The impact of discount rate choice in estimating the workout LGD

by

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Abstract

The workout approach to estimating the loss given default compares the actual value of the recovery flows with the exposure at default to measure the efficacy of the recovery process.
One of the main problems related to this approach is the selection of the proper discount rate for evaluating the portfolio. In the literature, there are different solutions proposed, but there is no evidence on the impact of the choice of one of these alternatives on the LGD measurement.
This paper looks at a proprietary database for the timeframe 1985-2005, evaluates the impact of the discount rate on the LGD value and studies the main determinants of LGDs computed using different approaches. Even if the explanatory variables are the same, LGDs defined using different discount rates show differences in the percentile distribution that could significantly affect the capital requirements of a financial intermediary.

JEL Codes: G21, G28

EFM Codes: 510, 130

Keywords: LGD, Discount rate, Workout approach

1. Introduction

Loss given default (henceforth LGD) is one of the main drivers of a portfolio's exposure to credit risk (Basel Committee on Banking Supervision, 2006). The LGD can be measured using different approaches: market LGD, implied market LGD and workout LGD (Schuermann, 2001).
One of the most important differences between these techniques is the assumption made in the first two approaches of an efficient market (Altman, Resti and Sironi, 2004); expectations of ultimate LGD are reflected in market prices, but other factors play a relevant role in the determination of ultimate losses (Carey and Gordy, 2005). International analyses have highlighted the difficulties in estimating the LGD with such approaches, especially for financial intermediaries (such as those in Europe), which do not frequently make public offerings of defaulted mortgage-backed securities (Araten, Jacobs and Varshney, 2004). The lack of market data for many countries forces to use the LGD workout approach for evaluating the intermediaries' credit portfolio exposure. Moreover, if the bank's policy is to service the defaulted assets, as European banks normally do, LGD estimation needs to be based on discounted workout recoveries (Brady, Chang, Miu, Ozdemir and Schwartz, 2006) .
This paper focuses on the workout approaches, and the aim is to study the relevance of the discount rate choice in determining the LGD estimates. Solutions proposed in the literature identify different proxies for the discount rate, such as the contractual rate (Asarnow and Edwards, 1995), the risk-free rate (Unal, Madan and Guntay, 2003) and some mono-factor models (Maclachlan, 2005). The available studies select only the discount rates that best fit the available data and compute the LGD without worrying about the impact of the discount rate choice on the LGD measurement.
Using a proprietary database of clients of a leading financial intermediary in Italy over a 20-year time horizon, we compare workout LGD estimates released using different discount rates. The results show that there is a difference between the LGDs computed using different discount rates, especially if the median value and the percentile distribution are taken into account, whereas, looking at the determinants of the LGD estimates, the drivers are unchanged when the different hypotheses about the discount rate are tested.
The paper presents a literature review for approaches to select the proper discount rate for estimation of the LGD (Section 2) and presents an empirical analysis for a proprietary database. In the empirical analysis, after a brief description of the sample considered (section 3.1), we present the methodology for computing the discount rate and the LGD (Section 3.2), and we present the results of the comparison between the different

LGD estimates (Section 3.3). In the last section, some brief conclusions and implications are presented (Section 4).

2. Literature review

The workout approach allows estimation of the LGD not only for bonds but also for lending solutions offered by financial intermediaries. The information for construction of the cash flow prospectus of the recovery process has to be sufficiently detailed to identify the amount and time of each cash flow related to the recovery procedure (Frye, 2004).
Once the timeline of the cash flows is evaluated, the evaluator has to select the proper discount rate to make flows at different times comparable. In the literature, different solutions are proposed:

- the contract rate applied to the customer;
- the risk-free rate;
- the correct yield rate, estimated using a single-factor approach.

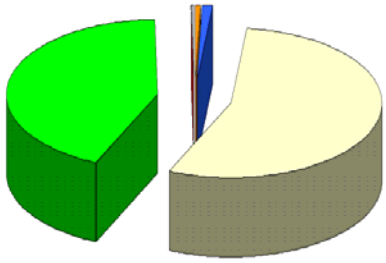
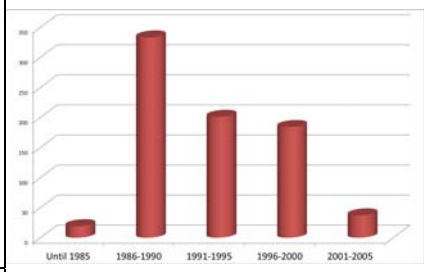
The contractual loan rate approach envisages that the flows recovered by the intermediary, after the state of insolvency has disclosed be discounted at the contract rate defined at the start of the relationship or at the last contractual rate renegotiated with the customer. The adoption of this approach can be deemed to be reasonable only if it is believed that the opportunity cost of the missing recovery of the sums at the contract due date can be correctly identified by this rate; it is assumed, then, that the appearance of the insolvency event does not modify the risk of the operation. The contractual rate approach makes it necessary to gather a complete internal information set, as any differences in the stipulated contracts have significant repercussions on the capacity to renegotiate the rates and, therefore, on their time for development. The use of mean or aggregate rates does not, then, represent a reasonable solution for estimating the LGD with this approach, and a complex information database has to be constructed that is primarily fed by the internal data gathered by the intermediary (Asarnow and Edwards, 1995).
The difficulties tied to the identification of the possible yield of the investment having characteristics similar to the financing granted can push the intermediary to choose to use the minimum opportunity cost for the time deferment of the repayments as the discount rate, i.e., the risk-free rate. The applicability of this approach is thus subordinate only to the identification of the reference market and to that of the best proxy available for the risk-free activity yield (Unal, Madan and Guntay, 2003). The appearance of the default event brings about the impossibility of foreseeing *ex ante* the amounts and the dates of appearance of the flows tied to the recovery process and thus brings about an increase in the variability of the repayment flows tied to the financing that is paid out. Even under the assumption that the risk-free rate represents a correct value for discounting future flows coming in for the intermediary before the appearance of the default, it is difficult to believe that the use of this rate is also correct when the flows lose their characteristic certainty. The decision to estimate the LGD with the risk-free rate approach can result in underestimation, as the current value of the flows generated by the recovery process would be computed without considering the greater degree of uncertainty that characterises the recovery flows.
The use of a risk-free rate can lead to underestimation of the loss in cases of insolvency because it is unlikely that an investment by the financial intermediary offers a yield no higher than the risk-free rate, as characterised by a non-zero risk of loss. A more credible solution relies on the use of a discount rate corrected for the estimated risk computed as the risk free rate plus a spread for the higher risk profile (Maclachlan, 2005). This approach assumes the possibility of identifying an index representative of the market risk for all debtors considered in the estimate of the LGD (Duellmann and Trapp, 2005). The analyses carried out with these approaches usually use indicators related to the average behaviour of the defaulted bonds negotiated in the market as a proxy for the market index (Altman, Brady, Resti and Sironi, 2005) or a proxy for the economy growth due to the strict relationship between the LGD and the economic cycle (Frye, 2000).

3. Empirical analysis

3.1 Sample

Data are collected from a proprietary database of a large Italian specialised financial intermediary and cover all the customers served in the timeframe 1984-2005. The number of customers served by the lender during the 20-year time horizon varies significantly over time due to growth; in 2005, the number of existing contracts was higher than 10,000. The mean default rate in Italy is normally lower than 2% (Bank of Italy, 2009), but we cannot consider all defaulted counterparties because the workout approach assumes that the recovery process has ended. The overall sample includes around 950 transactions for which all information about the amount and time scheduling of recovery process are available (Figure 1).

Figure 1 – The sample

Number of transactions considered for each area in Italy	*Number of transactions classified for type of customers*	*Number of transactions classified for vintage of the relationship*
		

Source: Data processed by the authors

Considering the geographical area, there is a high concentration in the more developed areas (such as Lombardia), and the role of small and not industrial areas (such as Valle d'Aosta, Trentino Alto Adige, Molise, and Basilicata) is residual.
Around 98% of the analysed counterparties come from the corporate and the retail sector, and the role of the other types of counterparties is residual. The sample composition is consistent with industry forecasts for these two sectors (plus project finance and structured finance) as consisting of high-risk portfolios; as a result, in LGD studies, the probability of obtaining an unbalanced sample on the basis of sector features is significantly high (BIS, 2005).
Looking at the starting date of the relationship between the customer and financial intermediary, more than 40% of the contracts analysed were signed from 1986 to 1990, and less than 6% of them can be attributed to the years before 1985 or after 2001.
Information given by the data provider enables the identification of the amount of inflows and outflows related to the recovery process for each customer in the portfolio and the time scheduling of these monetary flows. Even if the personal details of each customer are not available due to privacy law, some features of the customer (such as the vintage of the relationship, the sector of activity and the firm type) and some characteristics of the recovery process (such as the duration, the type of recovery process, the exposure to default, guarantees and the type of transaction) are provided.

3.2 Methodology

Coherently with the Italian supervisory guidelines for the financial intermediaries (Bank of Italy, 2001), we compute the result of the recovery process using the workout approach, and for each transaction, we measure the LGD on the basis of the following formula:

$$LGD_j^i = 1 - \frac{\sum_{t=j}^{n} \frac{F_t^i}{\left(1+r_t^i\right)^t}}{EAD_j^i}, \qquad (1)$$

where the LGD for the i[th] contract is defined on the basis of the ratio between the value at the time of default (j) of all flows related to the recovery process in the years from to j to n $\left(\sum_{t=j}^{n}\frac{F_t^i}{(1+r)^t}\right)$ with respect to the exposure at the time of default $\left(EAD_j^i\right)$.

To study the impact of the discount rate on the estimation of the LGD, we test different hypotheses proposed in the literature to define the current value of the recovery process. In detail, the discount rates considered are the following:

$$r_t^i = r^{CR}(i), \quad \text{(a)}$$

$$r_t^i = r_t^{RF}, \quad \text{(b)}$$

$$r_t^i = r_t^{RF} + \beta_2\left(GDP_t\right), \quad \text{(c)}$$

$$r_t^i = r_t^{RF} + \beta_1\left(r_t^{MKT} - r_t^{RF}\right), \quad \text{(d)}$$

where

$r^{CR}(i)$ = the last rate applied to the j[th] contract before the customer's default;

r_t^{RF} = the rate of return of Italian bonds with a one-year duration for the year t;

r_t^{MKT} = the rate of return of a defaulted corporate bond index (Lehman index – defaulted only) for the year t;

GDP_t = the Italian Gross Domestic Product for the year t;

β_1 = the Beta of the relationship between defaulted corporate bond index and the amount of recovery in the customers portfolio on the overall time horizon;

β_2 = the Beta of the relationship between the GDP and the amount of the recovery in the customer's portfolio on the overall time horizon.

Using the different approaches presented for the estimation of the discount rate, the LGD is computed using the following formulae:

$$LGD_j^i(CR) = 1 - \frac{\sum_{t=j}^{n}\frac{F_t^i}{\left(1+r^{CR}(i)\right)^t}}{EAD_j^i} \quad \text{(1a)}$$

$$LGD_j^i(RF) = 1 - \frac{\sum_{t=j}^{n}\frac{F_t^i}{\left(1+r_t^{RF}\right)^t}}{EAD_j^i} \quad \text{(1b)}$$

$$LGD_j^i(MF(MKT)) = 1 - \frac{\sum_{t=j}^{n}\frac{F_t^i}{\left[1+r_t^{RF}+\beta_1\left(GDP_t\right)\right]^t}}{EAD_j^i} \quad \text{(1c)}$$

$$LGD_j^i(MF(GDP)) = 1 - \frac{\sum_{t=j}^{n}\frac{F_t^i}{\left[1+r_t^{RF}+\beta_1\left(r_t^{MKT}-r_t^{RF}\right)\right]^t}}{EAD_j^i} \quad \text{(1d)}$$

A preliminary analysis of the impact of the discount rate on the estimation of the LGD is released using some summary statistics on the overall sample.

Once differences in the distribution of different types of LGDs for the same sample are identified following other studies proposed in the literature (e.g. Dermine and De Neto Carvalho, 2006), a study of the determinants of the recovery process performance is performed to test whether the choice of a different discount rate could affect the LGD determinants. The relationship studied is the following:

$$LGD_j^i = \alpha + \sum_{k=1}^{m} \chi_k AreaGeo_{ki} + \sum_{l=1}^{o} \delta_l Sector_{li} + \sum_{p=1}^{s} \phi_p Type\,of\ recovery_{pi} + \sum_{v=1}^{z} \varphi_v Guarantee_{vi} + \gamma Guarantee\,value / EAD_j^i + ,$$
$$+ \kappa \ln(EAD)_i + \lambda \,\mathrm{Recovery}\,duration_i + \theta V\mathrm{int}\,age_i + \tau Limited\ Partnership_i \qquad (2)$$

with

$$\sum_{k=1}^{m} \chi_k = 0 \qquad \sum_{l=1}^{o} \delta_l = 0 \qquad \sum_{p=1}^{s} \phi_p = 0 \qquad \sum_{v=1}^{z} \varphi_v = 0 ,$$

where

$\sum_{k=1}^{m} AreaGeo_{ki}$ = a set of 4 dummy variables (Northeast, Northwest, Central and South and Islands) that assume the value 1 if the defaulted customer is placed in this geographical area and 0 otherwise;

$\sum_{l=1}^{o} Sector_{l_i}$ = a set of 4 dummy variables (Public sector, Finance sector, Corporate and Retail) that assume the value 1 if the defaulted customer works in the l-sector and 0 otherwise;

$\sum_{p=1}^{s} Type\ of\ recovery_{pi}$ = a set of 3 dummy variables (Court, Out of court and Mixed) that assume the value 1 if the recovery process can classified as a p^{th}-type;

$\sum_{v=1}^{z} Guarantee_{vi}$ = a set of 2 dummy variables (Personal Guarantee and Real Guarantee) that assume the value 1 if the defaulted customer has the v type of guarantee;

$Gurantee\,value_i / EAD_i$ = the ratio between the value of the guarantee (real or personal) and the exposure at default for the i^{th} contract;

$\ln(EAD)_i$ = the natural logarithm of the value (in thousands) of the exposure at default;

$\mathrm{Recovery}\,duration_i$ = duration (in number of years) of the recovery process;

$V\mathrm{int}\,age_i$ = the starting year of the relationship between the customer and the financial intermediary;

$Limited\ Partnership_i$ = a dummy variable that assumes the value 1 if the customer is a limited partnership.

The sum of the coefficients of dummy variables related to the geographical area, sector, type of recovery process and guarantee are constrained to one. The approach adopted allows for these features to study the relative impact of each possible event with respect to all possible events.

The geographical features could affect the economic value of the LGD because the efficacy of judicial procedures is closely tied to the length of the recovery process and to the costs that the intermediary must sustain to enforce the guarantees he claims (Carey and Gordy, 2005). Empirical analyses proposed in the literature have also indicated for the Italian system the presence of a relationship between the geographic location of the court and the efficacy of the recovery process, thus supporting the need to make differentiated estimates depending on the geographic area to obtain correct evaluations of the LGD (De Laurentis and Riani, 2005).

The impact of the economic cycle is not independent of the debtor's sector of business and, for some business sectors, structural differences in the efficacy of the recovery processes can be brought out (Frye, 2000). 1998). The balance sheet assets of counterparties belonging to different sectors are not similar; consequently, the size of the recovery flows may be significantly influenced by the type of business carried on by the debtor (Carthy, Hamilton, Keenan, Moss, Mulvaney, Marshella and Subhas, 1998).

The effectiveness of the recovery process cannot be deemed to be independent of the type of recovery action because the mean duration of recovery processes differs significantly depending on the type of channels activated and on the activities that are potentially the subject of pursuit by creditors (Schuerman, 2004). Due to the availability of data, we distinguish only between court, out–of-court and mixed recovery channels.

Guarantees requested by the lender could be classified as personal or real guarantees, and the different types of guarantee offered could affect the usefulness in the recovery process (Peter, 2006). The relevance of each type of guarantee for the LGD estimates depends on the law and the procedure established for the recovery process. The role of the guarantee also depends on its value with respect to the exposure at default, with a higher ratio indicating a lower LGD for the contract analysed.

Due to some economies of scale in the recovery process and due to the type and time scheduling of the recovery process, the efficacy of the recovery process is normally higher for the largest exposures (Couwenberg and De Jong, 2008).
The duration of the recovery process could affect the results achieved, and normally the less time-consuming processes are those that ensure the better performance (Bank of Italy, 2001). Some authors support an alternative thesis that the duration of the recovery process is not relevant for itself but becomes relevant due to external macroeconomic factors that could affect the performance of the recovery process (Grunert and Weber, 2009).
The vintage represents a proxy of the strength of the relationship between the customer and the financial intermediary. As the duration of the relationship increases, the debtor will face greater problems in finding in the market other lenders who will offer him credit under the same conditions (Berger and Undell, 1995); moreover, during the course of the relationship, the lender is able to collect more information about the customer (Longhofer and Santos, 2000).
In the case of limited partnerships or listed companies, the lender is able to assert rights only on the firm through a judicial trial: this specificity is important if it is assumed that the recovery rate of an individual exposure is a function of the aggregate recovery rate of the company as a whole (Carey and Gordy, 2004). If the firm is a limited partnership, the risk after default is lower, and the recovery process is normally easier and less time consuming.

3.3 Results

The value of the LGD computed using different approaches for the discount rate for the overall sample is preliminarily analysed using summary statistics. (Table 1)

Table 1 – Summary statistics of LGD computed under different discount rate hypothesis

		LGD (CR)	LGD (RF)	LGD (MF(MKT)	LGD (MF(GDP))
Mean		0.5292	0.5051	0.5327	0.5085
Median		0.5994	0.5135	0.6208	0.5262
Variance		0.1932	0.1995	0.1923	0.1980
Minimum		0.0000	0.0000	0.0000	0.0000
Maximum		1.0000	1.0000	1.0000	1.0000
Skewness		-0.0948	-0.0102	0.1038	-0.0201
Kurtosis		1.1947	1.1696	1.1963	1.1741
Percentiles	10^{th}	0.0000	0.0000	0.0000	0.0000
	20^{th}	0.0000	0.0000	0.0000	0.0000
	30^{th}	0.0670	0.0299	0.0866	0.0332
	40^{th}	0.2289	0.1273	0.2392	0.1541
	50^{th}	0.5994	0.5135	0.6208	0.5262
	60^{th}	0.8904	0.847	0.9001	0.8503
	70^{th}	1.0000	1.0000	1.0000	1.0000
	80^{th}	1.0000	1.0000	1.0000	1.0000
	90^{th}	1.0000	1.0000	1.0000	1.0000

Source: Data processed by the authors

The mean value of the LGD is comparable with the international empirical evidence (such as Bruche and Gonzalez-Aguado, 2010) and coherent with the results obtained by other authors studying the same market using a proprietary database of another bank in a comparable time horizon (Caselli, Gatti and Querci, 2008). The choice of the discount rate could impact the mean value and the variance of the LGD distribution, but, due to the leptokurtic and asymmetric feature of the distribution, the impact is more clear on the median value with respect to the mean. Considering the percentile distribution, the analysis shows that more than 50% of cases in the extreme values (0 or 1) are consistent with the evidence obtained by other studies available in literature (Calabrese and Zenga, 2010). The contractual rate and the single-factor approach based

on the market index approach define a median value that is significantly higher with respect to other models and show a higher degree of asymmetry and skewness. The percentile distribution of the LGDs computed using different discount rates is not strictly comparable due to the significant differences in the value assigned to each percentile from the 30^{th} to the 60^{th}.
Looking at the determinants of the LGD estimated using different discount rates does not allow the identification of any significant differences in the explanatory features of the recovery process results (Table 2).

Table 2 – Explaining variables of the LGDs computed using different types of discount rate

		LGD (CR)	LGD (RF)	LGD (MF(MKT)	LGD (MF(GDP))
Geographical area	North-East	-0.0349	-0.0400	-0.0354	-0.0390
	North-West	-0.0121	-0.0067	-0.0121	-0.0066
	Center	-0.0201	-0.0274	-0.0194	-0.0270
	South and Islands	0.0670*	0.0741*	0.0669*	0.0725*
Sector	Public sector	0.0038	-0.0079	0.0064	-0.0054
	Finance sector	-0.0016	0.0257	-0.0039	0.0226
	Corporate	0.0641	0.0639	0.0629	0.0624
	Retail	-0.0663	-0.0817	-0.0654	-0.0796
Recovery process	Court	0.0876**	0.0894**	0.0907**	0.0897**
	Out of the court	-0.0527*	-0.0496*	-0.0534*	-0.0496*
	Mixed	-0.0349	-0.0398	-0.0374	-0.0401
Guarantee	Real	0.0114	-0.0654	0.0242	-0.0408
	Personal	-0.0105	-0.0118	-0.0070	-0.0088
Guaratee value/EAD		-0.0189*	-0.0186*	-0.0193*	-0.0187*
Ln(EAD)		-0.0447**	-0.0462**	-0.0442**	-0.0461**
Recovery duration		0.0035	-0.0004	0.0044	-0.0001
Limited partnership		-0.0468	-0.0635	-0.0455	-0.0616
Vintage		-0.0264**	-0.0290**	-0.0246**	-0.0289**
Constant		53.2753**	58.5808**	49.8097**	58.4034**
N° observation		792	792	792	792
F		12.44	12.8	11.83	12.78
Prob > F		0	0	0	0
MSE		0.3959	0.4011	0.3969	0.3996
R^2		0.1885	0.1933	0.1806	0.1932
Notes: * Significant at 95% level **Significant at 99% level					

Source: Data processed by the authors

The statistical fitness of the model is coherent with other studies on the loss given default; normally, this result is justifiable due to the high heterogeneity of recovery rates for the non-collateralised loans that could not be fully explained (Dermine and De Neto Carvalho, 2006).
Among geographical feature dummies, the only dummy that is statistically significant is the one that represents the South Island, where the relationship is identified is positive. The results support the thesis that customers in this area are riskier due to the lower efficiency of the recovery process (Giannotti and Gibilaro, 2009).
The sector does not significantly affect the results achieved by the recovery process; therefore, it is impossible to define the type of customer that riskier for the bank. The lack of significance of the results could be explained on the basis of the sample composition (around 98% of the contracts represent the corporate and retail sector) and due to the lack of detailed data necessary to define a more detailed cluster of the sample on the basis of the type of activity.

Due to the mean higher duration and cost of the court recovery process (Djakanov, La Porta, De Silanes and Schleifer 2002), the choice to adopt an out–of-the-court solution enables the reduction of the LGD, whereas the court solution negatively affects the efficacy or the recovery process.
Looking at the guarantees, the type of guarantee is not statistically relevant in explaining the LGD, whereas the amount guaranteed is significant. The results demonstrate that a higher ratio between the amount granted and the EAD leads to a lower LGD and supports the thesis that in the case of default, when provided, the guarantee payments are the main driver of the recovery process (Caselli, Gatti and Querci, 2008).
The LGD is affected by the size of the exposure; normally, a larger EAD leads to a better action of the recovery process. This result is consistent with the theory of economies of scale in the recovery process and with a standard incentive scheme applied to the recovery function for the financial intermediary (Malinconico, 2008).
The duration of the recovery process seems to be unrelated to the results of the recovery process; also, the existence of a limited partnership form does not affect the LGD.
The vintage negatively and significantly affects the results of the recovery process; consequently, the newest contracts are those that show a more efficient recovery process in the default case. This result can be considered reasonable if we assume that over the years, there was a change in the approach adopted for the LGD risk (Belotti, 2010) and, more specifically, that over the last year, the attention given to this type of risk is increasing over time.

4. Conclusions

The choice of the discount rate in estimating the LGD could significantly impact its value, and the evaluator can reduce the mean value of the risk factor by around 3% by selecting the proper discount rate. Due to the characteristics of the LGD distribution, the main differences among different estimates attain the median value and the percentile distribution: the median value could differ up to 10%, and the percentile distribution in normally non-comparable.
Considering the explanatory factor for the LGD, no differences could be assigned to only the choice of the discount rate. An evaluator could work on the discount rate to modify (residually) the value of the LGD, but the main drivers of this risk are always related to geographic features, the type of recovery process, the amount of the guarantee, the amount of exposure and the vintage.
On the basis of the current regulatory framework, the capital requirement is linearly related to the mean value of the LGD (Basel Committee of Banking Supervision, 2006). Due to the direct relationship, all changes (also the smallest ones) in the LGD estimates for a customer's portfolio of the bank could significantly affect its performance because there is a direct increase in the stock of capital required for the business.
A further development of the work leads to the possibility of using more complex approaches to construct the discount rate using a multi-factor approach that takes into account the drivers of the recovery process in the formula Gibilaro and Mattarocci, 2007). This solution could allow to allow to identify a LGD distribution with lower asymmetry and lower kurtosis and probably could also impact on the drivers of the recovery risk exposure.

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