Manipulation and Information Acquisition*

António Barbosa†
Department of Finance
ISCTE Business School - Lisbon

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Abstract

I analyze the behavior of an agent (the manipulator) who makes announcements based on his private information in order to maximize the profit from short-term trades. Truthful announcement strategies can be supported with positive probability but only if investors have another source of information in addition to the manipulator's announcement and/or if manipulation is occasionally punished. It is shown that the manipulator benefits from announcing/trading on more regulated markets with better informed investors, because both help the manipulator commit to announce more truthfully. The presence of a manipulator increases price efficiency and decreases risk premium, even if the manipulator manipulates the announcement. Therefore, regulation to prevent manipulation is only beneficial if it forces the manipulator to announce more truthfully, and not if it forces the manipulator to stop announcing. I also analyze how the presence of the manipulator impacts investors' decisions to purchase information. Some investors substitute the costly information for the manipulator's announcement, even though that decreases the manipulator's incentive to announce truthfully. Nevertheless, price efficiency improves and risk premium decreases with the presence of the manipulator.

EFM classification: 350

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†ISCTE Business School, Av. Forças Armadas, 1649-026 Lisbon, Portugal. Phone: +351 21 790 39 16. E-mail: antonio.barbosa@iscte.pt

1 Introduction

Information is essential for investors trading in financial markets. Better informed investors are able to take better decisions and obtain higher profits when trading against less informed investors. However, gathering and processing information is costly. It usually requires considerable monetary and time costs and, above all, expertise that only a few possess. Yet, everyday we see valuable information, such as trading recommendations, price targets and research reports, being given away to the general public through the media. The usual explanation for this phenomenon is that, once information has been used to initiate a trading strategy and/or has been sold to other investors, keeping the information private provides no additional benefit to those who possess it, whereas making the information public allows them to cash-in quickly after prices fully reflect that information and move on to other investment opportunities.

Although there is this honest motive to give away information, there is also a darker motive: to manipulate the market. The Securities Exchange Act of 1934 mostly eradicated those blatant cases of manipulation common before its enactment[1]. However it is not very effective against more subtle attempts of market manipulation. First, equity valuation is not an exact science and, to some extent, can be tweaked to produce the desired numbers. And then there is always the possibility of honest mistakes. Thus, most often it is up to investors who receive the information to judge its merits and, in doing so, discipline those who release information. Investors can do this essentially in two ways: analyzing the track record, or reputation, of the information issuer; and comparing the information released with other contemporaneous sources of information.

Several papers have analyzed information-based manipulation in settings where manipulators are kept in line due to reputation concerns (e.g. Benabou and Laroque 1992, van Bommel 2003 and Fishman 2007). However, to the best of my knowledge, none considers the availability of other information sources that investors can use to assess the credibility of the information announced. In this paper I develop a model that attempts to fill this void.

The model is based on Grossman and Stiglitz's (1980) noisy rational expectations model. I add an agent, the manipulator, who has imprecise private information about the liquidation value of the risky asset. The manipulator uses his information to open a position in the risky asset, and then makes an announcement based on his information. He may choose to announce truthfully or to manipulate his announcement. After the price incorporates the information contained in the announcement, he closes his position. Fully rational investors have the opportunity to purchase imprecise information, independent of the manipulator's information. They can use their own information to infer the probability with which the announcement was manipulated, and extract information from the announcement accordingly.

Both manipulator and investors face trade-offs. The manipulator wants to maximize his profit by strategically choosing what to announce. That is, he wants to maximize the price impact of his announcement which sometimes requires him to manipulate the announcement. However, investors

[1]See Allen and Gale (1992) and Benabou and Laroque (1992) for some historical accounts of manipulation prior to the Securities Exchange Act of 1934.

are aware of the manipulator's incentives and can assess the credibility of the announcement. The less credible the announcement, the less weight investors put on it and so, the smaller the price reaction to the announcement. In turn, investors wish to substitute costly information by the announcement's costless information. However, by increasing their dependence on the announcement, investors decrease the incentive for the manipulator to announce truthfully, resulting in more frequent manipulation and in the deterioration of the informativeness of the announcement. In face of these trade-offs, it is not obvious what the equilibrium outcome should be.

In this paper I focus on the analysis of the manipulator's announcement strategy and how it impacts: investors' decision to purchase information; price efficiency; and risk premium. In essence, is the presence of the manipulator welcomed or not? If investors are rational and understand that announcements may be manipulated, is manipulation bad enough so that it should be eradicated at all costs?

Another set of questions is related to the regulation changes of 2003. In the bull market of the nineties, Wall Street observers and regulators grew increasingly suspicious that analysts employed by investment banks and brokerage firms were facing a conflict of interest that resulted in overoptimistic recommendations.[2] Investigations led by the U.S. Congress, the New York Attorney General Elliot Spitzer and the SEC culminated with the April 28, 2003, Global Research Analyst Settlement (GRAS) involving ten of the leading investment banks.[3] With the objective of solving the conflict of interests among analysts employed by investment banks, the GRAS imposed two sets of measures. First, it imposed the separation between investment banking and research activities, increased the amount of disclosure in research reports and imposed a payment of $875 million in penalties. Second, it imposed the payment of $80 million for investor education and $432.5 million to fund independent research to be distributed along with their own research reports. In broad terms, the first set of measures aims at reducing the incentives to produce biased research, whereas the second set of measures aims at improving investors' ability to judge the credibility of that research. A natural question to ask is then how effective are these kinds of measures? Do they produce the intended results? Are there any undesirable side effects? And will a manipulator be pushed away from more regulated markets to less regulated markets?

The main results are as follows. First, I show that the availability of an additional source of information to investors is crucial to support truthful announcements. In this case, prices (more specifically the average price over the asset's supply realization) depend both on the announcement and on the investors' alternative source of information. Since the manipulator's own source of information is correlated with that of investors', this gives the manipulator an incentive to align the announcement with his own information, resulting in truthful announcements. By the contrary, when investors do not have access to an alternative source of information, prices depend exclusively

[2]Examples of articles in the financial press exposing the conflict of interest and revealing the unethical Wall Street practices are Siconolfi (1992, 1995a,b), Feldman and Caplin (2002), Byrne (2002a,b), Morgenson (2002) and Gasparino (2003). Quoting from Morgenson (2002), "[analysts] had become salesmen and saleswomen for their investment banking departments in their routine communications" and, from Byrne (2002a), "Historically, "sell" ratings have constituted fewer than 1% of analysts' recommendations, according to Thomson Financial/First Call".

[3]This process also produced NASD rule 2711 and NYSE rule 472, in the same spirit of the GRAS.

on the announcement. Therefore, if investors give full credibility to the announcement, the optimal announcement does not depend in any way on the manipulator's information.

Second, I show that manipulation *per se* is not bad. Because I assume that investors are sophisticated, they are aware of the manipulator's incentives and are able to correctly assess the credibility of his announcements. Therefore investors are not mislead by manipulated announcements, in the sense of giving them more credibility than they have. By the contrary, they are able to extract valuable information from potentially manipulated announcements. This means that, in terms of price efficiency and risk premium, a manipulated announcement is preferable to no announcement at all (better price efficiency and smaller risk premium). Manipulation is just not as good as truthful announcements.

Third, because investors are not mislead by manipulated announcements, manipulation is not a good deal for the manipulator. The more he manipulates, the less credibility investors give to the announcement, and the smaller the manipulator's ability to influence prices. Therefore, the manipulator would actually prefer to announce truthfully. The problem is that he cannot commit to announce truthfully if investors rely exclusively on his announcement as a source of information. The existence of an alternative source of information available to investors provides this commitment device, as does the existence of penalties for manipulation. This leads to the surprising conclusion that the manipulator prefers to operate in markets where investors are well informed and where manipulation is fought more vigorously.[4] The exception is when the manipulator fears prosecution for making a truthful announcement based on wrong information. This can happen if the regulator prosecutes based on a poor track record of recommendations, which may be due to manipulation but also to bad luck, rather than based on hard evidence of manipulation. In this case, if penalties are too harsh, the manipulator may be better off by not announcing at all, which has a negative impact in terms of price efficiency. In face of these results, the 2003 GRAS was a step in the right direction that might have gone too far.[5]

Fourth, the existence of a free source of information, in the form of the announcement, decreases the information disadvantage of those investors who do not purchase extra information relative to those who do. As a result, there is a substitution of costly information for the costless announcement, even if the latter is manipulated. Nonetheless, the existence of announcements still has a positive impact on price efficiency and contributes to a smaller risk premium. A curious pattern that

[4]O'Brien and Bhushan (1990) provide evidence supporting the former. They find that changes in analyst coverage are positively related with the degree of regulated disclosure. Since the quality of public information available is higher in industries with more regulated disclosure, this suggests that, as predicted by the model, analysts prefer to issue recommendations about industries where investors are already well informed.

[5]Clarke, Khorana, Patel, and Rau (2009) and Kadan, Madureira, Wang, and Zach (2009) find that although the GRAS succeeded in reducing the conflict of interest of affiliated analysts, it brought unintended consequences: the overall informativeness of recommendations both by affiliated and independent analysts decreased in the post-GRAS period. Moreover, there was a widespread move from a five-tier (strong buy, buy, hold, sell and strong sell) to a three-tier rating system (buy, hold and sell) by investment banks. This indicates that investment banks became more fearful of ill-founded prosecution in case of honest mistakes, being the move to a three-tier rating system a defensive move, since the likelihood of issuing a wrong recommendation is smaller. Reputation concerns may explain why investment banks moved to a three-tier rating system instead of stopping issuing recommendations as predicted by the model.

emerges is that unusually high or low prices leads to more investors purchasing information. This might look like investors believe price changes are informationally-driven, in which case they would purchase information to learn about the cause of the observed price change. But in reality, investors are simply reacting to the anticipated decrease in the informativeness of the announcement, since extreme prices lead to a higher probability of manipulation.

Finally, I point out that the interpretation of some empirical results on manipulation is inconsistent. Papers focusing on short-window event studies (e.g. Lin and McNichols, 1998; Frankel, Kothari, and Weber, 2006; Cliff, 2007 and Malmendier and Shanthikumar, 2007) find that the immediate price response to buy recommendations of analysts affiliated to investment banks is smaller than the response to their sell recommendations. In turn, papers focusing on long-horizon performance studies (e.g. Michaely and Womack, 1999; Barber, Lehavy, and Trueman, 2007 and Cliff, 2007) find that portfolios formed based on buy recommendations by affiliated analysts have a negative performance, just like those based on sell recommendations. Both findings are interpreted as evidence supporting upward manipulation of recommendations by affiliated analysts, which implies that in the former type of study investors are assumed to be sophisticated and not mislead by analysts' manipulation, whereas in the latter type of study the opposite assumption is made. The model predicts that, when investors are sophisticated, all portfolios should have positive long-run performance in order to reward risk. However, if there is upward manipulation, the buy portfolio should outperform the sell portfolio since a sell recommendation resolves more uncertainty than a buy recommendation. And this is exactly what one finds when revisiting Cliff's (2007) results.

This paper adds to the literature on information-based manipulation. The closest references are Benabou and Laroque (1992), van Bommel (2003) and Fishman (2007). Benabou and Laroque (1992) and Fishman (2007) analyze the strategic disclosure of information by insiders in models of repeated interaction where investors learn about the type of the insiders. In Benabou and Laroque (1992) the insider always have inaccurate information and may be honest or opportunistic, in which case he occasionally manipulates the announcement. In Fishman (2007) the insider may be a charlatan, who does not have information, or a genuine leader, who has inaccurate information with positive probability. In both papers, insiders restrict the frequency of manipulation in order to build and maintain a reputation that allow them to influence prices. Similarly, in van Bommel (2003) an insider spreads rumors and, once again, with repeated interaction he refrains from spreading untruthful rumors for reputation concerns. In all these papers, investors do not have the possibility to obtain information from other sources in order to evaluate the truthfulness of the announcement. Therefore, insiders restrict their manipulation only because there is repeated interaction and reputation building. In one-shot games, insiders cannot credibly commit to be truthful and so the announcement is disregarded by investors. In the model developed in this paper, the existence of another source of information acts as a commitment device to announce truthfully. As a result, informative announcements are credible in a one-shot game.

The paper proceeds as follows. I describe the model in Section 2. Section 3 is dedicated to the analysis of the manipulator's announcement strategy and its effect on price efficiency, risk premium

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- Manipulator observes his information (s_M), chooses his announcement strategy ($\theta_{a\|s_M}$) and announcement(a), and opens a position in the risky asset. - Investors choose their portfolio based on prior beliefs. - P_0 is formed.	- Manipulator announces a and closes his position. - Investors choose whether to observe s for a cost c. - Informed investors observe s, all investors observe a. - Investors choose their portfolio based on posterior beliefs. - P_1 is formed.	- Manipulator discloses his tradings to the regulator. - Regulator probes for manipulation and applies penalty.	- State of nature is revealed and risky asset is liquidated.

Figure 1: Time line of events.

and price response to the announcement when all investors have free access to an additional source of information. I also compare the model predictions with the extant empirical evidence. In Section 4 I endogenize the decision to purchase information, and analyze how the manipulator's strategy impacts information purchases. In Section 5 I present two extensions to the model. Section 6 concludes. All proofs are provided in the Appendix.

2 Model Description

In this section I develop a noisy rational expectations model based on the Grossman and Stiglitz's (1980) model, with two modifications. First, I allow for the existence of a manipulator, whose announcements provide an additional source of information to investors. Second, I assume that the risky asset's liquidation value follows a binomial distribution, instead of a normal distribution. This distributional assumption significantly simplifies the analysis since the manipulator can only be untruthful in one way.

In the remainder of this section I describe the model and discuss the assumptions made. The structure of the model and its parameters are common knowledge to all agents. Figure 1 provides a time line of events and summarizes the model.

2.1 Investment Opportunities

There are two assets available for trading at dates 0 and 1: a riskless asset with infinitely elastic supply and gross rate of return normalized to 1; and a risky asset liquidated at date 2. The risky asset's liquidation value (V) is either V_H, with probability q, or V_L, with probability $1-q$. Without loss of generality I set $V_H = 1$ and $V_L = 0$.[6] For simplicity I will focus on a symmetric distribution for V, i.e. $q = 1/2$.

To avoid a fully revealing equilibrium, I make the usual assumption of random risky asset's supply (see e.g. Grossman and Stiglitz, 1980). Specifically, the risky asset's supply at date $t \in \{0, 1\}$, denoted by z_t, follows the distribution $z_t \sim N\left(\bar{z}, \sigma_z^2\right)$, with $\bar{z} \geq 0$, and z_1 independent of z_0.

[6] Let $P\left(V_H, V_L, z\right)$ denote the price as a function of V_H, V_L and z (the asset's supply). With CARA preferences we have $P\left(V_H, V_L, z\right) = V_L + \left(V_H - V_L\right) P\left[1, 0, z\left(V_H - V_L\right)\right]$. That is, the asset that pays $\left(V_H, V_L\right)$ is equivalent to a portfolio with a riskless asset paying V_L and $\left(V_H - V_L\right)$ units of a risky asset that pays $\left(V_H = 1, V_L = 0\right)$.

2.2 Information

Nature determines V at date 0, but its value remains unobservable until date 2. However, there are two signals providing information about V before date 2.

The first signal is denoted by $s \in \{H, L\}$ and has accuracy $\rho \in [1/2, 1]$, i.e. the signal is correct with probability ρ.[7] This signal becomes available just prior to date 1 trading, and can be observed by any agent who chooses to pay a cost c. The second signal is denoted by $s_M \in \{H, L\}$ and has accuracy $\rho_M \in [1/2, 1]$. This signal is available immediately before date 0 trading occurs, but only to one agent, which I will refer to as the manipulator. Both signals are independent, which means that, unless $\rho = \rho_M = 1$, both $s = s_M$ and $s \neq s_M$ may occur.

In addition, before date 1 trading takes place, the manipulator makes a public announcement based on his signal, denoted by $a \in \{H, L, N\}$ with $a = N$ meaning that the investor does not announce. Although the manipulator bases the announcement on his information, he is free to announce something other than what he observes, in which case there is manipulation. All agents costlessly observe this announcement. The observation of the announcement a and the decision to observe signal s occurs simultaneously.[8] Figure 2 depicts the event tree associated with this information structure.

The triple (s, a, s_M) completely characterizes the informational state. To normalize notation, variables associated with or conditional on an informational state are subscripted by (s, a, s_M). If the variable is independent of, say, s_M, then it is indexed by $(s, a, -)$.

One way to interpret this information structure is the following. The manipulator is an analyst affiliated to an investment bank, a hedge fund or a trading desk. He has superior technical resources which allow him to produce information (s_M) faster than all other agents, and he has financial resources to use his research primarily for trading purposes. Less resourceful independent analysts take longer to produce research (s). The delay in producing information and lack of financial resources leads them to specialize in selling information to individual investors.[9] O'Brien, McNichols, and Lin (2005) and Cliff (2007) find that affiliated analysts issue recommendations for recently listed firms significantly sooner than independent analysts, which supports the idea that analysts with more resources produce research significantly faster than those with less resources. This provides a justification for why the manipulator enjoys an informational advantage at date 0.

2.3 Investors

There is a continuum of risk averse investors in the interval $[0, 1]$. All investors have the same CARA preferences over date 2 wealth (W_2). At date 1 each investor chooses his demand for the risky asset

[7] Because there are only two states of nature, an accuracy of $\frac{1}{2}$ means that the signal provides no information. Notice that a signal with accuracy below $\frac{1}{2}$ is equivalent to a signal with accuracy equal to its complement and the opposite outcome (e.g. observing $s' = H$ when $\rho' = \frac{1}{4}$ is the same as observing $s = L$ when $\rho = \frac{3}{4}$).

[8] In Section 5.2 I consider an extension where agents decide whether to observe signal s after observing the announcement.

[9] Alternatively, signal s can be seen as the result of research made by individual investors on their own. In this context, c represents the opportunity cost of the time spent in research. All investors are assumed to have the same ability so that their research produces the same signal.

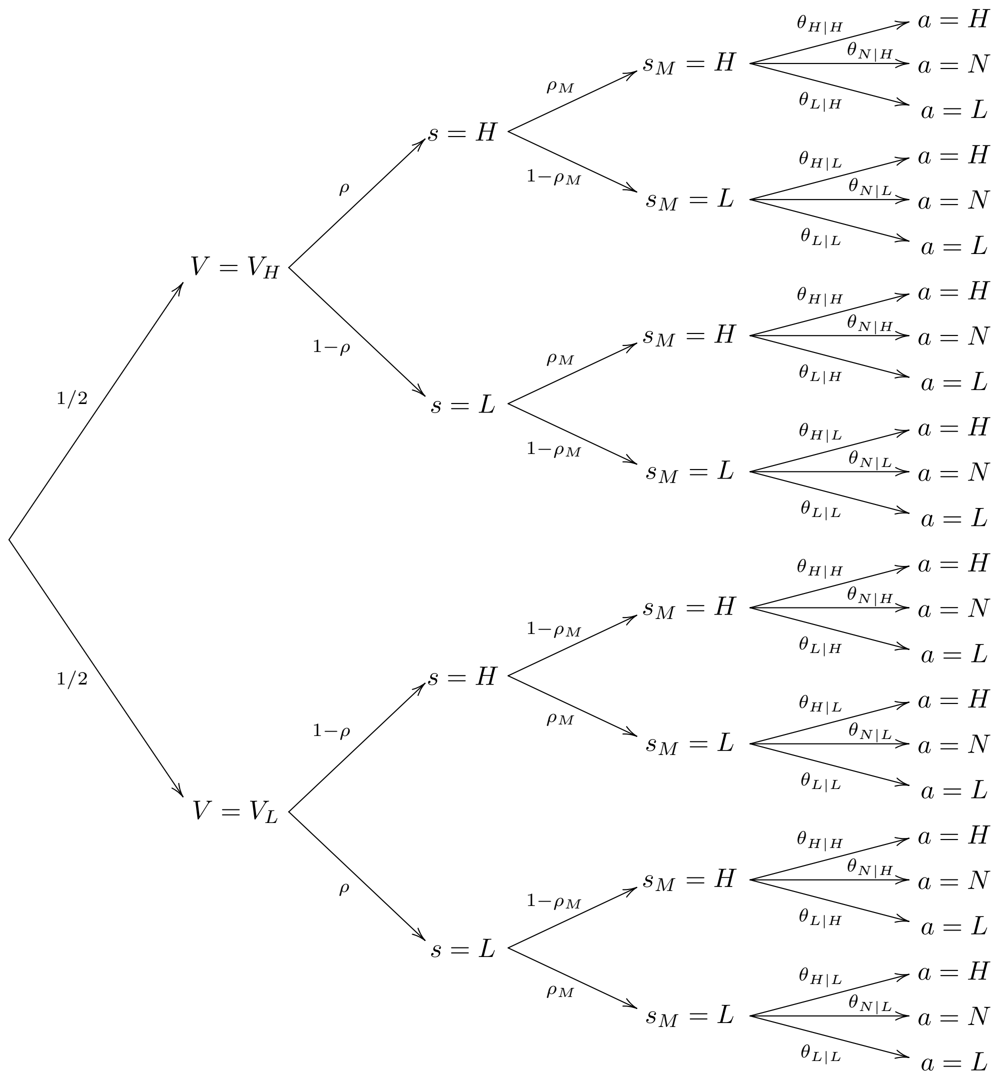

Figure 2: Event tree.

(X_1) in order to maximize the expected utility of W_2 given his budget constraint, solving

$$\begin{aligned} \max_{X_1} \mathbb{E}\left[U\left(W_2\right)\middle|\,\mathcal{F}_1\right] &= \mathbb{E}\left(-e^{-\alpha W_2}\middle|\,\mathcal{F}_1\right) \\ & s.t. \quad W_2 = W_1 + X_1\left(V - P_1\right), \end{aligned} \tag{1}$$

where $\alpha > 0$ is the coefficient of absolute risk aversion and $\mathcal{F}_1$ denotes the information available to the investor at date 1. The choice of CARA preferences is made for tractability. The only essential feature is that investors be risk averse.

Investors are identical in every aspect except for the information they possess at date 1. Some investors pay the cost to observe signal s and so are better informed than those who don't. I will call the former informed investors and the latter uninformed investors, denoting them by I and U, respectively. The fraction of informed investors is λ.

Uninformed investors are sophisticated enough to extract information from the equilibrium price, which partially reflects the information contained in signal s. Therefore, their date 1 information set is $\mathcal{F}_1^U = \{a, P_1\}$, where P_1 denotes the date 1 equilibrium price. Informed investors possess all the information available to investors, and so there is no additional information for them to extract from the equilibrium price.[10] Therefore their date 1 information set is $\mathcal{F}_1^I = \{a, s\}$. It is straightforward to see that a sufficient statistic for the investors' beliefs is the conditional probability they assign to state $V = V_H$, which I denote by $p^I \equiv \mathbb{P}\left(V_H | \mathcal{F}_1^I\right)$ and $p^U \equiv \mathbb{P}\left(V_H | \mathcal{F}_1^U\right)$.

2.4 Manipulator

The manipulator is the agent who privately observes the signal s_M at date 0. The manipulator is further characterized as follows: (i) he is risk neutral; (ii) he has measure zero; (iii) he has limited wealth and borrowing constraints;[11] (iv) he faces short sale constraints that limit the maximum size of his short positions to $\delta \in [0, 1]$ times the maximum size of his long positions; and (v) he faces a liquidity constraint that forces him to close his position in the risky asset at date 1.

Assumptions (ii) and (iii) imply that the manipulator trades without being noticed (a similar assumption is made by van Bommel 2003). Consequently, P_0 provides no information about s_M, which simplifies the computation of posterior beliefs. This assumption, however, is not crucial for the results in the paper. All that is needed, is that investors cannot exactly learn the manipulator's signal s_M from the price impact of his trade at dates 0 and 1, which would make the announcement irrelevant.

Assumption (iv) imposes short sale constraints by restricting the leverage of short positions relative to the leverage of long positions. When $\delta = 1$, the same leverage is allowed for long and short positions, and there is no short sale constraint. Whenever $\delta < 1$, short positions cannot be as leveraged as long positions, which imposes a short sale restriction. Without loss of generality I normalize the manipulator's wealth so that the maximum size of a long (short) position to be 1 (δ).

Finally, the assumption of early liquidation provides an honest motive for the manipulator's announcement.[12] The manipulator wants P_1 – the price at which he is forced to close his position – to be as far apart from P_0 – the price at which he opens his position – as possible, and in the direction of his trade. That is, if he takes a long position at date 0 he wants $P_1 > P_0$, otherwise he wants $P_1 < P_0$. To that end, the manipulator can use his announcement to influence P_0, P_1, or both.

To profit from influencing P_0 in a favorable way, the manipulator engages in post-announcement speculation. This type of speculation consists of making an announcement at date 0 and then opening a position in the asset at the manipulated price P_0. Clearly, the manipulator can only

[10] Informed investors do not observe the manipulator's signal s_M. However, the assumptions that I will make on the manipulator imply that his trading has no impact on the equilibrium price. Therefore investors cannot infer the manipulator's signal from the equilibrium price.

[11] It may seem counter intuitive that the manipulator has borrowing constraints but investors do not. One can always assume that investors face borrowing constraints but that their risk aversion is high enough and their borrowing constraints loose enough, even if tighter than those of the manipulator, such that their borrowing constraints will never bind.

[12] The strong assumption of certain forced liquidation is made for simplicity. The manipulator will behave similarly if forced liquidation is possible but not certain.

profit from this type of speculation by misleading investors. For instance when the manipulator observes good news ($s_M = H$), he increases his profit by announcing bad news ($a = L$) to depress the price at which he then opens a long position. If the manipulator announced truthfully ($a = H$) he would actually decrease his profit, and would be better by not announcing at all.

In turn, to profit from influencing P_1 in a favorable way the manipulator engages in pre-announcement speculation. In this type of speculation the manipulator opens a position in the asset at date 0, and then makes an announcement at date 1 to influence the price at which he then closes the position. In contrast with the case of post-announcement speculation, in this case the manipulator is able to increase his profit even if he announces truthfully ($a = s_M$). For example, consider that the manipulator observes $s_M = L$. If he opens a short position at date 0, announcing $a = L$ at date 1 incorporates his private information into P_1, thus lowering P_1 and boosting his date 1 profit. Hence, the manipulator has an honest motive to announce truthfully. However, the optimal announcement strategy is not necessarily the truthful one. Continuing the example, a low P_0 (due to a positive supply shock at date 0) or a tight short sale constraint will significantly reduce the profit from the truthful strategy (short at date 0, announce $a = L$ at date 1) and will tempt the manipulator to do the opposite: take a long position at date 0 and announce $a = H$ in order to manipulate the date 1 price upward.

The crucial difference between these two types of speculation strategies, and associated price manipulation, is that post-announcement speculation is easy to detect and punish a posteriori, whereas pre-announcement speculation is not. Post-announcement speculation requires the manipulator to announce and then *open* a position *inconsistent* with the announcement (e.g. announce bad news and then take a long position), exposing the manipulative behavior. In contrast, pre-announcement speculation requires the manipulator to announce and then *close* his position, which can always be justified by liquidity constraints. Besides, the kind of speculation may not even require price manipulation. In this case, to prove the existence of manipulation it is necessary to uncover s_M, which may not be feasible in practice. Therefore, the existence of a regulator who punishes manipulation (introduced in the next subsection) rules out post-announcement speculation and leaves pre-announcement speculation as the only viable way of profiting from manipulating prices.[13] With these assumptions in place, if the manipulator announces, he does so only at date 1 and to influence P_1.

I will use $\theta_{a|s_M} \equiv \mathbb{P}\left(a|\, s_M\right)$ to denote the probability with which the manipulator announces a conditional on the observation of signal s_M. If the manipulator announces a with the same probability conditional on $s_M = H$ and $s_M = L$ (i.e. $\frac{\theta_{a|H}}{\theta_{a|L}} = 1$), then signal a is uninformative, since $\mathbb{P}\left(s_M = H\right) = \mathbb{P}\left(s_M = L\right) = \frac{1}{2}$. If, instead, a is announced more frequently conditional on $s_M = H$ than on $s_M = L$, a becomes a signal for $s_M = H$. Without loss of generality, I will focus on equilibria where the announcement strategy satisfies the following assumptions.

Assumption 1. *(i) The manipulator uses $a = H$ to signal $s_M = H$ and $a = L$ to signal $s_M = L$, that is,*

[13] See Benabou and Laroque (1992) and van Bommel (2003) for models with both types of speculation.

$$\frac{\theta_{H|H}}{\theta_{H|L}} \geq 1 \geq \frac{\theta_{L|H}}{\theta_{L|L}};$$

(ii) The manipulator uses $a = N$ to make uninformative announcements, or as a substitute for signaling $s_M = H$ or $s_M = L$, that is,

$$\frac{\theta_{H|H}}{\theta_{H|L}} \geq \frac{\theta_{N|H}}{\theta_{N|L}} \geq \frac{\theta_{L|H}}{\theta_{L|L}}.$$

Because the manipulator is risk neutral, he always take positions with a size equal to the maximum allowed. Therefore, $\mathcal{T}_{(-,a,s_M)} \in \{1, -\delta\}$ characterizes his trading strategy when he announces a and observes signal s_M.

Notice that, even though signal s provides additional information to the manipulator, it is useless to him. This is so because manipulator has to decide his trading and announcement strategy at date 0, before s is released.

The assumption of forced liquidation, central to the results in this paper, is also made by Fishman (2007) in his model of strategic disclosure of information. More generally, the assumption of early liquidation risk appears in a large literature (e.g. DeLong, Shleifer, Summers, and Waldmann, 1990 and Dow and Gorton, 1994). Several justifications for early liquidation have been presented in the literature. Here, I assume that early liquidation arises as a consequence of large opportunity costs (Shleiver and Vishny, 1990) stemming from: (i) long time interval between dates 1 and 2; and (ii) availability of alternative investment opportunities at date 1.

2.5 Regulator

There is an agent, which I call the regulator, whose function is to identify and punish information-based manipulation. After date 1, the regulator investigates the existence of manipulation. At that time the regulator observes the signal s, equilibrium prices P_0 and P_1, and the manipulator's portfolio holdings at date 0 and 1, which the manipulator is required to disclose after date 1. The penalty for manipulation is $K > 0$ per unit traded, and it is assumed to be sufficiently large to deter manipulation if punishment is certain.

As discussed above, post-announcement speculation is *inconsistent* with truthful announcements. Therefore, the certain punishment of this type of speculation rules it out. In contrast, forced early liquidation makes pre-announcement speculation *consistent* with truthful announcements. Without further evidence, manipulation associated to this kind of speculation cannot be punished, reason why it may subsist. However, this does not mean that manipulation associated to pre-announcement speculation will always escape unpunished; it will depend on whether the regulator is skilled or unskilled.

The skilled regulator can uncover the private information of the manipulator (s_M) with some probability. He punishes the manipulator only if he learns that $s_M \neq a$, $a \in \{H, L\}$. The expected penalty for manipulation when the regulator is skilled is denoted by k, corresponding to K times the probability of punishment. On the other hand, the unskilled regulator is unable to uncover s_M, and so he never has hard proof of manipulation. However, because he is under the pressure of public opinion to perform his work and punish manipulation, he occasionally punishes the manipulator based on the discrepancy of the manipulator's announcement a and the liquidation value V.[14] I will also use k to denote the expected penalty when the regulator is unskilled.

Notice that, whereas a skilled regulator forces the manipulator to manipulate less frequently, an unskilled regulator may force the manipulator to stop making announcements. In the latter case, even though the manipulator announces truthfully, he may still be punished if he is unlucky and $a = s_M \neq V$.

For both types of regulator, whenever $k = 0$ it means that the regulator punishes manipulation associated to pre-announcement speculation with zero probability, and not that $K = 0$. That is, post-announcement speculation is still punished and therefore ruled out.

2.6 Equilibrium Definition

The equilibrium price P_0 is the only date 0 equilibrium variable that influences the date 1 equilibrium. Long (short) positions are more profitable for the manipulator the smaller (larger) P_0 is, and so P_0 plays an important role in the manipulator's optimal strategy. Because I want to focus on the date 1 equilibrium, where all the action is concentrated, I will consider P_0 as given throughout most of the paper. For now, all we need to know about P_0 is that it can take any value in the interval $[V_L, V_H]$ due to the normally distributed random supply z_0. The only thing lost by proceeding in this way is the likelihood of each P_0 and its corresponding equilibrium. This will be addressed in Section 5.1 where I endogenize P_0 and determine its distribution. Since I will focus only on date 1 variables from now on, I drop the time subscripts on all date 1 variables to simplify the notation.

The equilibrium demands and price are functions of the random variables s, a, P and z, and I denote them by $X^I(s, a, P)$, $X^U(a, P)$ and $P(s, a, z)$, respectively. Investors' date 1 strategies are then fully described by $\mathcal{I} = \left\{X^I, X^U, \lambda\right\}$; λ aggregates the individual choices of each investor about whether to observe s or not. In turn, the manipulator's date 1 strategy is described by $\mathcal{M} = \left\{\theta_{a|s_M}, \mathcal{T}_{(-,a,s_M)} : a \in \{H, L, N\}, s_M \in \{H, L\}\right\}$. The definition of the date 1 noisy rational expectations equilibrium ($NREE$) is the following.

Definition 2. A $NREE$ with manipulator is a triple $(\mathcal{I}^*, \mathcal{M}^*, P^*)$ such that:

(i) X^{I*} (X^{U*}) maximizes the expected utility of informed (uninformed) investors given their posterior beliefs p^I (p^U) and P^*;

(ii) Informed (uninformed) investors form their posterior beliefs p^I (p^U) from the observation of signal s (price P^*) and announcement a, while taking into account the manipulator's optimal announcement strategy $\theta^*_{a|s_M}$, using Bayes rule; posterior beliefs in zero-probability events are obtained by considering that the manipulator trembles independently of s_M;

[14] In this case we can think of the punishment as litigation and settlement costs rather than a fine charged by the regulator. One piece of anecdotal evidence is the case of Merrill Lynch, who in May 2002 agreed to pay $100 million in a settlement with the State of New York to end an investigation into their research practices following the exposure of e-mails trashing stocks they publicly promoted.

(iii) λ^* is such that no investor can improve his expected utility by changing his decision on whether to observe s or not;

(iv) P^* clears the market for the risky asset given X^{I*}, X^{U*} and λ^*;

(v) $\theta^*_{a|s_M}$ and $\mathcal{T}^*_{(-,a,s_M)}$ for all $a \in \{H, L, N\}$ and $s_M \in \{H, L\}$ maximize the manipulator's expected utility given P^*, P_0, k and the type of regulator.

As we will see in the next sections, there are multiple equilibria. Some of them are equivalent, in the sense that the payoff to all agents is the same. I call the collection of all equivalent equilibria an equilibrium type. Because the manipulator is monopolistic and there is an infinite number of investors, I consider that all agents coordinate in the type of equilibrium that maximizes the manipulator's expected utility, the focal type of equilibrium.

3 Equilibrium without Information Acquisition

In this section I analyze the date 1 equilibrium when all investors costlessly observe signal s. This case is akin to independent analysts making their recommendations available to the general public for free. I defer the analysis of the general case, where investors decide whether to purchase s or not, until the next section. The reason to proceed in this way is twofold. First, it allows me to focus on the manipulator's equilibrium strategy, which is essentially the same as in the general case, in a simpler setting. The next section will then focus on investors' equilibrium strategies, specifically their information acquisition decision.

Second, this setting is the closest to the one considered in the empirical literature (e.g. Dugar and Nathan, 1995; Lin and McNichols, 1998; Barber et al., 2007; Cliff, 2007) that tests the differential response to recommendations of analysts affiliated to investment banks and independent analysts. Affiliated analysts (here the manipulator) face conflict of interests which biases their recommendations. The conflict of interest stems from the pressure: to issue favorable reports on current and prospective clients of their investment banking business; to perform bullish research that stimulates trading and generate brokerage commissions; and to stay on friendly grounds with firms to have access to timely information. By the contrary, independent analysts (here whoever sends signal s) do not face any conflict of interests or, at least, do not face them to the same extent as affiliated analysts do.[15] Therefore, they issue recommendations that truthfully, or more closely, reflect their opinions. The analysis of the equilibrium in this section can then be used to validate the research designs commonly used and suggest other ways of testing for manipulation under the assumption that investors anticipate the effects of the conflict of interest of affiliated analysts.

[15] In this paper I consider only one manipulator. Therefore, there is at most one affiliated analyst for each stock. By the contrary, each stock may have more than one independent analyst issuing recommendations. In that case, signal s is then the aggregation of all recommendations issued by those independent analysts.

3.1 Solving for the Equilibrium

Substituting the budget constraint into the objective function and dropping date 1 subscripts, the optimization problem 1 becomes

$$\max_{X^I} \mathbb{E}\left(-e^{-\alpha W-\alpha X^I(V-P)} \middle| \mathcal{F}^I\right) = -p^I e^{-\alpha W-\alpha X^I(1-P)} - \left(1-p^I\right) e^{-\alpha W-\alpha X^I(0-P)}.$$

The first order condition of the optimization problem gives us the demand function

$$X^I = \frac{1}{\alpha}\ln\frac{p^I(1-P)}{(1-p^I)P}.$$

Market clearing then implies that

$$P = \frac{1}{1+\frac{1-p^I}{p^I}e^{z\alpha}}.$$

Note that $P \in (0,1)$. The demand and price functions satisfy the usual properties: demand decreases in prices and risk aversion, and increases in the expected liquidation value, i.e. in the probability of V_H (p^I); prices increase in p^I, and decrease in risk aversion and in the asset's supply (z).

For each of the 6 information scenarios $(s,a,-)$, investors form different beliefs about p^I, denoted by $p^I_{(s,a,-)}$. These beliefs are straightforward to obtain from the event tree of Figure 2, and are presented in Appendix A. As a result, the price function is indirectly a function of the information scenario $(s,a,-)$ and can be written as

$$P(s,a,z) = \frac{1}{1+\frac{1-p^I_{(s,a,-)}}{p^I_{(s,a,-)}}e^{\alpha z}}. \tag{2}$$

In turn, the manipulator chooses his strategy in order to maximize expected utility conditional on his private information, which is given by

$$\mathbb{E}(U_M) = \frac{1}{2}\mathbb{E}(U_M|\, s_M = H) + \frac{1}{2}\mathbb{E}(U_M|\, s_M = L)$$

with

$$\mathbb{E}(U_M|\, s_M) = \sum_{a\in\{H,L,N\}} \theta_{a|s_M}\Pi_{(-,a,s_M)},\ s_M \in \{H,L\}.$$

$\Pi_{(-,a,s_M)}$ denotes the (normalized) profit from trading in state (a, s_M). When the regulator is skilled it is given by

$$\Pi_{(-,a,s_M)} = \begin{cases} \max\limits_{\mathcal{T}_{(-,a,s_M)}\in\{1,-\delta\}} \mathcal{T}_{(-,a,s_M)}\left(\bar{P}_{(-,a,s_M)} - P_0\right) & if\ a = s_M \vee a = N \\ \max\limits_{\mathcal{T}_{(-,a,s_M)}\in\{1,-\delta\}} \mathcal{T}_{(-,a,s_M)}\left(\bar{P}_{(-,a,s_M)} - P_0\right) - \left|\mathcal{T}_{(-,a,s_M)}\right| k & otherwise \end{cases}$$

whereas when the regulator is unskilled it is given by

$$\Pi_{(-,a,s_M)} = \begin{cases} \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)} \left(\bar{P}_{(-,a,s_M)} - P_0\right) & if\, a = N \\ \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)} \left(\bar{P}_{(-,a,s_M)} - P_0\right) - \left|\mathcal{T}_{(-,a,s_M)}\right| k \left(1-\rho_M\right) & if\, a = s_M \\ \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)} \left[\bar{P}_{(-,a,s_M)} - P_0\right] - \left|\mathcal{T}_{(-,a,s_M)}\right| k \rho_M & otherwise \end{cases},$$

where $\bar{P}_{(-,a,s_M)} \equiv \mathbb{E}\left(P|\, a, s_M\right)$ is the expected price conditional on a and s_M, which is readily obtained from $P\left(s,a,z\right)$ as shown in Appendix A.

Notice that when the regulator is unskilled, the manipulator may be punished even when he is truthful but, unluckily, is information is wrong, which occurs with probability $1-\rho_M$. However, he is not punished when he lies but is information his wrong, which occurs with probability ρ_M. Therefore, unlike in the case of a skilled regulator, being truthful does not guarantee that there is no punishment. The only way to avoid punishment when the regulator is unskilled is by not announcing.

There is no closed form solution for the equilibrium, which is computed numerically using McKelvey's (1992) algorithm.[16]

3.2 The Manipulator's Optimal Strategy

Before I characterize the manipulator's optimal strategy, I need to introduce some additional notation. $\bar{P}^T_{(-,a,s_M)}$ will denote $\bar{P}_{(-,a,s_M)}$ when investors believe that the manipulator announces truthfully. And $\bar{P}^N_{(-,-,s_M)}$ will denote $\bar{P}_{(-,-,s_M)}$ when the manipulator never announces or, equivalently, investors believe that his announcement is completely uninformative.[17]

The next theorem provides a characterization of the manipulator's optimal announcement strategy when the regulator never punishes manipulation associated to pre-announcement speculation, i.e. $k=0$. The associated optimal trading strategy is not relevant for the analysis and so is omitted. Interested readers can find it in the proof of the theorem.

Theorem 3. *If the regulator never punishes manipulation associated to pre-announcement speculation ($k=0$), and the manipulator is informed ($\rho_M > \frac{1}{2}$), then there exist $\bar{P}^1 \leq \bar{P}^2 \leq \bar{P}^3 \leq \bar{P}^4$ defined by*

$$\bar{P}^1 = \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta}, \bar{P}^2 = \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta},$$
$$\bar{P}^3 = \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta}, \bar{P}^4 = \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)}}{1+\delta}$$

[16] See Judd (1998) pp. 133-135 for a description of McKelvey's algorithm.

[17] An announcement strategy is uninformative when the manipulator follows the same strategy conditional on $s_M = H$ or $s_M = L$.

such that the optimal announcement strategy is

$$\begin{cases} \theta_{H|H} = \theta_{L|L} = 1, & if\, P_0 \in \left[\bar{P}^2, \bar{P}^3\right] \; (Truthful\ Announcement) \\ \theta_{H|H} = 1,\, \theta_{H|L} = \omega_1,\, \theta_{L|L} = 1-\omega_1 & if\, P_0 \in \left(\bar{P}^1, \bar{P}^2\right) \; (Upward\ Manipulation) \\ \theta_{L|L} = 1,\, \theta_{L|H} = \omega_2,\, \theta_{H|H} = 1-\omega_2 & if\, P_0 \in \left(\bar{P}^3, \bar{P}^4\right) \; (Downward\ Manipulation) \\ \theta_{N|H} = \theta_{N|L} = 1 & if\, P_0 \in \left[0, \bar{P}^1\right] \cup \left[\bar{P}^4, 1\right] \; (Never\ Announce) \end{cases}$$

where $\omega_1, \omega_2 \in (0,1)$ satisfy $\lim_{P_0 \uparrow \bar{P}^2} \omega_1 = \lim_{P_0 \downarrow \bar{P}^3} \omega_2 = 0$ *and* $\lim_{P_0 \downarrow \bar{P}^1} \omega_1 = \lim_{P_0 \uparrow \bar{P}^4} \omega_2 = 1$.[18]

The main result of the theorem is that the manipulator may find it optimal to announce truthfully for some values of P_0, despite the fact that manipulation is not punished by the regulator. With the exception of the knife-edge case that will be identified below, this is possible only because investors have an additional source of information (signal s).

When s does not exist or, equivalently, is completely uninformative (i.e., $\rho = \frac{1}{2}$), the manipulator's announcement induces the same expected equilibrium price regardless of s_M, that is, $\bar{P}_{(-,a,H)} = \bar{P}_{(-,a,L)}$. This means that $\bar{P}^T_{(-,a,H)} = \bar{P}^T_{(-,a,L)}$ and so $\bar{P}_2 = \bar{P}_3$. This happens because investors do not observe s_M nor any signal whose outcome can be predicted by s_M.[19] Consequently, whatever announcement strategy the manipulator finds optimal when $s_M = H$, it is also optimal when $s_M = L$, and so the manipulator deviates from the truthful announcement: either he lies when $s_M = H$ or when $s_M = L$. The only exception is in the knife-edge case where the manipulator is indifferent between $a = H$ and $a = L$. This happens only on the zero-probability event that $P_0 = \bar{P}_2 = \bar{P}_3$. Panel A of Figure 3 illustrates this.

On the other hand, when s exists and is informative ($\rho > \frac{1}{2}$), the expected price will naturally be larger when $s = H$ than when $s = L$ all else equal, i.e. $\bar{P}_{(H,a,-)} > \bar{P}_{(L,a,-)}$. Because it is more likely that $s_M = s$ than otherwise (provided that $\rho_M > \frac{1}{2}$), it is also true that $\bar{P}_{(-,a,H)} > \bar{P}_{(-,a,L)}$. Therefore, the optimal announcement strategy when $s_M = H$ is no longer necessarily the same as when $s_M = L$, and a truthful announcement strategy can be supported for a range of P_0 values in the interval $\left[\bar{P}^2, \bar{P}^3\right]$. $\bar{P}^2$ and $\bar{P}^3$ define the indifference points between announcing $a = H$ and $a = L$ conditional on $s_M = L$ and $s_M = H$, respectively, when investors believe the announcement to be truthful. For P_0 smaller (larger) than any of the indifference points, the manipulator strictly prefers to announce $a = H$ ($a = L$). Therefore, the manipulator prefers to announce truthfully conditional on $s_M = H$ only when $P_0 \leq \bar{P}^3$, and conditional on $s_M = L$ only when $P_0 \geq \bar{P}^2$. When $P_0 \in \left[\bar{P}^2, \bar{P}^3\right]$ the manipulator always announces truthfully. This can be seen on Panel B of Figure 3.

Obviously, if P_0 is below $\bar{P}^2$ (above $\bar{P}^3$), the manipulator will find optimal to deviate from the truthful announcement strategy even if s is informative, and will manipulate the announcement upward (downward). For such P_0 values, if investors believe that the manipulator's announcement

[18] ω_1 and ω_2 correspond to twice the frequency of manipulation.

[19] When $\rho > 1/2$ and $\rho_M > 1/2$, s can be predicted by s_M, and vice-versa, because when $s_M = H$ it is more likely that $s = H$ than $s = L$.

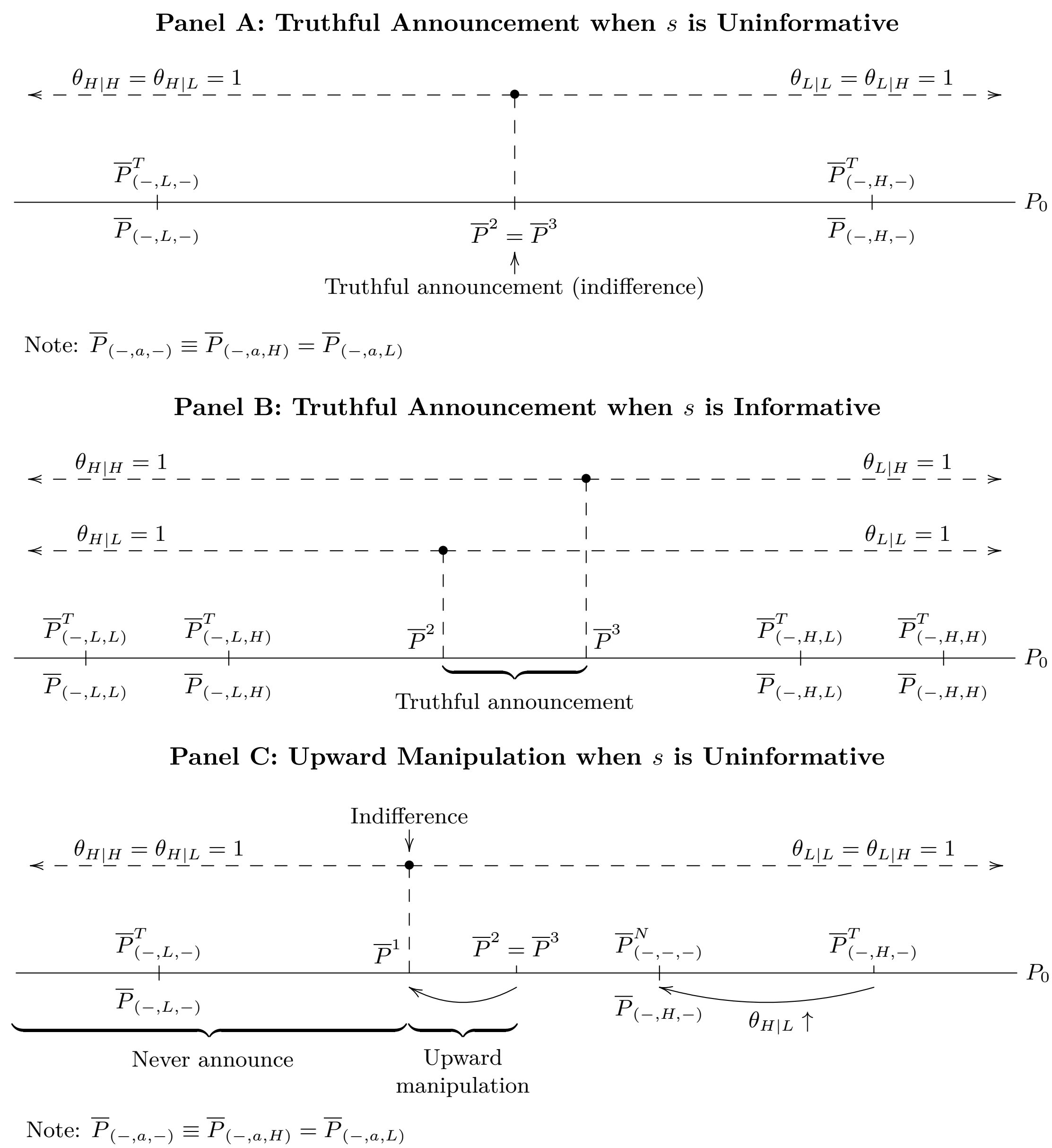

Figure 3: Equilibrium announcement strategies. In all panels it is assumed that $\delta = 1$ (no short sale constraints). Therefore, conditional on s_M, the manipulator prefers to announce $a = H$ if P_0 is farther away from $\bar{P}_{(-,H,s_M)}$ than from $\bar{P}_{(-,L,s_M)}$, and $a = L$ otherwise. Indifference is attained when P_0 is the midpoint between $\bar{P}_{(-,H,s_M)}$ and $\bar{P}_{(-,L,s_M)}$, where any announcement strategy can be supported in equilibrium. In Panel A, investors do not observe s. This means that $\bar{P}_{(-,a,H)} = \bar{P}_{(-,a,L)} \equiv \bar{P}_{(-,a,-)}$ and so the manipulator does not condition his announcement strategy on his signal. As a result, a truthful equilibrium is supported only at the indifference point. In Panel B, investors observe an informative s, which implies that $\bar{P}_{(-,a,H)} > \bar{P}_{(-,a,L)}$. As a result, the indifference point when $s_M = H$ is larger than the indifference point when $s_M = L$. A truthful announcement is supported for any P_0 in the interval defined by the two indifference points. In Panel C, the more the announcement is manipulated (i.e., the larger $\theta_{H|L}$ is), the smaller $\bar{P}_{(-,H,-)}$. $\bar{P}_{(-,L,-)}$ remains unchanged because the manipulator only announces $a = L$ when $s_M = L$. As a result, the indifference point at which the UM equilibrium is supported moves to the left. In the limit, the manipulator always announces $a = H$ and $\bar{P}_{(-,H,-)}$ reaches its lower bound $\bar{P}^N_{(-,-,-)}$, defining the lower indifference point that supports a UM equilibrium ($\bar{P}^1$). If $P_0 \leq \bar{P}^1$, the announcement is completely uninformative, and it is the same as never announcing.

is truthful, then the manipulator would like to always announce $a = H$ ($a = L$), as we can see from Panel B of Figure 3. However, if he does so, his announcement is uninformative and has no impact on expected equilibrium prices (equivalent to not announcing).

Although an uninformative announcement, or equivalently not announcing, is always an equilibrium, the manipulator can do better than that. He can maximize his expected utility by playing the most informative announcement strategy to which he can commit to, since the profit from his long (short) position on the asset increases whenever he influences the equilibrium price upward (downward).

In the case of upward manipulation this equates to announcing truthfully conditionally on $s_M = H$ (always open a long position and announce $a = H$) and occasionally manipulating conditionally on $s_M = L$ (mix between opening a long position and announcing $a = H$, and opening a short position and announcing $a = L$). Thus, when investors observe $a = L$ they know that the manipulator is being truthful and give more credibility to the announcement than they do when they observe $a = H$, which may have been manipulated. As a consequence, the expected price conditional on $a = H$ (and hence the profit from a long position) decreases with the frequency of manipulation, whereas the expected price conditional on $a = L$ (and hence the profit from a short position) remains unchanged. That is, $\bar{P}_{(-,H,s_M)} < \bar{P}^T_{(-,H,s_M)}$, but $\bar{P}_{(-,L,s_M)} = \bar{P}^T_{(-,L,s_M)}$. Hence, if P_0 is not too low (specifically, if it is larger than $\bar{P}^1$), there is an informative upward manipulation equilibrium where the frequency of manipulation induces prices such that the manipulator is indifferent between $a = H$ and $a = L$ when $s_M = L$. The smaller P_0 is, the higher the frequency of manipulation, and the less informative the announcement becomes. At some point, P_0 becomes so low that the manipulator always announces $a = H$, rendering the announcement uninformative. For any P_0 below this threshold, defined by $\bar{P}^1$, the uninformative announcement is equivalent to not announcing, and we are in the never announce equilibrium. This situation is depicted in Panel C of Figure 3 for the case where investors do not observe s.

Next, I look at the impact of penalties for manipulation on the optimal announcement strategy. The optimal announcement strategy when a skilled regulator punishes manipulation associated to pre-announcement speculation is the following.

Theorem 4. *If the regulator is skilled and punishes manipulation associated to pre-announcement speculation ($k > 0$), then there exist $\bar{P}^1 \leq \bar{P}^2 \leq \bar{P}^{2.5} \leq \bar{P}^3 \leq \bar{P}^4$ defined by*

$$
\begin{aligned}
\bar{P}^1 &= \begin{cases} \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}^*_{(-,H,H)}}{1+\delta} & if\, \bar{P}^1 > \bar{P}^T_{(-,L,L)} \\ 0 & otherwise \end{cases} \\
\bar{P}^2 &= \begin{cases} \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} & if\, k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ 0 & otherwise \end{cases} \\
\bar{P}^{2.5} &\in \left[\frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta}, \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta} \right]
\end{aligned}
$$

$$\bar{P}^3 = \begin{cases} \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^T_{(-,L,H)}+\delta k}{1+\delta} & if\, k < \bar{P}^N_{(-,-,H)}-\bar{P}^T_{(-,L,H)}+\bar{P}^N_{(-,-,L)}-\bar{P}^T_{(-,L,L)} \\ \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^N_{(-,-,H)}-\delta\bar{P}^T_{(-,L,L)}+\delta\bar{P}^N_{(-,-,L)}}{1+\delta} & otherwise \end{cases}$$

$$\bar{P}^4 = \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^N_{(-,-,H)}-\delta\bar{P}^*_{(-,L,L)}+\delta\bar{P}^N_{(-,-,L)}}{1+\delta},$$

where $\bar{P}^*_{(-,H,H)}$ ($\bar{P}^*_{(-,L,L)}$) is the equilibrium value of $\bar{P}_{(-,H,H)}$ ($\bar{P}_{(-,L,L)}$) in the Upward (Downward) Manipulation equilibrium when $P_0 = \bar{P}^1$ ($P_0 = \bar{P}^4$), such that that the optimal announcement strategy is

$$\begin{cases} \theta_{H|H}=1,\, \theta_{L|L}=1-\varepsilon,\, \theta_{N|L}=\varepsilon & if\, P_0 \in \left[\bar{P}^2, \bar{P}^{2.5}\right] \ (TA) \\ \theta_{H|H}=1-\varepsilon,\, \theta_{N|H}=\varepsilon,\, \theta_{L|L}=1, & if\, P_0 \in \left[\bar{P}^{2.5}, \bar{P}^3\right] \ (TA) \\ \theta_{H|H}=1,\, \theta_{H|L}=\omega_1,\, \theta_{L|L}=1-\omega_1-\varepsilon,\, \theta_{N|L}=\varepsilon & if\, P_0 \in \left(\bar{P}^1, \bar{P}^2\right) \ (UM) \\ \theta_{L|L}=1,\, \theta_{L|H}=\omega_2,\, \theta_{H|H}=1-\omega_2-\varepsilon,\, \theta_{N|H}=\varepsilon & if\, P_0 \in \left(\bar{P}^3, \bar{P}^4\right) \ (DM) \\ \theta_{N|H}=\theta_{N|L}=1 & if\, P_0 \in \left[0, \bar{P}^1\right] \cup \left[\bar{P}^4, 1\right] \ (NA) \end{cases}$$

where $\omega_1, \omega_2 \in (0,1)$, $\lim_{P_0 \uparrow \bar{P}^2} w_1 = \lim_{P_0 \downarrow \bar{P}^3} w_2 = 0$ and $\varepsilon \gtrapprox 0$.

The introduction of a penalty for manipulation weakly increases the informativeness of the manipulator's announcement: for a given P_0, either manipulation is less frequent; or the manipulator switches from not announcing to making an informative, although possibly manipulated, announcement; or the announcement stays truthful as it was before the introduction of the penalty. This is formalized in the next theorem, but it is easy to observe from the expressions for $\bar{P}^2$ and $\bar{P}^3$: when k is small, increases in k expand the region of P_0 values for which a truthful announcement is supported in both directions, which means that truthful announcements become more likely. Panel A of Figure 4 shows how the region of P_0 values that support each of the four types of equilibrium changes with k when the regulator is skilled.

The same is true for $\bar{P}^1$ and $\bar{P}^4$, although that is not so immediate. In the case of $\bar{P}^1$, when $k = 0$, $\bar{P}^*_{(-,H,H)} = \bar{P}^N_{(-,-,H)}$, because the manipulator always announces $a = H$ when $P_0 = \bar{P}^1$, rendering the announcement uninformative. But when $k > 0$, at some point it is better to not announce if the alternative is to manipulate the announcement so much that it becomes almost uninformative. By not announcing, the manipulator relinquishes the opportunity to influence prices a little, but avoids the penalty for manipulation. Therefore, $\bar{P}^*_{(-,H,H)} > \bar{P}^N_{(-,-,H)}$ with the former increasing in k, which implies that $\bar{P}^1$ decreases in k . Likewise, $\bar{P}^4$ increases in k.

When k becomes very large, any manipulation becomes prohibitively costly. Then, not surprisingly, the manipulator either announces truthfully or does not announce. In both cases he avoids the penalty altogether. The interesting aspect is that the manipulator chooses to announce truthfully for smaller values of P_0 and to not announce for large values of P_0. This is due to investors' risk aversion, the existence of a positive average amount of risk and the reduction in uncertainty associated to a truthful announcement. If P_0 is sufficiently small, the manipulator always takes a

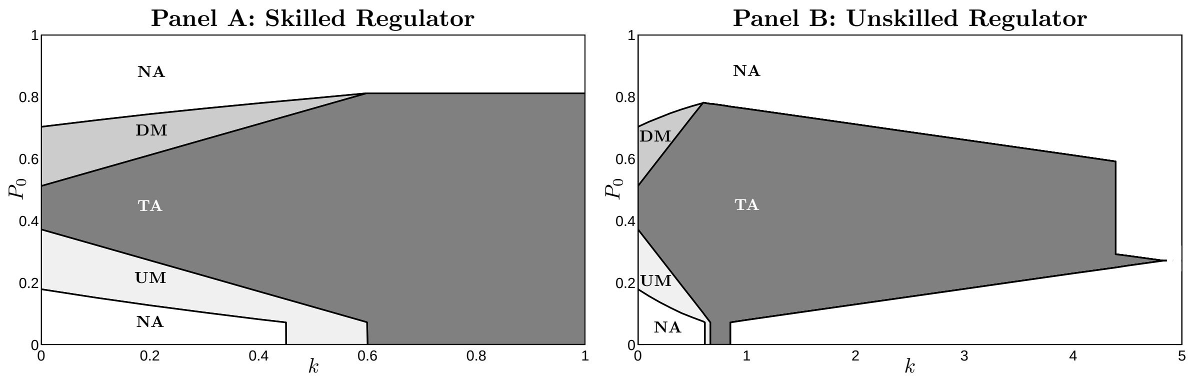

Figure 4: Effect of manipulation punishment on the announcement strategy. This figure shows how the region of P_0 values that support UM, DM, TA and NA announcement strategies (defined by $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$) changes in response to changes in the expected penalty for manipulation (k). In Panel A the regulator is skilled, whereas in Panel B the regulator is unskilled. In both panels the parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\rho = 0.8$, $\lambda = 1$, $\delta = 1$, $\rho_M = 0.9$.

long position in the asset regardless of what he announces. As a result, he is better off by announcing truthfully in order to benefit from the price increase associated to the reduction in uncertainty. Obviously, when P_0 is very large, he always takes short positions, and so he is not interested in reducing uncertainty and, consequently, does not announce.

The main result of the theorem, however, is that the manipulator chooses to use the penalty for manipulation as a *commitment device* to make more informative announcements. The manipulator could engage in upward (downward) manipulation and still avoid the penalty simply by using $a = N$ instead of $a = H$ ($a = L$) as a signal of $s_M = H$ ($s_M = L$). Out sophisticated investors would then interpret a no announcement as a possibly manipulated signal for $s_M = H$. But since there would be no formal announcement, the regulator would not be able to punish it. However, the manipulator chooses to forgo this opportunity to avoid the penalty. This suggests that the manipulator prefers to operate in markets that are more heavily regulated. The next theorem tells us that this is always the case when the regulator is skilled. Moreover, the more heavily the manipulation is punished, the more informative the manipulator's announcement.

Theorem 5. *If the regulator is skilled, increases in k weakly increase the manipulator's expected utility and weakly improve the informativeness of the manipulator's announcement* $\forall P_0$. *If k is small enough so that the subset of* P_0 *values that support either a UM or a DM equilibrium is nonempty, then increases in k strongly increase the manipulator's expected utility and strongly improve the informativeness of the manipulator's announcement for at least the* P_0 *values in that subset.*

This surprising result is actually quite simple to understand. Because investors can rationally anticipate the manipulator's equilibrium announcement strategy, manipulation is *not a good deal* for the manipulator. The more he manipulates, the less credibility investors attach to his announcement, and thus the less he can influence equilibrium prices. The only reason he manipulates is because he cannot credibly commit to do otherwise. If the manipulator could commit to be truthful, he would be better off doing so. To better understand this, recall the previous discussion about

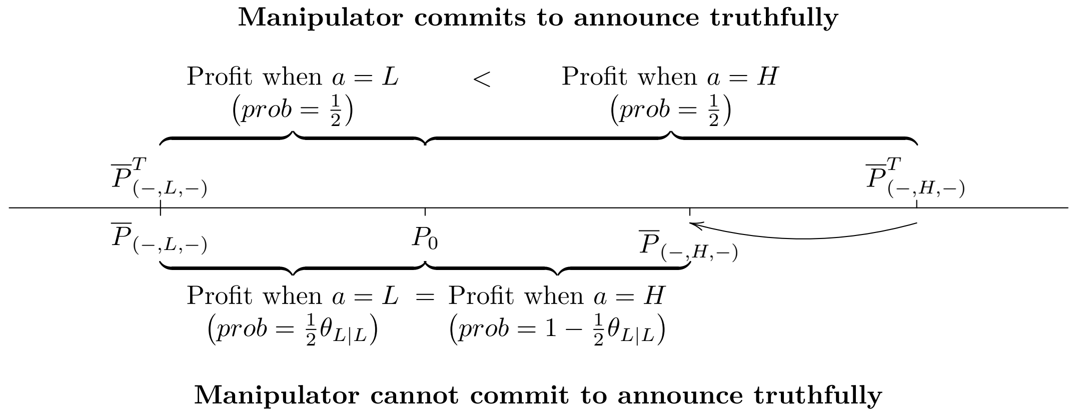

Figure 5: Benefit of commitment to announce truthfully. In the upper section of the figure, the manipulator is able to commit to announce truthfully (e.g. because of a penalty for manipulation). Because of the relatively low value of P_0, the manipulator obtains larger profits when $s_M = a = H$ than when $s_M = a = L$. In the lower section of the figure, the manipulator is not able to commit to announce truthfully, and tries to exploit the fact that, if his announcements are believed to be truthful, he obtains larger profits by announcing $a = H$. Rational investors will anticipate the manipulation of $a = H$, which leads to a decrease of $\bar{P}_{(-,H,-)}$ until the point where the manipulator becomes indifferent between manipulating or announcing truthfully. As a consequence, the manipulator obtains smaller profits conditional on $s_M = H$ than he would if he were able to commit to announce truthfully.

the upward manipulation announcement strategy. By occasionally manipulating when $s_M = L$ (announcing $a = H$), the manipulator lowers the expected utility when $a = H$ regardless of s_M, while keeping the expected utility when $a = L$ unchanged. The equilibrium is attained when he is indifferent between both announcements when $s_M = L$. Therefore, if $s_M = L$ the manipulator obtains the same expected utility he would obtain if he could commit to announce truthfully, but *less* expected utility when $s_M = H$. Figure 5 illustrates the point.

It is then quite obvious that the manipulator prefers to operate in heavily regulated markets, which help him commit to truthful announcements, provided he is punished only by his wrongdoing, and not by bad luck. However, if the regulator is unskilled, things are not so straightforward.

Theorem 6. *When the regulator is unskilled, if k is relatively small, and ρ_M is not too small, increases in k weakly increase the manipulator's expected utility and weakly improve the informativeness of the manipulator's announcement $\forall P_0$. However, if k becomes large enough, the manipulator never announces and his expected utility decreases.*

When the regulator is unskilled, the manipulator is punished in two situations: (i) when he manipulates and his information is correct and (ii) when he *does not* manipulate but unluckily his information is incorrect. In this case the only way to avoid the punishment is by not announcing. Hence, as k gets really large, the manipulator never announces. When k is small, however, increases in k improve the commitment device in the same way as in the case of a skilled regulator. Hence, the manipulator is better off and his announcement is more informative in a slightly regulated market. This is so because, although the manipulator is (on average) punished for announcing truthfully, he is punished more heavily for manipulating. Panel B of Figure 4 shows how the region of P_0 values that support each of the four types of equilibrium changes with k when the regulator is unskilled.

This shows that the easier/cheaper to implement penalties based on the comparison between what is announced and the realized outcome are only an imperfect substitute for penalties based on *de facto* manipulation. Both are successful in preventing manipulation. But the easier alternative only improves the informativeness of the announcement up to a certain point. After that, manipulation is reduced but at the cost of no announcements. In Section 3.4 we will investigate whether this is a desirable outcome or not.

3.3 Comparative Statics on the Manipulator's Announcement Strategy

In this subsection I analyze how $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$, which define the regions of P_0 values where each type of equilibrium occurs, change with the investors' signal accuracy (ρ), manipulator's signal accuracy (ρ_M) and short selling constraints (δ).

Theorem 7. *If the regulator does not punish manipulation associated to pre-announcement speculation ($k = 0$), then*

(i) $\frac{\partial \bar{P}^3}{\partial \rho} \geq 0$, $\frac{\partial \bar{P}^4}{\partial \rho} \geq 0$, $\frac{\partial \left(\bar{P}^3 - \bar{P}^2\right)}{\partial \rho} \geq 0$, $\frac{\partial \left(\bar{P}^4 - \bar{P}^1\right)}{\partial \rho} \geq 0$, $\bar{P}^3 = \bar{P}^2$ if $\rho = \frac{1}{2}$, and $\bar{P}^1 = \bar{P}^2 \wedge \bar{P}^3 = \bar{P}^4$ if $\rho = 1 > \rho_M$;

(ii) $\frac{\partial \bar{P}^1}{\partial \rho_M} \leq 0$, $\frac{\partial \bar{P}^4}{\partial \rho_M} \geq 0$, $\frac{\partial \left(\bar{P}^3 - \bar{P}^2\right)}{\partial \rho_M} \geq (\leq) \, 0$ if ρ_M is small (large), $\bar{P}^1 = \bar{P}^2 = \bar{P}^3 = \bar{P}^4$ if $\rho_M = \frac{1}{2}$ and $\bar{P}^2 = \bar{P}^3$ if $\rho_M = 1 > \rho$;

(iii) $\frac{\partial \bar{P}^j}{\partial \delta} < 0$, $j = 1, ..., 4$, $\frac{\partial \bar{P}^2 - \bar{P}^1}{\partial \delta} < 0$, $\frac{\partial \bar{P}^3 - \bar{P}^2}{\partial \delta} < 0$, $\frac{\partial \bar{P}^4 - \bar{P}^3}{\partial \delta} > 0$ and $\frac{\partial \bar{P}^4 - \bar{P}^1}{\partial \delta} < 0$.

The first part of theorem repeats in part what was discussed before: that investors' access to a source of information other than the announcement is crucial to support a truthful announcement strategy when manipulation is not punished. When there is no other source of information ($\rho = \frac{1}{2}$), and investors believe that announcements are truthful, the manipulator strictly prefers to announce the same regardless of his information (s_M), with a single exception: in the zero-probability event $P_0 = \bar{P}^2 = \bar{P}^3$. The problem here is that the date 1 expected price is a function of the announcement only. With the exclusive control over the date 1 expected price, the manipulator cannot avoid manipulating the announcement, unless he is indifferent about what to announce.

However, when investors have another source of information ($\rho > \frac{1}{2}$), the manipulator no longer has the exclusive control over date 1 expected prices. Now, the date 1 expected price depends both on the announcement and on the signal s observed by investors. This gives the manipulator an incentive to align his announcement with the information observed by investors, which is achieved by announcing truthfully, since $s = s_M$ is more likely than $s \neq s_M$. The idea is that if the manipulator observes $s_M = L$ but announces $a = H$, most likely investors will observe $s = L \neq a$ which contradicts the announcement. As a result, the date 1 expected price when $a = H$ is not as large as it would be if the manipulator had observed $s_M = H$, that is, $\bar{P}_{(-,H,H)} > \bar{P}_{(-,H,L)}$. Likewise, $\bar{P}_{(-,L,H)} > \bar{P}_{(-,L,L)}$. This means that there is a range of P_0 values for which announcing $a = H$ is optimal when $s_M = H$ but not when $s_M = L$, and announcing $a = L$ is optimal when $s_M = L$ but not when $s_M = H$, that is, for which a truthful announcement strategy is optimal.

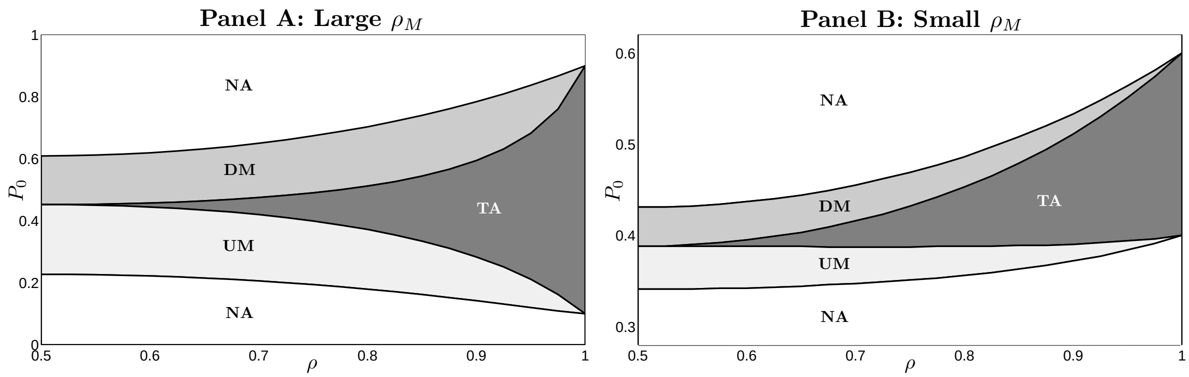

Figure 6: Effect of the accuracy of investors' information on the announcement strategy. This figure shows how the region of P_0 values that support UM, DM, TA and NA announcement strategies (defined by $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$) changes in response to changes in the accuracy of investors' information (ρ). In Panel A the manipulator is well informed ($\rho_M = 0.9$), whereas in Panel B he is poorly informed ($\rho_M = 0.6$). In both panels the parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\lambda = 1$, $\delta = 1$, $k = 0$.

This range of P_0 values expands as the spread between $\bar{P}_{(-,a,H)} - \bar{P}_{(-,a,L)}$ increases for $a = H$, $a = L$, or both, which is exactly what happens as ρ increases.

But even though the region of P_0 values that support a truthful announcement strategy expands as investors become better informed (i.e. as ρ increases), it might not expand in both directions: the upper bound $\bar{P}^3$ always increases in ρ, but the lower bound $\bar{P}^2$ does not necessarily decrease in ρ. The implication is that an increase in ρ does not guarantee that a truthful announcement is more likely. $\bar{P}^2$ may not decrease in ρ because it is a weighted average of $\bar{P}^T_{(-,H,L)}$ and $\bar{P}^T_{(-,L,L)}$. It can be shown that $\bar{P}^T_{(-,L,L)}$ increases in ρ. However, $\bar{P}^T_{(-,H,L)}$ may increase or decrease in ρ.[20] For $\bar{P}^2$ to decrease in ρ, $\bar{P}^T_{(-,H,L)}$ has to decrease by more than $\delta\bar{P}^T_{(-,L,L)}$ increases. This can be achieved if ρ_M is large enough and δ is small.[21] Figure 6 illustrates the impact of ρ on the announcement strategy, putting in evidence the differences when ρ_M is large (Panel A) and small (Panel B). The case of informative announcements (manipulated plus truthful announcements) is very similar to the case of truthful announcements.

Notice that when $\rho = 1$ (and as long as $\rho_M < 1$), the manipulator loses the ability to influence the date 1 expected price, since investors learn exactly the liquidation value from their signal alone. At that point, the manipulator's announcement strategy is irrelevant. Numerical results show that, as ρ converges to 1, the manipulator manipulates less frequently. This occurs because when ρ is

[20] $\bar{P}^T_{(-,a,s_M)}$ is a weighted average of $\bar{P}^T_{(H,a,-)}$, which increases in ρ, and $\bar{P}^T_{(L,a,-)}$, which decreases in ρ. Thus, depending on the weights and on how they change with ρ, it is possible for $\bar{P}^T_{(-,a,s_M)}$ to increase or decrease in ρ.

[21] This is so for two reasons. First, when ρ_M increases, the likelihood of $s = s_M$ increases. Thus, when $s_M = L$, the manipulator increases the weight on the prices conditional on $s = L$, which decrease in ρ. Second, when ρ_M is large, investors put a large weight on a relatively to s when forming their beliefs. This implies that when ρ increases, the change in beliefs is much larger when $a \neq s_M$ than when $a = s_M$. As a result, $\bar{P}_{(L,H,-)}$ decreases considerably more than what $\bar{P}_{(H,H,-)}$ increases and so $\bar{P}^T_{(-,H,L)}$ decreases more with ρ. At the same time, $\bar{P}_{(H,L,-)}$ increases more than what $\bar{P}_{(L,L,-)}$ decreases, which implies that $\bar{P}^T_{(-,L,L)}$ increases more with ρ. But, since prices conditional on $s = L$ are weighted more heavily, the net effect is that a larger ρ_M contributes to $\bar{P}^2$ decreasing in ρ. Finally, note that if δ is small, the weight of $\bar{P}^T_{(-,L,L)}$ on $\bar{P}^2$ is relatively small. Therefore $\bar{P}^T_{(-,H,L)}$ doesn't need to decrease by much to compensate for the increase in $\bar{P}^T_{(-,L,L)}$, which makes it easier to find that $\bar{P}^2$ decreases in ρ.

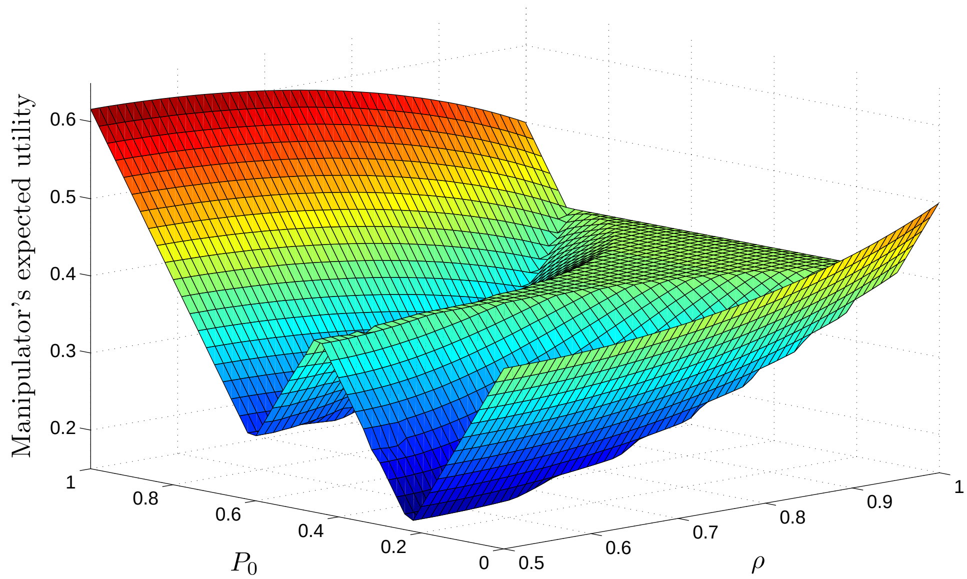

Figure 7: Effect of the accuracy of investors' information on the manipulator's expected utility. This figure shows how the accuracy of investors' information (ρ) impacts the manipulator's expected utility for a range of P_0 values. The throughs and peak seen in the figure correspond to the thresholds of the different announcement strategies. From smaller to larger values of P_0, the manipulator's announcement strategy is: NA and long position until the first through; UM from that point until the peak; TA in the peak; DM from the peak to the second through; and NA associated to a short position from the second through. The parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\lambda = 1$, $\delta = 1$, $k = 0$.

large, the signal available to investors is so accurate that the announcement provides little extra information. Moreover, investors can accurately identify manipulation. Therefore, the manipulator has very limited ability to influence prices through manipulation when investors are well informed, and he mainly resorts to announcing truthfully or not announcing at all.

As we have seen before, the manipulator prefers to operate in highly regulated markets because the penalty helps him commit to announce truthfully. Since an increase in ρ when ρ_M is large increases the frequency of truthful announcements just like an increase in k does when the regulator is skilled, a natural question to ask is "Does the manipulator prefer to enter in markets with better informed investors?". Figure 7 suggests an affirmative answer. The manipulator's expected utility when announcing truthfully remains essentially unchanged.[22] But manipulation is less frequent, which improves his expected utility. In addition, the expected utility when not announcing and taking a long position (small P_0) improves with ρ, because the risk is reduced. For the same reason, the manipulator's expected utility may decrease when not announcing and taking a short position, even if the manipulator switches from not announcing to announcing truthfully. However, this only happens for large values of P_0 which should have a small probability of occurrence. In Section 5.1 I will look at this question again after endogenizing P_0 and determining its distribution.

Turning the attention to what happens when ρ_M increases, the second part of Theorem 7 says that the probability of making an informative announcement increases in ρ_M. For any informative

[22]It increases slightly in ρ, except when ρ is very close to 1.

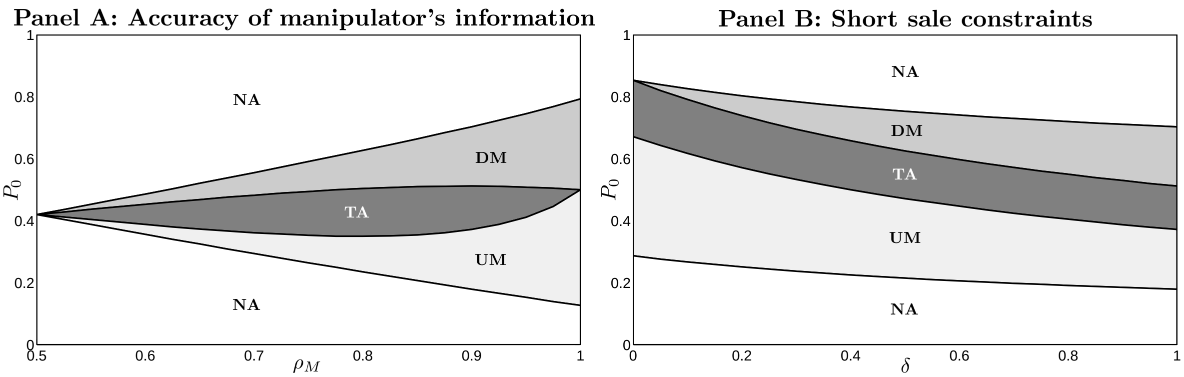

Figure 8: Effect of the accuracy of manipulator's information and short sales constraints on the announcement strategy. This figure shows how the region of P_0 values that support UM, DM, TA and NA announcement strategies (defined by $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$) changes in response to changes in the accuracy of manipulator's information (ρ_M, Panel A) and short sale constraints (δ, Panel B). In both panels the parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\rho = 0.8$, $\lambda = 1$ and $k = 0$. In Panel A $\delta = 1$ and in Panel B $\rho_M = 0.9$.

announcement strategy, the larger ρ_M, the more weight investors put on the announcement. Hence, the larger the manipulator's ability to influence prices. Naturally, this makes the manipulator more prone to take advantage of his ability by announcing.

On the other hand, truthful announcements do not necessarily become more likely as ρ_M increases. When ρ_M is small, the region of P_0 values that support a truthful announcement strategy expands, but the opposite happens when ρ_M is large. (In fact, when $\rho_M = 1$, there is a single P_0 value that supports a truthful announcement strategy.) This happens because there are two opposing forces at work when ρ_M increases. First, the difference between expected prices conditional on $s_M = H$ and $s_M = L$ tends to increase as the likelihood of $s = s_M$ increases. Second, the difference between these prices tends to decrease as investors focus more on the announcement instead of their signal when forming beliefs. When ρ_M is small relative to ρ, the first effect dominates, whereas when ρ_M becomes larger the second effect dominates. As discussed before, the difference between these prices is crucial for the existence of truthful announcement strategies. The larger it is, the larger the range of P_0 prices that support a truthful announcement strategy. Therefore, when ρ_M is small relative to ρ that range increases with ρ_M, but then at some point it starts to shrink as ρ_M increases. Panel A of Figure 8 provides an illustration.

One implication of all this is that manipulation is more likely for large ρ_M than it is for small ρ_M. It is possible to show that the single price that supports a truthful announcement strategy when $\rho_M = 1$ is larger than the one when $\rho_M = 1/2$, provided that $\delta \leq 1$. Thus, upward manipulation is guaranteed to be more likely for large values of ρ_M, whereas downward manipulation may or may not be more likely to occur. In general, upward manipulation becomes the most likely announcement strategy when ρ_M is large relative to ρ.

The third part of the Theorem 7 seems, at first sight, to indicate that tighter short sales constraints lead to more informative announcements, since the range of P_0 values for which there is a truthful or an informative announcement increase as δ decreases. However, this is only a second order effect, attributed to risk aversion. The main effect of a decrease in δ is that short positions become less attractive. As a result, the manipulator becomes more biased toward long positions. The implication is that the region of P_0 values that support TA, UM and DM equilibria shift toward larger values of P_0. Moreover, the region of upward manipulation expands, whereas the region of downward manipulation shrinks. Consequently, a smaller δ most likely decreases the average informativeness of the announcement and increases the probability that the announcement is manipulated upwards. Panel B of Figure 8 illustrates the case.

From the discussion above we can see that when tying a relatively small δ to a large ρ_M relatively to ρ, it is likely that upward manipulation becomes the most frequent announcement strategy among the informative ones. In such case, most of the times $a = L$ is a truthful signal for $s_M = L$, whereas $a = H$ is a manipulated signal which on average signals for $s_M = H$. This means that good information is spread through rumors (manipulated announcements) whereas bad information is spread through news (truthful announcements) which lends some credence to the old Wall Street saying "buy the rumor, sell the news". Not only does this parametrization seem plausible, but also its implications are supported by the vast empirical evidence suggesting that buy recommendations by affiliated analysts (here the manipulator) are less truthful than their sell recommendations (e.g. Dugar and Nathan, 1995; Lin and McNichols, 1998; Frankel et al., 2006; Barber et al., 2007; Cliff, 2007; Ljungqvist, Marston, Starks, Wei, and Yan, 2007; Agrawal and Chen, 2008).

Corollary 8. *If δ is small and/or ρ_M is large relative to ρ, then it is very likely that upward manipulation is the most frequent announcement strategy.*

3.4 Price Efficiency, Risk Premium and Price Response to Announcement

In this subsection I look at the effect of the manipulator's announcements on price efficiency, risk premium and price response to the announcement. All three are directly related to the informativeness of the manipulator's announcement. Intuitively, the more informative the manipulator's announcement, either because it is more truthful or because the manipulator is better informed, the higher the price efficiency, the higher the price response to the announcement and the smaller the risk premium.

Price efficiency, as usually in the literature (see e.g. Fishman and Hagerty, 1992), is a measure of the amount of information incorporated in the equilibrium price. In this model, this corresponds to the information available to investors, since the manipulator's trades have no impact on the equilibrium price. Therefore, I use the average probability investors assign to the true liquidation state, i.e. $p^I_{(H,a,-)}$ when V_H occurs (which happens with probability $p^I_{(H,a,-)}$) and $1 - p^I_{(H,a,-)}$ when V_L occurs (probability $1 - p^I_{(H,a,-)}$), as the measure of price efficiency. Of interest is the price efficiency conditional on the informational state $(s, a, -)$

$$\overline{eff}_{(s,a,-)} = \left(p^I_{(s,a,-)}\right)^2 + \left(1 - p^I_{(s,a,-)}\right)^2,$$

and the unconditional price efficiency,

$$\overline{eff} = \sum_{s,a} \overline{eff}_{(s,a,-)} \gamma_{(s,a,-)}$$

where $p^I_{(s,a,-)}$ and $\gamma_{(s,a,-)}$ are as defined in Appendix A. The following theorem presents the impact of the manipulator's announcement strategy, ρ and ρ_M on price efficiency.

Theorem 9. *Let $\widetilde{\theta} \equiv 1 - \frac{\theta_{H|H}+\theta_{L|L}}{2}$ denote the frequency with which manipulation occurs. Then, when all investors observe signal s:*

(i) The unconditional price efficiency decreases with the frequency of manipulation ($\widetilde{\theta}$);

(ii) The unconditional price efficiency increases with the accuracy of investors' information (ρ) and, except in the NA equilibrium, with the accuracy of manipulator's information (ρ_M);

In the simplified setting of this section, the manipulator's announcement strategy has only one simple impact on the unconditional average price efficiency: the more informative the announcement strategy, the better informed each and every investor becomes, and so the higher the price efficiency.[23] Therefore, there is a one-to-one positive relation between the informativeness of the announcement strategy and unconditional price efficiency (points i and ii). Moreover, numerical results suggest that unconditional average price efficiency and unconditional per share risk premium, defined as $\overline{RP} = \mathbb{E}\left(V - P\right)$, are inversely related.[24] Figure 9 provides an illustration.

I would like to emphasize that manipulation *per se* is not bad in terms of price efficiency (point (i) of the theorem). This is only the case if the manipulator can be made to announce truthfully (the first best). If the alternative to manipulation is to not announce, then manipulation (the second best) is preferable. This has a clear policy implication: any measure designed to mitigate manipulation has to create the conditions necessary for the manipulator to announce (more) truthfully, and not to silence him.

To conclude the analysis of the simplified version of the model, I look at the average price response to the manipulator's announcement. Since the initial price is independent of the announcement, I focus only on the post-announcement price. Typically $\bar{P}_{(s,L,-)} < P_0 < \bar{P}_{(s,H,-)}$ and so the price response is larger the smaller $\bar{P}_{(s,L,-)}$ and the larger $\bar{P}_{(s,H,-)}$ are.

Theorem 10. *In any equilibrium with informative announcement strategies the following holds:*

(i) $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{H|H}} \geq 0$, $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{L|L}} \geq 0$, $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{H|H}} \leq 0$, $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{L|L}} \leq 0$ *with* $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{L|L}} > \frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{L|L}} = 0$ *in UM equilibria and* $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{H|H}} < \frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{H|H}} = 0$ *in DM equilibria;*

(ii) $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \rho_M} > 0$, $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \rho_M} < 0$*;*

[23] As we will see in the general case, where investors have to decide whether to purchase signal s, the manipulator's announcement strategy will have an additional impact on price efficiency: in general, the more informative the announcement strategy, the less incentive there is to purchase the costly signal which, by itself, decreases price efficiency. Therefore, it is not obvious that the informativeness of the announcement strategy and price efficiency change in the same direction.

[24] The average risk premium cannot be expressed algebraically, which makes it difficult to prove this result generically. Nevertheless, it is possible to prove the result for the case of TA and NA announcement strategies. The proof for these special cases are omitted because the expressions are too long and complicated.

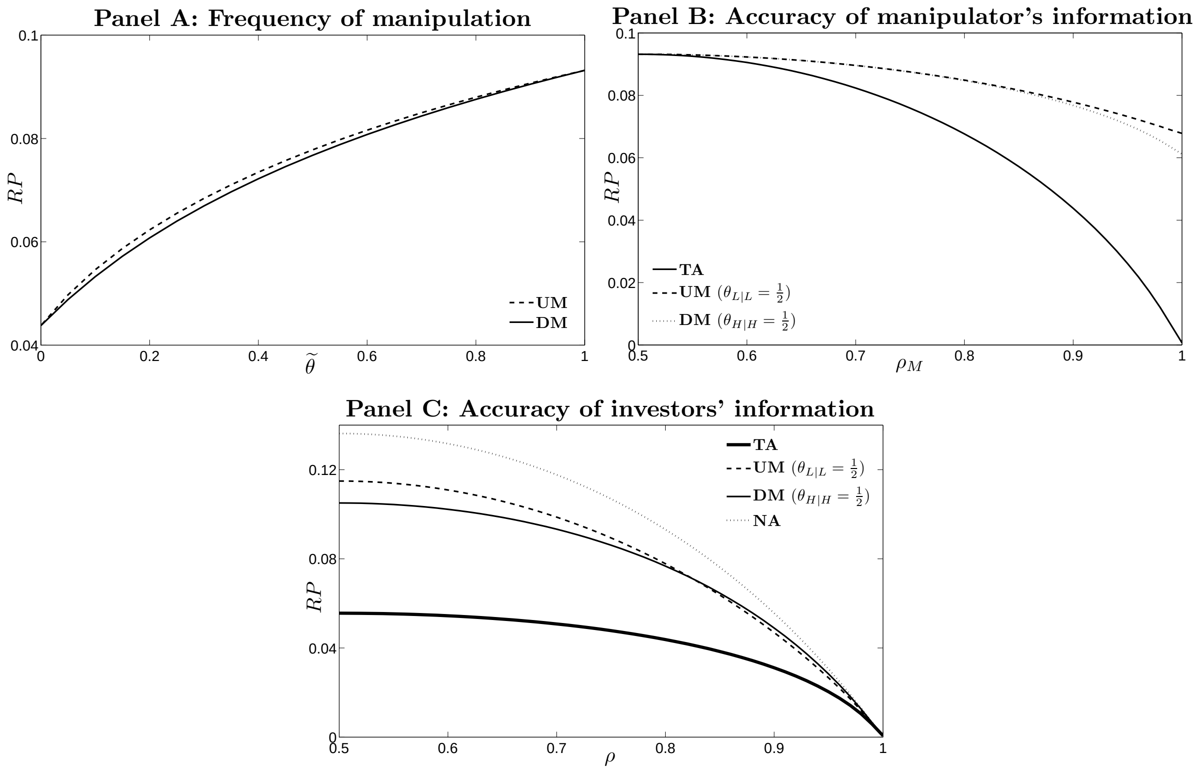

Figure 9: Effect of the frequency of manipulation, accuracy of investors' information and accuracy of manipulator's information on the unconditional risk premium. This figure plots the unconditional risk premium as a function of the frequency of manipulation ($\widetilde{\theta}$, Panel A), accuracy of manipulator's information (ρ_M, Panel B) and accuracy of investors' information (ρ, Panel C). In all panels, the parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\lambda = 1$ and $k = 0$. In Panel A $\rho = 0.8$ and $\rho_M = 0.9$, in Panel B $\rho_M = 0.9$ and in Panel C $\rho = 0.8$.

(iii) $\frac{\partial \bar{P}_{(H,a,-)}}{\partial \rho} > 0$, $\frac{\partial \bar{P}_{(L,a,-)}}{\partial \rho} < 0$.

In face of what we have seen up to this point, these results are rather intuitive. The higher the probability that the announcement was manipulated, the smaller the price reaction to that announcement (point i). For example, in a UM equilibrium $a = L$ is known to be a truthful announcement, whereas $a = H$ might have been manipulated. Hence, the higher the frequency of manipulation, the smaller the price reaction to $a = H$, whereas the price reaction to $a = L$ remains unchanged. In addition, the price reaction to the announcement increases with the quality of the manipulator's information (point ii). Similarly, prices are more sensitive to the investors' signal the more accurate it is (point iii). Summing up, the price reaction to the announcement is stronger the more uncertainty the announcement resolves.

An obvious consequence of a stronger initial price response to the announcement is that, later on when the liquidation value is revealed, the surprise is smaller. Also, since investors are rational, the average (over date 1 supply shocks, z) post-announcement return will always be positive so that the residual risk is rewarded. The following corollary summarizes these results.

Corollary 11. *The average initial price response to the manipulator's announcement increases with the truthfulness of his announcement and with the accuracy of his information, while the average* *price response to the revelation of the liquidation value (the post-announcement return) decreases. The post-announcement return is always positive.*

3.5 Linking model predictions with empirical evidence

3.5.1 How effective was the 2003 GRAS attempt to reduce manipulation?

During the bull maket of the 90's, it became clear that the conflict of interests among analysts employed by investment banks led to overoptimistic recommendations.[25] The April 28, 2003, Global Research Analyst Settlement (GRAS) between ten of the leading investment banks and a host of regulators (SEC, NASD and NYSE) represented an attempt to resolve this conflict of interests. To that end, the GRAS imposed the separation between investment banking and research activities, increased the amount of disclosure in research reports and imposed a payment of $875 million in penalties. The increased scrutiny over the activities of affiliated analysts effectively corresponds to an increase in k in the model. In addition, the GRAS imposed the payment of $80 million for investor education and $432.5 million to fund independent research to be distributed along with their own research reports. In the model this translates into an increase of ρ.[26]

Curiously, increasing k and ρ are exactly the only two ways of decreasing the frequency of manipulation and improving informativeness of announcements in the model. However, the model also tells us that reducing manipulation and improving the level of information may be conflicting goals. A manipulated announcement provides more useful information than no announcement at all. Therefore, reducing manipulation only improves the level of information in the economy if the manipulator announces more truthfully, and not if he stops announcing at all.

In the model, whether increasing k or increasing ρ is more effective in decreasing manipulation *and* improving the level of information depends on the circumstances. On the one hand, an increase in ρ is more effective if the manipulator is well informed. Otherwise, the frequency of manipulation is reduced but at the expense of forcing the manipulator to remain silent, instead of making the manipulator announce more truthfully. On the other hand, an increase in k is highly effective but only if the manipulator can avoid punishment by not manipulating (skilled regulator). If, by the contrary, the manipulator is occasionally punished despite being truthful (unskilled regulator), an increase in the penalty may force him to not announce. Unfortunately, the latter is more likely to be the case in practice, and increasing k is a double-edged sword. Increasing ρ seems to be a safer bet.

Cliff (2007), Clarke, Khorana, Patel, and Rau (2009) and Kadan, Madureira, Wang, and Zach (2009) provide empirical evidence suggesting that GRAS indeed succeeded in reducing the conflict

[25]Examples of articles in the financial press exposing the conflict of interest and revealing the unethical Wall Street practices are Siconolfi (1992, 1995a,b), Feldman and Caplin (2002), Byrne (2002a,b), Morgenson (2002) and Gasparino (2003). Quoting from Morgenson (2002), "[analysts] had become salesmen and saleswomen for their investment banking departments in their routine communications" and, from Byrne (2002a), "Historically, "sell" ratings have constituted fewer than 1% of analysts' recommendations, according to Thomson Financial/First Call".

[26]It also translates into a decrease in the cost of information (c) which, as we will see in Section 4, is essentially the same as an increase in ρ when all investors are informed.

of interest of affiliated analysts: affiliated analysts became less biased toward issuing buy recommendations and, accordingly, the informativeness of their buy recommendations increased. However, Clarke et al. (2009) and Kadan et al. (2009) also find that the GRAS brought unintended consequences: the overall informativeness of recommendations both by affiliated and independent analysts decreased in the post-GRAS period. Moreover, there was a widespread move from a five-tier (strong buy, buy, hold, sell and strong sell) to a three-tier rating system (buy, hold and sell) by investment banks. This suggests that investment banks became more fearful of ill-founded prosecution in case of honest mistakes, since the move to a three-tier rating system decreases the likelihood of issuing a wrong recommendation.[27]

In addition, the attempts at increasing ρ seem to have failed: the decrease in the quality of independent analysts' recommendations suggests that the new independent research firms created in the post-GRAS period produce less informative recommendations. Possible explanations for this failure are the suspicion of the true independency of independent analysts funded by investment banks, and the entry of mediocre research firms into the market financed by the funds made available by the GRAS, which caused a dilution of the overall quality of independent research.

All in all, it looks like the GRAS measures accomplished the easiest goal, to decrease manipulation, but failed the most important one, to improve the level of information. The increased scrutiny (higher k) increased the fear of ill-founded prosecution, leading to less informative announcements. On top of that, the attempt to improve the quality of independent recommendations seems to have failed.

Supposing that the GRAS suceeded in increasing ρ, would it really help to decrease manipulation and improve the level of information, as predicted by the model? There is some empirical evidence that suggests an affirmative answer. Ljungqvist et al. (2007) find that affiliated analysts are less biased when issuing recommendations for stocks with high institutional ownership. Arguably institutional ownership correlates positively with how well informed investors are, which corroborates the link between higher quality of investors' information and informativeness of the manipulator's announcement. Finally, O'Brien and Bhushan (1990) find that changes in analyst coverage are positively related with the degree of regulated disclosure. Since the quality of public information available is higher in industries with more regulated disclosure, this suggests that, as predicted by the model, analysts prefer to issue recommendations about industries where investors are already well informed.

3.5.2 How to detect manipulation?

The model suggests that we can test for manipulation in two ways: (i) by comparing the price response to announcements made by affiliated analysts to those of independent analysts (a short-window event study); or (ii) by comparing the long-run returns following announcements made by affiliated analysts to those following announcements made by independent analysts (a long-window

[27] Reputation concerns may explain why investment banks moved to a three-tier rating system instead of stopping issuing recommendations as predicted by the model.

post-event study). The second alternative is likely less robust. As Kothari (2001 pp. 187-192) argues: "The inferential issues for the short-window event studies are straightforward, but they are quite complicated for the long-horizon performance studies.". In addition to the usual problem of misestimation of abnormal returns, here, as Cliff (2007) points out, we also have the problem of determining the holding period over which to measure the returns associated to a recommendation.

Detecting manipulation from the initial price response to the announcement, however, is not a simple task. If manipulators are not biased toward upward or downward manipulation, and use both strategies equally likely, we will not be able to detect any bias in the price response to good or bad announcements. We will only observe a relatively small average price response to the announcement. But the same could be a result of a relatively small ρ_M, which is unobservable. If this is the case, then the only realistic possibility to detect manipulation is to take advantage of an event with predictable impact on manipulation, such as the 2003 GRAS.

However, there are good reasons to expect affiliated analysts to be biased toward issuing buy recommendations, which makes manipulation easier to detect. Investment banks may want to please current or prospective clients to gain their business by publishing optimistic recommendations on their stocks (e.g. Dugar and Nathan, 1995; Lin and McNichols, 1998 and Michaely and Womack, 1999). Brokerage firms have an incentive to issue bullish recommendations to boost brokerage fees due to investors' short sale constraints (e.g. Irvine, 2004; Jackson, 2005; Agrawal and Chen, 2005 and Cowen, Groysberg, and Healy, 2006). Analysts may fear the loss of access to timely information essential to perform their activity if they make sell recommendations (e.g. Francis and Philbrick, 1993; Das, Levine, and Sivaramakrishnan, 1998 and Lim, 2001). And finally, investment banks themselves may face short sale constraints.

Cliff (2007) uses both approaches in his search for evidence of upward manipulation. He finds that prices are more responsive to sell recommendations of affiliated analysts than to buy or hold recommendations in the pre-GRAS period. In contrast, price reactions to announcements of independent analysts, who are assumed to have no incentive to manipulate (or no bias toward upward of downward manipulation), do not exhibit such pronounced asymmetry. Moreover, in the post-GRAS period, the price reaction to buy recommendations of affiliated analysts increased and the asymmetry between the reaction to buy and sell recommendations decreased. These empirical results are in line with the predictions of the model and suggest that (i) affiliated analysts, on average, manipulate their recommendations upward and (ii) that the GRAS contributed to a decrease in that manipulation. Kadan et al. (2009) reach the same conclusions and several other short-window event studies confirm that affiliated analysts issue overoptimistic recommendations (e.g. Lin and McNichols, 1998; Frankel et al., 2006 and Malmendier and Shanthikumar, 2007).

In turn, using post-announcement returns of portfolios formed based on the recommendations, Cliff (2007) finds that (i) all portfolios formed based on the recommendations of affiliated analysts (buy, sell and hold portfolios) underperform and (ii) all portfolios formed based on the recommendations of independent analysts have a neutral performance. He takes these results as evidence of upward manipulation by affiliated analysts and poorly informed but honest independent analysts. Several other long-window post-event studies reach the same conclusions (e.g. Michaely and Womack, 1999 and Barber et al., 2007).

However, if investors are rational and sophisticated, finding a negative abnormal return for the buy portfolio is not evidence of upward manipulation.[28] This is because investors are able to discount the biases of affiliated analysts and are not mislead by them. In fact, according to the model, it is more truthful buy recommendations that lead to smaller post announcement returns, since there is less residual uncertainty. For Cliff's findings to be interpreted as a sign of upward manipulation, one needs to assume that investors are naive. In that case they are systematically mislead by manipulated buy recommendations, implying a subsequent poor return. However, the naiveness assumed here stands in contrast with the sophistication assumed in the interpretation of the short-term reaction to the announcement.[29]

To correctly interpret the long-run performance of recommendations when investors are sophisticated, we need to compare the relative performance of each portfolio, and not their absolute performance. This is because when the announcement is informative, some uncertainty is resolved which results in a smaller post-announcement return. Thus, portfolios formed based on less truthful recommendations will outperform those formed based on more truthful recommendations. But if the model for normal returns fails to capture this, negative absolute risk-adjusted returns may result. Based on the results of Cliff (2007), we observe that buy and hold portfolios outperform sell portfolios based on the recommendations of affiliated analysts, both in terms of raw returns and risk-adjusted returns. According to the model, this asymmetry does suggest upward manipulation of buy and hold recommendations. In contrast, all portfolios formed based on independent analysts' recommendations perform similarly, which indicates that these recommendations are not systematically biased.

4 Equilibrium with Information Acquisition

In this section I drop the assumption that all investors costlessly observe signal s. Instead, it is assumed that investors have the option to observe s at date 1 for a cost c. This will lead to the existence of two types of investors: those who observe s (informed investors, indexed by I) and those who don't (uninformed investors, indexed by U). We can think that signal s comes from independent analysts who sell their recommendations for a price c, or that it is the result of research by investors which incur time and monetary costs of c.

The main point of interest in this section is in how the existence of a manipulator and his

[28]Cliff implicitly assumes that the underperformance of the buy portfolio results from investors realizing that they were misled by the manipulated recommendation. However, if investors are rational and anticipate the manipulator's strategy they are not misled by the manipulator. For a correction to exist, (some) investors have to be naive. But if we assume that investors are not rational, then the test for manipulation becomes a joint test for manipulation and investors' rationality. Moreover, the conclusions taken from the reaction of prices to the announcement rely on the assumption that investors are rational.

[29]Several studies suggest that institutional investors are sophisticated enough to discount the analysts' biases, whereas individual investors do not (e.g. Boni and Womack, 2002, 2003; De Franco, Lu, and Vasvari, 2007 and Malmendier and Shanthikumar, 2007), which partially validates these common interpretations of the results.

announcement strategy affects investors' choice of information acquisition. On the one hand, the more informative the manipulator's announcement is, the less incentive there is to purchase the costly information. But, on the other hand, the smaller the fraction of informed investors, the more the announcement is manipulated, and so the less informative it is.

4.1 Solving for the Equilibrium

The demand function for each investor retains the same form of the demand function determined in the previous section where all investors observe the same information:

$$X^I = \frac{1}{\alpha} \ln \frac{p^I (1-P)}{(1-p^I) P}, \; X^U = \frac{1}{\alpha} \ln \frac{p^U (1-P)}{(1-p^U) P}.$$

The demand of informed and uninformed investors differs only because of their different beliefs about the probability of V_H occurring, p^I vs. p^U. With a fraction λ of informed investors, the average per capita demand of the risky asset is given by $\lambda X^I + (1-\lambda) X^U$. Market clearing then implies that

$$P(s,a,\lambda,z) = \frac{1}{1 + \left(\frac{1-p^I_{(s,a,-)}}{p^I_{(s,a,-)}}\right)^{\lambda} \left(\frac{1-p^U_{(s,a,-)}}{p^U_{(s,a,-)}}\right)^{1-\lambda} e^{\alpha z}}. \tag{3}$$

Uninformed investors form their conditional belief about the probability of V_H ($p^U_{(s,a,-)}$) by estimating the likelihood of $s = H$, denoted by $\hat{\gamma}$, from the information contained in the public announcement a and in the equilibrium price. To do this, uninformed investors start by computing the beliefs of informed investors conditional on $s = H$ and $s = L$. With this information they can invert the price function to determine the asset supply needed to support the observed price in the two possible scenarios: informed investors observed good news and the supply was relatively large, $\left\{p^I_{(H,a,-)}, z_{(H,a,-)}\right\}$; or they observed bad news and the supply was relatively small, $\left\{p^I_{(L,a,-)}, z_{(L,a,-)}\right\}$. The likelihood of the scenario $s = H$ is then obtained from the likelihood ratio

$$\hat{\gamma} = \frac{\phi\left(z_{(H,a,-)}\right)}{\phi\left(z_{(H,a,-)}\right) + \phi\left(z_{(L,a,-)}\right)} \in (0,1), \tag{4}$$

where $\phi(\cdot)$ if the density function for $z \sim N(\bar{z}, \sigma_z^2)$. In Appendix B, I show that $\hat{\gamma}$ depends on the realizations of s and z, but not a. Once uninformed investors obtain $\hat{\gamma}$, they can compute the probability of V_H conditional on the information they extracted from prices as

$$\begin{aligned} \mathbb{P}(V_H | P) &= \mathbb{P}(V_H | s=H)\,\mathbb{P}(s=H | P) + \mathbb{P}(V_H | s=L)\,\mathbb{P}(s=L | P) \\ &= \rho\hat{\gamma} + (1-\rho)(1-\hat{\gamma}) \equiv \hat{\rho}. \end{aligned}$$

Finally, $p^U_{(s,a,-)}$ is computed in the same way as $p^I_{(s,a,-)}$ (see Appendix A) with $\hat{\rho}$ in place of ρ.

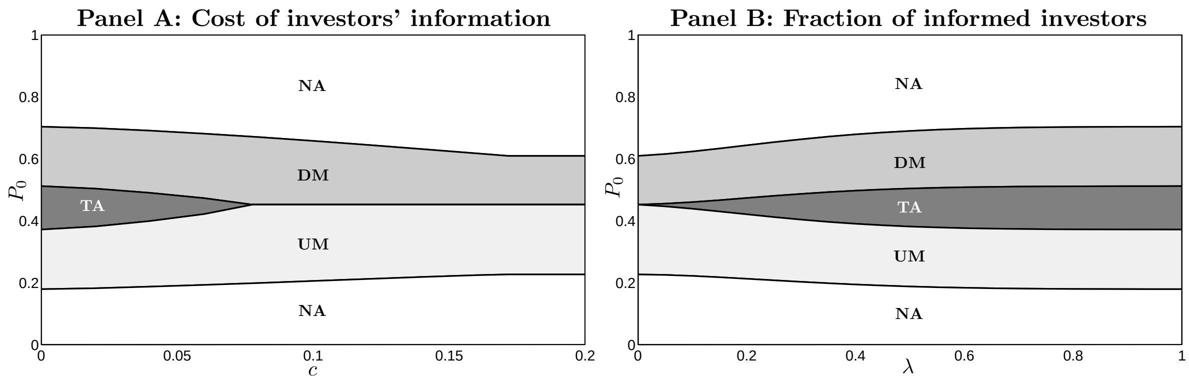

Figure 10: Effect of the cost of investors' information and fraction of informed investors on the manipulator's announcement strategy. This figure shows how the region of P_0 values that support UM, DM, TA and NA announcement strategies (defined by $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$) changes in response to changes in the cost of investors' information (c, Panel A) and in the fraction of informed investors (λ, Panel B). In both panels the parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\rho = 0.8$, $\delta = 1$, $k = 0$, $\rho_M = 0.9$.

One thing I would like to point out is that it is possible that $\hat{\gamma}_{(H,-,-)}(z) < \frac{1}{2}$ (or $\hat{\gamma}_{(L,-,-)}(z) > \frac{1}{2}$). That is, it is possible that uninformed investors think that $s = L$ ($s = H$) is more likely to have occurred when in fact $s = H$ ($s = L$) occurred. Nonetheless, on average uninformed investors get it right and assign a higher probability to the correct state.

From the equilibrium definition (Definition 2), λ is such that no investor wants to change his information acquisition decision in equilibrium. This implies that, ex-ante, the expected utility of informed investors is the same, larger and smaller than that of uninformed investors if $\lambda \in (0,1)$, $\lambda = 1$ and $\lambda = 0$, respectively.

The manipulator chooses his optimal announcement and trading strategy exactly in the same way as in the previous section. Once again, the equilibrium has to be solved numerically using McKelvey's (1992) algorithm.

4.2 The Manipulator's Optimal Strategy

Dropping the assumption that all investors observe signal s has no direct impact on the manipulator's announcement and trading strategies analyzed in the previous section. The only impact is indirectly through a decrease in the amount of independent information observed by investors. When less investors are informed, the impact of signal s on the equilibrium price is naturally smaller, since uninformed investors can only extract a weaker version of signal s from the equilibrium price. From the point of view of the manipulator, a smaller fraction of informed investors is qualitatively the same as a decrease in ρ when all investors are informed. All that matters to the choice of the announcement strategy is how sensitive equilibrium prices are to signal s. The more sensitive they are, be it because more investors observe signal s or because it is more accurate, the more informative the announcement becomes (see Figure 10). Therefore, all the analysis on the manipulator's announcement strategy performed in the previous section is still valid here.

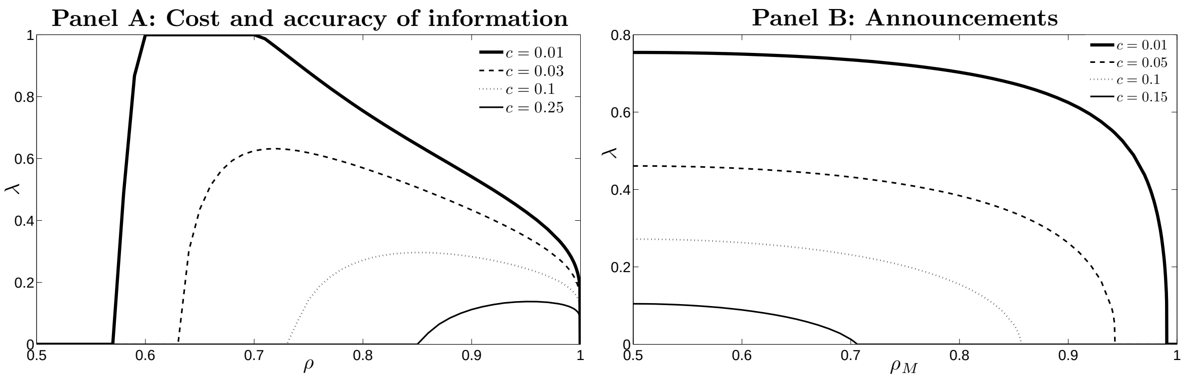

Figure 11: Effect of announcements, cost and accuracy of investors' information on the fraction of informed investors. Panel A plots the fraction of informed investors as a function of the accuracy of investors' information (ρ), for 4 levels of the cost of their information (c), when there is no manipulator. Panel B plots the fraction of informed investors as a function of the accuracy of manipulator's information (ρ_M), when the manipulator announces truthfully. When $\rho_M = \frac{1}{2}$ announcements are uninformative and it is as if no announcement is made. Similar results are obtained if the manipulator follows different announcement strategies (TA, UM or DM in Panel A, UM or DM on Panel B). In both panels the parametrization is the following: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\delta = 1$, $k = 0$. In Panel B, $\rho = 0.8$.

4.3 Investors' Acquisition of Information

The fraction of informed investors cannot be determined algebraically since there is no algebraic expressions for investors' expected utility. For this reason, the following results come from numerical simulations using a wide range of parameters, and cannot be proved formally.

In general, the optimal fraction of informed investors (λ) satisfies the following properties:

(i) λ decreases in the cost of information (c) with $\lambda = 1$ if $c = 0$ and $\lim_{c\to+\infty} \lambda = 0$;

(ii) There is a threshold ρ^* such that λ weakly increases in ρ for $\rho < \rho^*$ and weakly decreases in ρ for $\rho > \rho^*$;

(iii) λ decreases in the informativeness of the manipulator's announcement (i.e. the presence of a manipulator decreases λ).

The first result, illustrated in Panel A of Figure 11, is very intuitive and reflects the fact that, putting the cost aside, being informed is always desirable to being uninformed because informed investors use their informational advantage over uninformed investors to make better trading decisions and exploit the latter. Therefore, if s can be observed costlessly, all investors choose to observe s.

The second result, also illustrated in Panel A of Figure 11, puts in evidence the problem of information leakage. The more accurate the signal s is, the more aggressively informed investors trade on their information. On the one hand, this allows them to exploit uninformed investors more, which increases the benefit of being informed. But, on the other hand, more of their information is leaked through prices, which benefits uninformed investors. For smaller values of ρ, the first effect dominates and λ increases in ρ, but after a certain point, further increases in ρ result in too much information leakage, which decreases λ.

The third and final result, illustrated in Panel B of Figure 11, tells us that the manipulator's announcement helps uninformed investors by decreasing their informational disadvantage relative to informed investors. Hence, the incentive to purchase information decreases.[30] This happens because the incremental information content of the announcement is larger for uninformed investors than it is for informed investors, who are already better informed. This is clear in the extreme case of $\rho = 1$ (s is perfectly accurate), where the announcement provides additional information *only* to uninformed investors. Naturally, the announcement is more effective in reducing information asymmetry between the two groups of investors the more informative it is, that is, the better informed the manipulator is (higher ρ_M) and the more truthful his announcement is.

The latter means that the fraction of informed investors increases with the frequency of manipulation. From Theorem 3, we know that the frequency of manipulation is higher when P_0 is high or low. Therefore, the purchase of information increases when P_0 deviates significantly from intermediate values (see Panels A and B of Figure 13 in Section 5.1). This phenomenon may look like investors react to large date 0 price movements (relative to prior belief of $\frac{1}{2}$) by purchasing more information because they believe the price movement was informationally driven, in which case investors would purchase information to learn more about the news behind the price movement. However, in this model investors are simply adjusting their information acquisition decisions in response to (non-informationally driven) price movements in order to compensate for the anticipated changes in the quality of information provided by the manipulator.

4.4 Price Efficiency and Risk Premium

We just saw how the cost and precision of information, and the presence of a manipulator influence the fraction of informed investors. Now, we turn our attention to how these factors influence the price efficiency and risk premium when the fraction of informed investors is determined endogenously. The answer is not obvious since informative announcements decrease the fraction of informed investors which may deteriorate price efficiency and increase the required risk premium.

Both price efficiency and the risk premium are computed based on the beliefs of the representative investor. Comparing expressions (2) and (3) we can obtain these beliefs as

$$p^R_{(s,a,-)} = \frac{1}{1 + \left(\frac{1-p^I_{(s,a,-)}}{p^I_{(s,a,-)}}\right)^{\lambda} \left(\frac{1-p^U_{(s,a,-)}}{p^U_{(s,a,-)}}\right)^{1-\lambda}}.$$

In general, the following holds:

[30] In a previous version of the paper with mean-variance preferences, the announcement could benefit informed investors relative to uninformed investors, leading to an increase in λ. The reason for this odd result is that with mean-variance preferences the information extracted by uninformed investors from prices depended on the announcement made, which is not the case with CARA preferences. Specifically, uninformed investors believed that $s = H$ ($s = L$) was more (less) likely to have occurred when $a = H$ than when $a = L$. This meant that uninformed investors could become confused by the announcement. If the announcement were not informative enough to compensate for its negative impact on the quality of information extracted from prices, uninformed investors would benefit less than informed investors from the announcement, and λ would increase. This happened when uninformed investors extracted a significant amount of information from the equilibrium price (large ρ and small c and σ_z^2) and the informativeness of the manipulator's announcement (function of ρ_M and the frequency of manipulation) was not too large.

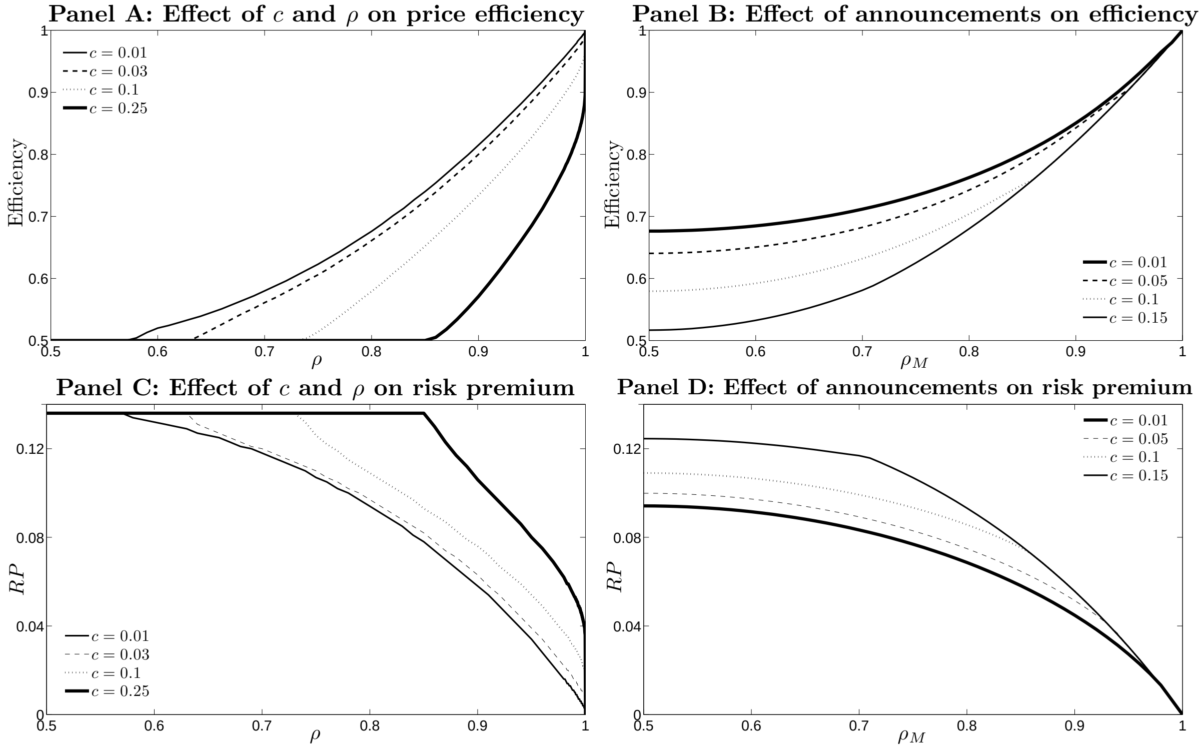

Figure 12: Effect of announcements, cost and accuracy of investors' information on price efficiency and risk premium. Panel A plots the price efficiency as a function of the accuracy of investors' information (ρ), for 4 levels of the cost of their information (c), when there is no manipulator. Panel B plots the price efficiency as a function of the accuracy of manipulator's information (ρ_M), when the manipulator announces truthfully. Panels C and D plot the same, but for the unconditional risk premium. When $\rho_M = \frac{1}{2}$ announcements are uninformative and it is the same as if no announcement is made. Similar results are obtained if the manipulator follows different announcement strategies (TA, UM or DM in Panels A and C, UM or DM on Panels B and D). In all panels the parametrization is as follows: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\delta = 1$, $k = 0$. In Panels B and D $\rho = 0.8$.

(i) Unconditional price efficiency increases in ρ and decreases in c, except when $\lambda = 0$;

(ii) Unconditional price efficiency increases with the presence of a manipulator, and with the informativeness of his announcement (i.e. with ρ_M and the truthfulness of the announcement);

(iii) The opposite of (i) and (ii) holds for the unconditional risk premium.

These results are identical to those we obtained in the previous section when λ was set exogenously to 1. Price efficiency improves (and the risk premium decreases) whenever signal s becomes more accurate, and the manipulator's announcement becomes more informative, even though that implies a decrease in the fraction of informed investors. This shows that the market as a whole is better informed even though *less* investors choose to become informed.[31] Figure 12 illustrates the results.

[31] In the previous version of the paper with mean-variance preferences, price efficiency could deteriorate, and the risk premium increase, with the presence of the manipulator. This was a consequence of the fact that the manipulator's announcement confused uninformed investors. In some circumstances, this resulted in a deterioration of uninformed investors' beliefs and of those of the market as a whole.

5 Extensions

In this section I consider two extensions to the model. In the first extension I endogenize P_0. The objective is to obtain the price function $P_0(z_0)$. With $P_0(z_0)$ and the likelihood of z_0 in hand, I can then determine which types of announcement strategies, which depend on $P_0(z_0)$, are more likely. In particular, I will be able to determine whether ex-ante the manipulator benefits from trading against better informed investors, as conjectured in Section 3.3, and whether the manipulator is indeed biased toward upward manipulation, as suggested in Corollary 8.

In the second extension, I consider a sequential version of the model, where investors take their decision to purchase information after observing the announcement, highlighting the differences to the simultaneous model considered up to this point.

5.1 Endogenizing P_0

To endogenize P_0, I consider that there are two generations of investors who invest for only one period of time. The initial generation opens a position in the risky asset at date 0 and liquidates it at date 1, whereas the second generation takes a position in the asset at date 1 and holds it until date 2 when the risky asset is liquidated. The second generation corresponds exactly to those investors considered in the previous sections.

The optimal date 0 demand of first-generation investors is determined, similarly to that of second-generation investors, by solving

$$\begin{aligned} \max_{X_0} \mathbb{E}\left[U\left(W_1\right)\middle|\,\mathcal{F}_0\right] &= \mathbb{E}\left(-e^{-\alpha W_1}\middle|\,\mathcal{F}_0\right) \\ s.t.\quad & W_1 = W_0 + X_0\left(P_1 - P_0\right). \end{aligned}$$

From equation (2), we know that $P_1(s, a, z_1)$ is a non-normally distributed random variable. Therefore, it is not possible to determine the optimal demand function associated to the above optimization problem in closed form. I have to resort to numerical integration and differentiation to solve the optimization problem and determine the optimal demand for a given price P_0.

First-generation investors are fully rational and know the structure of the economy. Therefore, when determining their demand, first-generation investors take into account the impact of P_0 on the manipulator's announcement strategy, and the impact of the latter on the decision to purchase information by second-generation investors and date 1 prices. The optimal date 0 price is determined by market clearing. The asset's supply at any date is i.i.d normal $N\left(\overline{z}, \sigma_z^2\right)$.

Panel A of Figure 13 plots the optimal price function $P_0(z_0)$, comparing it with the price function that would obtain if we considered myopic long-run investors instead.[32] The shape of the two price functions is the same, but the assumption of short-run investors results in less volatile prices since these investors face less risk.

Panel B plots the fraction of informed investors and the frequency of manipulation as a function

[32] The price function with myopic investors is given by equation (2) with $p = \frac{1}{2}$

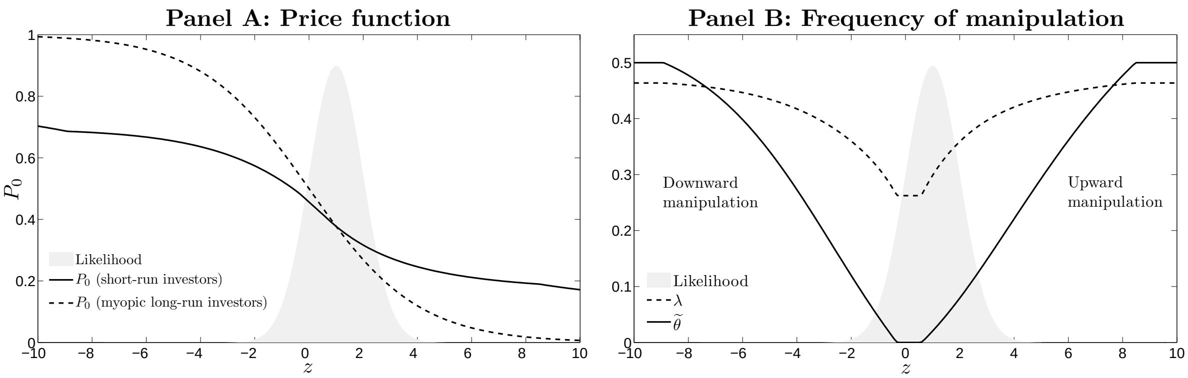

Figure 13: Endogenous P_0 and the likelihood of announcement strategies. Panel A plots the optimal P_0 with short-run investors and the P_0 with long-run myopic investors as a function of the asset's supply (z). Panel B plots the probability of manipulation and the fraction of informed investors (λ) as a function of z, when P_0 is determined endogenously (assuming short-run investors). In all panels it is plotted the likelihood associated with each z. The parametrization used in all panels is: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\delta = 1$, $\rho = 0.8$ and $\rho_M = 0.9$.

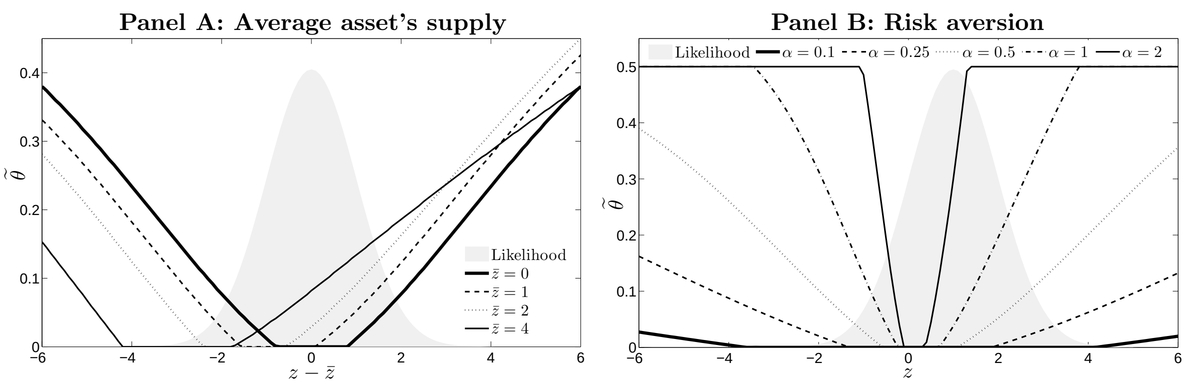

Figure 14: Effect of asset's average supply and risk aversion on the bias toward upward manipulation. Panel A shows how the frequency of manipulation changes with the average net supply ($\overline{z}$) and Panel B how it changes with the coefficient of risk aversion (α). In Panel A (Panel B) it is plotted the likelihood associated with each z ($z - \overline{z}$). The base parametrization used in all panels is: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\delta = 1$, $\rho = 0.8$, $\rho_M = 0.9$ and $c = 0$.

of z_0, which can be determined after knowing $P_0(z_0)$. We can see that there is indeed a bias toward upward manipulation (higher z_0 implies smaller P_0 which induces upward manipulation) even though there are no short sale constraints ($\delta = 1$). This is due to investors' risk aversion and the existence of an average positive amount of risk ($\bar{z} > 0$), which imply a positive average risk premium. The risk reduction that occurs with the release of information at date 1 then reduces the risk premium, pushing prices up. This creates an incentive for the risk neutral manipulator to take a long position at date 0, similarly to what a short sale constraint would do, which ultimately leads to upward manipulation. Panels A and B of Figure 14 show that the bias toward upward manipulation disappears as $\bar{z}$ converges to zero and as risk aversion decreases.

The final question to be answered is whether on average the manipulator benefits from playing against better informed investors. Figure 15 provides the answer. In Panel A we can see that when all investors are informed, the manipulator is in general better off by trading in a market populated by well informed investors. Panel B shows that the manipulator is also in general better off by

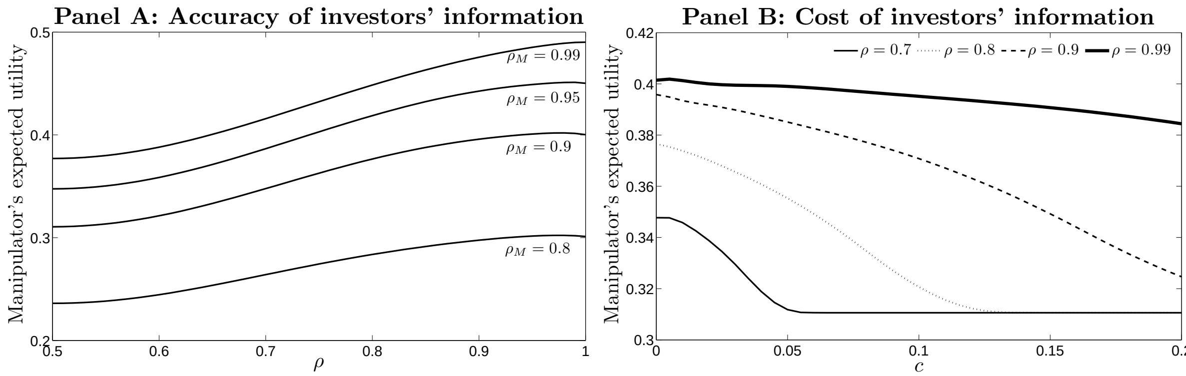

Figure 15: Effect of the cost and accuracy of investors' information on the manipulator's expected utility. This figure shows the manipulator's ex-ante expected utility as a function of the accuracy of investor's information (ρ) when all investors are informed (Panel A) and as a function of the cost of investors' information (c, Panel B). In both panels the parametrization is: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\delta = 1$. In Panel A $c = 0$ and in Panel B $\rho_M = 0.9$.

acting in a market with lower costs of information, which result in a higher fraction of informed investors. In both cases, the exception is when investors are extremely well informed (almost to the point where they learn the liquidation value and the asset becomes riskless) *and* better informed than the manipulator.

5.2 Equilibrium in the Sequential Model

Until now it was assumed that investors' decision to purchase information and the manipulator's announcement occur simultaneously. Here, I will briefly discuss what changes if investors make their decision only after observing the announcement.

For now, assume that $\overline{z} = 0$ and that the manipulator manipulates his announcement upwards (the case of downward manipulation is similar). As we saw in Section 3, when the announcement is manipulated upwards, $a = L$ is always truthful, but $a = H$ is not, which makes the former more informative than the latter. As a result, the marginal benefit of observing s is higher when $a = H$ than when $a = L$ leading to more investors purchasing information when $a = H$ than when $a = L$ in the sequential model. In addition, the fraction of informed investors is larger when $a = H$ than what it would be if investors could not condition their decision on the announcement, since the marginal benefit of observing s conditional on $a = H$ is larger than the unconditional marginal benefit of observing s. Similarly, the fraction of informed investors when $a = L$ is smaller than in the simultaneous model. The smaller fraction of informed investors that obtains in the sequential model when $a = L$ means that $\bar{P}_{(-,L,L)}$ (the price conditional on $a = L$ and $s_M = L$) is not as small as in the simultaneous model; ceteris paribus, this decreases the incentive to announce truthfully when $s_M = L$. However, the larger fraction of informed investors when $a = H$ implies that $\bar{P}_{(-,H,L)}$ is not as large as in the simultaneous model which, ceteris paribus, decreases the incentive to manipulate when $s_M = L$. In general, the second effect dominates and manipulation (both upward and downward) is less frequent in the sequential model than in the simultaneous model. That is,

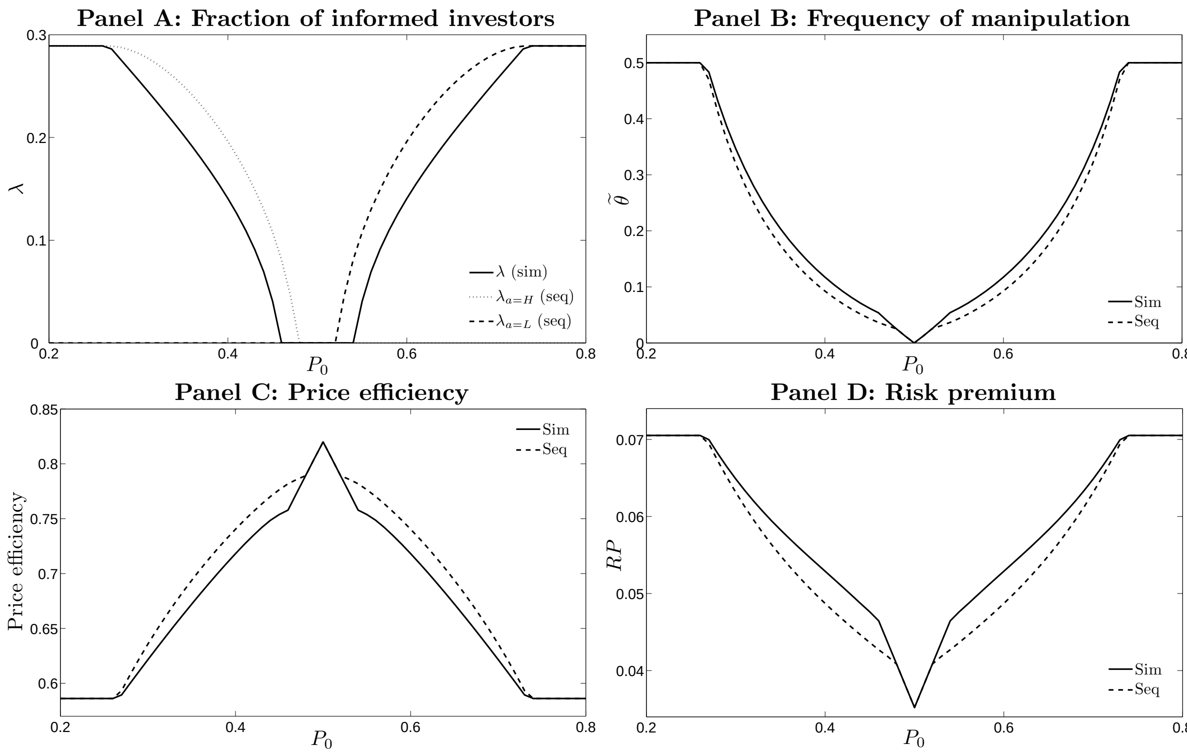

Figure 16: Sequential vs. simultaneous model with $\overline{z} = 0$. This figure compares the fraction of informed investors (λ, Panel A), the frequency of manipulation ($\widetilde{\theta}$, Panel B), the price efficiency (Panel C) and risk premium (Panel D) in the sequential model, where investors decide whether to purchase additional information after observing the announcement, with those in the simultaneous model, where investors purchase information and observe the announcement simultaneously, for a range of P_0 values . The parametrization used in all panels is: $\alpha = 0.5$, $\bar{z} = 0$, $\sigma_z = 1$, $\delta = 1$, $k = 0$, $\rho = 0.8$, $\rho_M = 0.9$, $c = 0.1$.

the investors' flexibility to purchase more information when most needed helps the manipulator to commit to announce more truthfully, which has a positive impact on price efficiency and reduces the risk premium. Figure 16 illustrates these results.

Dropping the assumption that $\overline{z} = 0$, we find that investors are biased toward purchasing information when $a = H$ ($a = L$) if $\overline{z} > 0$ ($\overline{z} < 0$). To understand why this occurs, notice that the informational advantage of informed investors over uninformed investors is highest when the asset's supply misleads uninformed investors, for this is when informed investors can trade aggressively with minimal leakage of their information.[33] For example, if $s = H$, a very large z will mislead uninformed investors into believing that $s = L$ and informed investors can take large long positions without worrying so much about information leakage. What happens when $\overline{z} > 0$ is that the realization of a very large z, which boosts the informational advantage of informed investors when $s = H$, is more likely than the realization of a very small z, which benefits informed investors

[33]Uninformed investors have to assess the likelihood of the only two possible scenarios: $\left\{p^I_{(H,a,-)}, z_{(H,a,-)}\right\}$ and $\left\{p^I_{(L,a,-)}, z_{(L,a,-)}\right\}$. If z is far into the right (left) tail of the distribution, then $z_{(L,a,-)}$ will be more (less) likely than $z_{(H,a,-)}$, regardless of which one is the actual realization. In contrast, when the true z is in the middle of the distribution, the value of z in the wrong scenario falls farther into the tails, making the wrong scenario less likely than the realized one.

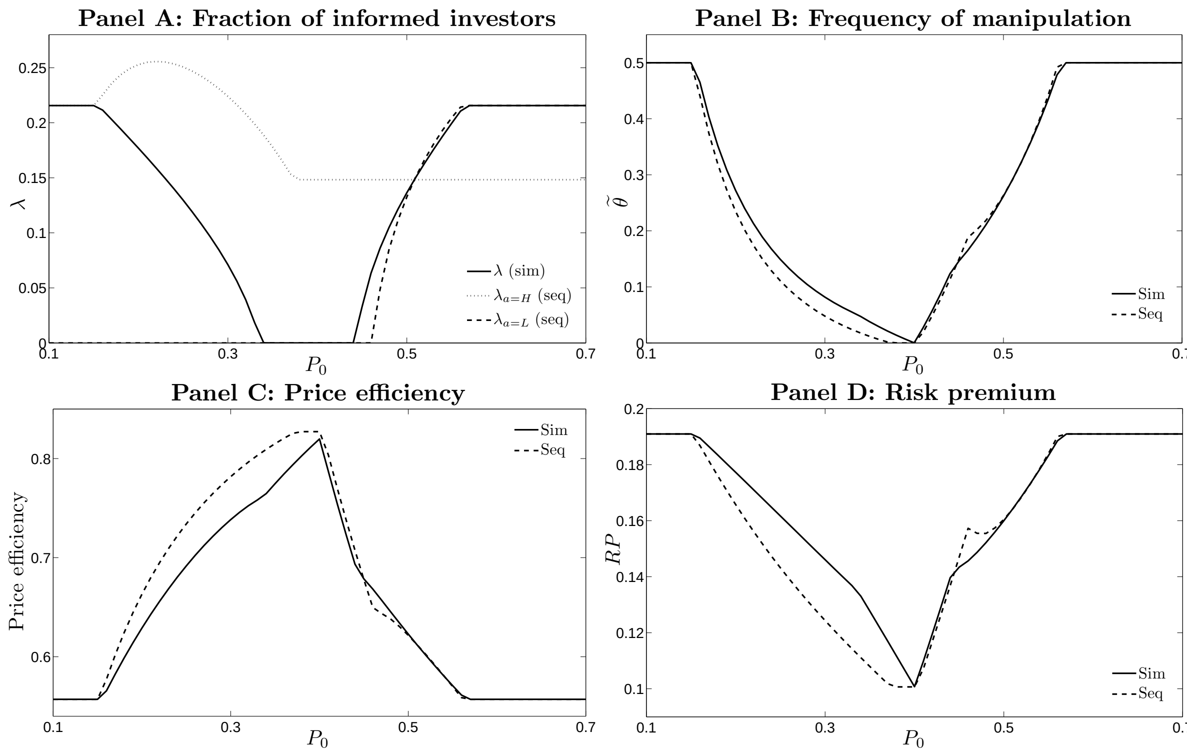

Figure 17: Sequential vs. simultaneous model with $\bar{z} > 0$. This figure compares the fraction of informed investors (λ, Panel A), the frequency of manipulation ($\widetilde{\theta}$, Panel B), the price efficiency (Panel C) and risk premium (Panel D) in the sequential model with those in the simultaneous model for a range of P_0 values. The parametrization used in all panels is: $\alpha = 0.5$, $\bar{z} = 2$, $\sigma_z = 1$, $\delta = 1$, $k = 0$, $\rho = 0.8$, $\rho_M = 0.9$, $c = 0.1$.

when $s = L$. As a result, when $\bar{z} > 0$ investors prefer to become informed when they expect to observe $s = H$. This is precisely what happens when investors observe $a = H$: for any informative announcement strategy, $s_M = H$ is more likely than $s_M = L$ when $a = H$; and $s = H$ is more likely than $s = L$ when $s_M = H$, provided that both s and s_M are informative.

Figure 17 replicates Figure 16 with $\bar{z} = 2$ illustrating the impact of $\bar{z} > 0$ in the fraction of informed investors (Panel A), frequency of manipulation (Panel B), price efficiency (Panel C) and risk premium (Panel D). The first thing to notice is that, even in the simultaneous model, there is an asymmetry in the response of all these four variables to changes in P_0. Negative deviations of P_0 from the unique value that supports a truthful strategy when no investor is informed result in a smaller fraction of informed investors, frequency of manipulation and risk premium, and a larger price efficiency than positive deviations of the same magnitude. These asymmetries are attributed to the strict concavity of the price function with respect to the probability p when $z > 0$.[34] In

[34] For simplicity, suppose that no investor observes s, and set $\delta = 1$. From equation (2) we obtain

$$\frac{\partial^2 P(s, a, z)}{\partial p^2_{(s,a,-)}} = \frac{2e^{\alpha z}\left(e^{\alpha z} - 1\right)}{\left[1 + p_{(s,a,-)}\left(e^{\alpha z} - 1\right)\right]^3} > 0 \Leftrightarrow z > 0.$$

Let P_0^* be the price that supports $\theta^*_{H|H} = \theta^*_{L|L} = 1$, in which case $p^*_{(-,H,-)} = 1 - p^*_{(-,L,-)} = \rho_M$. Consider $P_0 = P_0^- < P_0^*$ such that $\theta^-_{H|H} = 1$ and $\theta^-_{L|L} = \omega$. Then $p^-_{(-,L,-)} = 1 - \rho_M$ and, from equation (5), $p^-_{(-,H,-)} = \rho_M - \frac{(1-\omega)(2\rho_M - 1)}{2-\omega} < \rho_M$. Now suppose that $P_0 = P_0^+ = 2P_0^* - P_0^- > P_0^*$ supports $\theta^+_{H|H} = \omega$ and $\theta^+_{L|L} = 1$, in which the sequential model, the bias toward purchasing information when $a = H$, evident in Panel A, exacerbates these asymmetries. As a result, upward manipulation is less frequent in the sequential model than in the simultaneous model, but the same does not necessarily apply to downward manipulation; and price efficiency (risk premium) is larger (smaller) in the sequential model than in the simultaneous model when the announcement is manipulated upwards, but not necessarily when the announcement is manipulated downwards.

Panel A also shows that, unlike in the simultaneous model, in the sequential model more informative announcements can lead to a larger fraction of informed investors. In the case of $\bar{z} > 0$ this occurs when $a = H$ and the announcement is manipulated upwards. As we saw in the context of the simultaneous model, more informative announcements benefit more uninformed investors, since the marginal information content of the announcement is higher for these investors. This effect by itself contributes to a negative relation between announcement informativeness and the fraction of informed investors. However, in the sequential model, the announcement provides a signal for s that investors will use when deciding whether to purchase information or not; the more informative the announcement, the more accurately it signals s. In this case, $a = H$ becomes a stronger signal for $s = H$ as the informativeness of the announcement improves in an upward manipulation equilibrium. As we just saw, investors prefer to become informed when they expect to observe $s = H$ because $\bar{z} > 0$. Therefore, ceteris paribus the expected benefit of being informed conditional on $a = H$ increases as the announcement is manipulated upwards less frequently. The latter effect is stronger and dominates when ρ_M is small and the announcement's informativeness is low.

6 Conclusion

In this paper I have shown the crucial role of investors' access to additional sources of information in providing the commitment device the manipulator needs to announce truthfully. Without this independent source of information, prices depend exclusively on the announcement, and the manipulator has no incentive to be truthful. Prices need to depend also on other sources of information for the manipulator to have an incentive to align his announcement with those sources of information and, ultimately, with his information.

The existence of this and other commitment devices is in the manipulator's best interest. This is so because investors are not mislead by his manipulation, and so manipulation reduces the ability of the manipulator to influence prices and, consequently, his profits. Manipulation only occurs because the manipulator cannot credibly commit to announce truthfully. Therefore, the manipulator

case $p^+_{(-,H,-)} = \rho_M$ and $p^+_{(-,L,-)} = 1 - \rho_M + \frac{(1-\omega)(2\rho_M - 1)}{2-\omega} > 1 - \rho_M$. For a variable v, define $\Delta v^- \equiv v^* - v^-$ and $\Delta v^+ \equiv v^* - v^+$. Then we have that $\Delta p^+_{(-,L,-)} = -\Delta p^-_{(-,H,-)} = \frac{(1-\omega)(2\rho_M - 1)}{2-\omega}$, $\Delta p^+_{(-,H,-)} = \Delta p^-_{(-,L,-)} = 0$. In turn, this implies that $\Delta \bar{P}^-_{(-,L,-)} = \Delta \bar{P}^+_{(-H,-)} = 0$ and, given the strict concavity of $P(s, a, z)$ with respect to $p_{(s,a,-)}$, that $\Delta \bar{P}^-_{(-,H,-)} < -\Delta \bar{P}^+_{(-,L,-)} < 0$. Therefore, if $\bar{P}^*_{(-,H,-)} - P^*_0 = P^*_0 - \bar{P}^*_{(-,L,-)}$ and $\bar{P}^-_{(-,H,-)} - P^-_0 = P^-_0 - \bar{P}^-_{(-,L,-)}$, which is the case in equilibrium, then it must also be the case that $\bar{P}^+_{(-,H,-)} - P^+_0 < P^+_0 - \bar{P}^+_{(-,L,-)}$, which contradicts the initial assumption. As a consequence, the manipulator will deviate from $\theta^+_{H|H} = \omega$ since $a = L$ is strictly preferred to $a = H$. To attain an equilibrium it is necessary that $\theta^+_{H|H} < \omega$ so that $\bar{P}^+_{(-,L,-)}$ increases and $\bar{P}^+_{(-,H,-)} - P^+_0 = P^+_0 - \bar{P}^+_{(-,L,-)}$.

actually prefers to operate in markets where investors are better informed and manipulation is prosecuted more vigorously. However, if there is the risk of being penalized simply for a poor track record of recommendations, the manipulator may be better off by not announcing.

These results suggest that, on paper, the 2003 GRAS was an adequate medicine to treat the conflict of interests faced by affiliated analysts, but with big risks of side effects. Empirical evidence brought by Clarke et al. (2009) and Kadan et al. (2009) suggests that the patient suffered the side effects: the overall informativeness of recommendations both by affiliated and independent analysts decreased in the post-GRAS period. Moreover, there was a widespread move from a five-tier to a three-tier rating system by investment banks. This indicates that investment banks are now more fearful of ill-founded prosecution in case of honest mistakes, being the move to a three-tier rating system a defensive one. The result of the 2003 GRAS was less biased, but also less informative, announcements.

Even more worrying is the decrease in the informativeness of the recommendation by independent analysts. Possible explanations for this are: the suspicion of the real independency of independent analysts funded by investment banks, specially when those reports are distributed together with the research reports produced by those same investment banks; and the entry of mediocre research firms into the market financed by the funds made available by the GRAS, which caused a dilution of the overall quality of independent research. In hindsight, it appears that the GRAS should have aimed at improving public awareness directly through investor education, which received a mere 5.7% of the total settlement.

Appendix

A Solving for the Equilibrium without Information Acquisition

Here I provide details on how to compute the equilibrium when all investors are informed. The probability investors assign to V_H given the signal s and announcement a they observe, $p^I_{(s,a,-)}$, is straightforward to obtain from the event tree of Figure 2. These probabilities are given by

$$
\begin{aligned}
p^I_{(H,a,-)} &= \rho \frac{\theta_{a|H}\rho_M + \theta_{a|L}(1-\rho_M)}{2\gamma_{(H,a,-)}}, &(5)\\
p^I_{(L,a,-)} &= (1-\rho) \frac{\theta_{a|H}\rho_M + \theta_{a|L}(1-\rho_M)}{2\gamma_{(L,a,-)}}, &(6)
\end{aligned}
$$

for $a \in \{H, L, N\}$, where

$$
\begin{aligned}
\gamma_{(H,a,-)} &= \frac{\rho\left[\theta_{a|H}\rho_M + \theta_{a|L}(1-\rho_M)\right] + (1-\rho)\left[\theta_{a|H}(1-\rho_M) + \theta_{a|L}\rho_M\right]}{2},\\
\gamma_{(L,a,-)} &= \frac{\rho\left[\theta_{a|L}\rho_M + \theta_{a|H}(1-\rho_M)\right] + (1-\rho)\left[\theta_{a|L}(1-\rho_M) + \theta_{a|H}\rho_M\right]}{2},
\end{aligned}
$$

denote the ex-ante probability of the information state $(s, a, -)$.

With these probabilities in hand, we can use the expression (2) to obtain the expected equilibrium price in the information scenario $(s, a, -)$, denoted $\bar{P}_{(s,a,-)} \equiv \mathbb{E}(P|\, s, a)$. Note that there is no closed form solution for these expected prices. Therefore, I compute them numerically via Gauss-Hermite quadrature with 15 nodes.

Since prices are independent of s_M, the expected price in the information state $(-, a, s_M)$, necessary to compute the manipulator's expected utility, is obtained from $\bar{P}_{(s,a,-)}$ as follows:

$$
\bar{P}_{(-,a,s_M)} \equiv \mathbb{E}(P|\, a, s_M) = \mathbb{P}(s = H|\, s_M)\,\bar{P}_{(H,a,-)} + \left[1 - \mathbb{P}(s = H|\, s_M)\right]\bar{P}_{(L,a,-)} \qquad (7)
$$

where

$$
\mathbb{P}(s = H|\, s_M = H) = 1 - \mathbb{P}(s = H|\, s_M = L) = \rho\rho_M + (1-\rho)(1-\rho_M) \geq \frac{1}{2}.
$$

B Solving for $\hat{\gamma}$

Uninformed investors observe neither s nor z, but they observe P. From the price function (3), they determine that

$$
P = \frac{1}{1 + \left(\frac{1-p^I_{(s,a,-)}}{p^I_{(s,a,-)}}\right)^{\lambda}\left(\frac{1-p^U}{p^U}\right)^{1-\lambda} e^{\alpha z_{(s,a,-)}}} \qquad (8)
$$

and, inverting for $z_{(s,a,-)}$,

$$
z_{(s,a,-)} = \frac{1}{\alpha}\ln\left[\frac{1-P}{P}\left(\frac{p^I_{(s,a,-)}}{1-p^I_{(s,a,-)}}\right)^{\lambda}\left(\frac{1-p^U}{p^U}\right)^{1-\lambda}\right].
$$

In the expression above consider that $s=H$. Substituting P for equation (8) with $s=L$ and simplifying we obtain

$$z_{(L,a,-)}=z_{(H,a,-)}-\frac{\lambda}{\alpha}\ln\frac{\left(1-p^I_{(L,a,-)}\right)p^I_{(H,a,-)}}{p^I_{(L,a,-)}\left(1-p^I_{(H,a,-)}\right)}.$$

Using the definitions of $p^I_{(L,a,-)}$ and $p^I_{(H,a,-)}$, given by equations (5) and (6), we can simplify the expression further to obtain

$$z_{(L,a,-)}=z_{(H,a,-)}+\frac{2\lambda}{\alpha}\ln\frac{1-\rho}{\rho}. \tag{9}$$

Starting from equation (4), writing the normal density $\phi(\cdot)$ explicitly and simplifying, the expression for $\hat{\gamma}$ becomes

$$\hat{\gamma}=\frac{1}{1+\exp\left[\frac{\left(z_{(H,a,-)}-\overline{z}\right)^2-\left(z_{(L,a,-)}-\overline{z}\right)^2}{2\sigma_z^2}\right]}. \tag{10}$$

If $s=H$, then $z_{(H,a,-)}=z$, the realized supply. Using equation (9) to substitute for $z_{(L,a,-)}$ in the expression above, we finally obtain the probability uninformed investors assign to informed investors observing $s=H$ when effectively $s=H$ as a function of z as

$$\hat{\gamma}_{(H,-,-)}(z)=\frac{1}{\exp\left[\frac{(z-\overline{z})^2-\left(z-\overline{z}+\frac{2\lambda}{\alpha}\ln\frac{1-\rho}{\rho}\right)^2}{2\sigma_z^2}\right]+1}.$$

and, similarly, the probability of investors observing $s=H$ when in fact they observed $s=L$ as

$$\hat{\gamma}_{(L,-,-)}(z)=\frac{1}{\exp\left[\frac{\left(z-\overline{z}-\frac{2\lambda}{\alpha}\ln\frac{1-\rho}{\rho}\right)^2-(z-\overline{z})^2}{2\sigma_z^2}\right]+1}.$$

C Proofs

The following lemmas collects some results necessary to prove the theorems.

Lemma 12. The partial derivative of $\bar{P}_{(s,a,-)}$ with respect to ρ, ρ_M, $\theta_{a|s_M}$ has the same sign of the partial derivative of $p^I_{(s,a,-)}$ with respect to the same variables.

Proof. Taking the derivative of $\bar{P}_{(s,a,-)}$ with respect to $p^I_{(s,a,-)}$, we obtain that

$$\frac{\partial\bar{P}_{(s,a,-)}}{\partial p^I_{(s,a,-)}}=\mathbb{E}\left(\left.\frac{\partial P(a,s,z)}{\partial p^I_{(s,a,-)}}\right|s,a\right)=\mathbb{E}\left(\left.\frac{e^{z\alpha}}{\left[p^I_{(s,a,-)}+\left(1-p^I_{(s,a,-)}\right)e^{z\alpha}\right]^2}\right|s,a\right)>0. \tag{11}$$

□

Define the operator $sign(.)$ as

$$sign(x)=\begin{cases}1 & if\,x>0\\ 0 & if\,x=0\\ -1 & if\,x<0\end{cases}.$$

Lemma 13. *When all investors observe the signal s ($\lambda = 1$), $\frac{1}{2} < \rho < 1$ and $\frac{1}{2} < \rho_M < 1$, then the following holds for $s \in \{H, L\}$, $a \in \{H, L, N\}$ and $s_M \in \{H, L\}$:*

(i) $\bar{P}_{(H,a,-)} > \bar{P}_{(L,a,-)}$ and $\bar{P}_{(-,a,H)} > \bar{P}_{(-,a,L)}$;

(ii) sign $\left(\bar{P}_{(s,H,-)} - \bar{P}_{(s,N,-)}\right) =$ sign $\left(\bar{P}_{(-,H,s_M)} - \bar{P}_{(-,N,s_M)}\right) =$ sign $\left(\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}\right)$;

(iii) sign $\left(\bar{P}_{(s,N,-)} - \bar{P}_{(s,L,-)}\right) =$ sign $\left(\bar{P}_{(-,N,s_M)} - \bar{P}_{(-,L,s_M)}\right) =$ sign $\left(\frac{\theta_{N|H}}{\theta_{N|L}} - \frac{\theta_{L|H}}{\theta_{L|L}}\right)$;

(iv) $\begin{cases} \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} = 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|H}} = 0 & if\, \theta_{a|L} = 0 \vee \rho = 1 \vee \rho = \frac{1}{2} \vee \rho_M = \frac{1}{2} \\ \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} > 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|H}} > 0 & otherwise \end{cases}$ *and*

$\begin{cases} \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|L}} = 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|L}} = 0 & if\, \theta_{a|H} = 0 \vee \rho = 1 \vee \rho = \frac{1}{2} \vee \rho_M = \frac{1}{2} \\ \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|L}} < 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|L}} > 0 & otherwise \end{cases}$;

(v) $\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} = (>)\, \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$ if $\bar{z} = 0$ $(\bar{z} > 0)$;

(vi) $\bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} = (>)\, \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$ if $\bar{z} = 0$ $\left(\bar{z} > 0 \wedge \rho > \frac{1}{2} \wedge \rho_M > \frac{1}{2}\right)$.

If one of the ratios $\frac{\theta_{a|H}}{\theta_{a|L}}$ is undefined because a is a zero-probability event, then it is considered to be equal to 1 (equally likely trembles).

Proof. **(i)** Making use of Lemma 12, to show that $\bar{P}_{(H,a,-)} > \bar{P}_{(L,a,-)}$ it suffices to show that $p^I_{(H,a,-)} > p^I_{(L,a,-)}$. Using the definitions of $p^I_{(H,a,-)}$ and $p^I_{(L,a,-)}$, equations (5) and (6), we can write $p^I_{(H,a,-)} - p^I_{(L,a,-)}$ as

$$p^I_{(H,a,-)} - p^I_{(L,a,-)} = \frac{(2\rho - 1)\left[\theta_{a|H}(1-\rho_M) + \theta_{a|L}\right]\left[\theta_{a|L}(1-\rho_M) + \theta_{a|H}\right]}{\left[\theta_{a|H} + \left(\theta_{a|L} - \theta_{a|H}\right)(\rho + \rho_M - 2\rho\rho_M)\right]\left[\theta_{a|L} + \left(\theta_{a|H} - \theta_{a|L}\right)(\rho + \rho_M - 2\rho\rho_M)\right]}.$$

Since $\frac{1}{2} < \rho < 1$, $\frac{1}{2} < \rho_M < 1$ and $\theta_{a|s_M} > 0$, the numerator is positive. In the denominator, $\frac{1}{2} < \rho + \rho_M - 2\rho\rho_M < 1$. Regarding the denominator, if $\theta_{a|L} - \theta_{a|H} > 0$, then the first term in the denominator is clearly positive. Under the same assumption, it is straightforward to determine that the second term is also positive, since

$$\theta_{a|L} + \left(\theta_{a|H} - \theta_{a|L}\right)(\rho + \rho_M - 2\rho\rho_M) > \theta_{a|L} + \left(\theta_{a|H} - \theta_{a|L}\right) = \theta_{a|H} > 0.$$

Therefore, $p^I_{(H,a,-)} > p^I_{(L,a,-)}$. If, by the contrary we assume that $\theta_{a|L} - \theta_{a|H} < 0$ it is easy to see that, by symmetry, the denominator is also positive. Hence, $p^I_{(H,a,-)} > p^I_{(L,a,-)}$ and $\bar{P}_{(H,a,-)} > \bar{P}_{(L,a,-)}$.

Finally, using equation (7), we can write $\bar{P}_{(-,a,H)} - \bar{P}_{(-,a,L)}$ as

$$\bar{P}_{(-,a,H)} - \bar{P}_{(-,a,L)} = \left[2\mathbb{P}\left(s = H \mid s_M = H\right) - 1\right]\left(\bar{P}_{(H,a,-)} - \bar{P}_{(L,a,-)}\right)$$

which is positive, since $\mathbb{P}\left(s = H \mid s_M = H\right) > \frac{1}{2}$. This concludes the proof of part (i).

(ii) Again, to show that the sign of $\bar{P}_{(s,H,-)} - \bar{P}_{(s,N,-)}$ equals the sign of $\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}$, it suffices to show that the same holds for the sign of $p^I_{(s,H,-)} - p^I_{(s,N,-)}$. We can write $p^I_{(H,H,-)} - p^I_{(H,N,-)}$ and $p^I_{(L,H,-)} - p^I_{(LN,-)}$ as

$$\begin{aligned} p^I_{(H,H,-)} - p^I_{(H,N,-)} &= \frac{\theta_{H|L}\theta_{N|L}\left(\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}\right)\rho(1-\rho)(2\rho_M - 1)}{\left[\theta_{H|H} + \left(\theta_{H|L} - \theta_{H|H}\right)(\rho + \rho_M - 2\rho\rho_M)\right]\left[\theta_{N|H} + \left(\theta_{N|L} - \theta_{N|H}\right)(\rho + \rho_M - 2\rho\rho_M)\right]} \\ p^I_{(L,H,-)} - p^I_{(L,N,-)} &= \frac{\theta_{H|L}\theta_{N|L}\left(\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}\right)\rho(1-\rho)(2\rho_M - 1)}{\left[\theta_{H|L} + \left(\theta_{H|H} - \theta_{H|L}\right)(\rho + \rho_M - 2\rho\rho_M)\right]\left[\theta_{N|L} + \left(\theta_{N|H} - \theta_{N|L}\right)(\rho + \rho_M - 2\rho\rho_M)\right]}. \end{aligned}$$

The denominators of both expressions are similar to the denominator analyzed in the proof of part (i), and are positive. In turn, the sign of the numerator is the sign of $\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}$, which proves that the sign of $\bar{P}_{(s,H,-)} - \bar{P}_{(s,N,-)}$ equals the sign of $\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}$.

In turn, we can write $\bar{P}_{(-,H,s_M)} - \bar{P}_{(-,N,s_M)}$ as

$$\bar{P}_{(-,H,s_M)} - \bar{P}_{(-,N,s_M)} = \mathbb{P}\left(s = H|\, s_M\right)\left(\bar{P}_{(H,H,-)} - \bar{P}_{(H,N,-)}\right) + \left[1 - \mathbb{P}\left(s = H|\, s_M\right)\right]\left(\bar{P}_{(L,H,-)} - \bar{P}_{(L,N-)}\right)$$

which clearly has the same sign as $\frac{\theta_{H|H}}{\theta_{H|L}} - \frac{\theta_{N|H}}{\theta_{N|L}}$.

(iii) The proof of point (iii) is similar to the proof of point (ii). Simply substitute $s_M = H$ for $s_M = N$ and $s_M = N$ for $s_M = L$.

(iv) To prove that $\frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} > 0$ it suffices to prove that $\frac{\partial p^I_{(s,a,-)}}{\partial \theta_{a|H}} > 0$. These derivatives are given by

$$\begin{aligned}
\frac{\partial p^I_{(H,a,-)}}{\partial \theta_{a|H}} &= \frac{\theta_{a|L}\rho\left(1-\rho\right)\left(2\rho_M - 1\right)}{\left[\theta_{a|H} + \left(\theta_{a|L} - \theta_{a|H}\right)\left(\rho + \rho_M - 2\rho\rho_M\right)\right]^2} \geq 0 \\
\frac{\partial p^I_{(L,a,-)}}{\partial \theta_{a|H}} &= \frac{\theta_{a|L}\rho\left(1-\rho\right)\left(2\rho_M - 1\right)}{\left[\theta_{a|L} + \left(\theta_{a|H} - \theta_{a|L}\right)\left(\rho + \rho_M - 2\rho\rho_M\right)\right]^2} \geq 0.
\end{aligned}$$

It is immediate that $\frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} > 0$ if $\theta_{a|L} > 0$ and $\frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} = 0$ otherwise. Proceeding in the same way for the case of $\frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|L}}$, it is easily obtained that

$$\begin{aligned}
\frac{\partial p^I_{(H,a,-)}}{\partial \theta_{a|L}} &= -\frac{\theta_{a|H}\rho\left(1-\rho\right)\left(2\rho_M - 1\right)}{\left[\theta_{a|H} + \left(\theta_{a|L} - \theta_{a|H}\right)\left(\rho + \rho_M - 2\rho\rho_M\right)\right]^2} \leq 0 \\
\frac{\partial p^I_{(L,a,-)}}{\partial \theta_{a|L}} &= -\frac{\theta_{a|H}\rho\left(1-\rho\right)\left(2\rho_M - 1\right)}{\left[\theta_{a|L} + \left(\theta_{a|H} - \theta_{a|L}\right)\left(\rho + \rho_M - 2\rho\rho_M\right)\right]^2} \leq 0
\end{aligned}$$

which proves that $\frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|L}} < 0$ if $\theta_{a|H} > 0$ and $\frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} = 0$ otherwise.

Finally, since $\bar{P}_{(-,a,s_M)}$ is a weighted average of $\bar{P}_{(H,a,-)}$ and $\bar{P}_{(L,a,-)}$ (with weights independent of $\theta_{a|s_M}$), it follows immediately that the derivatives of $\bar{P}_{(-,a,s_M)}$ with respect to $\theta_{a|s_M}$ have the same sign of the derivatives of $\bar{P}_{(H,a,-)}$ and $\bar{P}_{(L,a,-)}$.

(v) Using equation (7), $\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)}$ and $\bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$ are given by

$$\begin{aligned}
\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} &= \mathbb{P}\left(s = H|\, s_M = H\right)\left(\bar{P}^T_{(H,H,-)} - \bar{P}^N_{(H,-,-)}\right) \\
&\quad + \left[1 - \mathbb{P}\left(s = H|\, s_M = H\right)\right]\left(\bar{P}^T_{(L,H,-)} - \bar{P}^N_{(L,-,-)}\right) \\
\bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} &= \mathbb{P}\left(s = H|\, s_M = H\right)\left(\bar{P}^N_{(L,-,-)} - \bar{P}^T_{(L,L,-)}\right) \\
&\quad + \left[1 - \mathbb{P}\left(s = H|\, s_M = H\right)\right]\left(\bar{P}^N_{(H,-,-)} - \bar{P}^T_{(H,L,-)}\right).
\end{aligned}$$

Let $p^T_{(s,a,-)}$ and $p^N_{(s,-,-)}$ denote the probability of V_H that are associated to $\bar{P}^T_{(s,a,-)}$ and $\bar{P}^N_{(s,-,-)}$. The former probability is obtained by setting $\theta_{H|H} = \theta_{L|L} = 1$ (announcement is truthful) and the latter by setting $\theta_{H|H}=\theta_{L|L}=\frac{1}{2}$ (announcement is uninformative),

$$p^T_{(H,H,-)}=\frac{\rho\rho_M}{1-(\rho+\rho_M-2\rho\rho_M)},\ p^T_{(H,L,-)}=\frac{\rho\left(1-\rho_M\right)}{\rho+\rho_M-2\rho\rho_M},\ p^N_{(H,-,-)}=\rho,$$
$$p^T_{(L,L,-)}=\frac{(1-\rho)\left(1-\rho_M\right)}{1-(\rho+\rho_M-2\rho\rho_M)},\ p^T_{(L,H,-)}=\frac{(1-\rho)\,\rho_M}{\rho+\rho_M-2\rho\rho_M},\ p^N_{(L,-,-)}=1-\rho.$$

It follows that $p^T_{(H,H,-)}+p^T_{(L,L,-)}=p^T_{(L,H,-)}+p^T_{(H,L,-)}=p^N_{(H,-,-)}+p^N_{(L,-,-)}=1$.

We can write $P^T_{(s,a,-)}(z)$ as

$$P^T_{(s,a,-)}(z)\quad=\quad\Psi\left(-z,\frac{1}{\alpha}\ln\frac{1-p^T_{(s,a,-)}}{p^T_{(s,a,-)}},\frac{1}{\alpha}\right)$$

where $\Psi(z,l,s)$ is the CDF of a logistic distribution with location l and scale s. The expression for $P^N_{(s,-,-)}(z)$ is identical, with $p^N_{(s,-,-)}$ in place of $p^T_{(s,a,-)}$.

Since $p^T_{(L,L,-)}=1-p^T_{(H,H,-)}$, we find that

$$\begin{aligned}P^T_{(H,H,-)}(z)&=\Psi\left(-z,\frac{1}{\alpha}\ln\frac{1-p^T_{(H,H,-)}}{p^T_{(H,H,-)}},\frac{1}{\alpha}\right)\\ P^T_{(L,L,-)}(z)&=\Psi\left(-z,-\frac{1}{\alpha}\ln\frac{1-p^T_{(H,H,-)}}{p^T_{(H,H,-)}},\frac{1}{\alpha}\right).\end{aligned}$$

This means that $P^T_{(H,H,-)}(z)$ and $P^T_{(L,L,-)}(z)$ are the CDFs of logistic distributions for $-z$ with the same scale factor but centered at symmetric. Consequently, $P^T_{(H,H,-)}(z)+P^T_{(L,L,-)}(-z)=1$. The same applies for the pairs $P^T_{(L,H,-)}(z)$ and $P^T_{(H,L,-)}(z)$, and $P^N_{(H,-,-)}(z)$ and $P^N_{(L,-,-)}(z)$. Therefore, $P^T_{(H,H,-)}(z)-P^N_{(H,-,-)}(z)=P^N_{(L,-,-)}(-z)-P^T_{(L,L,-)}(-z)$ and $P^T_{(L,H,-)}(z)-P^N_{(L,-,-)}(z)=P^N_{(H,-,-)}(-z)-P^T_{(H,L,-)}(-z)$.

Moreover, we can verify that $P^T_{(H,H,-)}(z)-P^N_{(H,-,-)}(z)$ is a positive, symmetric and unimodal function of z, centered at $-\frac{1}{2\alpha}\left(\ln\frac{1-p^T_{(H,H,-)}}{p^T_{(H,H,-)}}+\ln\frac{1-p^N_{(H,-,-)}}{p^N_{(H,-,-)}}\right)>0$, and likewise $P^T_{(L,H,-)}(z)-P^N_{(L,-,-)}(z)$ is positive, symmetric, unimodal and centered at $-\frac{1}{2\alpha}\left(\ln\frac{1-p^T_{(L,H,-)}}{p^T_{(L,H,-)}}+\ln\frac{1-p^N_{(L,-,-)}}{p^N_{(L,-,-)}}\right)>0$.

Taking expectations, if $\bar{z}=0$, then it is immediate that $\bar{P}^T_{(H,H,-)}-\bar{P}^N_{(H,-,-)}=\bar{P}^N_{(L,-,-)}-\bar{P}^T_{(L,L,-)}$ and $\bar{P}^T_{(L,H,-)}-\bar{P}^N_{(L,-,-)}=\bar{P}^N_{(H,-,-)}-\bar{P}^T_{(H,L,-)}$, since z is as likely as $-z$. As a result, $\bar{P}^T_{(-,H,H)}-\bar{P}^N_{(-,-,H)}=\bar{P}^N_{(-,-,L)}-\bar{P}^T_{(-,L,L)}$.

However, if $\bar{z}>0$, positive values of z are more likely than negative values, and so $\bar{P}^T_{(H,H,-)}-\bar{P}^N_{(H,-,-)}>\bar{P}^N_{(L,-,-)}-\bar{P}^T_{(L,L,-)}$ and $\bar{P}^T_{(L,H,-)}-\bar{P}^N_{(L,-,-)}>\bar{P}^N_{(H,-,-)}-\bar{P}^T_{(H,L,-)}$, since the left (right) hand side differences are centered around positive (negative) values of z. Thus, $\bar{P}^T_{(-,H,H)}-\bar{P}^N_{(-,-,H)}>\bar{P}^N_{(-,-,L)}-\bar{P}^T_{(-,L,L)}$.

(vi) Using equation (7), $\bar{P}^T_{(-,H,H)}-\bar{P}^T_{(-,L,H)}$ and $\bar{P}^T_{(-,H,L)}-\bar{P}^T_{(-,L,L)}$ are given by

$$\begin{aligned}\bar{P}^T_{(-,H,H)}-\bar{P}^T_{(-,L,H)}&=\mathbb{P}\left(s=H|\,s_M=H\right)\left(\bar{P}^T_{(H,H,-)}-\bar{P}^T_{(H,L,-)}\right)\\&\quad+\left[1-\mathbb{P}\left(s=H|\,s_M=H\right)\right]\left(\bar{P}^T_{(L,H,-)}-\bar{P}^T_{(L,L,-)}\right)\\ \bar{P}^T_{(-,H,L)}-\bar{P}^T_{(-,L,L)}&=\mathbb{P}\left(s=H|\,s_M=H\right)\left(\bar{P}^T_{(L,H,-)}-\bar{P}^T_{(L,L,-)}\right)\\&\quad+\left[1-\mathbb{P}\left(s=H|\,s_M=H\right)\right]\left(\bar{P}^T_{(H,H,-)}-\bar{P}^T_{(H,L,-)}\right).\end{aligned}$$

From the proof of the point (v), we know that $P^T_{(H,H,-)}(z) - P^T_{(L,H,-)}(z) = P^T_{(H,L,-)}(-z) - P^T_{(L,L,-)}(-z)$. The term in the left hand side is a positive, symmetric and unimodal function of z, centered at a positive value.

Averaging over z, we obtain that $\bar{P}^T_{(H,H,-)} - \bar{P}^T_{(L,H,-)} = \bar{P}^T_{(H,L,-)} - \bar{P}^T_{(L,L,-)}$ when $\bar{z} = 0$, since z and $-z$ are equally likely); and $\bar{P}^T_{(H,H,-)} - \bar{P}^T_{(L,H,-)} > \bar{P}^T_{(H,L,-)} - \bar{P}^T_{(L,L,-)}$ when $\bar{z} > 0$, since positive values of z are more likely than negative ones and the left hand side of the expression is centered around positive values of z and the left side around negative ones.

If $\bar{z} = 0$, then it is immediate that $\bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} = \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$. We have $\bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} > \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$ if, in addition to $\bar{z} > 0$, $\Pr(s = H | s_M = H) > \frac{1}{2}$. This holds if and only if $\rho > \frac{1}{2}$ and $\rho_M > \frac{1}{2}$. □

C.1 Proof of Theorem 3

Truthful announcement (TA): $P_0 \in [\bar{P}^2, \bar{P}^3]$

A TA strategy is characterized by $\theta_{H|H} = \theta_{L|L} = 1$, which implies that $\frac{\theta_{H|H}}{\theta_{H|L}} = \infty > \frac{\theta_{N|H}}{\theta_{N|L}} = 1 > \frac{\theta_{L|H}}{\theta_{L|L}} = 0$. Using parts (ii) and (iii) of Lemma 13 we then obtain that

$$\bar{P}_{(-,H,s_M)} > \bar{P}_{(-,N,s_M)} > \bar{P}_{(-,L,s_M)},\ s_M \in \{H, L\}. \tag{12}$$

In any informational scenario $(-, a, s_M)$, the manipulator's trading strategy $\mathcal{T}_{(-,a,s_M)}$ is given by

$$\begin{cases} \mathcal{T}_{(-,a,s_M)} = 1 & if\ P_0 \le \bar{P}_{(-,a,s_M)} \\ \mathcal{T}_{(-,a,s_M)} = -\delta & otherwise \end{cases}.$$

The first step is to determine which trading strategies can hold in a TA equilibrium. Let us start by considering that $\mathcal{T}_{(-,H,H)} = -\delta$, which implies that $P_0 > \bar{P}_{(-,H,H)}$. From (12) we know that $\bar{P}_{(-,H,H)} > \bar{P}_{(-,L,H)}$. Therefore, we have $\mathcal{T}_{(-,L,H)} = -\delta$ and $\Pi_{(-,L,H)} > \Pi_{(-,H,H)}$. This means that the manipulator deviates from $\theta_{H|H} = 1$, and so $\mathcal{T}_{(-,H,H)} = -\delta$ cannot hold in a TA equilibrium. Similarly, $\mathcal{T}_{(-,L,L)} = 1$ cannot hold in a TA equilibrium, otherwise the manipulator would deviate from $\theta_{L|L} = 1$.

Therefore, it must be the case that $\mathcal{T}_{(-,H,H)} = 1$ and $\mathcal{T}_{(-,L,L)} = -\delta$, which implies that $\bar{P}_{(-,L,L)} \le P_0 \le \bar{P}_{(-,H,H)}$. In this case, $\theta_{H|H} = 1$ is optimal if and only if $\Pi_{(-,H,H)} \ge \Pi_{(-,L,H)}$ and $\Pi_{(-,H,H)} \ge \Pi_{(-,N,H)}$. It is easy to see that the manipulator deviates from $\theta_{H|H} = 1$ only if P_0 is so high that a short position and the announcement of $a = L$ his more profitable that a long position and the announcement of $a = H$. Therefore, the manipulator does not deviate from $\theta_{H|H} = 1$ if and only if $\bar{P}_{(-,H,H)} - P_0 \ge \delta\left(P_0 - \bar{P}_{(-,L,H)}\right)$. Likewise, the manipulator does not deviate from $\theta_{L|L} = 1$ if and only if $\delta\left(P_0 - \bar{P}_{(-,L,L)}\right) \ge \bar{P}_{(-,H,L)} - P_0$.

Bringing both conditions together and noting that in a TA strategy $\bar{P}_{(-,a,s_M)} = \bar{P}^T_{(-,a,s_M)}$, the TA strategy $\theta_{H|H} = \theta_{L|L} = 1$ is optimal whenever

$$\bar{P}^2 \equiv \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta} \le P_0 \le \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta} \equiv \bar{P}^3.$$

This condition can be satisfied sine, from part (i) of Lemma 13, $\bar{P}_{(-,a,H)} \ge \bar{P}_{(-,a,L)}$.

Upward manipulation (UM): $P_0 \in \left(\bar{P}^1, \bar{P}^2\right)$

Suppose that $P_0 \lessapprox \bar{P}^2$. From what we saw above for the case of a TA strategy, the manipulator will deviate from $\theta_{L|L} = 1$, but not from $\theta_{H|H} = 1$. Suppose he deviates from $\theta_{L|L}$ to $\theta_{H|L}$.[35] Because $\theta_{H|H} = 1$, we have that $\theta_{L|H} = 0$. Thus, from part (iv) of Lemma 13, the decrease in $\theta_{L|L}$ has no impact on $\bar{P}_{(-,L,H)}$ and $\bar{P}_{(-,L,L)}$, whereas the increase in $\theta_{H|L}$ decreases $\bar{P}_{(-,H,H)}$ and $\bar{P}_{(-,H,L)}$. This implies that both $\frac{\bar{P}_{(-,H,L)}+\delta\bar{P}_{(-,L,L)}}{1+\delta}$ and $\frac{\bar{P}_{(-,H,H)}+\delta\bar{P}_{(-,L,H)}}{1+\delta}$ decrease. Therefore, at some point

$$\frac{\bar{P}_{(-,H,L)} + \delta\bar{P}^T_{(-,L,L)}}{1+\delta} = P_0 \leq \frac{\bar{P}_{(-,H,H)} + \delta\bar{P}^T_{(-,L,H)}}{1+\delta}$$

and $\theta_{H|H} = 1$, $\theta_{H|L} = \omega_1$, $\theta_{L|L} = 1-\omega_1$, $\omega_1 \in (0,1)$ is an equilibrium announcement strategy, with associated trading strategy $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,H,L)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$.

The more P_0 decreases, the more ω_1 has to increase for the condition above to be satisfied. As $\omega_1 \uparrow 1$, the announcement becomes uninformative, since the manipulator always announces $a = H$, and so it is effectively the same as not announcing at all. If P_0 decreases enough, the condition no longer holds and the manipulator switches to not announcing (or announcing a completely uninformative announcement). Therefore, $\bar{P}^1 \equiv \frac{\bar{P}^N_{(-,-,L)}+\delta\bar{P}^T_{(-,L,L)}}{1+\delta}$. On the other hand, as $P_0 \uparrow \bar{P}^2$ we have $\omega_1 \downarrow 0$.

Downward manipulation (DM): $P_0 \in \left(\bar{P}^3, \bar{P}^4\right)$

This case is identical to the previous one and is omitted.

Never announces (NA): $P_0 \in \left[0, \bar{P}^1\right] \cup \left[\bar{P}^4, 1\right]$

A NA strategy is characterized by $\theta_{N|H} = \theta_{N|L} = 1$, which implies that $\frac{\theta_{H|H}}{\theta_{H|L}} = \frac{\theta_{N|H}}{\theta_{N|L}} = \frac{\theta_{L|H}}{\theta_{L|L}} = 1$ (trembles are equally likely to occur when $s_M = H$ or $s_M = L$). Then, using parts (ii) and (iii) of Lemma 13 we have

$$\bar{P}_{(-,H,s_M)} = \bar{P}_{(-,N,s_M)} = \bar{P}_{(-,L,s_M)},\ s_M \in \{H, L\}\,. \tag{13}$$

This implies that the manipulator does not deviate from $\theta_{N|H} = \theta_{N|L} = 1$ which is the equilibrium announcement strategy. Notice that the announcement strategy does not depend on P_0, which implies that not announcing is always an equilibrium. However, any equilibrium where the announcement is (at least) partially informative is preferred by the manipulator, since he is able to influence prices in a favorable way. Therefore, when there is no penalty, never announcing is the focal equilibrium only when it is the only equilibrium. P_0 only determines the trading strategy. We have $\mathcal{T}_{(-,N,H)} = 1$ if $P_0 \leq \bar{P}^N_{(-,-,H)} \leq \bar{P}^4$ and $\mathcal{T}_{(-,N,H)} = -\delta$ otherwise; and $\mathcal{T}_{(-,N,L)} = 1$ if $P_0 \leq \bar{P}^1 \leq \bar{P}^N_{(-,-,L)}$ and $\mathcal{T}_{(-,N,L)} = -\delta$ otherwise. Therefore, $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$ if $P_0 \in \left[0, \bar{P}^1\right]$ and $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ if $P_0 \in \left[\bar{P}^4, 1\right]$.

C.2 Proof of Theorem 4

The proof is in everything similar to the proof of Theorem 3. The only two differences introduced by the existence of a penalty are: the manipulator may prefer to never announce instead of manipulating excessively, even if the latter is an equilibrium; and if $a = N$ is a zero-probability event, at some point it is optimal to deviate from lying to $a = N$, which causes a jump in the price associated to $a = N$. To avoid the latter, I focus on strategies where $a = L$ ($a = H$) and $a = N$ are substitutes for signaling $s_M = L$ ($s_M = H$) when P_0 is below (above) some threshold. Equilibria where $a = N$ is never played, if they exist, are equivalent. Without loss of generality, I focus on the case where $a = N$ is played with almost zero probability (i.e. $\varepsilon \gtrapprox 0$ in what follows).

[35] Deviation to $\theta_{N|L}$ will make $a = N$ and $a = L$ substitute signals for $s_M = L$ and will not lead to an equilibrium.

I start by identifying the different types of equilibria and the region of P_0 values, given by the boundaries , $\tilde{P}^1$, $\tilde{P}^2$, $\tilde{P}^3$ and $\tilde{P}^4$, for which they exist. Finally, I identify the subset of P_0 values for which each equilibrium is preferred, i.e., $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$.

Truthful announcement (TA1): $P_0 \in \left[\tilde{P}^2, \bar{P}^{2.5}\right]$

In a TA1 equilibrium, we have $\theta_{H|H} = 1$, $\theta_{L|L} = 1-\varepsilon$, $\theta_{N|L} = \varepsilon$, which implies that $\frac{\theta_{H|H}}{\theta_{H|L}} = \infty > \frac{\theta_{N|H}}{\theta_{N|L}} = \frac{\theta_{L|H}}{\theta_{L|L}} = 0$. Then, from parts (ii) and (iii) of Lemma 13 we have

$$\bar{P}_{(-,H,s_M)} > \bar{P}_{(-,N,s_M)} = \bar{P}_{(-,L,s_M)},\ s_M \in \{H, L\}\,.$$

It is easy to verify that, as in the proof of Theorem 3, $\mathcal{T}_{(-,H,H)} = -\delta$ is inconsistent with $\theta_{H|H} = 1$ in equilibrium. Therefore, it must be the case that $\mathcal{T}_{(-,H,H)} = 1$, which implies that $\bar{P}_{(-,H,H)} > P_0$. The manipulator does not deviate from $\theta_{H|H} = 1$ if and only if $\Pi_{(-,H,H)} \geq \Pi_{(-,N,H)}$ (if he deviates, he does so to $a = N$ in order to avoid the penalty), which is equivalent to $\bar{P}_{(-,H,H)} - P_0 \geq \delta\left(P_0 - \bar{P}_{(-,N,H)}\right) = \delta\left(P_0 - \bar{P}_{(-,L,H)}\right)$.

Now, assume that $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = -\delta$, which implies that $\bar{P}_{(-,L,L)} < P_0$. The manipulator does not deviate from $\theta_{L|L} = 1-\varepsilon$ and $\theta_{N|L} = \varepsilon$ if and only if $\Pi_{(-,L,L)} = \Pi_{(-,N,L)} \geq \Pi_{(-,H,L)}$ which is equivalent to $\delta\left(P_0 - \bar{P}_{(-,L,L)}\right) \geq \bar{P}_{(-,H,L)} - P_0 - k$. If, by the contrary, $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = 1$ (which implies that $\bar{P}_{(-,L,L)} > P_0$), then the manipulator does not deviate if and only if $\bar{P}_{(-,L,L)} - P_0 \geq \bar{P}_{(-,H,L)} - P_0 - k$. This condition holds true if k is large enough, that is, if $k \geq \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$.

Bringing all conditions together we have that the TA1 equilibrium is supported for values of P_0 in the interval

$$\tilde{P}^2 \leq P_0 \leq \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta},\ \tilde{P}^2 = \begin{cases} \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} & if\, k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ 0 & otherwise \end{cases}$$

and the optimal trading strategy is given by

$$\mathcal{T}_{(-,H,H)} = 1,\ \mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = \begin{cases} -\delta & if\, k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ 1 & otherwise \end{cases}.$$

Truthful announcement (TA2): $P_0 \in \left[\bar{P}^{2.5}, \tilde{P}^3\right]$

Following the same steps as above, we determine that the TA2 equilibrium is supported for values of P_0 in the interval

$$\frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta} \leq P_0 \leq \tilde{P}^3,\ \tilde{P}^3 = \begin{cases} \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)} + \delta k}{1+\delta} & if\, k < \bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} \\ 1 & otherwise \end{cases}$$

and the optimal trading strategy is

$$\mathcal{T}_{(-,L,L)} = 1,\ \mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,N,H)} = \begin{cases} 1 & if\, k < \bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} \\ -\delta & otherwise \end{cases}$$

It then follows that $\bar{P}^{2.5}$ can be any value in the interval

$$\frac{\bar{P}^T_{(-,H,L)} + \delta\bar{P}^T_{(-,L,L)}}{1+\delta} \leq \bar{P}^{2.5} \leq \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^T_{(-,L,H)}}{1+\delta}.$$

Upward manipulation (UM): $P_0 \in \left(\tilde{P}^1, \tilde{P}^2\right)$

Suppose that $\bar{P}^T_{(-,L,L)} < P_0 < \tilde{P}^2$. In this case, the trading strategy is $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = 1$, $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = -\delta$. Following the same steps as in the proof of Theorem 3, we obtain that $\theta_{H|H} = 1$, $\theta_{H|L} = \omega_1$, $\theta_{L|L} = 1 - \omega_1 - \varepsilon$, $\theta_{N|L} = \varepsilon$, $\omega_1 \in (0,1)$ is an equilibrium announcement strategy when

$$\frac{\bar{P}_{(-,H,L)} + \delta\bar{P}^T_{(-,L,L)} - k}{1+\delta} = P_0 \leq \frac{\bar{P}_{(-,H,H)} + \delta\bar{P}^T_{(-,L,H)}}{1+\delta}$$

When P_0 decreases, ω_1 has to increase so that $\bar{P}_{(-,H,L)}$ decreases and the condition above continues to hold (as $P_0 \uparrow \tilde{P}^2$ we have $\omega_1 \downarrow 0$). When $\omega_1 = 1$, $\bar{P}_{(-,H,L)} = \bar{P}^N_{(-,-,L)}$ and the lower bound attains its minimum value. Therefore, we must have $P_0 \geq \frac{\bar{P}^N_{(-,-,L)} + \delta\bar{P}^T_{(-,L,L)} - k}{1+\delta} > \bar{P}^T_{(-,L,L)}$ for the condition above to hold and an upward manipulation equilibrium to exist. The second inequality in the latter expression holds whenever $k < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$.

If, instead, $P_0 \leq \bar{P}^T_{(-,L,L)} < \tilde{P}^2$, then the trading strategy is $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = \mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = 1$. In this case, the manipulator becomes indifferent between $a = H$ and $a = L$ conditional on $s_M = L$ (hence $\omega_1 \in (0,1)$) when $\bar{P}_{(-,H,L)} - P_0 - k = \bar{P}^T_{(-,L,L)} - P_0 \Leftrightarrow \bar{P}_{(-,H,L)} = \bar{P}^T_{(-,L,L)} + k$, which does not depend on P_0. Therefore, as long as $\bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \leq k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$, the UM equilibrium is supported for $P_0 \leq \bar{P}^T_{(-,L,L)}$. Notice that if $k \geq \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$, there is no UM equilibrium, since we are back in the case where the manipulator announces truthfully ($\bar{P}^2 = 0$). It then follows that

$$\tilde{P}^1 = \begin{cases} \frac{\bar{P}^N_{(-,-,L)} + \delta\bar{P}^T_{(-,L,L)} - k}{1+\delta} < \tilde{P}^2 & if\, k < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \\ 0 < \tilde{P}^2 & if\, \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \leq k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}. \\ 0 = \tilde{P}^2 & otherwise \end{cases}$$

Downward manipulation (DM): $P_0 \in \left(\tilde{P}^3, \tilde{P}^4\right)$

Proceeding in the same way as above, we obtain

$$\tilde{P}^4 = \begin{cases} \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^N_{(-,-,H)} + \delta k}{1+\delta} > \tilde{P}^3 & if\, k < \bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} \\ 1 > \tilde{P}^3 & if\, \bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} \leq k < \bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)}. \\ 1 = \tilde{P}^3 & otherwise \end{cases}$$

Never announces (NA): $P_0 \in \left[0, \tilde{P}^1\right] \cup \left[\tilde{P}^4, 1\right]$

Like in the case of $k = 0$, never announcing is always an equilibrium.

Preferred equilibria: determining $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$

Unlike in the case where $k = 0$, there are circumstances where the never announce equilibrium is preferred to the other equilibria. To determine the threshold prices $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$ I need to compare the expected utility of never announcing with that of other equilibria.

Starting with $\bar{P}^1$, there are three cases to be considered, corresponding to the combinations of possible trading strategies in both equilibria. The first case is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = 1$, $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the UM equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\begin{aligned} & \bar{P}^N_{(-,-,H)} - P_0 + \delta\left(P_0 - \bar{P}^N_{(-,-,L)}\right) > \bar{P}_{(-,H,H)} - P_0 + \omega_1\left(\bar{P}_{(-,H,L)} - P_0 - k\right) \\ & \qquad\qquad + (1-\omega_1)\,\delta\left(P_0 - \bar{P}^T_{(-,L,L)}\right) \\ \Leftrightarrow\quad & \bar{P}^N_{(-,-,H)} - P_0 + \delta\left(P_0 - \bar{P}^N_{(-,-,L)}\right) > \bar{P}_{(-,H,H)} - P_0 + \delta\left(P_0 - \bar{P}^T_{(-,L,L)}\right) \\ \\ \Leftrightarrow\quad & \delta\left(\bar{P}^T_{(-,L,L)} - \bar{P}^N_{(-,-,L)}\right) > \bar{P}_{(-,H,H)} - \bar{P}^N_{(-,-,H)}, \end{aligned}$$

where the second equivalence follows from the fact that if $\omega_1 \in (0,1)$ holds in equilibrium, then the utility from $a = H$ or $a = L$ when $s_M = L$ is the same. Noting that $\bar{P}^N_{(-,-,s_M)} = \bar{P}_{(-,N,s_M)}$ when $\frac{\theta_{N|H}}{\theta_{N|L}} = 1$, from parts (ii) and (iii) of Lemma 13 the condition above is impossible since the left hand side is negative whereas the right hand side is positive. Therefore, the UM equilibrium is preferred to the NA equilibrium.

In the second case, the trading strategy in the UM equilibrium is as above, but the trading strategy in the NA equilibrium is $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$. Comparing expected utilities, the NA equilibrium is preferred if P_0 satisfies

$$P_0 < \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}_{(-,H,H)}}{1+\delta}.$$

For the UM equilibrium to be the one assumed above, we need $P_0 \geq \bar{P}^T_{(-,L,L)}$, which implies that

$$\frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}_{(-,H,H)}}{1+\delta} > \bar{P}^T_{(-,L,L)} \Leftrightarrow \bar{P}_{(-,H,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)}. \quad (14)$$

Notice that $\bar{P}_{(-,H,H)}$ is the UM equilibrium value, and so it is implicit that the UM equilibrium exists.

The third and final case, is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = \mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = 1$ in the UM equilibrium, which means that $P_0 < \bar{P}^T_{(-,L,L)}$. From point (iii) of Lemma 13 we have $\bar{P}^T_{(-,L,L)} < \bar{P}^N_{(-,-,L)}$, and so the trading strategy in the NA equilibrium is $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$. In this scenario, the manipulator prefers the NA equilibrium if

$$\bar{P}_{(-,H,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)}. \quad (15)$$

Since conditions (14) and (15) are identical, the preference of the NA equilibrium over the UM equilibrium is independent of P_0, and it follows that

$$\bar{P}^1 = \begin{cases} \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}^*_{(-,H,H)}}{1+\delta} & if\ \bar{P}^1 > \bar{P}^T_{(-,L,L)} \\ 0 & otherwise \end{cases}$$

where $\bar{P}^*_{(-,H,H)}$ is the equilibrium value in the UM equilibrium when $P_0 = \bar{P}^1$.

Next I look at $\bar{P}^2$. Here, there are another three cases to consider. The first is when $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$ in the TA1 equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. In this case the NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} < \bar{P}^T_{(-,L,L)} - \bar{P}^N_{(-,-,L)}$$

which, from parts (ii) and (iii) of Lemma 13, is impossible.

In the second case, the trading strategy in the NA equilibrium changes to $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$. The NA equilibrium is preferred if

$$\begin{aligned} P_0 &< \frac{\bar{P}^N_{(-,-,H)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,H,H)}}{1+\delta} \\ &= \bar{P}^T_{(-,L,L)} + \frac{\bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,H,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}}{1+\delta} \\ &\leq \bar{P}^T_{(-,L,L)} \end{aligned}$$

which cannot hold in a TA1 equilibrium. The last inequality follows from part (v) of Lemma 13 and is due to the risk aversion and reduction of uncertainty associated to the TA1 equilibrium.

In the third and final case, $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = 1$ in the TA1 equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$ in the NA equilibrium. In this case the NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$$

which once again is impossible due to part (v) of Lemma 13. Therefore, the NA equilibrium is never preferred to the TA1 equilibrium and $\bar{P}^2 = \tilde{P}^2$, that is,

$$\bar{P}^2 = \begin{cases} \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} & if\, k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ 0 & otherwise \end{cases}.$$

Turning the attention to $\bar{P}^3$, there are another three possible cases. The first one is when $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$ in the TA2 equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. We have just seen that in this case the NA equilibrium is never preferred to the TA1 equilibrium, and the same holds for the TA2 equilibrium. The second case is when $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$ in the TA2 equilibrium and $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. In this case the NA equilibrium is preferred if P_0 satisfies

$$P_0 > \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^T_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)}}{1+\delta}. \tag{16}$$

For the trading strategy assumed in the TA2 equilibrium to be optimal, we need $P_0 < \bar{P}^T_{(-,H,H)}$. From part (v) of Lemma 13 we have

$$\begin{aligned} & \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^T_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)}}{1+\delta} \\ = \;& \bar{P}^T_{(-,H,H)} + \delta \frac{\bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,H,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}}{1+\delta} \\ < \;& \bar{P}^T_{(-,H,H)} \end{aligned}$$

which means that condition (16) can be verified.

The third case is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = -\delta$ in the TA2 equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} > \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$$

which holds true due to part (v) of Lemma 13.

$\bar{P}^3$ is then given by the threshold (16) or by $\tilde{P}^3$, whichever is smaller,

$$\bar{P}^3 = \begin{cases} \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^T_{(-,L,H)}+\delta k}{1+\delta} & if\, k < \bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,L,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \\ \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^N_{(-,-,H)}-\delta\bar{P}^T_{(-,L,L)}+\delta\bar{P}^N_{(-,-,L)}}{1+\delta} & otherwise \end{cases}.$$

Finally, to determine $\bar{P}^4$ we need to look at another three cases. In the first case, we have $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,L,H)} = -\delta$, $\mathcal{T}_{(-,H,H)} = 1$ in the DM equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$$

which is impossible due to part (v) of Lemma 13.

In the second case, the trading strategy in the NA equilibrium changes to $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = -\delta$. In this case the NA equilibrium is preferred if P_0 satisfies

$$P_0 > \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^N_{(-,-,H)} - \delta\bar{P}_{(-,L,L)} + \delta\bar{P}^N_{(-,-,L)}}{1+\delta}.$$

Due to part (v) of Lemma 13 the threshold above is below $\bar{P}^T_{(-,H,H)}$ and so there are P_0values that satisfy the condition when the assumed strategies are optimal. Notice that $\bar{P}_{(-,L,L)}$ is the DM equilibrium value and so it is implicit that it exists.

Finally, the third case is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = -\delta$ in the DM equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. In that case, the NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} > \bar{P}^N_{(-,-,L)} - \bar{P}_{(-,L,L)}$$

which always holds true due to parts (ii), (iii) and (v) of Lemma 13. It then follows that

$$\bar{P}^4 = \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^N_{(-,-,H)} - \delta\bar{P}^*_{(-,L,L)} + \delta\bar{P}^N_{(-,-,L)}}{1+\delta}$$

where $\bar{P}^*_{(-,L,L)}$ is the equilibrium value in the DM equilibrium when $P_0 = \bar{P}^4$.

This concludes the proof of Theorem 4.

C.3 Proof of Theorem 5

Consider a UM equilibrium with $\theta_{H|L} = \omega_1$. The expected utility in this equilibrium is given by

$$\frac{\Pi_{(-,H,H)} + \omega_1\Pi_{(-,H,L)} + (1-\omega_1)\,\Pi_{(-,L,L)}}{2} = \frac{\bar{P}_{(-,H,H)} - P_0 + \omega_1\left(\bar{P}_{(-,H,L)} - P_0 - k\right) + (1-\omega_1)\,\Pi_{(-,L,L)}}{2}.$$

All else equal, if k increases, then $\Pi_{(-,H,L)}$ decreases. As a result, $\Pi_{(-,H,L)} < \Pi_{(-,L,L)}$ and $\theta_{H|L} = \omega_1$ is no longer an equilibrium. From Lemma 13, as ω_1 decreases, $\bar{P}_{(-,H,L)}$ and $\bar{P}_{(-,H,H)}$ increase, while $\bar{P}_{(-,L,L)}$ remains constant. Consequently, $\Pi_{(-,H,L)}$ and $\Pi_{(-,H,H)}$ increase whereas $\Pi_{(-,L,L)}$ remains constant. An equilibrium is reached when either $\omega_1 \in (0,1)$ is such that $\Pi_{(-,H,L)} = \Pi_{(-,L,L)}$ (a UM equilibrium with less manipulation) or $\omega_1 = 0$ and $\Pi_{(-,H,L)} < \Pi_{(-,L,L)}$ (a TA equilibrium). In both cases the increase in k increases the manipulator's expected utility conditional on $s_M = H$ and has no impact on the expected utility conditional on $s_M = L$. Therefore, unconditionally the manipulator is better off with the increase in k.

The case of a DM equilibrium is very similar. An increase in k leads to a decrease in ω_2. In the new equilibrium the manipulator's expected utility conditional on $s_M = L$ increases while the expected utility conditional on $s_M = H$ remains constant, and so the manipulator is also better off with the increase in k.

The cases of a TA and NA equilibria are trivial. In both cases the manipulator pays no penalty and so the increase in k has no impact on the manipulator's expected utility in any of these equilibria. But if the NA equilibrium was initially preferred to a UM or DM equilibrium, the increase in the expected utility in UM and DM equilibria may make one of these preferable over the NA equilibrium after an increase in k. In such a case the manipulator's expected utility increases.

Therefore, an increase in k may increase or have no impact on the manipulator's expected utility, depending on the value of P_0 and the initial level of k, but never decrease it. If k is small enough so that there exist P_0 that support a UM or DM equilibrium, the expected utility averaging over P_0 increases in k. Otherwise, it remains constant. This concludes the proof.

C.4 Proof of Theorem 6

The last part of the theorem is an obvious consequence of the fact that the manipulator can only avoid the penalty by not announcing. Therefore, when k becomes very large the expected profit for any strategy other than not announcing becomes negative, and thus smaller than the expected utility of not announcing. Not announcing is then the optimal strategy for any P_0.

To prove the first part of the theorem, I start by deriving the optimal announcement strategies for the case of a small k. Using the same method as in the proof of Theorem 4, we obtain that the optimal strategy, when k is sufficiently small, is

$$\begin{cases} \theta_{H|H} = 1,\, \theta_{N|L} = 1 & if\, P_0 \in \left[\bar{P}^2, \bar{P}^{2.5}\right]\ (Truthful\ Announcement) \\ \theta_{N|H} = 1,\, \theta_{L|L} = 1 & if\, P_0 \in \left[\bar{P}^{2.5}, \bar{P}^3\right]\ (Truthful\ Announcement) \\ \theta_{H|H} = 1,\, \theta_{H|L} = \omega_1,\, \theta_{N|L} = 1-\omega_1 & if\, P_0 \in \left(\bar{P}^1, \bar{P}^2\right)\ (Upward\ Manipulation) \\ \theta_{L|L} = 1,\, \theta_{L|H} = \omega_2,\, \theta_{N|H} = 1-\omega_2 & if\, P_0 \in \left(\bar{P}^3, \bar{P}^4\right)\ (Downward\ Manipulation) \\ \theta_{N|H} = \theta_{N|L} = 1 & if\, P_0 \in \left[0, \bar{P}^1\right] \cup \left[\bar{P}^4, 1\right]\ (Never\ Announce) \end{cases}$$

where

$$\begin{aligned}
\bar{P}^1 &= \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,N,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}^*_{(-,H,H)} + k\left(1-\rho_M\right)}{1+\delta} \\
\bar{P}^2 &= \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,N,L)} - k\rho_M}{1+\delta} \\
\bar{P}^{2.5} &\in \left[\frac{\bar{P}^T_{(-,N,L)} + \delta \bar{P}^T_{(-,L,L)} + \delta k\left(1-\rho_M\right)}{1+\delta}, \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,N,H)} - k\left(1-\rho_M\right)}{1+\delta}\right] \\
\bar{P}^3 &= \frac{\bar{P}^T_{(-,N,H)} + \delta \bar{P}^T_{(-,L,H)} + \delta k\rho_M}{1+\delta} \\
\bar{P}^4 &= \frac{\bar{P}^T_{(-,N,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^*_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)} - k\left(1-\rho_M\right)}{1+\delta},
\end{aligned}$$

Notice that because truthful announcements may also be punished (if manipulator's information is wrong), the manipulator uses $a = N$ to signal $s_M = H$ ($s_M = L$) when P_0 is large (small). By doing so the manipulator avoids being punished while signaling truthfully. The manipulator prefers to avoid the punishment when signaling truthfully, even though truthful signaling occurs less frequently than manipulated signaling in the UM and DM equilibria. This happens because if he is punished while sending the manipulated signal, he can commit to manipulate less frequently which, as we saw previously, benefits the manipulator. If the manipulator chose to avoid punishment while sending the manipulated signal, the occasional punishment of truthful announcements would increase the incentive to manipulate. As a result, the credibility of the announcement would decrease, decreasing manipulator's expected utility.

Because the manipulator always strictly prefers to signal truthfully by not announcing, I have to make an additional assumption on the trembles: when $P_0 \leq \bar{P}^{2.5}$ the manipulator only trembles at $s_M = L$, and when $P_0 > \bar{P}^{2.5}$ only trembles at $s_M = H$. This assumption is needed to avoid the problem that the manipulator may want to deviate to a zero probability announcement whose payoff changes when he does that.

Looking that the expression for $\bar{P}^2$ and $\bar{P}^3$ it is immediate that the former decreases and the latter increases in k. In turn, at first sight it looks like $\bar{P}^1$ increases in k. However, when k increases, $\bar{P}^*_{(-,H,H)}$ increases by more than k, and so $\bar{P}^1$ actually decreases in k. To see why this is true, recall from the proof of Theorem 5 that if k increases the manipulator decreases the probability with which he manipulates. As a result, both $\bar{P}^*_{(-,H,L)}$ and $\bar{P}^*_{(-,H,H)}$ increase. $\bar{P}^*_{(-,H,L)}$ increases by $\Delta k\rho_M$ in order to restore indifference between $a = H$ and $a = L$ when $s_M = L$. It can be shown numerically that $\bar{P}^*_{(-,H,H)}$ in general increases by more than $\bar{P}^*_{(-,H,L)}$. Even when that is not the case, the difference is small. Since $\Delta k\left(1-\rho_M\right) < \Delta k\rho_M$ it follows that in general $\bar{P}^1$ decreases. Using a similar argument, we have that in general $\bar{P}^4$ increases in k. Therefore, when k is small, an increase in k results in a weak increase in the announcement informativeness for all P_0. The improvement in the manipulator's expected utility can be proved following the same steps as in the proof of Theorem 5. This concludes the proof.

C.5 Proof of Theorem 7

A formal proof for the sign of the derivatives in points (i) and (ii) is difficult to construct, since there is no closed form solution for expected prices. Numerical results suggest that those results hold generically. Figure 18 provides an illustration, plotting the values of $\bar{P}^1$ to $\bar{P}^4$ as a function of ρ and ρ_M, for the following parametrization: $\alpha = \frac{1}{2}$, $\bar{z} = 1$ and $\sigma_z^2 = 1$. The same qualitative results are obtained for different parametrizations, provided that $\alpha > 0$, $\bar{z} > 0$ and $\sigma_z^2 > 0$.

(i) When $\rho = \frac{1}{2}$, it is as if investors do not observe signal s. Since they do not observe s_M, it follows

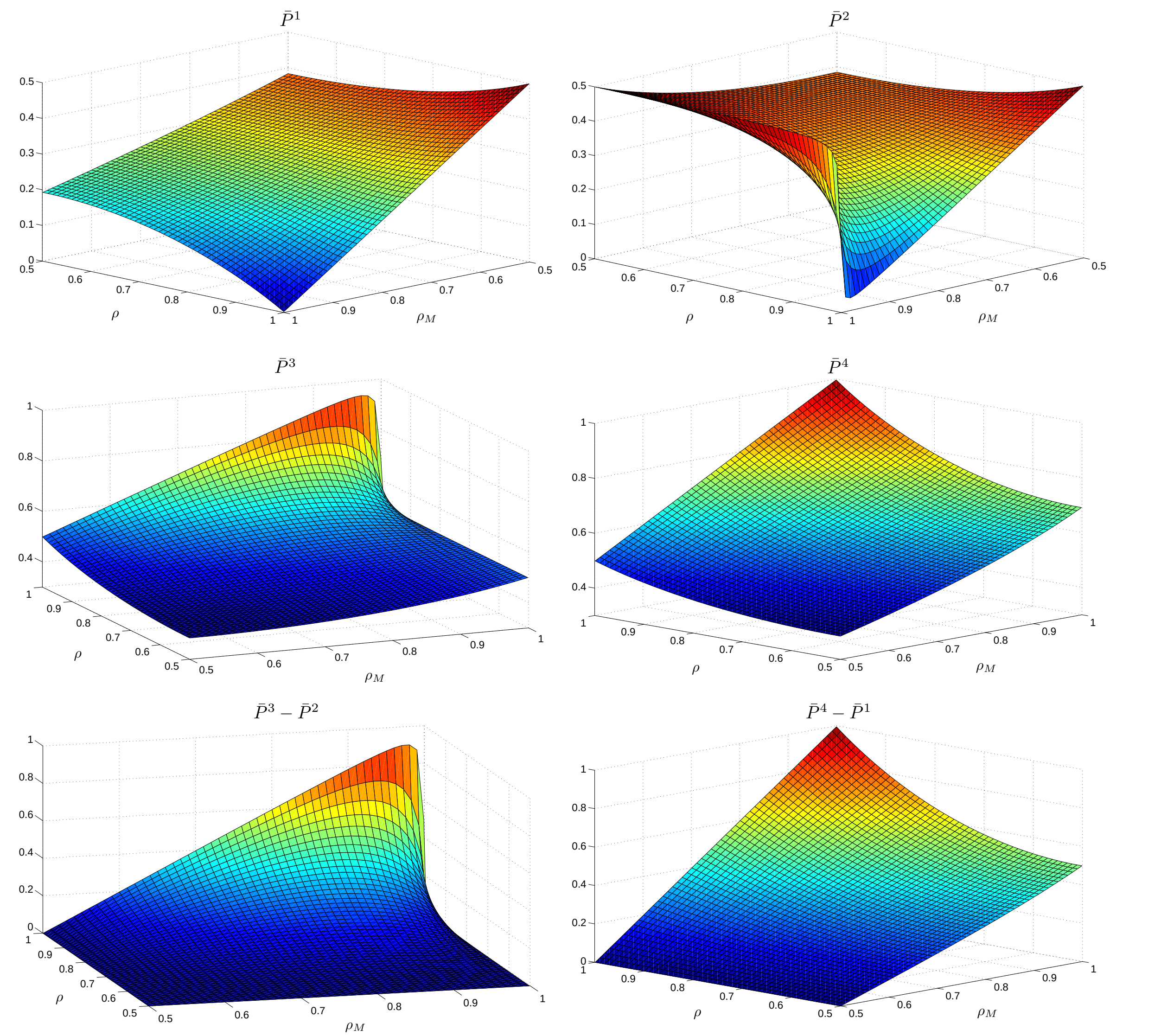

Figure 18: Effect of the accuracy of investors' information and of the manipulator's information on the announcement strategy. This figure shows how the limits that define each type of announcement strategy change with the accuracy of investors' information (ρ) and of manipulator's information (ρ_M). The parametrization used was: $\alpha = \frac{1}{2}$, $\bar{z} = 1$ and $\sigma_z^2 = 1$.

immediately that the price only depends on the announcement and its credibility. Therefore, $\bar{P}^T_{(-,a,L)} = \bar{P}^T_{(-,a,H)}$ which implies that $\bar{P}^2 = \bar{P}^3$.

In turn, if $\rho = 1$, investors learn the liquidation value exactly from the observation of s. This means that the price is independent of the announcement and its credibility. However, it depends on the manipulator's signal, since it is more likely that $s_M = s$ than otherwise. Therefore, $\bar{P}^N_{(-,-,L)} = \bar{P}^T_{(-,H,L)} < \bar{P}^T_{(-,L,H)} = \bar{P}^N_{(-,-,H)}$ implying that $\bar{P}^1 = \bar{P}^2$ and $\bar{P}^3 = \bar{P}^4$.

It is possible to prove the sign of the derivative $\frac{\partial \bar{P}^3 - \bar{P}^2}{\partial \rho}$. Using the definitions of $\bar{P}^2$ and $\bar{P}^3$ from Theorem 3, and using equation (7), we can write

$$\bar{P}^3 - \bar{P}^2 = \left[2\mathbb{P}\left(s = H \middle| s_M = H\right) - 1\right]\left(\bar{P}^T_{(H,H,-)} - \bar{P}^T_{(L,H,-)} + \delta\bar{P}^T_{(H,L,-)} - \delta\bar{P}^T_{(L,L,-)}\right).$$

The derivative with respect to ρ is then given by

$$\begin{aligned}\frac{\partial \bar{P}^3-\bar{P}^2}{\partial \rho} &= 2\left(2\rho_M-1\right)\left(\bar{P}^T_{(H,H,-)}-\bar{P}^T_{(L,H,-)}+\delta\bar{P}^T_{(H,L,-)}-\delta\bar{P}^T_{(L,L,-)}\right)\\ &\quad +\left[2\mathbb{P}\left(s=H\middle|\, s_M=H\right)-1\right]\left(\frac{\partial \bar{P}^T_{(H,H,-)}}{\partial\rho}-\frac{\partial \bar{P}^T_{(L,H,-)}}{\partial\rho}+\delta\frac{\partial \bar{P}^T_{(H,L,-)}}{\partial\rho}-\delta\frac{\partial \bar{P}^T_{(L,L,-)}}{\partial\rho}\right)>0.\end{aligned}$$

By definition the first and third terms are positive (strictly if $\rho_M>\frac{1}{2}$). Part (i) of Lemma 13 implies that the second term is positive as well. The sign of the last term is obtained using Lemma 12 and the results in Appendix C.7 where I show that $\frac{\partial p_{(H,a,-)}}{\partial\rho}>0$ and $\frac{\partial p_{(L,a,-)}}{\partial\rho}<0$.

(ii) When $\rho_M=\frac{1}{2}$ the manipulator observes an uninformative signal. As a result, the price is independent of his information and of his announcement. Thus, $\bar{P}^N_{(-,-,s_M)}=\bar{P}^T_{(-,a,s_M)}$ and $\bar{P}^N_{(-,-,H)}=\bar{P}^N_{(-,-,L)}$, which implies that $\bar{P}^1=\bar{P}^2=\bar{P}^3=\bar{P}^4$.

In turn, when $\rho=1$, the manipulator learns the liquidation value from the observation of s_M. Then, if the announcement is seen as truthful, investors ignore the information provided by signal s. This implies that the price is independent of s and s_M. Therefore, $\bar{P}^T_{(-,a,L)}=\bar{P}^T_{(-,a,H)}$ and so $\bar{P}^2=\bar{P}^3$.

(iii) Taking the derivative of $\bar{P}^1$ to $\bar{P}^4$ as given by Theorem 3, with respect to δ, we obtain

$$\frac{\partial \bar{P}^1}{\partial\delta}=\frac{\bar{P}^T_{(-,L,L)}-\bar{P}^N_{(-,-,L)}}{\left(1+\delta\right)^2}<0,\ \frac{\partial \bar{P}^2}{\partial\delta}=\frac{\bar{P}^T_{(-,L,L)}-\bar{P}^T_{(-,H,L)}}{\left(1+\delta\right)^2}<0,$$

$$\frac{\partial \bar{P}^3}{\partial\delta}=\frac{\bar{P}^T_{(-,L,H)}-\bar{P}^T_{(-,H,H)}}{\left(1+\delta\right)^2}<0,\ \frac{\partial \bar{P}^4}{\partial\delta}=\frac{\bar{P}^N_{(-,-,H)}-\bar{P}^T_{(-,H,H)}}{\left(1+\delta\right)^2}<0.$$

The inequalities follow from the application of points (ii) and (iii) of Lemma 13, noting that the announcement strategy associated to prices $\bar{P}^T_{(-,a,s_M)}$ satisfies $\frac{\theta_{H|H}}{\theta_{H|L}}=\infty$, $\frac{\theta_{L|H}}{\theta_{L|L}}=0$ and the announcement strategy associated to prices $\bar{P}^N_{(-,-,s_M)}$ satisfies $\frac{\theta_{N|H}}{\theta_{N|L}}=1$. From these derivatives we can determine that

$$\begin{aligned}\frac{\partial \bar{P}^2-\bar{P}^1}{\partial\delta} &= \frac{\bar{P}^N_{(-,-,L)}-\bar{P}^T_{(-,H,L)}}{\left(1+\delta\right)^2}<0\\ \frac{\partial \bar{P}^3-\bar{P}^2}{\partial\delta} &= \frac{\bar{P}^T_{(-,L,H)}-\bar{P}^T_{(-,H,H)}-\bar{P}^T_{(-,L,L)}+\bar{P}^T_{(-,H,L)}}{\left(1+\delta\right)^2}<0\\ \frac{\partial \bar{P}^4-\bar{P}^3}{\partial\delta} &= \frac{\bar{P}^N_{(-,-,H)}-\bar{P}^T_{(-,L,H)}}{\left(1+\delta\right)^2}>0\\ \frac{\partial \bar{P}^4-\bar{P}^1}{\partial\delta} &= \frac{\bar{P}^N_{(-,-,H)}-\bar{P}^T_{(-,H,H)}-\bar{P}^T_{(-,L,L)}+\bar{P}^N_{(-,-,L)}}{\left(1+\delta\right)^2}<0.\end{aligned}$$

The sign of all derivatives follow from a direct application of points (ii), (iii), (v) and (vi) of Lemma 13.

C.6 Proof of Theorem 9

The unconditional average price efficiency can be written as

$$
\begin{aligned}
\overline{eff} &= \sum_{s,a} \gamma_{(s,a,-)} - 2\sum_{s,a} p_{(s,a,-)}\gamma_{(s,a,-)} + 2\sum_{s,a} p^2_{(s,a,-)}\gamma_{(s,a,-)} \\
&= 2\sum_{s,a} p^2_{(s,a,-)}\gamma_{(s,a,-)}
\end{aligned}
$$

$$
= \sum_a \left[\frac{\rho^2 \left[\theta_{a|L}\left(1-\rho_M\right)+\theta_{a|H}\rho_M\right]^2}{\theta_{a|H}+\left(\theta_{a|L}-\theta_{a|H}\right)\left(\rho+\rho_M-2\rho\rho_M\right)} + \frac{\left(1-\rho\right)^2 \left[\theta_{a|L}\left(1-\rho_M\right)+\theta_{a|H}\rho_M\right]^2}{\theta_{a|L}+\left(\theta_{a|H}-\theta_{a|L}\right)\left(\rho+\rho_M-2\rho\rho_M\right)} \right]
$$

since the first term of the first equation equals 1 by definition and the second term is twice the probability of $V = V_H$ which is $\frac{1}{2}$.

(i) Start by considering a UM equilibrium. Substituting $\theta_{a|s_M}$ by the optimal values in a UM equilibrium, using $\theta_{H|L} = 1 - \theta_{N|L} - \theta_{L|L}$ and taking the limit as $\theta_{N|L} \to 0$, the average efficiency becomes

$$
\begin{aligned}
\overline{eff} = &\frac{\rho^2\left[\left(1-\theta_{L|L}\right)\left(1-\rho_M\right)+\rho_M\right]^2}{1-\theta_{L|L}\left(\rho+\rho_M-2\rho\rho_M\right)} + \frac{\left(1-\rho\right)^2\left[\left(1-\theta_{L|L}\right)\left(1-\rho_M\right)+\rho_M\right]^2}{1-\theta_{L|L}+\theta_{L|L}\left(\rho+\rho_M-2\rho\rho_M\right)} \\
&+\frac{\theta_{L|L}\rho^2\left(1-\rho_M\right)^2}{\rho+\rho_M-2\rho\rho_M} + \frac{\theta_{L|L}\left(1-\rho\right)^2\left(1-\rho_M\right)^2}{1-\left(\rho+\rho_M-2\rho\rho_M\right)}.
\end{aligned}
$$

Taking the derivative with respect to $\theta_{L|L}$, we obtain

$$
\frac{\partial\overline{eff}}{\partial\theta_{L|L}} = \frac{\rho^2(1-\rho)^2(2\rho_M-1)^2\theta_{L|L}\left[1+3\left(1-\theta_{L|L}\right)\right]\left[\left(1-\theta_{L|L}\right)^2+\rho(1-\rho)+\rho_M(1-\rho_M)(2\rho-1)^2\right]}{\left[1-\theta_{L|L}+\theta_{L|L}(\rho+\rho_M-2\rho\rho_M)\right]^2(\rho+\rho_M-2\rho\rho_M)^2[1-(\rho+\rho_M-2\rho\rho_M)]^2\left[1-\theta_{L|L}(\rho+\rho_M-2\rho\rho_M)\right]^2} \geq 0
$$

with strict inequality when $\theta_{L|L} > 0$.

Similarly, in a DM equilibrium the average efficiency is given by the expression above with $\theta_{H|H}$ instead of $\theta_{L|L}$. Therefore, $\frac{\partial\overline{eff}}{\partial\theta_{H|H}} \geq 0$. This proves that the average efficiency decreases in $\widetilde{\theta}$.

(ii) Starting with the case of a TA equilibrium, the average efficiency is given by

$$
\overline{eff} = \frac{\rho\left(1-\rho\right)+\rho_M\left(1-\rho_M\right)-6\rho\left(1-\rho\right)\rho_M\left(1-\rho_M\right)}{\left(\rho+\rho_M-2\rho\rho_M\right)\left[1-\left(\rho+\rho_M-2\rho\rho_M\right)\right]}
$$

and its derivative with respect to ρ and ρ_M are given by

$$
\begin{aligned}
\frac{\partial\overline{eff}}{\partial\rho} &= \frac{2\left(2\rho-1\right)\rho_M^2\left(1-\rho_M\right)^2}{\left(\rho+\rho_M-2\rho\rho_M\right)^2\left[1-\left(\rho+\rho_M-2\rho\rho_M\right)\right]^2} \geq 0 \\
\frac{\partial\overline{eff}}{\partial\rho_M} &= \frac{2\left(2\rho_M-1\right)\rho^2\left(1-\rho\right)^2}{\left(\rho+\rho_M-2\rho\rho_M\right)^2\left[1-\left(\rho+\rho_M-2\rho\rho_M\right)\right]^2} \geq 0
\end{aligned}
$$

with strict inequalities when $\rho_M < 1$ and $\rho < 1$, respectively.

In the case of a NA equilibrium, we have

$$
\begin{aligned}
\overline{eff} &= 1-2\rho\left(1-\rho\right) \\
\frac{\partial\overline{eff}}{\partial\rho} &= 2\left(2\rho-1\right) \geq 0
\end{aligned}
$$

with strict inequality for $\rho > \frac{1}{2}$.

In the case of UM equilibrium (in a DM equilibrium substitute $\theta_{L|L}$ for $\theta_{H|H}$), the average efficiency is as determined in the proof of (i). Its derivatives with respect to ρ and ρ_M are

$$
\begin{aligned}
\frac{\partial \overline{eff}}{\partial \rho} &= \frac{2}{(2\rho-1)^3} - \frac{\theta_{L|L}\rho(1-\rho)\left[\rho(1-\rho)+2\rho_M(1-\rho_M)(2\rho-1)^2\right]}{(\rho+\rho_M-2\rho\rho_M)^2\left[1-(\rho+\rho_M-2\rho\rho_M)\right]^2} \\
&\quad + \frac{\left(2-\theta_{L|L}\right)^3\rho(1-\rho)\left\{2\left(\theta_{L|L}-1\right)\left(1+2\rho+2\rho^2\right)-\theta_{L|L}^2\left[\rho(1-\rho)+2(2\rho-1)^2\rho_M(1-\rho_M)\right]\right\}}{\left[1-\theta_{L|L}+\theta_{L|L}(\rho+\rho_M-2\rho\rho_M)\right]^2\left[1-\theta_{L|L}(\rho+\rho_M-2\rho\rho_M)\right]^2} \geq 0 \\
\frac{\partial \overline{eff}}{\partial \rho_M} &= \frac{\theta_{L|L}\rho^2(1-\rho)^2(2\rho_M-1)}{(\rho+\rho_M-2\rho\rho_M)^2\left[1-(\rho+\rho_M-2\rho\rho_M)\right]^2} \\
&\quad + \frac{\rho^2(1-\rho)^2(2\rho_M-1)\theta_{L|L}^2\left(2-\theta_{L|L}\right)^3}{\left[1-\theta_{L|L}+\theta_{L|L}(\rho+\rho_M-2\rho\rho_M)\right]^2\left[1-\theta_{L|L}(\rho+\rho_M-2\rho\rho_M)\right]^2} \geq 0.
\end{aligned}
$$

It is easy to see that $\frac{\partial \overline{eff}}{\partial \rho_M} \geq 0$. The sign of $\frac{\partial \overline{eff}}{\partial \rho}$, however, is much more difficult to determine. I use Mathematica to algebraically determine that $\frac{\partial \overline{eff}}{\partial \rho} \geq 0$. The same result is obtained in a DM equilibrium. This concludes the proof for the unconditional average price efficiency.

C.7 Proof of Theorem 10

(i) Part (iv) of Lemma 13 establishes that $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{H|H}} \geq 0$ and $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{L|L}} \leq 0$. It is straightforward to obtain, from its definition, that $\bar{P}_{(s,H,-)}$ does not depend directly on $\theta_{L|L}$. It depends on $\theta_{L|L}$ only indirectly through the relation $\theta_{L|L}+\theta_{N|L}+\theta_{H|L}=1$, and we can write $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{L|L}} = \frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{H|L}}\frac{\partial \theta_{H|L}}{\partial \theta_{L|L}}$. It is immediate that the second term is weakly negative and from part (iv) of Lemma 13 we know that the first term is also weakly negative. Therefore, $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{L|L}} \geq 0$. Following similar steps, it is easily determined that $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{H|H}} \leq 0$.

In UM equilibria we have that $\theta_{L|H} = \theta_{N|H} = \theta_{N|L} = 0$. From part (iv) of Lemma 13 we obtain $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{L|L}} = 0$ and $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{H|L}} < 0$. Since $\frac{\partial \theta_{H|L}}{\partial \theta_{L|L}} < 0$ we then obtain that $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{L|L}} > 0$. The proof for the case of DM equilibria is similar and is omitted.

(ii) The derivatives of $p_{(s,a,-)}$ with respect to ρ_M are

$$
\begin{aligned}
\frac{\partial p_{(H,H,-)}}{\partial \rho_M} &= \frac{\theta_{H|L}\left(\frac{\theta_{H|H}}{\theta_{H|L}}-1\right)\left(\theta_{H|H}+\theta_{H|L}\right)\rho(1-\rho)}{\left[\theta_{H|H}+\left(\theta_{H|L}-\theta_{H|H}\right)(\rho+\rho_M-2\rho\rho_M)\right]^2} > 0 \\
\frac{\partial p_{(L,H,-)}}{\partial \rho_M} &= \frac{\theta_{H|L}\left(\frac{\theta_{H|H}}{\theta_{H|L}}-1\right)\left(\theta_{H|H}+\theta_{H|L}\right)\rho(1-\rho)}{\left[\theta_{H|L}+\left(\theta_{H|H}-\theta_{H|L}\right)(\rho+\rho_M-2\rho\rho_M)\right]^2} > 0 \\
\frac{\partial p_{(H,L,-)}}{\partial \rho_M} &= \frac{\theta_{L|L}\left(\frac{\theta_{L|H}}{\theta_{L|L}}-1\right)\left(\theta_{L|H}+\theta_{L|L}\right)\rho(1-\rho)}{\left[\theta_{L|H}+\left(\theta_{L|L}-\theta_{L|H}\right)(\rho+\rho_M-2\rho\rho_M)\right]^2} < 0 \\
\frac{\partial p_{(L,L,-)}}{\partial \rho_M} &= \frac{\theta_{L|L}\left(\frac{\theta_{L|H}}{\theta_{L|L}}-1\right)\left(\theta_{L|H}+\theta_{L|L}\right)\rho(1-\rho)}{\left[\theta_{L|L}+\left(\theta_{L|H}-\theta_{L|L}\right)(\rho+\rho_M-2\rho\rho_M)\right]^2} < 0
\end{aligned}
$$

where the inequalities follow from Assumption 1. From Lemma 12 we know that $\frac{\partial \bar{P}_{(s,a,-)}}{\partial p_{(s,a,-)}} > 0$. The proof of point (ii) is now immediate.

(iii) The derivatives of $p_{(s,a,-)}$ with respect to ρ are given by

$$\begin{aligned}\frac{\partial p_{(H,a,-)}}{\partial \rho} &= \frac{\left[\theta_{a|H}\left(1-\rho_M\right)+\theta_{a|L}\rho_M\right]\left[\theta_{a|L}\left(1-\rho_M\right)+\theta_{a|H}\rho_M\right]}{\left[\theta_{a|H}+\left(\theta_{a|L}-\theta_{a|H}\right)\left(\rho+\rho_M-2\rho\rho_M\right)\right]^2}>0 \\ \frac{\partial p_{(L,a,-)}}{\partial \rho} &= -\frac{\left[\theta_{a|H}\left(1-\rho_M\right)+\theta_{a|L}\rho_M\right]\left[\theta_{a|L}\left(1-\rho_M\right)+\theta_{a|H}\rho_M\right]}{\left[\theta_{a|L}+\left(\theta_{a|H}-\theta_{a|L}\right)\left(\rho+\rho_M-2\rho\rho_M\right)\right]^2}<0.\end{aligned}$$

The proof of point (iii) is now immediate.

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