High Speed Rail Transport Valuation and Conjuncture Shocks

Gualter Couto[1], Cláudia Nunes[2], Pedro Pimentel[1]

[1] *Universidade dos Açores, Departamento de Economia e Gestão, CEEAplA, R. Mãe de Deus, 9501-801 Ponta Delgada, Portugal,*
[2] *Department of Mathematics and CEMAT, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*

October 7, 2010

Abstract

In the present paper we derive the optimal investment policy of investment in the high speed rail (HSR) project, under uncertainty, using the real options analysis (ROA) framework. We assume that the HSR demand, the main source of uncertainty, follows a geometric brownian motion with random jumps, complying with abrupt change of level caused by random events. The occurrence of such events is caused by external shocks.

We assess the impact of these shocks in the HSR demand threshold, along with the investment opportunity value, and the option to differ. We consider several distributions for these jumps, and we compare with the base-line case (where exogeneous jumps are not considered).

JEL classification: D81, D83,D92.
Keywords: real options, uncertainty timing, waiting, investment, high speed rate.

1 Introduction

Transportation investments are the most critical ones for economic sustainable growth, Banister and Berechman (2001). According to Wilson (1986), since 1970, economists have drawn their attention to transportation, where rail transport assumes an important role. The same author suggested that poor transportation policies and investment mistakes in transport infrastructures may compromise economic growth. Fulfil this compromise requires improvements in suitable decision criteria supported on cost/benefit analysis in uncertainty environments. Infrastructure investments, such as in seaports, airports, railways, energy networks, and road systems, have provided huge economic benefits and have leveraged economic growth. The size, budget and impact of these investments on the global economic activity led transportation investments to assume the role of strategic options. Almost all transportation investments include a portfolio of options to protect the enormous funds required to implement the investments.

Rose (1998) valued the concession of a toll road in Australia, considering the existence of two options interacting with each other. Smit (2003), in an empirical ROA study, focused on the valuation of structural investments in the transportation sector, regarding the expansion of a European airport. Under uncertainty, investment opportunity value claim for variables showing significant impact on cash flows variability. Shilton (1982) found empirical evidence that improvements in speed contribute significantly to increase railway traffic, enhanced by aggressive marketing strategies.

The decision to invest in HSR requires huge funds, that are in most case sunk costs due to the irreversible nature of the investment. The investment success depends on the optimal timing to invest. The option to defer drives the HSR investment opportunity value, as well the optimal timing to invest. The decision to invest instead of delay will be function of the HSR demand level, since this variable represents the main source of uncertainty, Rose (1998). An overview about real options and investments under uncertainty may be found in Dixit and Pindyck (1994) and Trigeorgis (1996).

Related to real options analysis in high speed train investment valuation is Bowe and Lee (2004), who applied binomial analysis to evaluate high speed train investment in Taiwan. They combine three different options (expand, reduce and defer), and the according interactions between options.

Also related is Pimentel et al. (2007), with a HSR investment valuation model using the real options framework, when only HSR demand faces uncertainty. We extend their model in order to allow positive or negative chocks in HSR demand level. In financial literature, ROA appears mostly in natural resources investments. Transportation investment analysis rarely incorporates real option theory. When does, use discrete time frameworks. As a result, this paper will introduce the transportation investment analysis of the HSR investment valuation in continuous time with stochastic demand facing random shocks, providing some closed form solutions. Our aim is to fill in this gap in the literature. Although Pereira et al. (2006) studied these issues, their work focused on airport construction. Our ROA framework will support the utility balance for the user between different rail speed services.

This model was developed based on the project of HSR in Portugal. Although it's flexible enough to be extended to other similar investment projects.

The remainder of the paper is organized as follows. Section 2 presents the framework rationale. Section 3 derives the demand process facing random shocks. Section 4 develops the valuation framework. Section 5 focuses on the optimal policy given the ability to defer the investment. Section 6 focuses on the value of the investment opportunity. Section 7 provides the corresponding numerical results and the parallel economic rationale. The paper primary conclusions and recommendations for future extensions are in Section 8.

2 The model

In a HSR project, the owner of the investment opportunity can claim, in any moment in time, the expected cash flow in exchange for the payment of the investment expenditures. Thus, we are dealing with an option to invest.

The investment in HSR line can be seen as an optimal stopping problem, with an embedded option to defer. Following McDonald and Siegel (1986) and Salahaldin and Granger (2005), the optimal timing to invest rule is derived.

The HSR demand is the main source of uncertainty. However, unexpected conjuncture shocks caused by random events, forces abrupt changes in HSR demand. In this context, it is important to incorporate in the ROA framework this feature. The continuous time ROA framework assumes that the option to defer is unlimited over time. The number of users interested in a certain connection of the HSR may register some sudden changes caused by unexpected conjuncture changes. These changes, originated by unexpected random events, cause fluctuations at the demand level.

For instance, in a HSR link, in which there is a regular airline service, may come up events which cause changes at the demand level for both transportations. In the HSR business, these shocks, often positive on the demand for the railway service, may outcome, for instance, from the increase of insecurity and fear to travel on airplanes, or even for preferring the HSR over the airline service, motivated by the loss of competitiveness of the airline service in a certain connection. In one of these scenarios, the demand for the HSR may easily register a sudden increase.

The modeling of this reality will only consider the main uncertainty factor inherent to the HSR investment. We intend to avoid that the mathematical complexity resultant from the inclusion of the remaining uncertainty factors to hamper the analysis of the impact of changes introduced in the evaluation framework.

3 Modelling the Demand Process

In this paper we assume that the demand for the new HSR, here denoted by $X = \{X_t, t \in \Re^+\}$, follows the following equation:

$$\frac{dX_t}{X_t} = \mu + \sigma dW_t + dN_t \tag{1}$$

where $\{W_t, t \in \Re^+\}$ is a Brownian motion, $\{N_t, t \in \Re^+\}$ is a Poisson process with rate λ, independent of $\{X_t, t \in \Re^+\}$, such that:

$$dN(t) = 1 + U, \quad \text{with probability } \lambda \tag{2}$$

Thus we assume that the demand level follows a geometrical Brownian motion, with random jumps occurring according to a Poisson process. This dynamics express our intuition that the level of future HSR demand can be affected by conjuncture shocks, as it was the case with the oil crisis in 2008, that increased significantly the demand for such service all around Europe. We note also that here we assume that these shocks are exogeneous to the demand process, although this assumption may be relaxed, as we will discuss latter.

The parameters of Equation (1) have the following meaning: μ represents the growth rate and σ represents the volatility of HSR demand. We assume that both parameters are constant in time.

Moreover, we assume that the conjuncture shocks are modelled by a collection of independent and identically distributed random variables, identically to U, taking positive values, independent of the Brownian motion and of the Poisson process. The resulting sample path for $\{X_t\}$ will be continuous most of the time, with finite jumps occurring at discrete points in time.

Similar assumptions with U deterministic were found in Rose (1998), who modelled highway traffic; Salahaldin and Granger (2005), who modelled the dynamics of a city population; and Marathe and Ryan (2005) and Pereira et al. (2006), who modelled airline demand. Emery and McKenzie (1996), on the other hand, implicitly assumed that income from the railway followed a geometric Brownian motion process . Finally, Bowe and Lee (2004) assumed the discrete time analogue of a geometric Brownian motion for the operational cash flows of a HSR investment.

Following Merton (1976), the solution of Equation (1) is as follows:

$$X_t = x_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t} \left(\prod_{j=1}^{N(t)} (1 + U_j) \right) \tag{3}$$

where x_0 is the initial HSR demand (that we assume to be known), $N(t)$ is the number of jumps of the Poisson process till time t and U_j denotes the magnitude of the jth jump.

Furthermore, it is a simple consequence of the fact that the conjuncture shocks are exogeneous that one can derive the following expectation value for X_t:

$$\begin{aligned} E[X_t^\theta] &= x_0^\theta e^{\theta(\mu - \frac{\sigma^2}{2})t} E[e^{\theta\sigma W_t} \prod_{j=1}^{N(t)} (1 + U_j)^\theta] \\ &= x_0^\theta e^{\left(\theta\mu + \frac{\sigma^2}{2}\theta(\theta-1) + \lambda E[(1+U)^\theta] - \lambda\right)t} \end{aligned} \tag{4}$$

for $\theta \in \Re^+$. We note that Equation (4) follows from the moment generator function of a normal distribution, the moment generator function of a Poisson distribution and the Wald equation, Ross (1996).

4 The valuation model

The users will only choose to travel in the HSR if they can at least maintain their level of utility, at the same level as the one they have in the conventional railway service. Otherwise, it will always be better to pay the fare for the conventional railway service even if they spend more time travelling. The fact that the investment has an infra-structuring nature and a governmental scale, allows us to consider this problem as an economic welfare problem, based upon the utility balance for users between two similar railway services.

In this section we explain the valuation model, and in particular we address the total cost of travel for each user, that we denote by $\Psi(.)$, as a function of time t. This cost is a function of the total value of travel time for the user, $\eta(.)$, and the travel fare, $p(.)$. Here we assume that these costs are functions of the demand process, that evolves with time, and therefore they also vary with time.

Following Owen and Phillips (1987) and Wardman (1994), we assume that the total value of travel time at time t, given that the demand is X_t, is given by

$$\eta(t) = \beta X_t^{\theta_\beta} \tag{5}$$

The parameter β depends on the stage of construction of the HSR, as the travel time varies depending on:

1. The HSR is still not in service (i.e., before and during the investment);
2. The HSR is already in service (i.e., after the investment implementation).

We denote by β_0 (β_1) the value of the parameter β for stage 1 (2), with $\beta_0 \geq \beta_1$. The difference between β_0 and β_1 reflects the decrease in travel time due to the implementation of the HSR.

Similarly, we also assume a variable travel fare function, $p(.)$, given by:

$$p(t) = \alpha X_t^{\theta_\alpha} \tag{6}$$

Moreover, we assume that there are operation costs related with the HSR. There are two sorts of operating costs: costs per user (here denoted by ωX_t, where X_t is the HSR demand at time t) and fixed (here denoted by φ).

Our model also considers the time-to-build effect. In this paper we assume that the time to build is fixed, and denoted by n.

Using the objective function of Ramsey-Koopmans to compute the net benefits generated by the HSR investment, given that the HSR demand is X_t, the value that each user associates, at time t, to a railway trip before (V_{before}) and after (V_{after}) the implementation of the investment is:

$$\begin{aligned} V_{\text{before}}(X(t)) &= m(t) - \beta_0 X(t)^{\theta_\beta} - \alpha X(t)^{\theta_\alpha} \\ V_{\text{after}}(X(t)) &= m(t) - \beta_1 X(t)^{\theta_\beta} - \omega X_t - \varphi \end{aligned}$$

where $m(t)$ is the individual disposable income at time t. Thus considering n building periods before the HSR begins to operate and a fixed discount rate ρ, the investment opportunity value for arbitrary $X_0 = x$, here denoted by $v(x)$, is given by:

$$\begin{aligned} v(x) &= \int_0^\infty e^{-\rho(t+n)} E\left[V_{\text{after}}(X_{t+n}) - V_{\text{before}}(X_{t+n})\right] dt \\ &= \int_0^\infty e^{-\rho(t+n)} \left[(\beta_0 - \beta_1) E[X_{t+n}^{\theta_\beta}] + \alpha E[X_{t+n}^{\theta_\alpha}] - \omega E[X_{t+n}] - \varphi\right] dt \end{aligned} \tag{7}$$

where the expected values in Equation (7) are given by Equation (4).

The current valuation framework implicitly assumes that each user will bear his/her part of the investment expenditure, plus the corresponding operating costs per user. Hence, along with the benefit from decrease in travel time, Equation (7) considers the saved conventional travel fare, p, as a fair HSR travel fare is already implicitly considered in the valuation framework.

The purpose of the model is to calculate the optimal timing to invest preserving utility function balance. For that, it is necessary to locate the HSR demand threshold, here denoted by $x^\star$, above which it will be optimal to invest. Thus one wants to find the optimal value $x^\star$ such that:

$$v(x^\star) = \sup_x v(x) \tag{8}$$

meaning that the investment opportunity value is the largest possible at level $x^\star$.

Finally, we assume that the HSR investment benefit will last for an unlimited time horizon.

5 Optimal Policy

It follows from Equations (4) and (7), that if $\rho < \theta\mu + \frac{1}{2}\sigma^2\theta(\theta-1) + \lambda E[(1+U)^\theta] - \lambda$, then the investment is never optimal (and therefore one would wait forever). Therefore we assume that the following condition holds:

$$\rho > \theta\mu + \frac{1}{2}\sigma^2\theta(\theta-1) + \lambda E[(1+U)^\theta] - \lambda \tag{9}$$

This condition also imposes the HSR demand growth rate to be lower than the discount rate, thus providing a rational economic interpretation to the mathematical developments.

Rewriting Equation (7), computing the integral and simplifying, we obtain the following equation:

$$v(x) = Ax^{\theta_\beta} + Bx^{\theta_\alpha} + Cx + D \tag{10}$$

where

$$A = \frac{2(\beta_0 - \beta_1)e^{(\theta_\beta\mu + \frac{1}{2}\theta_\beta(\theta_\beta - 1)\sigma^2 + \lambda E[(1+U)^{\theta_\beta}] - \lambda - \rho)n}}{2\rho - 2\theta_\beta\mu - \theta_\beta(\theta_\beta - 1)\sigma^2 - 2\lambda E[(1+U)^{\theta_\beta}] + 2\lambda} \tag{11}$$

$$B = \frac{2\alpha e^{(\theta_\alpha\mu + \frac{1}{2}\theta_\alpha(\theta_\alpha - 1)\sigma^2 + \lambda E[(1+U)^{\theta_\alpha}] - \lambda - \rho)n}}{2\rho - 2\theta_\alpha\mu - \theta_\alpha(\theta_\alpha - 1)\sigma^2 - 2\lambda E[(1+U)^{\theta_\alpha}] + 2\lambda} \tag{12}$$

$$C = \frac{-we^{(\rho - \mu + \lambda E[U])n}}{\rho - \mu + \lambda E[U]} \tag{13}$$

$$D = -\frac{\varphi e^{-\rho n}}{\rho} \tag{14}$$

To include economic intuition, note that A reflects the present value of travel time savings; B reflects the present value of conventional railway travel fare; C reflects the present value of variable operating costs, and, finally, D reflects the present value of fixed operating costs.

Now, as $v(.)$ is a function of the demand process $\{X_t\}$, that follows a geometric Brownian motion with random jumps, if we apply Ito's lemma to $v(X_t)$, we end up with the following ordinary differential equation (Merton (1976)):

$$\frac{1}{2}\sigma^2 v''(x) + \mu x v'(x) - (\rho+\lambda)v(x) + \lambda E[v((1+U)x)] = 0 \tag{15}$$

subject to the boundary equations:

$$v(0) = 0 \tag{16}$$
$$v(x^\star) = A(x^\star)^{\theta_\beta} + B(x^\star)^{\theta_\alpha} + Cx^\star + D \tag{17}$$
$$v'(x^\star) = \theta_\beta A(x^\star)^{\theta_\beta - 1} + \theta_\alpha B(x^\star)^{\theta_\alpha - 1} + C \tag{18}$$

We note that the first condition means that the process is absorving when the HSR demand is 0; the second is the value-matching condition and the third is the smooth-pasting condition.

Therefore the investment opportunity function, $v(.)$, considering the current HSR demand, is given by the supremum of Equation (8), that satisfies the differential equation (15).

We note that Equation (15) is a Cauchy-Euler equation, whose solution is known to be given by:

$$v(x) = a_1 x^{r_1} + a_2 x^{r_2} \tag{19}$$

where the constants a_1 and a_2 are derived using the boundary conditions previously presented, and r_1 and r_2 are, respectively, the positive and negative solution of:

$$\frac{1}{2}\sigma^2 r(r-1) + \mu r - (\rho+\lambda) + \lambda E[(1+U)^r] = 0 \tag{20}$$

which can be found numerically. We remark that as $r_2 < 0$, then $x^{r_2} \to \infty$ when $x \to 0$. From the condition $\mu(0) = 0$, it follows then that $a_2 = 0$.

Moreover, using the value-matching condition, we conclude that:

$$a_1 = A(x^\star)^{\theta_\beta - r_1} + B(x^\star)^{\theta_\alpha - r_1} + C(x^\star)^{1-r_1} + D(x^\star)^{-r_1} \tag{21}$$

Thus, if $x^\star$ can be explicitly derived, then the solution of Equation (15) is:

$$v(x) = \left[A(x^\star)^{\theta_\beta - r_1} + B(x^\star)^{\theta_\alpha - r_1} + C(x^\star)^{1-r_1} + D(x^\star)^{-r_1}\right] x^{r_1} \tag{22}$$

Thus, for a given $X_0 = x$, the value of $x^\star$ that maximizes $v(.)$ is given by the solution of the following equation:

$$A(\theta_\beta - r_1)(x^\star)^{\theta_\beta - r_1} + B(\theta_\alpha - r_1)(x^\star)^{\theta_\alpha - r_1} + C(1-r_1)(x^\star)^{1-r_1} - Dr_1(x^\star)^{-r_1} = 0 \tag{23}$$

i.e., $x^\star$ is a zero of the first derivative of $v(.)$. But in the most general cases, the solution of Equation (23) can only be founded numerically.

5.1 Special case

Although in general the optimal value $x^\star$ can only be founded numerically, there is one special case where one can derive the critical value explicitly. Assume that:

- The variable costs are negligible (i.e., $\omega = 0$, so that C, in Equation (13), is equal to zero)
- $\theta_\beta = \theta_\alpha$
- $r_1 > \theta$

We note that although the variable costs are never zero, in fact they are negligible when compared with the HSR total operating costs. Moreover, the assumption that $\theta_\alpha = \theta_\beta = \theta$ means that the value of the travel time for the user and the value of the travel fare grow up at the same rate. This can certainly be assumed in stable economies, for instance.

In this particular situation Equation (23) simplifies:

$$(A + B)(\theta - r_1)(x^\star)^\theta = -Dr_1 \tag{24}$$

and therefore the HSR threshold is given explicitly by:

$$x^\star = exp\left\{\frac{1}{\theta}ln\frac{-r_1 D}{(A+B)(r_1-\theta)}\right\} \tag{25}$$

When the HSR demand threshold, $x^\star$, is reached, it justifies an immediate implementation of the HSR investment, which will begin to operate n periods afterwards. This solution preserves the utility balance for users between the HSR and conventional railway, making the optimal solution independent of the original income, m , and the initial HSR demand, x_0. Because the framework deals with an economic welfare issue, based on the utility balance for users between two similar transportations, this framework is specially adequate to analyze governmental scale investment decisions.

6 Valuation of an HSR Investment Using ROA Framework

Considering the investment's value function solution given by Equation (23), for a certain initial value $X_0 = x$, the investment opportunity value when $x < x^\star$ is given by:

$$(\frac{x}{x^\star})^{r_1}\left[A(x^\star)^{\theta_\beta} + B(x^\star)^{\theta_\alpha} + Cx^\star + D\right]$$

whereas for $x > x^\star$, the investment opportunity value is given by:

$$x^{r_1}\left[Ax^{\theta_\beta - r_1} + Bx^{\theta_\alpha - r_1} + Cx^{1-r_1} + Dx^{-r_1}\right] \tag{26}$$

In the special situation of Section (5.1), if we plug in the HSR demand threshold, $x^\star$, given by Equation (25), the investment opportunity value may be rewritten in the following terms:

$$v(x) = \begin{cases} \left(\frac{x}{x^\star}\right)^{r_1} \left[\frac{\theta D}{\theta - r_1}\right] & x < x^\star \\ (A+B)x^\theta + D & x \geq x^\star \end{cases} \tag{27}$$

In accordance to previous studies, as Dixit and Pindyck (1994), from the moment in which the HSR demand threshold is reached, $x^\star$, the value of the option to defer is zero. As a result, it is always better to invest and receive in exchange the NPV - given by Equation (26). As long as the optimal timing to invest has not been reached, there is always an inherent value of waiting for new information about the HSR demand. In this case, the value of the option to defer is given by the difference between the investment opportunity value, $v(x)$, and the NPV calculated, using the expected HSR demand at that moment. In addition, for allowing the inclusion of the i) time-to-build, ii) fixed operating costs and iii) variable operating costs, in the investment opportunity value, these developments take into consideration the elasticity between the value of travel time/travel fare and demand.

7 Numerical Illustration

In this section we present numerical values that illustrate the influence of the shocks in the decision about the investment in HSR. The parameter values that we present, as shown in Table (1), are supported by the released Portuguese Government data on the HSR investment project. We note, however, that in this present paper this numerical illustration is also used as a case study, and therefore could be used in other situations, with another set of parameter values.

Additionally, we assume that the construction should need 5 years, and the present value of the investment expenditures is 5 billion Euros. According to demand studies, the actual demand for HSR services is around 3 million passengers and should rise 3.5% per year, with 15% standard deviation.

The conventional railway service operates in the same link. According to engineering studies, the new HSR service will reduce the travel time to one third comparatively to the conventional railway service moving from around 3 hours to around 1 hour needed to complete the journey. Using official data provided by Transport Analysis Guidance (UK) and OECD, one estimates that the value of travel time per hour in Portugal is 20 Euros.

Fixed operating costs in the first year amounts for 90 million Euros, and the variable operating costs are neglected. It is assumed a 9% discount rate, which represents the required return for an investment with a similar risk level.

Elasticity between the value of travel time and HSR demand and cross-elasticity between conventional railway service fare and HSR demand are both set to 0.5, based on statistical data regarding the railway sector in Portugal. This elasticity and cross elasticity value lead to an expected annual growth in the value of travel time and conventional service fare, half of the one expected for HSR demand. In each case those expected growth rates are acceptable in a stable economic environment.

Table 1: Base-case parameters for the project

Parameter	Numerical value
x (present demand)	3M
γ (present value of investment expenditures)	5,000 M€
η_0 (value of travel time in conventional railway)	30 €
η_1 (value of travel time in HSR)	20 €
p_0 (Conventional railway fare)	25 €
φ (fixed operating costs)	90 M€
ρ (discount rate)	0.09
μ (expected growth rate of the demand process)	0.035
σ (standard deviation of the demand process)	0.15
n (number of years for the construction)	5
$\sigma_\alpha = \sigma_\beta = \sigma$	0.5

Concerning the jumps, we assume different situations, in order to assess the impact of the jumps and its distribution in the optimal investment policy. In particular we consider the following cases:

- No jumps
- Deterministic and constant jumps, with magnitude of 0.10
- Magnitude of the jumps distributed according to an uniform distribution in the interval $(0.05, 0.15)$
- Magnitude of the jumps distributed according to an exponential distribution, with mean value equal to 0.10

The occurrence probability, formally denoted by λ, is set to 10%. Note that the jumps have equal (expected) value, but increasing variance.

In Table (2) we present the different values obtaining for these 4 situations, regarding the level of demand that justifies the investment ($x^\star$), as well as the investment opportunity value, the net present value and the value of the option to differ.

Table (2) presents the HSR line investment valuation results for the base-case parameters in two cases: when in presence of an uncertain demand subject to the effect resultant from unexpected events, and when we assume that there are no shocks (which can be easily obtained by setting the λ parameter, the probability of occurrence of shocks, equal to 0 in Equation (20)).

If we consider demand shocks, then the results suggest that the construction of the HSR line should start when the demand reaches around 9.20 million passengers, whereas in the absence of

Table 2: HSR Investment Valuation Results with positive shocks

	No jumps	Determ.	Unif.	Expon.
$x^\star$ (HSR Demand Threshold)	9.847	9.188	9.20403	9.13104
$v(x^\star)$ (Invest. Opport. Value, in M €)	1769.18	6566.63	6655.46	6636.24
Net Present Value (in M €)	-1620.72	3568.63	3655.03	3655.03
Value of the Option to Defer (in M€)	3389.88	2997.99	3000.44	2981.21

shocks this value increases to 9.847 million passengers (demand shocks cause a decrease around 7.16%). Moreover, the value $x^\star$ is roughly the same independently of the distribution that we assume. Note that this conclusion is not surprising, as we specifically choose distributions with small variance (considerably smaller than the expected value).

With a negative NPV, the project shouldn't be implemented at the current time, concerning the uncertainty regarding the number of passengers of the new service. This case only happens when we do not consider the possible presence of shocks. Maintaining "alive" this investment opportunity has a value of 3389 million Euros, if we do not consider the presence of shocks, whereas in the presence this value drops to around 3000 million Euros (with small variations according to the particular distribution of the jumps magnitude).

Figure (1) shows the impact in HSR demand volatility in the optimal threshold when we consider the previous assumptions for the jumps, assuming that the drift parameter, μ, is equal to 10%. Note that in this situation the behaviour of $x^\star$, the trigger value, has similar values for the random cases (uniform and exponential cases), and that in all the situations the HSR threshold for the model without jumps is larger than in the models with jumps. Furthermore, the absolute difference of these thresholds is roughly constant with σ.

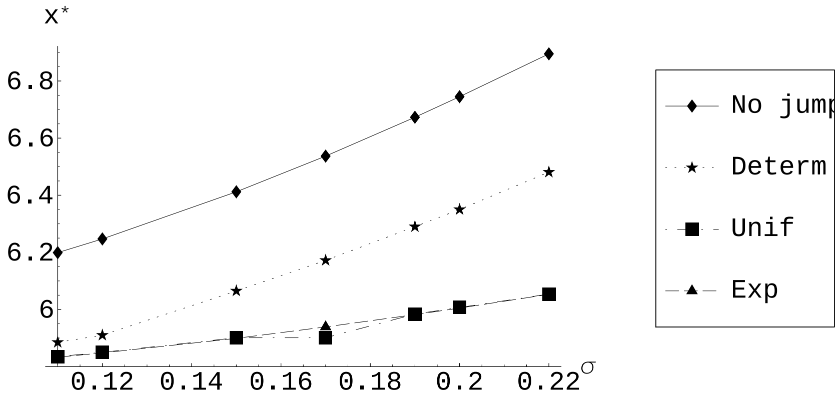

Figure 1: Behaviour of $x^\star$ as a function of the volatility (σ).

Furthermore, we can assess the impact of σ in the investment opportunity value (see Figure (2)), both for the base-line case (no jumps) and for the model with deterministic jumps. In the presence of jumps, the investment opportunity value increases, in an exponential way, with

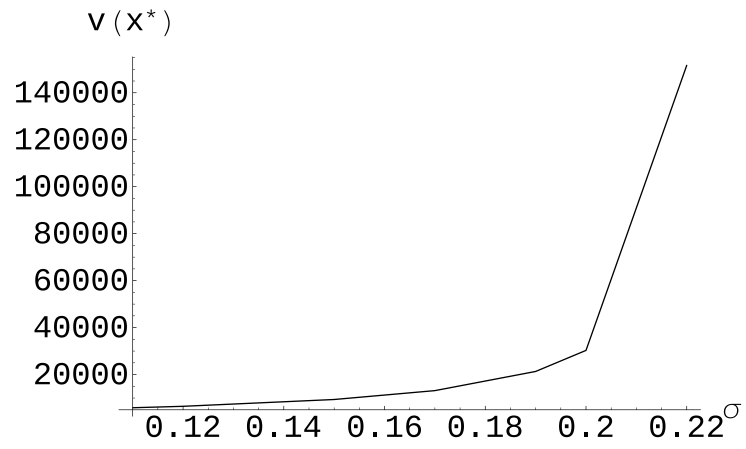

Figure 2: Behaviour of $v(\theta^{\star})$ as a function of the volatility (σ).

increasing volatility, and much faster than in the absence of jumps. Therefore the difference between a model without shocks and with shocks differs highly, in term's of investment opportunity value, specially with larger values of σ, the volatility parameter. In this scenario the investment's implementation should be anticipated. The possibility of unexpected shocks on demand increases the importance of the lost cash-flows by delaying the investment's implementation. The model results with stochastic demand subject to random positive shocks results in lower investment opportunity value whenever demand evolves in time, comparatively to the values registered on the base-case.

In Figures (3) and (4) we plot the influence of the jump magnitude and the demand drift in the optimal demand threshold.

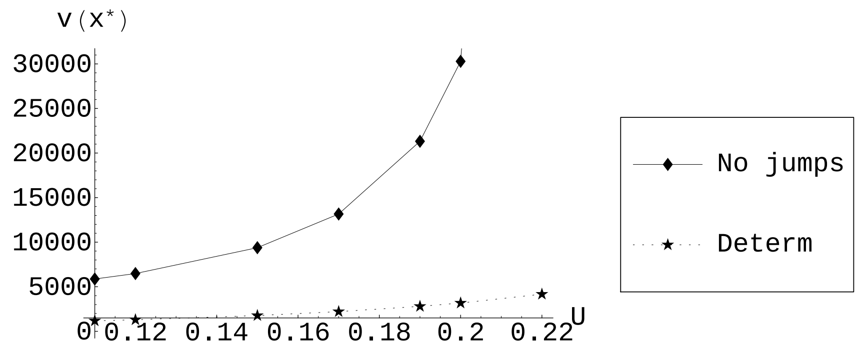

Figure 3: Behaviour of $v(x^{\star})$ as a function of the jump magnitude (U).

When we compare the critical demand level, $x^{\star}$ as a function of the magnitude of the jumps, we conclude that for small magnitudes of the jump the investment decision takes place latter in the model with jumps, but for larger jumps we observe the opposite (3).

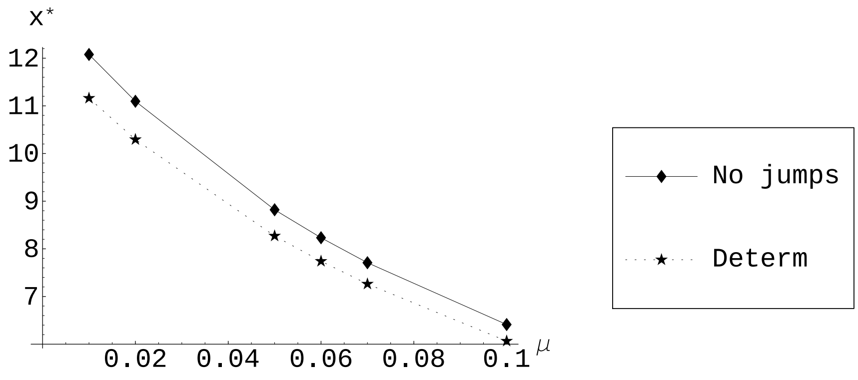

Figure 4: Behaviour of $x^\star$ as a function of the drift of the demand (μ).

8 Conclusions and Extensions

This paper developed a framework to determine the optimal timing to invest in HSR and the investment opportunity value considering a stochastic HSR demand facing negative or positive shock depending on random conjuncture changes.

As far as our knowledge is concerned closed form solutions for ROA in railway investments is still left out in literature. The HSR investment analysis was incremental regarding conventional railways. The users' utility balance between the HSR and the conventional railway quantified the benefits to the HSR users. The optimal timing to invest was calculated with the HSR demand threshold model. The developments regarding the optimal timing to invest and the investment opportunity value present the advantage of offering a clear way to evaluate the HSR Investment opportunity in each moment in time, for a set of potential users. The numerical example and simulation of some important input parameters demonstrated the consistency of the framework. Positive shocks in demand cause a decrease in HSR demand threshold and in the value of the option to defer around 7,16% and 10,34%, accordingly, in comparison with the inexistence of such shocks. The optimal timing and the value of the option to defer the HSR investment shows an inverse behaviour against positive random shocks, as well as against its occurrence probability. HSR demand uncertainty facing random shock leads to early investment decision.

Our numerical results suggest that, at least with distributions highly concentrated around the mean value, the optimal investment policy is robust, as the optimal demand level ($x^\star$), as well as the value of the project do not change significantly with the particular distributions that we consider for U.

Ignoring the conditions related to elasticity equality and neglected variable costs result in the increase of the HSR demand threshold and the value of the option to defer. In an investment scenario where both conditions can't be neglected, we find no significant change in results.

In an investment decision process, higher uncertainty means more relevance for ROA's application, as shown in this research. Without uncertainty and flexible management the ROA framework results matches with the traditional investments' analysis. The numerical example and simulation of some important input parameters demonstrates the framework consistency. Behind a decision to invest that is not "now or never", it's important to optimize the decision of "when" to invest. Our research provides an economic basis and financial rationale to the decision to invest in HSR project, in an environment exposed to deep conjuncture impacts.

Several extensions of the model in order to incorporate different probability distribution functions on positive shocks and probability may be conducted in the future, at expenses of additional complexity and the use of deeper numerical methods.

References

Banister, D. and Y. Berechman (2001). Transport investment and the promotion of economic growth. *Journal of Transport Geography 9*, 209–218.

Bowe, M. and D. Lee (2004). Project evaluation in the presence of multiple embedded real options: Evidence from the taiwan high-speed rail project. *Journal of Asian Economics 15*, 71–98.

Dixit, A. and R. Pindyck (1994). *Investment Under Uncertainty.* Princeton University Press.

Emery, H. and K. McKenzie (1996). Damned if you do, damned if you don't: An option value approach to evaluating the subsidy of the CPR mainline. *The Canadian Journal of Economics 29*(2), 255–270.

Marathe, R. and S. Ryan (2005). On the validity of the geometric brownian motion assumption. *The Engineering Economist 50*, 159–192.

McDonald, R. and D. Siegel (1986). The value of waiting to invest. *The Quarterly Journal of Economics 101*(4), 707–728.

Merton, R. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics 3*.

Owen, A. and G. Phillips (1987). The characteristics of railway passengers demand. *Journal of Transport Economics and Policy 21*, 231–253.

Pereira, P., A. Rodrigues, and M. Armada (2006). The optimal timing for the construction of an international airport: A real options approach with multiple stochastic factors and shocks. 10th Real Options Conference, New York.

Pimentel, P., J. Pereira, and G. Couto (2007). High speed rail transport valuation. 11th Real Options Conference, Berkeley.

Rose, S. (1998). Valuation of interacting real options in a tollroad infrastructure project. *The Quarterly Review of Economics and Finance*, 711–723.

Ross, S. (1996). *Stochastic Processes.* New York: Wiley.

Salahaldin, L. and T. Granger (2005). Investing in sustainable transport to relieve air pollution under population-growth uncertainty. 9th Real Options Conference, Paris.

Smit, H. (2003). Infrastructure investment as real options game: The case of european airport expansion. *Journal of Transport Geography 32*.

Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. The MIT Press. Cambridge, MA.

Wardman, M. (1994). Forecasting the impact of service quality changes on the demand for inter-urban rail travel. *Journal of Transport Economics and Policy 28*, 287–306.

Wilson, G. (1986). Economic analysis of transportation: A twenty-five year survey. *Transportation Journal Fall*, 33–44.