New efficient frontier: Can structured products really improve risk-return profile?

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Abstract

In this paper we investigate the contribution of structured bonds to the efficient frontier. We conduct our analysis by simulating the term structure according to a no-arbitrage multifactor model (G2++) and comparing the performance of basic products (like zero-coupon bond, coupon bond and floating rate notes) with respect to more sophisticated products (like cms, collars, spread and volatility notes). In particular, our analysis considers different initial market environment like interest rate term structure shapes, as well as volatility and correlation in its changes and takes into account how the combined effect of risk-premium required by investors and fees that they have to pay can change the portfolio allocation respect to the one made only of basic securities. Our simulation results show that structured products can be an interesting investment only under particular scenarios. However, in general the return net of the fees in these securities is in average lower than the return in basic securities.

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1 Introduction

Structured products are a special type of financial products that may go by various names: principal protected notes, accelerated return notes, range notes, barrier notes are just a few example. There is not a unique definition of structured products. In a broad sense structured products can be defined as a combination of financial products in a new structure that provides original payoff. Structured products can also be defined as those products built to tailor specific investment purposes of clients. Another definition considers structured products as those instruments whose performance are linked to the performance of an underlying security such as stocks, basket of stocks, interest rates, commodities. Structured products are offered by an issuer (usually a large bank) as medium term notes with a term that can vary from a few months to several years. Considering all the different elements together, we could define structured products as debt instruments with embedded derivatives designed so to tailor made risk/return profiles for investors.

Structured products have known, in the last years, a significant growth in many countries and they became retail investment products. Structured products are in fact now owned by a large spectrum of clients ranging from institutional to retail individuals.

But what can explain the growth of structured products? A first possible explanation is that structured products, with their combination of different instruments allow to generate any desired payoff and to match any desired wealth distribution. In this view structured products increase the investment opportunities available. Very risk avers investors, who prefer certainty above all, invest all their money in a state deposit. Risk taker investors select risky assets. But some other investors may wish to seek for capital protection while taking advantage of increasing markets as well. These investors may prefer a product that protects their initial capital increase by a given percentage of the growth (if any) of an underlying asset. So structured products can be designed for risk avers investors aiming at investing in risky assets but with a protected capital. From a theoretical point of view this kind of structured products can be considered a redundant asset. In fact a structured product linked to an index that gives at maturity the capital protection plus a performance linked to the underlying index is built as a combination of bonds and options. It can be replicated by an investor that purchase the bond by himself and replicate the option with a combination of the risk-free asset and a short position in a stock. If continuous hedging was possible the investor could create structured products by himself combining stocks and bonds. But this is not the case. Furthermore concave payout structured products that combines a position in the underlying asset with a short position in an option cannot be easily replicated by investors. So structured products can create value by offering risk return profiles that cannot be easily replicated with traditional financial instruments. On one side structured products provide exposure to non-traditional asset classes, such as commodities, to which investors might not have direct access. On the other side the payoff functions provided by structured products have special characteristics such as minimum or maximum payoff or non-linear payoff otherwise not available to investors. Traditional products are built on a long/short view: the investor buy traditional products (stocks, bonds, funds) if an increase of the price is expected and sell in the opposite case. However other variables have recently shown their importance on portfolios. An example can be considered volatility that in recent times has received an increased interest by investors. Volatility is known to be negatively correlated with stock index return. Adding volatility exposure to a portfolio should improve diversification. Traditional financial products do not offer an exposure to volatility. Structured products built as options combination and incorporating derivatives allow designing different payoff and new strategies. If this is true structured products should be considered as a way to diversify portfolios adding new strategies. Despite this view, structured products are generally sold as separate financial products without taking into consideration the impact that they might have on portfolio risk and return. They are placed to investors no matter what is the underlying portfolio and without taking into consideration the correlations between these products and others and the impact that they might have on the total portfolio risk. But if one of the benefits of structured products is instead to offer exposure to new assets or to new payoff strategies selling them as separated instruments give away all the potential benefits of payoff diversification. Also clients and financial advisors claims that is difficult to understand how structured products might contribute to an existing portfolios. The construction of tools to optimize the overall portfolio, including structured products is then of particularly importance to fully understand the potential benefits of these products.

Another important issue related to structured products is the impact of fees charged on the final performance of the instrument. These products are in general perceived as being costly, overly complex and low transparent. Investors might have difficulties on understanding all relevant characteristics of complex products. Furthermore, financial institutions and retail clients have strong information asymmetries on pricing the products. For clients is very difficult to price complicated exotic options. This allows financial institutions to charge high fees that are not fully displayed to investors but that significantly reduce the final performance of the instrument. The premium of structured products is very relevant but often not disclaimed to investors. An estimation of the impact of different fee level is then another important issue to consider when valuing the net advantages of investing in structured products.

Finally, we remark that the market for structured products is quite different across Europe. For example Hens and Rieger (2009) noted that in 2007 alone structured products amounted to almost seven percent of total market capitalization in Germany, and they amounted to more than seven percent of market capitalization in Switzerland. Instead, as reported in Kjos (2010), the Financial Supervisory Authority of Norway wrote a circular (4/2008) that to a large extent put an end to structured products in Norway. In the circular the FSA Norway presupposed that institutions should not sell structured products to customers who could not be regarded as professional investors. Moreover, the FSA Norway advised institutions against offering debt financing when selling structured products (Finanstilsynet, 2008). The FSA Norway emphasized that the financial institutions have a duty to do an assessment of the client, and have a duty to inform about all costs related to the investment[1]. In Italy, the

[1] Kjos also reports that in a press release related to the circular, the Director General at the FSA stated: "The new regulations mean in practice a complete stop to the purchase of structured products financed by loans. Further, the regulations mean that banks and other financial institutions will normally not be selling such products to normal savers, who cannot be regarded as professionals in this context."

insurance regulator Isvap (Istituto per la Vigilanza sulle Assicurazioni Private e di Interesse Collettivo), and the securities market regulator Consob (Commissione Nazionale per le Società e la Borsa) have introduced new rules that make it harder to sell structured products. Consob set out new rules in March 2009 that require product distributors to formally distinguish between products considered liquid and those considered illiquid. For products considered illiquid - which includes over-the-counter derivatives-based investments - new documentation, pricing and reporting standards have been introduced for the distributors. As reported in Ferry (2009), whilst before a small regional bank would be free to sell structured products it had sourced from an investment bank as long as it provided the buyers with the appropriate documentation, risk warnings and so on, the new ruling from Consob, however, means that the bank may only sell such products if it has the in-house ability to price each component of the underlying structure and monitor that pricing on a continuing basis.

In this paper we investigate the contribution of structured products to the efficient frontier. If structured products are created to offer new payoff not otherwise available, they should give a positive contribution to the efficient frontier construction allowing an improvement of the risk return profile. We focus only on interest rate linked since our goal is to analyze the contribution of how new payoff profiles, created with structured products, can eventually improve the efficient frontier. We are not interested to study the contribution of structured products in term of asset diversification since there are many papers investigating how a new underlying (commodities, exchange rates, etc.) can contribute to the efficient frontiers. Our interest is on the payoff strategies diversification. We consider a base portfolio made of traditional interest rate products (zero coupon, fixed and floating coupon bonds) and we add structured products to this portfolio. The structured products we consider are a constant maturity swap, a collared floating rate note that is a floating rate note with a minimum and a maximum coupon, a constant maturity swap with a collar, a spread note with a performance linked to the difference between two swap rates and a volatility note with a performance linked to the absolute value of the difference between a swap rate and a fixed amount. In spite of its simplicity our base portfolio is very significant for Italian retail investors that are traditionally bondholders and allocate to the equity component only small portion of their wealth. Furthermore, at the best of our knowledge there are no previous studies on the contribution of interest rate linked structured products to the efficient frontier. We also investigate how this convenience is robust to different initial market environment like interest rate term structure shapes, as well as volatility and correlation in its changes. We also examine how the combined effect of risk-premium required by investors and fees that they have to pay can change the portfolio allocation respect to the one made only of basic securities.

2 Literature review

Most academic papers studying structured products have focused on pricing related issues (Chen and Kesinger (1990), Wasserfallen and Schenk (1996), Burth Kraus and Wohlwend (2001), Stoimenov and Wilkens (2005) and Baule, Entrop and Wilkens (2007) between others). These studies focus on the difference between the quoted structured product price and the theoretical fair value and they arrive at the conclusion that structured products are on average mispriced.

More recently academics have started to investigate the question of what could explain the growth of structured products and which benefits structured products offer.

Rieger (2007) analyses the properties that a product should have to maximize the utility function of an investor. Assuming that financial markets are efficient, that market participants have homogeneous beliefs and maximize their utility and finally that they allocate all their total wealth into structured products, Rieger seeks for the optimal portfolio without imposing mean variance rational optimization but allowing also for behavioral based models. Results obtained show that optimal products follow the market, that is they are co-monotone with the market portfolio (in the case of the CAPM) or with the inverted state price function (in the general case). If this is true on designing new products financial engineers should look for co-monotonocity. They apply their conclusion to the case of down and out barrier products. These are investments that offer capital protection plus a positive performance (if the underlying asset increases) as long as the underlying does not hit a given barrier. In the case of down and out products the barrier is below the initial underlying price. If the barrier is hit the capital protection is gone and the final result of the investment will depend on the underlying performance. These products are not co-monotone with the CAPM and so the author concludes that they are not optimal in spite of their huge diffusion. Finally Rieger explains the market success of these products in terms of investors underestimating the probability to hit the barrier.

Branger and Bruer (2008) analyze if retail investors with a buy and hold trading strategy can benefit from an investment in structured products. They focus on the German market and the investment on Certificates that are structured products designed to allow retail investors to access the derivatives market. In Branger and Bruer model, investors can allocate their money into stocks, a money market account and one certificate (bonus, discount, spring, turbo). Using a constant relative risk aversion utility function and under the hypothesis that investors maximize their expected utility in a market with stochastic volatility and jumps, they show that the benefit of investing in typical retail products is equivalent to an annualized risk-free excess return of at most 35 basis points. Taking into consideration transaction costs the benefits are reduced to 14 basis points. They conclude that if investors have a constant relative risk aversion than the growing demand for retail derivatives is not explained.

Jessen and Lochte (2008) develop an optimal portfolio choice model to describe the role of structured bonds in holdings of small retail investors making reference to the Danish market. The set of investment opportunities available to investors are a risk free asset a risky reference fund and a structured product. Jessen and Lochte model is based on the hypothesis that small retail investors are very risk averse but rational, maximize their expected terminal utility and hold the investment through the time period. They first show then if a constant relative risk aversion (CRRA) utility function is assumed then structured products are redundant assets. More specifically, Jessen and Lochte show that with a CRRA utility the presence of structured products in retail investors optimal portfolios is a function of the correlation between the risky asset available to retail investors and the risky asset not available to retail investors if not with structured products. When the correlation increase the relevance of structured products in optimal portfolios decreases. The demand for structured products is also a function of risk aversion. Very risk avers investors deposit large part of their wealth on a bank account and the relevance of structured products in their portfolio is limited. Low risk adverse investors choose to allocate their wealth to the risky asset. Structured products have the highest relevance for medium risk aversion level. But when a decreasing relative risk aversion utility function is assumed (DRRA) then structured products remain relevant in portfolios also for high levels of risk aversion. Jessen and Lochte also show that the portion of structured products is very sensitive to change in cost of construction: when the total cost of construction reach a level of 6%-7% structured products are pushed out from portfolios.

Henderson and Pearson (2009) investigate the dark side of financial innovation concluding that if some groups of investors misunderstand financial markets or suffers from cognitive biases that make them to assign incorrect probability weights to events, financial institutions can exploit these biases creating products that pay off in the states that investors overweight and do not payoff in the states that investors underweight leading investors to misprice the new instruments and assign a value that is greater than the fair value. In this view structured products are created to allow financial institution to gains from the willingeness of investors to overapay.

Hens and Rieger (2009) analyze the benefits in term of utility gains that can be achieved using structured products to deviate from a linear exposure. They show that some of the most used structured products are not optimal for rational retail investors if the utility function is concave. Using different, non concave utility function the gains become significant but still too small to compensate premium costs. They conclude that behavioral factors such as loss-aversion or probability mis-estimation more than utility gains explain the growing demand for structured products.

3 The products

The paper focus on interest rate structured products having a maturity of five years with different structures. We assume that the bonds are default free. The structure of the different bonds consists in a periodic payment (fixed or variable) $N \times \alpha_{i-1,i} \times c_i$ at times t_i and the payment of the notional N at maturity t_n

$$\pi(t_i) = \begin{cases} N \times c_i, & i = 1, ..., n-1, \\ \\ N \times (1 + \alpha_{n-1,n} \times c_i), & i = n, \end{cases}$$

where $\alpha_{i-1,i}$ is the accrual factor between dates t_{i-1} and t_i.

In the following we denote by $P(t, t_n)$ the time t discount factor for maturity t_n and with $\alpha_{i-1,i}$ the fraction of time (computed according to a given day convention) between two successive payment dates, i.e. t_{i-1} and t_i, $i = 1, ...n$ and $t_0 = t$. In general, $P(t, t_i)$ is constructed using market quotations of Euribor rates and swap rates trough a bootstrapping procedure. Euribor and swap rates are often also used as reference rates in the determination of the coupon payment. We denote with $E(t, t + \tau)$ the Euribor rate quoted at time t with tenor τ. It is related to $P(t, t + \tau)$ by the relationship

$$P(t, t+\tau) = \frac{1}{1 + \alpha_{t,t+\tau} E(t, t+\tau)} \iff E(t, t+\tau) = \frac{1}{\alpha_{t,t+\tau}} \left(\frac{1}{P(t, t+\tau)} - 1 \right).$$

We denote by $S(t;\tau_n)$ the swap rate quoted at time t with tenor τ_n. It is related to the term structure of discount factors by the well known relationship

$$S(t;\tau_n) = \frac{1-P(t,t+\tau_n)}{\sum\limits_{i=1}^{n}\alpha_{t+\tau_{i-1},t+\tau_i}P(t,t+\tau_i)}$$

$$\Updownarrow$$

$$P(t,t+\tau_n) = \frac{1-S(t;\tau_n)\times\sum\limits_{i=1}^{n-1}\alpha_{t+\tau_{i-1},t+\tau_i}P(t,t+\tau_i)}{1+\alpha_{t+\tau_{n-1},t+\tau_n}S(t;\tau_n)}.$$

In particular, we consider three basic products and five structured products. In our analysis we do not consider path-dependent products like range-accruals, where the payoff depends on the time spent by the reference index inside a corridor, because as shown by Dybvig (1988), in a complete market the most efficient way to achieve a wealth distribution is by purchasing 'simple' structured products. whose payoffs only depend on the value of the underlying asset at maturity not at intermediate times. Similar results have been obtained also in incomplete markets, see for example Vanduffel et al. (2009a).

The description of the notes considered is given in the next sections.

3.1 Basic products

As basic products, we consider a zero-coupon bond, a coupon bond and a floating rate note. By construction, the issue price of these bonds is equal to the par value N. The description of these bonds is as follows.

- **zero-coupon bond (zcb)**: given a initial investment of amount N at maturity the payoff is given by

$$c_i^{zc} = 0, i = 1, ...n-1,$$
$$c_n^{zc} = c,$$

  where the amount c is equal to

$$c^{zc} = \frac{1}{P(t,t_n)} - 1.$$

- **coupon bond (cb)**: given a initial investment of amount N we receive at times t_i, $i = 1, ..., n$ (here $n = 5$) a periodic and constant amount equal to $\alpha_{i-1,i} \times c \times N$ and the notional at maturity, so that

$$c_i^{cb} = c, i = 1, ...n.$$

  where c is the constant coupon. The coupon c here is chosen such that the present value of all payments is equal to N, i.e. it is a par coupon rate and is given by

$$c^{cb} = \frac{1-P(t,t_n)}{\sum\limits_{i=1}^{n}\alpha_{i-1,i}P(t,t_i)}.$$

- **floating rate note (frn)**: given a initial investment of amount N and a reference rate (here the 12m Euribor rate), this note pays at times t_i, $i = 1, ..., n$ (here $n = 5$) a periodic and variable amount equal to $\alpha_{i-1,i} \times E(t_{i-1}, t_i) \times N$ and the notional at maturity

$$c_i^{frn} = E(t_{i-1}, t_i), i = 1, ...n.$$

3.2 Structured products

The structured notes we are considering are floating rate notes in which the coupon is set according to: a) a swap rate (constant maturity swap), b) a Euribor or a swap rate, with a collar structure, c) a difference of two swap rates (spread note), d) the absolute value of the difference between a swap rate and a fixed rate (volatility note). The detailed description of these notes follows

- **constant maturity swap (cms)**: given a initial investment of amount N and a reference rate (here the 5 years swap rate) we receive at times t_i, $i = 1, ..., n$ (here $n = 5$) a periodic and variable amount equal to

  $$c_i^{cms} = m \times S(t_i; \tau), i = 1, ...n$$

  where $S(t_i; \tau)$ is the swap rate with constant tenor τ and m is the partecipation factor (or multiplicator) chosen to ensure that the issue price is N. The main differences with respect to the floating rate are: a) in this case the reset and payment dates are the same, whilst in the frn case there is time lag between reset and payment dates, b) the CMS issue price in general is not equal to the par value, so that we need to introduce m to guarantee that the fair price is equal to the par value. Constant maturity swaps allow investors to take a position on the different evolution of the short term (Euribor/Libor rate) and long term (swap rate) rates of interest.

- **floating rate note with a collar (frnc)**: the payoff here has a cap (maximum rate) c, a floor (minimum rate) f and a spread component δ

  $$c_i^{\text{frn col}} = \min(\max(E(t_{i-1}, t_i) + \delta, f), c), i = 1, ...n.$$

  Here the three components are adjusted to ensure that the issue price is equal to the notional N. Several combinations are possible. We usually set the floor equal to 90% of the expected value of the Euribor rate and we adjust the other two components to ensure a par value.

- **constant maturity swap with a collar (cmsc)**: the payoff here is like for the cms with a cap c, a floor f and a spread component

  $$c_i^{\text{cms col}}(t_i) = \min(\max(S(t_i; \tau) + \delta, f), c).$$

  Here the three components δ, f and c are adjusted to ensure that the issue price is equal to the notional N. We set the floor equal to 90% of the expected value of the swap rate and we adjust the other two components to ensure a par value.

- **spread note (spread)**: the payoff here depends on the difference between two swap rates wiht a collar structure, a multiplicator m and a spread component δ

  $$c_i^{spread} = \min(\max((S(t_i; \tau_1) - S(t_i; \tau_2)) \times m + \delta, f), c).$$

  In general, the shorter tenor is subtracted from the longer tenor. In our simulations the tenor of the two swap rate is taken to be $\tau_1 = 10yrs$ and $\tau_2 = 2yrs$. This SP is a bet on changes of the slope of the swap curve.

- **volatility note (vol)**: the payoff here depends on the absolute value of the difference between a swap rate and a fixed amount c times a multiplicator m
$$c_i^{vol} = m \times |S(t_i;\tau) - c|.$$
In our simulations the tenor of the swap rate is taken to be $\tau = 10yrs$. The coupon will be large when the swap rate will deviate from the reference value c. On the other side, the coupon will be very low when there is not much volatility in the market.

The different elements δ, m, f and c in the above coupon formula are adjusted to ensure that the fair price is equal to the notional N. The fair price is set equal to the expected discounted payoff computed under the risk-neutral measure

$$\pi(t) = \sum_{i=1}^{n} \alpha_{i-1,i} \times \widetilde{E}_t \left(\frac{c_i}{B(t,t_i)} \right) \times N + P(t,t_n) \times N,$$

where $B(t,t_i)$ is the so called money market account, i.e. the t_i value of a unit initial investment in a risk-free account[2].

For example, in the CMS case the multiplicator factor is chosen so that

$$m \times \sum_{i=1}^{n} \alpha_{i-1,i} \times \widetilde{E}_t \left(\frac{S(t_i;t_i,t_i+\tau_1)}{B(t,t_i)} \right) \times N + P(t,t_n) \times N = N,$$

i.e

$$m = \frac{1 - P(t,t_n)}{\sum_{i=1}^{n} \alpha_{i-1,i} \times \widetilde{E}_t \left(\frac{S(t_i;t_i,t_i+\tau_1)}{B(t,t_i)} \right)}.$$

Where possible (like for the frn with a collar) the risk-neutral expectation, given certain assumptions on the evolution of the stochastic factors, can be computed analytically otherwise we use Monte Carlo simulation[3], as described in the next section.

4 The dynamics of the term structure

As a reference model, for conducting our simulations we have adopted the Two-Additive Factor Gaussian G2++ Model, see Brigo and Mercurio (2006). The main features of this model are

- the short rate is given as sum of two mean-reverting correlated Gaussian factors plus a deterministic function allowing the user to fit the current term structure of spot factors. This allows the user to take into account current in the simulations the current market environment.

- given that the distributional properties of the model are known, it allows for an efficient and fast Monte Carlo simulation for pricing different payoffs.

[2] This account earns instant by instant an (instantaneous) interest rate $r(t)$, so that its t_i value is $B(t,t_i) = \exp\left(\int_t^{t_i} r(s)\,ds\right)$.

[3] For example, the pricing formula for a caplet/floorlet can be found in Brigo and Mercurio (2006), page 155.

- in addition, the model provides closed form expression for discount bonds, European options on zcb and caps, easing the parameters calibration to market quotations;
- the model is more suitable to describe the movements in the term structure, allowing a non perfect correlation between changes of rates of different maturity as we observe empirically, a feature that cannot be captured by one factor term structure models. Different SP's react differently to the movements of different parts of the interest rate curve.
- on the negative side, the model can allow for negative interest rates, albeit this problem can be partially resolved if there is enough mean reversion in the two driving factors.

In the G2++ short rate model, the instantaneous short rate $r(t)$ is given by

$$r(t) = x(t) + y(t) + \phi(t), r(0) = r_0,$$

where the processes $\{x(t), t \geq 0\}$ and $\{y(t), t \geq 0\}$ satisfy the Ornstein-Uhlembeck dynamics. These dynamics, specified under the risk-neutral measure, are

$$\begin{aligned} dx(t) &= -ax(t)\,dt + \sigma d\widetilde{W}_1(t), x(0) = x_0, \\ dy(t) &= -by(t)\,dt + \eta d\widetilde{W}_2(t), y(0) = y_0. \end{aligned}$$

Here $d\widetilde{W}_1$ and $d\widetilde{W}_2$ are the increments of two correlated Brownian motions

$$\widetilde{E}_t\left(d\widetilde{W}_1 d\widetilde{W}_2\right) = \rho dt,$$

where ρ is the correlation coefficient.

The parameter restrictions are

$$r_0, a, b, \sigma > 0, \eta > 0, \rho \in [-1, 1].$$

Parameters a and b are interpreted as mean-reversion coefficients of the two stochastic factors $x(t)$ and $y(t)$. In particular these factors revert to 0 under the risk-neutral measure. The mean-reverting property in the two factors, imply that the short rate will revert toward the deterministic function $\phi(t)$. The role of this function is to guarantee that the time 0 model zero-coupon bond prices are equal to the market ones. This is guaranteed if the following restriction is satisfied

$$\int_t^T \phi(s)\,ds = -\ln\frac{P^{mkt}(0,T)}{P^{mkt}(0,t)} + \frac{1}{2}\left(V(0,T) - V(0,t)\right). \tag{1}$$

with $P^{mkt}(0,T)$ market price of a $T - zcb$ and the expression for the function $V(t,T)$ is given in formula (4.10) in Brigo and Mercurio (2006). Formula (1) suggests that the only curve needed is the market discount curve $P^{mkt}(0,t)$, $t > 0$. Zcb prices $P(t,T)$ at a future date t depend on the forward price $P^{mkt}(0,T)/P^{mkt}(0,t)$,and are an exponential function of the two stochastic factors

$$P(t,T) = \frac{P^{mkt}(0,T)}{P^{mkt}(0,t)} \times \exp\left(-\frac{1-e^{-a(T-t)}}{a} x(t) - \frac{1-e^{-b(T-t)}}{b} y(t) + \frac{1}{2}\left(V(t,T) - V(0,T) + V(0,t)\right)\right). \tag{2}$$

In addition, we observe that the risk-neutral dynamics of the zcb price is

$$\frac{dP(t,T)}{P(t,T)} = r(t)\,dt + \sigma D(T-t;a)\,d\widetilde{W}_1 + \eta D(T-t;b)\,d\widetilde{W}_2,$$

where the function $D(\tau;\theta)$ is related to the (stochastic) duration of the zcb price

$$D(\tau;\theta) = \frac{1-e^{-\theta\tau}}{\theta},$$

and $\sigma D(T-t;a)$ and $\eta D(T-t;b)$ represent the contribution to the bond price volatility from the volatility in the two factors, x and y.

The above dynamics are relevant for pricing the structured products at the initial time. However, to compare the performance of the different products we need the dynamics under the so called physical or risk natural measure. This requires a specification of the risk premium required by the market for taking the risk given by the two Brownian motions. The literature on the specification of this risk-premium is quite large. A discussion can be found in Singleton (2006). However, for aim of simplicity and also for a better understanding of our results we will assume that the risk premium is constant, but we will run the simulations assuming different values for it. In practice the specification of the risk-premium consists in replacing $d\widetilde{W}_i(t)$ by a new Brownian motion $dW_i(t)$, by using

$$d\widetilde{W}_i(t) = \lambda_i dt + dW_i(t), i = 1,2$$

so that the dynamics under the new measure are

$$dx(t) = a\left(\frac{\lambda_1\sigma}{a} - x(t)\right)dt + \sigma dW_1(t), x(0) = x_0,$$
$$dy(t) = b\left(\frac{\lambda_2\eta}{b} - y(t)\right)dt + \eta dW_2(t), y(0) = y_0.$$

Now, under the true measure, the two factors will revert toward $\lambda_1\sigma/a$ and $\lambda_2\eta/b$. Depending on the sign of λ_i, these long-run values can be positive, null or negative. Therefore the forward curve will be a unbiased forecast of future rates only under the exceptional case of a zero risk-premium. The dynamics of zcb prices under the true measure become

$$\frac{dP(t,T)}{P(t,T)} = \left(r(t) + \lambda_1\sigma D(T-t;a) + \lambda_2\eta D(T-t;b)\right)dt + \sigma D(T-t;a)\,dW_1 + \eta D(T-t;b)\,dW_2.$$

In particular, the excess return, in the unit of time, with respect to the instantaneous investment is given by

$$E_t\left(\frac{dP(t,T)}{P(t,T)}\right) - r(t) = \left(\underbrace{\lambda_1\sigma D(T-t;a) + \lambda_2\eta D(T-t;b)}_{\text{term premium}}\right)dt. \tag{3}$$

This excess return, computed now using the true measure (this is why we use E_t rather than $\widetilde{E}_t$), is named term premium, see Duffee (2002): longer term notes are riskier and require a premium to compensate for this extra risk. However, notice that in the specification (3), the term premium for a given time to maturity $T-t$ is assumed to be time homogeneous. Instead, as noted by Fama and French (1993), the sign of predicted excess returns changes over time, mainly because These term premia vary over time as interest rate risk and investors' risk tolerance fluctuate. However, we believe that this does not represent a problem for our simulations that are performed under different parametrization for the parameters λ_i and comparing bonds with the same maturity.

We stress that portfolio allocation aims at modelling the probability distribution of the market prices at a given future investment horizon under the "true" probability distribution of the market prices, as opposed to the risk-neutral probability measure used for derivatives pricing. Based on this distribution, the buy-side community takes decisions on which securities to purchase to improve the prospective payout profile of their position. In practice, the estimation of the true probability distribution (as opposed to the calibration procedure required to obtain the risk-neutral distribution) represents the main quantitative challenge in risk and portfolio management. The differences between the two approaches are examined in Meucci (2010).

4.1 Model Calibration

The implementation of the G2++ requires

1. **Term structure of market discount factors** $P^{mkt}(0, t_i)$: We have considered four different scenarios (labelled A, B, C and D) representative of different shapoes taken from the term structure observed in the market: negatively sloped (A), positively sloped (B), near flat (D). In particular, these curves were observed at the following dates: June 6th, 2008, September 28, 2007 and May 20th, 2009. Case C refers to the average level that we observed in the period 1/1/2005 to 30/09/2010. The four different curves are represented in Figure (1) and are given in Table (1).

2. **Parameters of the G2++ model**: We have calibrated the model following two different procedures: a) historical: i.e. we choose the parameters that give the best fit to the historical covariance matrix of changes in spot rates with maturities from 1 to 5 years. The analysis by De Jong et al. () suggests that the volatility implied by the options is a poor predictor, because it consistently overestimates realised volatility. That means, we have to use historical volatilities and correlations in our simulations. The covariance matrix has been estimated with reference to the period period 1/1/2005 to 30/09/2010. However, the De Jong paper refers to fairly old data, so that, also for giving robustness to our analysis, we consider a market implied calibration, i.e.we have chosen the parameters that give the best fit to the implied volatility swaption surface adopting the procedure as in Brigo and Mercurio (2006), page 166. Following both procedures we have fitted the parameters a, b, σ, η and ρ.

3. Henceforth, we have chosen **the risk-premium parameters** λ_1 and λ_2 that allow us to generate different shapes of the term premium curve. As

previously discussed the size and sign of these parameters will determine the performance of the SP's. The different parametrizations, labelled from I to V, are provided in Table (2), whilst Figure 2 shows how the term premium given in (3) behaves for different time to maturities. In particular, the different parametrizations allows for different shapes and sign of the term premium. Among these, we also consider the case where both parameters λ_1 and λ_2 are zero, that is market participants in average are risk-neutral (scenario II).

4.2 Monte Carlo simulation

The G2++ model is markovian in the two state variables $x(t)$ and $y(t)$. This fact implies that MC simulation can be performed in a straigthforward manner.

1. Calibrate the model and assign $x(0)$ and $y(0)$, the time step Δ, the option maturity $T = n\Delta$ and the initial value of the money market account $MMA(0) = 1$.

2. Simulate according to the true probability measure from a bivariate normal distribution

$$\left[\begin{array}{c} x^{(k)}(i\Delta) \\ y^{(k)}(i\Delta) \end{array} \middle| \mathcal{F}_{(i-1)\Delta} \right] \sim \mathcal{N}\left(M^{(k)}(i\Delta), V(\Delta)\right), i = 1, ..., n$$

   where

$$M^{(k)}(i\Delta) = \left[\begin{array}{c} x^{(k)}((i-1)\Delta)e^{-a\Delta} - \lambda_1\sigma_1\left(1 - e^{-a\Delta}\right) \\ y^{(k)}((i-1)\Delta)e^{-b\Delta} - \lambda_2\eta\left(1 - e^{-b\Delta}\right) \end{array} \right],$$
$$V(\Delta) = \left[\begin{array}{cc} \frac{\sigma^2}{2a}\left(1 - e^{-2a\Delta}\right) & \rho\sigma\eta\frac{1-e^{-(a+b)\Delta}}{a+b} \\ \rho\sigma\eta\frac{1-e^{-(a+b)\Delta}}{a+b} & \frac{\eta^2}{2b}\left(1 - e^{-2b\Delta}\right) \end{array} \right]$$

3. Compute the short rate according to

$$r^{(k)}(i\Delta) = x^{(k)}(i\Delta) + y^{(k)}(i\Delta) + \phi(i\Delta),$$

   and the discount curve according to

$$P^{(k)}(i\Delta, T) = \frac{P^{mkt}(0, T)}{P^{mkt}(0, i\Delta)} \exp\left(A^{(k)}(i\Delta, T)\right); \tag{4}$$

   where $A^{(k)}$ is given by

4. Update the money market account according to

$$MMA^{(k)}(i\Delta) = MMA^{(k)}((i-1)\Delta) \exp\left(r^{(k)}(i\Delta)\Delta\right);$$

5. At each coupon date, given the values of the stochastic factors $x^{(k)}(i\Delta)$ and $y^{(k)}(i\Delta)$ and the discount curve (4), compute the coupon $c(i\Delta)^{(k)}$ and discount it at the initial date

$$\frac{c(i\Delta)^{(k)}}{MMA^{(k)}(i\Delta)};$$

6. For each product, in the simulation k we obtain the present value $PV^{(k)}$ of its cashflows

$$PV^{(k)} = \sum_{i=1}^{n} \frac{c\,(i\Delta)^{(k)}}{MMA^{(k)}\,(i\Delta)} + \frac{N}{MMA^{(k)}\,(n\Delta)}.$$

7. Repeat steps 1-6 for $k = 1, ..., K$, where K is the number of simulations (we have set $K = 50000$).

We have perform the steps 1-7 twice: once under the risk-neutral measure in order to price each note, and another under the true measure. If the price is not equal to the notional we adjust the bond characteristics (floor, cap, spread, gearing) in order to obtain a fari value equal to the par. Therefore, we resimulate the stochastic factors under the true measure and we obtain an array of dimension $K\times$ 8, (here 8 is the number of SP's considered), containing the present value of the (random) cash flows we can achieve investing in a particular SP. We correct this present value by the amount invested (by construction all products are worth N at the issue date) and for the fees that the investor has to pay for investing in the SP. The fee amount is discussed in next section.

5 Fees

The choice of the amount of the fees that an retail investor has to pay for investing in SP is a quite delicate issue. In particular, higher the complexity of the product, higher the overpricing. Stoimenov and Wilkens (2005) find out a lack of transparency in the German market of SP's, in the sense that these products appears to be overpriced and thus favor issuing institutions. Wilkens and Stoimenov (2007), considering leverage products in the German retail market, show that these products near guarantee risk-free profits for their issuers. Similarly, significant mispricing in favor of issuers has been found by Benet, Gianetti and Pissaris (2005) with reference to reverse-exchangeable securities, which are traded on the AMEX (American Stock Exchange). A recent analysis has been performed on the Italian retail market by Billi and Fusai (2010). These authors have considered fixed income products issued in the Italian retail market in the year 2009. They amount to around 500 collocations. Their preliminary results seem to confirm the previous insights and in addition they find in the primary market typically an average premium over theoretical values of about 2% to 6%. The mispricing has usually a positive relation with product complexity and is more pronounced in less developed markets.

By conversations with practitionners and according to the results of previous literature it appears that subscription fees (implicit and explicit) can have large variations depending on the issuer, on the underlyng (interest, index, equity, commodity, etc.), on the presence of exotic components and on the maturity of the contract. However, the smallest fees are paid for zcb, coupon bond and plain vanilla floating rate notes. According to these facts and taking into account that zcb, cb and frn are in general very liquid, largely traded in many markets, typically issued by governments, and therefore are largely available to the vast majority of investors that can buy them paying very little commissions, we have decided of setting to zero the fee for basic products like zcb, cb and frn. For the remaining products, our analysis will be conducted under different assumptions, with different levels of (percentage) fee. In practice, the initial investment required to the subscriber amounts to

$$N \times (1 + g)\,,$$

where g = 0.5%, 0.1%, 1.5%, 2%, 2.5% and 3%. Taking into account that our experiments consider a time horizon of five years, and no fee is charged at the coupon dates, these commissions vary between 0.1% and 0.6% per year that appears to be an underestimate of the current situation in the Italian SP market.

6 Portfolio allocation

The problem the investor is facing is to decide if to select a portfolio of looklike government bonds (i.e. zcb, cb and frn) or to diversify its investments considering also SP on which on the other hand he has to pay a fee. In both cases, the choice of its optimal portfolio is done as follow. Let us define $PV_j^{(k)}$ the present value of future the cash flows of the products j, $j = 1...8$ in simulation k, $j = 1...K$ (here $K = 50,000$). For example, Figure (3) shows the density function of the different products in Scenario D-IV. It is evident the non-gaussianity of SP's bonds. The net return, in present value terms, of the investment is

$$r_j^{(k)} = \begin{cases} \frac{PV_j^{(k)}}{N} - 1 & j = 1, 2, 3. \\ \\ \frac{PV_j^{(k)}}{N\times(1+g)} - 1 & j = 4, ..., 8. \end{cases}$$

Assuming mean-variance preferences, the investor computes the expected (across the K simulations) vector μ

$$\mu' = \left[\begin{array}{ccc} \frac{1}{K}\sum\limits_{k=1}^{K} r_1^{(k)} & \underbrace{\frac{1}{K}\sum_{k=1}^{K} r_j^{(k)}}_{\mu_j} & \frac{1}{K}\sum\limits_{k=1}^{K} r_8^{(k)} \end{array} \right],$$

and the covariance matrix $\mathbf{V}$ with elements $V_{m,n}$, $m, n = 1...8$:

$$\begin{aligned} V_{m,n} &= cov\left(r_m^{(k)}, r_n^{(k)}\right) \\ &= \frac{1}{K}\sum_{k=1}^{K}\left(r_m^{(k)} - \mu_m\right)\left(r_n^{(k)} - \mu_n\right). \end{aligned}$$

Henceforth, the investor solves with respect to the vector of holdings $\mathbf{w}$, $\mathbf{w} \in R_+^n$, the following mean-variance problem with no-short selling constraint

$$\begin{array}{l} \min \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w} \\ sub \\ \mu'\mathbf{w} = m \\ \mathbf{1}'\mathbf{w} = 1 \\ \mathbf{w} \geq \mathbf{0}, \end{array} \tag{5}$$

where m is the target expected return required by the investor. If the investor is considering only basic securities, he is solving the same problem but with the additional constraint

$$w_n = 0, n = 4, ..., 8, \tag{6}$$

i.e. a zero investment in the more "exotic" structures. The target expected return m in (5) is taken considering 30 equally spaced points in the range $\left[m^{low}, m^{high}\right]$ where m^{low} and m^{high} are the smallest and largest expected return we can achieve investing only in basic securities[4].

The comparative analysis we conduct is related to the performance of the optimal portfolio that is solution of (5) and that we call $\mathbf{w}_{all}$ and the portfolio of the less sophisticated investor that solves (5) with the additional constraint (6). This portfolio is denoted $\mathbf{w}_{basic}$.

To evaluate the two different approaches, namely the investment with a restrictive pool of assets versus the investment in a larger market, we use the two-step mean variance approach in Meucci (2005) and proceed as follow:

1. First, we compute the mean-variance efficient frontier in euro terms considering the two investment possibilities (only basic products or basic and SP's). We emphasize that to do so we do not need to assume normality. This step reduces the dimension of the market to a one-parameter family of portfolios.

2. Second, for a given utility function u, we compute the maximum expected utility ensuing from the two efficient frontiers. In particular, if we define the certainty equivalent associated to the mean-variance portfolio s,

$$ce\left(s\right) = u^{-1}\left(\frac{1}{K}\sum_{k=1}^{K} u\left(\sum_{j=1}^{n} w_j\left(s\right)\left(r_j^{(k)} - g\right)\right)\right),$$

where $w_j\left(s\right)$ is the weight of bond j in this portfolio, we can compare the basic and sophisticated investement by to the difference between the maximum certainty equivalent the investor can achieve. We define this difference as $\Delta score$

$$\Delta score = ce^*_{all} - ce^*_{basic},$$

where

$$ce^*_{all} = \max_{s \in MV_{all}} ce\left(s\right),$$
$$ce^*_{basic} = \max_{s \in MV_{basic}} ce\left(s\right),$$

where MV_{basic} and MV_{all} stand for the set of mean-variance portfolios given that the investment set is limited or not to basic securities.

For example, if we consider a quadratic utility function $u\left(x\right) = x - \left(1/2\right) bx^2$, the certainty equivalent is given by

$$ce_b = \frac{1 - \sqrt{1 - b \times eu}}{b}, \tag{7}$$

[4]In practice m^{low} is the expected return on the portfolio that solves $\min \frac{1}{2}\mathbf{w'Vw}$, sub $\mathbf{1'w} = 1$ and $\mathbf{w} \geq \mathbf{0}$ whilst μ^{high} is the largest element in μ. In both cases, we give a non-zero weight only to basic securities.

where

$$eu = \mathbf{w}'\mu - \frac{1}{2}b\mathbf{w}'\mathbf{V}\mathbf{w}.$$

We also refine our analysis, considering an investor with preferences that cannot be described in the space mean-variance. The idea of this approach is that ex-ante the investor, for each investment vehicle, evaluates the distribution function of its wealth according to its own preference structure, summarized by the utility function. In practice, the problem the investor is facing is

$$\begin{array}{l} \max \frac{1}{K}\sum_{k=1}^{K} u\left(\mathbf{w}'\left(\mathbf{r}^{(k)} - g\mathbf{1}\right)\right) \\ sub \\ \mathbf{w} \geq \mathbf{0}, \end{array} \tag{8}$$

In practice, we have solved this problem assuming an exponential utility function, i.e.

$$u\left(r\right) = -e^{-\lambda r}, \lambda > 0$$

where λ is the (constant) risk-adversion parameter. Similarly, to the previous case we can compute the $\Delta score$ given that the certainty equivalent is given by

$$ce_{\lambda}\left(s\right) = -\frac{1}{\lambda}\ln\left(\frac{1}{K}\sum_{k=1}^{K} e^{-\lambda\sum_{j=1}^{n} w_j(s)\left(r_j^{(k)} - g\right)}\right).$$

7 Results

A preliminary analysis is conducted having a look to the mean vector μ, to the correlation matrix and the standard deviations of the different products, across the 20 scenarios (four initial curves and five different G2++ parametrization).

The most important remarks are relative to the following facts

- Table (5) illustrates the expected return, before fees are paid, for each product in each scenario. The most interesting insigths from these Table are that:
  - there are parameters scenarios (I and V), where the expected return for basic products is negative. These scenarios correspond to a market situation where the term premium is monotone increasing (I) or decreasing (V); in this market setting, among the SP's only the cms and the volatility note are characterized by a positive expected return, except when the initial term structure is increasing (curve B).
  - if the term premium is zero (II), i.e. the average investor in the market is risk-neutral and does not require risk-compensation, the expected return for all products is zero: this is not surprising at all, given that the true measure is equal to the risk-neutral measure and the expected performance is equal to the paid price;
  - according to these results, we can compute the maximum fee that can be charged to an investor holding an equally weighted portfolio: this fee, given in Table (6), is obtained as difference between the mean expected return of structured notes and the average expected

return of basic products, provided that these averages are positive. It appears that only for term premium scenario V, and sometimes for scenario IV, the net expected return of the investor is positive even when the fee is as large as 3%. This is related to the very good performance of the volatility note in these scenarios.

- Table (4) illustrates the correlations, standard deviations and the expected returns of the different products, averaged across scenarios. The standard deviations in each scenario are given in Table (7). The frn appears to be the less risky investment (in terms of standard deviation) followed by the cms and by the two collar structures. This can appear natural because the uncertainty relative to his cash flows is compensanted by the uncertainty in the value of the numeraire asset (the money market account). The cms note shares similar characteristics but it is slightly more volatile. The two collar structures, combining floating and fixed characteristics, are comparable but with an higher volatility. The assets with the highest risk appear to be the spread and the volatilty notes.

- Absolute volatility is not all, and we need to compare the securities in terms of their contribution to the portfolio diversification. However, also in this case the frn and the cms show the lowest correlation with the remaining assets, see bottom part of Table (8). For example, the frn has an average correlation with the remaining assets that is negative (-0.08), followed by the cms with an average correlation of 0.04. Very interesting diversification properties characterize the volatility note.

- Figures (4)-(8) provide some insigth about the joint distribution of the frn with the remaining products. As it can be seen from the scatter plot, the frn appears scarcely correlated with the remaining products, explaining its relevance in the allocation decision.

- If we examine the average portfolio composition across all 20 scenarios considered, see Table (9), we observe that in average, if there are no fees, basic securities account around 63% of the portfolio composition. This is shown in columns 1 and 2 of Table (9). Respect to the basic portfolio, there is a sizeable reduction in the allocation to zcb and cb that are replaced in large part by collar and spread structures (16% and 14%) and in part by the cms and by the volatility note. The percentage of wealth allocated to SP' vary from 25% for an investor with high risk-adversion[5] to 47% for an individual with low risk-adversion, see columns 1 and 2 in Tables (12)-(10). This appears a significant change in the portfolio composition with respect to a basic investor.

- However, the results change as we consider fees. The percentage invested in basic securities, even with a fee as low as 0.5% over the five years, increases to 96%, and the size of the investment in SP's appears to be marginal: a high risk adverse investor will invest 3% of its wealth in the

[5] In this preliminary analysis, in order to classify the investor we proceed as follows. Once we have built the mean-variance (mv) portfolios, the low risk-adverse investors consider the investment in first third of mv portfolios with the highest expected return (and risk), the high risk-adverse investors is considering the last third, and the average investors are investing in the portfolios having an intermediate level of expected return and risk.

spread note; a low risk adverse investor will consider a 1% investment in the volatility note, even with fees as large as 3%.

- This case is shown in Figures (9) and (10), where we present the efficient frontier when different assumptions on the fees are made. In particular, these Figures refer to the scenario combinations B-II and D-II. As it can be shown, if no fees are paid for investing in structured products, we can improve considerably the trade-off expected return-risk: the efficient frontier with SP is above the one where we invest only in basic securities. However, as we increase the fee, this advantage is completely lost and the investor moves back to a portfolio made only of basic securities.

In general the above results are quite disappointing showing that: without fees, the investor can improve its tradeoff risk-return, but in presence of fees, this convenience disappears. The reason is in large part explained from the small differences in expected returns between bonds, see Table (5).

However, a deeper analysis shows that under particular scenarios, like the IV and V in the G2++ settings, the investment in structured products appears to be relevant even when the fees are large. Indeed in these cases, the outperformance of the volatility note appears to be so significant that Table (6) shows that in these market settings, the convenience is robust to large fees. This is also illustrated in Table (14), where, in the G2++ setting V, we compute the certainty equivalent ce_b, formula (7), for different values of the risk-adversion coefficient b and for different fee levels g. Figure (12) shows a similar in the setting C IV: here we illustrate the percentage of the wealth invested in SP's for different fee levels and different volatility levels: the interesting point is that also for high risk-adverse investors (low volatility) the percentage can be still interesting. The entire weigth in this case is allocated to the volatility note.

A similar result is shown in figure (13) where we report the percentage invested in SP and the certainty equivalent ce_λ changing the risk-adversion coefficient λ.

8 Conclusions

In the present paper we have discussed the relative convenience of investing in a portfolio of fixed income structured products. We have shown that, without fees, the structured product can improve the risk-return trade off for a retail investor. This result is in general not robust to the presence of fees: in this case the optimal portfolio of the investor is made only of basic products and the percentage invested in structured products appears to be marginal. However, under particular configurations of the term premium, this investment can still appear to be convenient, mainly with reference to products like volatility and spread notes.

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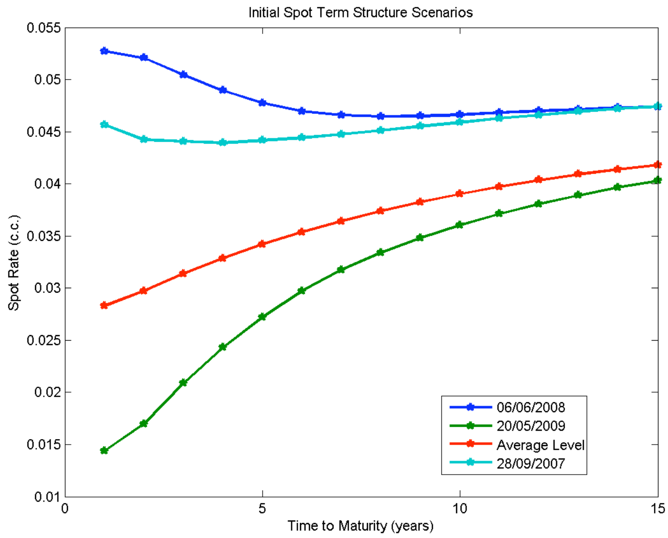

Figure 1: Initial term structures used in the G2++ model, see Table (1)

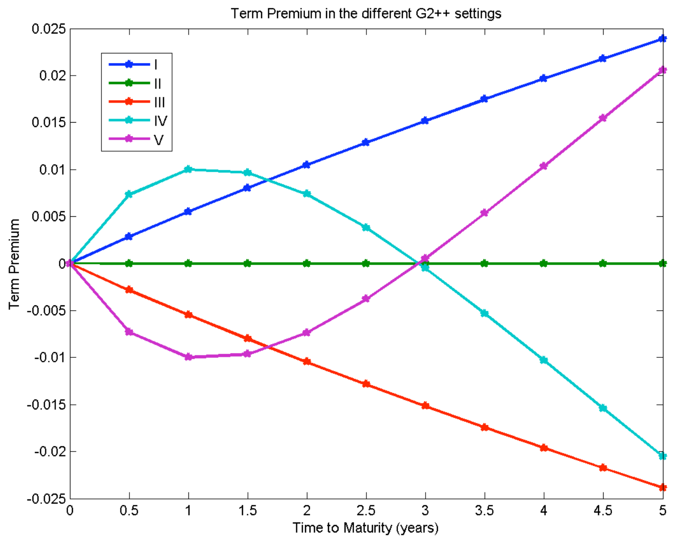

Figure 2: Behavior of the term premium under different G2++ parametrization, see Table (2)

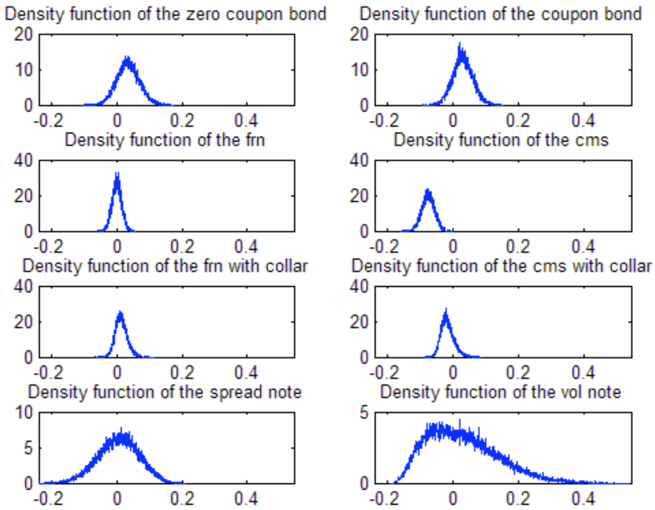

Figure 3: Density function of the return of the different bonds. Scenario D IV

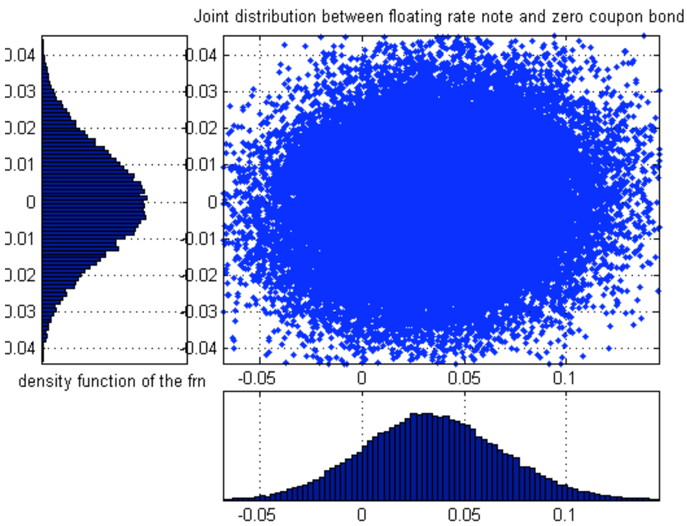

Figure 4: Joint distribution of the frn and zcb

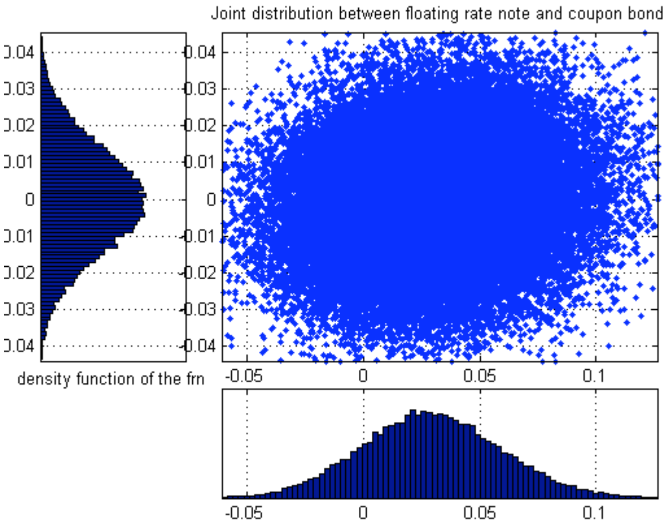

Figure 5: Joint distribution of the frn and zcb

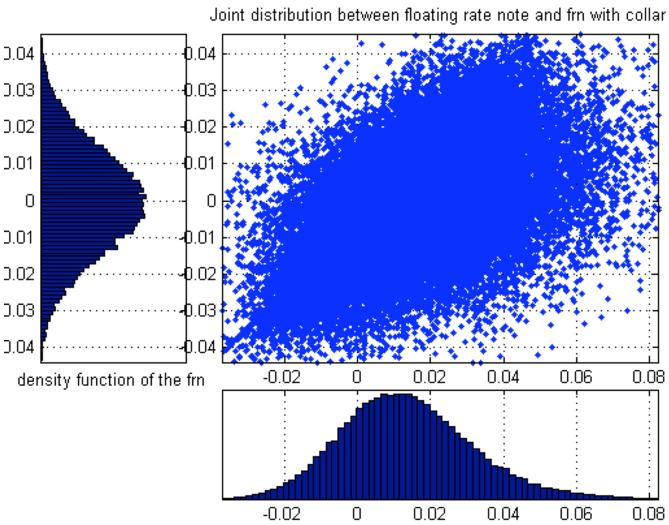

Figure 6: Joint distribution of the frn and the frn with a collar

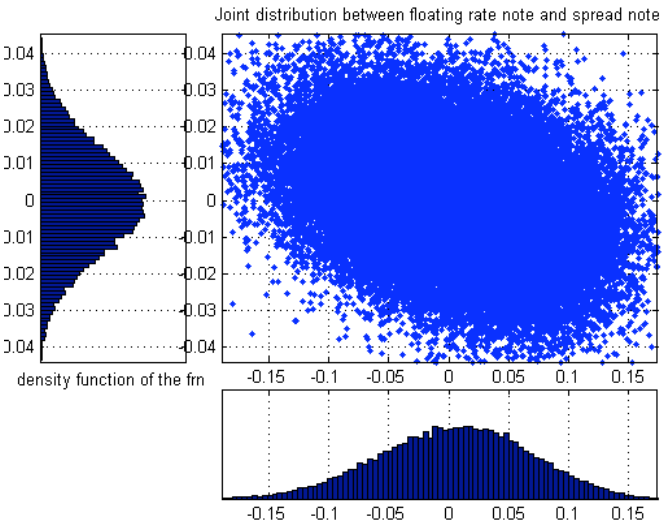

Figure 7: Joint distribution of the frn and the spread note

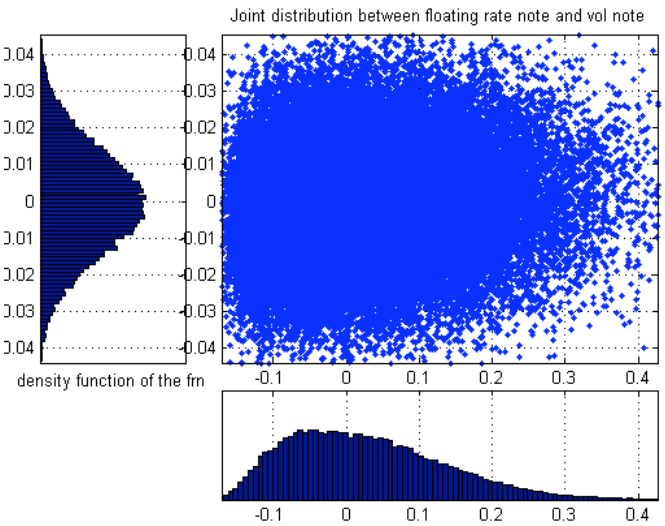

Figure 8: Joint distribution of the frn and the volatility note

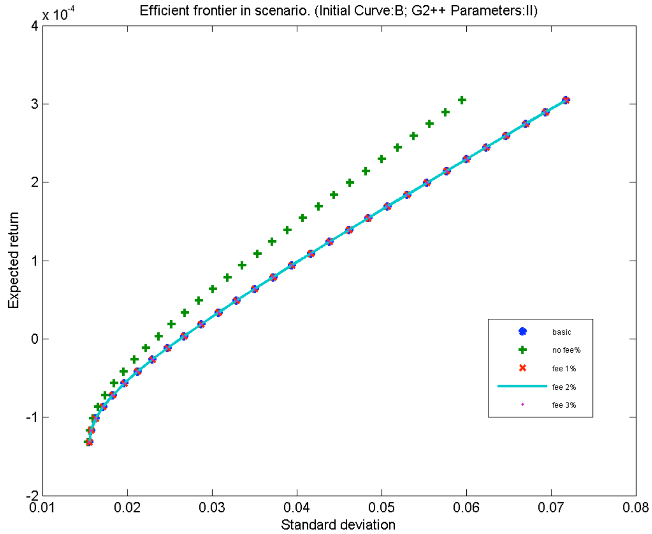

Figure 9: Efficient frontier with basic securities and with structured products under different assumptions on the fees. Scenario B II.

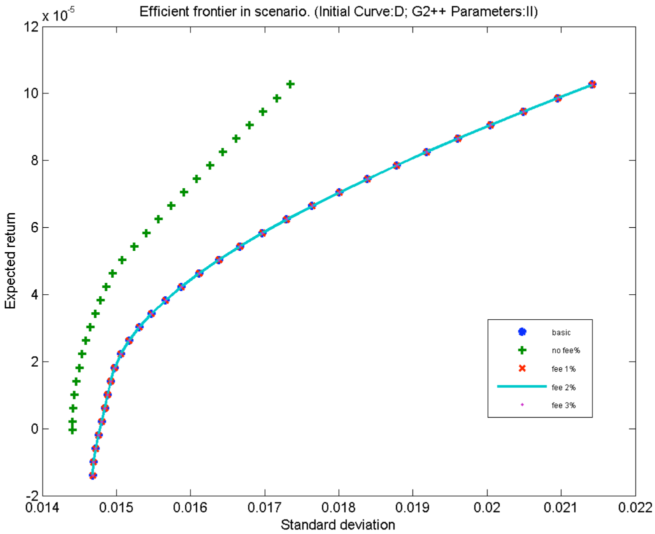

Figure 10: Efficient frontier with basic securities and with structured products under different assumptions on the fees. Scenario D II.

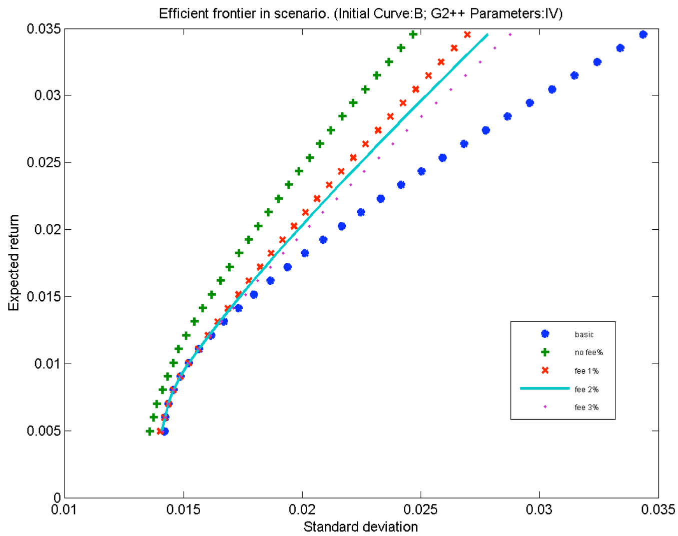

Figure 11: Efficient frontier with basic securities and with structured products under different assumptions on the fees. Scenario B IV.

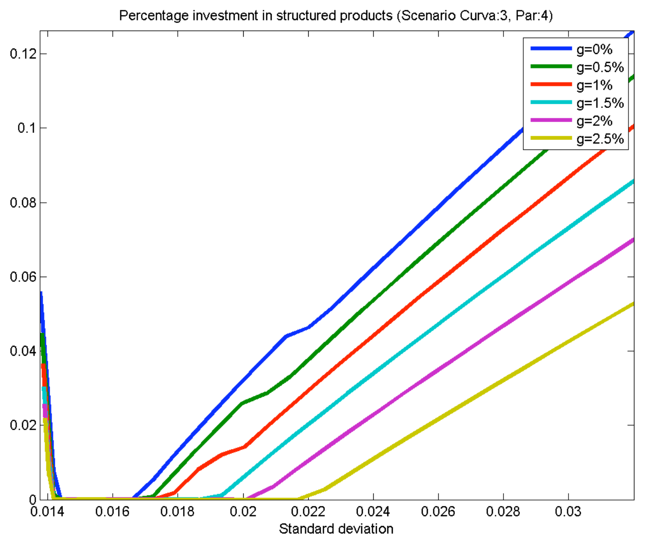

Figure 12: Percentage investment in SP's changing the fee level $g$and the riskness of the portfolio. Scenario setting C IV.

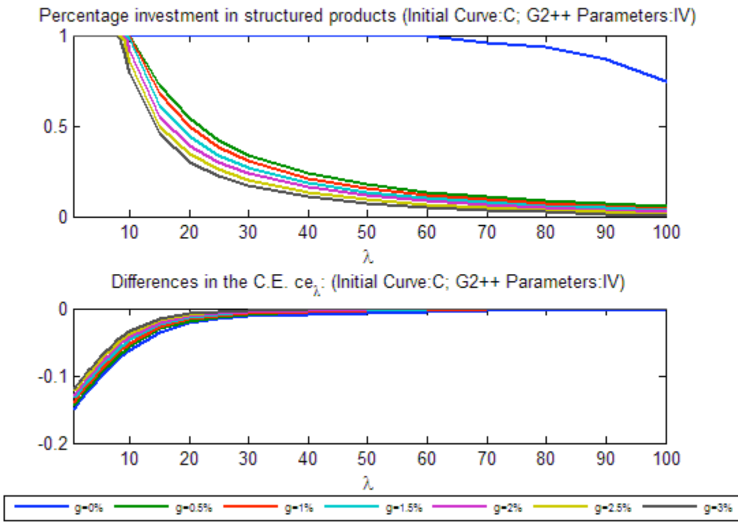

Figure 13: Percentage investment in SP's changing fee level and risk adversion λ for an individual with exponential utility

Years	A (6 jun 08)	B (28 sep. 07))	C average	D (20 may 09)
1	5.2716%	1.4357%	2.8280%	4.5662%
2	5.2083%	1.6978%	2.9718%	4.4234%
3	5.0438%	2.0882%	3.1379%	4.4075%
4	4.8936%	2.4309%	3.2864%	4.3928%
5	4.7736%	2.7213%	3.4166%	4.4154%
6	4.6964%	2.9731%	3.5348%	4.4423%
7	4.6591%	3.1743%	3.6423%	4.4750%
8	4.6450%	3.3418%	3.7391%	4.5114%
9	4.6506%	3.4801%	3.8255%	4.5506%
10	4.6615%	3.6008%	3.9028%	4.5893%
11	4.6818%	3.7101%	3.9726%	4.6264%
12	4.6981%	3.8064%	4.0355%	4.6591%
13	4.7143%	3.8889%	4.0907%	4.6926%
14	4.7333%	3.9649%	4.1379%	4.7202%
15	4.7374%	4.0310%	4.1790%	4.7403%

Table 1: Initial term structure of spot rates in the four interest rate scenarios

	I	II	III	IV	V
λ_1	-0.403	0.000	0.403	-1.660	1.660
λ_2	-0.281	0.000	0.281	1.580	-1.580
a	1.792	1.792	1.792	0.774	0.774
b	0.052	0.052	0.052	0.082	0.082
σ	0.002	0.002	0.002	0.022	0.022
η	0.019	0.019	0.019	0.010	0.010
ρ	-0.644	-0.644	-0.644	-0.702	-0.702

Table 2: Parameters for the G2++ model in the five different scenarios

	Coupon	Floor	Cap	Partecipation	Spread
ZCB	21.48%				
CB	3.95%				
FRN					
CMS				84.71%	
FRN Coll		1.77%	5.59%		0.10%
CMS Coll		3.11%	4.47%		0.33%
SPREAD			5.72%	304.81%	2.97%
VOL NOTE	3.98%			259.37%	

Table 3: Average across scenarios of the components of the different structured products

	ZCB	CB	FRN	CMS	FRN Coll	CMS Coll	Spread	Vol Note
ZCB	1	0.9997	-0.3077	-0.1351	0.7674	0.9477	0.9623	0.1345
CB	0.9997	1	-0.2972	-0.1277	0.7700	0.9485	0.9608	0.1358
FRN	-0.3077	-0.2972	1	0.5606	0.0783	-0.1952	-0.4752	0.0710
CMS	-0.1351	-0.1277	0.5606	1	-0.0400	-0.0105	-0.1732	0.2165
FRN Coll	0.7674	0.7700	0.0783	-0.0400	1	0.7876	0.6578	0.0598
CMS Coll	0.9477	0.9485	-0.1952	-0.0105	0.7876	1	0.9075	0.1383
Spread	0.9623	0.9608	-0.4752	-0.1732	0.6578	0.9075	1	0.0801
Vol Note	0.1345	0.1358	0.0710	0.2165	0.0598	0.1383	0.0801	1
Std. Dev.	6.42%	5.69%	1.93%	2.86%	3.05%	4.58%	8.41%	11.16%
Ex Return	0.07%	0.06%	-0.01%	0.01%	-0.16%	-0.24%	0.01%	9.44%

Table 4: Average correlations, standard deviations and expected returns of different structured products across all scenarios

Discount Curve	G2++ Parameters	ZCB	CB	FRN	CMS	FRN Coll	CMS Coll	SPREAD	VOL NO
A	I	-4.77%	-4.15%	-0.03%	1.34%	-2.49%	-1.96%	-6.69%	2.24%
A	II	0.00%	0.00%	-0.01%	0.01%	0.01%	0.00%	-0.01%	0.02%
A	III	5.03%	4.37%	0.00%	-1.33%	2.58%	1.90%	6.28%	2.28%
A	IV	3.45%	2.88%	-0.02%	-8.63%	1.16%	-3.39%	1.18%	3.08%
A	V	-3.36%	-2.82%	-0.02%	8.61%	-1.38%	2.06%	-1.65%	32.76%
B	I	-4.75%	-4.36%	-0.01%	-0.69%	-1.59%	-4.36%	-6.13%	-0.95%
B	II	0.02%	0.03%	-0.02%	-0.01%	-0.09%	0.03%	0.03%	0.04%
B	III	4.98%	4.59%	0.00%	0.78%	1.23%	4.56%	6.49%	3.93%
B	IV	3.46%	3.12%	-0.01%	-4.66%	0.51%	3.14%	2.02%	24.34%
B	V	-3.34%	-3.02%	-0.02%	4.70%	-1.81%	-3.00%	-1.45%	26.68%
C	I	-4.79%	-4.31%	-0.01%	0.17%	-2.70%	-3.95%	-6.54%	0.03%
C	II	0.01%	0.00%	-0.01%	0.01%	0.02%	0.00%	0.01%	-0.03%
C	III	5.00%	4.50%	0.01%	-0.07%	2.75%	3.99%	6.78%	3.46%
C	IV	3.45%	3.03%	-0.02%	-6.38%	0.48%	2.50%	1.69%	19.13%
C	V	-3.35%	-2.95%	-0.01%	6.39%	-1.62%	-2.86%	-1.47%	34.13%
D	I	-4.77%	-4.18%	-0.02%	0.86%	-2.63%	-2.73%	-5.54%	1.90%
D	II	-0.05%	-0.05%	0.01%	0.04%	-0.01%	-0.01%	-0.05%	0.02%
D	III	5.00%	4.39%	-0.01%	-0.81%	2.70%	2.56%	5.15%	2.13%
D	IV	3.47%	2.94%	-0.02%	-7.62%	1.36%	-1.70%	0.45%	2.88%
D	V	-3.34%	-2.83%	-0.01%	7.60%	-1.73%	-1.61%	-0.37%	30.73%
	min	-4.79%	-4.36%	-0.03%	-8.63%	-2.70%	-4.36%	-6.69%	-0.95%
	mean	0.07%	0.06%	-0.01%	0.01%	-0.16%	-0.24%	0.01%	9.44%
	max	5.03%	4.59%	0.01%	8.61%	2.75%	4.56%	6.78%	34.13%

Table 5: Expected return of the different structured products in each scenario. The expected return is computed simulating the G2++ under the physical measure

	I	II	III	IV	V
A	0.00%	0.01%	0.00%	0.00%	10.15%
B	0.30%	0.00%	0.21%	2.88%	7.15%
C	0.44%	0.00%	0.22%	1.33%	9.02%
D	1.36%	0.02%	0.00%	0.00%	8.99%

Table 6: Maximum fee that can be charged to the investor investing in the structured note with maximum expected return

Discount Curve	G2++ Parameters	ZCB	CB	FRN	CMS	FRN Coll	CMS Coll	SPREAD	VOL NO
A	I	7.47%	6.43%	2.12%	4.27%	3.76%	3.50%	11.26%	11.43%
A	II	7.83%	6.74%	2.13%	4.26%	3.59%	3.24%	10.75%	10.80%
A	III	8.22%	7.06%	2.14%	4.21%	4.01%	3.55%	10.04%	13.48%
A	IV	3.43%	2.97%	1.45%	2.18%	1.77%	1.93%	4.39%	11.00%
A	V	3.21%	2.79%	1.44%	2.20%	1.87%	2.18%	3.51%	10.19%
B	I	7.41%	6.79%	2.22%	2.48%	2.57%	6.79%	9.56%	6.34%
B	II	7.83%	7.17%	2.25%	2.56%	2.21%	7.17%	10.21%	8.19%
B	III	8.20%	7.51%	2.27%	2.65%	2.44%	7.51%	10.78%	11.63%
B	IV	3.44%	3.17%	1.52%	1.51%	1.50%	3.17%	4.99%	15.45%
B	V	3.21%	2.96%	1.52%	1.56%	2.01%	2.96%	4.51%	10.03%
C	I	7.46%	6.69%	2.17%	2.83%	4.05%	6.16%	10.25%	7.91%
C	II	7.78%	6.98%	2.19%	2.85%	3.90%	6.20%	10.71%	8.83%
C	III	8.24%	7.39%	2.22%	2.87%	4.42%	6.47%	11.17%	12.40%
C	IV	3.45%	3.12%	1.50%	1.66%	1.47%	2.99%	5.21%	15.85%
C	V	3.21%	2.91%	1.49%	1.71%	1.97%	2.90%	4.48%	11.79%
D	I	7.46%	6.49%	2.13%	3.58%	3.93%	4.55%	9.27%	10.37%
D	II	7.81%	6.79%	2.14%	3.54%	3.79%	4.13%	8.54%	9.81%
D	III	8.20%	7.12%	2.18%	3.56%	4.26%	4.16%	8.00%	12.70%
D	IV	3.44%	3.02%	1.47%	1.93%	1.83%	2.01%	6.46%	10.79%
D	V	3.23%	2.84%	1.46%	1.94%	1.99%	2.78%	5.28%	9.51%
	min	3.21%	2.79%	1.44%	1.51%	1.47%	1.93%	3.51%	6.34%
	mean	6.03%	5.35%	1.90%	2.72%	2.87%	4.22%	7.97%	10.93%
	max	8.24%	7.51%	2.27%	4.27%	4.42%	7.51%	11.26%	15.85%

Table 7: Standard deviation of the return of the different structured products.

Discount Curve	G2++ Parameters	ZCB	CB	FRN	CMS	FRN Coll	CMS Coll	SPREAD	VOL NO
A	I	29%	29%	-13%	-14%	33%	35%	23%	-19%
A	II	36%	36%	-16%	-17%	42%	44%	30%	7%
A	III	45%	45%	-24%	-24%	51%	52%	40%	33%
A	IV	59%	60%	13%	-29%	57%	61%	49%	46%
A	V	39%	40%	13%	-5%	40%	43%	35%	-38%
B	I	49%	49%	-20%	41%	39%	49%	45%	11%
B	II	55%	55%	-25%	43%	42%	55%	51%	38%
B	III	60%	60%	-30%	45%	48%	60%	57%	52%
B	IV	65%	65%	19%	29%	26%	65%	53%	53%
B	V	50%	50%	11%	36%	44%	50%	42%	-45%
C	I	41%	42%	-20%	7%	42%	42%	37%	-3%
C	II	50%	51%	-26%	6%	53%	51%	46%	26%
C	III	57%	57%	-32%	4%	59%	58%	53%	45%
C	IV	60%	60%	18%	0%	32%	61%	48%	48%
C	V	44%	45%	11%	14%	40%	45%	37%	-43%
D	I	32%	33%	-15%	-9%	35%	35%	28%	-20%
D	II	40%	40%	-19%	-10%	45%	44%	36%	6%
D	III	49%	49%	-26%	-16%	54%	54%	46%	34%
D	IV	61%	61%	12%	-19%	58%	63%	46%	48%
D	V	41%	42%	9%	1%	40%	42%	31%	-39%
	min	29%	29%	-32%	-29%	26%	35%	23%	-45%
	mean	48%	48%	-8%	4%	44%	50%	42%	12%
	max	65%	65%	19%	45%	59%	65%	57%	53%

Table 8: Average correlation of each product with the remainings in the different scenarios

	Fees							
	only basic	0%	0.5%	1%	1.5%	2%	2.5%	3%
ZCB	29%	6%	24%	24%	25%	25%	25%	25%
CB	15%	2%	14%	15%	15%	15%	15%	15%
FRN	56%	55%	60%	60%	60%	59%	59%	59%
CMS	0%	2%	0%	0%	0%	0%	0%	0%
FRN Coll	0%	16%	0%	0%	0%	0%	0%	0%
CMS Coll	0%	3%	0%	0%	0%	0%	0%	0%
Spread	0%	14%	1%	0%	0%	0%	0%	0%
Vol Note	0%	2%	1%	1%	1%	1%	1%	0%

Table 9: Average portfolio composition across scenarios for different level of fees

	Fees							
	only basic	0	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%
ZCB	5%	0%	5%	5%	5%	5%	5%	5%
CB	22%	6%	16%	20%	21%	21%	21%	21%
FRN	73%	69%	75%	74%	74%	74%	73%	73%
CMS	0%	0%	0%	0%	0%	0%	0%	0%
FRN Coll	0%	12%	0%	0%	0%	0%	0%	0%
CMS Coll	0%	2%	0%	0%	0%	0%	0%	0%
Spread	0%	11%	3%	1%	1%	0%	0%	0%
Vol Note	0%	0%	0%	0%	0%	0%	0%	0%

Table 10: Average portfolio composition across scenarios for an individual with **high risk adversion** and for different level of fees

	Fees							
	only basic	0	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%
ZCB	31%	7%	26%	27%	27%	28%	28%	28%
CB	12%	2%	13%	13%	12%	12%	12%	12%
FRN	57%	55%	60%	60%	60%	60%	59%	59%
CMS	0%	1%	0%	0%	0%	0%	0%	0%
FRN Coll	0%	17%	0%	0%	0%	0%	0%	0%
CMS Coll	0%	3%	0%	0%	0%	0%	0%	0%
Spread	0%	13%	0%	0%	0%	0%	0%	0%
Vol Note	0%	2%	1%	1%	1%	0%	0%	0%

Table 11: Average portfolio composition across scenarios for an individual with **medium risk adversion** and for different level of fees

	Fees							
	only basic	0	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%
ZCB	49%	12%	40%	41%	41%	41%	42%	42%
CB	12%	0%	12%	12%	12%	12%	12%	12%
FRN	39%	41%	47%	46%	46%	45%	45%	44%
CMS	0%	4%	0%	0%	0%	0%	0%	0%
FRN Coll	0%	18%	0%	0%	0%	0%	0%	0%
CMS Coll	0%	4%	0%	0%	0%	0%	0%	0%
Spread	0%	18%	0%	0%	0%	0%	0%	0%
Vol Note	0%	3%	1%	1%	1%	1%	1%	1%

Table 12: Average portfolio composition across scenarios for an individual with low risk adversion and for different level of fees

	Fees							
	only basic	0	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%
ZCB	0%	0%	0%	0%	0%	0%	0%	0%
CB	9%	0%	0%	0%	0%	0%	0%	0%
FRN	91%	73%	73%	73%	73%	73%	73%	73%
CMS	0%	0%	0%	0%	0%	0%	0%	0%
FRN Coll	0%	0%	0%	0%	0%	0%	0%	0%
CMS Coll	0%	0%	0%	0%	0%	0%	0%	0%
Spread	0%	19%	19%	19%	19%	19%	19%	19%
Vol Note	0%	8%	8%	8%	8%	8%	8%	8%

Table 13: Portfolio composition for an individual with high risk adversion in the scenario D-IV when the term premium is changing sign

G2++	V	V	V	V	V	V	V	V	V	V	V	V
Initial Curve	A	A	A	B	B	B	C	C	C	D	D	D
b	0.1	1	10	0.1	1	10	0.1	1	10	0.1	1	10
basic	-0.01%	-0.02%	-0.06%	-0.01%	-0.02%	-0.07%	-0.01%	-0.01	-0.06	-0.01	-0.01	-0.06
no fee	1.12	1.13	1.16	0.82	0.82	0.82	1.00	1.01	1.03	1.16	1.17	1.21
g=0.5	1.03	1.03	1.05	0.74	0.74	0.73	0.93	0.93	0.94	1.10	1.10	1.13
g=1	0.93	0.93	0.95	0.65	0.65	0.64	0.85	0.85	0.86	1.03	1.03	1.06
g=1.5	0.84	0.84	0.84	0.57	0.57	0.55	0.78	0.78	0.78	0.96	0.96	0.98
g=2	0.74	0.74	0.74	0.48	0.48	0.46	0.70	0.70	0.69	0.89	0.89	0.91
g=2.5	0.65	0.64	0.64	0.40	0.40	0.38	0.62	0.62	0.61	0.83	0.83	0.83
g=3	0.55	0.55	0.53	0.32	0.31	0.29	0.55	0.55	0.53	0.76	0.76	0.76

Table 14: Certainty equivalent ce_b, in percentage terms, for an individual with quadratic utility changing the parameter b in the utility function and under different initial curve and G2++ model parametrization