Macroeconomic Uncertainty, Difference in Beliefs, and Bond Risk Premia

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ABSTRACT

In this paper we study empirically the implications of macroeconomic disagreement for bond market dynamics. If there is a source of heterogeneity in the belief structure of the economy then differences in beliefs can affect equilibrium asset prices. Using survey data on a unique data set we propose a new empirically observable proxy to measure macroeconomic disagreement and find a number of novel results. First, consistent with a general equilibrium model, heterogeneity in beliefs affect the price of risk so that belief dispersion regarding the real economy, inflation, short and long term interest rates predict excess bond returns with $\overline{R}^2$ between 21%- 43%. Second, macroeconomic disagreement explains the volatility of stock and bonds with high statistical significance with an $\overline{R}^2 \sim 26\%$ in monthly projections. Third, disagreement also contains significant information trading activity: dispersion in beliefs explains the growth rate of open interest on 10 year treasury notes with $\overline{R}^2$ equal to 21%. Fourth, while around half the information contained in the cross-section of expectations is spanned by the yield curve, there remains large unspanned component important for bond pricing. Finally, we control for an array of alternative predictor variables and show that the information contained in the belief structure of the economy is different from either consensus views or fundamentals.

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Introduction

This paper investigates the empirical implications of macroeconomic disagreement for the time variation in bond market risk premia. When moving from single agent to heterogeneous agent models several important properties of asset prices change. If there is a source of heterogeneity in the belief structure on the endowment process then differences in beliefs can affect the stochastic discount factor, thus equilibrium asset prices. This is important since the dynamics of macroeconomic disagreement may become a source of predictable variation in excess bond returns. A growing body of evidence indicates that heterogeneity plays an important role in a variety of settings, including equity, foreign exchange, and derivative markets, however, little is known about its affect on bond markets. In this paper we test the link between macroeconomic disagreement and expected bonds returns using the BlueChip data set of macroeconomic forecasts that allows us to directly look at market participants expectations regarding real, nominal, and monetary components of the economy.

The term structure literature is truly vast. Traditional reduced-form and structural models have provided significant insights that have improved our understanding of the dynamics of interest rates and are used in a number of applications, including risk management, trading, and monetary policy. At the same time, however, the literature has highlighted several empirical regularities that are difficult to reconcile with traditional homogeneous economies with no frictions. First, long term bond yields appear too volatile to accord with standard representative agent models (Shiller (1979)). Second, model implied Sharpe ratios appear difficult to reconcile with those observed in the data (Duffee (2011)). Third, while unconditional excess returns on bonds are close to zero, there appears to be a large degree of predictability in returns by several yield curve factors (Fama and Bliss (1987) , Cochrane and Piazzesi (2005)). In addition, term structures contain information on future term structures in a direction and magnitude that is difficult to explain within some classes of models (Campbell and Shiller (1991)). Fourth, the dynamics of bond market risk compensation are complex and demand a rich specification for the price of risk. For example, Duffee (2002) proposes the 'essentially affine' class that allows for a flexible specification for the price of risk. However, while the essentially affine class can better match some salient features of the data, they are unable to match at the same time first and second moments of yields. Furthermore, in their canonical form, essentially affine models imply that primitive shocks underlying the economy are perfectly spanned by the yield curve so that macroeconomic aggregates contain no incremental information useful for bond pricing. Finally, interest rate dynamics display unspanned stochastic volatility: bond portfolios appear unable to hedge interest rate derivatives, thus suggesting some form of market incompleteness. It appears that the set of state variables driving volatility is not the same set driving yields.

Solutions posed in the literature can be roughly sorted into three strands: *i*) statistical models which include either extensions to the price of risk (essentially affine, extended affine, quadratic models) or extensions to the state space (time-varying covariances, Wishart or multi-frequency dynamics) ; *ii*) reduced form economic models which either use new econometric methods for measuring state variables (dynamic factor analysis, least absolute shrinkage and selection operator (lasso) approaches), or introduce observable information (monetary policy shocks extracted from high frequency data, spanned and/or unspanned risk factors); or *iii*) structural macro-finance models that include a richer preference structure (habit formation, ambiguity aversion, or recursive preferences). This paper takes a different route by focusing on the cross-sectional and time series relationship between heterogeneity in beliefs and *a*) bond market risk premia; *b*) volatility; *c*) and trading activity. Our empirical approach takes a reduced form approach within the framework of difference in beliefs model to provide the first set of comprehensive empirical results relating investor heterogeneity to bond market dynamics.

One of the first results showing the potential role played by heterogeneous beliefs is discussed in Harris and Raviv (1993), who developed a model of speculative trading based on difference of opinion in which investors receive common information but differ in the way in which they interpret information.[1] All investors in their economy agree on the nature of the information, be it positive or negative, but disagree on its importance. They show that the heterogeneity in beliefs has important implications for asset prices. Similar settings have been employed by Detemple and Murthy (1994) and Zapatero (1998) in the context of a continuous time economy. Buraschi and Jiltsov (2006) consider a general equilibrium economy with Bayesian learning and show how heterogeneous beliefs affect the equilibrium stochastic discount factor and become a source of variation for option prices and trading volumes. As disagreeing agents engage in risk sharing, prices of options are affected in equilibrium and even small changes in the differences in beliefs can generate an option-implied volatility smile and help to explain the dynamics of option prices. Disagreement can be shown to arise for agents (even when they posses common priors) due to different degrees of confidence on the data generating process. In times of high economic

[1] Recent equilibrium treatments of heterogeneity in beliefs include Bhamra and Uppal (2011); David (2008); Gallmeyer and Hollifield (2008) and Buraschi, Trojani, and Vedolin (2011)

uncertainty agents who observe noisy realisations of the dividend process disagree on the precision of the empirically generated probability measure and form different opinions regarding the model of expected future cash flows. Therefore, higher economic uncertainty is directly linked to difference in beliefs. Scheinkman and Xiong (2003) couple the assumption of overconfidence with short-selling constraints and show that these two assumptions can lead to equilibrium asset price bubbles. More recently, Xiong and Yan (2010) provide a theoretical treatment of bond risk premia in a heterogeneous agent economy. The authors develop a model of speculative trading in which two types of investors hold different beliefs regarding the central bank's inflation target. In the model, the inflation target is unobservable so investors form inferences based on a common signal. Although the signal is actually uninformative with respect to the inflation target heterogeneous prior knowledge causes investors to react differently to the signal flow. Investor trading drives endogenous wealth fluctuations that amplify bond yield volatilities and generates a time varying risk premium. They provide a calibration exercise and show that a simulation of their economy can reproduce the Campbell and Shiller (1991) regression coefficients and the tent shaped linear combination of forward rates from Cochrane and Piazzesi (2005). Our results build on those by Xiong and Yan (2010) as no empirical evidence on this topic is yet available in the literature.

Framing our empirical study, we derive testable hypotheses for the role of macroeconomic disagreement for (*a*) bond risk premia; (*b*) volatility; and (*iii*) trading activity. Considering a multiple agent Lucas tree economy with some minimal structure on the endowment process we show that under logarithmic preferences the local diffusion of the Radon-Nikodym derivative of agents' likelihood functions is proportional to differences in belief regarding a non-observable stochastic component that enters the growth rate of the economy. The Radon-Nikodym derivative completely characterises risk sharing and thus equilibrium risk premia which covary positively with disagreement. Moreover, in this economy the formation of expectations directly affects bond volatility, even if in a (fictitious) homogeneous economy volatility were to be constant. Finally, we show that optimal portfolio holdings depend on the level of disagreement and thus heterogeneity in beliefs represent a direct source of trading activity

Our empirical results proceed along several fronts. First, casting our tests in the context of classic return predictability regressions we show that the cross-section of agents expectations contains economically important and statistically significant information on expected excess bond returns on a 1-year horizon. The combination of real, inflation and monetary disagreement measures forecast excess bond returns with $\overline{R}^2$ equal to 43% and 21% on 2-year and 10-year bonds, respectively. We find that disagreement about the real economy is highly statistically significant in a number of specifications and loads positively on expected excess returns, while disagreement about inflation appears less important and is subsumed by monetary components. Disagreement about short term, and long term interest rates loads positively, and negatively, respectively, and is always highly statistically significant. Controlling for consensus views and realisations of fundamentals we test whether the information content in belief dispersion is subsumed by more traditional predictor variables and find the results are robust to the inclusion of a number of alternatives. Additionally, we recast our return predictability tests in terms of reverse regressions a la Hodrick (1992) and confirm its statistical significance. These findings are important since they shows that information contained in the belief structure of the economy that is key for asset pricing is not contained in representative expectations or in macro aggregates; thus, single agent homogeneous economies are incapable of fully explaining the term structure puzzles highlighted above.

Second, we examine the role of heterogeneity for second moments by running regressions of stock and bond volatility measured from squared daily returns between $t \to t+1$ on disagreement recorded at t and find a strong result. Consistent with our theoretical framework, relative wealth fluctuations between agents who 'agree to disagree' generate a source of endogenous return volatility. In monthly projections disagreement about the real economy and inflation load positively on realised future volatility of stocks and bonds, with t-stats significant at the 1% level, and $\overline{R}^2$ of 26% and 23%, respectively. Symmetrically to the results on return predictability, in specifications including monetary components we find no marginal increase on the explanatory power above pure macro disagreement. Controlling for macro expectations and fundamentals has no effect on real disagreement while disagreement about inflation loses some significance for stock return volatility. Overall, the results on volatility are striking, statistically significant, and consistent with economic theory.

Third, we focus on the relationship between investor heterogeneity and trading activity by running regressions of the growth rate of open interest on belief dispersion. To summarise, the results strongly suggest an economically important and statistically robust positive correlation between investor heterogeneity and trade. Considering open interest from options and futures on 10-year treasury notes, including only disagreement on the right hand side, we find that heterogeneity explains the time variation in open interest growth with $\overline{R}^2$ equal to 21% while adding macro fundamentals raises this $\overline{R}^2$ to 35%. Importantly, the t-stats on real disagreement and inflation are significant at the 5% level or higher. This result is consistent with the intuition that optimal portfolio holdings are determined by subjective beliefs so that changes in the belief structure of the economy generate trade until the equilibrium relative weights of logarithmic investors equal the Radon-Nikodym derivative of their beliefs.

Fourth, we study the spanning properties of macroeconomic disagreement. We find that time variation in the shape of the forward curve in part represents heterogeneity in the belief structure of the economy, thus lending economic support to the empirical results Cochrane and Piazzesi (2005). Our findings suggest that cross-sectional price information reveals properties of the stochastic discount factor which are affected by disagreement. This is consistent with intuition derived in the motivating framework below that is independent of specific assumptions about agents' learning behaviour and/or the dynamics of the state variables. In this case, the information embedded in the cross-section of yields is correlated with the time variation in the price of risk, thus proving to be a forecasting factor for excess bond returns. However, disagreement is only partially spanned by the yield curve in the sense that important components of disagreement, which are orthogonal to the first 5 principle components of yields, contain economically and statistically important information on expected returns. In a return predictability regression including the unspanned components of disagreement as right hand variables we find that disagreement about inflation and real GDP are unimportant but that disagreement about short and long ends of the yield curve are statistically significant at the 5% level, forecasting bond returns with an $\overline{R}^2$ of between 27% and 29%, on $2-5$ year bonds. Furthermore, exploring the relationship between the time-series dynamic of yields and disagreement we project the hidden risk premium factor from Duffee (2011) on unspanned components and find that disagreement about short term interest rates is statistically significant at the 1% level with an $\overline{R}^2$ statistic of 6%. Finally, using information orthogonal to a space spanned by both the cross-section and time-series dynamics of yields we document an 'above' component linked to disagreement about long term interest rates which retains economically important forecasting power for expected returns.

I A Motivating Framework

Consider an economy with two agents with different subjective conditional probability measures dQ_t^a and dQ_t^b. When absolutely continuous, the difference in subjective measures, or 'disagreement', between the two agents can be conveniently summarized by the Radon-Nikodym derivative $\eta = \frac{dQ^b}{dQ^b}$; let X_t be an $\Im_t$-measurable random variable, then

$$E^b(X_T|\Im_t) = E^a\left(\frac{\eta_T}{\eta_t}X_T|\Im_t\right). \tag{1}$$

In the context of an economy in which agents agree to disagree and X_t is observable, $\Im_t$ is common knowledge among all agents, so that as $T \to t$ agents agree on X_t. In the difference in beliefs literature, Scheinkman and Xiong (2003) and Buraschi and Jiltsov (2006) study economies in which a process η_t arise from investors' different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables. Kurz (1994) argues that non-stationarity of economic systems and limited data make it difficult for rational investors to identify the correct model of the economy from alternative ones. If agents use signals to learn about a random variable, Acemoglu, Chernozhukov, and Yildiz (2008) show that if agents are uncertain about the informativeness of the signal, then the observation an infinite sequence for these signal does not guarantee that disagreement will disappear asymptotically. This is because investors have to update beliefs about two sources of uncertainty using one sequence of signals. Other important examples of economies with rational learning and disagreement include Dumas, Kurshev, and Uppal (2010), Buraschi, Trojani, and Vedolin (2011), and Xiong and Yan (2010).

Independent of the structural economy supporting the dynamics for disagreement, equilibrium conditions give rise to an important link between the dynamics of disagreement and relative wealth ratios. For instance,

Lemma 1. *If agents have logarithmic preferences, $u(c^i) = e^{-\rho(T-t)} \ln c_t^i$, they will trade until their wealth ratio is equal to η_T, i.e. $\eta_T = W_T^b/W_T^a$.*

Furthermore, the assumption of logarithmic preferences implies further restrictions on the link between (fictitious) homogeneous economies that would be populated by only one type of investor and multiple agent economies populated by heterogeneous investors.

Theorem 1. *In equilibrium the price of a zero-coupon bond $B_t^{(T-t)}$ with time to maturity $T-t$ is equal to the η_t-weighted average of the zero-coupon bonds prices prevailing in the (fictitious) homogeneous economies populated only by each of the two agents, $B_t^{(T-t),a}$ and $B_t^{(T-t),b}$, with*

$$B_t^{(T-t)} = \frac{1}{1+\eta_t}B_t^{(T-t),a} + \frac{\eta_t}{1+\eta_t}B_t^{(T-t),b} \tag{2}$$

the result holds independently of any specific assumption on the endowment market structure or learning process. As differences in beliefs affect the dynamics of the wealth ratio of the two agents, the formation of expectations plays a direct role in the relative weight of the two agents in equilibrium bond prices. This generates important implications for both the link between the cross-sectional shape of the term structure and η_t and the time-series properties of bond prices (e.g. expected returns). The following implication emerges:

Corollary 1. *Suppose that a log utility homogeneous economy supports an affine solution for B_t^a and B_t^b in some subjective state variable $\hat{f}_t^i$, i.e. $B_t^i = \exp(A(t,T) + B(t,T)\hat{f}_t^i)$, the equilibrium bond price in an economy with differences in beliefs would not be affine. To see this it is sufficient to notice that the bond yield $\frac{1}{T-t} ln(B_t) = \frac{1}{T-t}\ln(\frac{1}{1+\eta}[A(t,T)\ \exp(C(t,T)\hat{f}_t^a] + \frac{\eta}{1+\eta}[A(t,T)$ $\exp(C(t,T)\hat{f}_t^b]$ is not linear in the subjective state variables $\hat{f}_t^a$ and $\hat{f}_t^b$.*

This shows that, while convenient for many purposes, the affine framework for bond prices does not easily allow for aggregation with respect to heterogeneous information sets. An example is provided by Xiong and Yan (2010) in the context of a monetary economy. They consider the case of agents disagreeing on the inflation target rates set by the central bank. The result, however, applies more generally.

To motivate the empirical exercise we now add some minimal structure to the endowment process and study the implications on: (a) bond risk premia, (b) bond volatility, and (c) bond trading volume. Suppose that the growth rate of the endowment follows a process that reverts to a stochastic target θ_t

$$dD_t/D_t = \delta_t dt + \sigma_d dW_t^d, \tag{3}$$

$$d\delta_t = -\lambda_\delta(\delta_t - \theta_t)dt + \sigma_\delta dW_t^\delta. \tag{4}$$

where W_t^d is a scalar Brownian motion on filtered probability space $\{\Omega, \mathcal{F}, \mathcal{F}(t)\}, \mathcal{P}\}$, and σ_ϵ and δ_t, but not θ_t are adapted to $\{\mathcal{F}^d(t)\}$. The evolution of θ_t may depend on incompletely observed state variables such as, for instance, policy decisions M or on the unknown effect of these policies, i.e. $\theta_t = \theta(M, dW_t^\theta)$.

$$d\theta_t = -\lambda_\theta(\theta_t - \bar{\theta}(M))dt + \sigma_\theta dW_t^\theta. \tag{5}$$

Let $\theta_t^i(M^i, dW_t^{i,\theta})$ be the agent specific assumption and $\hat{\theta}_t^i$ be the subjective growth rate. The incomplete information about θ_t gives rise to agent disagreement about the value of δ_t thus affecting the optimal demand of long-term bond of different agents. Previous literature has described the implications of learning when agents update their beliefs using both observations on D_t and imperfectly correlated signals (see Buraschi and Jiltsov (2006)).

A Bond Risk Premia

Imposing the log-utility case in the above framework shows that the equilibrium stochastic discount factors of the two agents must be proportional to $\xi_t^i \propto e^{-\rho t}\frac{1}{c_t^i}$.[2] Substituting the optimal consumption equations given in the appendix yields $\xi_t^a \propto e^{-\rho t}\frac{(1+\eta_t)}{D_t}$ and $\xi_t^b \propto e^{-\rho t}\frac{(1+\eta_t)}{\eta_t D_t}$. The subjective pricing kernel for either agent is then obtained by applying Ito's lemma to the first order condition of the individual agents problem:

$$\frac{d\xi_t^a}{\xi_t^a} = -(\rho + \delta_t - \sigma_d^2)dt - \underbrace{\sigma_d}_{k_d} dW_d(t) - \underbrace{\frac{-\eta_t}{1+\eta_t}}_{k_\eta^a(t)} \frac{d\eta(t)}{\eta(t)} \tag{6}$$

$$\frac{d\xi_t^b}{\xi_t^b} = -(\rho + \delta_t - \sigma_d^2)dt - \underbrace{\sigma_d}_{k_d} dW_d(t) - \underbrace{\frac{+1}{1+\eta_t}}_{k_\eta^b(t)} \frac{d\eta(t)}{\eta(t)} \tag{7}$$

Three observations emerge. First, the process η_t is a direct source of priced risk. By solving for $\frac{d\eta(t)}{\eta(t)}$ it is possible to show that since the local diffusion of the factor of $d\eta(t)/\eta(t)$ is proportional to $DiB_t \equiv (\hat{\theta}_t^b - \hat{\theta}_t^a)$, the price of risk itself is proportional to differences in beliefs. For homogeneous agents, $\hat{\theta}_t^b = \hat{\theta}_t^a$, one obtains the standard representation for the stochastic discount factor, with the price of risk being simply equal to dividend volatility. When agents are heterogeneous, $\hat{\theta}_t^b > \hat{\theta}_t^a$, the price of risk of agents b is greater than a as his relative consumption is lower in bad states of the world due to the ex-ante optimal risk sharing agreement. Second, the elasticities of the discount factors of the two agents with respect to changes in η are different and depend on their wealth ratios, which depend on the realisations of their speculative trades. This implies that it is a non-diversifiable source of risk: shocks increasing the discount factor of one agent are not canceled out by the reduction in the discount factor of the other agent. Third, one can observe that the price of risk $k_\eta(t)$ is time-varying, as long as disagreement is time-varying. This is important since it implies that expected excess return covary with disagreement $DiB_t = (\hat{\theta}_t^b - \hat{\theta}_t^a)$ making disagreement a priced state variable that could help explain expected excess bond returns.

Corollary 2. *If bond prices can be inverted to reveal the underlying state dynamics, then the cross-section of bond prices reveals DiB_t, thus expected excess bond returns. To see this, note*

[2] The proportionality factor is equal to the Lagrange multiplier of the static budget constraint in the associated martingale representation of the investment-consumption problem.

that equation 29 in the appendix implies that observing the cross-section of bond yields should be sufficient to reveal the disagreement in the economy since $\theta_t^a - \theta_t^b = [I_a(B_t^a) - I_b(B_t^b)]$ *where* $I(B)$ *and is the inverse function to the bond pricing formulas in the homogeneous economies.*

This property shows the possibility (as well as the limits) of using the cross-sectional features of the yield curve to reveal characteristics of the stochastic discount factor that may influence expected bond returns. For disagreement to be spanned by the cross-section of yields it is necessary that bond prices in the homogeneous economies to be invertible with respect to θ. While this is indeed the case for an affine economy, it is known that in several specifications yield inversion is not unique. Moreover, if risk aversion or prudence were time-varying, the representative investor would have time-varying parameters loadings on interest rates. In this case, the representation for DiB_t would require information from both the cross-section and the time-series properties of yields.

B Bond Volatility

In an homogeneous economy, the volatility of bond yields are affected by the volatility of the factors driving the marginal productivity of capital. In an heterogeneous economy, the process of formation of expectations can directly affect bond volatility. To see how this occurs, consider the simple case in which the homogeneous economy has an affine factor structure. In this case, it is easy to see that $\sigma(y_t^{(T-t)}) = \sigma(\frac{1}{T-t}\ln(\frac{1}{1+\eta_t}[\exp(A(t,T)+B(t,T)\hat{f}_t^a)] + \frac{\eta_t}{1+\eta_t}[\exp(A(t,T)+B(t,T)\hat{f}_t^b)]))$. Since $d\eta_t/\eta_t = (\hat{\theta}_t^b - \hat{\theta}_t^a)\frac{\lambda_\theta}{\sigma_\theta}d\hat{W}_\theta^a(t)$, it is possible to show that $\sigma(y_t^{(T-t)})$ is increasing in $(\hat{\theta}_t^b - \hat{\theta}_t^a)$.

C Trading Volume

It is possible to show that optimal portfolio holdings depend on the level of differences in belief. Assume there exists a risk free security and a risky default free security (in zero net supply) with terminal payoff B_T with subjective price dynamics $dB_t/B_t = \mu_t^B(\theta_t^i)\, dt + \sigma_B\, dW_t^\delta$. For each agent, the optimal portfolio holding follows from Merton's and it is proportional to the ratio of the bond risk premium (the expected excess bond return) over it variance:

$$q_t^i = W_t^i \frac{\frac{\partial B}{\partial d}\kappa_d\sigma_d + \frac{\partial B}{\partial \theta}\kappa_\theta^i\sigma_\theta}{\sigma_B^2(\hat{\theta}_t^b - \hat{\theta}_t^a)}. \tag{8}$$

Since it is possible to show that $\kappa_\theta^i = -\eta_t/(1-\eta_t)(\hat{\theta}_t^b - \hat{\theta}_t^a)\lambda_\theta/\sigma_\theta$, and $\kappa_\theta^i = +1/(1-\eta_t)(\hat{\theta}_t^b - \hat{\theta}_t^a)\lambda_\theta/\sigma_\theta$, then the component of bond holding sensitive to the beliefs on $\hat{\theta}_t^i$ are

$$q^b(\hat{\theta}_t) = \frac{\eta_t(\hat{\theta}_t^b - \hat{\theta}_t^a)\lambda_\theta/\sigma_\theta,}{(1-\eta_t)(1+\eta_t)} \text{ and } q^a(\hat{\theta}_t) = -q^b(\hat{\theta}_t). \tag{9}$$

When $\hat{\theta}_t^b > \hat{\theta}_t^a$, because of the larger subjective risk premium, the optimist holds more of the riskier (although default free) security. Changes in DiB_t affect bond trading volume.

While the previous restrictions suggest a monotonic relationship between DiB_t and bond risk premia, volatility, and trading volume it is important to stress that they rely on the assumption of log-preferences. There is an emerging literature studying more general preference specifications, or a non trivial dynamics for the distribution of optimists and pessimists that can generate non-linear implications. Interesting recent studies include Bhamra and Uppal (2011) and Borovicka (201). The first studies the interaction of heterogeneity in both preferences and beliefs and provides closed form solutions for the representative discount; the second study compares the long-run properties of a two-agent heterogeneous beliefs economy with separable and recursive preferences. He derives conditions under which recursive preferences admit equilibria in which both agents survive in the long run. Ultimately, whether differences in beliefs play a significant role to help explaining bond puzzles is an empirical question.

D The Dimensionality of the State-Space

For the empirical exercise, it is important to carefully specify the dimensionality of the state-space and disagreement. Buraschi and Jiltsov (2006) show that if agents use multiple signals z_1 and z_2 to help improve inference about $\hat{\theta}_t$ but disagree on their informativeness, then disagreement on the growth rates of both $z_1$and z_2 will directly affect the price of risk. Disagreement is a source of market incompleteness that can increase the dimensionality of the asset pricing state-space beyond the number of observable fundamental variables. Buraschi and Jiltsov (2006) use this feature to support a simple economy that makes options not redundant even if the endowment process has constant volatility under the physical measure, so that an option would be redundant in the respective single agent economy. Motivated by these observations we allow for multiple type of disagreements and we let the data decide which of them is relevant in explaining bond excess returns.

First, we consider disagreement on fundamental variables, such as gdp and inflation. Second, we consider disagreement on short-term interest rates since agent may disagree on the policy decisions about the Fed Fund rate, which is decided at the FOMC meetings by the Federal Reserve Banks. Finally we consider a proxy of disagreement about future *risk-adjusted* interest rates (long-term bond yields), i.e. $E_{dQ_t^i}(E_{d\tilde{Q}_{t+1}^R}(B_T))$ where $d\tilde{Q}_{t+1}^R$ is the risk-adjusted measure of the representative agent at time $t+1$. This is important since it allows us to investigate whether disagreement on future equilibrium discount factors can affect prices today after controlling for disagreement in fundamentals under the physical measure.

II Data

A Disagreement Data

We obtain measures of heterogeneity directly from market participants' expectations of future fundamentals. Survey data provides a rich source to learn how agents form beliefs about economic variables but few sources exist with large sample periods or appropriate frequencies; BlueChip Economic Indicators (BCEI) does provide an extensive panel of data on expectations by agents who are woking at institutions who are active in financial markets and importantly it allows a simple aggregation procedure (discussed below) that mitigates problems associated with rolling forecast horizons. Unfortunately, digital copies of BCEI are only available since 2007. We obtained, however, the complete BCEI paper archive directly from Wolters Kluwer and proceeded to manually enter the data. The digitisation process required inputting around 350,000 entries of named forecasts plus quality control checking. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations in asset pricing.

Each month BlueChip carry out surveys of professional economists from leading financial institutions and service companies regarding a large set of economic fundamentals covering real, nominal, and monetary variables. While exact timings of the surveys are not published, the survey is conducted over the first two days of the beginning of each month and mailed to subscribers on the third day our empirical analysis is therefore not affected by biases induced by overlapping observations of returns and responses [3]. The sample period for which we have a fully digitised dataset is 1.1.1990 - 1.12.2011. Forecasts are available for:

[3] An exception to the general rule was the survey for the January 1996 issue when non-essential offices of the U.S. government were shut down due to a budgetary impasse and at the same time a massive snow storm covered Washington, DC: www.nytimes.com/1996/01/04/us/battle-over-budget-effects-paralysis-brought-shutdown-begins-seep-private-sector.html. As a result, the survey was delayed a week.

1. Real: Real GDP, Disposable Personal Income, Non-residential Fixed Investment, Unemployment, Industrial Production, Corporate Profits, Housing Starts, Auto/Truck Sales.

2. Nominal: Consumer Price Inflation, Nominal GDP.

3. Monetary: 3 Month Treasury Rate, 10 year Treasury, AAA corporate Bond.

Furthermore, for each variable two types of forecast are made:

1. Short-Term: an average for the remaining period of the current calendar year;

2. Long-Term: an average for the following year.

For example, in July 2003 each contributor to the survey made a forecast for the percentage change in total industrial production for the remaining two quarters of 2003 (6 months ahead), and an average percentage change for 2004 (18 months ahead). The December 2003 issue contains forecasts for the remaining period of 2003 (1 month ahead) and an average for 2004 (13 months ahead). The moving forecast horizon induces a seasonal pattern in the survey which can be adjusted in two simple ways: i) one can adjust cross-sectional statistics for both long and short term forecasts using an X-12 ARIMA filter and subsequently take some linear combination of the resulting seasonally adjusted measures; or ii) one can compute an implied constant maturity forecast for each individual forecaster, compute summary statistics, then adjust any residual seasonality with an X-12 ARIMA filter. Testing both methods we find that combining long and short term forecasts at the individual level removes the vast majority of the observable seasonality and we proceed with this method which is outlined in detail in the appendix. On average 51 respondents are surveyed for short term forecasts and 49 for long term forecasts with standard deviations of 1.6 and 3.3 respectively. Figures 2 and 3 plot the distributions and time series properties of respondent numbers which show that only on rare occasions are survey numbers less than 40 and no business cycle patterns are visible. For comparison, figure 4 plots the time series and distribution of respondents to the more traditional 'Survey of Professional Forecasters'.[4] Compared with the BlueChip dataset the distribution of respondents displays significant variability; for example, while the mean number of respondents is around 40, the standard deviation is 13 and in some years the number of contributors is as low as 9. Furthermore, while forecasts are available since the 4th quarter of 1968 the survey is conducted quarterly meaning that including the 4th quarter of 2011, 169 observations are available compared with 252 from BlueChip over the sample period

[4]available here:
www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/.

considered here. The survey has been administered by different agencies over the years. While at the beginning of the sample the number of forecasters was around 60, it decreased in two major steps in the mid 1970s and mid 1980s to as low as 14 forecasters in 1990 and if one restricts the attention to forecasters who participated to at least 8 surveys, this limits the number of data point considerably. There are additional problems, for example, it is suggested that quarters may not be comparable across years since the forecasting horizon shifted in a non-systematic way. For a detailed discussion on the issues related to the Survey of Professional Forecasters see D'Amico and Orphanides (2008) and Giordani and Soderlind (2003). Other well known surveys, such as the 'University of Michigan Survey of Consumers' do not provide point estimates from individual survey respondents.

[Insert figure 2 , 3 and 4 here.]

Macroeconomic disagreement is then measured as the cross-sectional mean-absolute-deviation (MAD) in forecasts. Finally, we proxy for disagreement about the real economy from the first principle component of the filtered MAD regarding Industrial Production Growth and Real GDP, and disagreement about inflation from the first principle component of filtered MAD about the GPD deflator and the Consumer Price Index.

[Insert figure 5,6,7,8]

Figure 5 plots the first principle component from the individual disagreement measures shown in figures 6 - 8 [5]. One can observe observe a general decline in the level of disagreement since the early 1990's accompanied by economically interesting periods characterised by large spikes. Swanson (2006) makes a similar observation using a different measure of cross-sectional uncertainty on the same data set. The purpose of Swanson (2006) was to study the effect of central bank transparency with respect to private sector interest rate forecasts. Using various measures of forecast accuracy the author shows that since the 80's private sector agents have a) improved projections of the federal funds rate; and b) are more unanimous (cross-sectionally) in forming expectations. In unreported results we find that not only are agents more unanimous regarding interest rate forecasts but are more unanimous regarding real, nominal, and monetary elements of the economy. Figures 6 - 8 plot the time series dynamics for inflation, real, and monetary disagreement measures [6]. Comparing the plots with the summary statistics from table I we

[5]As usual the 1st PC is essentially a level factor which in this instance explains $\sim 45\%$ of the variance from the underlying measures

[6]In constructing disagreement about long term rates we use forecasts of AAA rated corporate bonds before 1996 since 10 year Treasury rate forecasts are were unavailable before then.

find that while measures of real and inflation disagreement are highly correlated, disagreement across real and inflationary components is not especially high. Furthermore, the dynamics of disagreement regarding long and short ends of the yield curve appear quite distinct.

[Insert table I here.]

B Stock and Bond Data

For Treasury bonds data, we use both the (unsmoothed) Fama-Bliss discount bonds dataset, for maturities up to five years, and the (smoothed) Treasury zero-coupon bond yields dataset of Gürkaynak, Sack, and Wright (2006) (GSW). The GSW data set includes daily yields for longer maturities: 1-15 years pre-1971 and 1-30 years post-1971.[7] We introduce notation along the lines of Cochrane and Piazzesi (2005) by defining the date t log price of a n-year discount bond as:

$$p_t^{(n)} = \text{log price of n-year zero coupon bond.} \tag{10}$$

The yield of a bond, the known annual interest rate that justifies the bonds price is given by $y_t^{(n)} = -\frac{1}{n}p_t^{(n)}$.The date t 1-year forward rate for the year from $t+n-1$ and $t+n$ is $f_t^{(n)} = p_t^{(n)} - p_t^{(n+1)}$.The log holding period return is the realised return on an n-year maturity bond bought at date t and sold as an $(n-1)$-year maturity bond at date $t+12$:

$$r_{t,t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}. \tag{11}$$

Excess holding period returns are denoted by:

$$rx_{t,t+12}^{(n)} = r_{t,t+12}^{(n)} - y_t^{(1)}. \tag{12}$$

The realised second moments of stock and bond returns are measured at daily frequency following Schwert (1990) and Viceira (2007) among many others. Integrated instantaneous volatility is proxied by realised volatility between month t and $t+1$ as

$$\widehat{\sigma}_{S,B}^2(t) = \frac{1}{n-1}\sum_{i=1}^{n} r_{S,B}^2(t,i). \tag{13}$$

[7] The dataset is available at: www.federalreserve.gov/econresdata/researchdata.htm.

Integrated instantaneous covariance is then proxied by realised covariance on stocks and bonds between month t and $t+1$:

$$\widehat{\sigma}_{S,B}(t) = \frac{1}{n-1}\sum_{i=1}^{n} r_S(t,i) \times r_B(t,i), \tag{14}$$

and stock-bond correlation then estimated as:

$$\widehat{\rho}_{S,B}(t) = \frac{\widehat{\sigma}_{S,B}(t)}{\widehat{\sigma}_S(t)\widehat{\sigma}_B(t)}. \tag{15}$$

All estimates are then annualised appropriately. For volatility and correlation estimates, we use squared daily returns from the GSW dataset. As proxy for the equity market portfolio we take the value-weighted index from the daily CRSP files which consists of stocks traded in NYSE, AMEX, and NASDAQ. Stock and Bond data sample periods run from 31.12.1989 - 30.11.2011.

C Open Interest

In order to gauge a measure of trade we collect total combined (options and futures) open interest on CBOT 10-year Treasury Notes and the CME S&P 500 index from the 'Commitments of Traders in Commodity Futures' which is available here: `http://www.cftc.gov/OCE/WEB/index.htm`. Following Hong and Yogo (2011) we compute a 12-month geometrically averaged growth rate of bond and stock market open interest. Sample Period includes 28.03.96 - 30.11.11

D Macro Data

Dynamic macroeconomic theory suggests a small set of common factors are responsible for the co-movement of a large set of economic and financial time series. However, until recently the search for these factors has been carried out with limited success. The limited success of linking the macro economy to term premia led researchers to explore alternative empirical routes to pin down the state variables priced in bond markets. For example, Ang and Piazzesi (2003) estimate a VAR with identifying restrictions derived from the absence of arbitrage and find the combination of macro and yield curve factors improves performance over a model including yield factors only. More recently Ludvigson and Ng (2009b) find strong evidence linking variations in the level of macro fundamentals to time variation in the price of risk. We adopt the procedure of Ludvigson and Ng (2009b) by estimating macro-activity factors using static factor analysis on a large panel of macroeconomic data. The panel used in our estimation is an updated version of the one in Ludvigson and Ng (2009a), except that we exclude price based information in order to interpret factors as pure 'macro' and allow clearer distinction between information contained in agents' beliefs from that contained in macroeconomic aggregates [8]. After removing price based information from the panel we end up with a 99 macro series. Classical understanding of risk compensation for nominal bonds also says that investors should be rewarded for the volatility of inflation and consumption growth. We proxy for these by estimating a GARCH process for monthly log differences of CPI All Urban Consumers: Non-Durables (NSA) and Industrial Production and Capacity Utilisation: All Major Industry Groups (NSA). Finally, from Campbell, Sunderam, and Viceira (2009) we know that an important driver of bond risk premia is the real-nominal covariance which we proxy for by estimating a dynamic correlation MV GARCH process for inflation and consumption growth. All macro data is either from Global Insight or the Federal Reserve Economic Data (FRED) set. Sample Period includes 1.1.1990 - 1.12.2011.

III Empirical Results

In the following section we study the role of heterogeneity across a number of dimensions of asset pricing : i) risk premia; ii) volatility; iii) trade; iv) the spanning properties of bond prices. Specifically, we run multivariate regressions that focus on differences in belief about the real economy and inflation, and augment these measures with monetary measures that potentially reveal important information for bond pricing [9]. The estimated coefficients in the tests that follow are both economically and statistically significant and survive a host of robustness tests. Importantly, the signs on disagreement can be rationalised within the existing theoretical literature on investor heterogeneity.

A Disagreement and Bond Risk Premia

The expectations hypothesis says that nothing should forecast excess returns, or alternatively, in expected return regressions the factor loadings on right hand variables should not be different from zero. If the risk premium is time-varying, however, we can expect deviations from the expectation hypothesis in the form of return predictability. To investigate the potential link between macro economic disagreement and return predictability we run multivariate forecasting regressions of

[8] Examples of price variables removed include: S&P dividend yield, the Federal Funds (FF) rate; 10 year T-bond; 10 year - FF term spread; Baa - FF default spread; and the dollar-Yen exchange rate. A small number of discontinued macro series were replaced with appropriate alternatives or dropped.

[9] one may worry about the inclusion of persistent interest rates as right hand variables, *disagreement* about interest rates however is not that persistent (see table I) compared to, for example, dividend yields.

1-year excess returns from 2, 5 and 10 year maturity bonds and control for a number of factors known in the literature to contain information on expected returns. We run regressions of the following form:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{4} \beta_i^{(n)} DiB_{i,t}(\star) + \sum_{i=1}^{2} \gamma_i^{(n)} E_{i.t}(\star) + \sum_{i=1}^{3} \phi_i^{(n)} Macro_{i,t}(\star) + \varepsilon_{t+12}^{(n)},$$

where $DiB_t(\star)$ includes the set of disagreement measures as discussed above, $E_t(\star)$ is the consensus estimate of either expected inflation or expected RGDP, and $Macro_t(\star)$ includes a set of controls as outlined in section D.

[Insert table II here.]

Table II columns $(i) - (iii)$ report that disagreement about the real economy, long and short term interest rates are statistically significant with slope coefficients increasing in magnitude with bond maturity, indicating a larger change in the term premium for longer maturity bonds given a shock to any one factor. In terms of predictable variation, the results are striking: dispersion in beliefs forecasts excess returns with $\overline{R}^2$'s ranging from 21% on 10-year bonds to 43% on 2-year bonds. For 2-year bonds the t-statistics for the slope coefficient on the real economy (long rates, and short rates) is equal to 3.82 (-3.84, and -5.23) respectively. However, while disagreement about inflation appears significant in specification (i) for 2-year bonds, it does not appear important for expected bond returns elsewhere. The signs of the slope coefficients on disagreement about the real economy and short rates are positive, while the signs of the slope coefficient on disagreement about long rates is negative. To make clear the economic significance of the estimated loadings table **??** documents the effect on risk premia given a shock to any one factor compared to unconditional mean returns. According to the model expected excess returns are highly variable; for example, 10-year bond returns averaged 4.98% above the risk free 1-year bond return but with a standard deviation of 2.16%. A 1-standard deviation shock to disagreement about the real economy raises expected returns on these bonds by 1.42% while a 1-standard deviation shock to disagreement about short term rates raises expected returns by 2.39%

Columns (iv) and (v) control for information contained in macro expectations and in macro aggregates, respectively. The information contained in the cross section of agents expectations is largely orthogonal to either consensus views or realisations of fundamentals themselves. Specifically, the consensus does not enter significantly alongside disagreement and has virtually no effect on $\overline{R}^2$'s. Furthermore, while many of the macro factors enter significantly the only loss of statistical significance is for real disagreement for 2-year bonds only, while there is very little increase in $\overline{R}^2$'s. These results suggest that a sizeable proportion of time-variation in expected returns is due to changes in the level of macroeconomic disagreement and that this result is not subsumed by more traditional risk factors that have been studied recently in the fixed-income literature.

A.1 Robustness

The standard approach in the predictability literature relies on compounding returns and conducting significance tests of explanatory variables using overlapping observations. It is well known however that the use of overlapping returns in not innocuous from a statistical point of view. Compounding returns induces an MA(12) error structure under the null of no predictability which must be corrected for during estimation. In the above we conducted tests of return predictability using a robust GMM generalisation of Hansen and Hodrick (1983) with an 18-lag Newey-West correction. While most researchers agree that risk premia are time-varying the size of the observed predictability is a topical question. A good summary for the arguments against a 'large' predictable component in asset returns are given by Ang and Bekaert (2007) in the space of stock returns or by Wei and Wright (2010) in the space of bond returns. Ang and Bekaert find that the evidence of long horizon predictability using Hansen-Hodrick or Newey-West errors disappears once robust correction of heteroskedasticity and autocorrelation is conducted, while Wei and Wright argue that long-horizon predictive regressions using overlapping observations induce serious size distortions even after correction. Both sets of authors advocate use of an alternative inference procedure proposed by Hodrick (1992).

Hodrick (1992) proposes an alternative estimator for the point estimate, β, in return predictability regressions. The numerator of the estimator $\hat{\beta}^{(n)}$ is a covariance. Hodrick suggests to project 1-period returns on a lagged summation of right hand variables as opposed to the traditional projection of the future overlapping returns on time t observations. Covariance stationarity should lead to the same result:

$$cov(r_{t,t+k}, x_t) = cov(r_t + \dots r_{t+k}, x_t) \tag{16}$$

$$= \sum_{j=0}^{k-1} cov(r_{t+j}, x_t) = cov(r_t, \sum_{j=0}^{k-1} x_{t-j}). \tag{17}$$

The last term is the numerator of the slope coefficient of a regression of 1-period returns on a lagged summation of right hand variables. Long ($\hat{\beta}^{(n)}$) and short ($\hat{\gamma}$) horizon regression coefficients are therefore linked by the relation:

$$\hat{\beta}_k = V_o^{-1} cov(r_{t,t+k}, x_t) = V_0^{-1} V_n \hat{\gamma}. \tag{18}$$

where V_o is the parameter covariance matrix from the overlapping regression and V_n is the parameter covariance matrix from the non-overlapping regression. Therefore, a necessary and sufficient condition to reject the null of no-predictability using overlapping annual horizon returns is that the loading on a 12-period lagged sum of past disagreement measures be different from zero in a monthly forecasting regression. We call this the '*reverse regression*'.

In addition to testing the robustness of the above findings we also investigate the extent to which macroeconomic disagreement is exogenous to time t price innovations. One may worry that heterogeneity in beliefs might be correlated with contemporaneous return volatility so that disagreement would map risk premia associated with some other unobserved fundamental factor. If this were the case, date $t \to t+h$ returns and date t disagreement would be correlated by construction and no causal interpretation could be attached. Table IV addresses the issue of size and exogeneity simultaneously.

[Insert table IV here.]

We consider projections of 1-period returns on $h = 3,\ 6$ and 12 month summations of past disagreement measures (dropping disagreement about inflation) corresponding to implied forecast horizons of 3, 6 and 12 months. In addition to casting predictability tests in terms of reverse regressions we consider lags of $k = 1,\ 2$ and 3 of disagreement for each forecast horizon. Mindful of the so-called 'Richardson's Critique' who argues that interpretation of such results should take into account correlation in the test statistics, we estimate all the regressions simultaneously in a GMM framework and test the hypothesis that loadings on DiB^{RGDP}, DiB^{LR}, and DiB^{SR} are jointly different from zero. Considering the loadings on real disagreement, the t-statistics are significant at the 5% level or above for 8 out of 9 of the loadings. For example, consider a 2-month lag of disagreement for horizons $h = 3,\ 6$ and 12 the t-stats are 2.31, 2.70, and 2.52, respectively. Furthermore, considering a joint restriction for real disagreement we strongly reject the null of no predictability with asymptotic $\chi^2(3)$ values of 8.54 ($p = 0.04$), 8.78 ($p = 0.03$), and 8.66 ($p = 0.03$), respectively. The conclusion here then is that disagreement about real consumption growth contains substantial information on expected bond returns for horizons up to 1-year. Moving to robustness tests of monetary components the results are convincing: the estimated loadings are mainly individually significant at the 1% level, and jointly have $\chi^2(3)$ values that are always above the 5% threshold.

B Disagreement and Volatility

To study the role played by disagreement for the second moments of stock and bond returns we estimate stock / bond volatility and stock bond correlation according to equations 13 - 15 and run regressions of the type:

$$Vol/Corr_{t,t+1} = const + \sum_{i=1}^{4} \beta_i DiB_{i,t}(\star) + \sum_{i=1}^{2} \gamma_i E_{i,t}(\star) + \sum_{i=1}^{3} \phi_i Macro_{i,t}(\star) + \varepsilon_{t+1},$$

where as in the previous section $DiB_t(\star)$ is a set of disagreement measures, $E_t(\star)$ is consensus estimates, and $Macro_t(\star)$ is a set of controls estimated from fundamentals.

[Insert table V here.]

Table V reports estimates for second moment regressions. Considering first bond volatility, in contrast with the return predictability regressions where monetary disagreement was the strongest predicting factor, disagreement about long term rates is now insignificant, and while the loading on disagreement about short rates enters significantly in specifications (ii) - (iv) it does not survive the inclusion of $Macro_t(\star)$ and contributes very little in $\overline{R}^2$. However, symmetrically, the coefficients on inflation and real disagreement are both positive and highly statistically significant. In terms of $\overline{R}^2$ real and inflationary dispersion measures explain 26% of the time variation in 10-year treasury volatility. Table VI shows that the estimated loadings are also economically meaningful: compared to the sample mean a 1-standard deviation shock to disagreement about inflation or the real economy increases 10 year bond volatility by approximately 10%. Consensus views enter significantly with negative signs but add just 4% in $\overline{R}^2$ while real and inflationary disagreement remain significant. Finally, controlling for information in macro aggregates neither the level of macro activity or the volatility of inflation are significant. The volatility of consumption however, is positive and significant consistent with a standard single agent Lucas Tree economy where asset volatilities are equal to the volatility of the endowment process [10]. Consistent with heterogeneous agent Lucas Tree economies such as Buraschi and Jiltsov (2006), Xiong and Yan (2010),

[10]this evidence is in contrast with findings in Schwert (1990) who finds weak evidence to support the hypothesis that macroeconomic volatility can help predict stock and bond volatility

or Buraschi, Trojani, and Vedolin (2011), belief dispersion results in relative wealth fluctuations which amplify asset volatilities or can even generate heteroscedastic second moments when the endowment process is homoscedastic.

Considering now stock volatility, real and inflationary dispersion measures are again consistently positive and significant in specifications (*i*) and (*iii*). However, after controlling for consensus estimates which are negative and significant, and the volatility of consumption growth, which is positive and highly significant, we find that disagreement about inflation is driven out while disagreement on the real economy survives at the 10% level.

Finally, we find a number of results which contribute to the existing debate of the determinants of stock bond correlation. Firstly, consistent with the results David and Veronesi (2008) we do find a statistical relationship between dispersion in inflation expectations and the second moments of stocks and bonds [11]. However, like Viceira (2007) we also find that this result is not robust to the inclusion of other predicting factors, in our case the consensus view of inflation, disagreement about short term interest rates, and a macro activity factor [12]. In light of these findings, in unreported results, we also control for the level of the short term interest rate and find that the significance of expected inflation and its loading are cut in half ($\phi = 0.16$, t-stat$= 2.25$) in specification (*iii*), that the short rate does indeed drive out dispersion on inflation in specification (*i*), but that the significance and economic impact of disagreement about short rates is unaffected by its inclusion.

C Disagreement and Trade

In this section we examine the relationship between investor heterogeneity and trade by running regressions of opening interest (our gauge of trading activity) on belief dispersion. Since the level of open interest is non-stationary we follow Hong and Yogo (2011) and compute a 12-month geometrically averaged growth rate of bond and stock market open interest. We then run regressions of the type:

[11] David and Veronesi (2008) take a structural approach to forecasting second moments by specifying a joint macroeconomic relationship between nominal and real variables within a bayesian learning setting. Investor 'uncertainty' about fundamentals as proxied by dispersion in beliefs forecasts second moments with strong statistical significance after controlling for lags of second moments or macro aggregates.

[12] Viceira (2007) notes that inflation uncertainty proxied cross-sectional dispersion is driven out as a significant predictor once the short rate is included. Viceira suggests this result is because the level of the short rate is a general proxy for aggregate economic uncertainty.

$$\frac{OI(t+1) - OI(t)}{OI(t)} = const + \sum_{i=1}^{4} \beta_i DiB_t(\star) + \sum_{i=1}^{2} \gamma_i E_t(\star) + \sum_{i=1}^{3} \phi_i Macro_t(\star) + \varepsilon_{t+1}. \qquad (19)$$

Table VII reports the results. Column (i) reports the baseline specification including disagreement about inflation and the real economy on the right hand side. For the growth rate of open interest on treasury note futures and options disagreement on both inflation and the real economy loads positively with high statistical significance (4.38 and 3.62, respectively) with an $\overline{R}^2$ of 19%, while the results for the S&P are insignificant. Column (ii) introduces disagreement about monetary components which enter with statistically insignificant coefficients for disagreement on long term rates for treasury open interest with an $\overline{R}^2$ of 8% but again insignificant for the S&P. Moving to column (iii), which includes all disagreement measures as explanatory variables, we find a marginal contribution for disagreement about long term rates for treasury open interest (DiB^{LR} loads positively with a t-stat of 1.86) while the point estimates and t-stats for real and inflation dispersion measures on treasury open interest are largely unaffected. Finally, moving to columns (iv) and (v) we control for consensus expectations and then macro fundamentals. Including consensus views on treasury open interest, has little effect of disagreement about inflation (the point estimate is unaffected and with a t-stat of 3.79) while real disagreement becomes insignificant. However, one notices that real expectations themselves are contribute nothing: both the point estimate and its significance are almost zero. The result is therefore entirely driven by inflation expectations and thus have little theoretically to do with real uncertainty. In terms of predictable variation the addition of consensus view to disagreement raises the $\overline{R}^2$ just 4%, from 21% to 25%. Considering the inclusion of macro fundamentals in column (v) both disagreement about the real economy and inflation remain highly statistically significant, while the volatility of inflation and real consumption growth also enter significantly and raise the $\overline{R}^2$ to 35%. In summary, the results on disagreement and trade strongly suggest an economically important and statistically robust positive correlation between investor heterogeneity and trade, consistent with the intuition derived in the motivating example, that optimal portfolio holdings are determined by subjective beliefs so that changes in the belief structure of the economy generate portfolio rebalancing between until the relative weights of logarithmic investors equal the Radon-Nikodym derivate for their beliefs.

C.1 Disagreement and Economic Uncertainty

The theoretical origins of disagreement and uncertainty are distinct. The last refers to unknown unknowns and studies the role of the lack of knowledge regarding the reference model on the equilibrium demand *at the individual level*. The first focuses, instead, on the pricing implications of state-contingent trading *among disagreeing agents*. Empirically, while the last relies on proxies of dispersion of *individual* priors (or empirical measures of entropy) at the level of the individual agent, the first relies on the difference in the mean forecasts of *different agents*. While these concept are different, it is reasonable to argue, however, that they are conditionally correlated. In a world of certainty, after all, agents would not disagree.

Buraschi, Trojani, and Vedolin (2011) study explicitly the link between economic uncertainty and disagreement in a structural model with Bayesian learning and rational inattention. In the context of an economy in which agents disagree on the information content of a signal, used to form a posterior for the dividend growth rate, they show comparative static results linking: (a) the degree of average individual uncertainty and heterogeneity in perceived uncertainty across individuals; and (b) average individual uncertainty and belief disagreement. They show that in a linear model higher economic uncertainty leads to larger disagreement. It is sensible to argue, therefore, that observable measures of differences in beliefs can provide a forward looking proxy of economic uncertainty. D'Amico and Orphanides (2008) provide empirical evidence on this issue. They focus on inflation and report scatter plots for disagreement, uncertainty, and disagreement about uncertainty. They show that while disagreement and uncertainty are positively correlated, the correlation is imperfect thus suggesting that these two concepts require the use of explicitly distinct proxies.[13]

An important contribution to the literature related to ambiguity aversion is Ulrich (2010). He considers a single agent economy in which the investor has multiple priors about the inflation process and is ambiguity averse. The agent is assumed to observe the expected change in relative entropy between the worst-case and the approximate model for trend inflation. The observed set of multiple forecasts on trend inflation exposes the investor to inflation ambiguity. In the context of a min-max recursive multiple-prior solution, Ulrich (2010) shows that risk premia can be generated if changes in aggregate ambiguity are correlated with changes in the real value of a nominal bond. He uses the quarterly Survey of Professional Forecasters to obtain a measure of variance across individuals for inflation expectations which is then used to proxy for relative entropy at

[13] They report that "higher average expected inflation is associated with both higher average inflation uncertainty and greater disagreement about the inflation outlook. Disagreement about the mean forecast, however, may be a weak proxy for forecast uncertainty."

the individual level and fit the yield curve. He finds that the inflation ambiguity premium is upward sloping and peaked during the mid 1970s and early 1980s. The two approaches show that uncertainty and heterogeneity in beliefs can have a first order effect on bond prices and returns. An important distinguishing feature between these two approaches are their implications for trading volumes. As discussed in the previous section, heterogeneous beliefs models provide falsifiable implications in terms of trading volumes. In equilibrium, risk premia are the outcome of a risk transfer from pessimist to optimist. These implications are supported by the empirical results. Ambiguity models, however, refer to single agent economies, thus are silent about trading volumes. Given Ulrich (2010) and D'Amico and Orphanides (2008), however, it is reasonable to expect that uncertainty is a priced risk factor both at the individual level (ambiguity) and at the aggregate level (DiB).

IV The Information 'In', 'Not In', and 'Above' the Term Structure

Affine term term structure models are ones in which interest rates are modelled as an affine (linear) function of some state vector. The main advantage of this class over their non-linear counterparts is tractability, which led to their wide adoption by both academics and practioners.[14]. In an affine term structure model there are N state variables, denoted $X_t \equiv [X^Q_{t,1}, \ldots, X^Q_{t,N}]'$, which drive the instantaneous (short) interest rate as $r_t = \delta_0 + \delta X_t$, where δ_0 is a scalar and δ is an N-vector. The dynamics for the state variables follow an affine diffusion under the equivalent martingale measure Q:

$$dX_t = \kappa^Q(\Theta^Q - X_t)dt + \Sigma\sqrt{S_t}dB^Q_t, \tag{20}$$

where B^Q_t is an N-vector of independent standard brownian motions, and K^Q and Σ are $N \times N$ matrices. The matrix S_t is diagonal with ith element $S_{ii,t} = \alpha_i + \beta_i' X_t$, where β_i is an N-vector and α_i is a scalar. Denoting the time t price of a default free zero coupon bond maturing at $t+\tau$ as $P(X_t, \tau)$ we know from Duffie and Kan (1996) that bond prices are exponentially-affine:[15]

$$P(X_t, \tau) = \exp(a(\tau) + b(\tau)'X_t),$$

[14]The seminal work of Vasicek (1977) and Cox, Ingersoll Jr, and Ross (1985) are early examples of equilibrium structural affine models. More recent affine specification are discussed by Wachter (2006). Non-linear structural models include, for example, Buraschi and Jiltsov (2007) and Porchia and Trojani (2009)

[15]A complete characterisation of multi-factor affine term structure models was provided by Duffie and Kan (1996), while Dai and Singleton (2000) discuss the structural differences and empirical strengths/weaknesses among the completely affine class and provide parameter restrictions required to ensure S_t is non-negative for all i and in all states .

where $a(\tau)$ is a scalar function and $b(\tau)$ is an N-valued function. Continuously compounded yields are therefore affine in the state vector: $y(X_t, n) = -\frac{\log P(X_t,\tau)}{T} = A(\tau) + B(\tau)'X_t$, for coefficients $A(\tau) = -a(\tau)/\tau$ and $B(\tau) = -b(\tau)/\tau$. Two key ingredients for an affine term structure model are the dynamics of the short rate r under Q and the change from the equivalent martingale measure to the physical measure P. The dynamics for the pricing kernel, $\mathcal{M}$, are then written as $\frac{d\mathcal{M}}{\mathcal{M}} = -r_t dt - \Lambda_t' dB_t^P$, where B_t^P is an N-vector of independent standard Brownian motions under the physical measure and $\Lambda_t = \Lambda(Y_t, t)$ is the N-vector market prices of risk. Invoking Girsanov's theorem the dynamics of X_t under the physical measure can then be written as $dX_t = \kappa^Q(\Theta^Q - X_t)dt + \Sigma\sqrt{S_t}\Lambda_t dt + \Sigma\sqrt{S_t}dB_t^P$. From the fundamental pricing equation,

$$E_t\left(\frac{dP_t}{P_t}\right) - r_t dt = -E_t\left(\frac{dP_t}{P_t}\frac{d\mathcal{M}}{\mathcal{M}}\right), \tag{21}$$

noting that $P_t = P(X_t, \tau)$, and using Ito's lemma we obtain the instantaneous expected excess return to holding a T period bond:

$$rx_{t,t+dt}^{(T)} = -b(T)'\Sigma\sqrt{S_t}\Lambda_t. \tag{22}$$

The model is closed with a specification for the price of risk which, chronologically, resulted first in the 'completely affine' class, $\Lambda_t = \sqrt{S_t}\lambda_1$, in which expected excess returns are completely determined by factor variance. Dai and Singleton (2000) denote the admissible subfamily of completely affine models as $A_m(N)$ which are those with m state variables driving N conditional variances S_t. Although convenient, the completely affine specification imposes significant restrictions for the link between conditional first and second moments of bond yields and expected bond returns. Specifically, elements of the state vector X_t that do not affect factor volatility (and hence bond volatility) cannot affect expected returns, thus factor variance and expected returns go hand-in-hand. Motivated by this observation, Duffee (2002) extends the completely affine class to a set of 'essentially' affine models in which the risk factors in the economy enter the market price of risk directly and not just through their factor volatilities.[16] The essentially affine price of risk is written as

$$\Lambda_t = \sqrt{S_t}\lambda^0 + \sqrt{S_t^-}\lambda^X X_t,$$

where λ^X is an $n \times n$ matrix of constants and S^- is a diagonal matrix such that $[S_t^-]_{ii} = (\alpha_i + \beta' X_t)^{-1}$ if $\inf(\alpha_i + \beta_i' X_t) > 0$ and zero otherwise. The additional flexibility of non-zero

[16] Cheridito, Filipovic, and Kimmel (2007) extend even further this class to yield models that are affine under both objective and risk-neutral probability measures without permitting arbitrage opportunities.

entries in S^- translates into additional state dependent flexibility for the price of risk such that the tight link between risk compensation and factor variance is broken. Duffee (2002) estimates essentially affine $A_0(3)$ and $A_1(3)$ models showing that his specification for the price of risk provides better in and out-of-sample forecasts than the corresponding specifications in Dai and Singleton (2000). For example, $A_0(3)$ models do a good job at forecasting yields and predicting excess returns but impose the unattractive restriction of constant volatility, while $A_1(3)$ models prove more accurate measures of volatility but gives up ability to fit excess returns.

A shared characteristic of the $A_m(N)$ subfamily of affine term structure models is that the cross-section of bond yields follows a Markov structure so that all current information regarding future interest rates (and thus expected returns) is summarised in the shape of the term structure today. Linear combinations of date t bond yields thus suffice to characterise date t risk factors through so-called yield curve inversion.[17] Building on this notion Cochrane and Piazzesi (2005) show that the shape of the term structure embeds substantial information that explains the dynamics of bond excess returns. The Cochrane-Piazzesi return forecasting factor, CP_t, is a tent-shaped linear combination of forward rates that embeds all spanned information on 1-year risk premia predicts excess returns on bonds with R^2 statistics as high as 43% (in their sample period).[18]. More recently evidence presented by Ludvigson and Ng (2009b) and Cooper and Priestley (2009) suggest that yield inversion is not enough to reveal all relevant dynamics for underlying state variables and thus crucial ingredients for term structure models are unspanned by the space of yields. Recent work along these lines is found in Duffee (2011) and Joslin, Priebsch, and Singleton (2009) who independently develop the theme of hidden factor models, or unspanned macro risk, in which time variation in macro variables orthogonal to the cross-section of yields (and thus absent from date t prices) contains substantial forecasting power for future excess returns on bonds.

What are the deeper learning points with respect to returns predictability that we present here? In a single agent Gaussian economy term structure inversion reveals the dynamics of risk factors and thus expected returns. In a multiple agent economy this isn't necessarily true even if the above

[17]Specifically, assume N bond yields are measured without error. Then, stacking these yields into the vector $y^N = A^N + B^N X_t$, we can solve for the risk factors through inversion as $X_t = (B^N)^{-1}\left(y^N - A^N\right)$ so long as the matrix B^N is non-singular.

[18]For a detailed discussion of CP_t we refer the reader to Cochrane and Piazzesi (2005). Briefly, the single factor construction begins with projecting average excess return (across maturity) on a constant plus available forward rates: $\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+12}^{(n)} = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(1)} + \gamma_3 f_t^{(2)} + \gamma_4 f_t^{(3)} + \gamma_5 f_t^{(4)} + \bar{\varepsilon}_{t+12} = \gamma' f_t + \bar{\varepsilon}_{t+1}$. Next, the fitted regression coefficients are used as loadings in forming a linear combination of forward rates that serves as a state variable in restricted univariate and multivariate regressions: $rx_{t,t+12}^{(n)} = \beta(\gamma' f_t) + \phi X_t + \varepsilon_{t+1}^{(n)} = \beta CP_t + \phi X_t + \varepsilon_{t+12}^{(n)}$.

holds: a) risk sharing / market clearing may generate non-affine prices; or b) $E\left[\frac{M(T)}{M(t)}g(X_t)\right]$ may not reveal all relevant dynamics of risk factors, X_t, and thus disagreement may be unspanned by the cross-section of prices yet reveal information about expected returns. A natural question to ask is which component of disagreement relevant for expected returns is revealed by the cross-section of prices (Cochrane-Piazzesi) versus the time-series of prices (Duffee; Joslin, Priebsch, Singleton). Proceeding in two steps, we first define the information set $G_1 \subseteq \sigma\left(PC(1-5)\right)$ and compute the unspanned component of DiB which is not explained by the cross-section of bond prices (the first five principal component, as used in Cochrane and Piazzesi (2005)): $\mathcal{UN}_{DiB_t} = DiB_t - P_j\left[DiB_t \middle| G_1\right]$.[19] Then, we proceed to test the content of unspanned, i.e. 'Not-In', disagreement as follows:

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)}\mathcal{UN}_{DiB_t}^{INF} + \beta_2^{(n)}\mathcal{UN}_{DiB_t}^{RGDP} + \beta_3^{(n)}\mathcal{UN}_{DiB_t}^{LR} + \beta_4^{(n)}\mathcal{UN}_{DiB_t}^{SR} + \varepsilon_{t+12}^{(n)}. \tag{23}$$

Second, we define $G_2 \subseteq \left[G_1 \cup \sigma(y^{(n)})\right] \setminus G_1$ where $G_2 \sim \sigma(H_t)$ is the '$Hidden_t$' factor filtered from the time-series of prices from a 5-factor Gaussian term structure model studied in Duffee (2011).[20] Then, we estimate the component of disagreement unspanned neither by the cross-section of prices nor by information related to the hidden factor H_t. We define $\mathcal{AB}_{DiB_t} = \mathcal{UN}_{DiB_t} - P_j\left[\mathcal{UN}_{DiB_t} \middle| H_t\right]$ and test the predictive content of macroeconomic disagreement which is '*Above*' the yield curve as

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)}\mathcal{AB}_{DiB_t}^{INF} + \beta_2^{(n)}\mathcal{AB}_{DiB_t}^{RGDP} + \beta_3^{(n)}\mathcal{AB}_{DiB_t}^{LR} + \beta_4^{(n)}\mathcal{AB}_{DiB_t}^{SR} + \varepsilon_{t+12}^{(n)}. \tag{24}$$

Table VIII reports a contemporaneous projection of disagreement measures on the first 5 principle components from an eigenvalue decomposition of the unconditional covariance matrix of yields (from the Fama-Bliss data set as in Cochrane-Piazzesi). The results show that a substantial proportion of the time-variation in disagreement about the real economy and short term interest rates is spanned by the yield curve, specifically, DiB^{REAL} and DiB^{SR} both load significantly on PCs 1- 4 with $\overline{R}^2$'s of 36% and 38% respectively. The first learning point, then, is that time variation in the shape of the forward curve can in part represent heterogeneity in the belief structure of the economy, thus lending economic support to the empirical results of Cochrane and Piazzesi (2005) and the theoretical results of Xiong and Yan (2010). Panel A and B of table X documents the

[19] More specifically, G_1 is the sigma algebra (information set) generated by the eigenvalue decomposition of the unconditional covariance matrix of yields, or, alternatively, since there exists a linear mapping between yields and forward rates, G_1 is the space spanned by the return forecasting factor CP.

[20] We thank G. Duffee for providing the data on the hidden factor $Hidden_t$.

impact on return predictability when one removes the component of DiB spanned by the yield curve. Note first that repeating the return predictability regressions of section A on a different dataset, on a different sample period (pre- 2008 crisis), we obtain almost identical results both in terms of point estimates, t-statistics, and $\overline{R}^2$'s. The second learning point with respect to this section is that more than half of the time-variation in expected returns attributable to disagreement is unspanned and that component is entirely due to monetary disagreement. For example, in moving from spanned to unspanned disagreement the $\overline{R}^2$ for 2-year bonds goes from 42% to 27%.

Next, we examine the time-series characteristics of unspanned disagreement by running projections of unspanned disagreement on the $Hidden_t$ risk premium component from Duffee (2011). Table IX reports the following multivariate regression :

$$Hidden_t = const + \sum_{i=1}^{4} \beta_i \, \mathcal{UN}^i_{DiB} + \varepsilon^i_t,$$

The results show that information contained in dispersion in beliefs that is orthogonal to the yield curve explains time variation in the hidden factor with high statistical significance (t-stat: 2.96) with an $\overline{R}^2$ statistic of 8% [21]. The third learning point is that after controlling for information extracted from the time-series of prices there still exists a substantial proportion of information contained in the cross-section of agents expectations that is relevant for bond pricing. Table X documents the predictive power of the above components, as defined in equation 24, for expected bond returns. This particular unspanned component is specific to disagreement regarding the long end of the yield curve and is orthogonal to i) the cross-section of yields; and ii) a risk premium component embedded in the time series of yields. Still, it contains substantial information for future expected bond returns, with t-statistics significant at the 1% level, and $\overline{R}^2$ between 20% and 22%. Importantly, this component is also economically important for bond risk premia: a 1-standard deviation shock to $AB^{LR}_{DiB_t}$ lowers expected excess returns on 5-year bonds by 2.38%.

[Insert table VIII, IX , and X here.]

V Concluding Remarks

For a long time the empirical success of linking macroeconomic fluctuations to bond market dynamics was limited. Researchers typically resorted to reduced form models to reveal latent state

[21] This compares with an R^2 of 10% in a projection of H_t on the real activity factor (PC1) from Ludvigson and Ng (2009b).

variables through yield curve inversion. Increasingly sophisticated models led researchers to better fit the yield curve and match time-variation in risk premia yet a deeper understanding of the macroeconomic link to bond markets remained elusive.

This paper contributes to the debate from a new perspective by studying the implications of *macroeconomic disagreement* for time-variation in bond market risk premia, volatility, and trade. We focus our attention on macroeconomic disagreement through the lens of a special class of rational expectations models in which agents make subjective assessments of the the future path of the economy but 'agree to disagree' on its outcome. Equilibrium is supported by risk sharing whereby agents trade a set of ex-ante state contingent contracts until the Radon-Nikodym derivative of the agents likelihood functions equals the ratio of their subjective expected marginal utilities.

Constructing a unique dataset of market participants' expectations we propose a novel way to build constant maturity disagreement measures covering real, nominal and monetary components of economic activity. Our empirical results are then summarised as follows. First, we learn that jointly in classic return predictability regressions, disagreement about the real economy, short, and long ends of the yield curve are economically important and highly statistically significant with $\overline{R}^2$ as high as 43% for 1-year excess bond returns on 2-year maturity bonds. These results are robust to inclusion of a number of known predictor variables and consensus expectations and survive alternative specifications for statistical inference. Second, moving to second moments we document empirically large explanatory power for realised stock and bond volatilty above that contained in macro expectations or realisations of fundamentals such as consumption volatility. For example, in projecting 1-month return volatility on 10-year treasuries measured from $t \rightarrow t+1$ on real and inflation disagreement both factors are significant at the 1% level with an $\overline{R}^2$ of 23%. Third, we document that a large proportion of the time-variation in trading activity for futures and options on stocks and bonds can be accounted for by time-variation in the belief structure of the economy. Finally, exploring the spanning properties of disagreement, we find that disagreement regarding inflation and the real economy are highly correlated with the shape of the yield curve and will therefore be revealed through yield curve inversion. However, disagreement about short term rates is only partially spanned in the sense that information from both the cross-section and the time-series of yields is needed to capture its relevant pricing dynamics. These findings lend some economic support to Cochrane and Piazzesi (2005) who show that cross-sectional price information contains significant information on future bond returns in addition to that contained in the level or slope. Finally, we document the existence of a component linked to disagreement about long term interest rates that appears ‘above’ the yield curve: it is neither spanned by the cross-section or by the time-series dynamics of yields.

A Appendix A: Proofs

B Appendix A:

Proof. **Lemma 1.** First we prove that $\eta_T = W_T^b/W_T^a$, then we derive the bond pricing relation. Suppose there exists a tradable asset with terminal payoff B_T. In equilibrium, since this asset is observable, both agents must agree on its value. Under logarithmic preferences, this requires $E_t^b(\frac{c_t^b}{c_T^b}B_T) = E_t^a(\frac{c_t^a}{c_T^a}B_T)$. From Merton (1971) we know that myopic agents consume wealth at a rate proportional to their time preference, $c_t^i = \rho W_t^i$, so that $E_t^b(\frac{W_t^b}{W_T^b}B_T) = E_t^a(\frac{W_t^a}{W_T^a}B_T)$. Define $\frac{W_t^b}{W_T^b}B_T = \tilde{B}_T$, then $E_t^b(\tilde{B}_T) = E_t^a\left[(\frac{W_T^b/W_T^a}{W_t^b/W_t^a})\tilde{B}_T\right]$, so that $\frac{\eta_T}{\eta_t} = \frac{W_T^b/W_T^a}{W_t^b/W_t^a}$. □

Proof. **Theorem 1.** Since market clearing requires that $c_t^a + c_t^b = D_t$, the consumption share of each agent is equal to $c_t^a = \frac{1}{1+\eta_t}D_t$ and $c_t^b = \frac{\eta_t}{1+\eta_t}D_t$. Substitute in the Euler equation which needs to hold for each agent for a single nominal cash flow asset B_T (e.g. for a zero-coupon bond $B_T = 1$), $B_t/\pi_t = E_t^a\left(e^{-\rho(T-t)}\frac{u'(c_T^a)}{u'(c_t^a)}B_T/\pi_T\right)$. where π_t is the nominal price index. We obtain

$$\begin{aligned}
B_t/\pi_t &= E_t^a(e^{-\rho(T-t)}\frac{c_t^a}{c_T^a}B_T/\pi_T) && (25)\\
&= E_t^a(e^{-\rho(T-t)}\frac{1+\eta_T}{1+\eta_t}\frac{D_t}{D_T}B_T/\pi_T) && (26)\\
&= \frac{1}{1+\eta_t}E_t^a(e^{-\rho(T-t)}\frac{D_t}{D_T}B_T/\pi_T) + \frac{\eta_t}{1+\eta_t}E_t^a(e^{-\rho(T-t)}\frac{\eta_T}{\eta_t}\frac{D_t}{D_T}B_T/\pi_T) && (27)\\
&= \frac{1}{1+\eta_t}E_t^a(e^{-\rho(T-t)}\frac{D_t}{D_T}B_T/\pi_T) + \frac{\eta_t}{1+\eta_t}E_t^a(e^{-\rho(T-t)}\frac{\eta_T}{\eta_t}\frac{D_t}{D_T}B_T/\pi_T) && (28)\\
&= \frac{1}{1+\eta_t}B_t^{(T-t),a}/\pi_t + \frac{\eta_t}{1+\eta_t}B_t^{(T-t),a}/\pi_t && (29)
\end{aligned}$$

which proves the main theorem. One can notice that the result holds independently of any specific assumption on the endowment market structure or learning process.

□

C Appendix B: Figures

In order to construct a constant 1-year maturity disagreement measure for each forecaster, we take a weighted average of the short and long term forecasts from the BlueChip survey. Figure 1 gives a visual explanation to the construction of the constant maturity proxy. Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2..12$. A constant maturity disagreement is formed taking as weight $(1 - \frac{j}{12})$, for the short term disagreement (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term disagreement (the forecast for the following year). As an example, in April each year the approximate 1-year difference in belief is constructed from 9/12th's of the short term forecast and 3/12th's of the long term forecast.

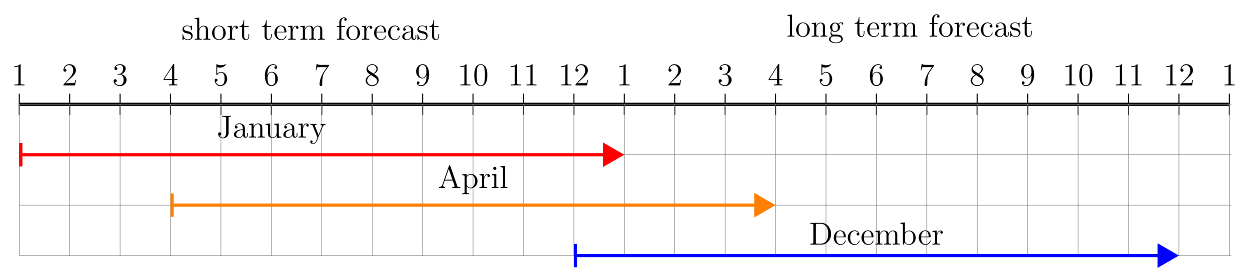
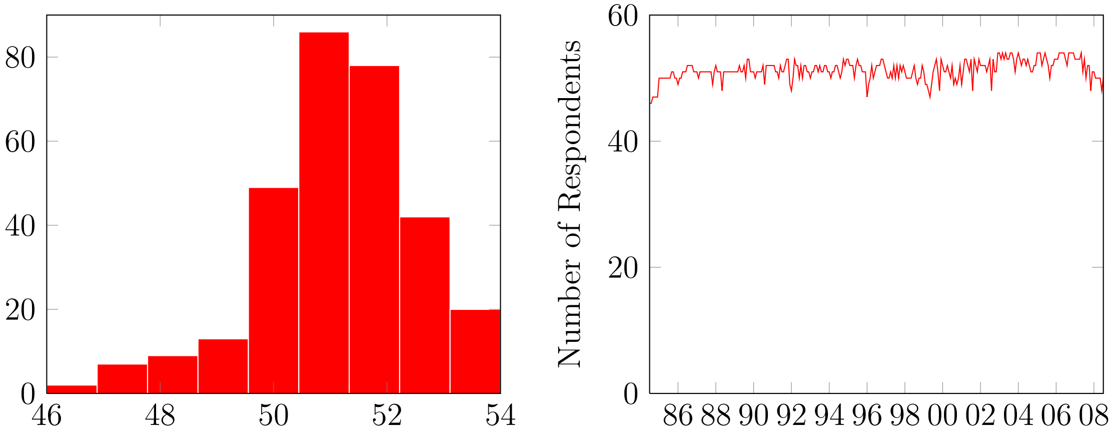


Figure 1 – Constant Maturity Disagreement

Diagram illustrating the construction of the constant maturity disagreement measures built from a moving weighted average of long term and short term disagreement measures.

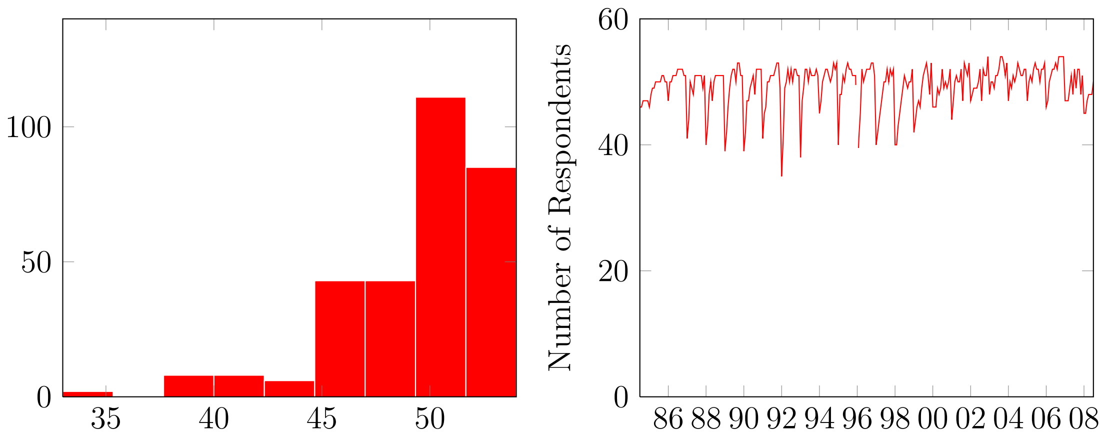


(a) Distribution of respondent numbers

(b) Time Series of number of respondents

Figure 2 – Short Term Forecast Respondent Numbers

Panel (a): histogram displaying the distribution of the number of respondents for short term forecasts (average for the remaining period of the current calendar year). Panel (b): time series of number of respondents contributing to the short term forecast.

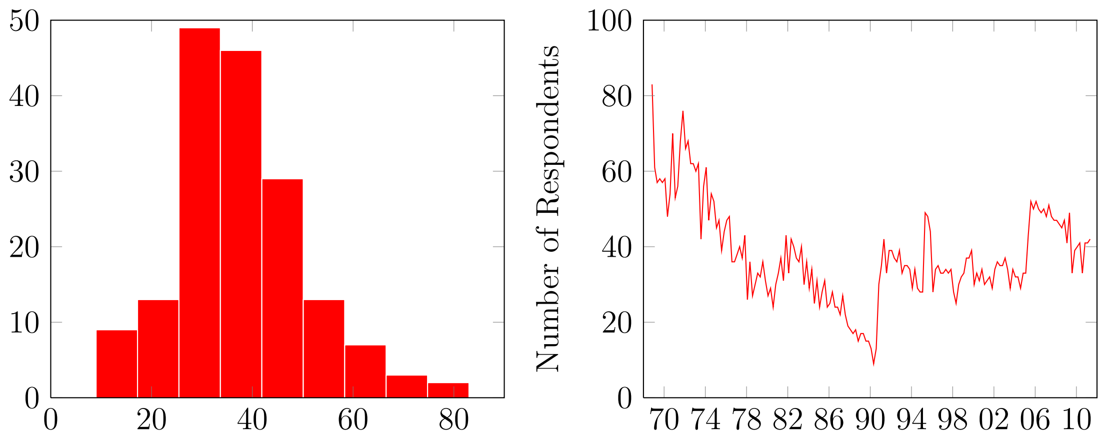


(a) Distribution of respondent numbers

(b) Time Series of number of respondents

Figure 3 – Long Term Forecast Respondent Numbers

Panel (a): histogram displaying the distribution of the number of respondents for short term forecasts (an average for the following year). Panel (b): time series of number of respondents contributing to the long term forecast.



(a) Distribution of respondent numbers

(b) Time series of number of respondents

Figure 4 – Forecast Respondent Numbers: Survey of Professional Forecasters

Panel (a): histogram displaying the distribution of the number of respondents for short term forecasts (an average for the following year). Panel (b): time series of number of respondents contributing to the long term forecast.

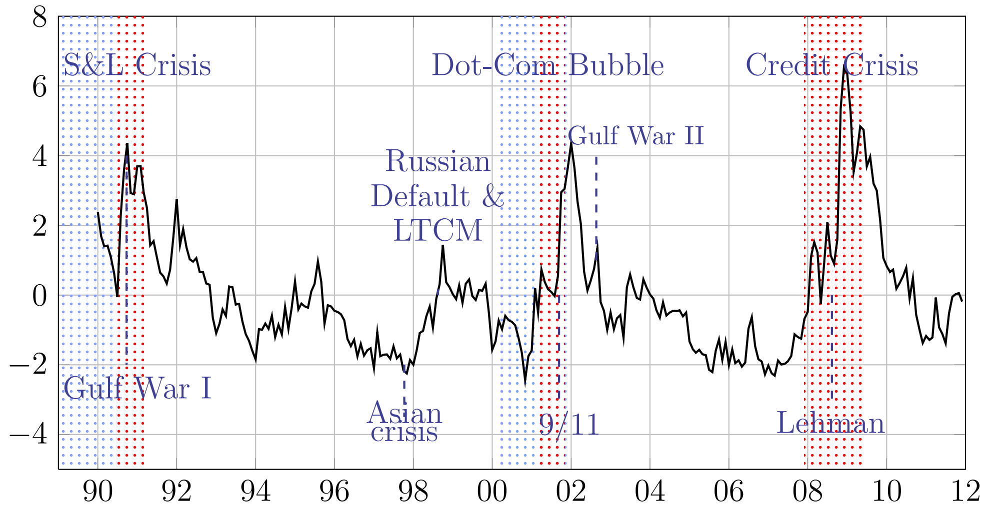


Figure 5 – Disagreement

Time series of first principle component of filtered DiB (cross-sectional mean absolute deviation) series discussed in section II A.

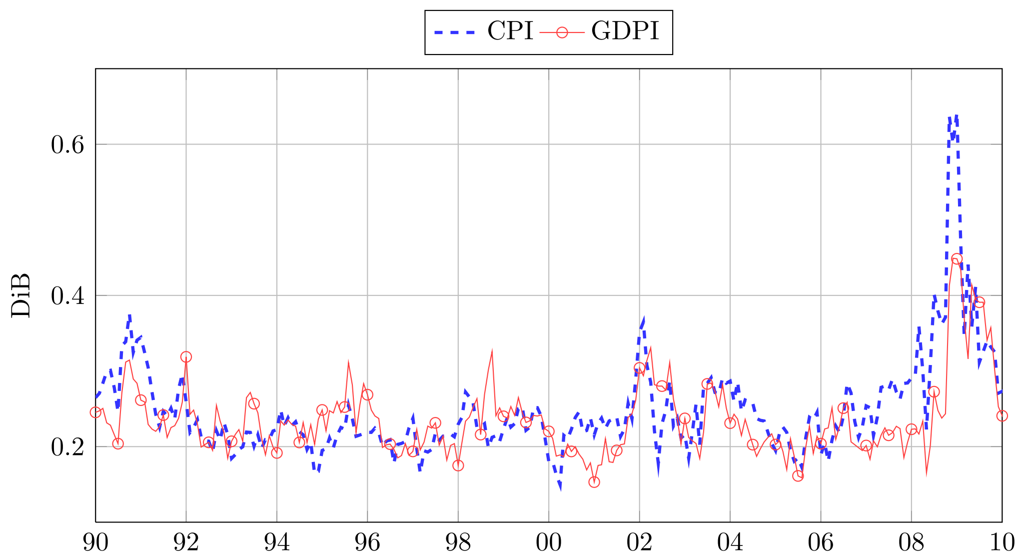


Figure 6 – Disagreement on Inflation

Time series of first principle component of filtered DiB (cross-sectional mean absolute deviation) series discussed in section II A.

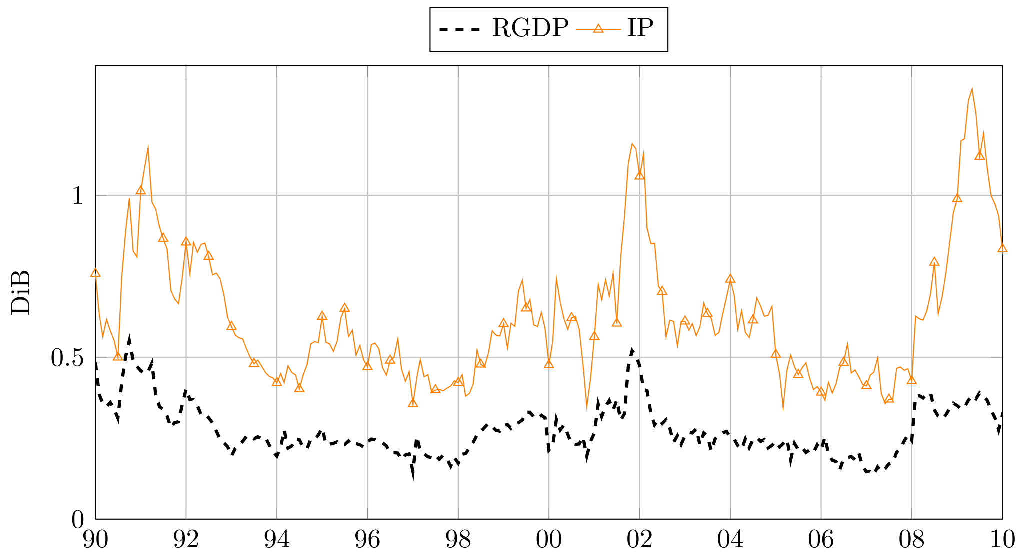


Figure 7 – Disagreement on Real Growth

Time series of first principle component of filtered DiB (cross-sectional mean absolute deviation) series discussed in section II A.

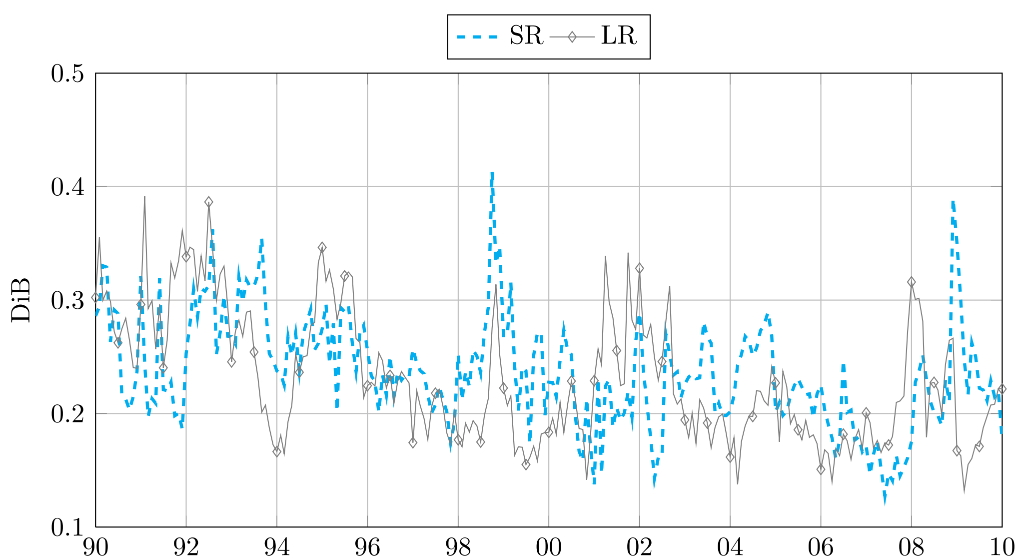


Figure 8 – Disagreement on Interest Rates

Time series of first principle component of filtered DiB (cross-sectional mean absolute deviation) series discussed in section II A.

D Appendix C: Tables

Table I – Summary Statistics: Disagreement

Table reports the summary statistics for mean-absolute-deviation in economist forecasts for real, nominal, and monetary components. Sample Period: 1990.1 - 2011.12 The abbreviations used in the column and row headings are above. Panel A reports constant maturity disagreement, Panel B the correlation matrix for each disagreement type.

	DiB^{RGDP}	DiB^{IP}	DiB^{CPI}	DiB^{GDPI}	DiB^{SR}	DiB^{LR}
Panel A:						
Mean	0.27	0.62	0.25	0.24	0.23	0.22
SDev	0.08	0.20	0.06	0.05	0.05	0.06
Skew	1.05	1.30	2.75	1.76	0.45	0.28
Kurt	4.10	4.47	15.18	7.43	3.42	2.88
AC(1)	0.90	0.93	0.85	0.82	0.71	0.86
Panel B:						
DiB^{RGDP}	1.00	0.79	0.45	0.42	0.19	0.42
DiB^{IP}	0.79	1.00	0.53	0.58	0.16	0.29
DiB^{CPI}	0.45	0.53	1.00	0.69	0.01	-0.07
DiB^{GDPI}	0.42	0.58	0.69	1.00	0.20	0.01
DiB^{SR}	0.19	0.16	0.01	0.20	1.00	0.38
DiB^{LR}	0.42	0.29	-0.07	0.01	0.38	1.00

Table II – Return Predictability Regressions

This table reports estimates from OLS regressions of annual ($t \rightarrow t+12$) excess returns of 2,5 and 10 year zero-coupon bonds on disagreement factors and consensus expectations:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{4} \beta_i^{(n)} DiB_t(\star) + \sum_{i=1}^{2} \gamma_i^{(n)} E_t(\star) + \sum_{i=1}^{3} \phi_i^{(n)} Macro_t(\star) + \varepsilon_{t+12}^{(n)},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of DiB_t variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

	$rx^{(2)}$					$rx^{(5)}$					$rx^{(10)}$				
regressor	*(i)*	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(i)*	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(i)*	*(ii)*	*(iii)*	*(iv)*	*(v)*
DiB_t^{INF}	-0.22		-0.03	-0.10	-0.21	-0.50		0.04	0.38	-0.23	-0.69		0.07	1.08	0.17
	(-1.85)		(-0.25)	(-1.19)	(-1.19)	(-1.46)		(0.11)	(0.91)	(-0.39)	(-1.18)		(0.13)	(1.36)	0.17
DiB_t^{REAL}	0.51		0.38	0.24	0.08	1.51		1.21	1.55	0.90	1.90		1.42	2.43	1.94
	(2.93)		(3.82)	(1.57)	(0.62)	(3.04)		(3.78)	(2.10)	(1.98)	(2.67)		(2.37)	(2.02)	(2.32)
DiB_t^{LR}		-0.36	-0.42	-0.46	-0.37		-1.28	-1.48	-1.91	-1.36		-1.64	-1.88	-3.01	-1.84
		(-3.22)	(-3.84)	(-4.44)	(-3.60)		(-3.09)	(-3.53)	(-3.66)	(-3.51)		(-1.83)	(-2.11)	(-3.05)	(-2.37)
DiB_t^{SR}		0.75	0.61	0.50	0.58		2.05	1.66	1.35	1.69		2.85	2.39	2.20	2.50
		(5.64)	(5.23)	(3.47)	(5.08)		(4.10)	(3.44)	(2.12)	(3.15)		(2.94)	(2.47)	(1.76)	(2.20)
E_t^{INF}				0.18					-0.91					-2.09	
				(1.60)					(-1.17)					(-1.60)	
E_t^{REAL}				-0.26					-0.11					1.03	
				(-1.27)					(-0.17)					(1.08)	
F_t^1					0.42					0.02					-1.76
					(2.66)					(0.04)					(-1.84)
F_t^2					-0.09					-0.59					-1.28
					(-1.39)					(-2.69)					(-3.38)
σ_t^{INF}					0.01					0.43					0.80
t					(0.08)					(0.78)					(0.73)
σ_t^{REAL}					-0.10					-0.44					-0.63
					(-2.18)					(-2.41)					(-1.40)
$\rho_t^{INF,REAL}$					0.12					0.63					1.36
					(1.36)					(2.03)					(2.57)
$\overline{R}^2$	0.16	0.37	0.43	0.46	0.47	0.12	0.29	0.35	0.39	0.37	0.06	0.18	0.21	0.32	0.24
χ^2	9.99	48.07	112.93	67.06	51.06	9.99	25.91	50.77	52.70	29.83	8.12	12.23	17.07	49.45	20.35
p-value	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00

Table III – Economic Significance: Expected Returns

Economic significance from return predictability regression of excess holding period returns on 2, 5, and 10-year maturity bonds on disagreement factors:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{4} \beta_i^{(n)} DiB_t(\star) + \varepsilon_{t+12}^{(n)},$$

$\sigma(E(rx^{(n)}))$ is the standard deviation of the model, i.e., $\sigma(\sum_{i=1}^{4} \beta_i^{(n)} DiB_t(\star))$, while the remaining columns are the response to a 1-standard deviation shock to each factor.

Maturity(n)	$E(rx^{(n)})$	$\sigma(E(rx^{(n)}))$	$\sigma(DiB_t^{INF})$	$\sigma(DiB_t^{REAL})$	$\sigma(DiB_t^{LR})$	$\sigma(DiB_t^{SR})$
2^{yr}	0.93	0.52	−0.03	0.38	−0.42	0.61
5^{yr}	3.03	1.74	0.04	1.21	−1.48	1.66
10^{yr}	4.98	2.16	0.07	1.42	−1.88	2.39

Table IV – Reverse Regressions

This table reports estimates from OLS estimates of monthly ($t \rightarrow t+1$) average returns across maturity (2 - 5 year bonds) in excess of the Fama 1-month Risk Free Rate (CRSP) on a lagged summation of right hand disagreement factors according to Hodrick (1992) :

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+1}^{(n)} = const^{(n)} + \sum_{i=1}^{4} \beta_i DiB_t^{h,k}(\star) + \varepsilon_{t+1}^{(n)},$$

where $\overline{DiB}_t^{h,k} = \sum_{i=1}^{h} DiB_{t-k-i}$. The regressors are pre-multiplied by $(1/k)$ for comparison reasons. t-statistics, reported in ( )'s, are adjusted for heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. The $\chi^2(3)$ computed with 6 Newey-West lags statistic tests the joint restriction that the three slope coefficients (across h) are different from zero. Data spans 1990.1 - 2010.1.

	lag $k=1$			lag $k=2$			lag $k=3$		
horizon(h)	DiB_t^{REAL}	DiB_t^{LR}	DiB_t^{SR}	DiB_t^{REAL}	DiB_t^{LR}	DiB_t^{SR}	DiB_t^{REAL}	DiB_t^{LR}	DiB_t^{SR}
3	0.09	-0.22	0.19	0.12	-0.21	0.16	0.13	-0.17	0.15
	(1.63)	(-3.16)	(3.08)	(2.31)	(-2.92)	(2.56)	(2.59)	(-2.30)	(2.59)
6	0.12	-0.24	0.19	0.14	-0.24	0.17	0.15	-0.22	0.16
	(2.26)	(-3.13)	(3.10)	(2.70)	(-2.96)	(2.92)	(2.87)	(-2.49)	(3.03)
12	0.13	-0.30	0.18	0.14	-0.30	0.15	0.15	-0.28	0.13
	(2.37)	(-3.31)	(2.84)	(2.52)	(-3.17)	(2.46)	(2.51)	(-2.85)	(2.04)
$\chi^2(3)$	8.54	14.55	10.31	8.78	11.06	8.61	8.66	8.15	9.69
p-value	0.04	0.00	0.02	0.03	0.01	0.04	0.03	0.04	0.02

Table V – Volatility and Correlation Regressions

This table reports estimates from OLS regressions of monthly ($t \rightarrow t+1$) realised volatility of stocks (CRSP value weighted index) and bonds (10 yr zero-coupon), or monthly ($t \rightarrow t+1$) realised stock bond correlation on disagreement factors and consensus expectations:

$$Vol/Corr_{t,t+1} = const + \sum_{i=1}^{4} \beta_i DiB_t(\star) + \sum_{i=1}^{2} \gamma_i E_t(\star) + \sum_{i=1}^{3} \phi_i Macro_t(\star) + \varepsilon_{t+1},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of DiB_t variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

	BondVol					*StockVol*					*StockBondCorr*				
regressor	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)	(i)	(ii)	(iii)	(iv)	(v)
DiB_t^{INF}	1.20		1.09	0.66	0.73	3.50		3.13	1.90	0.10	-0.11		-0.09	-0.03	-0.03
	(5.06)		(4.73)	(2.46)	(2.43)	(2.63)		(2.05)	(1.51)	(0.09)	(-2.80)		(-2.50)	(-0.64)	(-0.41)
DiB_t^{REAL}	1.08		1.16	0.58	0.97	2.95		3.70	2.10	1.61	-0.05		-0.12	-0.07	-0.04
	(3.17)		(4.19)	(2.06)	(3.42)	(3.49)		(4.43)	(1.90)	(1.75)	(-0.72)		(-1.67)	(-0.67)	(-0.44)
DiB_tLR		-0.07	-0.23	0.13	-0.19		-1.25	-1.84	-0.46	-2.89		0.14	0.17	0.02	0.16
		(-0.19)	(-1.06)	(0.55)	(-0.87)		(-1.37)	(-2.04)	(-0.48)	(-4.29)		(2.58)	(2.97)	(0.41)	(3.30)
DiB_t^{SR}		0.53	0.34	0.35	0.25		-0.44	-0.99	-0.98	-1.25		0.12	0.13	0.14	0.12
		(1.84)	(1.84	) (1.94)	(1.48)		(-0.39)	(-1.21)	(-1.58)	(-1.92)		(2.01)	(2.20)	(3.16)	(1.94)
E_t^{INF}				-0.85					-3.07					0.28	
				(-3.31)					(-5.57)					(5.56)	
E_t^{REAL}				-0.81					-2.11					0.05	
				(-2.28)					(-1.92)					(0.54)	
F_t^1					0.14					2.77					-0.15
					(0.39)					(2.71)					(-2.10)
F_t^2					0.06					-0.29					-0.01
					(0.26)					(-0.56)					(-0.20)
σ_t^{INF}					-0.17					-1.38					0.04
					(-0.45)					(-1.39)					(0.51)
σ_t^{REAL}					0.59					4.34					-0.03
					(2.72)					(9.92)					(-0.57)
$\rho_t^{INF,REAL}$					0.41					1.63					0.00
					(2.72)					(2.89)					(0.03)
$\overline{R}^2$	0.26	0.02	0.27	0.31	0.32	0.23	0.01	0.26	0.32	0.54	0.04	0.10	0.16	0.32	0.15
χ^2	51.54	3.43	101.96	12.96	17.90	15.50	2.08	34.88	9.31	50.68	10.97	11.39	29.06	10.80	17.35
p-value	0.00	0.18	0.00	0.01	0.00	0.00	0.35	0.00	0.05	0.00	0.00	0.00	0.00	0.03	0.00

Table VI – Economic Significance: Volatility

Economic significance from monthly ($t \rightarrow t+1$) realised volatility of stocks (CRSP value weighted index) and bonds (10 yr zero-coupon), or monthly ($t \rightarrow t+1$) realised stock bond correlation on disagreement factors:

$$Vol/Corr_{t,t+1} = const + \sum_{i=1}^{4} \beta_i DiB_t(\star) + \varepsilon_{t+1},$$

$\sigma(E(\cdot))$ is the standard deviation of the model, i.e., $\sigma(\sum_{i=1}^{4} \beta_i^{(n)} DiB_t(\star))$, while the remaining columns are the response to a 1-standard deviation shock to each factor.

Dependent	$E(\cdot)$	$\sigma(E(\cdot))$	$\sigma^{DiB_t^{INF}}$	σDiB_t^{REAL}	$\sigma^{DiB_t^{LR}}$	$\sigma^{DiB_t^{SR}}$
$BondVol$	9.51	1.59	1.09	1.16	-0.23	0.34
$StockVol$	15.97	4.97	3.13	3.70	-1.84	-0.99
$StockBondCorr$	0.03	0.21	-0.09	-0.12	0.17	0.13

Table VII – Open Interest Regressions

This table reports estimates from OLS regressions of the monthly ($t \rightarrow t+1$) growth rate of combined open interest (OI) of S&P and Treasury note options and futures on disagreement factors and consensus expectations:

$$\frac{OI(t+1) - OI(t)}{OI(t)} = const + \sum_{i=1}^{4} \beta_i DiB_t(\star) + \sum_{i=1}^{2} \gamma_i E_t(\star) + \sum_{i=1}^{3} \phi_i Macro_t(\star) + \varepsilon_{t+1},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

	S&P					Treasury Note				
regressor	*(i)*	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(i)*	*(ii)*	*(iii)*	*(iv)*	*(v)*
DiB_t^{INF}	0.09		0.10	-0.25	0.19	0.58		0.57	0.54	0.77
	(0.71)		(0.77)	(-1.56)	(1.35)	(4.38)		(5.04)	(3.79)	(6.55)
DiB_t^{REAL}	-0.10		-0.22	-0.97	-0.32	0.50		0.40	0.14	0.33
	(-0.48)		(-0.95)	(-2.09)	(-1.37)	(3.62)		(2.53)	(0.59)	(2.58)
DiB_t^{LR}		0.22	0.26	0.21	0.05		0.46	0.28	0.23	0.07
		(0.99)	(1.09)	(0.96)	(0.22)		(2.49)	(1.86)	(1.50)	(0.67)
DiB_t^{SR}		0.09	0.18	0.33	0.29		0.08	0.11	0.25	0.01
		(0.76)	(1.85)	(2.53)	(2.52)		(0.42)	(0.62)	(1.41)	(0.06)
E_t^{INF}				-0.66					-0.45	
				(-2.65)					(-2.27)	
E_t^{REAL}				-0.66					0.01	
				(-2.21)					(0.05)	
F_t^1					0.16					-0.10
					(0.72)					(-0.66)
F_t^2					-0.26					0.04
					(-1.75)					(0.50)
σ^{INF}					-0.40					-0.41
					(-1.57)					(-2.75)
σ^{REAL}					0.13					0.44
					(0.95)					(4.30)
$\rho_t^{INF,REAL}$					0.48					0.03
					(2.17)					(0.24)
$\overline{R}^2$	0.01	0.01	0.01	0.08	0.06	0.19	0.07	0.21	0.25	0.35
χ^2	0.54	1.91	4.52	7.91	7.91	28.19	8.49	58.20	25.21	68.97
p-value	0.76	0.39	0.34	0.10	0.09	0.00	0.01	0.00	0.00	0.00

Table VIII – Spanned Disagreement

This table reports contemporaneous regressions of disagreement factors on the 5 principle components from an eigenvalue decomposition of the yield covariance matrix. The yields are 1-5 years in maturity from the Fama-Bliss data set. PC1 is as usual a level factor, PC2 is a slope factor, and PC3 is a curvature factor. PC4 and PC5 are the additional principle components shown to be economically important for bond risk premia in Cochrane and Piazzesi (2005).

$$DiB_t^i = const + \sum_{i=1}^{5} \beta_i \; PC_t^i + \varepsilon_t^i,$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2010.12

regressor	DiB^{INF}	DiB^{REAL}	DiB^{LR}	DiB^{SR}
PC^1	-0.08	0.15	0.14	0.29
	-0.86	1.87	2.02	4.49
PC^2	-0.07	0.50	0.37	0.40
	-0.72	7.02	4.04	6.56
PC^3	0.18	0.20	0.02	0.23
	1.82	2.34	0.21	3.09
PC^4	-0.04	0.24	0.01	0.29
	-0.46	2.75	0.14	4.54
PC^5	0.80	-1.10	-0.96	1.45
	(2.97)	(1.51)	(2.23)	(-0.13)
$\overline{R}^2$	0.03	0.36	0.14	0.38

Table IX – Unspanned Disagreement and the Hidden Factor

This table reports contemporaneous regressions of the hidden factor from Duffee (2011) on the unspanned components of disagreement.

$$Hidden_t = const + \sum_{i=1}^{4} \beta_i \; \mathcal{UN}_{DiB}^i + \varepsilon_t^i,$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2007.12

	$\mathcal{UN}_{DiB}^{INF}$	$\mathcal{UN}_{DiB}^{REAL}$	$\mathcal{UN}_{DiB}^{LR}$	$\mathcal{UN}_{DiB}^{SR}$	$\overline{R}^2$
$Hidden_t$	0.03	-0.12	-0.03	0.27	0.06
	(0.36)	(-1.35)	(-0.37)	(2.81)	

Table X – Return Predictability, Spanned, Unspanned, and Above Disagreement

This table reports estimates from OLS regressions of annual ($t \rightarrow t+12$) excess returns of 2,3,4, and 5 year zero-coupon bonds from the Fama-Bliss data set 'spanned', 'unspanned', and 'above' disagreement factors:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{4} \beta_i^{(n)} DiB_t(\star) + \varepsilon_{t+12}^{(n)},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of DiB_t variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2007.12

regressor	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
Panel A: Spanned Disagreement				
DiB^{INF}	-0.03	-0.01	-0.05	0.04
	(-0.28)	(-0.05)	(-0.17)	(0.12)
DiB^{REAL}	0.38	0.74	0.98	1.14
	(3.80)	(4.13)	(3.82)	(3.50)
DiB^{LR}	-0.42	-0.87	-1.24	-1.50
	(-3.75)	(-3.92)	(-3.78)	(-3.48)
DiB^{SR}	0.62	1.11	1.55	1.80
	(4.85)	(4.26)	(4.19)	(3.76)
$\overline{R}^2$	0.42	0.41	0.40	0.36
Panel B: Unspanned Disagreement				
$\mathcal{UN}_{DiB}^{INF}$	0.03	0.09	0.17	0.16
	(0.16)	(0.31)	(0.43)	(0.33)
$\mathcal{UN}_{DiB}^{REAL}$	0.27	0.42	0.45	0.41
	(2.09)	(1.94)	(1.48)	(1.16)
$\mathcal{UN}_{DiB}^{LR}$	-0.51	-1.06	-1.51	-1.87
	(-5.05)	(-5.47)	(-5.37)	(-5.05)
$\mathcal{UN}_{DiB}^{SR}$	0.35	0.66	0.91	1.14
	(2.36)	(2.26)	(2.18)	(2.17)
$\overline{R}^2$	(0.27)	(0.29)	(0.28)	(0.28)
Panel C: Above Disagreement				
$\mathcal{AB}_{DiB}^{INF}$	0.02	0.04	0.11	0.01
	(0.10)	(0.10)	(0.22)	(0.01)
$\mathcal{AB}_{DiB}^{REAL}$	0.30	0.47	0.48	0.43
	(1.88)	(1.73)	(1.30)	(1.02)
$\mathcal{AB}_{DiB}^{LR}$	-0.50	-1.04	-1.47	-1.83
	(-4.12)	(-4.40)	(-4.29)	(-4.18)
$\mathcal{AB}_{DiB}^{SR}$	0.24	0.46	0.62	0.72
	(1.25)	(1.25)	(1.20)	(1.13)
$\overline{R}^2$	0.21	0.22	0.21	0.20

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