C. Illustration

We now present an illustration of how conglomeration can result in an increase in expected bankruptcy costs for empirically plausible parameter values. To this end, Figure 3 calibrates the four parameters of our baseline model (r_H , r_L , p , and β) using representative values obtained from the empirical literature. To identify our four parameters, we use the probability of bankruptcy, the internal rate of return, the loss given default, and the bankruptcy recovery rate. The key assumption for this calibration exercise is that returns are binary. The calibrated values are $r_H = 1.22$; $r_L = 0.49$; $p = 0.9$; $\beta = 0.65$, for which projects could be financed separately since $r_i^* = 1.076 < 1.22 = r_H$ and jointly $r_m^{**} = 1.107 < 1.22 = r_H$ but not at the rate below the crossing point $r_m^* = 1.006 > 0.855 = (r_H + r_L)/2$. In this illustration, the risk contamination effect identified in Proposition 2 is

$$p(1-p)(1-\beta)r_H = (.9)(1-.9)(1-0.65)1.22 \approx 3.8\%$$

of the investment outlay $I = 1$, corresponding to $.038/0.175 \approx 22\%$ of the project's net present value.

CALIBRATED VARIABLE	PARAMETRIZATION
1. Probability of bankruptcy	$1 - p$
2. Internal rate of return (IRR)	$\frac{pr_H + (1-p)r_L - 1}{1}$
3. Bankruptcy recovery rate	β
4. Bankruptcy costs as a fraction of firms' value prior to default	$\frac{(1-\beta)r_L}{pr_H + (1-p)r_L}$
SOURCE	VALUE
1. Longstaff et al. (2005): BBB-rated firms, 5 year horizon (10%)	0.10
2. IRR rules: go ahead if IRR >10-15% (depending on risk)	0.175
3. Alderson and Betker (1995) (65%)	0.65
4. Altman (1984): 11-17% of firms' value up to 3 years before default	0.10

Figure 3. Parameter Calibration.

D. Comparative Statics Predictions

We now derive comparative statics predictions with respect to changes in the characteristics of the projects: the recovery rates and the distribution of returns (mean, variability, and skewness). For each attribute, we study whether separate or joint financing is optimal for a larger range of the remaining parameters. Again, the key aspect is whether it becomes easier or more difficult to obtain a repayment rate for joint financing below the crossing point. In turn, this depends on how parameter changes affect the crossing point as well as the amount the firm can pledge to the creditors at that point.

At the same time, we contrast our predictions with those from existing theories and discuss how our predictions on joint and separate financing match existing empirical evidence. Support for the desirability of joint financing can be found on the occurrence of mergers, especially conglomerate mergers, and support for separate financing can be found in spinoffs. As argued by Leland (2007), structured finance, and in particular asset securitization and project finance, are other means to separate activities from originating or sponsoring organizations. A bankruptcy-remote special purpose or vehicle (SPE or SPV) raises funds to compensate the sponsor by selling securities that are collateralized by the cash flows of the transferred assets. These entities have the key features of a separate firm from our analytical perspective.

PREDICTION 1: For higher bankruptcy costs (lower β) then (a) financing, both jointly and

separately, can be obtained for a smaller region of parameters and (b) separate financing is preferred for a larger region of the remaining parameters.

A higher bankruptcy cost decreases the maximum pledgeable income both jointly and separately since the recovered returns in case of default are higher. With joint financing, this is represented in Figure 4(a) as a higher discount in the pink area.

Consistent with our prediction, Subramanian et al. (2009), by comparing the incidence of bank loans for project finance with regular corporate loans for large investments, show that project financing (separate financing) is more likely in countries with (i) less efficient bankruptcy procedures and (ii) weaker creditor rights in bankruptcy. Both variables should increase bankruptcy costs.

PREDICTION 2: For higher probability of high return (higher p) then (a) financing can be obtained for a larger region of parameters, both jointly and separately and (b) joint financing is optimal for a larger region of the remaining parameters.

If the probability of a high return increases it becomes easier to finance the project as well as to finance at a repayment rate that avoids intermediate bankruptcy. Graphically, this lowers all the horizontal lines in the graph increasing the expected value, the area above the distribution. Financing is eased and, in particular, financing at a rate that avoids intermediate bankruptcy is eased because the maximum expected return pledgeable to creditors (the sum of the yellow area, the red area, and a fraction $1 - \beta$ of the pink area) also increases.

Neither the bankruptcy recovery rate nor the probability of success affect the crossing point, $(r_H + r_L)/2$. Changes in the variability of the project's return instead also affect the crossing point when the distribution of returns is asymmetric, $p \neq 1/2$.

This prediction contrasts with that of Inderst and Muller (2003). In their model, better projects are better kept separate to avoid self-financing and thus commit to return to the capital market. The two effects might explain the conflicting empirical evidence in this issue. On the one hand, Maksimovic and Philips (2002) show that, consistent with the diversification discount, low-productivity firms diversify while high-productivity firms do not. On the other hand, Schoar (2002) shows that productivity of plants in conglomerate firms are higher than in stand-alone firms, despite a market diversification discount.

Further support for our prediction can be found on the consistent evidence on the differences between small and large firms that get acquired. Maksimovic and Philips (2002) find

that conglomerate firms are less productive for all but for the small firms. McGuckin and Nguyen (1995) specifically compare productivity of small and large firms before acquisition. The large firms that are bought have low productivity but the small firms have a high one. Our prediction is consistent with this evidence if bankruptcy concerns are important for small firms.

During booms, projects might have a higher expectation and therefore a higher p . Our prediction would then be consistent with a large body of empirical evidence that shows that merger activity usually heats up during economic booms and slows down in recessions (see e.g. Maksimovic and Phillips, 2001). Similarly, Cantor and Demsetz (1993) show that off-balance sheet activity (separate financing) grows following a recession.

Further evidence for our prediction can also be found in securitisations. Bannier and Hansel (2008) find that well performing banks (those which had a high p) securitize less than banks with low performance.

PREDICTION 3: Consider the effect of a mean-preserving spread in the project's return consisting in an increase in the high return r_H and a reduction in the low return r_L so as to maintain the mean return constant. Then, there exists $\bar{p} < 1/2$ such that the region of parameters for which separate financing is optimal increases if and only if $p > \bar{p}$.

To derive this result, first consider the effect of a mean preserving spread for the special case with a symmetric distribution ($p = 1/2$). In this case, the mean preserving spread consists in an increase in r_H exactly equal to the reduction in r_L . While the crossing point is clearly unaffected, according to equation (2), the joint financing rate that avoids intermediate bankruptcy becomes more difficult to obtain. Thus, a mean preserving spread in the distribution of returns tends to favor separate financing. Indeed, low returns are even lower and therefore the pledgeable returns before the crossing point are lower. In the graph, the pink area, and therefore the area to the right of the crossing point, shrinks.

Turning to the case of asymmetric distributions, $p \neq 1/2$, a mean-preserving spread also affects the crossing point, $(r_H + r_L)/2$. If the distribution of returns is negatively skewed and therefore the mean is higher than the crossing point ($p > 1/2$), the crossing point is decreased. Indeed, to maintain the mean constant, a given increase in r_H must be combined with a larger decrease in r_L , resulting in a reduction in the crossing point.¹¹ Thus, it becomes

¹¹Formally, from $r'_H = r_H + \varepsilon$ and $r'_L = r_L - \varepsilon p/(1 - p)$, we have $(r'_H + r'_L)/2 = (r_H + r_L)/2 - \varepsilon(2p - 1)/2(1 - p)$.

even more difficult to obtain joint financing below the crossing point. Thus, this second effect also favors separate financing.

The sign of the second effect is reversed if the distribution of returns is instead positively skewed ($p < 1/2$) and therefore the mean return lies below the crossing point. In this case, the two effects go in opposite directions. If the distribution is sufficiently skewed ($p < \bar{p} < 1/2$), the second effect is stronger than the first. In the graph, the increase in the yellow area more than compensates the decrease in the pink area.

Kleimeier and Megginson (2000) find that project finance loans are far more likely to be extended to borrowers in riskier countries, particularly countries with higher political and economic risks. They claim “As a whole, these geographic lending patterns are consistent with the widely held belief that project finance is a particularly appropriate method of funding projects in relatively risky (non-OECD) countries.” This confirms earlier evidence from Kensinger and Martin (1988), Smith and Walter (1990), and Brealey, Cooper, and Habib (1996)

Evidence on securitizations and off-balance sheet debt is also consistent with this prediction. Gorton and Souleles (2005) find that riskier firms generally securitize more, *ceteris paribus*. Firms with greater credit risks are more likely to use substantive off-balance sheet debt (Mills et al., 2004)

This prediction is consistent with Leland’s (2007)

PREDICTION 4: Consider the effect of a mean-preserving increase in negative skewness in the project’s return consisting in a reduction in the low return level r_L and an increase in the probability of high return p so as to maintain the mean return constant. Then, it becomes optimal to finance the projects separately for a larger region of parameters if and only if the high return level r_H is sufficiently large.

Consider first the case in which bankruptcy is extremely costly and the recovery rate is zero ($\beta = 0$). In this case, an increase in the negative skewness has two conflicting effects. On the one hand, as r_L decreases, the crossing point is reduced, so that joint financing at the rate avoiding intermediate bankruptcy becomes more difficult. On the other hand, as p increases so as to keep the mean constant, the probability that both projects’ returns are low is reduced, so that it becomes easier to finance the projects at the rate avoiding intermediate bankruptcy. Graphically, the yellow area representing the creditor’s expected returns at the crossing point is less wide (lower crossing point) but higher (higher probability of staying

afloat).

Which of these two effects dominates depends on the level of the high return r_H . For a larger r_H , the same reduction in the probability of high return p needs a higher reduction in the low return realization to ensure a constant mean. As a result, for the same increase in the probability of staying afloat (height of the rectangle), we have a higher reduction in the crossing point (width of the rectangle). Hence, the increase in negative skewness makes it more difficult to finance the projects jointly at a repayment rate below the crossing point.

For the general case with a positive recovery rate $\beta > 0$, there is a third effect that makes it even more difficult to finance the projects at the rate avoiding intermediate bankruptcy because the increase in negative skewness reduces the recovered returns. Indeed, the pink area (the expected returns conditional on default) shrinks by becoming less wide (lower r_L) and less high (lower $1 - p$). Therefore, the threshold level of r_H above which an increase in negative skewness favors separate financing decreases in β .

Esty (2002) offers a description of the distribution of returns of projects. Project finance is used when returns have large negative skewness due e.g. to environmental and expropriation risks (risks in which the possibility of total loss exists, as opposed to exchange rate, throughput, quantity or price risks, which can go either way). As Esty (2003) confirms, project finance is more appropriate for situations where the possibility of a total loss exists (i.e., the distribution of possible losses exhibits large negative skewness).

Further indirect evidence can be provided by hedge funds. Hedge funds returns have negative skewness and, at the same time, they make high use of off-balance sheet instruments. Indeed, Brooks and Kat (2002) found that the published hedge fund indices exhibited relatively low skewness and high kurtosis. Further, Malkiel and Saha (year) show that many hedge fund categories have considerable negative skewness. At the same time, hedge funds rely principally on off-balance sheet techniques. Long term capital balance sheet's, for example, amounted to \$125 billion but its off-balance sheet positions had notional amount of \$1.25 trillion.

E. Managerial Implications

We now show that the option with the lowest repayment rate does not necessarily have the lowest likelihood of bankruptcy and it might therefore not be optimal.

Figure 4 depicts the financing conditions and the repayment rates charged as a function

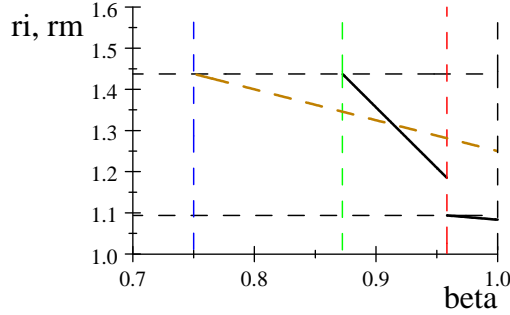


Figure 4. Rates. This figure plots the rates for separate and joint financing depending on the bankruptcy recovery rate. In this illustration, $r_L = 3/4$, $r_H = 23/16$, and $p = 1/2$. Separate financing corresponds to the brown line and joint financing to the black lines. Separate financing is obtained for $\beta > \beta_i^*$, represented by the blue line. Joint financing at a rate that does and does not avoid intermediate bankruptcy is obtained for $\beta > \beta_m^*$ (red line) and for $\beta > \beta_m^{**}$ (green line), respectively.

of the recovery rate β , for a given homogeneous combination of returns, r_H and r_L , and probability of high return, p . Separately, financing for each project can be obtained after the blue line threshold at the rate depicted by the brown line. Jointly, financing at a rate that does and does not avoid intermediate bankruptcy can be obtained after the red and green thresholds, respectively, at a repayment rate depicted by the black line.

After the red threshold, the borrower obtains lower repayment rates with joint financing and, as we have seen before, joint financing is optimal. Nevertheless, below this threshold the loan rates are not necessarily lower with separate financing, although this is the optimal financing choice. The following proposition formalizes this result; borrowers should not always accept the loan with the lowest repayment rate.

PROPOSITION 3: *Projects should be financed separately despite having higher repayment rates if and only if (i) the joint rate is r_m^{**} and (ii) the separate rate is such that $r_i^* < \beta r_H$.*

Suppose that the borrower has the choice of financing the projects independently and jointly, although only at a rate with intermediate bankruptcy. In this region with bad conglomeration, the low return project drags down the high return one. The borrower should finance the projects separately because the losses from bankruptcy are lower. However, if, at the same time, the returns recovered from a bankrupt high value project are higher

than what the creditor can charge for separate loans ($\beta r_H > r_i^*$), the creditor has higher returns if the projects are financed jointly, even though bankruptcy is more likely. As a result, the repayment rates are lower with joint than with separate financing. The borrower might feel tempted to finance the projects jointly, but this is suboptimal. Low interest rate associated with joint financing here are deceptively attractive—while it might look good, conglomeration is bad.

The logic can be further illustrated by Panel (b) of Figure 2. For an (exogenous) repayment rate above the crossing point, $r > (r_H + r_L)/2$, as the one depicted, the creditor's expected returns might be higher if projects are financed jointly in spite of the increased occurrence of bankruptcy. Indeed, with joint financing, the creditor obtains the part of the yellow area above the dashed line as well as a fraction β of the red and pink areas. With separate financing, the creditor obtains the yellow area and the upper part of the red area fully and a fraction β of the pink area. The creditor's returns at this interest rate are higher if proceeds from the fraction β of the red area, $p(1-p)\beta r_H$, are greater than the sum of the upper part of the red area and the part of the yellow area below the dashed line, $p(1-p)r$. This is the case if and only if $\beta r_H > r$, which holds if bankruptcy costs are sufficiently low. If the creditor breaks even at rate $r_i^* = r$ in the equilibrium with separate financing, the equilibrium rate with joint financing must be lower, so that $r_m^{**} < r_i^*$. In this case, equilibrium interest rate for joint financing is lower despite higher probability of bankruptcy. Intuitively, creditors can obtain higher expected proceeds from bankruptcy with joint financing, and so are forced by competition to offer lower interest rate—however, the borrower obtains higher expected payoff with separate financing at a higher interest rate.¹²

III. Correlation and Large Number of Projects

We now study the effects of correlation and a higher number of projects. We modify first the distribution of joint returns. Suppose that the probability of having two high returns is equal to $p[1 - (1-p)(1-\rho)]$, the probability of two low returns is equal to $(1-p)[1 - p(1-\rho)]$, whereas the probability that one of the projects yields a high return

¹²Note if the distribution of returns was continuous (rather than discrete, as in our model with binary returns), the extra losses from higher probability of bankruptcy if the equilibrium rate with joint financing was marginally above the crossing point will always be compensated by the increased proceeds from bankruptcy. Therefore, ugly conglomeration always appears when the project's returns are continuously distributed, because then there would be no discrete jump in the probability of bankruptcy at the crossing point (as there is with binary returns, for which ugly conglomeration therefore does not always arise).

whereas the other yields a low one is equal to $p(1-p)(1-\rho)$. In that case, ρ would be the correlation coefficient between the two projects. In order to be well-defined, it is necessary to assume that $\rho \geq \max\langle -(1-p)/p, -p/(1-p) \rangle$. Clearly, if $\rho = 0$ we are back to the case with independent returns.

PREDICTION 5: If the correlation between the projects increases (ρ is larger), then separate financing is preferred for a larger set of parameters.

The probability of having two high returns and the probability of having two low returns increase simultaneously with ρ . As a result, the repayment rate when intermediate bankruptcy is avoided is higher because the probability of two low returns is higher. When intermediate bankruptcy cannot be avoided, instead, the repayment rate is lower because the probability of two high returns also increases. As a consequence, the financing conditions avoiding intermediate bankruptcy are tighter and those not avoiding it looser.

The effects of correlation on the optimality conditions are also intuitive. In the extreme case in which one project has a high return the other necessarily has a low one (i.e., if $\rho = -1$ and $p = 1/2$), projects can always be jointly financed at a rate that avoids intermediate bankruptcy.¹³ Projects, therefore should always be financed jointly. As correlation increases above $\rho = -1$, conglomeration is less likely to be optimal. If both projects are perfectly correlated ($\rho = 1$), the conditions for joint and separate financing are identical and the firm is clearly indifferent between them. This prediction is similar to the one in Inderst and Muller (2003) and Leland (2007)

Second, consider a borrower with access to a large number of projects with independent returns. We show that if the number of projects is sufficiently large, it always becomes possible for the borrower to finance all the projects with a single loan. This result exploits the law of large numbers. Namely, as the number of projects n increases, the probability that the average number of projects with high returns differs from p , the probability of a high return, by more than a small amount ε tends to zero. We can then construct a rate offer to finance all projects jointly that is acceptable for the creditors. The borrower's returns when financing all projects jointly is then arbitrarily close to the first best as the number of projects grows large. Therefore, for a large number of projects financing all the projects jointly is approximately optimal for the borrower, because it yields a payoff that is close to

¹³This is not true for $p \neq 1/2$ because either the probability of two high realizations or the probability of two low realizations is greater than 0, even when the correlation is at the lowest possible level.

the highest possible level.

PROPOSITION 4: There exists n' and $q \in (0, p)$ such that for $n > n'$ a joint loan comprising all projects can be financed at a repayment rate that avoids bankruptcy when nq projects have high returns. The per-project return achieved in this way approaches the net present expected value of each project as n grows.

This leads to a new prediction.

PREDICTION 6: A large number of independent projects should be financed together.

IV. Heterogeneous Projects

In a recent paper, Leland (2007) stresses a different benefit of financial separation from ours. Financial separation allows firms with different return profiles to choose different capital structures. We have abstracted so far from this effect by assuming that projects are ex-ante symmetric. In this section, we extend the model to allow for heterogeneity across projects.

A. Financing Conditions

Focus on the case with $n = 2$ heterogeneous (and independently distributed) projects $i = 1, 2$. Project i yields returns r_H^i with probability p_i and r_L^i with probability $1 - p_i$. Without loss of generality, assume that $r_H^1 + r_L^2 > r_L^1 + r_H^2$, interchanging the indices if necessary. With asymmetric projects, four (rather than three) levels of combined returns are possible, adding an extra case to the conditions for joint financing. Now, the possibility arises that default is avoided if project 1 yields a high return and project 2 a low return, whereas default is not avoided if the reverse occurs (case (b) in the following proposition).

PROPOSITION 5: There exists r'_i such that project i can be financed separately if and only if $r'_i < r_H^i$, in which case, the equilibrium repayment rate is r'_i . If the firm seeks joint finance, there exist r'_m, r''_m and r'''_m such that

- (a) if $r_L^1 + r_H^2 > 2r'_m$, then the equilibrium rate is r'_m ;
- (b) if $r_H^1 + r_L^2 > 2r''_m$ and $r_L^1 + r_H^2 < 2r'_m$, then the equilibrium rate is r''_m ;
- (c) if $r_H^1 + r_H^2 > 2r'''_m$, $r_L^1 + r_H^2 < 2r'_m$, and $r_H^1 + r_L^2 < 2r''_m$, then the equilibrium rate is r'''_m ;
- (d) if $r_H^1 + r_H^2 < 2r'''_m$, $r_L^1 + r_H^2 < 2r'_m$, and $r_H^1 + r_L^2 < 2r''_m$, then the projects cannot be financed.

B. Good and Bad Conglomeration

We now turn to the question of whether the borrower should finance the projects jointly or separately when both financing modes are feasible. As in the symmetric case, if a rate that avoids bankruptcy in both intermediate situations can be obtained (case (a) in Proposition 5), then projects co-insure each other and therefore should be financed jointly. If, instead, the firm can only obtain a rate that does not avoid bankruptcy in any of the intermediate situations (case (c)), then the projects should be financed separately because they drag down each other. If bankruptcy can only be avoided for the more favorable intermediate situation, then both co-insurance and contamination effects are present at the same time. On the one hand, project 1, when it yields a high return, saves project 2 when project 2 yields a low return; on the other hand, project 1, when it yields a low return, contaminates project 2 when project 2 yields a high return. The optimality of separate or joint financing depends on whether the gains from co-insurance dominate the losses from risk contamination.

PROPOSITION 6: If the borrower can finance both projects separately and jointly, then

(a) If the joint rate is r'_m , then the borrower should finance the projects jointly because of the co-insurance effect. The gain in expected payoff from joint rather than separate financing is $(1 - p_1)p_2(1 - \beta)r_L^1 + p_1(1 - p_2)(1 - \beta)r_L^2$.

(b) If the joint rate is r''_m , then the borrower should finance the projects separately if and only if the risk contamination effect dominates the co-insurance effect: $(1 - p_1)p_2(1 - \beta)r_H^2 > p_1(1 - p_2)(1 - \beta)r_L^2$.

(c) If the joint rate is r'''_m , then the borrower should finance the projects separately because of the risk contamination effect. The gain in expected payoff from separate rather than joint financing is $p_1(1 - p_2)(1 - \beta)r_H^1 + (1 - p_1)p_2(1 - \beta)r_H^2$.

Note that if the two projects have the same probability of success, then the risk contamination effect always dominates the co-insurance effect in case (b). With joint financing, the probabilities of saving and dragging down project 2 are the same but the co-insurance gains are outweighed by the contamination losses, because the project is saved when it has a low return but it is dragged down following a high return. Hence, separation is optimal unless a joint-financing rate that avoids bankruptcy with both intermediate returns can be obtained.

C. Comparative Statics Predictions

For the case in which one project is a mean preserving spread of the other, the next result establishes that more risk typically induces even more separation.

PREDICTION 7: If project 1 second-order stochastically dominates project 2, and therefore $p_1 = p_2$ and $r_H^1 = r_H^2 + \varepsilon$ and $r_L^1 = r_L^2 - \frac{p_1}{1-p_1}\varepsilon$ for $\varepsilon > 0$, then projects should be financed separately unless the rate r'_m can be obtained with joint financing. The region of parameters for which separation is optimal increases with the spread of the risky project.

The area in which joint financing is optimal shrinks as the spread of the risky project increases, as the condition for obtaining the rate r'_m (case (a) in Proposition 5) becomes more stringent. Indeed, the less favorable intermediate returns ($r_L^1 + r_H^2$) decrease in the spread of project 1 and the repayment rate (r'_m) increases, as the creditor recovers less in the event of bankruptcy (when both projects yield low returns). In addition, it becomes easier to finance the projects separately as the increase in the high realization of the return is not compensated by the increase in the repayment rate (r'_i), making condition (a) in Proposition 5 easier to satisfy.

Gorton and Souleles (2005) provide evidence on which banks are more likely to use securitization, a form of separate financing especially designed to avoid bankruptcy procedures. They show that riskier originator banks are more likely to securitize, and therefore separate financing is more likely to be used when the risk of the riskier project increases. Bannier and Hansel (2008) also find that banks with higher expected credit risk are more likely to use loan securitizations.

When projects are heterogeneous, separation has the additional advantage of allowing for project-specific loans. If one of the projects has a low expected return, it might be better to finance only the other project rather than financing both of them with the same loan, even if this is possible. This reduces the attractiveness of joint financing, even when a rate that avoids bankruptcy for all intermediate returns can be obtained, as illustrated by the following result.

PREDICTION 8: If project 1 first-order stochastically dominates project 2, and in particular, $r_H^1 = r_H^2$ and $r_L^1 = r_L^2 = r_L$ and $p_1 > p_2$, then if both projects can be financed separately they should be financed separately unless the rate r'_m can be obtained. If only the high-mean project can be financed separately, then the borrower should only finance this project

unless (i) the rate r'_m can be obtained and (ii) the ex-post net present value of financing the low-mean project separately is compensated by the co-insurance effects, i.e. if and only if $[(1 - p_2)p_1 + (1 - p_1)p_2](1 - \beta)r_L > 1 - p_2r_H - (1 - p_2)\beta r_L$.

If both projects can be financed separately ($p_2r_H + (1 - p_2)\beta r_L - 1 > 0$) then they should be financed separately unless the rate joint rate r'_m can be obtained (here r''_m is never obtained, as there is only one level of intermediate returns). If, instead, the low-mean project has negative ex-post returns ($p_2r_H + (1 - p_2)\beta r_L - 1 < 0$), then this project cannot be financed separately. However, it might still be possible to finance this project jointly with the high-mean project if a joint rate r'_m can be obtained, as projects might save each other when they generate a low return. The borrower should indeed opt for joint financing rather than financing only the high-mean project if the co-insurance benefits more than compensate the ex-post negative returns of the low-mean project.

D. Managerial Implications

In Section E we show that the option with the lowest repayment rate does not need to have the lowest likelihood of bankruptcy and it is therefore not necessarily optimal. Here we show that the financing option with the lowest probability of bankruptcy might not be optimal either. This is due to the fact that, despite having a higher probability, the benefits of co-insurance might be outweighed by the costs of risk contamination.

PROPOSITION 7: Separate financing is optimal even though it results in higher probability of bankruptcy if and only if (i) the joint rate is r''_m and the contamination losses dominates co-insurance gains: $(1 - p_1)p_2(1 - \beta)r_H^2 > p_1(1 - p_2)(1 - \beta)r_L^2$; and (ii) the probability of the former is lower than that of the latter ($p_1 > p_2$).

When the joint rate is r''_m , we have that (i) if project 1 yields a low return, it drags down project 2's high return (that would have stayed afloat with separate financing) and (ii) if project 1 yields a high return, it saves project 2's low return (that would have defaulted with separate financing).

On the one hand, with separate financing, the probability of default is reduced when project 1 fails and project 2 succeeds—as the now separate project 2 is not dragged down by the failing project 1, as instead would not have happened with joint financing. According to this first effect, the probability of default with separate financing is reduced by $(1 - p_1)p_2$

compared to joint financing. On the other hand, with separate financing, the probability of default is increased when project 2 fails and project 1 succeeds—as the failing project 2 is not saved by the successful project 1, as instead would have happened with joint financing. According to this second effect, the probability of default with separate financing is increased by $p_1(1 - p_2)$ compared to joint financing. Overall, the probability of default with separate financing is higher than with joint financing if $(1 - p_1)p_2 < p_1(1 - p_2)$, i.e., if $p_2 < p_1$. Indeed, with joint financing project 2’s probability of staying afloat goes up from p_2 to p_1 , whereas project 1’s probability is the same.

Despite this, it might still be that the risk contamination effect dominates the co-insurance effect, and therefore the projects should be financed separately. We have that $p_1 > p_2$ and therefore $p_1(1 - p_2) > p_2(1 - p_1)$ but $p_1(1 - p_2)(1 - \beta)r_L^2 < (1 - p_1)p_2(1 - \beta)r_H^2$ provided that r_H^2 is sufficiently high compared to r_L^2 . Even though the probability of the co-insurance outcome is higher than that of risk contamination, if the level of bankruptcy costs conditional on default are sufficiently greater when project 2’s return is high, the risk contamination losses outweigh the co-insurance gains.

V. Debt, Equity, and Taxes

This section extends the model to allow the entrepreneur to use equity to finance part of the initial investment. Net equity proceeds are subject to corporate taxation at rate τ , whereas debt payments are tax deductible and therefore exempt from taxes. Equity and debt markets are assumed to be competitive. We denote the equilibrium fraction of the company j sold to the equity market as α_j , and the equilibrium equity and debt values of the company as E_j and D_j , respectively. Here, the promised repayment in the debt contract r_j shall depend on the optimal proportion of the initial investment outlay, 1, financed from debt, D_j , or equity markets, E_j , where $D_j + E_j = 1$.

A. Financing Conditions

For the case of separate financing, we now need to distinguish two separate cases, as it might be possible to obtain a rate that avoids bankruptcy altogether. Indeed, if it is possible to obtain a rate r'_i such that $r'_i < r_L$ by selling a fraction α of the company, r'_i should satisfy

$$\frac{\alpha(1 - \tau)}{1 + r_E} [p(r_H - r'_i) + (1 - p)(r_L - r'_i)] = E_i \quad \text{and} \quad r'_i = D_i.$$

Notice that the effect of a higher/lower tax is equivalent to that of a higher/lower equity premium. To simplify the notation, we normalise r_E to $r_E = 0$ and interpret the effects of a higher equity premium as the effects of a higher tax. Substituting into $E_i + D_i = 1$, this rate can be obtained if and only if

$$r'_i(\alpha) := 1 - \frac{\alpha(1-\tau)[pr_H + (1-p)r_L - 1]}{1 - \alpha(1-\tau)} < r_L. \quad (4)$$

Clearly, if the firm uses no equity ($\alpha = 0$) or equity is fully taxed ($\tau = 1$), then $r'_i = 1$ and the condition is never satisfied ($r_L < 1$), as in the baseline debt-only case. But, as more equity is offered, the debt repayment is lower ($r'_i(\alpha)$ is decreasing) and the condition might be satisfied.

Intuitively, the entrepreneur should choose the lowest possible amount of equity satisfying condition (4). Indeed, if the condition is satisfied, the entrepreneur would obtain $(1-\alpha)E$, which is equal to

$$\frac{(1-\alpha)(1-\tau)}{\tau + (1-\alpha)(1-\tau)} (pr_H + (1-p)r_L - 1).$$

She would get a fraction of the net present value, which corresponds to her (after-tax) equity holding – the other part is obtained by the government. Since this function is decreasing in α , the entrepreneur will offer α'_i such that $r'_i(\alpha'_i) = r_L$, and therefore

$$\alpha'_i := \frac{(1-r_L)}{(1-\tau)[pr_H + (1-p)r_L - 1 + (1-r_L)]}.$$

But of course, this is only possible if the proportion sold satisfies $\alpha'_i \leq 1$ and therefore

$$\tau < \tau_1^s := 1 - \frac{(1-r_L)}{(1-r_L) + pr_H + (1-p)r_L - 1}. \quad (5)$$

If this is not satisfied, then even by giving all the equity to the creditor ($\alpha = 1$), it is not enough to obtain this rate. If the condition is satisfied, the entrepreneur would get, $(pr_H + (1-p)r_L - 1) - \tau p(r_H - r_L)$.

Following the same procedure, a rate such that $r''_i < r_H$ can be obtained if and only if

$$r''_i(\alpha) := \frac{1 - \alpha(1-\tau)pr_H - (1-p)\beta r_L}{p[1 - \alpha(1-\tau)]} < r_H, \quad (6)$$

which, for the case $\alpha = 0$ it is exactly the same condition as in the baseline debt-only scenario, $r''_i(0) = r_i^* < r_H$. This condition is satisfied as long as the *ex-post* net present value is positive, $pr_H + (1-p)\beta r_L - 1 > 0$. Given that the entrepreneur would again hand out the lowest possible amount of equity (she would get the same fraction of the ex-post net present

value), the entrepreneur will prefer not to sell any equity, $\alpha'' = 0$, resulting in a payoff equal to $(1 - \tau)(pr_H + (1 - p)\beta r_L - 1)$. If the ex-post net present value is negative then it would be better not to fund the project at all rather than fund it at this rate.

As shown in the following proposition, it is not straightforward which rate will be optimal chosen if more than one is available.

PROPOSITION 8: There exists $\tau_{2,1}^s$ such that

- (i) if $pr_H + (1 - p)\beta r_L - 1 > 0$ then the projects will be financed at a rate $r'_i(\alpha'_i) = r_L$ if $\tau < \tau_{2,1}^s$ and at a rate $r''_i(0) = r_i^*$ if $\tau > \tau_{2,1}^s$.
- (ii) if $pr_H + (1 - p)\beta r_L - 1 < 0$ then the projects will be financed at a rate $r'_i(\alpha) = r_L$ if $\tau < \tau_1^s$ and cannot be financed if $\tau > \tau_1^s$.

This proposition shows that if taxes are high enough, the projects will be financed in the same situations (when ex-post net present value is positive, i.e. Condition (1)) at the same rate ($r_i^* > 1 > r_L$) as in the baseline case of no debt. Moreover, the project will, optimally, be financed entirely with debt. But here, if taxes are lower, the projects might be financed, using a positive amount of equity, at a rate that avoids bankruptcy altogether ($r'_i(\alpha) = r_L$), even if Condition (1) is not satisfied.

For the case of joint financing, we have three potential rates. The first, $r'_m < r_L$, in which we avoid bankruptcy altogether, is exactly equivalent to (and can be obtain under the same circumstances as) that of separate financing ($r'_m = r'_i$). Indeed, if the rate avoids bankruptcy, then the corporate structure does not matter.

Following the same procedure, a rate $r''_m < \frac{r_H + r_L}{2}$ can be obtained if and only if

$$r''_m(\alpha) := \frac{1 - \alpha(1 - \tau) [p^2 r_H + 2p(1 - p)\frac{r_H + r_L}{2}] - (1 - p)^2 \beta r_L}{[1 - (1 - p)^2] [1 - \alpha(1 - \tau)]} < \frac{r_H + r_L}{2}, \quad (7)$$

which is again a generalization for $\alpha \geq 0$ of Condition (2) of the baseline case, $r''_m(0) = r_m^* < (r_H + r_L)/2$. Substituting, the entrepreneur gets again the same proportion of the ex-post net present value. Again, provided that the ex-post net present value is positive, the entrepreneur will choose the minimum amount of equity to have condition (7) satisfied,

$$\alpha''_m := \max\left\{\frac{1 - [1 - (1 - p)^2] \frac{r_H + r_L}{2} - (1 - p)^2 \beta r_L}{(1 - \tau)p^2 \frac{r_H - r_L}{2}}, 0\right\}.$$

In this case, it might even be that the entrepreneur does not need to sell any equity at all, $\alpha''_m = 0$. This happens exactly if and only if Condition (2) of the baseline case is satisfied. Now, however, this rate can be obtained even if Condition (2) is not satisfied, by selling

some equity. At most, though, we can sell all the equity and therefore the rate can only be obtained if and only if $\alpha_m'' < 1$, i.e. if and only if

$$\tau < \tau_2^m := 1 - \frac{1 - [1 - (1 - p)^2] \frac{r_H + r_L}{2} - (1 - p)^2 \beta r_L}{p^2 \frac{r_H - r_L}{2}}.$$

A rate such that $r_m''' < r_H$ can be obtained as long as

$$r_m'''(\alpha) := \frac{1 - (1 - p)^2 \beta r_L - 2p(1 - p) \beta \frac{r_H + r_L}{2} - \alpha(1 - \tau)p^2 r_H}{p^2 [1 - \alpha(1 - \tau)]} < r_H, \quad (8)$$

which again, generalizes the condition of the baseline case for $\alpha \geq 0$, i.e. $r_m'''(0) = r_m^{**} < r_H$. The entrepreneur would again get the same proportion of the ex-post net present value, which is decreasing in α . But, Condition (8) is satisfied independently on the amount of equity sold. As a result, the entrepreneur should choose $\alpha_m''' = 0$.

PROPOSITION 9: There exist $\tau_{2,1}^{m,a}$, $\tau_{2,1}^{m,b}$, $\tau_{3,1}^m$ and $\tau_{3,2}^m$ such that, the optimal rates are

- (i) If $[1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L > 1$ then $r_m' = r_L$ if $\tau < \tau_{2,1}^{m,a}$ and $r_m''(0) = r_m^*$ if $\tau > \tau_{2,1}^{m,a}$.
- (ii) If $1 > [1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L > 1 - \frac{p}{2}(1 - r_L)$ then $r_m' = r_L$ if $\tau < \tau_{2,1}^{m,b}$, $r_m'' = \frac{r_H + r_L}{2}$ if $\tau_{2,1}^{m,b} < \tau < \tau_{3,2}^m$ and $r_m'''(0) = r_m^{**}$ if $\tau > \tau_{3,2}^m$.
- (ii) If $[1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L < 1 - \frac{p}{2}(1 - r_L)$, then $r_m' = r_L$ if $\tau < \tau_{3,1}^m$ and $r_m'''(0) = r_m^{**}$ if $\tau > \tau_{3,1}^m$.

B. Good and Bad Conglomeration

The profitability of merging is going to be driven by the cases in the cases in Proposition 9. Suppose that all rates are available.

In case (i), merging will always be profitable (at least weakly). This case is equivalent to the case of “good conglomeration” of our baseline case. The condition is exactly the same as the condition enabling the entrepreneur to obtain r_m^* in the baseline case.

In cases (ii) and (iii), conglomeration is “bad” in the baseline case. And indeed, if taxes are high enough conglomeration is still bad. If taxes are lower, however, it might still be possible to finance the projects with rates that avoid “intermediate” bankruptcy and even bankruptcy altogether (by financing with equity). In the following case, it is strictly optimal to merge when the rate that avoids intermediate bankruptcy can be obtained when merging, and the rate that avoids bankruptcy altogether cannot be obtained when non-merged.

In this case, conglomeration is good only when the rate that avoids bankruptcy altogether

can be obtained when merging, and the rate that avoids bankruptcy altogether when non-merged cannot be obtained.

PROPOSITION 10: When the borrower can finance both projects separately and jointly:

- (i) If $[1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L > 1$, the entrepreneur is indifferent if $\tau < \tau_{2,1}^{m,a}$ and the projects should be financed jointly if $\tau > \tau_{2,1}^{m,a}$.
- (ii) If $1 > [1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L > 1 - \frac{p}{2}(1 - r_L)$, the entrepreneur is indifferent if $\tau < \tau_{2,1}^{m,b}$, the projects should be financed jointly if $\tau_{2,1}^{m,b} < \tau < \tau_{2,1}^s$, and separately if $\tau > \tau_{2,1}^s$.
- (ii) If $[1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L < 1 - \frac{p}{2}(1 - r_L)$, the entrepreneur is indifferent if $\tau < \tau_{2,1}^s$, the projects should be financed jointly if $\tau_{2,1}^s < \tau < \tau_{3,1}^m$, and separately if $\tau > \tau_{3,1}^m$.

In sum, if taxes are high enough, we are exactly in the same situation as in the baseline case. That is, in case (i) it is profitable to merge and in cases (ii) and (iii), it is not. The condition separating case (i) from the other two is exactly the same as in the debt-only baseline case.

If taxes are intermediate, merging here can be profitable in cases in which it was not in the baseline case (cases ii and iii). This is because, by financing jointly, one can obtain rates that avoid intermediate bankruptcy or bankruptcy altogether by using equity. The former makes financing jointly sometimes optimal in case (ii) and the latter in case (iii).

Finally, if taxes are sufficiently low, then a merger has no effect because, in any case, bankruptcy can be avoided altogether.

VI. Conclusion

This paper addresses the classic question of the value of conglomeration with bankruptcy costs. By focusing on the simplest setting with binary returns, qualify the long-standing claim that joint financing generates financial benefits by economizing on bankruptcy costs. The same logic that allows conglomeration to create co-insurance savings in expected bankruptcy costs also results in additional risk contamination losses. We provide a full characterization of the conditions for which combining two (high-risk low-return) projects results in an increase in expected bankruptcy costs. We derive the following predictions:

- An increase in the bankruptcy recover rate favors joint financing.
- An increase in the probability of a high return favors joint financing.

- An increase in the riskiness of (sufficiently negatively skewed) projects favors separate financing.
- An increase in the negative skewness of projects (with sufficiently high return) favors separate financing.
- An increase in the correlation of projects favors separate financing.
- Joint financing of a sufficiently large number of independent projects is preferred.

In addition, our analysis uncovers additional advantages of separate financing when projects are heterogeneous. We also characterize situations in which projects should be financed in separate companies, even though this involves paying a higher interest rate than under joint financing. This is the case when the recovery rate in case of bankruptcy is sufficiently high, creditors are forced by competition to offer a more favorable interest rate for joint than separate financing. These results have clear implications for project finance and securitization.

Appendix A

Proofs

Proof of Proposition 5: Clearly, separate financing is not affected by correlation. The joint financing repayment rates, r_m^* and r_m^{**} in Proposition 1, and the corresponding financing conditions, are now replaced by $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$, respectively, where

$$r_{m,\rho}^* := \frac{1 - (1-p)[1-p(1-\rho)]\beta r_L}{1 - (1-p)[1-p(1-\rho)]} < \frac{r_H + r_L}{2},$$

and

$$r_{m,\rho}^{**} := \frac{1 - (1-p)\beta r_L}{p[1 - (1-p)(1-\rho)(1-\beta)]} < r_H.$$

Note that $r_{m,\rho}^*$ and $r_{m,\rho}^{**}$ are respectively increasing and decreasing in ρ . Q.E.D.

Proof of Proposition 4: First statement. Define $g(\gamma) := \gamma r_H + (1-\gamma)r_L$. We have that $g(p) > 1$ because of the positive net present value condition, and trivially $g(0) = r_L < 1$ and $g'(\gamma) > 0$. Then there exists a unique $\gamma^* \in (0, p)$ such that $g(\gamma^*) = 1$. For a fixed rational number ε (small) define $q := \gamma^* + \varepsilon$. Clearly, $qr_H + (1-q)r_L > 1$

Take any number of projects n such that nq is an integer number. Suppose that we were to finance all these n projects jointly at an interest rate that avoids bankruptcy when at least nq of them have high returns. This is possible if and only if the per-project repayment satisfies

$$r_n^* \leq qr_H + (1-q)r_L.$$

But, the creditor's zero profit condition implies that

$$r_n^* := \frac{1 - \beta \left(\sum_{k=0}^{nq-1} f(k) \frac{kr_H + (n-k)r_L}{n} \right)}{1 - F(nq-1)},$$

where $f(m)$ and $F(m)$ are the probability density and distribution that m out of the n projects have high returns, i.e.

$$f(m) := \binom{n}{m} p^m (1-p)^{n-m} \quad \text{and} \quad F(m) := \sum_{k=0}^m f(k).$$

Given that the returns recovered in the event of bankruptcy are positive we have that

$$r_n^* \leq \frac{1}{1 - F(nq-1)} < \frac{1}{1 - F(nq)}.$$

From the law of large numbers we have that $F(nq)$ tends to 0 as n grows large (remembering that $q < p$). Therefore r_n^* is bounded above by a number that is arbitrarily close to 1. Given

that $qr_H + (1 - q)r_L > 1$, there exists n' such that for all $n > n'$ then r_n^* is such that

$$r_n^* \leq qr_H + (1 - q)r_L,$$

as was to be shown.

Second statement: From the loan described above, the borrower obtains a per-project gross profit

$$\pi_n = \beta \sum_{k=0}^{nq-1} f(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right] + \sum_{k=nq}^n f(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right].$$

Fix a small rational number ε and an integer n such that $n(p - \varepsilon)$ and $n(p + \varepsilon)$ are integer numbers. Then, given that $q < p - \varepsilon$, and that all terms in the first and in the second sum are positive, we have that

$$\pi_n \geq \sum_{k=n(p-\varepsilon)}^{n(p+\varepsilon)} f(k) \left[\frac{k}{n} r_H + \left(1 - \frac{k}{n}\right) r_L \right].$$

Given that the terms in the second factor in the sum are larger for larger k , the sum is reduced by replacing the summand of a given k by that of $n(p - \varepsilon)$, the smallest term. Then, rearranging,

$$\pi_n \geq [(p - \varepsilon)r_H + [1 - (p - \varepsilon)]r_L] [F[n(p + \varepsilon)] - F[n(p - \varepsilon)]].$$

From the law of large numbers, $F[n(p + \varepsilon)] - F[n(p - \varepsilon)]$ tends to 1 as n grows. Indeed, from elementary statistics we know that

$$F[n(p + \varepsilon)] - F[n(p - \varepsilon)] \geq 1 - \frac{(p + \varepsilon)(1 - p)}{n\varepsilon^2} - \frac{(1 - p + \varepsilon)p}{n\varepsilon^2} = 1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}$$

and therefore

$$\pi_n \geq [pr_H + (1 - p)r_L - \varepsilon(r_H - r_L)] \left(1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}\right).$$

That is for n large, the gross per-project profit differs from the (gross) present value of each project by an amount that is arbitrarily small, $\varepsilon(r_H - r_L)$. Similarly,

$$\frac{\pi_n}{\pi^*} \geq \left(1 - \frac{\varepsilon(r_H - r_L)}{pr_H + (1 - p)r_L}\right) \left(1 - \frac{2p(1 - p) + \varepsilon}{n\varepsilon^2}\right)$$

where π^* is equal to first-best gross profits, $\pi^* = pr_H + (1 - p)r_L$. Q.E.D.

Proof of Proposition 5: Following the same procedure as in the symmetric case, the repayment rate should satisfy $1 < r'_i < r_H^i$. The creditor's zero profit condition is now

$$pr'_i + (1 - p_i)\beta r_L^i - 1 = 0, \tag{A1}$$

and project i can be financed (at r'_i) if and only if

$$r'_i := \frac{1 - (1 - p_i)\beta r_L^i}{p_i} < r_H^i. \quad (\text{A2})$$

There are three cases in which joint financing is feasible depending on whether bankruptcy can be avoided in both cases with intermediate returns, or only when project 1 yields a high return and 2 a low return, or in neither case. In the former case, competitive credit markets imply that

$$[1 - (1 - p_1)(1 - p_2)]2r'_m + (1 - p_1)(1 - p_2)\beta(r_L^1 + r_L^2) - 2 = 0, \quad (\text{A3})$$

and therefore this is possible if and only if

$$r'_m := \frac{1 - (1 - p_1)(1 - p_2)\beta\frac{r_L^1 + r_L^2}{2}}{1 - (1 - p_1)(1 - p_2)} < \frac{r_L^1 + r_H^2}{2}. \quad (\text{A4})$$

If default can be avoided with high intermediate returns but not with low intermediate returns, then

$$p_1 p_2 2r''_m + p_1(1 - p_2)2r''_m + (1 - p_1)p_2\beta(r_L^1 + r_H^2) + (1 - p_1)(1 - p_2)\beta(r_L^1 + r_L^2) - 2 = 0, \quad (\text{A5})$$

and therefore this is possible if and only if

$$\frac{r_L^1 + r_H^2}{2} < r''_m := \frac{1 - (1 - p_1)p_2\beta\frac{r_L^1 + r_H^2}{2} - (1 - p_1)(1 - p_2)\beta\frac{r_L^1 + r_L^2}{2}}{p_1} < \frac{r_H^1 + r_L^2}{2}.$$

If default cannot be avoided with either intermediate returns, then

$$p_1 p_2 2r'''_m + p_1(1 - p_2)\beta(r_H^1 + r_L^2) + (1 - p_1)p_2\beta(r_L^1 + r_H^2) + (1 - p_1)(1 - p_2)\beta(r_L^1 + r_L^2) - 2 = 0, \quad (\text{A6})$$

and therefore this is possible if and only if

$$\frac{r_H^1 + r_L^2}{2} < r'''_m < \frac{r_H^1 + r_H^2}{2}, \quad (\text{A7})$$

where

$$r'''_m := \frac{1 - p_1(1 - p_2)\beta\frac{r_H^1 + r_L^2}{2} - p_2(1 - p_1)\beta\frac{r_L^1 + r_H^2}{2} - (1 - p_1)(1 - p_2)\beta\frac{r_L^1 + r_L^2}{2}}{p_1 p_2}.$$

Again, since the borrower obtains all the ex-post net present value, rate r'_m is preferred to r''_m and r''_m is preferred to r'''_m . To complete the proof we only need to show that the lower bound conditions for r''_m and r'''_m are irrelevant. From (A3) and (A5), and rearranging, we

have

$$p_1(r'_m - r''_m) = p_2(1 - p_1) \left[\beta \left(\frac{r_L^1 + r_H^2}{2} \right) - r'_m \right],$$

and therefore if $r'_m > \frac{r_L^1 + r_H^2}{2}$ then the right hand side is negative. As a consequence, we have $r''_m > r'_m > \frac{r_L^1 + r_H^2}{2}$. Similarly, from (A5) and (A6) and rearranging, we have

$$p_2(r''_m - r'''_m) = (1 - p_2) \left[\beta \left(\frac{r_H^1 + r_L^2}{2} \right) - r''_m \right]$$

and therefore if $r''_m > \frac{r_H^1 + r_L^2}{2}$ then the right hand side is negative. As a consequence, we have $r'''_m > r''_m > \frac{r_H^1 + r_L^2}{2}$. Q.E.D.

Proof of Proposition 6: Substituting r'_m in the right hand side of (A3) and r'_i in the right hand side of (A1) and subtracting the latter from the former, we have

$$p_2(1 - p_1)(1 - \beta)r_L^1 + p_1(1 - p_2)(1 - \beta)r_L^2 (> 0).$$

Similarly, substituting r''_m in the right hand side of (A5) and subtracting again the ex-post net present value of financing the two projects separately from this, we obtain

$$-(1 - p_1)p_2(1 - \beta)r_H^2 + p_1(1 - p_2)(1 - \beta)r_L^2,$$

which can be positive or negative. Lastly, substituting r'''_m in the right-hand side of (A6) and subtracting the ex-post net present value of financing the two projects separately from this, we have

$$-p_1(1 - p_2)(1 - \beta)r_H^1 - p_2(1 - p_1)(1 - \beta)r_H^2 (< 0),$$

as desired. Q.E.D.

Proof of Prediction 7: Given that one project is obtained from an elementary increase in risk from the other and returns should still be binary, we must have that $p_1 = p_2$. Letting ε be such that $r_H^1 = r_H^2 + \varepsilon$, we have $r_L^1 = r_L^2 - \frac{p}{1-p}\varepsilon$. Indeed, $p(r_H^2 + \varepsilon) + (1-p)r_L^1 = pr_H^2 + (1-p)r_L^2$. We can also check that $r_L^1 + r_H^2 = r_L^2 - \frac{p}{1-p}\varepsilon + r_H^2 < r_L^2 + \varepsilon + r_H^2 = r_H^1 + r_L^2$.

As shown in the previous proposition, given that the probabilities of success are equal, we have that, when both projects can be financed separately as well as jointly, joint financing is only optimal if a rate r'_m can be obtained. Moreover, the region for which joint financing is optimal shrinks as the repayment rate r'_m is more difficult to obtain if ε increases. Indeed, the left-hand side of condition (A4) decreases in ε and the repayment rate (the right-hand

side) increases in ε .

On the other hand, the region for which separate financing is possible expands if ε increases. Indeed, the derivative of the left-hand side of condition (A2) is equal to β whereas the right hand-side is equal to 1. Hence, this condition is more easily satisfied as ε increases. Q.E.D.

Proof of Prediction 8: If both projects can be financed separately then, Proposition 5 implies that the borrower should finance them jointly if and only if a repayment rate r'_m can be obtained. Indeed, case (b) never occurs if the ex-post returns are the same as there is only one level of intermediate returns.

Suppose now that only one project can be financed separately, i.e. $p_i r_H + (1 - p_i) \beta r_L > 1$ and $p_j r_H + (1 - p_j) \beta r_L < 1$ for $i \neq j$, where we denote again $r_H := r_H^i$ and $r_L := r_L^i$ for $i = 1, 2$. Then the expected surplus from funding the project separately is $p_i r_H^i + (1 - p_i) \beta r_L^i - 1$. Subtracting this from the expected surplus from joint financing in the case in which the repayment rate r'_m can be obtained and simplifying, we have

$$p_j r_H + (1 - p_j) \beta r_L - 1 + [(1 - p_j) p_i + (1 - p_i) p_j] (1 - \beta) r_L.$$

On the other hand, if we subtract this from the expected surplus from joint financing in the case in which the repayment rate r'_m can be obtained and simplifying, we obtain

$$p_j r_H + (1 - p_j) \beta r_L - 1 - [(1 - p_j) p_i + (1 - p_i) p_j] (1 - \beta) r_H,$$

so that separate financing is optimal because both terms are negative.

Suppose that none of the two projects can be financed separately, if a rate r'_m can be obtained, i.e. if condition (A4) is satisfied, then from (A3), we have that the ex-post net present value of the joint combination is positive. On the other hand, it cannot be that a rate that does not avoid intermediate bankruptcy is obtained since (the second) inequality in (A4) implies that

$$p_1 p_2 2r_H + [p_1(1 - p_2) + p_2(1 - p_1)] \beta (r_L + r_H) + (1 - p_1)(1 - p_2) \beta 2r_L - 2 > 0,$$

which implies that the ex-post net present value is positive. This condition is equivalent to

$$p_1 r_H + (1 - p_1) \beta r_L - 1 + p_2 r_H + (1 - p_2) \beta r_L - 1 - [p_1(1 - p_2) + (1 - p_1) p_2] (1 - \beta) r_H > 0,$$

which contradicts the fact that the two projects cannot be financed independently. Q.E.D.

Proof of Proposition 3: To prove this, suppose first that a rate below the crossing point can be obtained. We have that

$$r_m^* = \frac{1 - (1-p)^2 \beta r_L}{1 - (1-p)^2} < \frac{1 - (1-p) \beta r_L}{p} = r_i^*,$$

because $1 > \beta r_L$. Suppose now that only a rate r_m^{**} can be obtained and therefore the probability of bankruptcy is higher with joint financing. r_m^* associated with joint financing is, nevertheless, lower than r_i^* associated with separate financing whenever

$$r_m^{**} = \frac{1 - (1-p) \beta (pr_H + r_L)}{p^2} < \frac{1 - (1-p) \beta r_L}{p} = r_i^*,$$

or equivalently

$$\beta r_H > \frac{1 - (1-p) \beta r_L}{p} = r_i^*.$$

Proof of Proposition 8: After the analysis above, it only remains to be shown which rate will be chosen if both are possible. If this is the case, comparing the respective payoffs, the entrepreneur will choose the first over the second rate if and only

$$\tau < \tau_{2,1}^s := 1 - \frac{(1 - r_L)}{[1 - \beta](1 - p)r_L + (1 - r_L)},$$

that is if bankruptcy costs $(1 - \beta)$ are high enough and/or taxes are small. It is straightforward to show that $\tau_1^{*,s} > \tau_{2,1}^s$.

Proof of Proposition 9: We first compute the payoffs of the entrepreneur in each case. First, if we can finance the projects at a rate r_m'' , the entrepreneur gets,

$$(1 - \tau) \left[p^2 r_H + 2p(1-p) \left(\frac{r_H + r_L}{2} \right) + (1-p)^2 \beta r_L - 1 \right]$$

if condition (??) is satisfied (financed at $r_m''(0) = r_m^*$), and

$$(1 - \tau) \left[p^2 r_H - p^2 \frac{r_H + r_L}{2} \right] + [1 - (1-p)^2] \frac{r_H + r_L}{2} + (1-p)^2 \beta r_L - 1$$

if condition (??) is not satisfied (financed at $r_m'' = \frac{r_H + r_L}{2}$). Similarly, if a rate r_m'' can be chosen, the entrepreneur gets

$$(1 - \tau) \left[p^2 r_H + 2p(1-p) \beta \frac{r_H + r_L}{2} + (1-p)^2 \beta r_L - 1 \right].$$

For part (i): If r_m'' can be obtained using no equity, then it will always be better than the third rate (ex-post net present value is higher). It might still be, though, that the first rate

is better. This is the case if and only if

$$\tau < \tau_{2,1}^{m,a} := 1 - \frac{(1 - r_L)}{(1 - r_L) + (1 - p)^2(1 - \beta)r_L}.$$

In parts (ii) and (iii), to obtain the second rate, it is necessary to sell some equity. In part (ii) the first is preferred to the second as long as

$$\tau < \tau_{2,1}^{m,b} := 1 - \frac{[1 - (1 - p)^2] \frac{r_H + r_L}{2} + (1 - p)^2 \beta r_L - r_L}{(1 - \frac{p}{2})p[r_H - r_L]}.$$

On the other hand, the second rate would still be preferred over the third as long as

$$\tau < \tau_{3,2}^m := 1 - \frac{1 - [1 - (1 - p)^2] \frac{r_H + r_L}{2} - (1 - p)^2 \beta r_L}{1 - p^2 \frac{r_H + r_L}{2} - 2p(1 - p)\beta \frac{r_H + r_L}{2} - (1 - p)^2 \beta r_L},$$

whereas, the first would be preferred over the third as long as

$$\tau < \tau_{3,1}^m := 1 - \frac{(1 - r_L)}{(1 - \beta) [2p(1 - p) \frac{r_H + r_L}{2} + (1 - p)^2 r_L] + (1 - r_L)}.$$

It can be shown that the previous cutoffs should be ordered as follows: $(1 - \tau)_{3,2}^m < (1 - \tau)_{3,1}^m < (1 - \tau)_{2,1}^{m,b}$. Therefore, we have the ordering in the text.

For part (iii), we havat that, the first is always preferred to the second. The ordering of the third with respect to the first or second is the same as in case (ii). As a result, we have the ordering in the text.

Proof of Proposition 10: Part (i): Indeed, if the second rate is optimal, then the profits of the entrepreneur will be higher than under the first rate of non-merging (which are the same than under the first rate of merging). If the first rate of merging is optimal, then the profits are the same (indeed, it cannot be that the second rate of non-merging is optimal because the profits are lower than under the second rate of merging, which in turn are lower than under the first rate of merging).

Part (ii): Remember that in this case, after merging, the third is optimal if $(1 - \tau) < (1 - \tau)_{3,2}^m$, the second is optimal if $(1 - \tau)_{3,2}^m < (1 - \tau) < (1 - \tau)_{2,1}^{m,b}$ and the first is optimal if $(1 - \tau)_{2,1}^{m,b} < (1 - \tau)$. We can further show that the changing point for non-merged is in between the two cutoffs, $(1 - \tau)_{3,2}^m < (1 - \tau)_{2,1}^s < (1 - \tau)_{2,1}^{m,b}$.

Therefore, if the third of merging is optimal, it is not profitable to merge because the ex-post profits of the second rate of separate financing are higher (this rate is optimal here because $(1 - \tau)_{3,2}^m < (1 - \tau)_{2,1}^s$). If the second of merging is optimal, then it is possible to show that it is not profitable to merge if the second rate of non-merged is used, i.e. if

$(1 - \tau)_{3,2}^m < (1 - \tau) < (1 - \tau)_{2,1}^s$, but, it is profitable if the first rate of non-merge is used, i.e. if $(1 - \tau)_{2,1}^s < (1 - \tau) < (1 - \tau)_{2,1}^{m,b}$ (because in the second merged we get more profits than in the first merged, which is the same as in the non-merged case). Finally, if the first rate of merging is optimal, then it is the same.

Part (iii): In that case, if the third rate of merging is optimal, $(1 - \tau) < (1 - \tau)_{3,1}^m$, it is not profitable to merge (indeed, the ex-post profits of the second case of non-merging are higher, and given that $(1 - \tau)_{3,1}^m < (1 - \tau)_{2,1}^s$, the second rate is optimal if projects are financed separately). If, on the other hand, the first of merging is optimal, then not-merging cannot be worse (at least we can get the same). In fact, $(1 - \tau)_{3,1}^m < (1 - \tau) < (1 - \tau)_{2,1}^s$ it is strictly better and indifferent for $(1 - \tau)_{2,1}^s < (1 - \tau)$.

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Appendix B: OMITTED SUPPLEMENTARY MATERIAL

Optimal Contracting with Non-Verifiable Returns

The debt contract we have adopted in this paper is the optimal financial arrangement in the “costly state verification” model (see Townsend, 1978, and Gale and Hellwig, 1985), when creditors can verify company returns at a cost equal to the bankruptcy cost. As it is well known, however, debt is no longer optimal when the possibility of renegotiation is introduced, because verification is ex-post suboptimal in equilibrium.

This appendix revisits our analysis of conglomeration in an alternative model in which debt is not only the optimal financial arrangement, but is also robust to the introduction of renegotiation. This model is a two-project extension of Bolton and Scharfstein’s (1990) dynamic model of debt with non-verifiable returns.

Suppose that (1) projects generate unverifiable returns not for one but potentially for two periods and (2) creditors can threaten continuation from the first to the second period by withholding required intermediate funding. Formally, assume that each of the $n = 2$ projects available requires an up-front investment I at time 0 and generates income R_H with probability p and R_L with probability $1 - p$, with $R_H > R_L$, at time 1. Conditional on an additional investment of L at time 1, each project generates, at time 2, an expected income R_H^2 if the first-period income was R_H and expected income R_L^2 if it was R_L , where $R_H^2 > R_L^2$. Although projects’ returns are correlated across periods, the additional investment required is independent from the first-period return. Termination is inefficient in any case, as $R_H^2 > R_L^2 > L$. Therefore, in this setting bankruptcy costs are defined as the loss in expected net present value from early termination of the project.¹⁴

Returns are not verifiable and therefore the borrower will repay nothing in the second (and last) period. In other words, R_j^2 is a private benefit for the borrower. Creditors, however, can induce a truthful report of the first period by committing (ex-ante) to provide additional funds at time 1. However, it is not possible to always guarantee extra funding, as $R_L < I + L =: I^n$. As in the baseline model, we assume away discounting and assume that the borrower has all the bargaining power.

When deciding whether to finance the two projects separately or jointly, the borrower faces the following trade off. On the one hand, by financing the two projects jointly, the bor-

¹⁴As in the baseline model, more resources are lost when a good project is terminated. However, the creditor obtains the same amount whether the project is good or bad.

rower can achieve the benefits of coinsurance. On the other hand, by financing each project separately in a stand-alone company, the borrower reduces the possibility of risk contamination, the phenomenon whereby a failing asset drags an otherwise healthy sponsoring firm into distress. We assume that, when the projects are financed jointly, it is impossible to terminate one project without terminating the other or, equivalently, that a common refinancing probability must be used. If this was not the case, then it would always be optimal to finance the projects jointly as, at the very least, one could replicate the separate contracts. Indeed, if the projects are financed separately, one of them can be terminated without terminating the other and therefore separate refinancing probabilities can be used. Finally, to abstract from the problems of internal financing, studied by Inderst and Müller (2003), we assume that self-refinancing is not possible.

We proceed by first deriving the optimal contracts for separate and joint financing, and then comparing the expected benefits of these two options. The proofs are collected at the end of the section.

Optimal Contracts

For each separate project, the borrower maximizes the expected surplus subject to the individual rationality and the incentive compatibility constraints. Denote as D_1 the payment at time 1 if a high return is announced and D_0 if a low is announced. Let the continuation probabilities be y_1 in case of a high announcement and y_0 in case of a low. The borrower's problem is given by

$$\begin{aligned} & \underset{y_0, y_1, D_1, D_0}{Max} \quad p(R_H - D_1 + y_1 R_H^2) + (1-p)(R_L - D_0 + y_0 R_L^2) \\ \text{s.t.} \quad & (IC_h) \quad (y_1 - y_0) R_H^2 \geq D_1 - D_0 \\ & (IR) \quad p(D_1 + (1 - y_1)L) + (1-p)(D_0 + (1 - y_0)L) \geq I^n, \end{aligned}$$

where we are implicitly assuming that $D_1 \leq R_H$, $D_0 \leq R_L$ (and $D_1 > D_0$) and that IC_l , $(y_0 - y_1)R_L^2 \geq D_0 - D_1$, is satisfied.

LEMMA B1: IC_h and IR constraints are binding and $y_1 = 1$.

In the high state, it is better to have a higher continuation probability than a lower payment (better for efficiency and for incentives). In contrast, in the low state, it might be better to have a low probability and a high payment to improve incentives. As stated in the

IC_h constraint, the difference in probability should be high enough relative to the difference in payments to induce truth-telling.¹⁵

PROPOSITION 11: Assume that

$$\frac{I^n - R_L}{pR_H^2 + (1-p)L} R_H^2 < R_H - R_L \quad (\text{B8})$$

then the optimal contract satisfies

$$D_1 = \frac{I^n - R_L}{pR_H^2 + (1-p)L} R_H^2 + R_L; \quad D_0 = R_L;$$

$$1 - y_1 = 0; \quad 1 - y_0 = \frac{I^n - R_L}{pR_H^2 + (1-p)L}.$$

To minimize inefficiencies, the borrower tries to set the continuation probability in the low state as high as possible. With respect to the payments, it is optimal to have a high payment in the low state and a not so high payment in the high state. If the difference is small, then the continuation probability difference can also be small and therefore the probability of continuation in the low state can be high.

Similarly, the optimal contract for joint financing should maximize the borrower's expected surplus subject to individual rationality and the incentive compatibility constraints. Denote $D_{1,1}$ the payment in period 1 if two high returns are announced, $D_{1,0}$ if a high and a low are announced and $D_{0,0}$ if two lows are announced and the respective continuation probabilities as $y_{1,1}$, $y_{1,0}$ and $y_{0,0}$. The problem becomes

$$\max_{y_{0,0}, y_{1,0}, y_{1,1}, D_{1,1}, D_{0,0}, D_{1,0}} p^2(2R_H - D_{1,1} + y_{1,1}2R_H^2) +$$

$$2(1-p)p(R_L + R_H - D_{1,0} + y_{1,0}[R_H^2 + R_L^2]) + (1-p)^2(2R_L - D_{0,0} + y_{0,0}2R_L^2)$$

$$\text{s.t. } (IC_{h,h}) \quad (y_{1,1} - y_{1,0})2R_H^2 \geq D_{1,1} - D_{1,0}$$

$$(IC_{h,l}) \quad (y_{1,0} - y_{0,0})(R_H^2 + R_L^2) \geq D_{1,0} - D_{0,0}$$

$$(IR) \quad p^2(D_{1,1} + (1 - y_{1,1})2L) + 2p(1-p)(D_{1,0} + (1 - y_{1,0})2L) +$$

$$+(1-p)^2(D_{0,0} + (1 - y_{0,0})2L) \geq 2I^n,$$

where, we are again implicitly assuming that the limited liability ($D_{1,1} \leq 2R_H, D_{0,0} \leq$

¹⁵We show that we can still view the optimal contract should as a debt contract since, in the sense that the probability of termination is 0 in case of a repayment.

$2R_L, D_{1,0} \leq R_L + R_H$) and the other IC constraints are satisfied.¹⁶

The following proposition outlines the optimal contract.

PROPOSITION 12: If $2p(1-p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2) > 0$ then the optimal contract satisfies

$$(1 - y_{1,1}) = 0;$$

$$(1 - y_{1,0}) \text{ as low as possible subject to } D_{1,0} < R_H + R_L \text{ and } D_{1,1} < 2R_H$$

$$(1 - y_{0,0}) = \frac{2I^n - 2R_L + (1 - y_{1,0}) [2p(1-p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2)]}{(R_H^2 + R_L^2)(1 - (1-p)^2) + (1-p)^2 2L}$$

$$D_{1,1} = (1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + 2R_L$$

$$D_{1,0} = (1 - y_{0,0})(R_H^2 + R_L^2) - (1 - y_{1,0})(R_H^2 + R_L^2) + 2R_L; \quad D_{0,0} = 2R_L.$$

Note that in the case of independent projects we assumed that the limited liability constraint is satisfied for the high repayment. Here we do not make that assumption a priori, but it must be checked that this condition holds.

Separate or Joint Finance?

Again, the expected inefficiency might be greater by pooling the projects and therefore separation might be optimal. It is possible that a good and a bad realization ends up with a joint failure, whereas if the projects had been independent, the good one would have been saved. As opposed to the baseline model, a high and a low realization is not a sure failure but a strictly positive probability of failure. Still, for other parameter values, a good and a bad never go bankrupt, the bad one is saved by the high one with probability one. As a result, the expected inefficiency is lower with joint projects and therefore they should be financed jointly.

For the comparison we will compare the inefficiency arising from joint projects, denoted as A , and that from separation, denoted by B , where

$$A := (1-p)^2(1-y_{0,0})2(R_L^2 - L) + 2p(1-p)(1-y_{1,0})(R_H^2 + R_L^2 - 2L),$$

$$B := 2(1-p)(1-y_0)(R_L^2 - L).$$

¹⁶We can easily check that $2[I + p^2L + 2p(1-p)L + (1-p)^2L] = 2I^n$.

The comparison is driven by the previous proposition. If bankruptcy does not result following a high and a low realization, joint financing is optimal. If instead bankruptcy does result in that instance, separate financing might dominate.

LEMMA B2: $y_{1,0}$ can be equal to 1 only if

$$\frac{2I^n - 2R_L}{(R_H^2 + R_L^2)(1 - (1-p)^2) + (1-p)^2 2L} (R_H^2 + R_L^2) < R_H - R_L. \quad (\text{B9})$$

This condition is more stringent (in terms of $R_H - R_L$) than condition (B8).

We analyze two cases, depending on whether this condition is satisfied, always assuming that condition (B8) in Proposition 11 for financing independent projects holds. Note that in the baseline case the key to decide whether to bundle the projects was whether bankruptcy results when one project yields a high return and the other low return. Here, if the condition is satisfied, the two projects are again saved when one of them yields a low return so that joint financing is optimal:

PROPOSITION 13: If condition (B9) is satisfied, then financing the projects jointly is optimal.

If, on the other hand, the condition is not satisfied, then it might be that separation is optimal. To show this, we focus on the special case in which L is close to R_L^2 . In this case, the condition in Proposition is equivalent to $p < 2/3$.

PROPOSITION 14: If condition (B9) is not satisfied, there are cases in which it is optimal to finance the two projects separately. For example, if L is close to R_L^2 and if the following two conditions are satisfied

$$p < 2/3 \text{ and } \frac{I^n - R_L}{p^2 R_H^2 + (1-p^2) R_L^2} < \frac{R_H^2 + R_L^2 - (1-p)^2 (R_H^2 - R_L^2)}{R_H^2 + R_L^2 - (1-2p^2) (R_H^2 - R_L^2)},$$

separate financing is optimal if and only if

$$\frac{I^n - R_L}{p + (1-p) \frac{R_L^2}{R_H^2}} < R_H - R_L < \frac{2I^n - 2R_L}{1 - (1-p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}}.$$

Proofs for Appendix B

Proof of Lemma B1: The IR constraint is binding because if not, we could lower D_1 and still IC_h would be satisfied while the optimum would be higher. $y_1 = 1$ because, if not ($1 > y_1 > y_0$) an increase in y_1 by a small amount ε and an increase in D_1 by εL would keep

IR and IC_h satisfied (the latter since $R_H^2 > L$) and the borrower's utility would increase by $p(R_H^2 - L)\varepsilon > 0$. IC_h is binding because, if not, we could raise y_0 by ε and increase D_1 by $\varepsilon L(1-p)/p$ so that IR is still satisfied. Borrower's utility would increase by $-p\varepsilon L(1-p)/p + (1-p)\varepsilon R_L^2 = (1-p)\varepsilon(R_L^2 - L) > 0$. Q.E.D.

Proof of Proposition 11: From the previous lemma, we have from the IR and the IC constraints

$$\begin{aligned} (1 - y_0) R_H^2 + D_0 &= D_1, \\ pD_1 + (1 - p)(D_0 + (1 - y_0)L) &= I^n. \end{aligned}$$

Substituting the first into the second and rewriting, we have that

$$\begin{aligned} D_1 &= (1 - y_0)R_H^2 + D_0 \\ 1 - y_0 &= \frac{I^n - D_0}{pR_H^2 + (1 - p)L}. \end{aligned}$$

(Notice that the constraint $(y_0 - y_1)R_L^2 \geq D_0 - D_1$ is satisfied. Indeed $D_1 = (1 - y_0)R_H^2 + D_0 \geq (1 - y_0)R_L^2 + D_0$.) Substituting the D_1 in the objective function

$$pR_H + (1 - p)R_L - D_0 + y_0 [pR_H^2 + (1 - p)R_L^2]$$

and substituting y_0 we have

$$pR_H + (1 - p)R_L - D_0 + \left[1 - \frac{I^n - D_0}{pR_H^2 + (1 - p)L}\right] [pR_H^2 + (1 - p)R_L^2].$$

Deriving with respect to D_0 we have

$$-1 + \frac{pR_H^2 + (1 - p)R_L^2}{pR_H^2 + (1 - p)L}$$

which is positive because L is lower than R_L^2 and therefore the denominator is lower than the numerator. That should be as high as possible, $D_0 = R_L$. This is possible because the assumption ensures that $D_1 < R_H$. Q.E.D.

Proof of Proposition 12: First, following the same reasoning as before, we can show that the constraints are binding and $y_{1,1} = 1$. Substituting them into the the IC constraints and rewriting

$$D_{1,1} = (1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + D_{0,0}, \quad (\text{B10})$$

$$D_{1,0} = (1 - y_{0,0})(R_H^2 + R_L^2) - (1 - y_{1,0})(R_H^2 + R_L^2) + D_{0,0}. \quad (\text{B11})$$

Substituting these and $y_{1,1} = 1$ into the IR constraint

$$(IR) \quad D_{0,0} + (1 - y_{0,0}) [(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L] \\ - (1 - y_{1,0}) [2p(1 - p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2)] = 2I^n$$

and then in the objective function, we have

$$p^2(2R_H) + 2(1 - p)p(R_L + R_H) + (1 - p)^2(2R_L) + p2R_H^2 + 2R_L^2(1 - p) - D_{0,0} \\ - (1 - y_{1,0})p^2(R_H^2 - R_L^2) - (1 - y_{0,0}) [(R_H^2 + R_L^2) - (1 - p)^2(R_H^2 - R_L^2)].$$

Now, by increasing $D_{0,0}$ by εs (where $s = (R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L$) and $y_{0,0}$ by ε we have the same IR and the objective function increases by $\varepsilon[(R_H^2 + R_L^2) - (1 - p)^2(R_H^2 - R_L^2) - (R_H^2 + R_L^2)(1 - (1 - p)^2) - (1 - p)^2 2L] = (1 - p)^2 [2R_L^2 - 2L] > 0$.

The IR can be written as

$$(1 - y_{0,0}) = \frac{2I^n - 2R_L + (1 - y_{1,0}) [2p(1 - p)(R_H^2 + R_L^2 - 2L) - p^2(R_H^2 - R_L^2)]}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L}$$

and therefore if $y_{1,0}$ is higher $y_{0,0}$ is higher and both the $y_{1,0}$ and $y_{0,0}$ terms increase the objective function. Provided that the condition on the statement of the proposition is true, we have that the numerator is positive. Q.E.D.

Proof of Lemma B2: We have that $y_{1,0}$ can be equal to 1 only if $D_{1,0} < R_H + R_L$, which simplifying is equivalent to

$$\frac{2I^n - 2R_L - (1 - y_{1,0}) [p^2 2R_H^2 + (1 - p^2) 2L]}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L} R_H^2 + R_L^2 < R_H - R_L.$$

Substituting $y_{1,0} = 1$, this condition is exactly as in the statement in the text.

The second part of the Lemma follows since the left hand side of the statement in the Lemma is higher than the left hand side of the statement in Proposition 2 as long as $(1 - p)L [R_L^2 + pR_H^2] + p^2 \frac{1}{2} (R_H^2 + R_L^2) R_H^2 > 0$, which is clearly true. Q.E.D.

Proof of Proposition 13: Substituting $y_{1,0} = 1$ into $(1 - y_{0,0}) (R_H^2 + R_L^2) + (1 - y_{1,0}) (R_H^2 - R_L^2) + 2R_L < 2R_H$ and simplifying

$$\frac{2I^n - 2R_L}{(R_H^2 + R_L^2)(1 - (1 - p)^2) + (1 - p)^2 2L} R_H^2 + R_L^2 < 2R_H - 2R_L,$$

and clearly if (B9) is satisfied this is also satisfied because this is less stringent.

Ex-ante inefficiencies from joint financing are

$$A = \frac{I^n - R_L}{\left(\frac{R_H^2 + R_L^2}{2}\right) \frac{(1 - (1 - p)^2)}{(1 - p)^2} + L} 2(R_L^2 - L),$$

Ex-ante inefficiencies from separate financing are

$$B = \frac{I^n - R_L}{\frac{p}{(1-p)}R_H^2 + L} 2(R_L^2 - L).$$

Clearly $B > A$ because B has higher numerator and a lower denominator than A . Q.E.D.

Proof of Proposition 14: Substituting $D_{1,0} = R_L + R_H$ into the equation of the proposition, we have that

$$(1 - y_{1,0}) = (1 - y_{0,0}) - \frac{(R_H - R_L)}{(R_H^2 + R_L^2)}.$$

If we have that $0 < (1 - y_{1,0}) < (1 - y_{0,0}) < 1$ and that $D_{1,1} < R_H$ we have that the ex-ante inefficiencies are

$$A = (1 - p)^2(1 - y_{0,0})2(R_L^2 - L) + 2p(1 - p) \left[(1 - y_{0,0}) - \frac{(R_H - R_L)}{(R_H^2 + R_L^2)} \right] (R_H^2 + R_L^2 - 2L)$$

and, for the case of separate projects,

$$B = \frac{I^n - R_L}{\frac{p}{(1-p)}R_H^2 + L} 2(R_L^2 - L).$$

Clearly if L is close to R_L^2 and $R_H^2 > R_L^2 = L$, we would have that

$$B = 0 \text{ and } A = 2p(1 - p)(R_H^2 - R_L^2)(1 - y_{1,0}) > 0$$

and therefore separation is optimal.

We now need to check that $(1 - y_{1,0}) > 0$ (clearly, then $(1 - y_{0,0}) > 0$). Substituting $R_L^2 = L$ in the expression above, we have that $(1 - y_{1,0}) > 0$ as long as

$$\frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}} > R_H - R_L. \quad (\text{B12})$$

Similarly, we have that $(1 - y_{0,0}) < 1$ (and therefore $(1 - y_{1,0}) < 1$) if and only if (remember that we have $(2 - 3p) > 0$),

$$\frac{2I^n - 2R_L - p^2 2R_H^2 - (1 - p^2) 2R_L^2}{p(2 - 3p) \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}} < R_H - R_L. \quad (\text{B13})$$

We also need that $D_{1,1} = (1 - y_{0,0})(R_H^2 + R_L^2) + (1 - y_{1,0})(R_H^2 - R_L^2) + 2R_L < 2R_H$, which substituting is equal to

$$\frac{2I^n - 2R_L}{p^2 + (1 - p^2) \frac{R_H^2}{R_H^2} + \left[1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2} \right]} < R_H - R_L. \quad (\text{B14})$$

Notice that if we have that the left-hand side of (B13) is lower than the one of independent

projects then, by assuming the latter the former becomes irrelevant. This is true if the second condition of the statement of the proposition is satisfied, i.e. the second condition is equivalent to

$$\frac{2I^n - 2R_L - p^2 2R_H^2 - (1 - p^2) 2R_L^2}{p(2 - 3p) \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}} < \frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}}.$$

It is easy to check that if the condition for the statement for separate projects is satisfied then the (B14) also becomes irrelevant. Finally, it is easy to check that the condition on separation can be satisfied simultaneously with (B14). Summarizing, it can indeed be that

$R_H - R_L$ satisfy

$$\frac{I^n - R_L}{p + (1 - p) \frac{R_L^2}{R_H^2}} < R_H - R_L < \frac{2I^n - 2R_L}{1 - (1 - p)^2 \frac{R_H^2 - R_L^2}{R_H^2 + R_L^2}},$$

and therefore all the conditions are satisfied. Q.E.D.