How Does A Firm's Default Risk Affect Its Expected Equity Return?

Kevin Aretz*

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Keywords Default risk, asset pricing, asset volatility

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Abstract

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1 Introduction

Many recent empirical studies, to be reviewed later, find a negative, flat or even hump-shaped relation between proxies for a firm's expected equity return and for its default risk. These results are so much at odds with the notion that high default risk stocks are risky to hold for investors and that they should therefore offer a risk premium that many academics interpret them as anomalous.[1] To explain the apparently anomalous results, more and more complicated stories have been developed, often based on investor irrationality (Avramov et al. (2009)) or market imperfections (Garlappi et al. (2008), George and Hwang (2010)).

In this article, I illustrate that, even in a *plain vanilla* asset pricing setup deliberately abstracting from irrationality and market imperfections, the expected equity return can decrease as default risk increases. The intuition is that, although a decrease in default risk always increases the chance of a positive equity payoff, it can sometimes shift equity payoff probability mass from states in which additional wealth is expected to be very desirable (recessions) to states in which it is expected to be less desirable (expansions), thereby increasing equity risk and thus the expected equity return. In the theoretical model, an increase in default risk can only decrease the expected equity return if default risk is high ($> 50\%$) and if it is driven by changing asset volatility, one of the three default risk drivers. The possibly non-positive relation is not just a theoretical curiosity. Cross-sectional asset pricing tests confirm that, while equity returns increase with default risk in other cases, this relation can become significantly negative if default risk is high and attributable to asset volatility.

The theoretical analysis is based on the two period model of Rubinstein (1976), which is itself a discrete-time equilibrium version of the Black and Scholes (1973) model. Firms are assumed to be financed with equity and debt claims, and both types of claims are held by one representative agent with a time-additive power utility function. The probability that a firm cannot honor its debt obligations, that is, its default risk, is a highly non-linear function of expected profitability (the expected log asset payoff), financial leverage (log debt) and business risk (the volatility of the log asset payoff).[2] Assuming a positive correlation between the log asset payoff and

[1] For example, Campbell et al. (2008) write that "the idea [of a positive relation between default risk and the expected equity return] has a certain plausibility". Anginer and Yildizhan (2010) even state that "financial theory suggests a positive relation", and George and Hwang (2010) claim that the former results are "puzzles under frictionless capital markets".

[2] While technically incorrect, I shall also often refer to these factors, which are the default risk drivers in my model, as the expected asset payoff, the debt repayment and asset volatility, respectively.

log consumption, my theoretical findings are as follows. First, if asset volatility stays constant, then changes in default risk due to changes in expected profitability and the debt level are always positively related to changes in the expected equity return. Second, if asset volatility varies, then changes in default risk due to varying asset volatility are also always positively related to changes in the expected equity return if default risk is below fifty percent. However, if default risk exceeds fifty percent, such induced changes can produce both equivalently- and oppositely-signed changes in the expected equity return, with the negative (positive) relation prevailing for levels of default risk closer to fifty percent (one-hundred percent).

Motivated by the prior analysis, I take the testable implications derived from the model to the data. Using an estimate obtained from the Merton (1974) model as my measure of default risk,[3] I show that the average equity returns of quintile portfolios sorted on default risk fail to increase significantly across these portfolios. However, when a second-order Taylor series expansion is used to decompose the default risk estimate into one component attributable to expected profitability and debt effects and another attributable to asset volatility effects, the cross-section of average equity returns increases significantly with the lagged first component. It also increases significantly with the lagged component capturing variations in default risk attributable to asset volatility, however, only until default risk reaches fifty percent. Starting from fifty percent, the relation becomes significantly negative up to the threshold level predicted by the theoretical model and then turns insignificant above the threshold level. In total, this is a striking collection of successes for the theoretical model.

My initial tests may be criticized for leaning too heavily on an over-stylized theoretical model. Hence, I also run tests on the more general insight of the model, namely, that the non-positive relation between default risk and equity returns may be attributable to firms with *relatively* high default risk (but not necessarily above fifty percent) driven by changing asset volatility. Once again, the data strongly support this prediction.

My empirical findings are robust to the inclusion of other variables known to affect equity returns, such as size, book-to-market, momentum, book leverage and stock price illiquidity. Controlling for these other variables is important, first, because they likely reflect cross-sectional variations in systematic risk captured in the model by the correlation between the log asset payoff and log consumption. Second, they should also take account of

[3] Merton (1974) default risk is a sufficient statistic for capturing default risk in the theoretical model.

endogeneity problems arising from the fact that the theoretical model abstracts from several important real-world issues (e.g., capital structure choice). Moreover, the control variables help me to show that my main conclusions survive controlling for the market imperfection-based explanations of the default risk premium puzzle advocated by Garlappi et al. (2008), George and Hwang (2010), and others. While I do not dispute that market imperfection-based explanations help to explain the puzzle, the effect unearthed in this article is distinct from those found in prior articles, and it remains important even when controlling for these other effects.

This paper contributes to a growing literature examining the relation between default risk and average equity returns. Many earlier studies offer indirect evidence of a positive relation, for example, by showing that exposure to aggregate default risk measures can explain the cross-section of US equity returns (Chan et al. (1985), Chen et al. (1986), Vassalou and Xing (2004)). If a high exposure indicates high firm-specific default risk, this evidence may suggest that default risk is a priced factor. Other studies show that market size is among the best predictors for bankruptcy, possibly implying that SMB (a size spread portfolio) attracts a significant risk premium to compensate investors for coping with high default risk (Chan and Chen (1991), Shumway (2001)). However, more recent studies question whether there is a positive relation. Using portfolio formation exercises, they show that the relation between the two variables can be flat, negative or even hump shaped.[4] Measures for default risk used in these studies include forecasts from a dynamic logit model (Campbell et al. (2008)), credit ratings (Avramov et al. (2009)), default risk implied by structural models (Zhang (2011)), credit spreads (Anginer and Yildizhan (2010)) and accounting-based bankruptcy scores (Dichev (1998), Griffin and Lemmon (2002)).

Recent theoretical studies try to explain the apparently anomalous findings using market imperfections. For example, Garlappi et al. (2008) argue that, if stockholders have high bargaining power, they will strategically default to extract economic rents from creditors. Using a simulation exercise based on the Fan and Sundaresan (2000) model, Garlappi et al. (2008) show that increasing default risk can in such cases decrease the expected equity return. Using a more tractable framework, in which firms endogenously determine their capital structure, Garlappi and Yan (2011) offer further evidence that high default risk translates into low market betas if stockholders' bargaining power is high. George and Hwang (2010) assume that firms trade off the tax shields of debt

[4]An interesting exception is Chava and Purnanandam (2010), who analyze the relation between expected equity returns derived from valuation models and default risk.

against the deadweight costs of distress to find their optimal capital structures. In this model, high deadweight costs of distress translate into high systematic risk, and this high systematic risk is only slightly reduced by managers optimally choosing low leverage ratios, implying a negative relation between default risk and expected equity returns.[5] Finally, O'Doherty (2010) shows that high uncertainty about asset values can decrease market betas, thereby also producing an inverse relation between default risk and expected equity returns.

My main contribution is to show that such additional model complexity is unnecessary to explain the default risk premium puzzle, and that the evidence is consistent with the notion that the puzzle is to some extent driven by high default risk firms and cross-sectional variations in default risk due to asset volatility.

The article is organized as follows. Section 2 develops the theoretical model. Section 3 describes the data and the empirical results. Section 4 concludes. Proofs are in the Appendix.

2 The model economy

2.1 Framework

Consider a frictionless two-period market economy with a representative agent, as in Rubinstein (1976).[6] The preferences of the representative agent are described by a time-additive power utility function:

$$u_0(z_0) + u_1(z_1) = \frac{1}{1-B} z_0^{1-B} + \rho \frac{1}{1-B} z_1^{1-B}, \tag{1}$$

[5]Johnson et al. (2011) argue that the theoretical results of George and Hwang (2010) are incorrect, and that the correct theoretical relation between default risk and the expected equity return induced through cross-sectional variations in the deadweight costs of distress is positive. However, they also show that the negative relation can be restored if allowing for cross-sectional variations in asset volatility instead of the deadweight costs of distress. These results are interesting, because, similar to those derived in this article, they point to asset volatility being the culprit behind the puzzle. However, the mechanism employed in their article differs from that used in mine. In theirs, it is optimal capital structure choices; in mine, it is cross-sectional variations in systematic risk unrelated to market imperfections. Notwithstanding these insights, the results of Johnson et al. (2011) do not invalidate the empirical tests of George and Hwang (2010). As a result, I follow George and Hwang (2010) in using the book leverage ratio to control for the optimal level of debt chosen by a firm (either because of the deadweight costs of distress or asset volatility).

[6]This model is a discrete-time, equilibrium version of the one proposed by Black and Scholes (1973). As a result, the two models produce identical closed-form solutions for the market value of equity, but only the one of Rubinstein (1976) yields a closed-form solution for the market value of assets. The goal of this paper is to find the partial derivatives of the expected equity return with respect to those firm characteristics related to default risk. As these partial derivatives depend in turn on the partial derivatives of the asset value with respect to the firm characteristics, I have to rely on the model of Rubinstein (1976) to derive any testable implications.

where B is relative risk aversion, and ρ is the time-preference parameter. Let p_A denote the market value of a firm's assets at time 0 and $\tilde{x}$ its random payoff at time 1, where a tilde is used to indicate a random variable. Aggregate consumption at time 0 and 1 is denoted by C_0 and $\tilde{C}_1$, respectively. Denoting the natural logarithm of the asset payoff by $\tilde{z} \equiv \ln(\tilde{x})$ and the natural logarithm of the marginal utility of consumption by $\tilde{y} \equiv \ln[(\tilde{C}_1/C_0)^{-B}]$, $\tilde{z}$ and $\tilde{y}$ are jointly normal with expectation vector $[\mu_x, \mu_C]' = [\mu_x, B(\ln C_0 - \hat{\mu}_C)]'$, where $\hat{\mu}_C$ is the expectation of $\ln(\tilde{C}_1)$. The covariance matrix of $\tilde{z}$ and $\tilde{y}$ can be written as follows:

$$\begin{bmatrix} \sigma_x^2 & \sigma_{x,C} \\ \sigma_{x,C} & \sigma_C^2 \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & -B\kappa\sigma_x\hat{\sigma}_C \\ -B\kappa\sigma_x\hat{\sigma}_C & B^2\hat{\sigma}_C^2 \end{bmatrix}, \tag{2}$$

where κ is the correlation between $\ln(\tilde{x})$ and $\ln(\tilde{C}_1)$ and $\hat{\sigma}_C^2$ is the variance of $\ln(\tilde{C}_1)$.

A firm may be financed by equity and debt. Denote the face value of the debt payable at time t by k. Then the equity payoff at time 1, $\tilde{x}_E$, is given by $\max[\tilde{x} - k, 0]$, and the default probability π at time 0 is given by:

$$\pi = \text{Prob}(\tilde{x} < k) = \text{Prob}\left[\frac{\tilde{z} - \hat{\mu}_x}{\sigma_x} < \frac{\ln k - \hat{\mu}_x}{\sigma_x}\right] = N\left[\frac{\ln k - \mu_x}{\sigma_x}\right], \tag{3}$$

where N[.] denotes the cumulative standard normal density function. Equation (3) shows that the default probability is a non-linear function of the expected asset payoff (μ_x), the debt repayment (k) and asset volatility (σ_x). We refer to μ_x, k and σ_x as default risk drivers. The partial derivatives of π with respect to these drivers are:

$$\frac{\partial \pi}{\partial \mu_x} = -\frac{1}{\sigma_x} n\left[\frac{\ln k - \mu_x}{\sigma_x}\right] < 0, \tag{4}$$

$$\frac{\partial \pi}{\partial k} = \frac{1}{k\sigma_x} n\left[\frac{\ln k - \mu_x}{\sigma_x}\right] > 0, \tag{5}$$

$$\frac{\partial \pi}{\partial \sigma_x} = -\frac{\ln k - \mu_x}{\sigma_x^2} n\left[\frac{\ln k - \mu_x}{\sigma_x}\right], \tag{6}$$

where n is the density function of the standard normal distribution. A higher expected asset payoff or a lower debt repayment shifts asset payoff probability from default states (i.e., states in which $\tilde{x} < k$) to surviving states, implying lower default risk. In contrast, asset volatility is non-monotonically related to default risk. If $(\ln k -$

$\mu_x) < 0$, implying $k < e^{\mu_x} = \text{median}(\tilde{x})$ (i.e., default risk is below fifty percent), then default risk increases with asset volatility. However, if $(\ln k - \mu_x) > 0$, then default risk falls as asset volatility rises.

The expected equity payoff is given by:

$$E[\tilde{x}_E] = E[\max(\tilde{x} - k, 0)] = e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] - kN\left[\frac{\mu_x - \ln k}{\sigma_x}\right]. \tag{7}$$

From the usual first-order conditions (e.g., Lucas (1978), Brock (1982)), the equity value p_E is given by:

$$p_E = \rho E\left[\max(\tilde{x} - k, 0)\left(\frac{\tilde{C}_1}{C_0}\right)^{-B}\right]. \tag{8}$$

Rubinstein (1976) shows that this expectation equals:

$$p_E = \rho e^{\mu_C + \frac{1}{2}\sigma_C^2}\left[e^{\mu_x + \frac{1}{2}(\sigma_x^2 + 2\sigma_{x,C})} N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] - kN\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\right]. \tag{9}$$

The expected equity return $E[\tilde{R}_E]$ is obtained by dividing Equation (8) by p_E and using the definition of covariance:

$$1 = E[\tilde{R}_E \rho(\tilde{C}_1/C_0)^{-B}] = E[\tilde{R}_E]E[\rho(\tilde{C}_1/C_0)^{-B}] + \text{cov}(\tilde{R}_E, \rho(\tilde{C}_1/C_0)^{-B}), \tag{10}$$

Rearranging:

$$E[\tilde{R}_E] = \frac{1 - \text{cov}(\tilde{R}_E, \rho(\tilde{C}_1/C_0)^{-B})}{E[\rho(\tilde{C}_1/C_0)^{-B}]}. \tag{11}$$

Equation (11) illustrates that cross-sectional variations in the expected equity return are completely captured by variations in the covariance between the equity return and the stochastic discount factor. Assuming that the representative agent is risk-averse ($B > 0$) and that the asset payoff and consumption are positively correlated ($\kappa > 0$), the covariance is negative, and a lower (i.e., more negative) covariance produces a higher expected equity return. As a result, changing default risk can only affect the expected equity return through the covariance between the equity return and the stochastic discount factor.

We can rewrite the former covariance as follows:

$$\operatorname{cov}(\tilde{R}_E, \rho(\tilde{C}_1/C_0)^{-B}) = \frac{1}{p_E} E[\tilde{x}_E \cdot (E[\rho(\tilde{C}_1/C_0)^{-B}|\tilde{x}_E] - E[\rho(\tilde{C}_1/C_0)^{-B}])]. \tag{12}$$

Equation (12) allows us to gain an intuitive understanding of the relation between default risk and the expected equity return. For example, consider a decrease in default risk reducing the probability of $\tilde{x}_E$ being equal to zero and increasing by an equal amount the probability of some positive $\tilde{x}_E$. By Equation (8), such a decrease would imply a higher equity price and therefore also a less negative covariance in Equation (12). However, it would also shift probability from states in which the product of $\tilde{x}_E$ and $E[\rho(\tilde{C}_1/C_0)^{-B}|\tilde{x}_E] - E[\rho(\tilde{C}_1/C_0)^{-B}]$ is zero to states in which it is non-zero. It is easy to show that, if $\kappa > 0$, then $E[\rho(\tilde{C}_1/C_0)^{-B}|\tilde{x}_E]$ decreases with $\tilde{x}_E$. As a result, if the decrease in default risk increases the probability of some low $\tilde{x}_E$, then the expectation in Equation (12) either slightly increases or slightly decreases. If the decrease in default risk instead increases the probability of some very large $\tilde{x}_E$, then the expectation must decrease. In certain cases, the decrease in the expectation may be sufficiently large to offset the increase in the equity price, thereby decreasing the covariance between the equity return and the stochastic discount factor and thus increasing the expected equity return. In the next subsection, I analyze whether such situations can occur in the Rubinstein (1976) model.

2.2 *Main results*

Plugging the closed-form solutions for the expected equity payoff and the equity value into the definition of the expected equity return, I can write the expected equity return as:

$$E[\tilde{R}_E] = \frac{E[\tilde{x}_E]}{p_E} = \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] - kN\left[\frac{\mu_x - \ln k}{\sigma_x}\right]}{\rho e^{\mu_C + \frac{1}{2}\sigma_C^2}\left[e^{\mu_x + \frac{1}{2}(\sigma_x^2 + 2\sigma_{x,C})} N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] - kN\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\right]}. \tag{13}$$

Assume first that variations in default risk are driven by variations in the expected asset payoff and the debt level, that is, that asset volatility is constant. In this case, there is a monotonic positive relation between default risk and the expected equity return, as shown by the following proposition:

Proposition 1: *If the correlation between asset payoff and consumption is positive ($\kappa > 0$) and asset volatility is a positive constant, then:*

(i) *both default risk (π) and the expected equity return ($E[\tilde{R}_E]$) are decreasing in the expected asset payoff (μ_x). As a result, variations in default risk induced by changes in the expected asset payoff are positively related to variations in the expected equity return.*

(ii) *both default risk (π) and the expected equity return ($E[\tilde{R}_E]$) are increasing in the debt repayment (k). As a result, variations in default risk induced by changes in the debt repayment are positively related to variations in the expected equity return.*

(iii) *both default risk (π) and the expected equity return ($E[\tilde{R}_E]$) are increasing in the difference between the natural logarithm of the debt repayment and the expected asset payoff ($\ln k - \mu_x$). As a result, variations in default risk induced by changes in both the expected asset payoff and the debt repayment are positively related to variations in the expected equity return.*

Proof: *See Appendix A.*

Proposition 1 indicates that a decrease in default risk due to changes in the expected asset payoff, the debt level or both always reduces equity risk and hence the expected equity return. Intuitively, this makes sense, as variations in these two factors cannot produce great shifts of equity payoff probability from low to high equity payoffs, and decreases in default risk attributable to them thus always increase the covariance between the equity return and the stochastic discount factor (recall Section 2.1).

Next assume that variations in default risk are attributable to variations in asset volatility. In this case, we no longer find a monotonic relation between default risk and the expected equity return, invalidating the widely-held belief that frictionless capital markets must necessarily produce a positive relation (see footnote 1).

Proposition 2: *Define the following function $F(\sigma_x)$:*

$$F(\sigma_x) = -H'[c^*] + H'[\alpha - \alpha^* + \beta\kappa] + \alpha H[\alpha - \alpha^* + \beta\kappa]\left[1 - H'[c^*]\right],$$

with α, α^*, β, c^*, the hazard function $H[x]$ and its first-order derivative $H'[x]$ defined in the Appendix. If the correlation between asset payoff and consumption is positive ($\kappa > 0$), then:

(i) *if default risk is below fifty percent, both default risk (π) and the expected equity return $(E[\tilde{R}_E])$ increase with asset volatility (σ_x). As a result, variations in default risk induced by changing asset volatility are positively related to variations in the expected equity return.*

(ii) *if default risk is between fifty percent and γ percent, where $\gamma > 50$ is the level of default risk consistent with the choice of asset volatility satisfying $F(\sigma_x) = 0$, default risk (π) decreases with asset volatility (σ_x), while the expected equity return $(E[\tilde{R}_E])$ increases with it. As a result, variations in default risk induced by changing asset volatility are negatively related to variations in the expected equity return.*

(iii) *if default risk is above γ percent, where $\gamma > 50$ is the level of default risk consistent with the choice of asset volatility satisfying $F(\sigma_x) = 0$, both default risk (π) and the expected equity return $(E[\tilde{R}_E])$ decrease with asset volatility (σ_x). As a result, variations in default risk induced by changing asset volatility are positively related to variations in the expected equity return.*[7]

Proof: *See Appendix A.*

Figure 1 helps to understand the intuition behind the possibly negative relation between changes in default risk due to variations in asset volatility and changes in the expected equity return.[8] The figure shows the density function of the asset payoff (left axis) of a highly distressed firm for an either low (0.40) or high (0.80) value

[7] My theoretical results do not allow me to formally rule out that the sign of the relation between changes in default risk driven by variations in asset volatility and changes in the expected equity return reverses more than once at levels of default risk above fifty percent. However, a close inspection of $F(\sigma_x)$, which determines the sign of the relation, suggests that this is unlikely. In the later empirical tests, I have also never found more than one value for σ_x that set $F(\sigma_x) = 0$. For simplicity, I have therefore stated the proposition as shown above. More correctly, it should state that at levels of default risk above fifty percent the relation is positive (negative) if and only if $F(\sigma_x) > 0$ ($F(\sigma_x) < 0$).

[8] In the figure, I assume that risk aversion (B) is one, the time preference parameter (ρ) is 0.95, the expectation ($\hat{\mu}_C$) and standard deviation ($\hat{\sigma}_C$) of log consumption are 0.15 and 0.10, respectively, and current consumption (C_0) is 1.10. The correlation between the log asset payoff and log consumption (κ) is 0.50. The expected asset payoff (μ_x) and the debt level (k) are 0.15 and 1.25, respectively, while asset volatility is either set to 0.40 (low sigma) or 0.80 (high sigma). As a result, expected consumption growth is around 5%, the expected net equity return is 19.0% (low sigma) or 19.5% (high sigma), default risk is 57.3% (low sigma) and 53.6% (high sigma), and the unconditional expectation of the stochastic discount factor ($E[\rho(\tilde{C}_1/C_0)^{-B}]$) is equal to 0.9039.

of asset volatility. Increasing asset volatility from 0.40 to 0.80 vividly shifts asset payoff probability mass from default states ($\tilde{x} < 1.25$) to surviving states, thereby decreasing default risk from 57.3% to 53.6%. However, it also strongly decreases the probability of low equity payoffs with a relatively high expected marginal utility (right axis), while increasing that of high equity payoffs with a relatively low expected marginal utility. The shift in equity payoff probability mass decreases the covariance between the equity *payoff* and the stochastic discount factor from -0.013 to -0.030. As the equity value increases at a lower rate (from 0.171 to 0.534) than the rate at which the expectation decreases, the covariance between the equity *return* and the stochastic discount factor drops from -0.076 to -0.080, causing the expected equity return to increase from 19.0% to 19.5%.

Intuitively, Proposition 2 does also make sense. In contrast to expected profitability (μ_x) and leverage (k), an increase in asset volatility pushes down the center of the asset payoff density, thereby inflating its tails. When the debt level exceeds the median asset payoff (i.e., when default risk exceeds fifty percent), limited liability implies that the fatter left tail is of no concern to stockholders. However, the shift of asset payoff probability mass from the center to the right tail concerns stockholders, as it increases the chance of receiving equity payoffs in states with low marginal utility (expansions) at the cost of receiving equity payoffs in states with high marginal utility (recessions). As of the three default risk drivers only asset volatility seems capable of shifting large amounts of asset payoff probability mass from desirable to undesirable states, it is the only default risk driver in the model that can produce a negative relation between default risk and the expected equity return.

A crucial ingredient in Proposition 2 is γ, the level of default risk at which the sign of the relation between default risk attributable to asset volatility and the expected equity return reverses back to positive. As it is difficult to gain any analytical insights into γ, I offer some numerical results in Table 1. The value of γ depends on the default risk drivers and the product of the correlation between the asset payoff and consumption, the risk aversion parameter and consumption volatility ($B\hat{\sigma}_C\kappa$). In the table, I thus report the value of asset volatility (σ_x, in bold) that fulfills $F(\sigma_x) = 0$ given specific values for log debt minus the expected asset payoff ($\ln k - \mu_x$) and $B\hat{\sigma}_C\kappa$. For each σ_x value, it also shows the corresponding level of default risk, γ. Varying $\ln k - \mu_x$ from 0.01 to 1.20 and $B\hat{\sigma}_C\kappa$ from 0.01 to 0.10, the table indicates that γ is almost entirely determined by the default risk drivers, that is, variations in $B\hat{\sigma}_C\kappa$ only produce minor variations in γ. Also, a higher value of $\ln k - \mu_x$ requires a higher value of

σ_x to ensure that $F(\sigma_x) = 0$, implying that the higher $\ln k - \mu_x$ the higher the level that σ_x needs to reach before further increases in σ decrease default risk, but increase the expected equity return.

2.3 Comparative statistics

In Figure 2, I offer comparative statistics.[9] In each panel, I plot the expected equity return on the y-axis against default risk driven by one of the three default risk drivers on the x-axis assuming different combinations of values for the risk aversion (B) and the time preference (ρ) parameters. Panels A and B consider variations in default risk due to expected profitability (μ_x) and the debt level (k), respectively, whereas Panels C and D consider variations due to business risk (σ_x). If the debt level lies below (above) the median asset payoff, then variations in asset volatility can only produce variations in default risk between zero and fifty (fifty and one-hundred) percent. To separately analyze these two cases, the debt level is set to one in Panels A to C, but to 1.25 in Panel D.

A comparison of Panels A and B suggests that μ_x and k play no independent roles in determining default risk and the expected equity return, that is, keeping σ_x constant, all combinations of values for μ_x and k producing the same value for $\ln k - \mu_x$ yield the same expected equity return. Consistent with Proposition 1, there is a strictly positive relation between default risk driven by μ_x and k and the expected equity return. This relation is highly non-linear, that is, it is concave at low levels of default risk, but becomes convex at higher ones. Consistent with Proposition 2, Panel C shows that there is again a highly non-linear, but strictly positive relation between default risk due to σ_x and the expected equity return if default risk falls short of fifty percent. However, if default risk exceeds fifty percent, Panel D indicates that this relation becomes non-monotonic, that is, the expected equity return decreases with default risk due to σ_x at levels of default risk closer to fifty percent, but increases with it at higher levels. Interestingly, the comparative statistics vividly show that the initial decrease in the expected equity return is far more pronounced than the later increase, which seems almost flat. I have found this to be the case not only for the combination of model values used in the figure, but also for other ones.

[9]Unless explicitly stated, I assume the same values for the model parameters as in Figure 1.

3 Empirical tests

3.1 Hypotheses

My theoretical results can be used to derive the following testable implications:

H1: Controlling for other factors, an increase in default risk attributable to a change in the expected asset payoff, the debt level or both increases the expected equity return.

H2: Controlling for other factors, an increase in default risk attributable to a change in asset volatility increases the expected equity return at levels of default risk below fifty percent, decreases it at levels of default risk between fifty and γ percent, and increases it again at levels of default risk above γ percent.

To test these hypotheses, I require estimates of default risk, the expected asset payoff, the debt level, asset volatility, the level of default risk above fifty percent at which the sign of the relation between default risk due to asset volatility and the expected equity return becomes positive again, and control variables capturing systematic risk and taking account of endogeneity problems arising from the fact that the theoretical model abstracts from several important real world issues. I also need to be able to decompose variations in default risk into variations due to the expected asset payoff and the debt level and variations due to asset volatility.

Hypotheses 1 and 2 may be criticized because they lean heavily on an over-stylized theoretical model. Also, they may be seen as unlikely to explain the default risk premium puzzle because only few firms attract a default risk above fifty percent. While these concerns are valid, the threshold of fifty percent is an artefact of the density function used to model the asset payoff. Using a non-log-normal density function can push this threshold below fifty percent.[10] Unfortunately, taking no firm stance on how the asset payoff is distributed implies that the range of default risk for which the relation should be negative can no longer be determined. Nevertheless, the intuition

[10] Assume that the asset payoff follows a mixture of two log-normal densities, with constant probabilities that the asset payoff is drawn from one of the two densities. The expectation and standard deviation of the log value drawn from the first density ($\mu_{x,1}$ and $\sigma_{x,1}$, respectively) are relatively low, while those of the log value drawn from the second density ($\mu_{x,2}$ and $\sigma_{x,2}$, respectively) are relatively high. Mixture densities such as these are often used to model random variables with high skewness. Also assume that log debt ($\ln k$) lies slightly above $\mu_{x,1}$, but far below $\mu_{x,2}$. Default risk, the probability-weighted average of the default risk associated with each of the two densities, must then be below fifty percent. Under these assumptions, an increase in $\sigma_{x,1}$ increases the volatility of the log asset payoff, but decreases default risk. Using the theoretical results in Section 2, it is easy to show that the increase in $\sigma_{x,1}$ also increases the expected equity return if the default risk associated with the first density is below the γ threshold obtained from this first density.

derived from the theoretical model dictates that a negative relation should only occur at relatively high levels of default risk. To test this more general insight, I also analyze a modified version of hypothesis 2:

H2a: Controlling for other factors, an increase in default risk attributable to a change in asset volatility increases the expected equity return at low levels of default risk, decrease it at moderately high levels of default risk, and increases it again at very high levels of default risk.

3.2 Proxy variables

Default risk: Equation (3), the default probability implied by the model, can be rewritten as follows:

$$\pi = N\left[\frac{\ln k - \mu_x}{\sigma_x}\right] = N\left[-\frac{\ln(p_A/k) + (E[\tilde{r}_A] - \frac{1}{2}\sigma_x^2)}{\sigma_x}\right], \tag{14}$$

where I have used the closed-form solutions for the expected asset payoff, $E[\tilde{x}]$, and the asset value, p_A, and the definition $E[\tilde{r}_A] \equiv \ln(E[\tilde{x}]/p_A)$ in the second equality. The second equality shows that the default risk implied by the theoretical model is identical to that implied by the Merton (1974) model, if we set the time-to-maturity in the Merton (1974) model equal to unity. Hence, it appears natural to use this structural model to measure default risk. I follow the approach suggested by Vassalou and Xing (2004) to imply default risk from the Merton (1974) model. This approach works as follows. The Black and Scholes (1973)-Merton (1974) (BSM) call option formula is applied to infer a firm's market value of assets at the end of each trading day over the prior twelve months. As inputs to the formula, I use equity volatility estimated over the prior twelve months as initial guess for asset volatility and several other variables (described below). The BSM formula is given by:

$$p_E = p_A N(d_1) - ke^{-r_f T} N(d_2), \tag{15}$$

where d_1=$[\ln(p_A/k) + (r_f + 0.5\sigma_x^2)T]/\sigma_x\sqrt{T}$ and d_2=$d_1 - \sigma_x\sqrt{T}$. Using the time-series of asset values, I update the guess of asset volatility, and then recreate the time-series of asset values. This process is repeated until the estimate of asset volatility converges. Upon convergence, I estimate the expected asset return by the mean log asset return over the prior twelve months. In combination with the other variables, the estimates of the market value of assets and asset volatility allow me to calculate estimates of log debt minus the expected asset payoff, $\ln k - \mu_x$, and of log debt minus the expected asset payoff scaled by asset volatility, $(\ln k - \mu_x)/\sigma_x$. The scaled variable, often referred to as the distance-to-default, is my empirical proxy for default risk.

The inputs for the iterative procedure are obtained as follows. The market value of equity, p_E, is measured by the stock price multiplied by the number of shares outstanding. The face value of debt, k, is measured by the sum of the book value of short-term debt and one-half that of long-term debt (Crosbie (1999)). The riskfree rate of return, r_f, is measured by the 3-month Treasury Bill yield.

Testing the hypotheses requires me to decompose the default risk proxy into one component attributable to the expected asset payoff and the debt level and another attributable to asset volatility. To perform the decomposition, I use a second-order Taylor series expansion of the default risk proxy:

$$\frac{\ln k - \mu_x}{\sigma_x} \approx \frac{\overline{pd}}{\overline{vol}} + \frac{1}{\overline{vol}}(pd - \overline{pd}) - \frac{\overline{pd}}{\overline{vol^2}}(vol - \overline{vol}) + \frac{\overline{pd}}{\overline{vol^3}}(vol - \overline{vol})^2 - \frac{1}{2}\frac{(pd - \overline{pd})(vol - \overline{vol})}{\overline{vol^2}}, \quad (16)$$

where, for notational convenience, $pd \equiv \ln k - \mu_x$, $vol \equiv \sigma_x$, and upperbars denote sample means. I will denote the second term on the right-hand side, $(pd - \overline{pd})/\overline{vol}$, by $f(pd)$, and the sum of the third and fourth terms, $-(\overline{pd}/\overline{vol^2})(vol - \overline{vol}) + (\overline{pd}/\overline{vol^3})(vol - \overline{vol})^2$, by $f(vol)$. Intuitively, $f(pd)$ captures variations in default risk (in excess of mean default risk, $\overline{pd}/\overline{vol}$) that can be attributed to variations in log debt minus the expected asset payoff, while $f(vol)$ captures such variations attributable to variations in asset volatility. The last additive term, $-(1/2)(pd - \overline{pd})(vol - \overline{vol})/\overline{vol^2}$, captures cross effects. Because $\overline{pd}/\overline{vol}$ and the cross effects never materially change any of the main conclusions reported later, I exclude them from the empirical analysis.[11] To ensure that extreme values of $f(pd)$ and $f(vol)$ do not confound the results, $f(pd)$ and $f(vol)$ are winsorized at their 1st and 99th percentiles, with the percentiles computed separately for each month.

To measure the level of default risk at which the relation between default risk attributable to asset volatility and the expected equity return reverses back to positive, γ, I set σ_x equal to its estimated value. However, because Section 2.2 shows that γ depends only very weakly on $B\hat{\sigma}_C\kappa$, I either set $B\hat{\sigma}_C\kappa$ to an arbitrary value (one) or to

[11] As $\overline{vol} > 0$, the approximation always increases with $\ln k - \mu_x$. Using the data also employed in the empirical tests, I have verified that, with only very few exceptions, the approximation also increases with σ_x if $\ln k - \mu_x < 0$, but that it decreases with σ_x if $\ln k - \mu_x > 0$. The approximation should hence be suitable to test hypotheses 1, 2 and 2a.

the product of estimates for $\hat{\sigma}_C$ and κ, again arbitrarily setting B equal to one. I then employ numerical methods to imply γ from the equality $F(\sigma_x) = 0$. As the resulting γ values are almost identical across the two choices for $B\hat{\sigma}_C\kappa$, I use the simpler ones based on $B\hat{\sigma}_C\kappa = 1$ in the empirical tests.[12]

Some empirical tests also rely on quintile or decile portfolios one-way sorted on Merton (1974) default risk, and these portfolios are formed as follows. In June of year t, I first extract all stocks with a zero book value of debt from the data and include them in portfolio 0. I then sort the remaining stocks into five or ten portfolios based on their breakpoints for the default risk measure in this month, with portfolio 1 containing the low default risk stocks and portfolio 5 or 10 the high default risk ones. As distressed stocks often experience a severe return reversal caused by a liquidity shock in the month after formation (Da and Gao (2010)), the portfolios are held from August of year t to July of year $t+1$, at which point they are reformed using the same algorithm. A similar strategy is followed by Garlappi et al. (2008) and Garlappi and Yan (2011).

Control variables: In the theoretical model, cross-sectional variations in the expected equity return are not only driven by variations in the default risk drivers, but also by variations in the correlation between the asset payoff and consumption, a measure of systematic risk. O'Doherty (2010) and Garlappi and Yan (2011) offer theoretical and empirical evidence suggesting that highly distressed stocks have relatively low conditional market betas, possibly explaining the flat or negative relation between default risk and the average equity return. In addition, it is well-known that portfolios sorted on default risk differ across several firm characteristic known to capture cross-sectional variations in the expected equity return, such as size, book-to-market, momentum and share price liquidity (Campbell et al. (2008)).[13] Finally, determinants of optimal capital structure (e.g., the deadweight costs of distress) might endogenously influence default risk *and* the expected equity return (George and Hwang (2010), Johnson et al. (2011)). Hence, the empirical tests include as control variables the conditional market beta, size, book-to-market, momentum, share price illiquidity and capital structure determinants.

The conditional market betas are estimated using either 60 month rolling window regressions of the monthly

[12] Results based on the other choice for $B\hat{\sigma}_C\kappa$ are virtually identical to those reported.

[13] I have also considered the exposures to spread portfolios sorted on size, book-to-market and momentum in the empirical tests, but similar to Daniel and Titman (1997) I have found that the explanatory power of the exposures of the spread portfolios is lower than that of the firm characteristics. Nevertheless, the main conclusions do not depend on whether the exposures to the spread portfolios or the firm characteristics are used.

stock return on the monthly market return or, similar to O'Doherty (2010) and Garlappi and Yan (2011), one-month rolling window regressions of the daily stock return on the daily market return and several of its lags:

$$\tilde{R}_{i,t} = \alpha_i + \beta_{i,1}\tilde{R}_{mkt,t} + \beta_{i,2}\tilde{R}_{mkt,t-1} + \beta_{i,3}(\tilde{R}_{mkt,t-2} + \tilde{R}_{mkt,t-3} + \tilde{R}_{mkt,t-4})/3 + \varepsilon_{i,t}, \tag{17}$$

where $\tilde{R}_{i,t}$ and $\tilde{R}_{mkt,t}$ are the daily returns of stock i and a value-weighted market index at time t, α_i, $\beta_{i,1}$, $\beta_{i,2}$ and $\beta_{i,3}$ are the parameters, and $\varepsilon_{i,t}$ is the residual (Lewellen and Nagel (2006)). The lagged market returns are included in the model to alleviate the impact of non-synchronic trading. The estimate of the conditional market beta for month t is then computed as the sum of the slope coefficients. Size is defined as the log of the share price times the number of shares outstanding, book-to-market as the log of the book value of a common share divided by its market price, and momentum as the compounded return over the prior twelve months. To proxy for share price illiquidity, I use the Amihud (2002) measure, defined as the ratio of the absolute daily return to daily trading volume averaged over the prior twelve months. The leverage ratio should reflect the factors determining capital structure. As a result, I follow George and Hwang (2010) in using book leverage, defined as the ratio of long-term debt plus long-term debt in current liabilities to total assets, to control for these factor. If one of these factors produces low optimal leverage ratios (and hence low default risk), but high systematic risk, the coefficient on book leverage should be negative. To ensure that outliers do not unduly influence the test results, the control variables are winsorized at the 1st and the 99th percentiles computed separately for each month.

3.3 Data

I study the monthly stock returns of firms traded on the NYSE, AMEX and NASDAQ with share codes equal to 10 and 11 in the sample period from August 1963 to December 2008. The market data are from CRSP. If available, I use the delisting return instead of the stock return. If the delisting code indicates a performance-related delisting (500; 520-584) but the delisting return is missing, I use a delisting return of -30 percent for firms traded on the NYSE and the AMEX and of -55 percent for firms traded on the NASDAQ (Shumway (1997), Shumway and Warther (1999)). As other studies in this literature (George and Hwang (2010)), I exclude firms with a stock price below one dollar to minimize the impact of market microstructure noise. I also exclude financial stocks, as their leverage ratios are not comparable with the leverage ratios of non-financial stocks. Accounting data are from COMPUSTAT. The accounting items are lagged by three months to ensure that they were known to investors at the time. Data on the market return are from Kenneth French's website.

3.4 Results

Descriptive statistics

Table 2 reports descriptive statistics on the value and equally-weighted default risk quintile portfolios. Panel A studies the sample period from August 1963 to December 2008, while, for reasons of comparability with Campbell et al. (2008), Panel B studies their sample period, January 1981 to December 2003.

The table suggests a complex relation between average equity returns and default risk. The quintile containing only stocks with a zero book value of debt in the formation month (0) often attracts a higher equity return than most other quintiles, possibly because firms in this portfolio are too risky to access debt markets. Equity returns then tend to increase from quintile 1 to 5, but only weakly and never monotonically. Only quintile 5 commands a substantial level of average default risk, exceeding those of the other quintiles by a factor larger than 4.5. Consistent with Campbell et al. (2008), who include firms with a zero book value of debt in their analysis, the spread in the equity return between quintiles 5 and 0 is close to zero in the extended sample period (Panel A), but becomes insignificantly negative in their sample period (Panel B). However, ignoring firms with a zero book value of debt, the spread in the equity return becomes more positive, but remains insignificant (see 5-1). Other studies using default risk measures implied by structural models also often find more positive relations, probably because structural models cannot be calibrated on stocks with a zero book value of debt. Hence, these studies likely also exclude the stocks in quintile 0 (Garlappi et al. (2008), Garlappi and Yan (2011)).[14]

Although the theoretical model cannot explain why quintile 0 attracts such a high equity return, it has the potential to explain why quintile 5 attract such a low equity return. Consistent with this claim, 13.20% (Panel A) and 18.20% (Panel B) of the firms in quintile 5 attract a default risk above fifty percent (% Default Risk $> 50\%$), of

[14] Garlappi et al. (2008) and Garlappi and Yan (2011) do not explicitly state that they exclude firms with a zero book value of debt, but the numbers of firms for which their default risk measure is available (reported in Table 1 of Garlappi et al. (2008)) strongly suggest that they do.

which two-third also attract a default risk above γ percent (% Default Risk $< \gamma\%|$Default Risk $> 50\%$). However, as increases in default risk due to changes in asset volatility only mildly increase the expected equity return above a default risk of γ percent (recall Figure 2), it is likely that the whole set of firms with a default risk above fifty percent deflate the expected equity return of quintile 5. Also, if the asset payoffs of firms in the real world are not log-normally distributed, then an even larger set of firms than suggested by the table can produce a negative relation between default risk and the expected equity return (recall footnote 10).

The table also illustrates that the market beta estimates computed from monthly data (Market Beta (60M)) increase strongly and significantly across the quintiles. In contrast, the increase in those computed from daily data (Market Beta (1M)) is far more attenuated and can reverse at higher levels of default risk, creating a hump-shaped relation between default risk and market betas. This result is consistent with those in O'Doherty (2010) and Garlappi and Yan (2011). However, in this context, it is noteworthy that, even when excluding stocks with a price below one dollar, quintile 3-5 still contain many stocks suffering from share price illiquidity (Illiquidity), perhaps implying that their Market Beta (1M) estimates are downward biases. While not the main objective of this paper, I test this conjecture by re-estimating Market Beta (1M) using only firms for which the Amihud (2002) illiquidity measure is below the monthly median. Upon this modification, Market Beta (1M) increases monotonically and significantly across the quintiles, casting doubt on the claim that highly distressed stocks command lower conditional market betas than less distressed stocks.

Cross-sectional tests

Table 3 reports the results of cross-sectional estimations of the stock return on several two month lagged pricing factors.[15] Panel A shows descriptive statistics on the decomposition of the default risk measure, $(\ln k - \mu_x)/\sigma_x$, including the mean, the standard deviation and several percentiles. Panels B and C offer the results of pooled OLS regressions with standard errors clustered by time and Fama and MacBeth (1973) (FM) regressions. Although Cochrane (2001) shows that pooled regressions adjusting for cross-sectional dependence in the residual should lead to results similar to those produced by FM regressions, they are rarely used in asset pricing studies. I rely

[15]Lagging the pricing factors by one or three months does not materially affect my results.

on them to test hypotheses 1 and 2, as some months generate few, if any, stocks with a default risk above fifty percent, rendering the FM regressions ill-suited to test these hypotheses. However, for reasons of comparability and because we are more familiar with them, I use FM regressions to test hypotheses 1 and 2a.[16]

Panel A shows that the approximated default risk measure is distributed in almost the same way as the original default risk measure, and that the absolute difference between them is mostly close to zero. Excluding those observations for which the absolute difference is large (by some standard) does not affect the main conclusions, and I therefore perform the cross-sectional tests on the whole set of observations with complete data. The standard deviations indicate that $f(pd)$ is slightly more important than $f(vol)$ in explaining variations in default risk, although both attract substantial standard deviations above 2.50. Finally, while cross-sectional means are used as expansion points in the approximation, the time-series correlation between the original and the approximated default risk measure (ρ), computed over all firms with more than 48 observations, is very high for the majority of firms, confirming that the approximation works well in capturing variations in default risk.

Equally-weighting the stocks in each month, as the OLS regressions necessarily do, the pooled estimations in Panel B suggest that, when only controlling for the market beta estimate, stock returns increase weakly, but significantly with default risk (model 1). However, this positive relation is due to distressed stocks also differing along other dimensions from less distressed ones, for example, they often have lower equity values, higher book-to-market ratios and lower prior returns (Campbell et al. (2008)). Controlling for these firm characteristics, the relation becomes highly insignificant (model 2). Substituting the original default risk measure for $f(pd)$, $f(vol)$, $f(vol)$ interacted with an indicator variable equal to one if default risk exceeds fifty percent and zero otherwise ($f(vol) \cdot D(DR > 50\%)$), and $f(vol)$ interacted with an indicator variable equal to one if default risk exceeds γ percent and zero otherwise ($f(vol) \cdot D(DR > \gamma\%)$), stock returns increase significantly with $f(pd)$ and $f(vol)$ when only controlling for the market beta estimate (model 3). However, the coefficient on f(vol) $\cdot$ D($DR > 50\%$) is not only significantly negative, but it is also larger in magnitude than the one on $f(vol)$, so that the Wald tests in Panel C indicate that the sum of the two coefficients is significantly negative. In contrast, the coefficient on $f(vol) \cdot D(DR > \gamma\%)$ is positive. Although it is insignificant, it renders the sum of the three coefficients related

[16]Testing hypotheses 1 and 2a with pooled regressions leads to identical conclusions.

to $f(vol)$ close to zero and insignificant, as the Wald tests in Panel C suggest.[17]

In sum, the evidence shows that stock returns increase with default risk attributable to expected profitability and leverage effects, but they only increase with default risk attributable to asset volatility effects up to a level of default risk of fifty percent. At higher levels of default risk, stock returns decrease with default risk attributable to asset volatility effects until default risk reaches γ percent, and then remain flat from γ percent onwards. Recalling that Figure 2 suggests that the increase in the expected equity return due to an increase in default risk driven by asset volatility is almost flat above γ percent, these results strongly support hypothesis 1 and 2.

Including the other control variables in the estimation drives out the significant effects of $f(pd)$ and $f(vol)$, but never the one of $f(vol) \cdot D(DR > 50\%)$ (models 4 & 5). These results might imply that size, book-to-market, momentum, share illiquidity and book leverage capture the more prevalent negative (positive) effect of expected profitability (leverage and business risk) on *both* default risk and the expected equity return. However, because they appear incapable of capturing the oppositely-signed effects of business risk on default risk and the expected equity return at high levels of default risk, other studies might find a negative relation between their default risk measures and equity returns upon controlling for the firm characteristics. A final interesting observation is that the coefficient on book leverage, while being negative, is insignificant, implying that optimal capital structure choice is unlikely to explain the default risk premium puzzle in these empirical tests.

The evidence so far is unlikely to explain why distressed stocks command relatively low equity returns, as only few highly distressed stocks exhibit a default risk above fifty percent. However, the fifty percent threshold is driven by the density used to model the asset payoff, and footnote 10 shows that one can easily find examples of densities implying a lower threshold. As a result, the final tests (the FM regressions) analyze the relation between default risk attributable to asset volatility and the expected equity return from a more general perspective. To achieve this goal, these tests use as interaction terms $f(vol)$ multiplied by an indicator variable equal to one if the stock belongs to either default risk decile portfolio 9 or 10 and zero otherwise ($f(vol) \cdot D(Q9, Q10)$) and $f(vol)$ multiplied by an indicator variable equal to one if the stock belongs to either default risk decile portfolio 10 and zero otherwise ($f(vol) \cdot D(Q10)$). The other exogenous variables are defined as before.

[17] In unreported tests, I have also interacted $f(pd)$ with the two indicator variables and found that the relation between $f(pd)$ and stock returns does not depend on the level of default risk.

Panel B shows that the inferences obtained from the FM regressions are very close to those obtained from the pooled regressions, although the pooled regressions seem to produce more conservative inference levels. Also, the coefficients on the interaction terms and their sums (in Panel C) show that, while the returns of the stocks in default risk decile 9 are insensitive to default risk due to asset volatility, an increase in default risk attributable to a change in asset volatility produces a significant decrease in the returns of the stocks in decile 10. Probably because only few stocks in decile 10 attract excessively high levels of default risk, the relation does not reverse back to positive in these tests. Other conclusions remain the same as before. Overall, the FM regressions vividly show that asset volatility can produce a negative relation between default risk and the expected equity return at high, but not necessarily above fifty percent, levels of default risk, thereby supporting hypothesis 2a.

Because the sample period studied in Campbell et al. (2008) is often used as a benchmark in the literature, I offer the results of the cross-sectional estimations for this period in Table 4. While there can be minor differences, for example, the coefficients on $f(pd)$ and $f(vol)$ have become slightly less significant, while that one on book leverage has become more significant, the general message is still that, at high levels of default risk, increases in default risk attributable to asset volatility can lead to a decrease in the expected equity return.

Robustness checks

January effects: The non-linear relation between default risk due to asset volatility and stock returns could also be explained by tax-loss selling (Rozeff and Kinney (1976), Grinblatt and Moskowitz (2004), etc.). The intuition behind this claim is that some of the distressed stocks may be distressed because they have suffered large capital losses over the recent past, making them ideal candidates for tax-loss selling. As asset volatility is the primary driver of $f(vol)$, $f(vol)$ might be able to distinguish these stocks from other distressed stocks, producing a negative relation between lagged $f(vol)$ and stock returns at high levels of default risk.[18] To test this conjecture, I include interaction terms between the variables related to $f(vol)$ and an indicator variable equal to one if the month equals January and zero otherwise, or I exclude January months from the estimation sample. The new interaction terms never attract significance at conventional levels. Also, excluding January months from the

[18] Recall that asset volatility and $f(vol)$ are negatively related at levels of default risk above fifty percent.

sample does not change the main conclusion. As a result, I can reject tax-loss selling as the main driver behind the non-linear relation between $f(vol)$ and stock returns found in the main tests.

Stockholder bargaining power: Garlappi et al. (2008) and Garlappi and Yan (2011) claim that high stockholder bargaining power can create a hump-shaped relation between default risk and the expected equity return. Hence, if stockholder bargaining power relates positively to $f(vol)$ at high levels of default risk (e.g., because firms with high business risk might be severely restricted by covenants, lowering the ability of stockholders to strategically default), then it might explain the concave part of the relation between $f(vol)$ and stock returns found in the main tests. To test this conjecture, I use two variables to proxy for stockholder bargaining power, the R&D and the tangible assets ratios. Garlappi et al. (2008) argue that R&D intensive firms usually face binding covenants, lowering stockholder bargaining power. In contrast, because intangible assets can only be sold at a great discount in a fire sale, stockholders can more easily force concessions from creditors in their presence. The R&D ratio is defined as R&D expenditures to total assets and, following Garlappi et al. (2008), the tangibles ratio as:

$$(0.715 \cdot \text{Receivables} + 0.547 \cdot \text{Inventories} + 0.535 \cdot \text{Capital} + \text{Cash + ST Investments})/\text{Total Assets}, \qquad (18)$$

where all variables used to construct the ratios are from COMPUSTAT. If the non-linear relation found in the main tests can be attributed to high stockholder bargaining power, then it should disappear in samples consisting of firms in which stockholders have low bargaining power. Motivated by this insight, I repeat the former tests on samples of firms with an R&D or tangibles ratio above the monthly median. The results from these tests are quantitatively identical to those reported previously, and I therefore reject the conjecture that high stockholder bargaining power drives the non-linear relation between $f(vol)$ and stock returns.[19]

4 Conclusions

Previous research often finds a flat, negative or hump-shaped relation between proxy variables for default risk and the expected equity return. In contrast to conventional wisdom, my theoretical results illustrate that a non-

[19]Tables featuring the results of the robustness checks are available upon request.

positive relation can arise in a plain-vanilla asset pricing setup. The intuition behind this result is that, if default risk is high and driven by asset volatility, then an increase in default risk can shift equity payoff probability mass from desirable (recessions) to less desirable states (expansions), thereby increasing equity risk and the expected equity return. Using a measure of default risk derived from the Merton (1974) model, which is consistent with default risk in the model, my empirical results confirm that average equity returns increase only insignificantly across portfolios sorted on default risk. Decomposing the default risk measure into one component attributable to expected profitability and leverage effects and another attributable to business risk effects, my empirical results further show that the weak increase in average equity returns can be attributed to a negative relation between default risk driven by asset volatility and stock returns at high levels of default risk.

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A Appendix: Proofs

Let $H[x]$ be defined as $n[x]/N[-x]$, with $n[x]$ being the probability density function and $N[x]$ the cumulative density function of the normally distributed random variable x. We can interpret this ratio as the hazard function for x. The following lemma will be useful throughout the subsequent proofs:

Lemma A.1: *The hazard function $H[x]$ exhibits the properties that (i) $H[x] > x$, (ii) $0 < H'[x] < 1$ and (iii) $H''[x] > 0$, where $H'[x]$ and $H''[x]$ denote the first and second derivatives of the hazard function with respect to its input argument x.*

Proof: *See Chechile (2003) and Freeman and Guermat (2006).*

Proof of Proposition 1:

(i) and (ii): Recall that default risk decreases as the expected asset payoff increases (i.e., $\partial\pi/\partial\mu_x < 0$) and increases as the debt level increases (i.e., $\partial\pi/\partial k > 0$). If the expected equity return does the same, there must necessarily exist a positive relation between changes in default risk induced by these two factors and changes in the expected equity return. If the expected equity return does the opposite, the relation is negative. The partial derivative of the expected equity return with respect to the expected asset payoff is given by:

$$\frac{\partial E[\tilde{R}_E]}{\partial\mu_x} = \frac{\partial E[\tilde{x}_E]/p_E}{\partial\mu_x} = \frac{(\partial E[\tilde{x}_E]/\partial\mu_x)p_E - (\partial p_E/\partial\mu_x)E[\tilde{x}_E]}{p_E^2}, \tag{19}$$

where I use the quotient rule in the second equality. The partial derivatives shown on the right hand side of the second equality are given by:

$$\frac{\partial E[\tilde{x}_E]}{\partial\mu_x} = e^{\mu_x+\frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right] > 0, \tag{20}$$

and

$$\frac{\partial p_E}{\partial\mu_x} = \rho e^{\mu_x+\mu_C+\frac{1}{2}[\sigma_x^2+2\sigma_{x,C}+\sigma_C^2]} N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right] > 0. \tag{21}$$

Plugging Equations (7), (9), (20) and (21) into Equation (19), I obtain:

$$\begin{aligned}
& \frac{1}{p_E^2}\left[e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]\left(\rho e^{\mu_x+\mu_C+\frac{1}{2}(\sigma_x^2+2\sigma_{x,C}+\sigma_C^2)}\right.\right. \\
& N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]-\rho k e^{\mu_C+\frac{1}{2}\sigma_C^2}N\left[\left.\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right) \\
& -\left(e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x-\ln k}{\sigma_x}\right]\right) \\
& \left.\rho e^{\mu_x+\mu_C+\frac{1}{2}(\sigma_x^2+2\sigma_{x,C}+\sigma_C^2)}N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right] \qquad (22) \\
= \quad & -\frac{\rho k e^{\mu_x+\mu_C+\frac{1}{2}(\sigma_x^2+\sigma_C^2)}}{p_E^2}\left[N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right. \\
& \left.-e^{\sigma_{x,C}}N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right], \qquad (23)
\end{aligned}$$

where, in the equality, I simplify the resulting partial derivative.

The partial derivative of the expected equity return with respect to the debt level equals:

$$\frac{\partial E[\tilde{R}_E]}{\partial k}=\frac{\partial E[\tilde{x}_E]/p_E}{\partial k}=\frac{(\partial E[\tilde{x}_E]/\partial k)p_E-(\partial p_E/\partial k)E[\tilde{x}_E]}{p_E^2}. \qquad (24)$$

The partial derivatives shown on the right hand side of the second equality are given by:

$$\frac{\partial E[\tilde{x}_E]}{\partial k}=-N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]=-\text{survival probability}<0, \qquad (25)$$

and

$$\frac{\partial p_E}{\partial k}=-\rho e^{\mu_C+\frac{1}{2}\sigma_C^2}N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]<0. \qquad (26)$$

Plugging Equations (7), (9), (25) and (26) into Equation (24), I obtain:

$$
\begin{aligned}
&\frac{1}{p_E^2}\left[-N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]\left(\rho e^{\mu_x+\mu_C+\frac{1}{2}(\sigma_x^2+2\sigma_{x,C}+\sigma_C^2)}N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right.\right.\\
&\left.-\rho k e^{\mu_C+\frac{1}{2}\sigma_C^2}N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right)-\left(-\rho e^{\mu_C+\frac{1}{2}\sigma_C^2}N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right)\\
&\left.\left(e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x-\ln k}{\sigma_x}\right]\right)\right] \qquad (27)\\
= \;&\frac{\rho e^{\mu_x+\mu_C+\frac{1}{2}(\sigma_x^2+\sigma_C^2)}}{p_E^2}\left[N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right.\\
&\left.-e^{\sigma_{x,C}}N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right], \qquad (28)
\end{aligned}
$$

which in turn is equal to:

$$-\frac{1}{k}\frac{\partial E[\tilde{R}_E]}{\partial \mu_x}, \qquad (29)$$

implying that the two partial derivatives must have opposite signs. As the time-preference parameter ρ, the debt level k and the range of the exponential function are strictly positive, the signs of the two partial derivatives hinge on the sign of the expression in the outer square parentheses in equations (23) and (28), which is identical across the two equations. For default risk and the expected equity return to be positively related, this expression must be positive, which holds if and only if the following inequality holds:

$$N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right] \;>\; e^{\sigma_{x,C}}N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]. \qquad (30)$$

As $N[\cdot]>0$, the last inequality is equivalent to:

$$\frac{N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]}{N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]} \;>\; e^{\sigma_{x,C}}\frac{N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}. \qquad (31)$$

If $\kappa=0$, then $\sigma_{x,C}=-B\kappa\sigma_x\hat{\sigma}_C=0$ and therefore:

$$\frac{N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]}{N\left[\frac{\mu_x-\ln k}{\sigma_x}\right]} \;=\; e^{\sigma_{x,C}}\left.\frac{N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}\right|_{\kappa=0}. \qquad (32)$$

Only the expression on the right hand side of inequality (31) depends on the correlation between the asset payoff and consumption (κ). If this expression were monotonically related to κ, then we could make the inequality hold by either setting κ to a positive number (if the right hand side of inequality (31) decreased in κ) or by setting it to a negative number (if it increased in κ). The natural logarithm of this expression is:

$$\sigma_{x,C} + \ln\left(N\left[\frac{\mu_x + \sigma_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\right) - \ln\left(N\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\right). \tag{33}$$

Taking the derivative of the above expression with respect to κ and re-arranging:

$$-B\sigma_x\hat{\sigma}_C\left[1-\left(\frac{n\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]} - \frac{n\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{N\left[\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]}\right)/\sigma_x\right]. \tag{34}$$

which can be written as:

$$\begin{aligned} & -B\sigma_x\hat{\sigma}_C\left[1-\left(\frac{n\left[-\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{N\left[-\left(-\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right)\right]} - \frac{n\left[-\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{N\left[-\left(-\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right)\right]}\right)/\sigma_x\right] \\ = \quad & -B\sigma_x\hat{\sigma}_C\left[1-\frac{H\left[-\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}+\sigma_x\right]-H\left[-\frac{\mu_x+\sigma_x^2+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{\sigma_x}\right]. \end{aligned} \tag{35}$$

Using the mean-value theorem:

$$-B\sigma_x\hat{\sigma}_C\left(1-H'[x^*]\right), \tag{36}$$

where $x^* \in (-(\mu_x + \sigma_x^2 + \sigma_{x,C} - \ln k)/\sigma_x, -(\mu_x + \sigma_x^2 + \sigma_{x,C} - \ln k)/\sigma_x + \sigma_x)$. As $H'[x] < 1$, the partial derivative of the right hand side of inequality (31) with respect to the correlation coefficient is negative. In turn, we can make inequality (31) hold by setting the correlation coefficient to a positive number. As a result, if the correlation coefficient is positive, the partial derivative of the expected equity return with respect to the expected asset payoff ($\partial E[\tilde{R}_E]/\partial\mu_x$) is negative, while the partial derivative with respect to the debt level ($\partial E[\tilde{R}_E]/\partial k$) is positive. As default risk has identical relations with the two factors, a change in default risk induced either by a change in the expected asset payoff or the debt level produces an equally-signed change in the expected equity return.

(iii) As the cumulative density function is monotonically increasing in its input argument, the sign of the partial derivative of default risk ($N[(\ln k - \mu_x)/\sigma_x]$) is identical to that of $(\ln k - \mu_x)/\sigma_x$. Define $\psi(\mu_x, k) \equiv (\ln k - \mu_x)/\sigma_x$. Treating asset volatility as a constant, the total derivative of $\psi(\mu_x, k)$ with respect to the expected asset payoff and the debt level can be written as:

$$d\psi(\mu_x, k) = \frac{\partial \psi(\mu_x, k)}{\partial \mu_x} d\mu_x + \frac{\partial \psi(\mu_x, k)}{\partial k} dk = -\frac{d\mu_x}{\sigma_x} + \frac{dk}{k\sigma_x}. \tag{37}$$

Without loss of generality, assume that default risk increases, that is, that $d\mu_x < dk/k$. Again treating asset volatility as a constant, the total derivative of the expected equity return is given by:

$$dE[\tilde{R}_E] = \frac{\partial E[\tilde{R}_E]}{\partial \mu_x} d\mu_x + \frac{\partial E[\tilde{R}_E]}{\partial k} dk. \tag{38}$$

Using Equation (29) gives:

$$dE[\tilde{R}_E] = \frac{\partial E[\tilde{R}_E]}{\partial \mu_x} d\mu_x - \frac{1}{k}\frac{\partial E[\tilde{R}_E]}{\partial \mu_x} dk = \frac{\partial E[\tilde{R}_E]}{\partial \mu_x}(d\mu_x - dk/k). \tag{39}$$

If the correlation coefficient (κ) is positive, the partial derivative of the expected equity return with respect to the expected asset payoff ($\partial E[\tilde{R}_E]/\partial \mu_x$) is less than zero. As default risk can increase if and only if $d\mu_x < dk/k$, an increase in default risk induced by changes in both the expected asset payoff and the debt level must therefore produce an increase in the expected equity return, and vice versa.

Proof of Proposition 2:

Recall that default risk increases with asset volatility if default risk is below fifty percent and decreases with asset volatility if default risk is above fifty percent. To obtain a positive relation between default risk driven by asset volatility and the expected equity return, the expected equity return must hence also increase with asset volatility if default risk is below fifty percent, but decrease with it if default risk is above fifty percent. The partial derivative of the expected equity return with respect to asset volatility is given by:

$$\frac{\partial E[\tilde{R}_E]}{\partial \sigma_x} = \frac{\partial E[\tilde{x}_E]/p_E}{\partial \sigma_x} = \frac{(\partial E[\tilde{x}_E]/\partial \sigma_x)p_E - (\partial p_E/\partial \sigma_x)E[\tilde{x}_E]}{p_E^2}, \tag{40}$$

where I use the quotient rule in the second equality. The partial derivatives shown on the right hand side of the second equality are given by:

$$\frac{\partial E[\tilde{x}_E]}{\partial \sigma_x} = \sigma_x e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] + kn\left[\frac{\mu_x - \ln k}{\sigma_x}\right] > 0, \tag{41}$$

and

$$\frac{\partial p_E}{\partial \sigma_x} = \rho e^{\mu_C + \frac{1}{2}\sigma_C^2}\left[(\sigma_x - B\kappa\hat{\sigma}_C)e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\sigma_{x,C}]} N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] + kn\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\right]. \tag{42}$$

Plugging Equations (7), (9), (41) and (42) into Equation (40), I obtain:

$$\begin{aligned}\frac{\partial E[\tilde{R}_E]}{\partial \sigma_x} \quad = \quad & \frac{1}{p_E^2}\Bigg[\Bigg(\sigma_x e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] + kn\left[\frac{\mu_x - \ln k}{\sigma_x}\right]\Bigg)\Bigg(\rho e^{\mu_x + \mu_C + \frac{1}{2}(\sigma_x^2 + 2\sigma_{x,C} + \sigma_C^2)} \\ & N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] - \rho k e^{\mu_C + \frac{1}{2}\sigma_C^2} N\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\Bigg) - \Bigg(\Bigg(e^{\mu_x + \frac{1}{2}\sigma_x^2} \\ & N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] - kN\left[\frac{\mu_x - \ln k}{\sigma_x}\right]\Bigg)\Bigg(\rho(\sigma_x - B\kappa\hat{\sigma}_C)e^{\mu_x + \mu_C + \frac{1}{2}(\sigma_x^2 + 2\sigma_{x,C} + \sigma_C^2)} \\ & N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] + k\rho e^{\mu_C + \frac{1}{2}\sigma_C^2} n\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\Bigg)\Bigg)\Bigg].\end{aligned} \tag{43}$$

The sign of the partial derivative is positive if and only if:

$$\begin{aligned}& \Bigg(\sigma_x e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] + kn\left[\frac{\mu_x - \ln k}{\sigma_x}\right]\Bigg)\Bigg(\rho e^{\mu_x + \mu_C + \frac{1}{2}(\sigma_x^2 + 2\sigma_{x,C} + \sigma_C^2)} \\ & N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] - \rho k e^{\mu_C + \frac{1}{2}\sigma_C^2} N\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\Bigg) > \Bigg(e^{\mu_x + \frac{1}{2}\sigma_x^2} \\ & N\left[\frac{\mu_x + \sigma_x^2 - \ln k}{\sigma_x}\right] - kN\left[\frac{\mu_x - \ln k}{\sigma_x}\right]\Bigg)\Bigg(\rho(\sigma_x - B\kappa\hat{\sigma}_C)e^{\mu_x + \mu_C + \frac{1}{2}(\sigma_x^2 + 2\sigma_{x,C} + \sigma_C^2)} \\ & N\left[\frac{\mu_x + \sigma_{x,C} + \sigma_x^2 - \ln k}{\sigma_x}\right] + k\rho e^{\mu_C + \frac{1}{2}\sigma_C^2} n\left[\frac{\mu_x + \sigma_{x,C} - \ln k}{\sigma_x}\right]\Bigg)\end{aligned} \tag{44}$$

Since $e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x-\ln k}{\sigma_x}\right]=E[\tilde{x}_E]>0$ and $e^{\mu_x+\frac{1}{2}\sigma_x^2+\sigma_{x,C}}N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]=(1/\rho)e^{-(\mu_C+\frac{1}{2}\sigma_C^2)}p_E>0$, the former inequality is equivalent to:

$$
\begin{aligned}
&\frac{\sigma_x e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]+kn\left[\frac{\mu_x-\ln k}{\sigma_x}\right]}{e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x-\ln k}{\sigma_x}\right]}\\
>\;&\frac{(\sigma_x-B\kappa\hat{\sigma}_C)e^{\mu_x+\frac{1}{2}\sigma_x^2+\sigma_{x,C}}N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]+kn\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}{e^{\mu_x+\frac{1}{2}\sigma_x^2+\sigma_{x,C}}N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}. \qquad (45)\\
=\;&\frac{(\sigma_x-B\kappa\hat{\sigma}_C)+e^{-(\mu_x+\frac{1}{2}\sigma_x^2+\sigma_{x,C}-\ln k)}n\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]/N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]}{1-e^{-(\mu_x+\frac{1}{2}\sigma_x^2+\sigma_{x,C}-\ln k)}N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]/N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]} \qquad (46)\\
=\;&\frac{(\sigma_x-B\kappa\hat{\sigma}_C)+n\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]/N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]}{1-\left(n\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]\right)/\left(n\left[\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]N\left[\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]\right)} \qquad (47)
\end{aligned}
$$

The expression on the right hand side of the last equality can be written as:

$$
\frac{(\sigma_x-B\kappa\hat{\sigma}_C)+H\left[-\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]}{1-H\left[-\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]/H\left[-\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}. \qquad (48)
$$

If $\kappa=0$, then $\sigma_{x,C}=0$. As a result:

$$
\frac{\sigma_x e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]+kn\left[\frac{\mu_x-\ln k}{\sigma_x}\right]}{e^{\mu_x+\frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x+\sigma_x^2-\ln k}{\sigma_x}\right]-kN\left[\frac{\mu_x-\ln k}{\sigma_x}\right]}=\left.\frac{(\sigma_x-B\kappa\hat{\sigma}_C)+H\left[-\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]}{1-H\left[-\frac{\mu_x+\sigma_{x,C}+\sigma_x^2-\ln k}{\sigma_x}\right]/H\left[-\frac{\mu_x+\sigma_{x,C}-\ln k}{\sigma_x}\right]}\right|_{\kappa=0}. \qquad (49)
$$

Only the right hand side of inequality (45), which is equivalent to the expression in formula (48), depends on the correlation between the asset payoff and consumption (κ). If this expression were monotonically related to κ, we could make inequality (45) hold by setting κ to a positive number (if the expression on the right hand side of the inequality decreased in κ) or by setting κ to a negative number (if the expression increased in κ).

For notational convenience, define $\alpha\equiv(\ln k-\mu_x)/\sigma_x$, $\alpha^*\equiv\sigma_x>0$ and $\beta\equiv B\hat{\sigma}_C>0$. Using these definitions, the expression in formula (48) can be written as:

$$\frac{\alpha^{*}-\beta\kappa+H\left[\alpha-\alpha^{*}+\beta\kappa\right]}{1-H\left[\alpha-\alpha^{*}+\beta\kappa\right]/H\left[\alpha+\beta\kappa\right]}. \tag{50}$$

The partial derivative of the expression above with respect to κ has the same sign as:[20]

$$\begin{aligned}&\beta\left[H'[\alpha-\alpha^{*}+\beta\kappa]-1\right]\left[1-H[\alpha-\alpha^{*}+\beta\kappa]/H[\alpha+\beta\kappa]\right]+\beta\left[\alpha^{*}-\beta\kappa+H\left[\alpha-\alpha^{*}+\beta\kappa\right]\right]\\&\quad\cdot\left[H'[\alpha-\alpha^{*}+\beta\kappa]/H[\alpha+\beta\kappa]-H[\alpha-\alpha^{*}+\beta\kappa]H'[\alpha+\beta\kappa]/H[\alpha+\beta\kappa]^{2}\right].\end{aligned} \tag{51}$$

Multiplying by $1/\beta>0$ and $H[\alpha+\beta\kappa]>0$, adding and subtracting α inside the third main expression, using the relationship $H'[x]=H[x][H[x]-x]$, and rearranging yields:

$$\begin{aligned}&\left[H'[\alpha-\alpha^{*}+\beta\kappa]-1\right]\left[H[\alpha+\beta\kappa]-H[\alpha-\alpha^{*}+\beta\kappa]\right]+H[\alpha-\alpha^{*}+\beta\kappa]\left[H\left[\alpha-\alpha^{*}+\beta\kappa\right]\right.\\&\left.-(\alpha-\alpha^{*}+\beta\kappa)+\alpha\right]\left[\left(H[\alpha-\alpha^{*}+\beta\kappa]-(\alpha-\alpha^{*}+\beta\kappa)\right)-\left(H[\alpha+\beta\kappa]-(\alpha+\beta\kappa)\right)\right] \qquad (52)\\=\;&\left[H'[\alpha-\alpha^{*}+\beta\kappa]-1\right]\left[H[\alpha+\beta\kappa]-H[\alpha-\alpha^{*}+\beta\kappa]\right]+H[\alpha-\alpha^{*}+\beta\kappa]\left[H\left[\alpha-\alpha^{*}+\beta\kappa\right]\right.\\&\left.-(\alpha-\alpha^{*}+\beta\kappa)+\alpha\right]\left[\alpha^{*}-\left(H[\alpha+\beta\kappa]-H[\alpha-\alpha^{*}+\beta\kappa]\right)\right]. \qquad (53)\end{aligned}$$

Dividing by $\alpha^{*}>0$, using the mean value theorem and $H'[x]=H[x][H[x]-x]$ gives:

$$\begin{aligned}&\left[H'[\alpha-\alpha^{*}+\beta\kappa]-1\right]H'[c^{*}]+H'[\alpha-\alpha^{*}+\beta\kappa]\left[1-H'[c^{*}]\right]+\alpha H[\alpha-\alpha^{*}+\beta\kappa]\left[1-H'[c^{*}]\right]\\=\;&-H'[c^{*}]+H'[\alpha-\alpha^{*}+\beta\kappa]+\alpha H[\alpha-\alpha^{*}+\beta\kappa]\left[1-H'[c^{*}]\right],\end{aligned} \tag{54}$$

where $c^{*}\in(\alpha-\alpha^{*}+\beta\kappa,\alpha+\beta\kappa)$. As $c^{*}>\alpha-\alpha^{*}+\beta\kappa$ and as $H[x]$ is a convex function, the sum of the first two terms in Equation (54) is negative. Also, $H[x]$ is positive and $H'[x]<1$. As a result, if default risk is below fifty percent (i.e., $\alpha=(\ln k-\mu_x)/\sigma_x<0$) or equal to fifty percent (i.e., $\alpha=0$), the third term is negative or zero, respectively, implying that the sign of the partial derivative of the right hand side of inequality (45) with respect

[20] The partial derivative is the term in Equation (51) divided by $(1-H\left[\alpha-\alpha^{*}+\beta\kappa\right]/H\left[\alpha+\beta\kappa\right])^{2}$.

to κ is negative. In this case, we can make inequality (45) hold by setting κ to a positive number. If inequality (45) holds, then the expected equity return increases with asset volatility. However, as default risk also increases with asset volatility when it is below fifty percent, there must then be a monotonic, positive relation between changes in default risk induced by changing asset volatility and changes in the expected equity return. For default risk equal to fifty percent, expected equity returns, *but not* default risk, increase with asset volatility.

If default risk exceeds fifty percent (i.e., if $\alpha = (\ln k - \mu_x)/\sigma_x > 0$), then the partial derivative of the expression on the right hand side of inequality (45) with respect to κ can have either sign. To see that it can be negative, note that the scaled partial derivative in Equation (54) is strictly negative at $\alpha = 0$. By continuity, the partial derivative must then also be negative for values of α slightly greater than zero. I use an example to illustrate that the partial derivative can be positive if default risk exceeds fifty percent. I set B, σ_x, $\hat{\sigma}_C$ and κ equal to 1, 0.40, 0.10 and 0.50, respectively. As a result, $\alpha^* (\sigma_x)$ equals 0.50 and $\beta (B\hat{\sigma}_C)$ equals 0.10. Under these assumptions, the partial derivative is negative of $\alpha < -0.07178$ and positive otherwise.

Table 1: The Gamma Parameter[a]

$B\hat{\sigma}_C\kappa$		Log Debt - Expected Asset Payoff							
		0.01	0.10	0.20	0.40	0.60	0.80	1.00	1.20
0.010	Asset Volatility	**0.1565**	**0.4590**	**0.6264**	**0.8495**	**1.0131**	**1.1472**	**1.2631**	**1.3665**
	Default Risk (in %)	52.5%	58.6%	62.5%	68.1%	72.3%	75.7%	78.6%	81.0%
0.025	Asset Volatility	**0.1568**	**0.4616**	**0.6292**	**0.8529**	**1.0169**	**1.1514**	**1.2675**	**1.3710**
	Default Risk (in %)	52.5%	58.6%	62.5%	68.0%	72.2%	75.6%	78.5%	80.9%
0.050	Asset Volatility	**0.1585**	**0.4649**	**0.6338**	**0.8586**	**1.0232**	**1.1581**	**1.2746**	**1.3784**
	Default Risk (in %)	52.5%	58.5%	62.4%	67.9%	72.1%	75.5%	78.4%	80.8%
0.100	Asset Volatility	**0.1613**	**0.4721**	**0.6428**	**0.8698**	**1.0358**	**1.1716**	**1.2888**	**1.3933**
	Default Risk (in %)	52.5%	58.4%	62.2%	67.7%	71.9%	75.3%	78.1%	80.5%

[a] In this table, I report several combinations of values for log debt minus the expected asset payoff ($\ln k - \mu_x$), asset volatility (σ_x) and the product of the risk aversion parameter, the volatility of future consumption, and the correlation between the log asset payoff and log consumption ($B\hat{\sigma}_C\kappa$) that satisfy the following equality:

$$-H'[c^*] + H'[\alpha - \alpha^* + \beta\kappa] + \alpha H[\alpha - \alpha^* + \beta\kappa]\left[1 - H'[c^*]\right] = 0.$$

To achieve this goal, I vary the value of $B\hat{\sigma}_C\kappa$ from 0.010 to 0.100 and the value of $\ln k - \mu_x$ from 0.01 to 1.20. I then use numerical methods to identify the value of σ_x (shown in bold in the table) that satisfies the equality above. The default probability corresponding to this choice of σ_x, that is, $N[(\ln k - \mu_x)/\sigma_x]$, is shown below the value for σ_x.

Table 2: Default Risk and Average Equity Returns[a]

	Default Risk Quintiles							
	No Debt	1	2	3	4	5	5-No Debt	5-1
PANEL A: Sample period from August 1963-December 2008								
# Stocks	365	471	468	465	465	455		
% Default Risk $>$ 50%	0.20	0.10	0.30	0.80	2.00	13.20		
% Default Risk $< \gamma\%$|Default Risk $>$ 50%	68.40	58.30	54.30	53.60	45.90	34.70		
Value-weighted firm characteristics								
Equity Return	0.95***	0.85***	0.84***	0.85***	0.87***	1.01***	0.06	0.15
Default Risk	0.03***	0.02***	0.22***	0.50***	1.60***	9.51***	9.48***	9.49***
Log Debt - Expected Asset Payoff	-4.30***	-3.70***	-2.20***	-1.80***	-1.53***	-1.09***	3.21***	2.61***
Asset Volatility	0.38***	0.25***	0.27***	0.29***	0.34***	0.42***	0.04***	0.17***
Market Beta (60M)	1.22***	0.89***	0.98***	1.06***	1.20***	1.34***	0.12***	0.45***
Market Beta (1M)	1.14***	0.94***	0.98***	1.04***	1.16***	1.23***	0.08**	0.28***
Share Price Illiquidity	2.49***	0.58***	1.16***	1.53***	2.37***	3.86***	1.36***	3.28***
Equally-weighted firm characteristics								
Equity Return	1.30***	1.09***	1.05***	1.05***	1.19***	1.36***	0.05	0.27
Default Risk	0.27***	0.15***	0.48***	1.27***	3.53***	16.69***	16.42***	16.54***
Log Debt - Expected Asset Payoff	-3.46***	-3.84***	-2.64***	-2.18***	-1.75***	-0.99***	2.48***	2.86***
Asset Volatility	0.55***	0.32***	0.36***	0.41***	0.47***	0.55***	0.00	0.24***
Market Beta (60M)	1.12***	0.94***	1.01***	1.08***	1.20***	1.27***	0.16***	0.33***
Market Beta (1M)	0.97***	0.89***	0.94***	0.96***	1.01***	1.02***	0.05***	0.13***
Share Price Illiquidity	11.36***	4.38***	5.48***	7.11***	10.30***	15.28***	3.92***	10.90***
PANEL B: Sample period from January 1981-December 2003								
# Stocks	446	596	591	587	586	572		
% Default Risk $>$ 50%	0.30	0.20	0.50	1.10	3.00	18.20		
% Default Risk $< \gamma\%$|Default Risk $>$ 50%	71.70	60.30	51.70	49.60	42.10	35.60		
Value-weighted firm characteristics								
Equity Return	1.31***	1.08***	1.11***	1.04***	0.88**	1.15***	-0.16	0.07
Default Risk	0.04***	0.03***	0.34***	0.63***	2.01***	11.56***	11.52***	11.53***
Log Debt - Expected Asset Payoff	-4.25***	-3.62***	-2.34***	-1.85***	-1.44***	-1.01***	3.24***	2.61***
Asset Volatility	0.45***	0.29***	0.31***	0.33***	0.37***	0.48***	0.03***	0.19***
Market Beta (60M)	1.21***	0.88***	0.98***	1.05***	1.12***	1.19***	-0.02	0.31***
Market Beta (1M)	1.27***	0.92***	0.99***	1.08***	1.11***	1.16***	-0.11**	0.24***
Share Price Illiquidity	1.61***	0.25***	0.38***	0.61***	1.20***	2.28***	0.67***	2.03***
Equally-weighted firm characteristics								
Equity Return	1.45***	1.25***	1.26***	1.19***	1.23***	1.44***	-0.02	0.19
Default Risk	0.42***	0.24***	0.69***	1.81***	4.93***	22.06***	21.64***	21.81***
Log Debt - Expected Asset Payoff	-3.44***	-3.88***	-2.85***	-2.32***	-1.72***	-0.81***	2.63***	3.07***
Asset Volatility	0.66***	0.36***	0.43***	0.49***	0.55***	0.66***	0.00	0.30***
Market Beta (60M)	1.03***	0.89***	0.98***	1.05***	1.07***	1.07***	0.05***	0.19***
Market Beta (1M)	0.93***	0.85***	0.91***	0.90***	0.86***	0.83***	-0.11***	-0.03
Share Price Illiquidity	8.22***	2.63***	3.74***	5.34***	8.29***	12.46***	4.24***	9.83***

[a] In this table, I report descriptive statistics on portfolios sorted on default risk. Panel A considers the sample period from August 1963 to December 2009, whereas Panel B considers that from January 1981 to December 2003. To construct the portfolios, I first sort the firms with no debt in June of year t into quintile portfolio 0. Using the remaining firms, I compute the quintile breakpoints of Merton (1974) default risk in June of year t, according to which I sort the firms with debt into portfolios. The portfolios are held from August of year t to June of year $t+1$, at which point they are re-shuffled following the same algorithm. The descriptive statistics include the number of stocks in each portfolio (# Stocks), the percent of stocks whose default risk exceeds 50% (%Default Risk $>$ 50%), and the percent of the former stocks whose default risk falls short of γ% (%Default Risk $< \gamma\%$|Default Risk $>$ 50%). They also include the time-series averages (value or equally-weighted) of the equity return, Merton (1974) default risk, log debt minus the expected asset payment, asset volatility, the market beta estimated using monthly data over the prior 60 months (Market Beta (60M)) or daily data over the prior month (Market Beta (1M)), and a share price illiquidity measure. ***, ** and * indicate statistical significance at the one, five and ten percent level, respectively.

Table 3: Cross-Sectional Asset Pricing Tests (1963-2009)[a]

PANEL A: Summary Statistics

	N	Mean	StDev	P1	P5	Q1	Median	Q3	P95	P99
Original Default Risk	1,222,165	-6.53	4.94	-23.57	-15.99	-8.85	-5.54	-3.01	-0.58	0.60
Approximated Default Risk	1,222,165	-6.46	4.11	-20.01	-13.80	-8.44	-5.87	-3.66	-1.18	0.15
abs(diff)	1,222,165	1.09	2.14	0.00	0.01	0.12	0.42	1.20	4.31	9.11
f(pd)	1,222,165	-2.49	3.45	-13.72	-9.38	-4.13	-1.72	-0.08	1.64	2.83
f(vol)	1,222,165	-4.10	2.72	-11.81	-8.73	-5.56	-3.59	-2.02	-1.07	-0.78
rho	10,036	0.85	0.31	-0.54	0.11	0.89	0.97	0.99	1.00	1.00

PANEL B: Regression Analysis

	Panel Data Regressions with Clustered Errors					**Fama & MacBeth (1973) Regressions**				
Variables	1	2	3	4	5	1	2	3	4	5
Intercept	**1.49**	**2.17**	**1.89**	**2.42**	**2.23**	**1.22**	**1.60**	**1.49**	**1.66**	**1.52**
	[4.84]	[6.35]	[4.26]	[5.87]	[3.57]	[4.88]	[5.10]	[5.07]	[4.84]	[3.89]
Market Beta	**0.05**	**0.06**	**0.03**	**0.04**	**0.04**	**0.03**	**0.03**	**0.00**	**0.01**	**0.02**
	[1.08]	[1.32]	[0.81]	[0.88]	[0.89]	[0.70]	[1.00]	[0.02]	[0.54]	[0.85]
Original Default Risk	**0.05**	**0.00**				**0.03**	**-0.01**			
	[2.65]	[-0.14]				[2.08]	[-1.43]			
f(pd)			**0.07**	**0.01**	**0.05**			**0.01**	**-0.01**	**0.03**
			[3.21]	[0.28]	[1.14]			[0.83]	[-0.75]	[1.87]
f(vol)			**0.12**	**0.08**	**0.07**			**0.10**	**0.04**	**0.00**
			[2.35]	[1.29]	[1.08]			[2.44]	[1.31]	[0.03]
f(vol) · D($DR > 50\%$)			**-0.24**	**-0.23**	**-0.31**					
			[-3.94]	[-3.45]	[-2.85]					
f(vol) · D($DR > \gamma\%$)			**0.06**	**0.11**	**0.16**					
			[0.68]	[1.06]	[1.25]					
f(vol) · D(Q9,Q10)								**-0.07**	**-0.03**	**-0.04**
								[-2.64]	[-1.36]	[-1.77]
f(vol) · D(Q10)								**-0.13**	**-0.12**	**-0.11**
								[-3.84]	[-3.58]	[-2.86]
Market Capitalization		**-0.17**		**-0.12**	**-0.06**		**-0.13**		**-0.10**	**-0.06**
		[-3.14]		[-2.10]	[-0.84]		[-3.71]		[-3.26]	[-1.89]
Book-to-Market		**0.43**		**0.48**	**0.43**		**0.29**		**0.28**	**0.19**
		[2.98]		[3.50]	[2.55]		[4.03]		[4.29]	[2.92]
Momentum		**0.00**		**0.00**	**0.00**		**0.01**		**0.01**	**0.01**
		[-0.28]		[-0.19]	[-0.02]		[4.22]		[4.51]	[4.91]
Illiquidity					**0.01**					**0.01**
					[0.94]					[2.64]
Book Leverage					**-1.04**					**-1.00**
					[-1.60]					[-3.53]

PANEL C: Wald Tests

f(vol) + f(vol) · D1			**-0.11**	**-0.15**	**-0.24**			**0.03**	**0.01**	**-0.04**
			(0.02)	(0.00)	(0.01)			(0.41)	(0.74)	(0.26)
f(vol) + f(vol) · D1 + f(vol) · D2			**-0.05**	**-0.05**	**-0.08**			**-0.10**	**-0.11**	**-0.15**
			(0.57)	(0.64)	(0.51)			(0.00)	(0.00)	(0.00)

[a] In this table, I report descriptive statistics on the decomposition of the default risk measure (Panel A) and the results of cross-sectional asset pricing tests (Panels B&C). The descriptive statistics include the number of firm-month observations (N), the mean, the standard deviation (StDev), the first percentile (P1), the fifth percentile (P5), the first quartile (Q1), the median, the third quartile (Q3), the ninety-fifth percentile (P95) and the ninety-ninth percentile (P99). I report descriptive statistics for the original default risk measure (i.e., $(\ln k - \mu_x)/\sigma_x$), its approximation, the difference between the original and the approximated measure (abs(diff)), the component of the approximation attributable to log debt minus the expected asset payoff (f(pd)), the component attributable to

asset volatility (f(cross)), and the time-series correlation coefficient between the original and the approximated measure per firm (ρ), where I use only firms with more than 48 months of data to compute the correlation coefficient. I perform both OLS panel data regressions with standard errors clustered across firms and Fama and MacBeth (1973) regressions of the stock return onto two month lagged pricing factors. Bold numbers are estimates, and those in square parentheses are t-statistics. The pricing factors include an estimate of the market beta obtained from monthly data over the prior six months, the original default risk measure, the component of the default risk measure approximation attributable to log debt minus the expected asset payoff (f(pd)), the component attributable to asset volatility (f(vol)), market capitalization, the book-to-market ratio, 11-month momentum, the share price illiquidity measure proposed by Amihud (2002) and the book leverage ratio. In the panel data estimation, I include two interaction terms between the component of the default risk approximation attributable to asset volatility and the level of default risk. The first of these interaction terms equals f(vol) multiplied by an indicator variable equal to one if default risk exceeds fifty percent and zero otherwise (f(vol) $\cdot$ D($DR > 50\%$)), and the second equals f(vol) multiplied by an indicator variable equal to one if default risk exceeds γ percent and zero otherwise (f(vol) $\cdot$ D($DR > \gamma\%$)). In the Fama and MacBeth (1973) regressions, I include two interaction terms between the component of the default risk approximation attributable to asset volatility and the default risk decile portfolio to which a firm belongs. The first of these interaction terms equals f(vol) multiplied by an indicator variable equal to one if the firm belongs to default risk decile portfolio 9 or 10 and zero otherwise (f(vol) $\cdot$ D(Q9,Q10)), and the second equals f(vol) multiplied by an indicator variable equal to one if the firm belongs to default risk decile portfolio 10 and zero otherwise (f(vol) $\cdot$ D(Q10)). The results of Wald tests on the sum of the slope coefficients on the component of the default risk approximation attributable to asset volatility and either the first interaction term (f(vol) + f(vol) $\cdot$ D1) or the sum of the two interaction terms (f(vol) + f(vol) $\cdot$ D1 + f(vol) $\cdot$ D2) are shown in Panel C, with f(vol) $\cdot$ D1 and f(vol) $\cdot$ D2 being equal to f(vol) $\cdot$ D($DR > 50\%$) and f(vol) $\cdot$ D($DR > \gamma\%$) (f(vol) $\cdot$ D(Q9,Q10) and f(vol) $\cdot$ D(Q10)) in the OLS panel data regressions (Fama and MacBeth (1973) regressions). The sample period ranges from August 1963 to December 2009.

Table 4: Cross-Sectional Asset Pricing Tests (1981-2003)[a]

PANEL A: Summary Statistics

	N	Mean	StDev	P1	P5	Q1	Median	Q3	P95	P99
Original Default Risk	775,905	-6.16	4.72	-22.49	-15.27	-8.45	-5.21	-2.77	-0.43	0.75
Approximated Default Risk	775,905	-5.93	3.63	-17.28	-12.50	-7.83	-5.46	-3.41	-1.04	0.27
abs(diff)	775,905	1.15	2.05	0.00	0.01	0.13	0.45	1.28	4.65	9.24
f(pd)	775,905	-2.27	3.13	-11.93	-8.56	-3.85	-1.59	-0.07	1.60	2.84
f(vol)	775,905	-3.76	2.34	-9.88	-7.71	-5.19	-3.38	-1.88	-1.00	-0.76
rho	8,370	0.86	0.30	-0.53	0.13	0.90	0.97	0.99	1.00	1.00

PANEL B: Regression Analysis

	Panel Data Regressions with Clustered Errors					**Fama & MacBeth (1973) Regressions**				
Variables	1	2	3	4	5	1	2	3	4	5
Intercept	**1.75**	**2.22**	**2.12**	**2.52**	**2.51**	**1.77**	**2.16**	**1.86**	**2.10**	**2.04**
	[5.37]	[5.87]	[4.44]	[5.70]	[3.09]	[5.97]	[5.76]	[5.33]	[5.06]	[3.69]
Market Beta	**0.03**	**0.06**	**0.02**	**0.03**	**0.04**	**-0.02**	**0.02**	**-0.02**	**0.01**	**0.02**
	[0.53]	[0.92]	[0.36]	[0.53]	[0.55]	[-0.42]	[0.52]	[-0.64]	[0.43]	[0.73]
Original Default Risk	**0.05**	**0.01**				**0.04**	**-0.01**			
	[2.28]	[0.17]				[2.07]	[-0.49]			
f(pd)			**0.07**	**0.00**	**0.08**			**0.02**	**-0.01**	**0.07**
			[2.26]	[-0.11]	[1.10]			[0.80]	[-0.27]	[2.42]
f(vol)			**0.13**	**0.11**	**0.11**			**0.12**	**0.05**	**0.00**
			[1.83]	[1.36]	[1.16]			[1.82]	[1.17]	[-0.04]
f(vol) · D($DR > 50\%$)			**-0.32**	**-0.33**	**-0.40**					
			[-3.98]	[-3.73]	[-2.95]					
f(vol) · D($DR > \gamma\%$)			**0.00**	**0.04**	**0.02**					
			[0.01]	[0.27]	[0.10]					
f(vol) · D(Q9,Q10)								**-0.07**	**-0.02**	**-0.04**
								[-1.83]	[-0.83]	[-1.29]
f(vol) · D(Q10)								**-0.20**	**-0.18**	**-0.16**
								[-3.69]	[-3.53]	[-2.66]
Market Capitalization		**-0.12**		**-0.07**	**0.00**		**-0.14**		**-0.11**	**-0.08**
		[-2.02]		[-1.09]	[0.01]		[-2.93]		[-2.61]	[-1.50]
Book-to-Market		**0.46**		**0.56**	**0.44**		**0.34**		**0.32**	**0.18**
		[3.06]		[4.08]	[2.48]		[3.52]		[4.16]	[2.26]
Momentum		**0.00**		**0.00**	**0.00**		**0.00**		**0.00**	**0.01**
		[-0.75]		[-0.70]	[-0.37]		[3.81]		[3.99]	[4.53]
Illiquidity					**0.01**					**0.01**
					[1.07]					[2.07]
Book Leverage					**-1.73**					**-1.74**
					[-2.07]					[-4.68]

PANEL C: Wald Tests

f(vol) + f(vol) · D1			**-0.19**	**-0.22**	**-0.29**			**0.05**	**0.03**	**-0.04**
			(0.00)	(0.00)	(0.01)			(0.41)	(0.53)	(0.45)
f(vol) + f(vol) · D1 + f(vol) · D2			**-0.19**	**-0.18**	**-0.27**			**-0.15**	**-0.15**	**-0.20**
			(0.09)	(0.13)	(0.08)			(0.00)	(0.00)	(0.00)

[a] In this table, I report descriptive statistics on the decomposition of the default risk measure (Panel A) and the results of cross-sectional asset pricing tests (Panels B&C). The descriptive statistics include the number of firm-month observations (N), the mean, the standard deviation (StDev), the first percentile (P1), the fifth percentile (P5), the first quartile (Q1), the median, the third quartile (Q3), the ninety-fifth percentile (P95) and the ninety-ninth percentile (P99). I report descriptive statistics for the original default risk measure (i.e., $(\ln k - \mu_x)/\sigma_x$), its approximation, the difference between the original and the approximated measure (abs(diff)), the component of the approximation attributable to log debt minus the expected asset payoff (f(pd)), the component attributable to

asset volatility (f(cross)), and the time-series correlation coefficient between the original and the approximated measure per firm (ρ), where I use only firms with more than 48 months of data to compute the correlation coefficient. I perform both OLS panel data regressions with standard errors clustered across firms and Fama and MacBeth (1973) regressions of the stock return onto two month lagged pricing factors. Bold numbers are estimates, and those in square parentheses are t-statistics. The pricing factors include an estimate of the market beta obtained from monthly data over the prior six months, the original default risk measure, the component of the default risk measure approximation attributable to log debt minus the expected asset payoff (f(pd)), the component attributable to asset volatility (f(vol)), market capitalization, the book-to-market ratio, 11-month momentum, the share price illiquidity measure proposed by Amihud (2002) and the book leverage ratio. In the panel data estimation, I include two interaction terms between the component of the default risk approximation attributable to asset volatility and the level of default risk. The first of these interaction terms equals f(vol) multiplied by an indicator variable equal to one if default risk exceeds fifty percent and zero otherwise (f(vol) $\cdot$ D($DR > 50\%$)), and the second equals f(vol) multiplied by an indicator variable equal to one if default risk exceeds γ percent and zero otherwise (f(vol) $\cdot$ D($DR > \gamma\%$)). In the Fama and MacBeth (1973) regressions, I include two interaction terms between the component of the default risk approximation attributable to asset volatility and the default risk decile portfolio to which a firm belongs. The first of these interaction terms equals f(vol) multiplied by an indicator variable equal to one if the firm belongs to default risk decile portfolio 9 or 10 and zero otherwise (f(vol) $\cdot$ D(Q9,Q10)), and the second equals f(vol) multiplied by an indicator variable equal to one if the firm belongs to default risk decile portfolio 10 and zero otherwise (f(vol) $\cdot$ D(Q10)). The results of Wald tests on the sum of the slope coefficients on the component of the default risk approximation attributable to asset volatility and either the first interaction term (f(vol) + f(vol) $\cdot$ D1) or the sum of the two interaction terms (f(vol) + f(vol) $\cdot$ D1 + f(vol) $\cdot$ D2) are shown in Panel C, with f(vol) $\cdot$ D1 and f(vol) $\cdot$ D2 being equal to f(vol) $\cdot$ D($DR > 50\%$) and f(vol) $\cdot$ D($DR > \gamma\%$) (f(vol) $\cdot$ D(Q9,Q10) and f(vol) $\cdot$ D(Q10)) in the OLS panel data regressions (Fama and MacBeth (1973) regressions). The sample period ranges from August 1981 to December 2003.

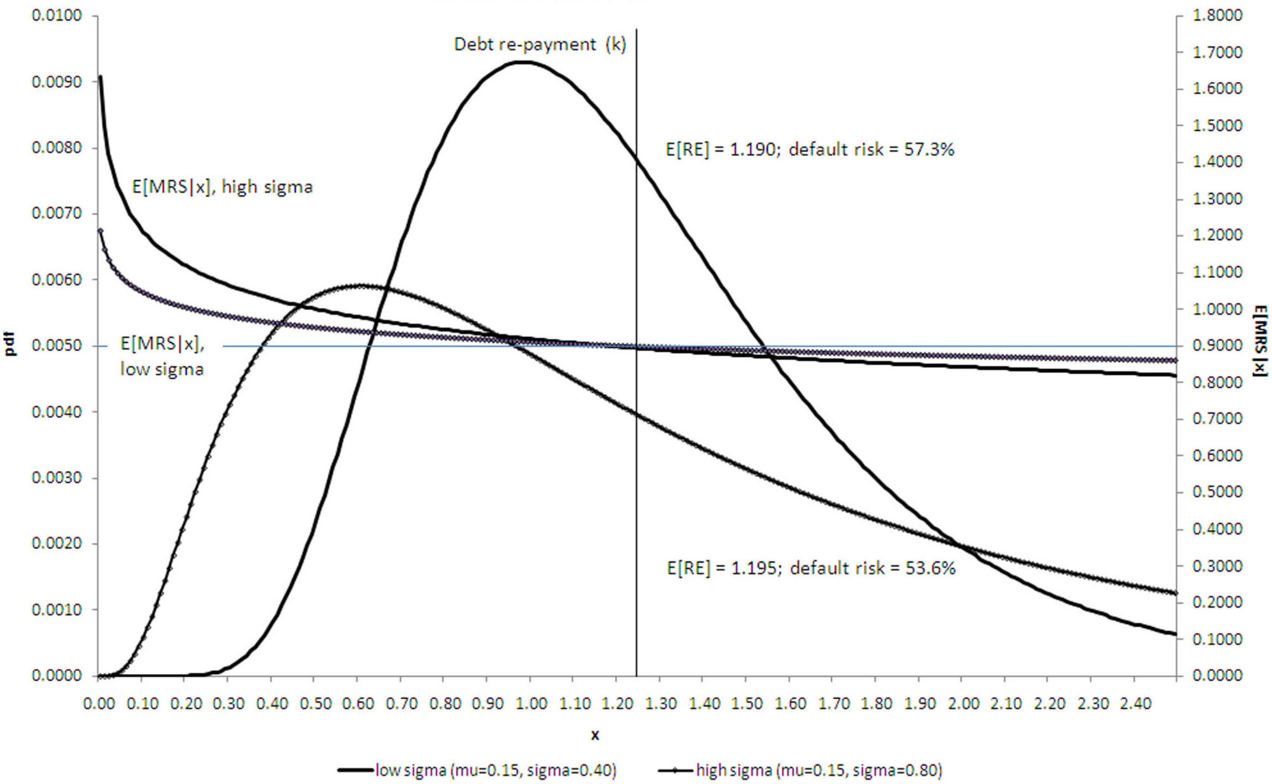

Figure 1: The figure shows the density function of the asset payoff conditional on two different values for asset volatility, together with the corresponding expected equity return ($E[R_E]$) and default risk. The value of the density function can be read off the left y-axis. The conditional expectation of the marginal rate of substitution ($E[MRS|x]$) is represented by the convex black line, and its value can be read off the right y-axis. The solid vertical line is the debt repayment, while the solid horizontal line is the unconditional expectation of the marginal rate of substitution. The base case values are as follows. Relative risk aversion (B) is 1, and the time preference parameter (ρ) is 0.95. The expectation ($\hat{\mu}_C$) and standard deviation ($\hat{\sigma}_C$) of log consumption are set to 0.15 and 0.10, respectively. Consumption at time 0 (C_0) is 1.10. The correlation between the log asset payoff and log consumption (κ) is 0.50. The expected asset payoff (μ_x), the debt level (k) and asset volatility (σ_x) are 0.15, 1.25 and 0.40 (low sigma) or 0.80 (high sigma), respectively.

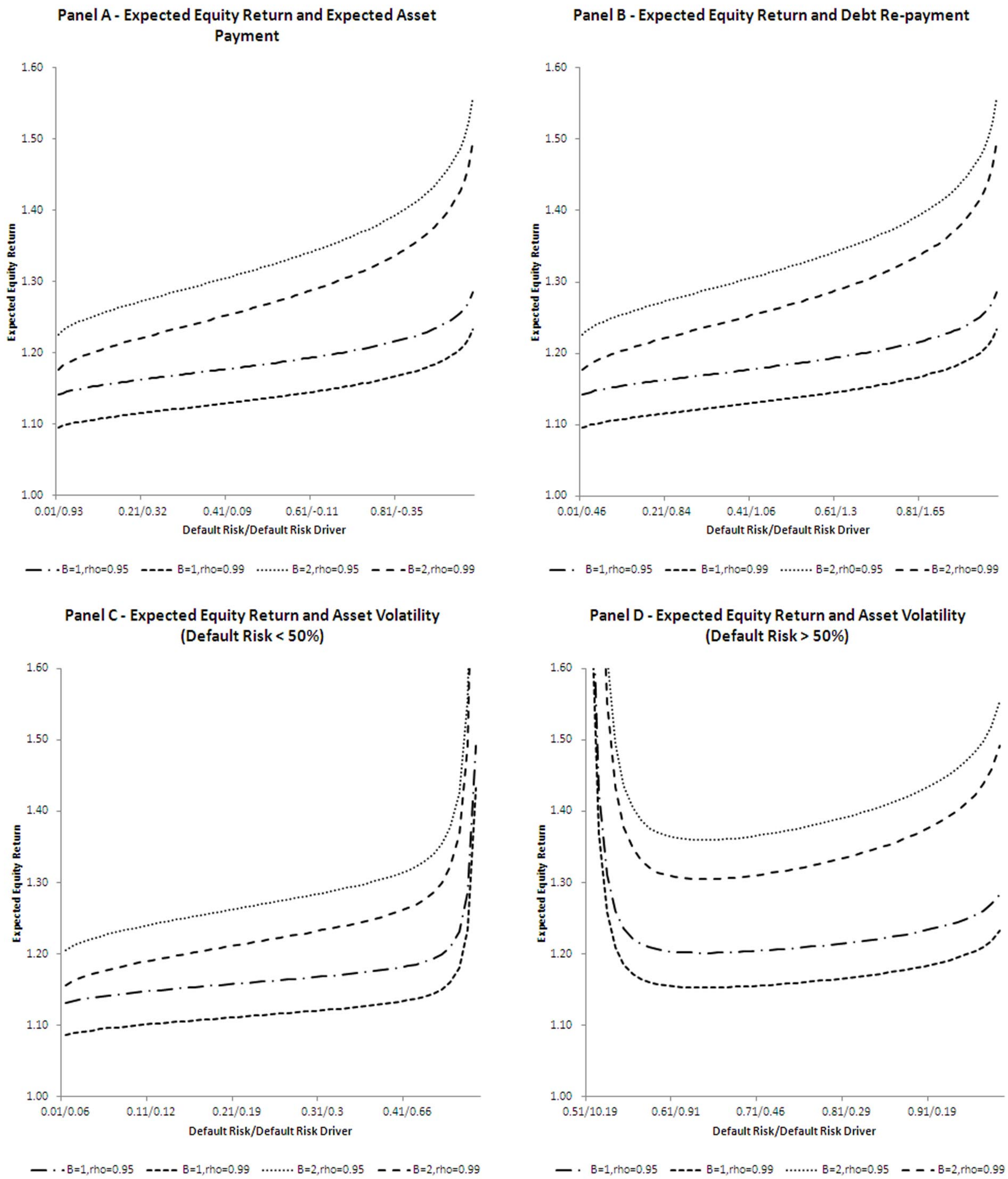

Figure 2: The figure shows the relation between default risk driven by changes in the expected asset payoff (Panel A), the debt level (Panel B) and asset volatility (Panels C & D) and the expected equity return. The effect of asset volatility on the expected equity return is examined for firms with a default risk below fifty percent (Panel C) and for firms with a default risk above fifty percent (Panel D). The base case values are as follows. The expectation ($\hat{\mu}_C$) and standard deviation ($\hat{\sigma}_C$) of log consumption are set to 0.15 and 0.10, respectively. Consumption at time 0 (C_0) is 1.10. The correlation between the log asset payoff and log consumption (κ) is 0.50. The expected asset payoff (μ_x), the debt level (k) and asset volatility (σ_x) are 0.15, 1.00 (Panels A-C) or 1.25 (Panel D) and 0.40, respectively. The values for the risk aversion parameter (B) and the time preference (ρ) parameters are indicated in the legend shown in the figure.