Private equity: strategies for improving performance[‡]

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Abstract

Evidence from the literature suggests the median private equity manager does not create excess returns over public markets net of fees. The major issues are the fees and the managers' ability to lag valuations (referred to as stale pricing). In this paper, we first confirm findings from literature using a more robust factor model that allows for leverage, illiquidity and volatility clustering. Our factor model explains 70 to 90 percent of the variation in returns. Second, based on the high explanatory power, we suggest two methods of applying these factor models to improve performance. We test the robustness of the methods using historical and simulated data. The findings from our paper can alter the views institutions currently hold on private equity in terms of asset allocation.

Keywords: private equity; conditional overlay; GARCH.

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1. INTRODUCTION

The use of private equity by institutional investors was pioneered by US endowments, notably by David Swensen when he joined Yale University in 1985 (Swensen, 2000), and Yale currently allocates 26 percent of its assets to private equity.[1] Following the success of the US endowment model, other institutions invested in the asset class. On average, 8 percent of institutional assets is invested in private equity globally (Bailie et al., 2008). Yet, few investors have been able to match the returns experienced by US endowments. In the private equity industry, institutional investors such as these endowments are referred to as Limited Partners (LPs), and fund managers as General Partners (GPs).[2] Lerner et al. (2007) measure the return differential between US endowments and other Limited Partners and find a difference of 21 percent per annum over the period from 1991 to 1998. Thus, the ability to access superior managers is considered key to successful investing in the private equity industry.[3]

A number of academic studies suggest the median private equity manager has not created excess returns after fees are taken into account.[4] Estimates for fees in the private equity industry start from 7 percent per annum by academics (Phalippou, 2009)[5] to 12 percent by experienced practitioners based on the Yale Endowment's

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[1] See http://www.yale.edu/investments/
[2] See appendix A for a full description of industry terms.
[3] For Endowments, alumni networks lead to access to superior General Partners (GPs) and the ability to co-invest (invest alongside the manager, thereby saving on fees). The discussion as to why institutions allocate to private equity if they cannot access these managers falls outside the scope of this paper. Moskowitz and Vissing Jorgensen (2002) suggest the private non-pecuniary benefits must be large.
[4] See Swensen (2000), Moskowitz and Vissing Jorgensen (2002), Kaplan and Schoar (2005), Phalippou and Zollo (2005), Conroy and Harris (2007) and Franzoni et al. (2009).
[5] The 7 percent fees estimate by Phalippou (2009) consists of 2 percent per annum base fee, 2.5 percent portfolio fees (transaction, advisory and director fees, taken directly out of the fund holdings) and 2.5 percent performance fees. Performance fees are 20 percent above an 8 percent hurdle rate.

experience (Swensen, 2000).[6] The rational model of financial intermediation by Berk and Green (2004) predicts that financial intermediaries provide zero excess returns to their investors while capturing a rent that is commensurate with their abilities. Empirical evidence suggests positive value add can even turn negative because of fees. French (2008) estimates the cost of active management at 67 basis points for U.S. public equity, and Phalippou et al. (2005, 2007) suggest it could be as high as 330 basis points for U.S. private equity after all the costs are taken into account.

In addition, the use of stale pricing (relying on lagged market valuations) overstates excess returns created by managers. Fund managers smooth reported quarterly returns, and during periods of sharp falls in public markets tend to lag valuations of non-traded assets which make up the majority of their illiquid portfolios so as to create stability in unit prices (Gompers and Lerner, 1997, and Anson, 2006).

Our paper makes the following contributions to extant literature. First, we tailor the methodology introduced by Bird et al. (2010) to improve our understanding of the fundamental drivers of private equity excess returns.[7] To substantiate findings from literature, we regress an asymmetric GARCH model to allow for volatility clustering and leverage in excess returns. To the best of our knowledge, we are the first to examine excess returns to private equity using this type of factor model. Our model is able to explain between 70 to 90 percent of the variation in private equity returns and confirm findings from literature.

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[6] In the case of Yale, the 12 percent fee estimate reflects the performance fees on the 21 percent return differential for US endowments. In addition, Lerner et al. (2007) estimate 20 percent of investors use fund of funds, which contributes an additional 1 percent base and 10 percent performance fees.

[7] The model is a general model for alternative investments, initially developed for hedge funds. For a more extensive discussion of the adopted methodology see Bird et al. (2010).

Second, based on the high explanatory power of our factor model, we introduce two methods to improve the performance of private equity. These methods can be applied independently or combined to increase reward to risk ratios for investors in the asset class.

The first method demonstrates how a passive factor based allocation to private equity reduces fees during the initial years, when the investor pays fees on committed capital, but the manager remains underinvested. Investors set aside capital for commitment for the next 10 to 14 years and management fees are paid on committed, rather than invested capital, typically at least for the first 5 years. On average only 50 percent of committed capital is invested over the commitment period as the fund manager searches for investment opportunities. Thus, investors tend to overcommit by a factor of 1.5 to 2 times to achieve the desired allocation.[8] As we shall see, the use of a factor model approach reduces the need for any such overcommitment.

The second method suggests that a mechanical overlay strategy has the ability to improve excess returns and reduce risk during periods of market stress when institutions are unable to sell holdings. Because of the illiquid nature of the underlying holdings, private equity managers are unable to quickly adjust portfolio positions to changes in market sentiment (unlike for example hedge funds). One option is to trade investments in the secondaries market at deep discounts. However, these actions can lead to large market distortions.[9] The method we apply in this paper overcomes illiquidity and has earlier been proposed for hedge funds by Healy and Lo (2009).

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[8] A 2 percent base fee is then effectively charged at 3 to 4 percent (Ljungqvist and Richardson, 2003 and Phalippou and Gottschalg, 2009).
[9] Refer http://www.preqin.com/item/secondaries-deluge-leads-to-asset-pricing-turmoil/101/1098.

The remainder of the article is structured as follows. In section 2, we review background literature on existing style models used in private equity prior to revisiting the model introduced by Bird et al. (2010) in section 3. Our private equity data source is examined in section 4. Section 5 presents the empirical results. Section 6 provides conclusive remarks and suggestions for further research.

2. BACKGROUND LITERATURE

A large body of academic evidence suggests that private equity as an industry, measured by returns provided either by managers or directly through analysis of the underlying holdings, does not outperform public markets once account is taken of fees. Table 1 shows the main studies in private equity in terms of style factor models used and summarises the results obtained.

<< INSERT TABLE 1 >>

As can be seen from table 1, most authors do not find evidence of excess returns for the private equity industry. Studies with evidence to the contrary exhibit either a time period or a sample selection bias. Peng (2001) measures up to the end of the tech bubble, while Ljungqvist and Richardson (2003) rely on data from a single undisclosed LP, thereby introducing a sample selection bias. In addition, the equity risk premium (commonly proxied by the S&P 500) continues to be the dominant factor for academic research. This is not surprising, given that many institutional investors continue to benchmark their private equity holdings against such a public benchmark targeting a 3 to 5 percent outperformance net of fees (Evans, 2008).

Phalippou (2009) concludes that literature provides us with an interesting puzzle: if the average performance of private equity funds is below the benchmark after fees are charged, why then does the marginal investor allocate to private equity? He suggests investors underestimate the impact of fees and rely on biased samples to form their judgement. He notes similar issues arise in other common investment vehicles like mutual funds and hedge funds.

Thus, the evidence supports the argument advanced by Berk and Green (2004) that one would expect managers to fully charge for any alpha that they generate in fees. Swensen (2000) is perhaps one of the most experienced private equity investors since his early investment days at Yale, and based on his practical experience as Chief Investment Officer notes that *"the large majority of funds fail to add sufficient value and only the upper quartile of managers are worth investing in."*

3. THE MODEL

Anson (2006) suggests private equity managers provide a package of returns consisting of market risk, liquidity risk, active skill and leverage, and a good model combines all these elements. In this section we review the multifactor model proposed by Bird et al. (2010) as an interpretation of the Intertemporal ICAPM (ICAPM), in which the equity risk premium is supplemented by additional state variables associated with alternative assets. Under ICAPM (Merton, 1973), state variables are defined as undesirable outcomes such as illiquidity in private equity, against which investors hedge when investing, thereby creating additional risk premia.

3.1 Adjustment for leverage

Within the private equity industry, buyout managers apply leverage.[10] Tobin's Separation Theorem (1958) suggests separating out the financing decision, and we do so by deleveraging the observed returns. Bertelli (2007) transforms leveraged returns to deleveraged returns as follows.[11]

$$R^{L}_{it} = R^{DL}_{it} + (R^{DL}_{it} - I_{it}) \cdot (\lambda\text{-}1) \iff R^{DL}_{it} = (R^{L}_{it} + I_{it} \cdot \lambda - I_{it}) / \lambda \quad (1)$$

Where R^{L}_{it} is the leveraged return of sector i at time t, R^{DL}_{it} is the deleveraged return of sector i at time t, I_{it} is the interest expense paid by sector i at time t set equal to a 50/50 mix of the high yield and mezzanine debt[12] rate consistent with observations by Anson (2006). λ represents the gearing level. For buyout funds we define λ as the sector exposure (equal to debt plus equity positions held by the sector) divided by the equity of the sector. We use a leverage ratio of 3.3, based on an average over the period from 1990 to 2009 using data collected by Anson (2006) and Griffin (2010).

3.2 Adjustment for illiquidity

For private equity, illiquidity adjustment methods based on serial correlation have been employed by Gompers and Lerner (1997), Jones and Rhodes-Kropf (2003) and Anson (2006), in general relying on regressions against the lagged equity market premium or multiple lags. Our adjustment builds on the work by Getmansky, Lo and Makarov (2004) who explore several sources of serial correlation in alternative

[10] Refer to appendix B for the differences between venture capital (VC) and buyout (BO) strategies. The growth in the private equity industry in recent years has come mainly from the buyout segment and industry practitioners at times use the terms ‘private equity’ and ‘buyout’ interchangeably.
[11] Leverage is implicit in buyout strategies to make opportunities profitable. An average over the period is used as investments stretch over 5 to 7 years. Private equity fund managers charge 20 per cent performance (‘skill’) based fees above an 8 percent hurdle rate, but can in theory achieve these excess returns by using a combination of beta and leverage, rather than skill. For example, a buyout manager may decide to leverage up 2 times by buying S&P futures and picks up 8 per cent over the hurdle (prior to financing cost). Investors can create the same payoff profile without having to pay performance based fees to the managers.
[12] A hybrid of debt and equity financing used to finance expansion of existing companies. Refer appendix A for more detail.

investments such as hedge funds, and show that the most likely explanation of serial correlation is illiquidity exposure.[13]

To remove the illiquidity effects we add the lagged dependent variable on the right hand side of the regression model as a liquidity factor. We find that to completely remove the serial dependence, we need to add up to three lags. In other words, private equity managers tend to rely on the previous three quarter valuations as an anchoring point for the present valuation. This is consistent with the number of lags in the multiperiod model suggested by Anson (2006), and reflects the 'stale pricing' methods employed by managers.

3.3 Risk factors

To test excess returns and based on the background literature review, we start with the Capital Asset Pricing Model (CAPM) introduced by Sharpe (1964). For venture capital, we add the returns to the NASDAQ. The NASDAQ is used as the main exit strategy for venture capitalists in terms of Initial Public Offerings for new startup companies in the technology/biotech sector. For completeness, we also test the Fama and French (1993) small cap and value factors, but do not find an improved fit, as the NASDAQ adequately captures the small cap/growth tilt in venture capital. For buyouts, we add a 50/50 mix of high yield and mezzanine debt, see Anson (2006).[14]

[13] The authors examine market inefficiencies, time varying expected returns, leverage and fee structures as possible sources of serial correlation before reaching their conclusions.

[14] High yield represents the returns on the Barclays Capital US high yield index. Mezzanine debt is represented by the returns on the Venture Economics Mezzanine debt index. In leveraged buyouts, mezzanine capital is used in conjunction with other securities to fund the purchase price of the company being acquired. Typically, mezzanine capital will be used to fill a financing gap between less expensive forms of financing (e.g., senior loans, second lien loan, high yield financings) and equity.

This factor captures the reliance of buyouts on the high yield credit markets which are the main source of their leverage financing.[15]

3.4 Conditional variable

Conditioning variables are introduced by Ferson and Schadt (1996) and examined for and found significant for alternative investments by Bird et al. (2010). Healy and Lo (2009) suggest applying the CBOE Volatility Index (VIX) as a conditioning variable. The VIX is a commonly used risk aversion indicator and readily observable.[16] We use the VIX to set up a dynamic overlay (hedge) whenever the VIX exceeds a certain threshold level to hedge out the portfolio betas. Such overlay strategies can be used to reposition the betas of an investor's entire portfolio, effectively rebalancing asset-class exposures without having to trade the less liquid underlying assets during periods of market dislocation.

3.5 Mean equation

Having identified the independent and conditioning variables, two different models as represented by equations (5) and (6) are used to measure the significance of excess returns created by private equity managers. Equations (5a) and (5b) represent the CAPM including an extension for leverage and illiquidity risk. For venture capital, the NASDAQ is added as an additional factor (5a). For buyouts, a 50/50 mix of mezzanine debt and high yield is added (5b).

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L} (R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,NQ} (R_{NQ,t} - R_{mt}) + \varepsilon_{it} \quad (5a)$$

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[15] In addition we test combined effects using both NASDAQ and high yield simultaneously for venture capital and buyouts, currency effects using the trade weighted U.S. dollar index (DXY), and international effects using the MSCI Europe, Australasia and Far East (EAFE). W do not find these factors significant.
[16] http://www.cboe.com/micro/vix/historical.aspx

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L} (R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,HY} (R_{HY,t} - R_{ft}) + \varepsilon_{it} \quad (5b)$$

Where R^{DL}_{it} is the deleveraged rate of return of manager i at time t, R_{ft} is the risk free rate at time t and α_i is the intercept term not explained by the multi-factor model. In terms of factor exposure, $(R_{mt} - R_{ft})$ represents the equity risk premium and $(R^{DL}_{i,t-j} - R_{f,t-j})$ the illiquidity premium based on the lagged dependent variables where we use up to three lags ($n=3$). Coefficients $\beta_{i,m}$ and $\beta_{i,L}$ measure the sensitivity of excess returns of manager i to the market and illiquidity premium.

The additional factors are represented by $(R_{NQ,t} - R_{mt})$ for equation (5a) reflecting the differential returns on NASDAQ stocks and larger capitalised companies, and $(R_{HY,t} - R_{ft})$ reflecting the credit premium obtained by investing in a mix of high yield and mezzanine debt for equation (5b).

Equations (6a) and (6b) extend equations (5a) and (5b) to include the Treynor and Mazuy (1966) market timing component $(R_{mt} - R_{ft})^2$. Under Treynor and Mazuy a significant positive coefficient ($\beta_{i,timing}$) suggests positive market timing ability. See Treynor and Mazuy (1966) or Bird et al. (2010) for a more detailed explanation.

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L} (R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,NQ} (R_{NQ,t} - R_{mt}) + \beta_{i,timing} (R_{mt} - R_{ft})^2 + \varepsilon_{it} \quad (6a)$$

$$R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m} (R_{mt} - R_{ft}) + \sum_{j=1}^{n} [\beta_{i,L} (R^{DL}_{ij,t-j} - R_{f,t-j})] + \beta_{i,HY} (R_{HY,t} - R_{ft}) + \beta_{i,timing} (R_{mt} - R_{ft})^2 + \varepsilon_{it} \quad (6b)$$

3.6 Variance equation

We find evidence of conditional clustering of variance in the residuals as the squared residuals exhibit long memory effects.[17] As private equity funds use leverage, we use maximum likelihood estimation to apply the GJR-GARCH version by Glosten, Jagannathan and Runkle (1993) which caters for asymmetry in the GARCH process in up and down markets.

Negative returns increase volatility more than positive returns due to the use of leverage. Under this model given the information set Ω at t-1 then $\varepsilon_{it} \mid \Omega_{t-1} \sim N(0,h_{it})$ and the conditional volatility is

$$h_{it} = \gamma_0 + \gamma_1 \varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3 \varepsilon^2_{i,t-1} I_{t-1} \quad (7)$$

where $I_{t-1} = 0$ if $\varepsilon_{i,t-1} \geq 0$, and $I_{t-1} = 1$ if $\varepsilon_{i,t-1} < 0$.

In this case, γ_0 represents a constant intercept impacting the long run unconditional volatility, γ_1 a weighting to the previous period's squared shock, γ_2 a weighting to the previous period's predicted volatility and γ_3 sensitivity to negative returns shocks. In private equity it is important to test the GJR model as we need to understand whether managers reduce volatility in returns as markets decline (the 'stale pricing' effect).

4. THE DATA

[17] For completeness, we also test more basic ARCH(1)..ARCH(4) models but find the squared error terms of the models retain the long memory effects. We are not aware of previous studies on the use of GARCH for private equity. Elyasiani, Getmansky and Mansur (2008) are among the first to investigate other alternative investments such as hedge funds using GARCH(1,1).

For our sample, we rely on quarterly data on the return on private equity funds using the Venture Economics indices[18], which cover over 2,184 private equity funds. We use aggregate performance of composites from January 1990 to December 2009, as prior to 1990 the different subindices are scarcely populated. In this case, the data is either not filled in or subject to the J-curve.[19] Venture Economics uses surveys sent to private equity funds and thus relies on self-reporting.

These surveys are voluntarily filled in on a confidential basis and not audited. A pooled return of all funds is then calculated by Venture Economics by treating all funds as a single fund by combining their monthly cash flows. This cash flow series is used to calculate a time-weighted rate of return.[20]

Chen et al. (2002) and Kaplan and Schoar (2005) suggest biases do not occur in the Venture Economics database, despite its voluntary reporting nature. First, Venture Economics' dataset is based on anonymous reporting. Thus there is no incentive to bias performance data upwards as funds cannot be marketed through the database. Second, private equity funds have a long and fixed lifespan (10 to 14 years). There is no incentive for a private equity manager to force early closure and no longer report returns, as long as fees can be collected. Third, Born et al. (2005) note successful and unsuccessful projects are intrinsically linked together within the performance of a fund. Thus, they argue unsuccessful investments are always included.

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[18] Venture Economics remains the most frequently used database in terms of academic research, see Kaplan and Schoar (2005), Conroy and Harris (2007) and Phalippou and Gottschalg (2009).

[19] The J-curve leads to a series of negative quarter returns for the first few years when the index is established, as the managers report losses during initial years when the fund is established but investments have yet to pay off.

[20] This method implicitly creates an asset weighted return and most closely matches the method that many investors use in measuring the return on their portfolio. Similar to a market-value weighted index in the equity market, this pooled method is considered the most appropriate method for presenting the aggregate performance of private equity funds. Venture Economics does not provide access to the cash flows themselves, but only aggregate measures.

The main biases in literature that have led to different conclusions on excess returns consist of sample selection bias and time period bias (for example, Peng, 2001, reports to the end of the tech bubble). We aim to minimise both biases by taking one of the largest possible datasets over as long as possible time period.

However, Venture Economics is estimated to cover only between 15 to 30 percent of funds in the US industry. Because of the private nature of the industry and its confidentiality, data is hard to come by. Apart from Venture Economics, researchers have tended to rely on self collected data, simulations or data from a limited set of undisclosed investors.

Grabenwarter and Weidig (2005) note that to fill out the database Venture Economics actively tries to complete data retrospectively, i.e. ask managers to add data on so far unreported funds and possibly only successful funds are reported. However, it can also be argued that since the database cannot be used for marketing, but is mainly used to measure performance against peers, there is an incentive to skew results downwards and thus also include poorly performing funds.

Despite all this, Venture Economics remains the industry standard for benchmarking. Increasing the limited coverage is beyond the control of Venture Economics as there is little incentive for funds to complete and return surveys of their performance as funds cannot be marketed through the database. This selection bias, as well as the anonymous nature of the database, are an important limitation of our research, and can mean results are still biased either upwards or downwards.[21]

[21] One other and unique bias to private equity is reported by Phalippou and Zollo (2005) and known as the "living dead" bias. The extent of this bias depends on the valuation of non-exited investments at the end of the sample period. Phalippou and Zollo were given access to data enabling them to calculate this bias by aggressively writing off residual values (rather than maintaining them at the last reported

5. EMPIRICAL RESULTS

5.1 Descriptive statistics

The descriptive statistics for the data are shown in table 2.

<< INSERT TABLE 2 >>

Based on the excess return column in table 2, the descriptive statistics suggest most strategies create excess returns when comparing deleveraged returns versus the risk free rate, but prior to adjustment for market and illiquidity risk factors. The skew and kurtosis adequately describe the experience investors have in these asset classes. Venture capital exhibits a large upside optionality (positive skew) which arises when portfolio companies go public at large multiples to their original investment. For venture capital, investing in smaller companies requires taking exposure to illiquidity, thereby leading to significant autocorrelation. Buyouts are more normally distributed than venture capital, as they invest in large and liquid companies. A negative skew comes from the leverage effect. When the buyout falls through, a large loss is absorbed by the investor.

5.2 Performance of GARCH models

Table 3 shows the estimated excess returns disappear under the GJR GARCH model, consistent with findings from literature from table 1. While excess returns (α) are detected for most strategies, in the range of 0.0034 percent per quarter (or 1.3 percent per annum) for venture capital and 0.0011 percent per quarter (or 0.44 percent per annum) for buyouts, these returns are not found to be statistically significant.

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values) on funds over 10 years old, over the period from 1980 to 1996. They argue the bias accounts for up to 2.5 to 3.5 percent of performance. However, it may be argued that Phalippou and Zollo's accounting methods are too aggressive. The evidence on the size of this bias is inconclusive at this stage and falls outside the scope of this paper.

<< INSERT TABLE 3 >>

As can be seen from table 3, equity (β_m) and illiquidity (β_L) exposure explain most of the returns. The significant of lagged valuations is also evident, $\beta_{L,t-1}$ to $\beta_{L,t-3}$ suggest that managers rely on up to three lags to value current assets. In addition, the NASDAQ (β_{NQ}) is a significant return driver for all venture capital type strategies. The high yield/mezzanine debt (β_{HY}) market is a significant driver for all buyout type strategies.

We find evidence of ARCH (γ_1) and GARCH (γ_2) effects, suggesting volatility persistence and clustering. These effects are more persistent in venture capital than buyout firms. More important, the leverage effect (γ_3) finds evidence that the reported volatility of the error terms (actual minus fitted observation) is higher during up markets. Specifically, we find at the 1999-2000 peak of the tech bubble, venture capital manager reported returns, which are based on self-valued assets, are much higher than those predicted by the factor model. On the other hand, during periods of market declines, the volatility of the errors decreases. This suggests an element of 'stale pricing': managers are more predisposed to revaluing assets upwards than they are to revaluing assets downwards.

The adjusted R^2 is high. The model is able to explain 71 percent of the variation in returns for all venture capital firms and 86 percent of the variation in returns for all buyout firms. This means that factor models are able to offer transparency in terms of understanding the return drivers, in an asset class that is commonly considered opaque in terms of the information disclosed by managers.

The results also offer a new angle on the fee debate. Private equity has traditionally been viewed as the domain of a handful of privileged institutions because of the expertise required to access private market opportunities, for which a commensurate fee is charged. If similar performance can be obtained using a mix of public markets instruments, factor models can offer a low cost alternative to direct investment.

5.3 Stability of factors

<< INSERT FIGURE 1>>

The seven year rolling averages for the betas in our model are presented in Figure 1. The stability demonstrated for these betas suggest the possibility of using these factors as the basis for replicating private equity returns. The other insight that we obtain from the information provided in Figure 1 is that the alphas for venture capital and buyouts have drifted around zero over most of our sample period. Positive excess returns during the 1990s tech bubble have been offset by the negative excess returns over 2000-2008. Thus, figure 1 suggests that alpha is episodic in nature for the private equity industry.

5.4 Strategies for improving performance

The high R^2 from our factor model and the stability of the betas has important implications for investors unable to access superior managers. Below we outline two strategies to replicate the returns generated by investments in both venture capital and buyouts.

5.4.1 Strategy 1: applying a factor approach during the commitment period

We rely on the illiquid asset model by Alexander and Takahashi (2001) to demonstrate the performance of directly investing in active managers versus passive factor models over a typical 12 year commitment period.[22] Prior to Swensen's arrival, Yale had been investing in buyouts since 1973 and venture capital since 1976. Based on Yale's experience, Alexander and Takahashi (2001) developed a model for assessing the impact of changing investment levels during the commitment period which has been fitted in terms of parameters based on Yale's actual experience.[23]

Figure 2 represents the investment pattern of a typical capital commitment. Figure 2 is based on Yale's default settings for contribution rates[24], fund life, committed capital and capital growth rates. When we cross reference Yale's settings with information provided by DeBrito et al. (2006) for the Venture Economics database from 1985 to 2004, we find that they substantiate the patterns provided by Yale. Figure 2 suggests that once capital is committed for the 12 year period, the majority of contributions by the investor are called up by the manager during the first 5 years of the commitment period. During this period the manager is searching for companies in which to invest, and starts distributing dividends and capital gains proceeds from year 3 onwards. Based on Yale's default settings for contribution rates, fund life, committed capital and capital growth rates, on average, 56 percent of committed capital is invested over the 12 years, which is shown by the horizontal line in Figure 2. Thus, an

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[22] For this analysis, we assume the fees to obtain factor exposure through e.g. futures or swaps in the derivatives markets are negligible as the number of required transactions is small. In general, transaction costs are low (a few basis points) and negotiable depending on investment size. Fees do not significantly alter the overall conclusions.

[23] Refer appendix C for a full description of the Yale model.

[24] Contribution refers to the capital called up each year from the investor whenever the manager finds suitable investments. Refer to appendix A for more detail.

overcommitment of at least 2 times is suggested and indeed practiced by many institutions.

<< INSERT FIGURE 2>>

The fact that fees are typically paid on committed, rather than invested capital, inflates the cost of investing in private equity, especially during the first 5 years when the overcommitment is greatest. The first strategy we propose enables investors to get exposure to private equity by creating a replicating portfolio which enables investors to reduce the costs associated with overcommitment. In the next paragraphs we describe the three steps used in a simulation where we compare the outcome from a direct investment in private equity versus obtaining the exposure by producing replicating portfolios based upon our factors.

1. **Simulation of direct investment returns.** A 'block' bootstrap method is employed by resampling with replacement from the available 20 year history (from 1990-2009) of active manager returns. This creates 1,000 runs of alternative 20 year paths of industry returns.[25] The 'block' refers to the resampling of continuous blocks of time (rolling 4 quarter periods) to capture the 3 period lagged dependencies detected in our factor model.

2. **Simulation of passive factor model returns.** Risk factors are resampled and then combined to form passive factor model returns at time t ($R_{clone,t}$).

[25] In effect, we create another 1,000 runs x 20 years x 4 rolling quarters = 80,000 data points to overcome the limited historical data set.

$$R_{clone,t} = \sum_{k=1}^{n} w_k F_{kt} \tag{8}$$

The portfolio weights w_k and the risk factors F_{kt} used to build up the clone returns are based on table 3. For venture capital the portfolio consists of 30 percent weight to the S&P 500, 40 percent to the NASDAQ and 30 percent cash. For buyouts we apply 10 percent weight to the S&P 500, 80 percent to high yield / mezzanine debt and 10 percent cash.

3. **Simulation of returns to investors**. We create 12 year commitment periods, whereby we replace the constant growth rate in the Yale model with the simulated results from either step 1 (to obtain the results from direct investment) or step 2 (to obtain the results from passive exposure). This affects the net asset value and distribution rates and the eventual rate of return experienced by the investor.

The factor model offers a number of immediate advantages. First, there is the absence of active management fees. Second, it is possible to instantly invest in all of the factors.[26] We exclude the illiquidity premium from the replication process, as there is no such instrument available.[27] Excluding illiquidity as a factor, the R^2 for venture capital reduces from 71 percent to 43 percent and for buyouts from 86 to 81 percent

[26] Mezzanine debt is less liquid. For the mezzanine debt component, for the first 2 years we use the liquid high yield credit market as a proxy investment, consistent with industry practice of the time allowed to get set in the asset class.
[27] In terms of creating an 'illiquidity' premium, Geltner (1993) suggests a formula to smooth returns to make it dependent on its previous lagged returns. This formula does not create any economic benefit, but it creates a smoothed return series with an autocorrelation in line with observed history.

which we still consider acceptable in terms of setting up a replicating portfolio as per Healy and Lo (2009).[28]

Table 4 examines the impact of a simulated allocation to private equity based on direct investment and the factor based approach using 1,000 simulated runs of 12 year commitment periods.

<<INSERT TABLE 4>>

As can be seen from Table 4, for venture capital, the factor model underperforms active managers by 14.2 – 8.0 = 6.2 percent. This comes as no surprise as we have given up two factors: the regression intercept and the illiquidity premium which exactly account for the 6 percent difference.[29] However, in return we obtain daily liquidity. Furthermore, 30 percent of the passive portfolio weight is allocated to cash and assumed to earn the risk free rate of return. In practice, the 30 percent invested in cash can be invested differently so as to increase the factor model returns closer to those of active managers.

The replicating portfolio produces a lower return. However, it produces a risk to reward ratio of 1.8 compared to 1.2 from a direct investment in venture capital. A 10.2 percent tracking error is observed between the factor based approach and the direct investment approach over the 12 year period due to the illiquidity (smoothed returns) of active managers. The factor model's worst possible run suggests a loss of minus 6.6 percent per annum over a 12 year period, compared to minus 1.2 percent

[28] Healy and Lo (2009) suggest the minimum R^2 needed to implement an effective overlay strategy is 25 percent, and they find a range of 25 to 75 percent for various hedge fund strategies.
[29] Under linear regression, the expected error (or $E[\varepsilon_i]$) is zero by definition.

per annum for the direct investment. Arguably, the increased downside risk is caused by inclusion of the NASDAQ as a risk factor, and its post tech bubble decline. On the other hand, it can also be argued that the factor model better represents risks than the smoothed returns obtained from the managers which are subject to stale pricing.

Table 4 furthermore suggests that the best 12 year period for active managers consists of a return of 123 percent per annum.[30] Once again, we argue that this is due to the overstatement of net asset values on the upside during the tech bubble (as we find from our GARCH model). We suggest the 21 percent per annum best 12 year period represented by the factor model represents a conservative and perhaps more accurate estimate.[31]

The results for buyouts in table 4 suggest the replicator is able to track buyouts with only a 1.9 percent tracking error. Compared to venture capital, we suggest this is because buyouts are more liquid and thus less prone to stale pricing issues. At the same time, the return dispersion is slightly wider: the worst possible period for the factor model is lower than that of the direct investment method, but the best possible period is also higher. This creates a somewhat lower reward to risk ratio than for direct investments. Again, capital market based factors are more likely to create higher volatility than smoothed returns provided by managers.

However, an additional benefit is that for buyout funds we can replicate the performance of equity in buyout firms using mainly credit instruments. Thus, a form

[30] This reflects four quarters in 1999 being resampled 12 times in a row as one of the possible outcomes.
[31] The ability to achieve 21 percent per annum after fees, for 12 years in a row would be considered remarkable, but not impossible by many investors.

of capital structure arbitrage (the arbitrage between different types of capital within a firm structure)[32] exists: debt ranks higher than equity for creditors claims, but in this case still offers higher returns than investing in the equity of buyout firms. This arbitrage opportunity may exist because of the long term nature of the investment and the capital required for this type of arbitrage.

In conclusion, the results from simulating strategy 1 are more promising for buyouts than for venture capital. The more liquid nature of buyout investments means a lower tracking error can be achieved. In addition, capital structure arbitrage can be considered.

5.4.2 Strategy 2: factor models as a conditional hedge

5.4.2.1 Usefulness of a hedging overlay

An alternative use of the high explanatory power in the factor model is the use of dynamic overlays to reduce downside risk. Our analysis starts with the question as to whether it is actually worthwhile to hedge downside risk, given that managers tend to smooth returns (offer stale pricing) during periods of market downturns and thus understate the downside risk associated with investment via active private equity managers.

<< INSERT TABLE 5 >>

Table 5 shows the performance of venture capital and buyout firms during up and down markets. The Treynor Mazuy coefficients (β_{timing}) suggest performance of

[32] Capital Structure arbitrage refers to mispricing within a firm's capital structure. Securities with more seniority in creditor claims should in theory offer less return. For example, shares are expected to return more than bonds based on the equity risk premium.

managers deteriorates during downmarkets. Although the coefficients are not statistically significant, they are overwhelmingly negative. Figure 3 depicts the relationship in a graphical format, whereby the curved line provides a quadratic estimation of a best fit line.

<< INSERT FIGURE 3 >>

From Figure 3, we see that the relationship is concave, rather than convex.[33] Thus, the suggestion is made that private equity funds underperform during down markets despite the ability to lag valuations. On a fundamental basis, small firms and levered buyouts tend to rely on sub investment grade debt, and financing becomes more difficult to obtain for these firms during periods of market stress than is the case for large investment grade firms with more established credit facilities. Figure 3 thus suggests how useful it might be to have the ability to hedge the exposure during periods of market stress.

5.4.2.2 Developing a hedging program

A successful hedging program requires that the risk factors in a linear risk model account for a significant fraction of the variability in the manager's returns, as we have found to be the case in our model. As Healy and Lo (2009) note, hedging these factors only during periods when the portfolio is deemed to be at higher risk and forgoing the overlay during other periods may seem like market-timing, but in fact is closer to volatility-timing, a considerably less daunting challenge. In fact, there is

[33] Under Treynor and Mazuy (1966) a convex relationship suggests a manager with good market timing ability: as the market declines, the manager's fund value should decline by less. As the market gains, the manager's fund value should increase by more. The outcome of figure 3 suggests the opposite.

evidence that volatility is both time-varying and persistent from our model, as we detect GARCH effects. Investors do respond dynamically to sharp changes in risk, which is consistent with a dynamic implementation of beta overlay strategies. Based on Healy and Lo, a hedge R_{ht} for strategy i can be constructed as follows:

$$R_{ht} = -\sum_{k=1}^{n} \beta_{ik} F_{kt} \tag{9}$$

Such that the sum of the R_{ht} and R_{it} contains no factor exposures.

$$R_{ht}+R_{it} = \alpha_i+\varepsilon_{it} \tag{10}$$

We apply an overlay method based on a number of assumptions regarding investor behaviour. First, we assume investors are concerned with volatility (VIX) as an important indicator of market conditions.[34] Second, we assume the hedge ratio used is determined by the investors' long term view on the asset class. We initially set the long term hedge ratios based on the information contained in Table 3, i.e. for venture capital we use 0.3 sensitivity to the S&P 500 and 0.4 to the NASDAQ. For buyouts we use a sensitivity of 0.1 to the S&P 500 and 0.8 to the high yield index as a base case.[35]

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[34] Various indicators exist to detect the presence of a 'risky' environment. Healy and Lo (2009) suggest a moving average system or an absolute VIX number. In our model, we test various hedge levels whenever the quarterly VIX in the preceding period exceeds long term historical stock volatility of 15 to 20 percent. We test 25, 30 and 35 levels.

[35] Our multifactor hedges require investors to be able to short the S&P 500 and NASDAQ futures for venture capital. In addition, for buyouts, only large investors may be able to purchase tailored credit default swaps (CDS) in the OTC market to obtain the short high yield /mezzanine credit spread exposure and it should be noted that CDS may not be always available to match these specifications.

The hedging process is easy to implement, especially given the limited number of parameters involved. We start by measuring the impact based on a single path: the historical returns realised under the market conditions existing from 1990 to 2009.

<< INSERT TABLE 6 >>

As can be seen from table 6, adopting the conditional hedge increases returns for both venture capital and buyout firms. At the same time risk, as defined by standard deviation, is reduced. The skew improves, as the distribution becomes more right tailed in both cases. For venture capital the kurtosis increases, for buyouts it decreases. In both cases we consider this an improvement. From table 2, we found venture capital to be right skewed (reflecting the possibility of exiting private companies at high earnings multiples in the public capital markets) and buyouts to be left skewed (reflecting the possibility of deal failure). Thus, the change in kurtosis under the conditional overlay increases the likelihood of positive tail events for venture capital, but at the same time decreases the possibility of negative tail events for buyouts. Specific downside risk measures such as semi-deviation (the standard deviation of down quarters only) and worst possible quarter also improve.

<< INSERT FIGURE 4 >>

Figure 4 illustrates these effects graphically. We find the conditional overlay turns the concave relationships we found earlier in figure 3 into more convex ones. As per figure 3, the curved line provides a quadratic estimation of a best fit line. Under

Treynor and Mazuy (1966) a positive quadratic relationship suggests applying the conditional hedge improves market timing ability for the hedged portfolio.

5.4.2.3 Robustness tests

Further robustness tests are performed by varying VIX trigger ranges for the hedge and betas to the risk factors to test the sensitivity of the hedge.

<<INSERT TABLE 7A>>

Table 7A measures the impact of changing the S&P betas under different VIX triggers in the hedging formula. The base case (default) hedge parameters are highlighted in the squared rectangle. A positive incremental return and risk reduction is noted if we vary the S&P beta over a wide range. The exception is the S&P beta to buyouts. Buyouts have a low base case beta of 0.1 to the S&P. Thus, shorting too many S&P futures (denoted in the table by an S&P beta above 0.5) will actually increase volatility as a short exposure to the S&P is taken which was not considered part of the original return drivers of buyouts as an asset class.

<<INSERT TABLE 7B>>

Table 7B provides comparable information to table 7A, but instead varies the NASDAQ factor for venture capital and the high yield premium for buyouts under different VIX trigger ranges. From the information provided in Tables 7A and 7B, the hedge is robust to a wide range of settings in the NASDAQ and high yield parameters.

So far we have discussed the hedge from a historical return perspective. As a final robustness test, we now compare the impact of the hedge under simulated market conditions. We simulate returns provided by the managers and the associated capital market factors required to set up the hedge (the S&P 500, the NASDAQ, the high yield premium and the VIX) for 1,000 runs of 20 years.[36] If the VIX is found to be above the hedge threshold during the current quarter, the hedging overlay is applied for the next quarter. At the end of each 20 year run, the difference in return and volatility between the unhedged and the conditionally hedged asset is compared and stored. In this manner a distribution of 1,000 runs is obtained as shown in table 7C.

<< INSERT TABLE 7C>>

Table 7C suggests that for the majority of runs, an increase in returns and at least some risk reduction is expected when applying the hedge across a large range of possible capital market conditions. For venture capital, we can be confident at the 90 percent level that the hedge improves excess returns while reducing risk. Only the 5th percentile suggests a lower return and higher risk from applying the hedge.

6. CONCLUSIONS

Our research confirms findings from literature in a more robust fashion. First, we do not find evidence of excess returns in private equity after adjusting for equity risk, illiquidity and leverage and volatility clustering. This is supportive of the arguments put forward by Berk and Green (2004) and the fee concerns raised by Phalippou (2009). It also concurs with recent findings by Ang et al. (2009) who based on an

[36] By simultaneously resampling the data across 4 rolling quarters (rows) as well as across economic variables (columns) we preserve the serial dependence within variables and also the relationships across variables.

extensive review conclude that *"there is little convincing evidence of superior risk adjusted returns to private equity and venture capital. Arguably some recent alternative vehicles simply repackage certain systematic factors in much more expensive forms."*[37] Confirming this argument, we find our proposed model suggests 70 to 90 percent of the variation in private equity returns is explainable by style tilts and that managers do engage in stale pricing up to three lags and more easily revalue fund values upwards than downwards.

Second, we suggest two strategies that can improve performance of this asset class to investors by reducing fees and creating liquidity. The strategies can be used independently or combined and can cause a rethink in asset allocation for what traditionally has been regarded by investors as a static commitment for a 10 to 14 year period. Our first proposed strategy helps reduce fees and increase liquidity during the commitment period. Furthermore, for buyouts, we suggest a form of capital structure arbitrage exists: similar returns to buyout managers can possibly be obtained by investing in a high yield credit. Our second proposed strategy suggests how excess returns, the reward to risk ratio, and liquidity can be improved by applying a conditional overlay to improve downside protection during periods of market stress. Both strategies are tested for robustness using historical data and simulation models.

A number of areas are of interest for further research. First, while literature suggests that the median private equity manager underperforms public markets, there is also

[37] Refer Ang, Goetzmann and Schaeffer (2009) in 'Evaluation of Active Management for the Norwegian Government Pension Fund'
http://www.regjeringen.no/upload/FIN/Statens%20pensjonsfond/rapporter/AGS%20Report.pdf

acknowledgement that U.S. Endowments have been very successful investors. A wide dispersion of manager returns and the persistence of top quartile performance by managers have been documented by e.g. Aigner et al. (2008). Thus, further research into the characteristics of these successful managers can substantially improve the excess returns accruing to investors, but the data limitations should be appreciated.

Second, investors who, unlike the U.S. endowments, find they cannot access top quartile managers can research ways of implementing fee reductions, e.g. through co-investment[38] or building in a style tilt exposure based on a factor model, rather than allocating to active private equity managers directly. Although our rolling beta analysis suggests these betas to be fairly constant, as the private equity industry continues to evolve, the weights and factors suggested by our model may reflect the contemporary nature of the industry. As more data points become available, more insight can be gained into the variability of these factors.

Third, biases have not been central to our discussion. Some biases are unique to private equity. The 'living dead' bias reported by Phalippou and Zollo (2005) reduces reported excess returns and investors may in reality experience an underperformance versus public markets. Apart from the lack of available data, some biases such as 'self selection' bias (whether or not funds decide to include themselves within a particular database) are notoriously hard to measure. Thus, our paper serves only as a starting point for further research on the fees, transparency and valuation methods currently offered by the industry.

[38] Typically, co-investors are existing limited partners. Co-investments are made outside of the existing fund and as such co-investors rarely pay management fees or carried interest on individual investments. Private equity firms offer co-investments to their largest and most important investors as an incentive to invest in future partnerships.

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Appendix A: Private equity industry terms

The following definitions are sourced from *the Thomson Reuters & The Australian Private Equity & Venture Capital Association Limited Yearbook 2009* and VC Experts' *The Glossary of Private Equity and Venture Capital (www.vcexperts.com/vce/library/encyclopedia/glossary.asp).*

Capital Call
When a venture capital firm has decided where it would like to invest, it approaches its investors in order to draw down the money. The money will already have been committed to the fund but this is the actual act of transferring the money.

Committed Capital
Capital committed by investors. Cash to the maximum of these commitments may be requested or drawn down by the private equity managers usually on a deal-by-deal basis. To the extent that capital invested does not equal capital committed, limited partners will have their private equity returns diluted by the much lower cash returns earned on the uninvested portion.

Early Stage Venture Capital
This is a fund investment strategy involving investment in companies for product development and initial marketing, manufacturing and sales activities. Revenues exist, but since this is a capital-intensive stage, profits are minimal if they exist at all.

General Partner (GP)
The partner in a limited partnership responsible for all management decisions of the partnership. The GP has a fiduciary responsibility to act for the benefit of the limited partners (LPs), and is fully liable for its actions.

Latter Stage Venture Capital
A fund investment strategy which provides financing for the growth of a company that has moved beyond the expansion stage to increase its sales volume and generate consistent growth. It is considered the last venture capital stage of financing prior to a liquidity event (i.e. an IPO or acquisition of the company).

Leveraged Buyout (LBO)
A takeover of a company, using a combination of equity and borrowed funds. Generally, the target company's assets act as the collateral for the loans taken out by the acquiring group. The acquiring group then repays the loan from the cash flow of the acquired company. For example, a group of investors may borrow funds, using the assets of the company as collateral, in order to take over a company. Or the management of the company may use this vehicle as a means to regain control of the company by converting a company from public to private. In most LBOs public shareholders receive a premium to the market price of the shares.

Limited Partner (LP)
An investor in a limited partnership who has no voice in the management of the partnership. LPs have limited liability and usually have priority over GPs upon liquidation of the partnership.

Mezzanine Debt
A hybrid of debt and equity financing that is typically used to finance the expansion of existing companies. Mezzanine financing is basically debt capital that gives the lender the rights to convert to an ownership or equity interest in the company if the loan is not paid back in time and in full. It is generally subordinated to debt provided by senior lenders such as banks and venture capital companies.

Seed Stage Venture Capital
An investment strategy involving portfolio companies at its earliest phase of development to promote a business concept before a company is started. Capital invested in companies at this point have not yet fully established commercial operations, and may also involve continued research and product development. Because it is the earliest stage of development, it is considered as the riskiest of the various financing stages.

Appendix B: Characteristics of venture capital and buyout firms

Private equity investments are defined as either venture capital (VC) or buyout (BO) funds. The following table from Anson (2006) summarises the key differences.

	Venture Capital (VC)	**Buyout (BO)**
Company	Start up	Mature
Competitive advantage	New technology	Distribution, marketing, production
Financing	Equity	Debt
Target IRR [1)]	40-50%	20-30%
Shareholder position	Minority	Control of company
Board Seats	1 or 2	All
Valuation	Compare to other companies	Discounted cash flow
Investment Strategy	Finance, innovation	Improve operating efficiency
Time to exit	2-5 years	4-7 years
Exit option	IPO, acquisition	IPO, acquisition or recapitalisation

IRR refers to the *Internal Rate of Return*, or the solution to the following equation:

$$-I_0 + \sum_{t=1}^{T} CF_t \;/\; (1+IRR)^t + NAV_T \;/\; (!+IRR)^T = 0$$

Where I_0 is the initial investment, CF_t is the net distribution at time t, and NAV_T is the estimated net asset value of yet to be liquidated holdings.

Appendix C: Simulation model

C1. Yale allocation model (standard settings)

The following allocation model to private equity is based on the Yale Endowment's experience. The model is able to incorporate and respond to actual experience. A more detailed description may be found in Alexander and Takahashi (2001).

Parameters	Description	Initial Settings (Yale)
RC	Rate of contribution	25% in year one, 33.3% in year 2, 50% in subsequent years
CC	Capital commitment ($)	$100
L	Life of fund (years)	12
B	Factor describing changes in the rate of distribution over time	2.5
G	Annual Growth rate	13%
Y	Yield (%)	0%
Variables		**Formula**
PIC	Paid in capital	$\sum_{0}^{t-1} C_t$
RD	Rate of distribution	Max[Y,(t/L)^B]
Outputs		**Formula**
C	Capital contributions ($)	$C_t=RC_t(CC-PIC_t)$
D	Distributions ($)	$D_t=RD*[NAV_{t-1}*(1+G)]$
NAV	Net Asset Value	$NAV_t=[NAV_{t-1}*(1+G)] + C_t-D_t$

The model is initially applied to a single path. C2 creates a simulation model (multiple paths).

C2. Combination with bootstrap (1000 runs)

G uses resampled output.

Venture Economics provides 20 years of quarterly observations (1990-2009), thereby offering 80 historical data points. An additional 1000 runs of industry growth during the 12 years of commitment are created by using resampling with replacement.

To arrive at the returns to investors, for each path the percentage invested capital is calculated based on the Yale model. The uninvested capital is assumed to be invested at the risk free rate. Thus, a weighted average return is calculated over the life of the partnership.

Figure 1 Seven year rolling beta and alpha experience with 95 percent confidence bands

Venture Capital **Buyouts**

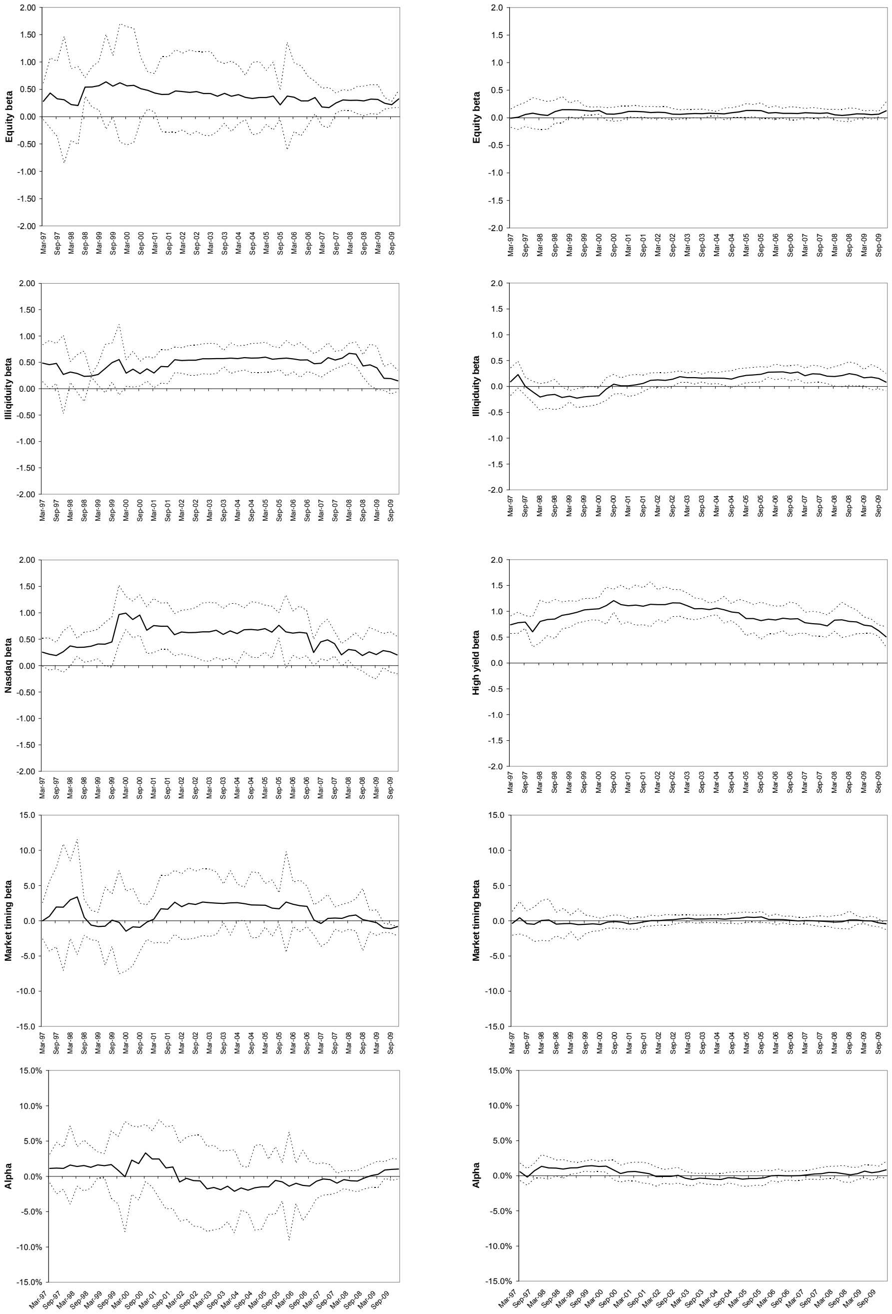


Figure 2 Base case for private equity allocation simulation model

Figure 2 suggests capital investment patterns based on Yale's default settings for contribution rates, fund life, committed capital and capital growth rates. On average, 56 percent of committed capital is invested over the 12 year period. A peak in invested capital is reached in year 5.

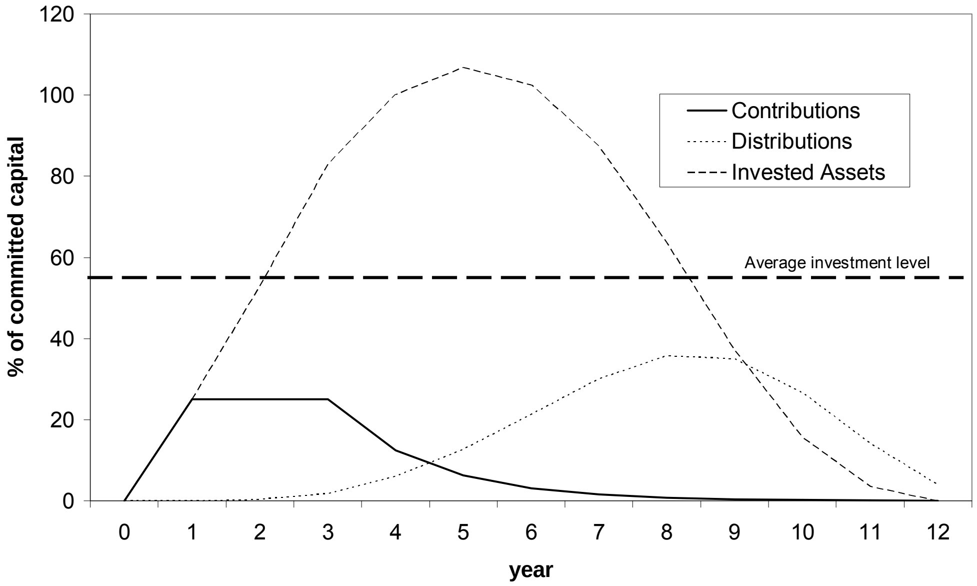


Figure 3 Evidence of market timing skill 1990 – 2009 (prior to conditional overlay)

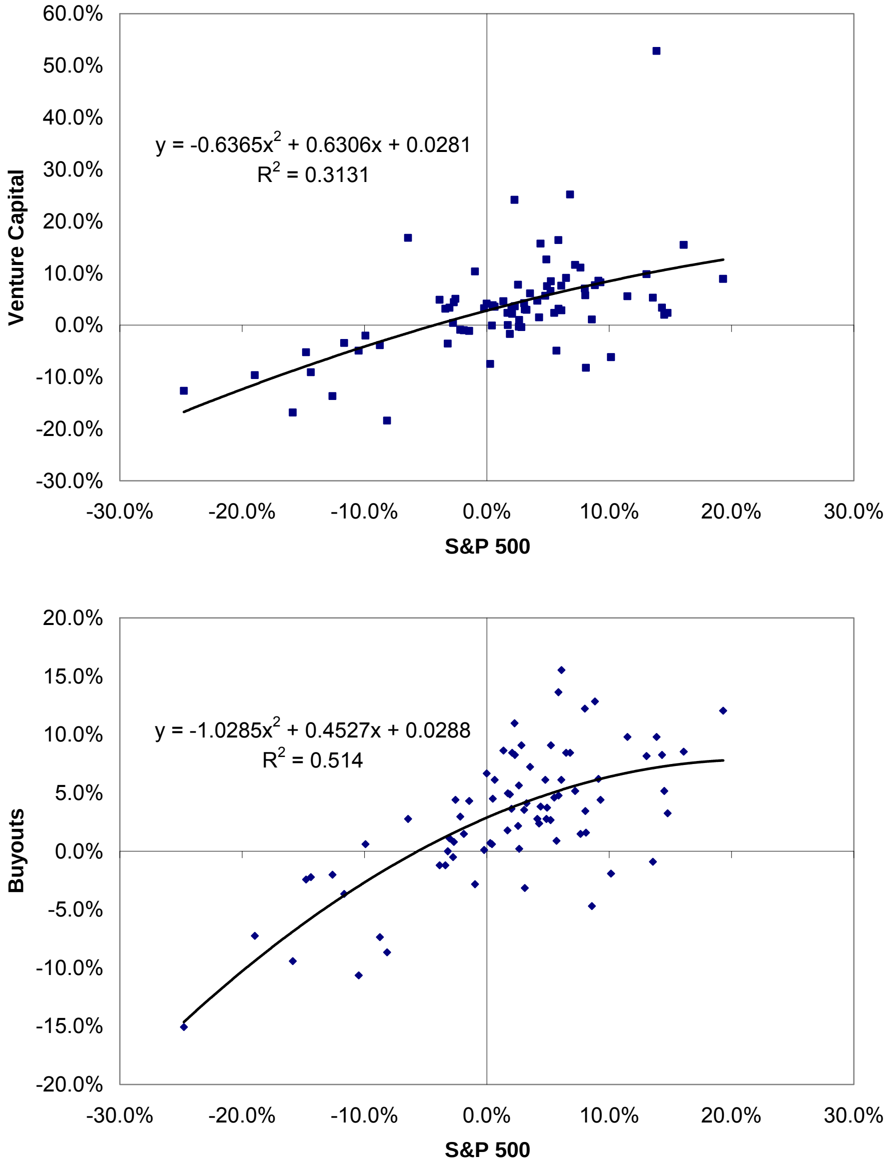
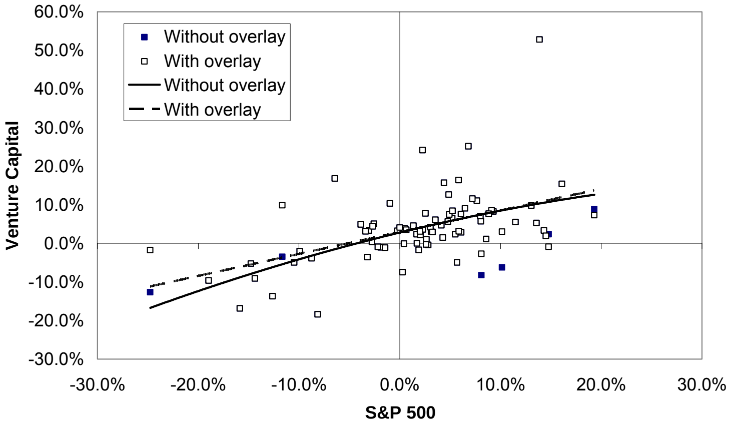


Figure 4 Impact of conditional overlay 1990-2009



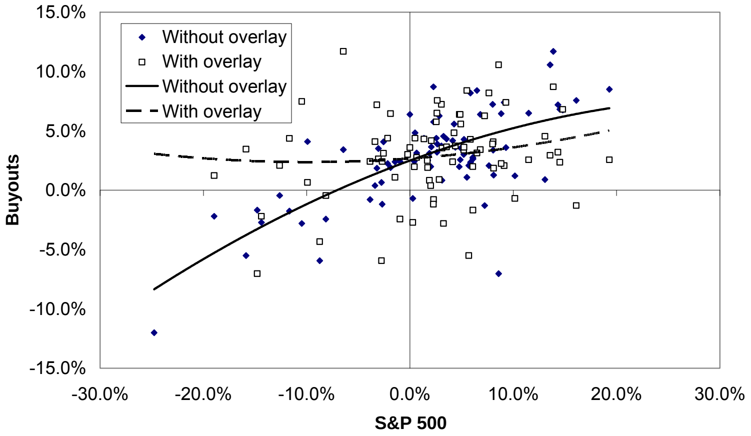


Table 1 An overview of private equity literature on factor (style) analysis

Paper	Factors (style benchmarks)	Results
Gompers and Lerner (1997)	1 factor: S&P 500.	Excess return found, but reduced after stale pricing adjustment.
Swensen (2000)	1 factor: S&P 500.	Buyouts underperform compared to an equally leveraged up version of the S&P 500.
Peng (2001)	1 factor: S&P 500 or NASDAQ.	Excess return found, but possibly reflects the sample period which ends with the tech bubble in 1999.
Moskowitz and Vissing-Jorgensen (2002)	1 factor: CRSP index.	No excess return found.
Quigley and Woodward (2003)	1 factor: S&P 500 or NASDAQ.	Excess return found, but based on self-built index, and performance to 1999 (time period bias).
Kaplan and Schoar (2005)	1 factor: S&P 500.	No excess return found.
Jones and Rhodes-Kropf (2003)	1 factor: CRSP value weighted index of NYSE, AMEX and NASDAQ stocks, lagged up to three periods.	Fama and French small cap and value premium. No excess return found.
Ljungqvist and Richardson (2003)	1 factor: S&P 500.	5 percent excess return found, but from a single undisclosed LP, with 73 investments (sample bias).
Ick (2005)	1 factor: S&P 500, NASDAQ, Russell 2000, Dow Jones Industrial Average, MSCI World.	No excess return found.
Anson (2006)	1 factor, S&P 500, lagged up to 4 periods.	Excess return found, but reduced after stale pricing.
Phalippou et al. (2005, 2007)	1 factor: S&P 500.	Negative 3 percent excess return after assuming biases.
Conroy and Harris (2007)	1 factor: S&P 500.	No excess return found.
Korteweg and Sorensen (2008)	3 factors: equity premium, Fama and French small cap and value premium, and Monte Carlo correction.	No excess return found.
Franzoni, Nowak and Phalippou (2009)	S&P 500, Fama and French small cap and value premium, liquidity factor corresponding to a long position in high liquidity stocks and a short position in low liquidity stocks (Pástor and Stambaugh, 2003).	No excess return found.

Table 2 Descriptive statistics of the private equity industry

Descriptive statistics of the private equity industry are based on quarterly data from March 1990 to December 2009. The data represents net of fees returns in USD on a lognormal basis based on the Venture Economics (VE) database. The total number of observations is 80 quarters. The total return is annualised by multiplying lognormal quarterly results times 4. Excess returns are calculated on a deleveraged basis versus the 3 month t-bill index. The standard deviation is annualised by multiplying quarterly standard deviation by the square root of 4. Best quarter represents the quarter with the highest quarterly return. Worst quarter represents the quarter with the lowest quarterly return. Beta is measured by calculating the slope of the strategy returns against the S&P 500 index. Skew represents the skew of the quarterly returns. Kurtosis refers to the excess kurtosis versus a normal distribution. Liquidity refers to the first order serial correlation.

	Total (%pa)	Excess (%pa)	Stddev(% pa)	Best quarter	Worst quarter	Beta	Skew	Kurtosis	Liquidity
Early/Seed VC	13.82%	10.0%	23.4%	58.8%	-22.8%	0.63	1.57	6.51	0.67
Seed Stage VC	6.70%	2.8%	17.3%	29.8%	-22.1%	0.36	0.69	1.81	0.25
Early Stage VC	14.06%	10.2%	23.7%	59.2%	-23.1%	0.64	1.56	6.40	0.67
Balanced VC	14.74%	10.9%	18.3%	53.9%	-17.1%	0.65	1.96	10.57	0.44
Latter Stage VC	14.76%	10.9%	14.9%	37.0%	-18.3%	0.57	0.50	4.80	0.42
All venture	14.41%	10.5%	19.0%	52.8%	-18.4%	0.63	1.60	8.45	0.57
Small buyouts	11.09%	6.9%	10.3%	13.8%	-12.0%	0.27	0.00	0.09	0.37
Medium buyouts	12.62%	8.3%	17.7%	55.7%	-14.5%	0.32	2.51	13.63	0.13
Large buyouts	10.94%	6.7%	12.6%	18.9%	-18.8%	0.30	-0.39	1.44	0.25
Mega buyouts	10.87%	6.6%	12.3%	18.8%	-15.7%	0.33	-0.32	1.13	0.13
All buyouts	11.17%	6.9%	11.3%	15.5%	-15.1%	0.32	-0.56	0.87	0.28

Model	(5a) $R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m}(R_{mt} - R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,NQ}(R_{NQ,t} - R_{mt}) + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1 \varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3 \varepsilon^2_{i,t-1} I_{t-1}$						(5b) $R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m}(R_{mt} - R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,HY}(R_{HY,t} - R_{mt}) + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1 \varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3 \varepsilon^2_{i,t-1} I_{t-1}$				
	Early/Seed Venture Capital	Seed Venture Capital	Early Stage Venture Capital	Balanced Venture Capital	Latter Stage Venture Capital	All Venture Capital	Small Buyout	Medium Buyout	Large Buyout	Mega Buyouts	All Buyouts
α	0.0007	0.0001	0.0003	0.0047	0.0038	0.0034	-0.0010	0.0004	-0.0003	-0.0020	0.0011
β_m	0.2630***	0.2356**	0.2443***	0.3937***	0.3105***	0.3300***	0.0071	0.0259	0.0341	0.0248	0.0728***
β_{t-1}	0.4262***	0.4329***	0.4297***	0.2166**	0.2521***	0.3060***	0.0584	0.0628	0.1169**	0.1096**	0.0346
β_{t-2}	0.2145***	0.2648***	0.2640***	0.2855***	0.3080***	0.2305***	0.1802***	0.2146***	0.1516***	0.1709***	0.1816***
β_{t-3}	0.0407	-0.0189	-0.0176	0.0076	0.0119	0.0593	0.2023***	0.1346***	0.1156***	0.0455	0.0434
β_{NQ}	0.4116***	0.4609***	0.4608***	0.4829***	0.5156***	0.3637***					
β_{HY}							0.8411***	0.7688***	0.7409***	0.8886***	0.7825***
γ_0	0.0004**	0.0008	0.0008*	0.0012	0.0001	0.0003***	0.0000	0.0001	0.0000	0.0001	0.0000
γ_1 (ARCH)	0.5418**	0.5001**	0.5071**	0.2883***	0.1194	0.4554**	0.0467	0.6828	0.2303	-0.0931	0.1088**
γ_2 (GARCH)	0.5557***	0.3759	0.3763	0.3796	0.8081	0.5273***	0.6494	0.2517	0.8210***	0.1884	0.9030***
γ_3	-0.6962***	-0.5994**	-0.6052**	-0.4337**	-0.0903	-0.5594***	0.0070	-0.1596	-0.01812	0.7389	-0.2781***
$\gamma_1 + \gamma_2 + 0.5\,\gamma_3$	0.7490	0.5763	0.5808	0.4511	0.8825	0.7030	0.6996	0.8547	0.9602	0.0953	0.8727
t-dist errors	14.46	16.02	15.88	18.92	6.57	13.88	15.64	3.89	3.30**	23.79	20.60
Adj R^2	0.7021	0.7128	0.7119	0.6227	0.7594	0.7142	0.8563	0.6851	0.7842	0.8431	0.8638
Log Likelihood	135.45	133.35	132.13	127.42	155.40	151.35	225.20	202.99	214.37	218.70	226.95

* significant at the 10 percent level ** significant at the 5 percent level *** significant at the 1 percent level

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Table 3 Results of GARCH model for quarterly deleveraged private equity fund returns (Augmented CAPM) 1990-2009

Table 3 presents the GARCH estimation of the augmented CAPM based on quarterly data from March 1990 to December 2009 based on mean equations (5a) and (5b) and variance equation (7). In the mean equation, the dependent variable (RDL_{it} - Rf_t) represents the deleveraged excess return of strategy i at time t and is regressed on the market risk premium ($Rm_t - Rf_t$), the illiquidity premium ($RDL_{i,t-}1$ - $R_{f,t-1}$), the NASDAQ premium ($RNQ_{,t} - Rm_t$) for venture capital or the credit premium($RHY_{,t} - Rf_t$) for buyouts. In the variance equation, the conditional variance of strategy i (h_{it}) is regressed on the error of the previous period ($\varepsilon^2_{i,t-1}$) and the variance of the previous period ($h_{i,t-1}$). The mean and variance equations used for each model are shown at the top of the respective tables. The R^2 has been adjusted to account for the degrees of freedom.

Table 4 Performance of direct investment versus factor model during commitment period

Table 4 presents a comparison of direct investment versus the factor based approach to private equity investing for 1000 simulation runs of 12 year commitment periods. Results are based on cash flow patterns as per the Yale allocation model. Simulated returns are generated by resampling (with replacement) over the period 1990 to 2009.

	Venture Capital		Buyouts	
	Direct investment	Factor mode'	Direct investment	Factor model
Return (%pa)	14.2%	8.0%	8.2%	8.8%
Risk (%)	11.8%	4.4%	2.0%	2.4%
Reward to risk	1.2	1.8	4.0	3.7
Best period (%pa)	123.0%	21.0%	15.1%	15.9%
Worst period (%pa)	-1.2%	-6.6%	2.6%	1.5%
Tracking error (%)	10.2%			1.9%

Returns are presented net of fees in USD and reflect the average annual return of possible return outcomes at the end of the 12 year commitment period. Risk reflects the standard deviation of average return outcomes at the end of the 12 year period. Reward to risk equals return divided by risk. Best period reflects the average annual return under the best possible 12 year outcome. Worst period reflects the average annual return under the worst possible 12 year period. Tracking error is calculated based on the standard deviation of the difference in outcome between the direct investment and the factor model at the end of the 12 year period.

Model	$(6a)\ R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m}(R_{mt} - R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,NQ}(R_{NQ,t} - R_{mt}) + \beta_{i,timing}(R_{mt} - R_{ft})^2 + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1}I_{t-1}$						$(6b)\ R^{DL}_{it} - R_{ft} = \alpha_i + \beta_{i,m}(R_{mt} - R_{ft}) + \sum (R^{DL}_{i,t-j} - R_{f,t-j}) + \beta_{i,HY}(R_{HY,t} - R_{mt}) + \beta_{i,timing}(R_{mt} - R_{ft})^2 + \varepsilon_{it}$ $h_{it} = \gamma_0 + \gamma_1\varepsilon^2_{i,t-1} + \gamma_2 h_{i,t-1} + \gamma_3\varepsilon^2_{i,t-1}I_{t-1}$				
	Early/Seed Venture Capital	Seed Venture Capital	Early Venture Capital	Balanced Venture Capital	Latter Stage Venture Capital^	All Venture Capital	Small Buyout	Medium Buyout	Large Buyout	Mega Buyouts	All Buyouts
α	0.0040	0.0042	0.0040	0.0069	-0.0020	0.0043	-0.0021	0.0015	-0.0004	-0.0038	0.0017
β_m	0.2073**	0.0175	0.2073**	0.3537***	0.2806***	0.3152***	0.0086	0.0291	0.0355	0.0348	0.0751***
β_{t-1}	0.4259***	0.2373***	0.4259***	0.2266***	0.3149***	0.2196***	0.0662	0.0454	0.1190**	0.1110***	0.0278
β_{t-2}	0.2659***	0.1823*	0.2659***	0.2371***	0.3042***	0.2212***	0.1825***	0.2150***	0.1508***	0.1785***	0.1717***
β_{t-3}	-0.0284	0.1883**	-0.0284	-0.0149	0.0947&	0.0717	0.2015***	0.1276**	0.1149***	0.0551	0.0269
β_{NQ}	0.4605***	0.5736***	0.4605***	0.3637***	0.4385***	0.4189***	0.8338***	0.7546***	0.7383***	0.8910***	0.7801***
β_{HY}											
β_{timing}	-0.4716	-0.3700	-0.4716	-0.2360	0.5433	-0.1336	-0.0002	-0.0767	0.0128	0.0655	-0.0646
γ_0	0.0007**	0.0011	0.0007**	0.0012	0.000	0.0001	0.0002	0.0001	0.0000	0.0001	0.0000***
γ_1 (ARCH)	0.4989**	0.7197	0.4989**	0.3291	0.0858	0.4413	-0.1218	0.7917	0.2495	-0.1307	0.0375
γ_2 (GARCH)	0.3724	0.4023	0.3724	0.3526	0.7011***	0.7720***	0.1388	0.2563	0.8053***	0.4677	0.9719***
γ_3	-0.5986**	-0.7798	-0.5986**	-0.4914	0.2431	-0.6563	-0.3843	-0.2953	-0.2071	0.5568	-0.2359***
$\gamma_1 + \gamma_2 + 0.5\ \gamma_3$	0.5720	0.7321	0.5720	0.4360	0.9085	0.8851	-0.4363	0.9003	0.9513	0.6154	0.8914
t-dist errors	15.66	662.07	15.66	3.69	7.54	2.82**	491.90	4.11	3.25	23.59	22.89
Adj R^2	0.7030	0.4310	0.7030	0.5853	0.7424	0.7056	0.8542	0.6752	0.7815	0.8321	0.8596
Log Likelihood	134.13	114.73	134.13	137.28	164.00	159.69	226.11	203.10	214.38	218.19	226.91

significant at the 10 percent level ** significant at the 5 percent level *** significant at the 1 percent level

^ A dummy variable is introduced as a variance regressor for the period from January 1995 to December 2009.to stabilise the GARCH model.

Table 5 Results of GARCH model for quarterly deleveraged private equity fund returns (Augmented Treynor and Mazuy model) 1990-2009

Table 5 presents the Treynor and Mazuy (market timing) model based on mean equations (6a) and (6b) and variance equation (8). The variables are as defined for table 3. In addition, in the mean equation, a market timing factor $(R_{mt} - R_{ft})^2$ is added.

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Table 6 Impact of conditional overlay 1990-2009

	without overlay		with overlay	
	VC	BO	VC	BO
Return (%pa)	14.4%	11.2%	16.1%	12.1%
Risk (%pa)	19.0%	7.8%	18.4%	7.2%
Skew	1.6	-0.8	1.7	-0.4
Kurtosis	8.5	2.2	9.5	0.6
Reward to risk	0.8	1.4	0.9	1.7
Downside risk measures				
Semi-deviation (%)	7.8%	3.7%	2.8%	1.7%
Worst quarter (%)	-18.4%	-12.0%	-18.4%	-7.0%

Table 6 represents the impact on Venture Capital (VC) and Buyouts (BO) of applying the conditional overlay on historical quarterly data. Returns are presented net of fees in USD based on annualised quarterly data. Risk is calculated by multiplying quarterly standard deviation by the square root of 4. Reward to risk is defined as the return divided by the risk. Skew represents the skew of the quarterly returns. Kurtosis refers to the excess kurtosis versus a normal distribution. Semi-deviation refers to the standard deviation of down quarters only. Worst quarter represents the worst possible quarter over the period measured.

Table 7 Robustness check for hedge overlay parameters (1990-2009)

7A Sensitivity versus the S&P 500

Venture Capital (VC)

Incremental Return (%pa) ^	VIX		
S&P beta	25	30	35
0.0	3.1%	2.0%	1.3%
0.1	2.8%	1.9%	1.3%
0.2	2.6%	1.8%	1.3%
0.3	2.3%	1.7%	1.3%
0.4	2.0%	1.6%	1.2%
0.5	1.7%	1.6%	1.2%
0.6	1.5%	1.8%	1.2%
0.7	1.2%	1.4%	1.1%
0.8	0.9%	1.3%	1.1%
0.9	0.6%	1.2%	1.1%
1.0	0.4%	1.2%	1.1%

Risk reduction (%pa)*	VIX		
S&P beta	25	30	35
0.0	4.5%	2.2%	1.5%
0.1	6.0%	2.8%	2.0%
0.2	7.0%	3.1%	2.3%
0.3	7.6%	3.2%	2.5%
0.4	7.8%	3.1%	2.4%
0.5	7.5%	1.8%	2.1%
0.6	6.8%	2.2%	1.6%
0.7	5.6%	1.5%	0.9%
0.8	4.1%	0.5%	0.1%
0.9	2.1%	-0.6%	-1.0%
1.0	-0.2%	-1.9%	-2.2%

Buyouts (BO)

Incremental Return (%pa) ^	VIX		
S&P beta	25	30	35
0.0	0.2%	1.0%	0.8%
0.1	-0.1%	0.9%	0.8%
0.2	-0.4%	0.8%	0.7%
0.3	-0.7%	0.8%	0.7%
0.4	-1.0%	0.6%	0.7%
0.5	-1.2%	0.6%	0.7%
0.6	-1.5%	0.5%	0.6%
0.7	-1.8%	0.4%	0.6%
0.8	-2.0%	0.3%	0.6%
0.9	-2.3%	0.3%	0.5%
1.0	-2.6%	0.2%	0.5%

Risk reduction (%pa)*	VIX		
S&P beta	25	30	35
0.0	0.7%	4.7%	4.7%
0.1	3.8%	7.3%	7.1%
0.2	4.4%	8.6%	8.2%
0.3	2.5%	8.6%	8.0%
0.4	-1.8%	6.4%	6.6%
0.5	-8.2%	6.3%	4.0%
0.6	-16.3%	-0.4%	20.0%
0.7	-25.9%	-5.5%	-4.3%
0.8	-36.4%	-11.6%	-10.7%
0.9	-48.0%	-18.3%	-15.8%
1.0	-60.4%	-25.6%	-23.6%

7B Sensitivity versus the NASDAQ (for VC) and the High Yield premium (for BO)

Venture Capital (VC)

Incremental Return (%pa) ^	VIX		
Nasdaq beta	25	30	35
0.0	-0.8%	-0.2%	-0.1%
0.1	0.0%	0.2%	0.2%
0.2	0.7%	0.7%	0.6%
0.3	1.5%	1.2%	0.9%
0.4	2.3%	1.7%	1.3%
0.5	3.1%	2.2%	1.6%
0.6	3.8%	2.7%	1.9%
0.7	4.6%	3.2%	2.3%
0.8	5.4%	3.7%	2.6%
0.9	6.2%	4.2%	2.9%
1.0	6.9%	4.6%	3.3%

Risk reduction (%pa)*	VIX		
Nasdaq beta	25	30	35
0.0	4.4%	0.6%	1.1%
0.1	6.5%	1.7%	1.7%
0.2	7.7%	2.5%	2.1%
0.3	8.1%	3.0%	2.4%
0.4	7.6%	3.2%	2.5%
0.5	6.3%	3.1%	2.4%
0.6	4.1%	2.8%	2.2%
0.7	1.2%	2.1%	1.8%
0.8	-2.4%	1.2%	1.2%
0.9	-6.6%	0.0%	0.5%
1.0	-11.4%	-1.5%	-0.3%

Buyouts (BO)

Incremental Return (%pa) ^	VIX		
High Yield Beta	25	30	35
0.0	-0.3%	-0.1%	-0.1%
0.1	-0.3%	0.0%	0.0%
0.2	-0.2%	0.2%	0.2%
0.3	-0.2%	0.3%	0.3%
0.4	-0.2%	0.4%	0.4%
0.5	-0.2%	0.5%	0.5%
0.6	-0.2%	0.6%	0.6%
0.7	-0.2%	0.8%	0.8%
0.8	-0.1%	0.9%	0.9%
0.9	-0.1%	1.0%	1.0%
1.0	-0.1%	1.2%	1.1%

Risk reduction (%pa)*	VIX		
High Yield Beta	25	30	35
0.0	7.1%	4.1%	4.1%
0.1	8.6%	5.1%	5.1%
0.2	9.9%	6.0%	6.0%
0.3	9.9%	6.8%	6.8%
0.4	9.8%	7.2%	7.2%
0.5	9.0%	7.6%	7.6%
0.6	7.8%	7.6%	7.6%
0.7	6.0%	7.6%	7.6%
0.8	3.8%	7.2%	7.2%
0.9	1.1%	6.7%	6.7%
1.0	-1.9%	5.9%	5.9%

^ Incremental return is calculated as the additional percentage return compared to the unhedged portfolio

* Risk reduction is calculated as the percentage reduction in annualised standard deviation compared to the unhedged portfolio

The numbers in the highlighted rectangle represent the base case for the hedge.

Table 7C Performance of conditional overlay (1000 runs of 20 years)

Venture Capital			Buyouts		
Percentile	Incremental Return (%pa)^	Risk Reduction (%pa)*	Percentile	Incremental Return (%pa)^	Risk Reduction (%pa)*
0.05	-0.2%	-0.2%	0.05	-0.3%	-1.5%
0.10	0.0%	0.2%	0.10	-0.2%	-0.7%
0.25	0.5%	1.2%	0.25	0.0%	0.4%
0.50	1.2%	2.3%	0.50	0.4%	3.8%
0.75	2.0%	4.0%	0.75	1.0%	7.8%
0.90	3.0%	6.2%	0.90	1.5%	10.6%
0.95	3.6%	7.9%	0.95	1.8%	12.0%

^ Incremental return is calculated as the additional percentage return compared to the unhedged portfolio

* Risk reduction is calculated as the percentage reduction in annualised standard deviation compared to the unhedged portfolio