Analysing the Difference between Forward and Futures Prices for the UK Commercial Property Market

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Abstract

The paper analyses the differences between forward and futures prices for the UK commercial property market, using both time series and panel data. A first battery of tests establishes that the observed differences are statistically significant. Further analysis considers the modelling of this difference using mean-reverting models. The proposed models are then estimated with a number of alternative estimation methods and second stage statistical tests are implemented in order to decide which model and estimation method best represent the data.

JEL: C12, C33, G13, G19

Key words: property derivatives, panel data, mean-reversion, martingale estimation, MCMC

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1. Introduction

The difference between forward and futures prices has been given considerable attention in the finance literature, both from a theoretical as well as from an empirical perspective, and for various underlying assets. On the theoretical side, Cox, Ingersoll and Ross (1981) (CIR) obtained a relationship between forward and futures prices based solely on no-arbitrage arguments[1]. A series of papers subsequently tested empirically the CIR result(s). Cornell and Reinganum (1981) investigated whether the difference between forward and futures prices in the foreign exchange market is different from zero. For several maturities and currencies, they found that the average forward-futures difference is not statistically different from zero. Furthermore, they reported very small values of the sample covariance between futures prices and discount bonds and concluded that their empirical findings are in agreement with CIR's theoretical results. In addition, they suggested that earlier studies identifying significant forward-futures differences for the Treasury bill markets ought to seek explanations elsewhere than in the CIR framework, since the corresponding covariance terms for this market were even smaller. French (1983) reported significant differences between forward and futures prices for copper and silver. Moreover, he conducted a series of empirical tests of the CIR theoretical framework and concluded that his results are in partial agreement with this theory. Park and Chen (1985) also investigated the forward-futures differences for a number of foreign currencies and commodities and they pointed out to significant differences for most of the commodities they analysed, but not for the foreign currencies. Also, their empirical tests confirmed that the majority of the average forward-futures price differences are in accordance with the CIR result.

Kane(1980) tried to explain the differences between futures and forward prices based on market imperfections such as asymmetric taxes and contract performance guarantees. Levy (1989) strongly argued that the difference between forward and futures prices arises from the marked-to-market process of the futures contract. Meulbroek (1992) investigated further the relationship between forward and futures prices on the Eurodollar market and suggested that the marked-to-market effect has a large influence. However, Grinblatt and Jegadeesh (1996) advocated that the difference between the futures and forward Eurodollar rates due to marking-to-market is small.

[1] Other early studies that considered the relationship between forward and futures prices in a perfect market without taxes and transaction costs are Margrabe(1978), Jarrow and Oldfield (1981) and Richard and Sundaresan (1981).

Alles and Peace (2001) concluded that the 90-day Australia futures prices and the implied forwards are not fully supported by the CIR model. Recently, Wimschulte (2010) showed that there is no significant statistical or economical evidence for price differences between electricity futures and forward contracts.

The relationship between forward and futures prices as developed under the CIR model makes the tacit assumption that futures are infinitely divisible. Levy (1989) starts with the same set of assumptions underpinning the CIR model except one. When considering interest rates, he advocates that, if only the next day's interest rate were deterministic, a perfect hedge ratio using fractional futures positions can be constructed to replicate the forward. Thus, for Levy (1989) it is only the interest rate for the next day that is important and not the entire time path of the stochastic rates. Consequently, for Levy (1989), the forward prices should be equal to futures prices and any empirical findings regarding actual price differentials can have only statistical explanations and they are non-systematic. On the other hand, Morgan (1981) studied the forward-futures differential assuming that capital markets are efficient and so concludes that forward and futures prices must be different. His conclusion is mainly based on the fact that current futures price depends on the joint future evolution of stochastic interest rates and futures prices. Polakoff and Diz (1992) argued that due to the indivisibility of the futures contracts[2], the forward prices should be different from futures prices even when interest rates and futures prices exhibit zero local covariances. Moreover, they show that the autocorrelation in the time series of the forward-futures price differences should be expected. Hence, testing must take into consideration the presence of autocorrelation. Polakoff and Diz (1992) offered a theoretical explanation that unifies the contradictory theoretical views originated in how interest rates are negociated in the model. Their main conclusion is that it is unnecessary for futures prices and interest rates to be correlated in order to imply that forward prices should be different from futures prices.

From the review discussed above it appears that the empirical evidence is mixed and asset class specific. Property derivatives are an emerging asset class of considerable importance for financial systems. Case and Shiller (1989, 1990) found evidence of positive serial correlation as well as inertia in house prices and excess returns. This implied that the U.S. market for single-family homes is inefficient. The use of derivatives for risk management in real estate markets has been discussed by Case et.al. (1993), Case and Shiller (1996), Shiller and Weiss (1999) with

[2] Although the vast majority of literature on futures is based on the assumption of infinite divisibility, Polakoff (1991) discusses the important role played by the indivisibility of futures contracts.

respect to futures and options. Fisher (2005) discussed NCREIF-based swap products, while Shiller (2008) described the role played by the derivatives markets in general for home prices.

For real-estate there has been a perennial lack of developments of derivatives products that could have been used for hedging price risk. The only property derivatives traded more liquidly in U.S. and U.K are the total return swaps (TRS), forward and futures. In the U.K. commercial property sector for example, all three types of contract have the Investment Property Databank (IPD) index as the underlying. Since February 2009 the European Exchange (Eurex) has listed the UK property index futures. The most liquid derivatives markets on IPD UK index are the TRS, which is an over-the-counter market, and the futures, both with five yearly market calendar December maturities. Any portfolio of TRS contracts can be decomposed into an equivalent portfolio of forward contracts. Hence, having data on TRS prices and futures prices opens the opportunity to compare, after some financial engineering, forward curves with futures curves on the IPD index. As remarked by Polakoff and Diz (1992) it is difficult to compare forward and futures prices on a daily basis when forwards are traded on a non-synchronous basis. By contrast, when forwards are derived on an implied basis from other instruments then matching the term-to-delivery is easy.

In this paper we investigate the forward-futures price differences for the UK commercial property market for all five end of the year market maturities. To our knowledge, this is the first study that considers the forward - futures price differences for this important asset class. Furthermore, all previous studies relied exclusively on time series analysis, whereas we take a step further and also conduct statistical tests for panel data.

The remainder of the paper is organised as follows: Section 2 next contains the modelling approach taken for the commercial property index, Section 3 focuses on describing the data and the testing methodology, Section 4 describes the alternative estimation methods for the proposed models and Section 5 presents our empirical findings. Section 6 concludes and finally some of the theoretical properties of the models described in Section 2 as well as a series of derivations are included in the Appendices.

2. Modelling the Relationship between Forwards and Futures

Let $S(t)$ be the spot value of the IPD index at time - *t*, $F(t,T_i)$, the associated time-*t* forward price with maturity T_i, $f(t,T_i)$ the time-*t* futures price with maturity T_i and $D(t,T_i)$ the stochastic discount factor at time *t* for maturity T_i. Then $B(t,T_i)=E_t^Q\left(D(t,T_i)\right)$ is the time-*t* zero-coupon bond price, with maturity T_i, where the expectation is taken under a risk-neutral measure *Q*.

There is a model-free relationship between forward and futures prices given by:[3]

$$F(t,T)-f(t,T)=\frac{cov_t^Q\left(S(T),D(t,T)\right)}{E_t^Q\left(D(t,T)\right)} \tag{1}$$

which holds for any maturity *T* and at any time $0 \leq t \leq T$, and where *Q* is a risk-neutral pricing measure. This fundamental relationship opens up the first line of investigation by testing whether the differences between forward and futures prices are statistically different from zero. Later on we shall investigate several models and estimation methods for the IPD index to see which ones best captures the IPD forward-futures price difference.

2.1 Mean-reverting models

Empirical properties of real-estate indices suggest that the family of mean-reverting models presented in Lo and Wang (1995) could be suitable for defining our modelling framework. Shiller and Weiss (1999) pointed out that the models advocated in Lo and Wang (1995) may not be appropriate for real-estate derivatives since the underlying asset is not costlessly tradable, and they advocated using a lognormal model combined with an expected rate of return rather than a riskless rate. Nevertheless, Fabozzi et. al (2011) designed a way to merge the best of the two worlds by completing the market with the futures contracts that are used directly to calibrate the market price of risk for the real-estate index and hence, indirectly fixing also the risk-neutral pricing measure which can be then applied for pricing other derivatives.

Real-estate prices exhibit serial correlation leading to a high degree of predictability, up to 50% R-squared for a short term horizon. Moreover, it has been documented that returns on real-estate indices are positively autocorrelated over short horizons and negatively correlated over

[3] See Shreve (2004), p. 247.

longer horizons, see Fabozzi et.al. (2011). A reasonable theoretical explanation of serial correlation for real-estate indices can be drawn from Polakoff and Diz (1992) since real-estate trades are not infinitely divisible neither in the spot market nor in the futures market. Furthermore, mean reversion is a characteristic that has a financial economics basis in commodity markets. Real-estate is viewed partly as a commodity although it also retains some characteristics of other investment financial assets.

We consider a slight variation of the trending OU process presented in Lo and Wang (1995), as follows: let $p(t) = ln(S(t))$; $p(t) = q(t) + (\mu_0 + \mu t)$,[4] where the dynamics of $q(t)$ under the physical measure P are:

$$dq(t) = -\gamma q(t)\,dt + \sigma dW(t) \tag{2}$$

where $\gamma \geq 0, \sigma \geq 0, \mu_0, \mu \in R$. The standard solution for the OU process given by (2) leads to the closed form solution

$$p(t) = \mu_0 + \mu t + \left[p(0) - \mu_0\right] exp(-\gamma t) + \sigma \int_{v=0}^{t} exp(-\gamma(t-v))\,dW(v) \tag{3}$$

for any $0 \leq t \leq T$. This model is very flexible and allows the study of logarithmic returns. The continuously compounded τ-period returns, computed at time t, are defined as $r_\tau(t) = p(t) - p(t-\tau)$. Following similar moment calculations as in Lo and Wang (1995), we get the autocorrelations,

$$corr_{univ}\left(r_\tau(t_1), r_\tau(t_2)\right) = -\frac{1}{2} exp\left[-\gamma(t_2 - t_1 - \tau)\right]\left[1 - exp(-\gamma\tau)\right] \leq 0, \tag{4}$$

for any $t_1, t_2,$ and τ such that[5] $t_1 \leq t_2 - \tau$. The models investigated in this paper are applied to price futures on the IPD index. This market is inherently incomplete given the fact that trading in the underlying index is not possible and the main role played by the futures contracts is to complete the market. From a technical point of view we need to consider the market price of risk η into our models and calibrate this important parameter from the market futures prices. This procedure will help to identify a risk-neutral pricing measure and then other derivatives on the same index can be priced under this measure consistently. The corresponding equation for $p(t)$ under the real-measure P is:

[4] To simplify notation, we suppress model subscripts, *univ* and *biv* for the univariate and bivariate models, respectively, unless where absolutely necessary.

[5] This condition ensures that returns are non-overlapping.

$$dp(t)=\left[\mu-\gamma\left(p(t)-(\mu_0+\mu t)\right)\right]dt+\sigma dW(t), \tag{5}$$

and upon risk neutralization it becomes:

$$dp(t)=\left[\mu-\gamma\left(p(t)-(\mu_0+\mu t)\right)-\eta\sigma\right]dt+\sigma dW^Q(t) \tag{6}$$

The solution to this modified equation is similar to (3):

$$p(t)=\mu_0+\mu t-\mu_0\, exp(-\gamma t)-\frac{\eta\sigma}{\gamma}+exp(-\gamma t)\left[p(0)+\frac{\eta\sigma}{\gamma}\right]+\sigma\int_{v=0}^{t} exp\left(-\gamma(t-v)\right)dW^Q(v) \tag{7}$$

for any $0\leq t\leq T$. Given the normality of *p*(*t*) the theoretical futures prices can be calculated in closed-form:

$$\begin{aligned} f_{univ}(0,T) &= E^Q_{0,univ}\left(S(T)\right)=E^Q_{0,univ}\left(exp\left(p(T)\right)\right) \\ &= \frac{\sigma^2}{4\gamma} exp\left(\mu_0+\mu T-\mu_0\, exp(-\gamma T)-\frac{\eta\sigma}{\gamma}+exp(-\gamma T)\left[ln\left(S(0)\right)+\frac{\eta\sigma}{\gamma}\right]\right)\left(1-exp(-2\gamma T)\right) \end{aligned} \tag{8}$$

As remarked in Lo and Wang (1995), although this specification is a valid modelling starting point, it has an important disadvantage in that the autocorrelation coefficients of continuously compounded τ-period returns can only take negative values[6].

A more flexible approach, also proposed in Lo and Wang (1995), is the bivariate trending OU process, a natural extension of the univariate version above. Here we implement the following version of their model:

$$dq(t)=\left[-\gamma q(t)+\lambda r(t)\right]dt+\sigma dW_S(t) \tag{9}$$

$$dr(t)=\delta\left(\mu_r-r(t)\right)dt+\sigma_r dW_r(t) \tag{10}$$

where $dW_S(t)dW_r(t)=\varrho dt$ and the second stochastic factor on which the log-price of the underlying depends is the short interest rate *r*(*t*). The solution to equation (10) is:

$$r(t)=\mu_r+exp(-\delta t)\left(r(0)-\mu_r\right)+\sigma_r\int_{v=0}^{t} exp\left(-\delta(t-v)\right)dW_r(v) \tag{11}$$

[6] See expression (4).

for any $0 \le t \le T$. Combining (11) and (9) gives the analytical solution for the log of the underlying index value:

$$p(t) = \mu_0 + \mu t + exp(-\gamma t)(p(0) - \mu_0) + \frac{\mu_r \lambda}{\gamma}(1 - exp(-\gamma t)) + \frac{\lambda}{\gamma - \delta}(r(0) - \mu_r)[exp(-\delta t) - exp(-\gamma t)]$$
$$+ \frac{\lambda \sigma_r}{\gamma - \delta} \int_{v=0}^{t} [exp(-\delta(t - v)) - exp(-\gamma(t - v))] dW_r(v) + \sigma \int_{v=0}^{t} exp(-\gamma(t - v)) dW_s(v) \tag{12}$$

As detailed in Appendix A, we obtain that under the risk neutral measure Q:

$$r_{bivar}(t) = C_1 + [r(0) - C_1] exp(-\delta t) + \sigma_r \int_{v=0}^{t} e(-\delta(t - v)) dW_r^Q(v) \tag{13}$$

with $C_1 = \mu_r - \frac{\sigma_r}{\delta}\left(\varrho \eta_1 + \sqrt{1 - \varrho^2}\eta_2\right)$, and

$$p(t) = \mu_0 + \mu t - \frac{\sigma \eta_1}{\gamma} + \left(p(0) - \mu_0 + \frac{\sigma \eta_1}{\gamma}\right) exp(-\gamma t) + \sigma \int_{v=0}^{t} exp(-\gamma(t - v)) dW_s^Q(v)$$
$$+ \lambda \left\{ \begin{aligned} & \frac{C_1}{\gamma}[1 - exp(-\gamma t)] + \frac{r(0) - C_1}{\gamma - \delta}[exp(-\delta t) - exp(-\gamma t)] \\ & + \frac{\sigma_r}{\gamma - \delta} \int_{v=0}^{t} [exp(-\delta(t - v)) - exp(-\gamma(t - v))] dW_r^Q(v) \end{aligned} \right\}. \tag{14}$$

The futures price for maturity T can be easily derived now as:

$$f_{bivar}(0, T) = E_{0,bivar}^Q(S(T)) = exp\left(C_2(T) + \frac{\sigma_y^2(T)}{2}\right) \tag{15}$$

where

$$C_2(T) = \mu_0 + \mu T - \frac{\sigma \eta_1}{\gamma} + \left(ln(p(0)) + \frac{\sigma \eta_1}{\gamma}\right) exp(-\gamma T)$$
$$+ \lambda \left[\frac{C_1}{\gamma}(1 - exp(-\gamma T)) + \frac{r(0) - C_1}{\gamma - \delta}(exp(-\delta T) - exp(-\gamma T))\right]$$

and

$$\sigma_y^2(T)=\frac{\sigma^2}{2\gamma}\left(1-exp(-2\gamma T)\right)+\frac{\lambda\sigma_r^2}{(\gamma-\delta)^2}\left[\frac{1-exp(-2\delta T)}{2\delta}+\frac{1-exp(-2\gamma T)}{2\gamma}-\frac{2\left(1-exp\left(-(\gamma+\delta)T\right)\right)}{\gamma+\delta}\right]$$
$$+\frac{2\varrho\lambda\sigma\sigma_r}{\gamma-\delta}\left[\frac{1-exp\left(-(\gamma+\delta)T\right)}{\gamma+\delta}-\frac{1-exp(-2\gamma T)}{2\gamma}\right]$$

The expression for the correlation of τ-period returns $corr_{bivar}\left(r_\tau(t_1),r_\tau(t_2)\right)$ for any $t_1, t_2,$ and τ such that $t_1\leq t_2-\tau,$ is more involved and thus excluded here. However, it can be shown that for certain values of the model parameters, the bivariate model, unlike the univariate model outlined above, is more flexible and can allow for both positive and negative autocorrelations.

3. Data and Testing Methodology

For our empirical analysis of the differences between the forward and futures prices on the IPD[7] UK property index we perform two types of tests. Firstly, we investigate whether the observed difference between the forward and futures prices is statistically different from zero. Secondly, we test which of a number of established continuous time models combined with various methods of estimation is able to best capture this difference. Using the previously defined notation, we have: $n = 5$ different maturities and $N = 71$ daily observations for each maturity.

3.1 Data

The data needed for our study contains IPD property futures prices, the IPD total return swap (TRS) rates, the IPD index, and also the GBP interest rates needed to calculate discount factors. Futures prices have been obtained from the European Exchange (Eurex[8]), the property TRS data (the fixed rate) has been provided by Tradition Group, a major dealer on this market and the IPD index was sourced from the Investment Property Databank (IPD[9]). In addition, the UK's interest rates have been downloaded from Datastream. Due to the availability of the property futures and TRS data, the sample period used is daily from 4 February 2009 until 7 July 2009. It generates 71 property futures daily curves and 71 sets of TRS rates with up to five years maturity (the first maturity date is 31 December 2009, the second maturity date is 31 December 2010, the

[7]IPD stands for Investment Property Databank. A detailed description of the data is given in Section 3.1 below.

[8] See www.eurexchange.com for more information on Eurex. IPD UK futures contracts started on 4 February 2009.

[9] See www.ipd.com for more information on IPD.

third maturity date is 31 December 2011, the fourth maturity date is 31 December 2012, and the fifth maturity date is 31 December 2013).

The evolution of the TRS series is depicted in Figure 1 and we could see that, for our period of investigation, most of the IPD TRS rates are negative for the first, second, and third maturity dates. For the fourth and fifth maturity dates, the values are higher or mostly positive. In addition, there is a dramatic increase of the fixed rate at the end of February 2009, possibly due to the rollover off the futures contracts in March combined with the publication of the IPD index for the year ending in December 2008. The property futures prices are illustrated in Figure 2. The property futures prices are quoted on a total return basis.

INSERT FIGURE 1 HERE

INSERT FIGURE 2 HERE

The descriptive statistics of the TRS rates are reported in Table 1. The mean values are mostly negative; the mean for the first maturity date is -17.80% and the means are increasing with maturity. The excess kurtosis is negative for all five futures contracts and the first year TRS contract and it is positive for the remaining four series of TRS rates. The skewness values have negative signs, except for the four year futures contract, implying that the distributions of the data are skewed to the left.

INSERT TABLE 1 HERE

It could be seen in Table 1 that the futures contract for the fifth maturity date appears to have the highest mean. The highest standard deviation is showed in the futures contract for the second maturity date. Similarly to TRS data, futures prices exhibit skewness and fat tails characteristics.

From the daily TRS prices for the market five yearly maturities one can reverse engineer the equivalent no-arbitrage forward prices for the same maturities. The equivalent fair property forward prices are derived daily from 4 February until 7 July 2009, with maturities matching the futures contracts maturities. The final engineered fair prices of property forwards are illustrated in Figure 3. The descriptive moments of the differences between forward and futures prices on IPD commercial index are also provided in Table 1. On average, the differences for the first three maturities are positive, while for the fourth and fifth maturities they are negative.

INSERT FIGURE 3 HERE

3.2 Testing Methodology

First, we test whether the difference between the market TRS equivalent forward prices and market futures prices is significantly different from zero. If this hypothesis is rejected, then in the second stage a series of models and estimation methods are employed for the terms on the right hand side of the fundamental relationship given by (1). The aim in the second stage is to decide on the capability of various models to appropriately capture the dynamics of the index *S* and the discount factor *D*.

For the first stage analysis, we run the following regression model for each maturity date T_i, $i \in \{1, 2, ..., 5\}$:

$$F(t, T_i) = \alpha_{0i} + \beta_{0i} f(t, T_i) + \varepsilon_{ti} \tag{16}$$

(with T_i fixed for each of the five time series regressions) and test whether $\alpha_{0i} = 0$ and $\beta_{0i} = 1$. If this null hypothesis cannot be rejected, then one can conclude that the difference between forward and futures prices is due to noise. If, however, the null is rejected, we then proceed to the second stage of our analysis. The same econometric analysis described above from a times series point of view, can also be performed using panel data. Using panel data has a series of advantages.[10] Firstly, it enables the analysis of a larger spectrum of problems that could not be tackled with cross-sectional or time series information alone. Secondly, it generally results in a greater number of degrees of freedom and a reduction in the collinearity among explanatory variables, thus increasing the efficiency of estimation. Furthermore, the larger number of observations can also help alleviate model identification or omitted variable problems.

The regression equation in (16) is rewritten for our panel data as:

$$F(t, T_i) = \alpha_0 + \beta_0 f(t, T_i) + \varepsilon_{ti} \tag{17}$$

with $i \in \{1, 2, ..., 5\}$ and $t \in \{1, 2, ..., 71\}$. More variations of a panel regression exist, the simplest one being the pooled regression, described above in (17), which implies estimating the regression equation by simply stacking all the data together, for both the explained and explanatory variables. Furthermore, the fixed effects model for panel data is given by:

$$F(t, T_i) = \alpha_0 + \beta_0 f(t, T_i) + \alpha_i + \upsilon_{it} \tag{18}$$

[10] See also Baltagi (1995), Hsiao (2003).

where α_i varies cross-sectionally (i.e. in our case it is different for each maturity date T_i), but not over time. Similarly, a time-fixed effects model can be formulated, in which case one would need to estimate:

$$F(t,T_i) = \alpha_0 + \beta_0 f(t,T_i) + \lambda_t + \upsilon_{it} \tag{19}$$

where λ_t varies over time, but not cross-sectionally. The fixed effects model and the time-fixed effects model, as well as a model with *both* fixed effects and the time-fixed effects, will be analysed. One can test whether the fixed effects are necessary using the redundant fixed effects LR test.

For the panel data random effects model the regression specification is given by:

$$F(t,T_i) = \alpha_0 + \beta_0 f(t,T_i) + \varepsilon_i + \upsilon_{it} \tag{20}$$

where ε_i is now assumed to be random, with zero mean and constant variance σ_ε^2, independent of υ_{it} and $f(t,T_i)$. Similarly, a random time-effects model can be formulated in the context of this paper as:

$$F(t,T_i) = \alpha_0 + \beta_0 f(t,T_i) + \varepsilon_t + \upsilon_{it} \tag{21}$$

Again, random effects and random time-effects models, as well as a two-way model which allows for both random effects and random time-effects, can be estimated. Furthermore, it is important to test whether the assumption that the random effects are uncorrelated with the regressors is satisfied.

For the second stage analysis we shall employ several models for the dynamics of the IPD index S. If the analysis is conditioned on knowing the bond prices, the RHS of identity (1) can be expressed as:

$$\frac{cov_t^Q\left(S(T), D(t,T)\right)}{E_t^Q\left(D(t,T)\right)} = \frac{S(t)}{B(t,T)} - E_t^Q\left(S(T)\right) \tag{22}$$

Based on (1) and (22), it is evident that for our testing purposes the following regression is useful:

$$F(t,T) - f(t,T) = \alpha + \beta \left[\frac{S(t)}{B(t,T)} - \underbrace{E_{t,m}^{Q}\left(S(T)\right)}_{f_m(t,T)} \right] + u_{t,m} \tag{23}$$

and test whether $\alpha = 0$ and $\beta = 1$, for each model *m*. For each model *m*, failing to reject the null hypothesis implies that this particular model is suitable for describing the dynamics of the underlying IPD index.

For a more comprehensive insight, we also consider a model for the interest rates that will lead to stochastic discount factors. In this paper, we assume a Vasicek one-factor model that is employed in conjunction with all models for the IPD index. Under this set-up we run the following regression:

$$F(t,T) - f(t,T) = \alpha + \beta \frac{cov_{t,m}^{Q}\left(S(T), D(t,T)\right)}{E_{t,m}^{Q}\left(D(t,T)\right)} + u_{t,m} \tag{24}$$

where again *m* is the model index, and the test whether $\alpha = 0$ and $\beta = 1$ is for each model *m*. If we fail to reject the null hypothesis, we then conclude that the model in question is suitable for describing the dynamics of S and D.

Upon estimation of the model parameters, including the market price of risk (vector) η, as described in previous sections, we can fit the regressions given in (23) and (24). The competing models and methods of estimation are compared with respect to whether β is significant and also considering the R^2 measure of goodness-of-fit.

4. Calibration of the models

In order to be able to use the models enumerated in the preceding section we have to first calibrate their parameters. The parameters of the continuous time models specified in (3) and (11)-(12) can be estimated from the monthly log prices on the IPD index, observed over the period between December 1986 and January 2009, and totalling 266 historical observations. The estimates then will be carried forward for analysing the differences between the forward and futures on IPD starting from February 2009.

4.1 Maximum Likelihood Estimation

When feasible, parametric inference for diffusion processes from discrete-time observations should employ the likelihood function, given its generality and desirable asymptotic properties of consistency and efficiency (Phillips and Yu, 2009). The continuous time likelihood function can be approximated with a function derived from discrete-time observations, obtained by replacing the Lebesgue and Ito integrals with Riemann-Ito sums. Remark that this approach gives reliable results only when the observations are spaced at small time intervals. When the time between observations is not small the maximum likelihood estimator can be strongly (upward) biased in finite samples.[11]

We first de-trend the log price data by estimating the regression:

$$p_{t_k} = \mu_0 + \mu t_k + u_{t_k} \tag{25}$$

and subsequently work with the residuals from this equation, where k = 1, 2, …, 266 and $t_k = k\tau$, with $\tau = \frac{1}{12}$ for monthly returns.

The (exact) discretization of equation (3) leads to:

$$u_{t_k} = c u_{t_{k-1}} + \varepsilon_{t_k} \tag{26}$$

where $c = exp(-\gamma\tau)$ and $\varepsilon_{t_k} = \sigma \int_{t_{k-1}}^{t_k} exp\left(-\gamma\left(t_k - s\right)\right) dW\left(s\right)$; $\varepsilon_{t_k} \sim N\left(0, \frac{\sigma^2}{2\gamma}\left(1 - exp\left(-2\gamma\tau\right)\right)\right)$.

Maximum likelihood estimation of the discrete-time model in (26) gives[12] $c = 0.995086$ and the standard deviation of ε_{t_k} as 0.011.

The exact discretization of the two-factor model given by (11)-(12) is:

$$\begin{aligned} q_{t_k} &= \alpha_q + \beta_q q_{t_{k-1}} + \varphi r_{t_{k-1}} + \varepsilon_{q,t_k} \\ r_{t_k} &= \alpha_r + \beta_r r_{t_{k-1}} + \varepsilon_{r,t_k} \end{aligned} \tag{27}$$

[11] Further discussion is given in Dacunha-Castelle and Florens-Zmirou (1986), Lo (1988), Florens-Zmirou(1989), Yoshida(1990) and Phillips and Yu (2009)

[12] To check the stability of the parameter estimates, the estimation above is repeated using a larger sample, namely Dec 1986 to Oct 2010, with an increased sample size of 287 monthly observations. The parameter estimates do not change much.

where, for reasons of space, the expressions for the parameters as well as the distribution of the error terms in (27) are only given in Appendix B.

4.2 *Alternative estimation methods: Martingale estimation and Markov Chain Monte Carlo (MCMC)*

Lo (1988) argued that maximum likelihood estimation does not produce consistent estimates for the parameters of the continuous time model, when a discrete data sample is used. Two alternative estimation techniques that are applied here in order to circumvent this problem are the martingale estimation method described in Bibby & Sorensen (1995) and Markov Chain Monte Carlo (MCMC) methodology (see Tsay, 2008).

Bibby and Sorensen (1995) have overcome this difficulty by developing a martingale estimating function estimator. A consistent estimator cannot be obtained from the discrete approximation of the likelihood function L because the associated pseudo-score function is biased. This bias is directly related to the time between observations being sizeable. The methodology proposed by Bibby and Sorensen (1995) – briefly described in Appendix D- compensates the pseudo-score function in order to obtain a martingale. Their estimator is consistent and asymptotically normal.

Following example 2.1 in Bibby and Sorensen (1995) and using previously defined notation, the estimator resulting from the martingale estimating function for the mean reversion parameter γ is:

$$\gamma = \frac{1}{\tau} ln \frac{\sum_{k=1}^{266} q_{t_{k-1}} q_{t_k}}{\sum_{k=1}^{266} q_{t_{k-1}}^2}$$

provided that the numerator is positive. It can be shown that this estimator is equal to the maximum likelihood estimator for the case when the volatility parameter σ is known.

Since the martingale approach is not suitable for deterministic time trending processes, we only apply it for models mean reverting towards a constant threshold. The parameters obtained with this method are $\mu_0 = 0.3382$ $\gamma = 0.0443$ $\sigma = 0.01$. These values will feed into formula (8) and lead to model futures prices.

MCMC techniques[13] are based on a Bayesian inference theoretical support and offer an elegant solution to many problems encountered with other estimation methods, at the cost of computational effort. The main advantage of employing this type of inferential mechanism is the

[13] For an excellent introduction see Tsay (2010). All MCMC inference in this paper has been produced with WinBUGS 1.4, from a sample of 100,000 iterations after a burn-in period of 500000 iterations.

capability to produce not only a point estimate but an entire posterior distribution for parameters of interest. Selecting various statistics from this distribution provides a more informed view on the plausible values of the parameters. Hence, for estimation purposes we select the mean, the 2.5% quantile and the 97.5% quantile of the posterior distribution of the mean reversion parameter. The estimates for the discretized version of the mean-reverting model given in (5) are reported in Table 3. One great advantage of the MCMC approach is that all parameters are estimated easily from the same output without additional computational effort.

4.3 *The Calibration of the Market Price of Risk*

To calibrate the market price of risk η, we follow standard practice and minimize the mean squared error function; for the univariate model we have:

$$\eta_t^* = \underset{\eta_t \in R}{arg\,min}\left\{\sum_{i=1}^{5}\left[f\left(t,T_i\right)-f_{univ}\left(\eta_t,t,T_i\right)\right]^2\right\} \tag{28}$$

where $f\left(t,T_i\right)$ represents the market futures price and $f_{univ}\left(\eta_t,t,T_i\right)$ is the theoretical futures price at time t for maturity T_i. This optimization exercise is performed for each day in our sample and for each of the estimation methodologies described above. The resulting time series for the market price of risk, for each set of estimates are given in Figure 4.[14]

INSERT FIGURE 4 HERE

All parameter estimates can be determined now and then the theoretical model can be used for producing property futures prices.

For the bivariate model, to calibrate the market price of risk vector $\boldsymbol{\eta} = \left(\eta_1 \quad \eta_2\right)'$, we solve:

$$\boldsymbol{\eta}_t = \underset{\boldsymbol{\eta}_t \in R^2}{arg\,min}\left\{\sum_{i=1}^{5}\left[\boldsymbol{g}\left(t,T_i\right)-\boldsymbol{g}_{bivar}\left(\eta_t,t,T_i\right)\right]^2\right\} \tag{29}$$

where $\boldsymbol{g}\left(t,T_i\right)=\left(f\left(t,T_i\right) \quad r\left(t,T_i\right)\right)'$ are the observed futures prices and the interest rates obtained from observable bond prices, respectively. For clarity,

[14] For the ML estimation we also investigate the calibration of a surface for the market price of risk, where we now allow η to vary across maturities as well as across time. The results are depicted in Figure 5.

$\boldsymbol{g}_{bi\,var}\left(\eta_t, t, T_i\right) = \left(f_{bi\,var}\left(t, T_i\right) \quad r_{bi\,var}\left(t, T_i\right)\right)'$ are their model counterparts, as given in (15) and (13), respectively.

Table 2 gives a list of the models investigated in this paper with various methods of estimation. In Table 3 we report the parameter estimation results for these models based on our data.

INSERT TABLE 2 HERE

INSERT TABLE 3 HERE

5. Empirical Analysis

Our empirical analysis is divided into a part related to plain tests of the market differences between forward and futures prices on IPD index and a more refined analysis looking at several models for the underlying IPD index dynamics, coupled with a model for interest rates, but also considering several estimation methods.

5.1 Model-free analysis

Having available both series of forward and futures prices allows us to test directly whether the forward – futures difference time series diverges significantly away from zero.

Before running the time-series regressions in (17-22) we test whether the forward and futures price series are stationary using the Augmented Dickey-Fuller (ADF) test. The results are reported in Table 4.

INSERT TABLE 4 HERE

As we could see from the Table 4, most of the ADF results show that the forward series for the first, second, third, and fifth maturity dates are non-stationary while the forward series for the fourth maturity date is stationary at 5% significance level. In addition, the ADF test indicates that the futures series for all maturity dates are non-stationary. Furthermore, we also investigated the stationarity of the first differenced data. According to Table 4, the forward and futures series for all maturity dates are stationary in the first differences.

Since most of the data is found to be non-stationary in levels and stationary in the first differences, we perform the remaining analysis on the first differenced data. We test H_0: $\alpha_{0i} = 0$ and $\beta_{0i} = 1$ vs. H_1: $\alpha_{0i} \neq 0$ or $\beta_{0i} \neq 1$, using an *F*-test and the results could be found in the Table 5.

INSERT TABLE 5 HERE

The F-test results presented in Table 5 show that the null hypothesis for all maturity dates could be rejected at 1% significance level. We could conclude that the difference between forward and futures is not just a noise. [15] The same conclusion is reached if we analyse the values of the t-statistics for the forward-futures differences reported in Table 6.

INSERT TABLE 6 HERE

Panel Stationarity tests

Levin and Lin (1993), Levin, Lin and Chu (2002), Im, Pesaran and Shin (1997) and Maddala and Wu (1999) have developed unit root tests for panel data[16]. The results of these tests are reported in Table 7.

INSERT TABLE 7 HERE

As it was the case with the time series data, the panel data is non-stationary in the levels, however, the first differenced data is stationary and hence we continue our analysis using the first differences. To choose an appropriate specification for our panel regression, we first test whether the fixed effects are necessary using the redundant fixed effects LR test. The results of this test are reported in Table 8.

INSERT TABLE 8 HERE

From the test results reported in Table 8, it appears that a model with fixed time effects only is most supported by our data. Furthermore, we also investigate whether a random effects model is appropriate using the Hausman test; the results of this test are also reported in Table8.
Based on these results, we arrive at the conclusion that the random effect model is to be preferred in this case.

Next, we compute the F-test statistic for multiple coefficient hypotheses using the panel regression random effect specification; the results are reported in Table 9.

INSERT TABLE 9 HERE

[15] In addition, we also investigate the diagnostic statistics for these regressions and report the Durbin-Watson test statistic results in Table 5. We note that for all but the fifth maturity date there is no autocorrelation in the regressions errors.
[16] For a description of the tests, see Hsiao (2003), p. 298-301.

According to Table 9, we could see that the F-values are significant at 1% level. We could strongly reject the null hypothesis (H_0: $\alpha_0 = 0$ and $\beta_0 = 1$) and conclude that the differences between forward and futures are not just noise in the panel data.[17]

5.2 Model-based analysis[18]

From a financial economics point of view, it has been established that even if the interest rates are constant then futures prices can differ from the associated forward prices. Assuming that interest rates are stochastic leads directly to the conclusion that the two series will diverge significantly over time. In this paper we would like to pose and answer the question, "which model and estimation method" most likely support the observed market differences. Furthermore, an additional level of complexity is generated from employing panel data tests.

Tables 10 and 11 summarize the results of our model comparison, for the time series and panel data, respectively. The martingale method and the MCMC methods perform better than the ML method. The R-square seems to increase with maturity overall hinting that stationarity problems may be more acute for near maturities. Please note that since maturities are fixed in the calendar by the market, the time to maturity of our series gets progressively smaller, across all five contracts.

INSERT TABLE 10 HERE

INSERT TABLE 11 HERE

Our results in Table 11 reveal that for panel data analysis all models employed here are well specified and the beta t-test is highly significant. The univariate model coupled with maximum likelihood estimation is by far the best approach, the R-squared being close to 93%.

6. Conclusions

In this paper we analyse the differences between forward and futures prices on a new asset class, commercial real-estate, using a battery of models, estimation methods and tests. The forward prices have been reversed engineered from total return swap rates using standard market practice. Testing is done on individual time series data but also in a panel data framework.

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[17] In addition, the values of Durbin Watson test show that there is no autocorrelation in the panel regression errors.

[18] The futures and engineered forward prices are given as percentages of the underlying IPD index $S(t)$. In order to be able to conduct the testing in this section, we need to first obtain the transformed futures and forward prices as follows: $\tilde{f}(t,T_i) = f(t,T_i)\frac{S(t)}{100}$ and $\tilde{F}(t,T_i) = F(t,T_i)\frac{S(t)}{100}$.

Our results provide evidence of significant differences between the implied forward and futures prices for the IPD UK index. One possible explanation could be the period of study, several months during 2009, in the aftermath of the subprime crisis.

Although our overall conclusion is that, for the period 4 February 2009 to 7 July 2009, the forward prices were different from futures prices, there is substantial variation in the strength of these results across contract maturities, methods of estimation and testing frameworks. Given the significance of our results even on a model-free basis, we organised a model race that best explains the relationship between synthetic forward prices derived from daily total return swap rates and the daily futures prices. The models were generated from using various methods of estimation for the mean-reverting OU continuous time process assumed for the underlying IPD index. Our models provided significant explanatory power for the relationship between forward and futures prices on commercial real-estate index in UK but the analysis of the error terms shows that there is more that can be explained. From a theoretical point of view our study can be expanded to two-factor models as detailed in the paper. Unfortunately, due to lack of space we could not report also the results from the two-factor models subset, but we hope to do that in the very near future.

Appendices

Appendix A: The Derivation of the Risk Neutral Dynamics for the Bivariate Process

Here we work directly with the de-trended process $\{q(t)\}$. In order to obtain the risk neutral *Q*-dynamics of the system

$$dq(t)=\left[-\gamma q(t)+\lambda r(t)\right]dt+\sigma dW_S(t) \tag{A1}$$

$$dr(t)=\delta\left(\mu_r-r(t)\right)dt+\sigma_r dW_r(t) \tag{A2}$$

where $dW_S(t)dW_r(t)=\varrho dt$ we first re-write the system under the physical measure *P*, but depending solely on the non-correlated Brownians[19] W_1 and W_2, where, for any t

$$\begin{aligned}W_1(t)&=W_S(t)\\ W_2(t)&=\frac{W_r(t)}{\sqrt{1-\varrho^2}}-\frac{\varrho W_S(t)}{\sqrt{1-\varrho^2}}\end{aligned} \tag{A3}$$

Then:

$$W_r(t)=\varrho W_1(t)+\sqrt{1-\varrho^2}\,W_2(t) \tag{A4}$$

Thus, system (A1-A2) can be re-written:

$$\begin{bmatrix}dp(t)\\ dr(t)\end{bmatrix}=\left\{\begin{bmatrix}\mu+\gamma(\mu_0+\mu t)\\ \delta\mu_r\end{bmatrix}+\begin{bmatrix}-\gamma & \lambda\\ 0 & -\delta\end{bmatrix}\begin{bmatrix}p(t)\\ r(t)\end{bmatrix}\right\}dt+\begin{bmatrix}\sigma & 0\\ \varrho\sigma_r & \sigma_r\sqrt{1-\varrho^2}\end{bmatrix}\begin{bmatrix}dW_1(t)\\ dW_2(t)\end{bmatrix} \tag{A5}$$

The risk neutral *Q*-dynamics of this system is given by

$$\begin{aligned}\begin{bmatrix}dp(t)\\ dr(t)\end{bmatrix}=&\left\{\begin{bmatrix}\mu+\gamma(\mu_0+\mu t)\\ \delta\mu_r\end{bmatrix}-\begin{bmatrix}\sigma & 0\\ \varrho\sigma_r & \sigma_r\sqrt{1-\varrho^2}\end{bmatrix}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}+\begin{bmatrix}-\gamma & \lambda\\ 0 & -\delta\end{bmatrix}\begin{bmatrix}p(t)\\ r(t)\end{bmatrix}\right\}dt\\ &+\begin{bmatrix}\sigma & 0\\ \varrho\sigma_r & \sigma_r\sqrt{1-\varrho^2}\end{bmatrix}\begin{bmatrix}dW_1^Q(t)\\ dW_2^Q(t)\end{bmatrix}\end{aligned} \tag{A6}$$

The above system can be solved and then we get:

$$r(t)=C_1+\left[r(s)-C_1\right]exp\left(-\delta(t-s)\right)+\sigma_r\int_{v=s}^{t}exp\left(-\delta(t-v)\right)dW_r^Q(v), \tag{A7}$$

[19] See also proposition 4.19 from Bjork (2009).

with $C_1 = \mu_r - \frac{\sigma_r}{\delta}\left(\varrho\eta_1 + \sqrt{1-\varrho^2}\eta_2\right)$, and $dW_r^Q(t) = \varrho W_1^Q(t) + \sqrt{1-\varrho^2}W_2^Q(t)$.

Appendix B: The Discretization of the Bivariate Model

The complete specification of the discretization for the two-factor model used in the paper is given as

$$\begin{aligned} q_{t_k} &= \alpha_q + \beta_q q_{t_{k-1}} + \varphi r_{t_{k-1}} + \varepsilon_{q,t_k} \\ r_{t_k} &= \alpha_r + \beta_r r_{t_{k-1}} + \varepsilon_{r,t_k} \end{aligned} \tag{B1}$$

where

$$\alpha_q = \mu_r \lambda \left[\frac{1-exp(-\gamma\tau)}{\gamma} - \frac{exp(-\delta\tau) - exp(-\gamma\tau)}{\gamma-\delta}\right], \beta_q = exp(-\gamma\tau),$$

$$\varphi = \frac{\lambda}{\gamma-\delta}\left[exp(-\delta\tau) - exp(-\gamma\tau)\right]; \alpha_r = \mu_r\left(1-exp(-\delta\tau)\right), \beta_r = exp(-\delta\tau),$$

$$\varepsilon_{q,t_k} = \frac{\lambda\sigma_r}{\gamma-\delta}\int_{t_{k-1}}^{t_k}\left[exp\left(-\delta\left(t_k - s\right)\right) - exp\left(-\gamma\left(t_k - s\right)\right)\right]dW_r(s) + \sigma\int_{t_{k-1}}^{t_k} exp\left(-\gamma\left(t_k - s\right)\right)dW_s(s)$$

$$\varepsilon_{r,t_k} = \sigma_r \int_{t_{k-1}}^{t_k} exp\left(-\delta\left(t_k - s\right)\right)dW_r(s).$$

We also notice that:

$$\alpha_q = \frac{\lambda}{\gamma}\alpha_r - \mu_r\varphi \tag{B2}$$

The error vector is bivariate normal, with covariance matrix:[20]

$$\begin{bmatrix} var\left(\varepsilon_{q,t_k}\right) & cov\left(\varepsilon_{q,t_k}, \varepsilon_{r,t_k}\right) \\ cov\left(\varepsilon_{q,t_k}, \varepsilon_{r,t_k}\right) & var\left(\varepsilon_{r,t_k}\right) \end{bmatrix}$$

[20] Lo and Wong (1995) obtained the same error covariance matrix although their model is not the same as ours.

where:

$$var\left(\varepsilon_{q,t_k}\right)=\frac{\sigma^2}{2\gamma}\left(1-exp\left(-2\gamma\tau\right)\right)+\frac{\lambda\sigma_r^2}{\left(\gamma-\delta\right)^2}\left[\frac{1-exp\left(-2\delta\tau\right)}{2\delta}+\frac{1-exp\left(-2\gamma\tau\right)}{2\gamma}-\frac{2\left(1-exp\left(-\left(\gamma+\delta\right)\tau\right)\right)}{\gamma+\delta}\right]$$
$$+\frac{2\varrho\lambda\sigma\sigma_r}{\gamma-\delta}\left[\frac{1-exp\left(-\left(\gamma+\delta\right)\tau\right)}{\gamma+\delta}-\frac{1-exp\left(-2\gamma\tau\right)}{2\gamma}\right]$$

(B3)

$$var\left(\varepsilon_{r:t_k}\right)=\frac{\sigma_r^2}{2\delta}\left(1-exp\left(-2\delta\tau\right)\right) \tag{B4}$$

$$cov\left(\varepsilon_{q,t_k},\varepsilon_{r,t_k}\right)=\frac{\lambda\sigma_r^2}{\gamma-\delta}\left[\frac{1-exp\left(-2\delta\tau\right)}{2\delta}-\frac{1-exp\left(-\left(\gamma+\delta\right)\tau\right)}{\gamma+\delta}\right]+\frac{\varrho\sigma\sigma_r}{\gamma+\delta}\left[1-exp\left(-\left(\gamma+\delta\right)\tau\right)\right]$$

(B5)

(B3) and (B5) represent a system of two equations in two unknowns, σ and ϱ.

Appendix C: Maximum Likelihood Estimation - Continuous Time Models Parameters in terms of the Discrete Time Models Parameters

Univariate Model

$$\gamma=-\frac{ln\left(c\right)}{\tau};\quad \sigma=\sqrt{\frac{2\gamma}{1-exp\left(-2\gamma\tau\right)}}\sigma_{\varepsilon_{t_k}} \tag{C1}$$

Bivariate Model

$$\gamma=-\frac{ln\left(\beta_q\right)}{\tau};\quad \delta=-\frac{ln\left(\beta_r\right)}{\tau};\quad \lambda=\frac{\varphi}{\tau}\frac{ln\left(\frac{\beta_r}{\beta_q}\right)}{\beta_r-\beta_q};\quad \sigma_r=\sqrt{\frac{2\delta}{1-exp\left(-2\delta\tau\right)}}\sigma_{\varepsilon_{r,t_k}} \tag{C2}$$

Appendix D: Martingale Estimation

If $X_{\Delta}, X_{2\Delta}, ..., X_{n\Delta}$ is a discrete observation sample from the path of the diffusion

$$dX(t) = b\left(X(t), \varphi\right)dt + \sigma\left(X(t), \varphi\right)dW(t) \tag{D1}$$

where φ is a parameter vector.

Then, denoting $F(x, \varphi) = E_{\varphi}\left[X_{\Delta} \mid X_0 = x\right]$, Bibby and Sorensen (1995) build the estimating function

$$\bar{G}_n(\varphi) = \sum_{i=1}^{i=n} \frac{\frac{\partial b\left(X_{(i-1)\Delta}, \varphi\right)}{\partial \varphi}}{\sigma^2\left(X_{(i-1)\Delta}, \varphi\right)} \left\{X_{i\Delta} - F\left(X_{(i-1)\Delta}, \varphi\right)\right\} \tag{D2}$$

This is a zero-mean martingale and thus it does not matter whether the diffusion coefficient σ depends on the parameter or not.

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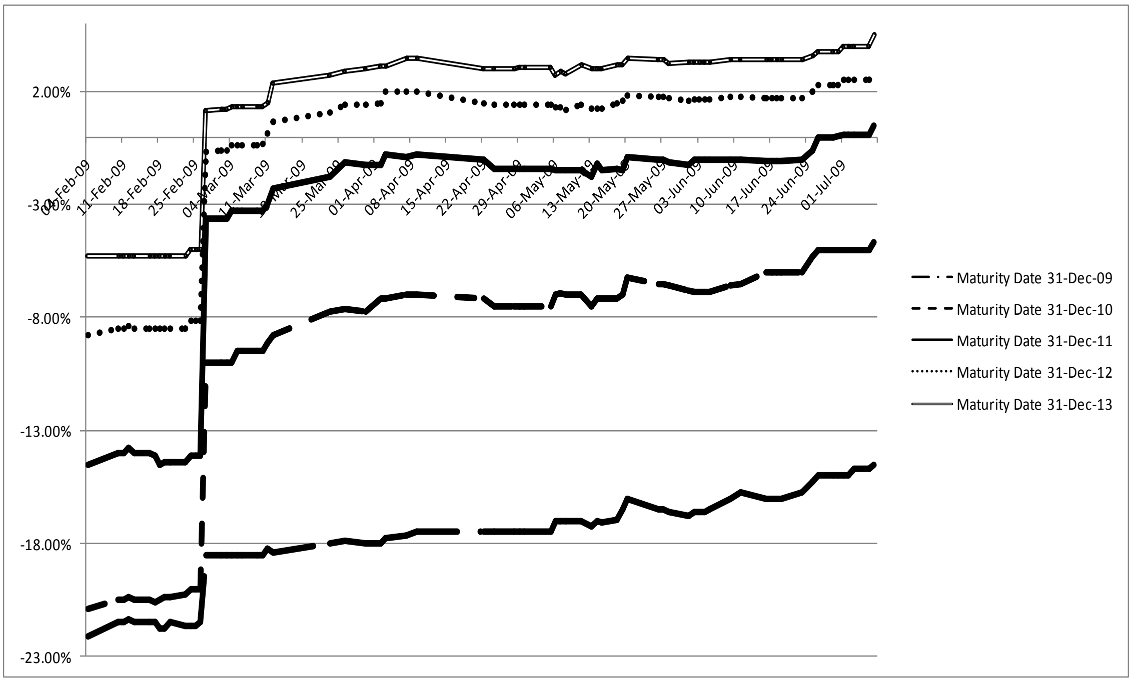
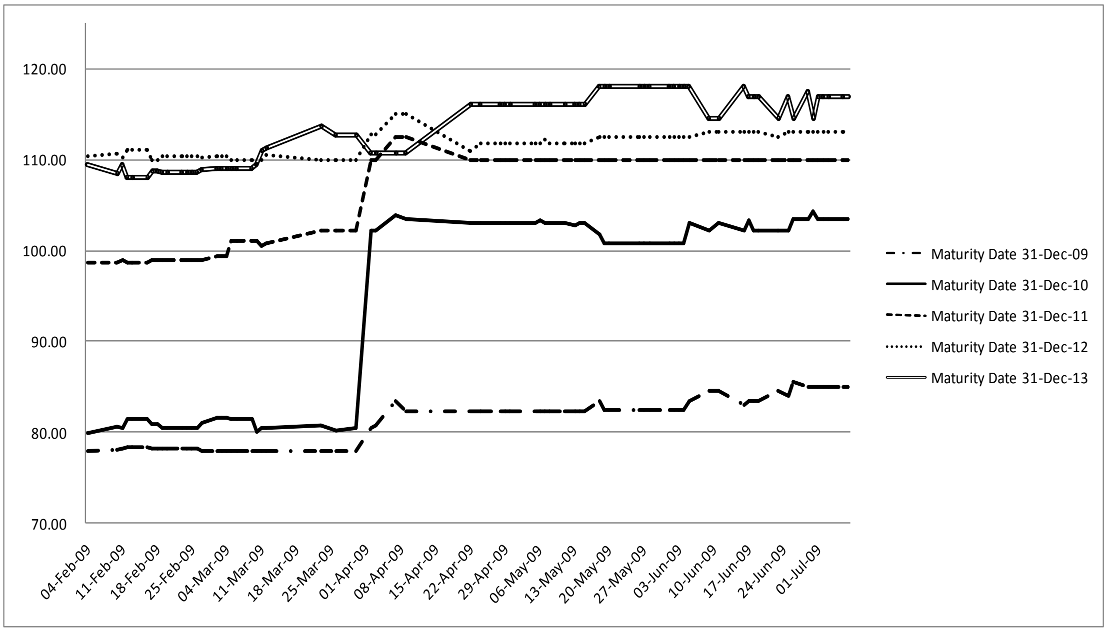
Figure 1 IPD Total Return Swap Rates (mid prices)



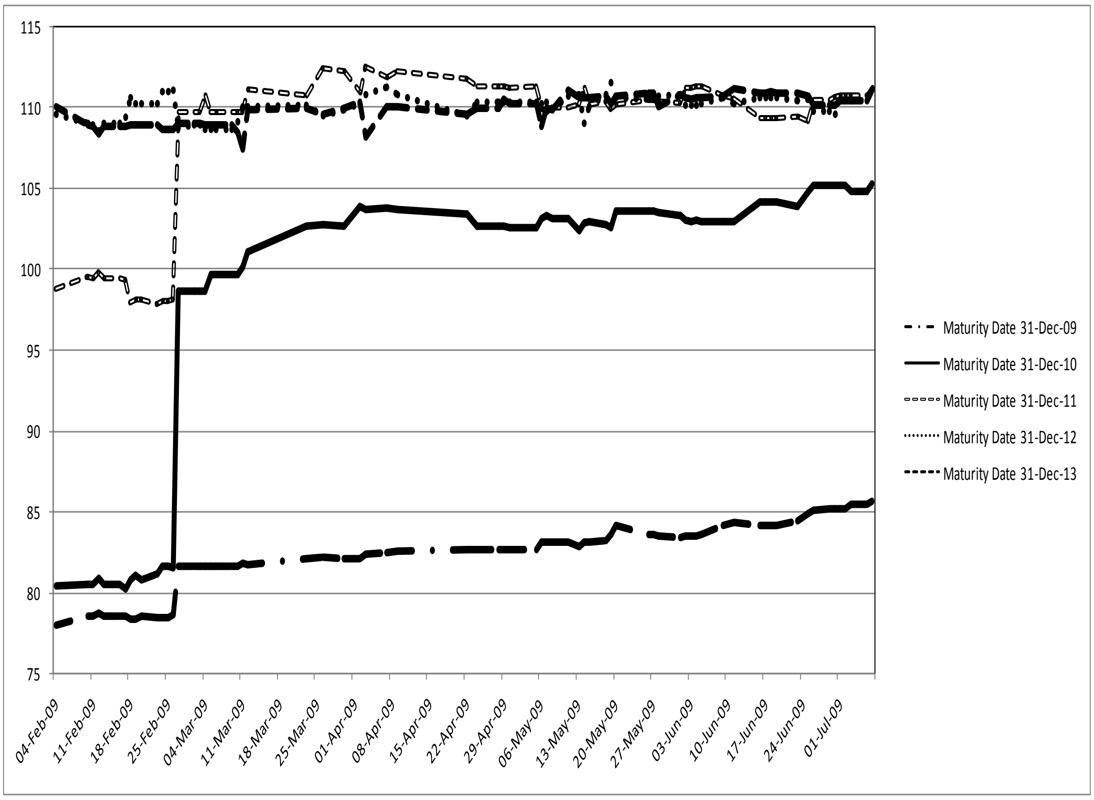
Notes: The plotted data is from 4 February until 7 July 2009 for the five maturity dates fixed in the market calendar, for the period of study. The total return swap rates are given as a fixed rate and not as a spread over Libor. A negative total return swap rate implies that the underlying commercial property market will depreciate over the period to the horizon indicated by the maturity of the contract.

Figure 2 Eurex Futures Prices



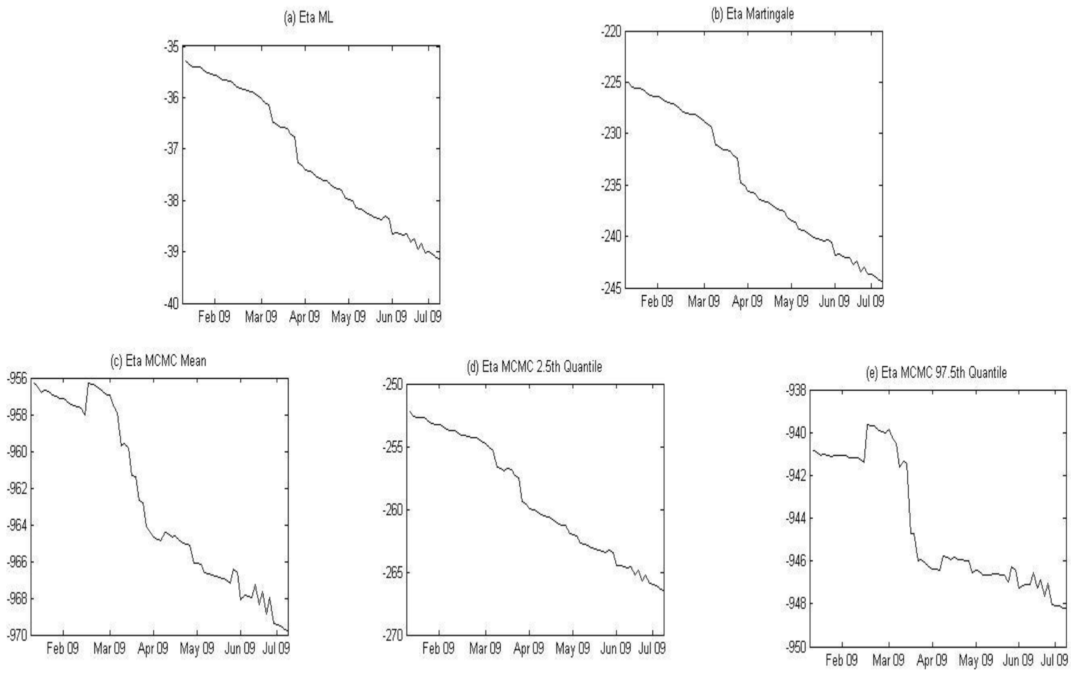
Notes: The plotted data is from 4 February until 7 July 2009 for the five maturity dates fixed in the market calendar. Futures prices are given on a total return basis so a futures price of 110 for December 2012 implies that the market expects a 10% appreciation of the commercial property in UK at this horizon.

Figure 3 The Fair Prices of Property Forwards



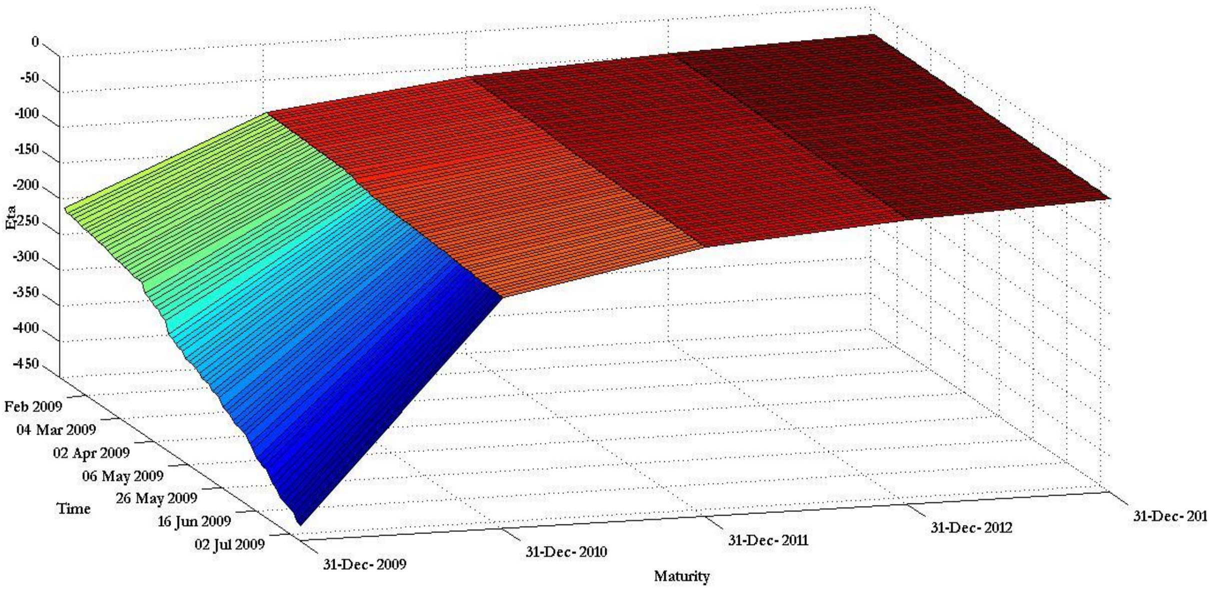
Notes: The plotted data is from 4 February until 7 July 2009 for the five maturity dates fixed in the market calendar, for the period of study. The fair property forward prices are reversed engineered from the corresponding portfolio of total return swaps.

Figure 4 Market Price of Risk



Notes: The figure plots the market price of risk for IPD commercial property index under the univariate OU model, with underlying model parameters estimated using maximum likelihood (ML) in panel (a), martingale estimation in panel (b), and Markov Chain Monte Carlo (MCMC) in panels (c)-(e), with posterior mean (panel c), posterior 2.5th quantile (panel d) and posterior 97.5th quantile (panel e) parameter estimates, using monthly log prices on the IPD index, observed over the period between December 1986 and January 2009. The market price of risk is fitted by minimising the mean squared error function, which measures the mean squared distance between the market and model futures prices, for each of the 71 days in the sample and across the five futures maturities. The IPD futures data is from 4 February until 7 July 2009, for the five maturities, namely December 2009, December 2010, December 2011, December 2012 and December 2013.

Figure 5 Market Price of Risk



Notes: The figure plots the market price of risk surface for IPD commercial property index and for the univariate OU model, with underlying parameters estimated using maximum likelihood (ML), using monthly log prices on the IPD index, observed over the period between December 1986 and January 2009. The market price of risk is fitted by minimising the mean squared error function, which measures the mean squared distance between the market and model futures prices, for each of the 71 days in the sample and each of the five futures maturities. The market price of risk is thus assumed vary both across time and across futures contracts maturities. The IPD futures data is from 4 February until 7 July 2009 for the five maturities, namely December 2009, December 2010, December 2011, December 2012 and December 2013.

Table 1. Descriptive Statistics for total return swap rates, Eurex futures prices and the forward-futures differences.

Maturity Dates					
	31-Dec-09	*31-Dec-10*	*31-Dec-11*	*31-Dec-12*	*31-Dec-13*
Total Return Swaps					
Mean	-0.178	-0.0971	-0.0389	-0.0056	0.0138
standard deviation	0.0217	0.0548	0.0521	0.0401	0.0336
Skewness	-0.6925	-1.3581	-1.4446	-1.4177	-1.3889
Kurtosis	-0.5131	0.1383	0.2633	0.2135	0.1585
Futures prices					
Mean	81.1982	94.275	106.1732	111.7035	113.5915
standard deviation	2.6558	10.6358	5.1369	1.2792	3.7762
Skewness	-0.0992	-0.4889	-0.5411	0.164	-0.2771
Kurtosis	-1.5313	-1.7844	-1.6426	-0.6302	-1.6544
Forward – Futures Differences					
Mean	1.1018	4.2432	2.0765	-1.599	-3.7103
Standard Deviation	1.3846	7.2574	3.7344	1.0573	3.0863
Kurtosis	0.3093	0.9912	0.8006	0.0824	-1.6454
Skewness	1.2702	1.6562	1.5582	0.3179	0.2274

Notes: The descriptive statistics are of the total return swap rates, futures prices and forward-futures differences on IPD UK All Property index. Daily mid prices are used for calculation for the period 4 February 2009 to 7 July 2009 for the five market calendar maturities, namely December 2009, December 2010, December 2011, December 2012 and December 2013. The forward prices used here are the synthetic fair prices derived from total return swap rates, synchronous with the futures prices.

Table 2 Models

Name	Model	Estimation method
univ_ML	univariate time-trending OU:	exact maximum likelihood
univ_mart	$dq(t) = -\gamma q(t)dt + \sigma dW(t)$	martingale estimation
univ_MCMC_mean		Markov Chain Monte Carlo (MCMC), mean parameter estimates
univ_MCMC_2.5q		MCMC, using the 2.5th quantile of the distribution of estimates
univ_MCMC_97.5q		MCMC, using the 97.5th quantile of the distribution of estimates
bivar_ML	bivariate time-trending OU:	exact maximum likelihood
biv_MCMC_mean	$dq(t) = \left[-\gamma q(t) + \lambda r(t)\right]dt + \sigma dW_S(t)$	MCMC, mean parameter estimates
biv_MCMC_2.5q	$dr(t) = \delta\left(\mu_r - r(t)\right)dt + \sigma_r dW_r(t)$	MCMC, using the 2.5th quantile of the distribution of estimates
biv_MCMC_97.5q	$dW_S(t)dW_r(t) = \varrho dt$	MCMC, using the 97.5th quantile of the distribution of estimates

Notes: $q(t)$ is the de-trended log price process for the underlying $S(t)$, the IPD UK commercial property price index: $p(t) = ln\left(S(t)\right)$; $p(t) = q(t) + \left(\mu_0 + \mu t\right)$. $r(t)$ denotes the short interest rate. For both models, the futures price is obtained as: $f_m(0,T) = E_{0,m}^{Q}\left(S(T)\right)$ where m = *univ* or *bivar*, for the two models, respectively, Q is the martingale pricing measure, and T is the futures maturity time.

Table 3 Parameter Estimates

Parameter \Model	univ_ML	univ_mart	univ_MCMC_mean	univ_MCMC_2.5q	univ_MCMC_97.5q
μ_0	-	0.0338	4.221	1.35	4.463
μ	0.4117	-	0.3121	0.2657	0.7051
γ	0.0591	0.04427	1.559	0.1892	1.979
σ	0.0384	0.01	0.0178	0.0131	0.0238
η (average)	-37.2076	-234.6772	-962.8572	-259.2317	-944.3792

Notes: Parameter estimates for the univariate OU model, obtained using likelihood (ML) (column 2), martingale estimation (column 3), and Markov Chain Monte Carlo (MCMC) (columns 4-6), with mean (column 4), 2.5th quantile (column 5) and 97.5th quantile (column 6) parameter estimates. The data used estimating the model parameters with these alternative estimation methods contains monthly log prices on the IPD index, observed over the period between December 1986 and January 2009, and totalling 266 historical observations.

Table 4 ADF Test for the Forward and Future Prices

Maturity Date	Forward		Futures	
	Level	First Differences	Level	First Differences
31 December 2009	-1.3453	-8.3646***	-0.8098	-5.1662***
31 December 2010	-1.9185	-8.4575***	-1.3019	-8.3168***
31 December 2011	-2.0106	-8.6799***	-1.3278	-4.3724***
31 December 2012	-3.4368**	-5.4320***	-2.3581	-5.9891***
31 December 2013	-0.9618	-6.4863***	-1.2087	-12.1882***

Notes: Augmented Dickey-Fuller (ADF) test results for the UK IPD commercial property index forward and futures prices. The test is performed for both the data in levels as well as for the first differenced data. The optimum number of lags used in the ADF test equation is based on the Akaike Information Criterion (AIC). *, **, and *** denote significance at the 10%, 5% and 1% level respectively. The data is from 4 February until 7 July 2009 for the five maturity dates given in the first column.

Table 5 *F*-Test for Time Series Data

Maturity Date	*F*-test	Durbin-Watson Statistic
31 December 2009	83.5072***	1.9905
31 December 2010	49.4309***	2.0595
31 December 2011	23.8320***	2.0613
31 December 2012	31.1717***	2.2589
31 December 2013	158.2534***	2.7247

Notes: *F*-Test and Durbin Watson statistic results for the regression in the equation (16) for the property forward and futures data from 4 February until 7 July 2009 for the five maturity dates given in the first column. For the *F*-test, the null hypothesis is that the difference between the forward and futures prices is just noise (i.e. α_{0i} = 0 and β_{0i} =1). *, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Table 6 *T*- statistics for the Differences between Forward and Futures prices

	Maturity Dates				
	31-Dec-09	*31-Dec-10*	*31-Dec-11*	*31-Dec-12*	*31-Dec-13*
t-statistic	6.7060***	4.9265***	4.6852***	-12.741***	10.1291***

Note: The values of the *t*-test are computed for the differences between forward and futures prices on the UK IPD commercial property index, using data from 4 February until 7 July 2009 for the five maturity dates given in the second row. *, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Table 7 Panel Unit Root Tests

Method	Forward		Futures	
	Level	First Differences	Level	First Differences
Levin, Lin & Chu t*	-1.8292	-6.7737***	-0.7206	-4.0345***
Im, Pesaran & Shin W-stat	-1.1578	-10.4686***	0.3880	-8.3918***

Note: Results for the panel unit root tests of Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) for forward and futures price data on the UK IPD commercial property index, from 4 February until 7 July 2009 for five maturity months, namely December 2009, December 2010, December 2011, December 2012 and December 2013.*, **, and *** denote significance at the 10%, 5% and 1% level respectively

Table 8 Tests for determining the most suitable panel regression model

Test	Value
Redundant Fixed Effects Test	
Cross-section F	1.0925
Cross-section Chi-square	5.5181
Period F	2.2062***
Period Chi-square	154.1939***
Cross-Section/Period F	2.1450***
Cross-Section/Period Chi-square	157.7444***
Hansen Test	
Cross-section random	3.0341*
Period random	0.0003
Cross-section and period random	0.1690

Notes: The redundant fixed cross-section effects test has a panel regression with fixed time (period) effects only under the null. Both the *F* and the chi-square version of the test are reported. The redundant fixed time (period) effects has a panel regression with fixed cross-section effects only under the null. Both the *F* and the chi-square version of the test are reported. For the random effects test (i.e. Hansen test) the null hypothesis in this case is that the random effect is uncorrelated with the explanatory variables. The panel data is from 4 February until 7 July 2009 for five maturities, namely December 2009, December 2010, December 2011, December 2012 and December 2013.*, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Table 9 The *F*-test for Panel Data

Test Statistic	Value
F-statistic	237.3960***
Durbin Watson Statistic	2.1419

Notes: F-test and Durbin Watson statistic results for the regression in the equation (20) for the property forward and futures panel data from 4 February until 7 July 2009, using the cross-section random effects specification. For the *F*-test, the null hypothesis is that the difference between the forward and futures prices is just noise (i.e. $\alpha_0 = 0$ and $\beta_0 = 1$).*, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Table 10 Model Comparison – Time Series Data

Test\Model	univ_ML	univ_mart	univ_MCMC_mean	univ_MCMC_2.5q	univ_MCMC_97.5q
			1^{st} Maturity		
F-test	120757.2***	122000.3***	112810.4***	121757.8***	99747.38***
t-test for β	1.4369	1.607	-0.8494	1.5724	-1.9758*
R_squared	0.0291	0.0361	0.0103	0.0346	0.0535
			2^{nd} Maturity		
F-test	5946.6***	6108.451***	2705.565***	6043.822***	411.4135***
t-test for β	2.0079**	2.1145**	0.5759	2.0585**	0.5991
R_squared	0.0552	0.0608	0.0048	0.0579	0.0052
			3rd Maturity		
F-test	30397.9***	36595.58***	4200.665***	31643.03***	63.6275***
t-test for β *t*-	4.2573***	5.1767***	4.2845***	4.3118***	15.7831***
R_squared	0.208	0.2797	0.2101	0.2123	0.7831
			4th Maturity		
F-test	216988.5***	360635.3***	2426.505***	214077.2***	2354.060***
t-test for β	2.2969**	4.5343***	7.6797***	0.9275	8.1801***
R_squared	0.071	0.2296	0.4608	0.0123	0.4923
			5th Maturity		
F-test	21886.5***	13565.37***	24054.33***	14288.12***	2822.825***
t-test for β	31.9967***	39.9288***	17.6248***	25.8627***	15.4195***
R_squared	0.9369	0.9585	0.8182	0.9065	0.7751

Notes: For the regression in (23), we report the value of the *F*-statistics for the null $\alpha = 0$ and $\beta = 1$, the value of the *t*-statistic for the beta coefficient and the R-squared of the regression, where the RHS, independent variable is based on the univariate OU model with parameters estimated using maximum likelihood (ML) in column 2, martingale estimation in column 3, and Markov Chain Monte Carlo (MCMC) in columns 4-6, with mean (column 4), 2.5^{th} quantile (column 5) and 97.5^{th} quantile (column 6) parameter estimates. The data used for fitting the model parameters with these alternative estimation methods contains monthly log prices on the IPD index, observed over the period between December 1986 and January 2009, and totalling 266 historical observations. The data used for the testing reported in this table is from 4 February until 7 July 2009 for five maturities, namely December 2009, December 2010, December 2011, December 2012 and December 2013 and the tests are performed for all five time series, corresponding to the five maturities. *, **, and *** denote significance at the 10%, 5% and 1% level respectively.

Table 11 Model Comparison – Panel Data

Test\Model	univ_ML	univ_mart	univ_MCMC_mean	univ_MCMC_2.5q	univ_MCMC_97.5q
F-test	21886.5***	66887.84***	26358.31***	64035.43***	10284.18***
t-test for β	31.9967***	9.9151***	11.3375***	10.6135***	6.8171***
R_squared	0.9369	0.2178	0.2669	0.2419	0.1163

Notes: For the regression in (23), we report the value of the *F*-statistics for the null $\alpha = 0$ and $\beta = 1$, the value of the *t*-statistic for the beta coefficient and the R-squared of the regression, where the RHS, independent variable is based on the univariate OU model with parameters estimated using maximum likelihood (ML) in column 2, martingale estimation in column 3, and Markov Chain Monte Carlo (MCMC) in columns 4-6, with mean (column 4), 2.5th quantile (column 5) and 97.5th quantile (column 6) parameter estimates. The data used for fitting the model parameters with these alternative estimation methods contains monthly log prices on the IPD index, observed over the period between December 1986 and January 2009, and totalling 266 historical observations. The panel data used for the testing reported in this table is from 4 February until 7 July 2009, for five maturities, namely December 2009, December 2010, December 2011, December 2012 and December 2013 *, **, and *** denote significance at the 10%, 5% and 1% level respectively.