Pricing Derivatives with modeling CO₂ Emission Allowance Using a Regime Switching Jump Diffusion Model:

Evidence from the European Trading Scheme

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Abstract

Extreme climate change in the global environment results in losses of human health and properties, and brings the greatest threats to humanity. Many countries and organizations have taken measures to reduce CO₂ against extreme climate change. The European Union Emissions Trading Scheme (EU ETS) is one of the most important initiatives ever taken to limit the greenhouse gas emission that causes climate change. Carbon markets trading with the spot European Union Allowance (EUA) is used to prevent from CO₂ emissions. Daskalakis et al. (2009) found that a jump diffusion model provides a better fit to the return of the spot EUAs than other models. In our paper, we find that there is a volatility clustering feature in the return of the EUAs. The jump diffusion model cannot capture the phenomenon of the volatility clustering. We propose a regime switching jump diffusion model to capture the dynamics of the EUAs return for the reason that the CO₂ emissions policy and the market economics just like a financial crisis affect the switching intensities of jump arrivals. Moreover, we find that the regime switching jump diffusion model not only produces the volatility clustering feature but also provide the theoretical prices that are closer to carbon market prices than the jump diffusion model in valuation of futures and futures options with stochastic convenience yield.

Key word: European Union Emissions Trading Scheme; Markov modulated Poisson process; Esscher transform; Black-Scholes formula; Jump diffusion model, Convenience yield.
1. Introduction

Since the industrial revolution originated in Britain in the eighteenth century, human beings excessively rely on petrochemical products, including the burning of fossil fuels. A large number of emissions of carbon dioxide (CO₂) and the greenhouse gas lead directly to the climate change, and thereby causing the extreme environmental changes such as an increasing the frequency of hurricanes, droughts and floods, colder or hotter weather. In addition, it also causes a huge loss of human life and properties. As a result, many countries and organizations have taken measures to reduce CO₂ for lessening the effect of extreme climate change. The Kyoto Protocol, the most crucial international commitments, was stipulated for lowering greenhouse gases CO₂. And carbon markets were born to reduce from CO₂ emissions.

The carbon trading markets are divided into two categories of carbon products. The one is the carbon trading program including European Union Emissions Trading (EU ETS), the New South Wales (NSW), the Chicago Climate Exchange (CCE), the Regional Greenhouse Gas Initiative (RGGI), and the Assigned Amount Units (AAU). The other one is the greenhouse gas reduction projects in Kyoto Protocol, including the Clear Development Mechanism (CDM), the Joint Implementation and the voluntary market. Table 1 presents the evolution of the trading value in the carbon markets from 2004 to 2010. Other allowances involve the trading carbon value in NSW, CCE, RGGI, and AAU, and other offsets cover the Joint Implementation and the voluntary market. The trading value of EUAs was the increasing from 2005 to 2010, and EUAs accounted for 84 percent of the global carbon market value in 2010.
There are some differences between the carbon markets of the EU ETS with the financial markets such as the provisions of cap-and-trade, the characteristic of different periods, and the largest existing emission trading scheme for the spot EUAs. Seifert et al. (2008) presented a tractable stochastic equilibrium model reflecting stylized features of the EU ETS, including the largest existing emission trading scheme, penalty cost, banking and borrowing, the trading period break, and increasing marginal abatement costs, to analyze the resulting CO₂ spot price dynamics. Their main findings are that CO₂ prices do not follow any seasonal patterns, discounted prices should possess the martingale property, and an adequate CO₂ price process should exhibit a time- and price- dependent volatility structure.

Hinz and Novikov (2010) explained the logical principles underlying risk-neutral modeling of emission certificate price evolution, and showed that within a realistic situation of risk-averse market equilibrium, but there is a useful feedback relation characterizing risk. The presence of jump events is particularly important to describe price shocks, which may result from possible discontinuities in the information flow. Moreover, Borovkov et al. (2010) extended to the view of Hinz and Novikov (2010) to assume the emission allowance certificates to be the jump diffusion models under the continuous time and develop the solution property of a stochastic partial difference equation (SPDE) by the finite difference method. Caromna et al. (2009) investigated in the dynamic price equilibrium and optimal market design and provided a mathematical analysis of market equilibrium and an optimal stochastic control to show social optimality. Then, Cetin (2009) considered the different stages of the spot along with the Markov chain in the local risk minimization to discuss
evaluation and hedging.

In theoretical pricing for the risk neutral measure and empirical finding for derivatives of the spot EUAs, Daskalakis et al., (2009) investigated the tree main markets for EUAs within the EU ETS: Powernext, Nord Pool and European Climate Exchange (ECX), and suggested that the prohibition of banking of emission allowances between distinct phases of the EU ETS has significant implications in terms of futures pricing. They develop an empirically and theoretically valid framework for the pricing and hedging of intra-phase and inter-phase futures and options on futures, respectively. Uhrig-Homburg and Wagner (2009) studied the relationship between the spot EUAs and the EUA futures, and found that these futures contracts lead the price discovery process of the spot EUAs. However, it is important to note that due to the market design, first-, and second-period EUAs (called Phase I and Phase II in our paper) are just two different goods. Therefore, the link between first-period spot and second-period futures is naturally very weak. Benz and Truck (2009) recommended that several Markov and AR-GARDH modes are proposed to fit the time series in the return of EUAs from ET EUS. Cetin and Cerschuere (2010) developed hidden Markov modes and a filtering approach to capture the impact of news releases, and derived the option pricing within the EU ETS.

Daskalakis et al., (2009) find that a jump diffusion model provides a better fit to the return of the spot EUAs than other models. In our paper, we find that there is a volatility clustering feature in the return of the EUAs. The jump diffusion model cannot capture the phenomenon of volatility clustering. We propose a regime switching jump diffusion model to capture the dynamics of the EUAs return based on the impact of economics and policies.
releases, because the policies of the EU ETS and the economics of the carbon market just like a financial crisis affect the switching intensities of jump arrivals. Moreover, we find that the regime switching jump diffusion model not only produce the volatility clustering feature but also the theoretical prices of futures and futures options are closer to the carbon market price than the jump diffusion model in the assumption of the stochastic convenience yield.

This article is organized as follows: Section 2 describes the EU emissions trading carbon emission allowances using statistic description and the economic analysis at BlueNext and EEX. In Section 3, we investigate financial econometrical models, including the Black-Scholes models and jump diffusion models, as well as regime switching jump diffusion models. The pricing formulas of futures and futures options under the EU ETS are derived in Section IV. Section V gives the empirical analysis of the futures option market. The final section concludes the results.

2. Economic analysis of EUAs in the EU ETS

This section investigates the dynamics of the spot EUAs, the EUA futures and the futures options, provides economic and policies analysis and show how to affect the abnormal price change of the carbon market based past results. The trading of spot emission allowances in Europe is mainly performed through two of the largest EUAs markets, the BlueNext and EEX. For 2009, the first market accounted for almost 63% of the spot market transactions in the exchanges of EUETS while the other for around 16%. The data used in this paper consist of daily closing prices for the period 09/03/2005-28/12/2007 and 26/02/2008- 30/12/2010 at BlueNext. Panel A and Panel B presents the daily closing price
and return dynamics of EUAs from BlueNext in Figure 1, respectively.

【Insert Figure 1】

According to the execution of the Kyoto Protocol at 2008, the dynamics of the EU ETS is divided into the three phases (Alberola et al., 2008; Chevalier, 2009 and 2010; Denny et al., 2010): the first phase, called Phase I, presents the previous Kyoto Protocol era from 2005 to 2007, the Phase II denotes the Kyoto Protocol era from 2008 to 2012, and the Phase III indicates the Post-Kyoto Protocol era from 2013 to 2020. During Phase II, the European Union target for the then fifteen member states was a reduction of 8 percent below 1990 emissions levels. The target in Phase III is that emission will be 21% lower than in 2005. In three phases, the dynamics, the statistics and the characters in the price of spot EUAs are described as following.


The EU ETS started on January 1, 2005. The Phase I was introduced as a warm up period and it operated in this phase in order to put in place the policy infrastructure of permits trading. Beginning at €8/ton on January 1, 2005 EUA prices rose to €25-30/ton until the release of 2005 verified emission on April 24, 2006 which had a depressive effect on EUA prices as shown by the sharp break in the EUA spot price. Based on this sharp break, and Alberola et al., (2008) divided the Phase I into the Period I from 24/06/2005 (data start in BlueNext) to 24/04/ 2006 and the Period II from 01/06/2006 to 28/12/2007. From data from the BlueNext, there are some economic features in EUAs as following.
Period I: Demand over Supply

In this period, the beginning at 8€/ton on January 1, 2005, EUA price increased to around 30€/ton on July 2005, fluctuated in range of 20-25€/ton during about the following six months, then rose to 30€/ton. In this period, demand comes mainly from power producers, while most other market participants did not take advantage of buying/selling carbon allowances. Therefore, the EUAs of demand continues come primarily from power operators, and its increases during the winter due to the rise of energy prices, especially gas price (Alberola et al., 2008; Kanen, 2006; Christiansen et al., 2005; Bunn and Fezzi, 2007; Convery and Redmond, 2007; Mansanet-Bataller et al., 2007). The main deriver of jumps is from energy price in the carbon prices in Period I. In Table 2, the number of jump events is 11 in Period II when the excess ± 5% in spot EUAs is regards as the jump events.

【Insert Table 2】

Period II: Excess of Supply over Demand

On the last week of April 2006 prices collapsed when operator disclosed 2005 verified emission data was oversupplied. EUA prices moved in the range from 15€/ton to 20€/ton until October 2006 (Alberola et al., 2008). In addition to the oversupplied 2005 verified emission, Period II are declining towards zero due to the banking restrictions, which allowances distributed during Phase I are not valid during Phase II; however, during Phase II and Phase III EUAs are fungible between the different phase. Therefore, the policy issue, including the oversupplied verified emission and the banking restrictions, is the main driver of EUA decreasing toward zero. The main deriver of jumps is from policy issue in the carbon
prices in Period II. Carbon price drivers vary depending on institutional events such as emissions cap and banking restrictions referred to Alberola et al., (2008). In Table 2, the number of jump events is 147 in Period II when the excess $\pm 5\%$ in spot EUAs is regards as the jump events, and the number of the decreasing jump events is around $2/3$ for the total number of jump events.

**Phase II: Kyoto Protocol Era (2008-2012)$^4$**

In Phase II, EUA prices are increasing to $20\,\text{€/ton}$ primarily due to the European Commission which has reaffirmed its will to enforce tighter targets. During Phase I, if an installation does not meet its emission target during the year, then penalty is equal to $40\,\text{€/ton}$ in excess, plus the restitution of one allowance during the next year. During Phase II, this amount corresponds to $100\,\text{€/ton}$, following the same principle. Carbon prices have been a more stable and healthy price pattern comparing to the phase I. As shown in Figure 1, the price has been oscillating between $10\,\text{€/ton}$ and $30\,\text{€/ton}$ of CO$_2$, depending on the levels of allowances demand due to industrial production, and the likely depressive impact of the economic recession crisis. Therefore, the main derivers of jump events are the economic business and the energy price in the carbon prices in Phase II. In Table 2, the number of jump events is 34 in Phase II when the excess $\pm 5\%$ in spot EUAs is regards as the jump events

Christiansen et al., (2005) and Alberola et al., (2008) lead to the identification of carbon

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$^4$ South Africa hosted the 17$^{th}$ Conference of Parties of the United Nations Framework Convention on Climate Change (UNFCCC COP 17) in 2011. There are two importance results from the UNFCCC COP 17 climate summit in Durban. First, the Kyoto Protocol will be on life support until it is replaced by a new agreement. Secondly, Ad Hoc Working Group on the Durban Platform for Enhanced Action will have protocol or legal instrument or agreed outcome with legal force after 2020.
prices main drivers being policy issues, energy prices, temperature events and economic activity. These main drivers also make the effects of the carbon price jumps. From the number of the EUAs jump events at Period I, Period II, and Phase II, the arrival rates are different in different periods. Therefore, the dynamic behavior of the spot EUAs and the spot EUAs derivative is analyzed by using Black-Scholes model, jump diffusion model, and regime switching jump diffusion model in this paper.

3. Financial econometrics analysis of emission allowances from the spot markets

In this section, we introduce the properties and characteristics of three models, including the Black-Scholes model, jump diffusion model and regime-switch jump diffusion model, and give the parameters estimation method with past literatures. Then, the carbon emission allowances are estimating and testing with the three models with the features of the spot market in BlueNext. Finally, the characteristics of the carbon emissions allowances markets are appeared by financial analyses which are consistent with the regime switch jump diffusion model.

3.1 Black-Scholes model

Black-Scholes (1973) model is used to compute the valuation of options and futures. Let $S_t$ be the spot EUAs at the time $t$. Then, the dynamics of the spot EUAs can be described by the Brownian motion process as follows,

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t),$$

(1)
where $\mu$ denotes the mean of the return at the instantaneous time $t$, $\sigma$ presents the volatility of the return at the instantaneous time $t$, and $dW(t)$ is the Brownian motion. By Itô’s Lemma, the log-return can be denoted as following equation,

$$R(t) = \mu + \sigma Z,$$

where $R(t)$ denotes the log-return at time $t$, $\mu$ denotes the mean of the log-return at the discrete time $\Delta t$, $\sigma$ presents the volatility of the log-return at the discrete time $\Delta t$, $Z$ is the standard normal distribution with zero mean and variance one.

Fama (1965) presented strong and voluminous evidence in favor of the random-walk hypothesis and leptokurtosis for the behavior of the stock price. Mandelbrot (1963) found that the “stable Paretian” is better to fit the underlying asset returns than normal distribution for the skewness and kurtosis. Kou (2002) also showed that BSM cannot capture leptokurtosis in the dynamic behavior of the stock return, and the volatility smile in the option markets. Merton (1976) and Kou (2002) propose a jump diffusion model to address the skewness and kurtosis to product the behavior of the stock return, and the volatility smile in option markets.

### 3.2 Jump diffusion model

Merton (1976) considered the normal variation and the abnormal variation and developed the jump diffusion model to capture the continuous process with the Brownian motion and the discontinuous variation with the compound Poisson process. More precisely, the dynamics of the EUAs price can be denoted as follows,
\[
\frac{dS(t)}{S(t-)} = \mu dt + \sigma dW(t) + d\left(\sum_{n=1}^{N(t)} (e^{Z_n} - 1)\right), \tag{3}
\]

where \( \mu \) denotes the mean of the return at the instantaneous time \( t \) conditional on no jump events happened, and \( \sigma \) presents the volatility of the return at the instantaneous time \( t \) conditional no jump events happened. The term, \( \sum_{n=1}^{N(t)} (e^{Z_n} - 1) \), denotes the compound Poisson, where \( N(t) \) presents a Poisson process with the mean \( \lambda t \) of the jump event over period from 0 to \( t \), and \( Z = \ln \frac{S(t)}{S(t-)} \) with the assumption of the normal distribution for the mean \( \mu_y \) and the variance \( \sigma_y^2 \) when jump events is happened. Similarly, using Itô Doléans, the return dynamics of the spot EUAs can be denoted as follows,

\[
R(t) = \mu + \sigma Z + \sum_{n=1}^{N(t)} Z_n \tag{4}
\]

where \( R_t \) denotes the log-return at time \( t \), \( \mu \) denotes the mean of the log-return at the time period \( \Delta t \), \( \sigma \) presents the volatility of the log-return at the time period \( t \), \( Z \) is the standard normal distribution. \( \sum_{n=1}^{N(t)} Z_n \) denotes the compound Poisson with the discontinuous time \( \Delta t \), where \( N(t) \) presents Poisson process with mean \( \lambda \) for the time period \( \Delta t \) and \( Z_i \) is the normal distribution with mean \( \mu_y \) and the variance \( \sigma_y^2 \).

A volatility clustering phenomena explored by Mandelbrot (1963) essentially implies that large values of volatility are usually followed by large values and that small values are
followed by small ones. Kou (2002) pointed out that the jump diffusion model cannot capture the clustering fluctuations. The JDM cannot address the volatility clustering to consistent with our observation of the carbon market returns, when the messages come along the high-frequency for the period or low frequency for the other period. Daskalakis et al., (2009) found that a jump diffusion model is better to capture the dynamics of the return of the spot European Union Allowance (EUAs) than lots of models by statistic tests, including Geometric Brownian motion model (GBM), mean reverting square-root model (MRSRM), mean reverting logarithmic model, constant elasticity of variance, GBM with jumps risks, and MRSRM with jump risks. However, the jump diffusion model cannot address the volatility clustering. Therefore, we use a regime switching jump diffusion model to fit the return of the EUAs and capture the feature of the volatility clustering.

3.3 Regime switching Jump diffusion model

Chang et al., (2011) propose a regime switching jump diffusion model to capture the leptokurtic return features, volatility smile, and volatility clustering by the Markov chain of the two intensities. More precisely, the dynamics of the EUAs price is assumed as follows as following,

$$\frac{dS(t)}{S(t-)} = \mu dt + \sigma dW(t) + d\left( \sum_{n=1}^{\Phi(t)} (e^{Z_n} - 1) \right),$$

(5)

where \( \Phi(t) \) denotes a Markov-modulated Poisson process with Markov process \( X(t) \) with finite state \( I = \{1, 2\} \) and \( Z_n \) indicates an independent sequence of jump sizes when the jump event occurs with the normal distribution with mean \( \mu_y \) and the variance \( \sigma_y^2 \).
Assume that the state of the carbon market is a homogeneous hidden Markov chain \( X = \{ X(t) \} \) with the continuous time, then the matrix of the transition probability with the continuous time can be written as following

\[
P^c(t) = \begin{bmatrix} p_{11}(t) & 1 - p_{11}(t) \\ 1 - p_{22}(t) & p_{22}(t) \end{bmatrix} = e^{Rt},
\]

where \( R = \begin{bmatrix} -\alpha_1 & \alpha_1 \\ \alpha_2 & -\alpha_2 \end{bmatrix} \) denotes the matrix of the transition rate, \(-\alpha_i\) presents the transition rate arriving from the other state to the state \( i \), and \( \alpha_i \) denotes the transition rate leaving from the state \( i \) to the other state. Let \( X(i) \) and \( \Phi(t) \) are defined the joint probability as follows, \( P_{ij}(n,t) \equiv \mathbb{P}(X(0) = i, X(t) = j, \Phi(t) = n) \), where \( \mathbb{P} \) denotes the physical measure.

A Laplace transformation of the Markov-modulated Poisson process is defined as

\[
P^c_t(\xi) = \sum_{n=0}^{\infty} P^c(n,t)\xi^n, \quad \forall 0 \leq \xi \leq 1 \quad (\text{cf. Last and Brandt, 1995}).
\]

Under the Kolmogorov's forward equation, the moment generating has a unique solution for

\[
P(\xi,t) = \exp\left( (R - (1-\xi)A) t \right), \quad \text{where } \exp(A) = \sum_{n=0}^{\infty} \frac{A^n}{n!} \text{ for the square matrix } A.
\]

Similarly, using Itô Doléans and simplifying the transition of the states, the return dynamics of the spot EUAs can be denoted

\[
R(t) = \mu + \sigma Z + \begin{cases} \sum_{n=1}^{N_{1}(t)} Z_n & \text{if } X(t) = 1 \\ \sum_{n=1}^{N_{2}(t)} Z_n & \text{if } X(t) = 2 \end{cases},
\]
where $N_1(t)$ denotes Poisson process with the intensity $\lambda_1$ for the time period $\Delta t$ in the state 1, and $N_2(t)$ denotes Poisson process with the intensity $\lambda_2$ for the time period $\Delta t$ in the state 2. Here, the transition probability of Eq. (7) can be denoted as Eq. (6) for the discrete time as following equation,

$$P^d(\Delta t) = \begin{bmatrix} p_{11}(\Delta t) & 1 - p_{11}(\Delta t) \\ 1 - p_{22}(\Delta t) & p_{22}(\Delta t) \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}. \quad (8)$$

Next, we estimate the parameters and test in BSM, JDM, and RSJM models by using maximum likelihood estimation and likelihood ratio test, where the parameters and its standard deviation of RSJM is estimated in maximum likelihood method by expectation maximization algorithm (EM, Dempster et al., 1977) with a gradient algorithm (Lange, 1995) and Supplemented Expectation maximization algorithm (SEM, Meng et al., 1991). The regime dynamics and of jump dynamics of the EUAs return can be presented and be consistent with the empirical result of the Phase I and Phase II. Then, we check whether the RSJM model can capture the feature of the volatility clustering by the autocorrelation function of squared return in spot EUAs or not.

### 3.4 Empirical analysis

**Parametric estimating and testing**

The parametric estimating and testing are listed in Table 3 in three different models: BSM, JDM, and RSJM. The mean and standard deviation of BSM is 0.0011 and 0.0251 in Period I, -0.0163 and 0.924 in Period II, respective. The price of EUAs is increasing in Period I and is decreasing in Period II and the volatility of EUAs is more violent in Period II than in
Period I. From LR test results, the null hypothesis of BSM are concluded to be reject, meaning that is significance with 95% confidence level, which is consistent with the results of Daskalakis et al., (2009) to find that JDM is better than competing models. The jump frequency, the mean and volatility of logarithm jump size of JDM are 0.4846, -0.0026, and 0.0314 in Period I, 0.9789, -0.1710 and 0.0879 in Period II, respective. The jump frequency of EUAs is increasing from Period I to Period II, the logarithm jump size with mean and volatility of JM are downward and violent from Period I to Period II.

Further, based on the LR test results of three different models in Phase I of Table 3, the JDM is better than BSM by the LR test result (632.38 is significant) with the null hypothesize of BSM, and the RSJM is better than JDM by the LR test results (134.21 is significant) with the null hypothesize of JDM. With the RSJM, the transition probabilities $p_{11}$ and $p_{22}$ are 0.9868 and 0.9830, respectively. Both probabilities are close to one, implying that the probabilities of switching from low frequency (0.0003) to high frequency (1.7970) and vice versa are very small. Carbon price drivers vary depending on institutional events such as emissions cap and banking restrictions, which affects the frequencies of information arrivals for good news or bad news. Finally, three different models are testing in Phase II of Table 3, the RSJM is also better than JDM by the LR test results (79.89 is significant) with the null hypothesize of JDM. Carbon price drivers vary depending on economic activity such as financial crisis, which also affects the frequencies of information arrivals for good news or bad news.
Figure 2 plots the dynamics of the spot EUAs price, its logarithm return, the probability of low frequency, and the probability of jumps. In panel A, the financial crisis is on July, 2008 and the EUAs prices went up because the emission of the CO\textsubscript{2} is low in the industrial production. Panel B shows that the volatilities from 2008 to 2009 are larger than the volatilities from 2009 to 2010, implying that the jump frequency is high in 2008. Volatility clustering can be observed in panel B. Panel C indicates that the probability of low frequency from February, 2008 to July, 2009 was high, because of the new CO\textsubscript{2} emission policy, and the probability of low frequency from October, 2008 to August, 2009 was low, because of the financial crisis caused the decreasing of the CO\textsubscript{2} emission. Hence, there is a transition of states in October, 2008 from the low frequency to the high frequency and August, 2009 from the high frequency to the low frequency. In around October, 2008, as the financial crisis occurred, the probability of low (high) frequency became lower (higher). There was also a switch of states around August, 2009, which the probability of low (high) frequency in August, 2009 also became higher (lower). There are switching features on September, 2009, November, 2009, and March, 2010, however, the high frequencies keep a short period. Panel D also shows the jump probability was large from October, 2008 to August, 2009, consisting of events of the financial crisis.

**Volatility Clustering**

Mandelbrot (1963) and Fama (1965) have pointed out that that large values of volatility are usually followed by large values and that small values are followed by small ones in the stock returns, called volatility clustering. This phenomenon of volatility clustering can be found out in the Panel B of Figure 1. The volatility clustering (Cont, 2007) is defined that the
The autocorrelation of squared returns is significant and slowly decreasing. The autocorrelation of the squared return in RSJM can refer to Chang et al. (2010).

The Panel A of Figure 3 denotes the autocorrelation of the data squared return in spot EUAs is significant and slowly decreasing. We use the parametric estimations in Table 3, and put the estimations into the autocorrelation of squared return in the RSJM model derived by Chang et al., (2010). The Panel B of Figure 3 presents the autocorrelation of the spot EUAs return from at the Phase II for RSJM model the in BlueNext. Therefore, there is the volatility clustering feature of the spot EUAs return at the Phase II in BlueNext. That is, based on the estimated parameters of the RSJM in EUAs return, the autocorrelation of the squared EUAs returns is significant, slowly decreasing, and consist with the autocorrelation of the data squared return.

4. Pricing futures and futures options on emission allowances market

This section is discussed that the general Esscher transformation is applied to the regime switching jump diffusion process in order to become martingale property processes such that the theoretical futures option is priced. In this paper, the empirical study is provided evidences for applying the mean reverting model in convenience yields form the relative mean square error (RMSE). Therefore, the futures and futures option are derived with stochastic convenience yields in mean reverting model and stochastic EUAs in regime switching jump diffusion process under no-arbitrage theorem.
4.1 Change of measure in regime switching jump diffusion

Generally speaking, it cannot find a unique martingale measure when we consider asset returns with jump risks, since it is an incomplete market. Gerber and Shiu (1994) used the Esscher transformation to price the options of insurance products in the incomplete market. The Esscher transformation has advantages in the dynamic processes after the measure transform to maintain the dynamic structure of invariance, which makes us easy to get in the pricing process closed form solution. It is only qualified the existing condition of the moment generating function no matter how much the sizes of the jump events are. Thus, the Esscher transformation is a kind of the general Girsanov transformation. In microeconomic ideal of Esscher transformation, Badescu (2009) pointed that Esscher transformation is kind of the exponential utility function under the Bühlmann's economy in an equilibrium condition. Furthermore, this paper is promoted more flexible and general Esscher transformation with the regime switching jump risks based on the Markov process of jump states.

Consider two independent variables with the dynamic price of EUAs $\{S(t)\}$, and the jump states are described by Markov Chain $\{X(t)\}$, which is the state variable to affect the jump events of the EUAs price for the main drives such as the energy policy, temperature events and economic activity. Assume the filtration of EUAs price denoted by $\{\mathcal{F}^S_t\}$ and the filtration of hidden Markov Chain denoted by $\{\mathcal{F}^X_t\}$. Define the join filtration of EUAs and hidden Markov Chain denoted by $\{\mathcal{F}_t\}$ to be the $\sigma$-algebra with $\mathcal{F}^X_t \lor \mathcal{F}^S_t$, and assume the transition of the state is known in the future $\mathcal{F}^X_t$, that is, there is no risk
premium when the current state changes into another state. Then, the Esscher transform referred Sui et al. (2008) is as follows,

$$\frac{d\mathbb{P}^h}{d\mathbb{P}}\bigg|_{\mathcal{F}_T} = \mathbb{E}^p \left[ \frac{\exp \left( h^C \sigma W(T) + h^J \sum_{n=1}^{\Phi(T)} Z_n \right)}{\mathbb{P} \left[ \exp \left( h^C \sigma W(T) + h^J \sum_{n=1}^{\Phi(T)} Z_n \right) \mathbb{1} \left\{ \mathcal{F}_0^S \cup \mathcal{F}_T^X \right\} \right]} \right]$$

(9)

where $h^C$ and $h^J$ are denoted as the risk premium in Brownian notion and jump risks under the Esscher transform, respectively, which is based on the Novikov’s condition,

$$\mathbb{E}^p \left[ \exp \left( h^C \sigma W(T) + h^J \sum_{n=1}^{\Phi(T)} Z_n \right) \mathbb{1} \left\{ \mathcal{F}_0^S \cup \mathcal{F}_T^X \right\} \right] < \infty.$$  

According to the definition of martingale for the discounted stock price under the risk neutral probability, the martingale condition is following

$$\mu + h^C \sigma^2 - r + \lambda_1 (\phi(h^J + 1) - \phi(h^J)) = 0$$

$$\mu + h^C \sigma^2 - r + \lambda_2 (\phi(h^J + 1) - \phi(h^J)) = 0$$

where $\phi(h^J)$ denotes the moment generating function with the normal distribution with mean $\mu_y$ and the variance $\sigma_y^2$. Since it is an incomplete market, there are infinite solutions for the parameters of Esscher transform satisfied with the martingale condition (Also see Bo et al., 2010). Assume that there are risk premiums in continuous part and discontinuous part. We can find a solution satisfied martingale condition as follows,

$$h^C_c = \frac{r - \mu}{\sigma^2}, \quad \text{and} \quad h^J = \frac{-\mu_y - \frac{1}{2} \sigma^2_y}{\sigma_y^2}.$$
Based on the solution of martingale condition, the dynamic process of Spot EUAs by Esscher transform under risk neutral $\mathbb{Q}$ as follows (Detail in appendix B),

$$
\frac{dS(t)}{S(t-)} = r dt + \sigma dW^Q(t) + d \left( \Phi^Q(t) \sum_{n=1} (e^{\sigma^2 n} - 1) \right),
$$

where $\{Z^Q_n\}$ follows the normal distribution with mean $-\frac{1}{2} \sigma^2$ and variance $\sigma^2$,

$N(-\frac{1}{2} \sigma^2, \sigma^2)$, $\Phi^Q(t)$ denotes a new Markov-modulated Poisson process with the new intensity matrix with parameters

$$
\Lambda^Q = \begin{bmatrix}
\lambda_1 \exp\left(\frac{\sigma^2}{8} \frac{\mu^2_y}{\sigma^2_y}\right) & 0 \\
0 & \lambda_2 \exp\left(\frac{\sigma^2}{8} \frac{\mu^2_y}{2\sigma^2_y}\right)
\end{bmatrix},
$$

and the invariant Markov Chain $X(t)$ with parameters $R = \begin{bmatrix} -\alpha_1 & \alpha_1 \\ \alpha_2 & -\alpha_2 \end{bmatrix}$. Therefore, based on the new intensity parameters and the invariant Markov Chain, the joint probability under the risk neutral probability, $Q_0(n,t)\equiv Q(X(0) = i, X(t) = j, \Phi^Q(t) = n)$, where $Q$ denotes the risk neutral probability measure. The joint probability under the risk neutral measure could be obtained by the moment generating function $P(\xi, t) = \exp\left( (R-(1-\xi)\Lambda^Q) t \right)$,

where $\exp(A) = \sum_{n=0}^{\infty} \frac{A^n}{n!}$ for the square matrix $A$. 

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It is worth noting, that the pricing futures option using Merton measure regarded jump term as non-systemic risk (Ballotta, 2005). Merton measure is the equivalent martingale measure obtained by shifting the drift of the Brownian motion but leaving the jump part unchanged by assuming that “jump risk” is diversifiable as Daskalakis et al. (2009). Nevertheless, Jarrow and Rosenfeld (1984) provided empirical evidences that the jump component does affect the equilibrium price of contingent claims, that is, the jump component is belong to the systemic risk. Moreover, Ballotta (2005) studied about the Merton measure, Esscher measure. He found that Esscher measure appears to be the most suitable one to capture the additional risk induced by the occurrence of crashes in the market. Therefore, we think the carbon market is system risk when the jump events happen in the carbon market. The investors shall require risk premiums for the jump risks each state. The Esscher transform also reduce to the Merton measure when the jump risk premium is zero, $h' = 0$.

4.2 The valuation of futures

Constant Convenience Yield

The futures and spot prices relationship are described by no-arbitrage, as following formula:

$$ F_c(t, T_1) = E^Q[S(T_1) | \mathcal{F}_t] = S(t)e^{(r-\delta(T_t-t))}, $$

(10)

where $F_c(t, T_1)$ is the daily futures price under constant convenience yield, $S(t)$ is the daily
spot of EUAs price, futures contracts expire in $T_1$, $r$ is the risk free interest rate, $\delta$ is called convenience yield. Since commodities spot generate income or benefits for holders as well known as energy commodities. Furthermore, Fama and French (1987) found that convenience yield was changed over time. If spot is a commodity of agriculture, then the convenience yield has seasonal effects. If the spot is a metal, oil commodity then convenience yield has a mean reverting of the phenomenon. Hilliard and Reis (1998) described commodities futures convenience yield random class and studied about pricing futures options. Casassus and Collin-Dufresen (2005) applied multi-factor jump models to describe convenience yield.

However, the carbon market is emerging markets trading with a fine and limit. The carbon market is different the common stock market, many scholars have diffidence views that are following. Joyeux and Milunovich (2010) furthermore found that emissions allowance spot for different maturities of the futures with significantly convenience yield in carbon market. Uhrig-Homburg and Wagner (2009) discussed the spot and futures in the different phases the cost of carry would not be hold, and Daskalakis et al., (2009) the relationship between spot and futures are also using the risk free interests rate $r$ and convenience rate $\delta = 0$, in the intra-phase. However, many scholars had maintained different ideals for convenience yield. Trück et al., (2006) proposed that spot and futures had relation with the convenience yield at EEX. They explained that the convenience yield $\delta$ was kind of the dividend yield of stock, and found the beginning of 10/2005 on the market to 05/2006 may Backwardation Market or Inverted Market of phenomena that is $S(t) > F(t)$. At the case, there is a negative convenience yield $\delta < 0$. However, we observe similar results in
empirical studying as show in Table 5. It is shown that the convenience yield $\delta$ alternate with positive and negative. We obtain the evidences show not only interest rate factors, but also have convenience yield $\delta$ on carbon allowances market. Furthermore economic analysis, as well known as the Spot EUAs is zero cost at the beginning, and the controlled manufacturers cannot submit enough allowance, or they are fined highly without enough EUAs. The EU will reduce allowance quantities in the next year. Hence, the Spot EUAs is appeared a convenience such as $\delta > 0$ at 2005. The convenience yield becomes $\delta < 0$ at the end of Phase I, since the spot is banned crossing used at Phase II. There are no benefits to the spot of holders. When the Phase II begins at 2008, the fines are increased year by year without enough EUAs. The fact implies that the futures are high cost after Phase III. Hence, the Spot EUAs is appeared a non-convenience for holders, which reduce to $\delta < 0$ after 2008. Thus, the convenience yields reveal that positive and negative are alternate in overall form 2005-2008. The empirical studying is followed to provide the evidences.

We define the empirical convenience yield $\delta(t_k)$ as the equation (14) to show that the $\delta(t)$ has phenomena of mean reverting as the equation (11). The empirical studying is followed

$$\delta(t_k) = r - \frac{1}{T_1-t} \ln \left( \frac{F(t_k, T_1)}{S(t_k)} \right)$$

(14)

where $F(t_k, T_1)$ at time $t_k$ are the EUA futures of ECX dailies closed prices, observed days are $t_k$, $t_k \leq T_1$, $\forall k$. We find empirical convenience yield under the conditions of no arbitrage. The Euribor (Euro Interbank Offered Rate) is an average interest rate of 57 European banks,
the interest rate is reference lending to each other. The Current Euribor rates are used as a reference interest rate for many financial products, and we list the average interest rate of 12-months maturity per month form 2005 to 2010 as shown in Table 4.

We find that variances of empirical convenience yield are difference to variances of risk free interest rate form the Table 4, especially after Phase II. The evidences show not only the interest rate factor to exist between the Spot EUAs and the EUA futures under the no-arbitrage market, but so have the factors of convenience yield existing. The carbon emission allowances market is revealed that the Spot EUAs commodities reveal convenience factors.

Stochastic Convenience Yield

The perspectives of each year convenience yields are alternating positive and negative. This is also consistent with Fama and French (1987) found that in some metals, oil commodities of convenience yield have phenomena of mean reverting. Therefore, observed in Table 5. Hillard and Reis (1998) had convenience yield as a mean reverting random process under risk-neutral measure $\mathbb{Q}$.

$$d\delta_s(t) = [\kappa_s(\theta_s - \delta_s(t)) - \lambda_s \sigma_s]dt + \sigma_s dW_s(t),$$

(11)
where $\kappa_s$ is the speed of mean-reversion, $\theta_s$ is long-term mean yield, $\sigma_s$ fluctuation of convenience yield, $\lambda_s$ is the convenience yield market price of risk.

Daskalakis, et al. (2009) pointed that the Spot EUAs and EUA futures can be held directly with the free risk interests in the same period that is different from many scholars. Then, we find that even in the same period of carbon emission allowances are presented phenomenon of the mean-reverting between the Spot EUAs and EUA futures.

The dynamic returns with the stochastic convenience yield is assumed under risk neutral measure at carbon allowances spot market as following

$$\frac{dS(t)}{S(t-)} = (r - \delta(t))dt + \sigma dW^s(t) + d\left(\sum_{n=1}^{q(t)} (e^{\lambda_n} - 1)\right)$$  \hspace{1cm} (12)

Supposed that the standard Brownian motion $dW_s(t)$ of stochastic convenience yield and the dynamic process of carbon emission allowances $dW^s(t)$ relationship is $\rho$ and the others’ assumption are unchanged. Therefore, we can get the theory future with stochastic convenience futures as following

$$F_s(t, T_1) = S(t)e^{r-H(t, T_1)S(t)}A(t, T_1)$$  \hspace{1cm} (13),

where $A(t, T_1) = \exp\left[\frac{H(t, T_1) - (T_1 - t)}{\kappa_s^2} (\kappa_s^2 \theta_s - \kappa_s \lambda_s \sigma_s / 2 + \rho \sigma_s \kappa_s) - \frac{\sigma_s^2 H^2(t, T_1)}{4 \kappa_s}\right]$ and

$$H(t, T_1) = \frac{1 - e^{-\kappa_s(T_1 - t)}}{\kappa_s} \text{ for all } t \leq T_1.$$

We can observe features of dynamic random futures with stochastic convenience yield, if volatility of futures is same the volatility of spots under fixed constant convenience yields
such as $F_c(t)$. Since the volatility of EUA futures and spot EUAs are differences from observations data. In this equation (13), features of dynamic random futures prices contain about parameters of mean reverting.

4.3 The valuation of options on emission allowances market

The symbol is denoted as following

$$C(F(t), K, r, T - t, \sigma, \sigma_j, n) = e^{-r(T-t)} E^Q \left[ (F(T) - K)^+ \Phi(t) = n \right] = e^{-r(T-t)} \left[ F(t)N(d_1) - KN(d_2) \right]$$

where

$$d_1 = \ln \frac{F(t)}{K} + \frac{1}{2} \left( \sigma^2(T-t) + \sigma_j^2 \right)$$

$$d_2 = d_1 - \sqrt{\sigma^2(T-t) + \sigma_j^2}$$

$F(t)$ is futures EUAs at time $t$, $K$ is strike price of the option, $r$ is riskless, $\sigma$ is volatility of $F(t)$, and survival time of call option is $T \leq T_i$ and $E^Q(\cdot | F_{T_i}) = E\left[ (\cdot)^+ | F_{T_i} \right]$ where the filter $F_{T_i}$ is known. In the paper, the $F_{T_i}(t)$ is endogenous for $F(t)$ into the formula of derivates. When $n$ is zero, we can obtain as the Black-Scholes futures option formula. Hence, generalized Black-Scholes formula futures option is the theorem as flowing:

**Carbon allowance futures option pricing formula**

$$C_{r}(F_{T_i}(t), K, r, T - t, V(t, T, T_i), Q) = \sum_{n=0}^{\infty} \sum_{\text{i, j, l}} \pi_i Q_j(n, \tau; \omega_i) C(F_{T_i}(t), K, r, T - t, V(t, T, T_i), \sigma_j, n)$$

(15)

where
\[
d_{1}(n) = \frac{\ln \frac{F_{s}(t)}{K} + \frac{1}{2}(V(t, T, T) + \sigma_{j}^2 n)}{\sqrt{V(t, T, T)^2 + \sigma_{j}^2 n}}
\]
\[
d_{2}(n) = d_{1}(n) - \sqrt{V(t, T, T)^2 + \sigma_{j}^2 n}
\]
\[
V(t, T, T)^2 = \sigma^2(T - t)
\]
\[
+ \frac{\sigma_{j}^2}{\kappa_{c}} \left[(T - t) - \frac{2}{k}(e^{-\kappa_{c}(T - T)} - e^{-\kappa_{c}(T - t)}) + \frac{1}{2\kappa_{c}}(e^{-2\kappa_{c}(T - T)} - e^{-2\kappa_{c}(T - t)})\right]
\]
\[
- \frac{2\rho\sigma_{c}\sigma}{\kappa_{c}} \left[(T - t) - \frac{e^{-\kappa_{c}(T - T)}}{\kappa_{c}} - \frac{e^{-\kappa_{c}(T - t)}}{\kappa_{c}}\right]
\]
\[
Q(n, t; \lambda') = \mathcal{P}^{*}(t)\big|_{\phi(h')}
\]

Proof in Appendix B

where expiration date of option futures is shorter than the futures expiration date. i.e., \( T \leq T_{1} \).

The formula is different that Hilliard and Reis (1998) adopt Merton measure under the fixed free-risk. The intensities of jump are equal but not zero, which is \( \lambda_{1} = \lambda_{2} \neq 0 \), the regime-switch model is simplified to jump diffusion model.

The equation (15) can be reduce following

\[
C_{JD}(F_{M}(t), K, \tau, r, \sigma, \sigma_{j}, \lambda^{*}) = \sum_{n=0}^{\infty} \left[e^{-\lambda^{*}\tau} \left(\frac{\lambda^{*}}{n!}\right)^{n} C(F_{M}(t), K, \tau, r, \sigma, \sigma_{j}; n)\right]
\]

where \( M = c \), or \( s \).

If intensities of jump are all equal to zero, which is \( \lambda_{1} = \lambda_{2} = 0 \), the generalized Black-Scholes remain Black-Scholes futures formula. Therefore, we can see that the generalized formula for the Back-Scholes is investment portfolio with the probabilities of Markov chain.
5. Empirical analysis of emission allowances market.

This section discusses about empirical studying of results in our formulae under the mean root squared error (MRSE). We estimate parameters of mean reverting stochastic processes using MRSE. Then $\bar{F}_c(t), \bar{F}_x(t)$ are represented as the actual futures prices of carbon allowances market in our formulae to gain values of futures option. The prices from formulae are comparison with the carbon market data from BlueNext with RMSE. The pricing performances are analyzed with out-the-money (OTM), at-the-money (ATM), and in-the-money (ITM), respectively.

5.1 Empirical analysis of futures emission allowances market.

The data is observed from 01/11/2010 to 30/12/2010 daily at futures Mar-11, Jun-11, Sep-11, Dec-11 in the ECX and spots EUAs of BlueNext. Let $\lambda_i$ be zero as Daskalakis et. al., 2009, and use RMSE to find the parameters of mean reverting model as shown in Table 5

![Insert Table 5](image)

The RMSE is defined as following:

$$\sqrt{\frac{1}{m_D} \sum_{k=1}^{m_D} \left( \frac{F_{m}(t_k) - F_{D}(t_k)}{F_{D}(t_k)} \right)^2} \quad \text{.........................(16)}$$

where $F_{m}(t_k)$ is theoretical price at time $t_k, F_{D}(t_k)$ is futures option of EUAs closed daily on ECX at time $t_k, m_D$ is total number of observes and $t_k \leq T \forall k$.

5.2 Empirical analysis of futures options emission allowances market.
Abate and Whitt (1992) provided the method that generate the probability matrix $P(t)$ to converted $Q(n,t)$ in the equation (15), and the empirical studying of the prices performance of formulae with RMSE that is similarly with the equation (16).

The estimated parameters of models are used the data where is 26/3/2008 to 30/12/2010 spot on BlueNext, riskless interest rate is average annual Euribor form 11/2010 to12/2011, which is $r = 1.9256\%$. The BSM is abbreviated from Black-Scholes model as the equation (1), the JDM is abbreviated from jump diffusion models as the equation (3). RSJM is abbreviated from regime switch jump diffusion processes as the equation (5), RSJMc is abbreviated from regime switch jump diffusion processes with stochastic convenience yield as the equation (13). We obtain that the average Spot EUAs price is 14.40€. Thus, the Call option observational data are futures options at ECX with strike price $K=12,12.5,13,13.5,14,14.5,15,15.5,16,16.5$ €, Mar-11, June-11, Sep-11 and Dec-11. There is 256 trading day per year.

The derivate of futures options formula with $F_t(t)$ as the equation (13) and $F_c(t,T_t)$ as the equation (10) under carbon market. Observations are division into the in-sample and out-sample different samples of area to test the formula. In sample period is 01/11/2010 to 30/12/2010. The out sample period is 03/01/2010 to 28/02/2011. We define OTM to be abbreviated from out-of-money subject to $S/K < 0.95$, and ATM is abbreviated from at-the-money subject to $0.95 \leq S/K \leq 1.05$, and ITM is abbreviated from In-the-money subject to $S/K > 1.05$ in each area of sample. During the in-sample period, the error of future options is 14.3634% under BSM, error of future options is 12.2328% under the JDM,
error of future options is 4.8755% under RSJM, and error of future options is 4.3004 % under RSJMc. During the out-sample area , error of future option is 24.5934% under BSM, error of future options is 22.1187% under the JDM, error of future options is 12.7232% under RSJM, and error of future option is 15.5524% under RSJMc . Total of data, error of future options is 19.8908% under BSM, error of future options is 17.6336% under the JDM, error of future options is 9.5004% under RSJM, and error of future options is 11.1843% under RSJMc. We organize in Table 6 for the others errors.

【Inserts Table 6】

We can observe performed of the three models during the in-sample period are better than themselves during out-sample .The errors at OTM are bigger than in the ITM under the three models. At different time for models testing, we can find that the results of jump diffusion model are better than the BSM model as well no matter at in-sample or out-sample. The performers of jump are also confirmed the assumption was supposed by Daskalakis et al., (2009) that the carbon market has jumping properties at difference the Phase. However it is more important that the RSJM and RSJM with stochastic convenience yield are better than the others’ models at difference period of sample. It is found that the regime switching phenomena are better approach included dynamic spot to value call option. Furthermore, it can be found that the RSJM with stochastic convenience yield is the best model at in-the-money and the RSJM is the best model at out-of money.

From data of BlueNext, we obtained parameters of regime switching jump diffusion model and find that when the state is one, \( \lambda_1 \) closed to zero. It means the RSJM has one kind
of dynamic states to close the BSM. On the other hand, the RSJM has strong a jump features such as state two. Since the intensity of RSJM $\lambda_2$ is closed to one, the messages are impact on the carbon market with high-frequency such that the carbon market has significant phenomena of jumps under EU ET. As the convenience yields, the parameters of mean-revering model are small, but they affect the volatilities of RSJM especially in the long maturity date such as Dec-11. The stochastic convenience yields also help volatilities of RSJM be corrected such that the volatilities of RSJM are not again orange volatilities of the dynamic Spot EUAs. In a sense, the volatilities are difference between futures market and spot market. In a word of overall, the performers of models are revealed that when macroeconomics is bumped by good or bad news, the carbon market has kind of innovation to influence the price of option.

6. Conclusion

This article has also made contribution in empirical and theoretical aspects under carbon emission allowance market. In empirical term, we test the dynamic process which descried the EUAs spot return of BlueNext in France. We not only find that the jump diffusion model is better than Black-Scholes model under the carbon market, but also find that jump diffusion model is better regime switching jump diffusion model for significant good explanatory statistic power under the carbon allowance market. Finally, the article analysis theoretical results of futures option pricing and compare with obverse data of ECX, under the Black-Scholes model, jump diffusion models and regime switching jump diffusion model and regime switching jump diffusion model with stochastic convenience yield. Those models are displayed error by RMSE at OTM, ATM, ITM each other conditions. We obtain that jump
regime switching jump diffusion model has the best performers with stochastic convenience yield, especially in OTM.

In theoretical term, we include the stochastic convenience yield view which is different with Daskalakis et al., (2009) proposed that spot between with futures relationship. The futures option closed-formula is derived by generalized Esscher transformation to reduce martingale property under the regime switching jump diffusion model with stochastic convenience yield. In the future, we will expand the convenience yield joined the factor of Markov chain and the results adding analysis static compare.
Appendix A

We consider that is martingale condition. Then the expectation \( E^Q \left[ S(t) \mid \mathcal{F}_t \right] \) is equal to the initial \( S(0) \) at the time \( t = 0 \) under the risk neutral measure \( Q \).

\[
E^P \left[ S(0) e^{-r t + \frac{1}{2} \sigma^2 t + \sigma W(t) + \sum_{i=1}^{n(t)} Z_n} \frac{e^{h c \sigma W(t)}}{E^P \left[ e^{h c \sigma W(t)} \right]} \frac{e^{h' \sum_{i=1}^{n(t)} Z_n}}{E^P \left[ e^{h' \sum_{i=1}^{n(t)} Z_n} \right]} \right] = S(0) \quad (A1)
\]

where
\[
\frac{e^{h c \sigma W(t)}}{E^P \left[ e^{h c \sigma W(t)} \right]} \frac{e^{h' \sum_{i=1}^{n(t)} Z_n}}{E^P \left[ e^{h' \sum_{i=1}^{n(t)} Z_n} \right]} = e^{(h c \mu \sigma)} \quad \text{is a pricing kernel of Esscher transform.}
\]

We divide the RSJM into two parts that are continuous dynamic process and jump dynamic process. Therefore, the martingale condition (A1) can be simply by the equations

\[
-rt + \mu t - \frac{1}{2} \sigma^2 t + \frac{1}{2} \left[ (h c + 1) \sigma \right]^2 t - \frac{1}{2} (h^2 \sigma)^2 t = 0
\]

\[
E^P \left[ e^{(1+h'') \sum_{i=1}^{n(t)} Z_n} \right] = e^0 = 1
\]

and it can be satisfied with \( h^c = \frac{r - \mu}{\sigma^2} \) and \( h' = \frac{-\mu t - \frac{1}{2} \sigma^2 t}{\sigma^2} \).

Since the continuous dynamic process and jump dynamic process are independent each other \( \{W(t)\} \perp \sum_{n=1}^{\Phi(t)} Z_n \). We apply Esscher transform to the continuous party \( \{W(t)\} \) and the jump party \( \sum_{n=1}^{\Phi(t)} Z_n \), respectively. The result of continuous party is similar to Girsanov
theorem. The jump party is detail as following

\textbf{Appendix B}

\[
\mathbb{P}^h \left( Z^* \in d\zeta_1, Z^*_2 \in d\zeta_2, \ldots, Z^*_m \in d\zeta_m, \Phi^* (t) = m, X^* (0) = i, X^* (t) = j \right) = \frac{e^{h^t \sum_{n=1}^{m} Z_n}}{e^{h^t \sum_{n=1}^{m} Z_n}} \cdot \mathbb{P} \left( Z^*_1 \in d\zeta_1, Z^*_2 \in d\zeta_2, \ldots, Z^*_m \in d\zeta_m, \Phi(t) = m, X(0) = i, X(t) = j \right)
\]

\[
= \frac{e^{h^t \zeta_1} \cdot e^{h^t \zeta_2} \cdots e^{h^t \zeta_m}}{\phi(h^t) \cdot \phi(h^t) \cdots \phi(h^t)} \cdot \frac{\phi(h^t)^m}{e^{h^t \sum_{n=1}^{m} Z_n}} \cdot P(m, t)
\]

Therefore, let \( f^*_Z \) be \( e^{h^t z} \) and \( Q(m,t; h^t) \) be \( \frac{\phi(h^t)^m}{e^{h^t \sum_{n=1}^{m} Z_n}} \cdot P(m, t) \). Hence, we have

the new probability density function of \( f^*_Z \) and the new Markov chain \( Q(m,t; h^t) \).

Thus, the new standard Browian motion \( W^*(t) \) under \( \mathbb{P}^h \) is given by

\[ dW^*(t) = dW(t) - \sigma h^t dt, \]

where the notation * mean the new processes with the new parameter under changed measure. The sizes of jump \{\( Z^*_n \)\} has a new probability density are

\[ N(\mu^*, \sigma^*_j) \] with \( \mu^*_j = \mu_j + h^t \sigma^*_j \) and \( \sigma^*_j = \sigma_j \). The new Markov modulate Poisson process \( \Phi^* (t) \) has the new intensity of jump \( \Lambda^*(h^t) = \Lambda \phi(h^t) \), and \( X^*(t) \) has new transformation matrix is
\[ P^* (t) = \frac{\exp \left( (R - [1 - \phi(h^k)]) \Lambda(t) \right)}{E^p \left[ \exp \left( (R - [1 - \phi(h^k)]) \Lambda(t) \right) \right]} \]

where \( E^p \left[ \cdot \right] = E^p \left[ \sigma(X(0)) \right] \), stable sates is initial value \( X(0) \), and \( \phi \) is moment generating function of \( N(\mu_j, \sigma_j^2) \).

Consider that the Europe called option

\[ E^p \left[ e^{-\alpha} (F_M(t) - K)^+ \right] = e^{-\alpha} E^p \left[ F_M(0) e^{\frac{1}{2} \sigma^2 \tau_q (t) + q(t) \chi} \frac{e^{\chi h \gamma_j \chi}}{E^p \left[ e^{\chi h \gamma_j \chi} \right]} 1_{\{s(t) \geq K\}} \right] - e^{-\alpha} K \mathbb{P}^p [ F_s(t) \geq K ] \]

\[ = F_M(0) \sum_{n=0}^{\infty} \sum_{j=1}^{I_{n_j}} N(d_1(n)) \pi_j Q_{ij}(n, t; 1 + h^j_t) - e^{-\alpha \tau} \sum_{n=0}^{\infty} \sum_{j=1}^{I_{n_j}} N(d_1(n)) \pi_j Q_{ij}(n, t; h^j_t) \]

where \( \chi = \left( \sigma W(t), \sum_{n=1}^{\Phi(t)} Z_n \right) \), \( h = (h^C, h^I) \) and \( M = s \) or \( c \).

Since \( \phi(1 + h^j_t) = \phi(h^I_t) \), we simplify notation of the equation by Black-Scholes formula as the equation (15). That is

\[ CF_r(F_s(t), K, r, T - t, V(t, T, T_1), \sigma_j, n) = \sum_{n=0}^{\infty} \sum_{\tau, j} \pi_j Q_{ij}(n, \tau; h^j_t) CF(F_s(t), K, r, T - t, V(t, T, T_1), \sigma_j, n) \]
Reference


Table 1: The evolution of the trading value in carbon market from 2004 to 2010

The carbon trading markets are divided into two categories of carbon products; one is the carbon emission trading system including European Union Emissions Trading (EU ETS), the New South Wales (NSW), the Chicago Climate Exchange (CCE), the Regional Greenhouse Gas Initiative (RGGI), and the Assigned Amount Units (AAU). The other one is the greenhouse gas reduction projects in Kyoto Protocol, including the Clear Development Mechanism (CDM), and the Joint Implementation and the voluntary market. Table 1 presents the evolution of the trading value in carbon market from 2004 to 2010. The other allowances involve the trading carbon value in NSW, CCE, RGGI, and AAU, and the other offset covers the Joint Implementation and the voluntary market. The trading value of EUAs is the increasing from 2005 to 2010, and EUAs accounts for 84 percent of global carbon market value in 2010. Data source comes from state and trends of the carbon market at 2011 in carbon finance by the World Bank.

<table>
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<tr>
<th>Value ($ Billion)</th>
<th>EU ETS Allowances</th>
<th>Other Allowances</th>
<th>Primary CDM</th>
<th>Secondary CDM</th>
<th>Other Offsets</th>
<th>Total</th>
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<tr>
<td>2005</td>
<td>7.9</td>
<td>0.1</td>
<td>2.6</td>
<td>0.2</td>
<td>0.3</td>
<td>11.0</td>
</tr>
<tr>
<td>2006</td>
<td>24.4</td>
<td>0.3</td>
<td>5.8</td>
<td>0.4</td>
<td>0.3</td>
<td>31.2</td>
</tr>
<tr>
<td>2007</td>
<td>49.1</td>
<td>0.3</td>
<td>7.4</td>
<td>5.5</td>
<td>0.8</td>
<td>63.0</td>
</tr>
<tr>
<td>2008</td>
<td>100.5</td>
<td>1.0</td>
<td>6.6</td>
<td>26.3</td>
<td>0.8</td>
<td>135.1</td>
</tr>
<tr>
<td>2009</td>
<td>118.5</td>
<td>4.3</td>
<td>2.7</td>
<td>17.5</td>
<td>10.7</td>
<td>143.7</td>
</tr>
<tr>
<td>2010</td>
<td>119.8</td>
<td>1.1</td>
<td>1.5</td>
<td>18.3</td>
<td>1.2</td>
<td>141.9</td>
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Table 2: Statistics of the spot EUAs return in BlueNext

<table>
<thead>
<tr>
<th>BlueNext</th>
<th>Phase I</th>
<th>Phase II</th>
<th>Phase III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period I</td>
<td>Period II</td>
<td>Period III</td>
</tr>
<tr>
<td>Number of Obs.</td>
<td>216</td>
<td>412</td>
<td>742</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0854</td>
<td>0.5108</td>
<td>0.1055</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.1343</td>
<td>-0.5108</td>
<td>-0.1029</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0011</td>
<td>-0.0163</td>
<td>-0.0006</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0006</td>
<td>0.0086</td>
<td>0.0006</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.0244</td>
<td>-0.1525</td>
<td>-0.2680</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.7496</td>
<td>9.2656</td>
<td>5.2771</td>
</tr>
<tr>
<td>Number of the return more than 5%</td>
<td>5</td>
<td>51</td>
<td>9</td>
</tr>
<tr>
<td>(mean)</td>
<td>(0.0643)</td>
<td>(0.1328)</td>
<td>(0.0723)</td>
</tr>
<tr>
<td>Number of return less than -5%</td>
<td>6</td>
<td>96</td>
<td>25</td>
</tr>
<tr>
<td>(mean)</td>
<td>(-0.0848)</td>
<td>(-0.1336)</td>
<td>(-0.0662)</td>
</tr>
<tr>
<td>Total</td>
<td>11</td>
<td>147</td>
<td>34</td>
</tr>
</tbody>
</table>

Note: 1. The EUAs spot price in BlueNext from 24, June, 2005 to 30, December, 2010.  
2. (••) denotes the mean of the return more than 5% or the return less than -5%. 

Table 3: Parameter of the BSM, JDM and RSJM estimate in spot market at BlueNext

<table>
<thead>
<tr>
<th>Time</th>
<th>Model</th>
<th>$p_{11}$</th>
<th>$p_{22}$</th>
<th>$\mu$</th>
<th>$\mu_J$</th>
<th>$\sigma$</th>
<th>$\sigma_J$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>LRT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period I</td>
<td>BSM</td>
<td>--</td>
<td>--</td>
<td>0.0011</td>
<td>--</td>
<td>0.0251</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>24/06/2005</td>
<td></td>
<td>(0.0017)</td>
<td>(0.0012)</td>
<td></td>
<td>(0.0012)</td>
<td>(0.0314)</td>
<td>(0.0018)</td>
<td>(0.3939)</td>
<td>(0.4846)</td>
<td></td>
</tr>
<tr>
<td>24/04/2006</td>
<td>JDM</td>
<td>--</td>
<td>--</td>
<td>0.0024</td>
<td>-0.0026</td>
<td>0.0112</td>
<td>0.0314</td>
<td>0.4846</td>
<td>--</td>
<td>61.89*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0013)</td>
<td>(0.0038)</td>
<td>(0.0013)</td>
<td>(0.0013)</td>
<td>(0.0018)</td>
<td>(0.3939)</td>
<td>(0.4846)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Period II</td>
<td>BSM</td>
<td>--</td>
<td>--</td>
<td>-0.0163</td>
<td>--</td>
<td>0.0924</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>01/06/2006</td>
<td></td>
<td>(0.0046)</td>
<td>(0.0032)</td>
<td></td>
<td>(0.0032)</td>
<td>(0.0013)</td>
<td>(0.0019)</td>
<td>(0.4902)</td>
<td>(0.4846)</td>
<td></td>
</tr>
<tr>
<td>28/12/2007</td>
<td>JDM</td>
<td>--</td>
<td>--</td>
<td>0.0004</td>
<td>-0.0171</td>
<td>0.0074</td>
<td>0.0879</td>
<td>0.9789</td>
<td>--</td>
<td>328.89*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0008)</td>
<td>(0.0046)</td>
<td>(0.0003)</td>
<td>(0.0019)</td>
<td>(0.4902)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase I</td>
<td>BSM</td>
<td>--</td>
<td>--</td>
<td>-0.0108</td>
<td>--</td>
<td>0.0805</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>24/06/2005</td>
<td></td>
<td>(0.0031)</td>
<td>(0.0022)</td>
<td></td>
<td>(0.0022)</td>
<td>(0.0013)</td>
<td>(0.0019)</td>
<td>(0.4902)</td>
<td>(0.4846)</td>
<td></td>
</tr>
<tr>
<td>28/12/2007</td>
<td>JDM</td>
<td>--</td>
<td>--</td>
<td>0.0005</td>
<td>-0.0180</td>
<td>0.0114</td>
<td>0.0947</td>
<td>0.6279</td>
<td>--</td>
<td>632.38*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0004)</td>
<td>(0.0051)</td>
<td>(0.0004)</td>
<td>(0.0014)</td>
<td>(0.1377)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>RSJM</td>
<td>0.9868</td>
<td>0.9830</td>
<td>0.0005</td>
<td>-0.0190</td>
<td>0.0168</td>
<td>0.0982</td>
<td>0.0009</td>
<td>1.0460</td>
<td>134.21*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0068)</td>
<td>(0.0087)</td>
<td>(0.0009)</td>
<td>(0.0052)</td>
<td>(0.0008)</td>
<td>(0.0065)</td>
<td>(0.1022)</td>
<td>(0.0068)</td>
<td></td>
</tr>
<tr>
<td>Phase II</td>
<td>BSM</td>
<td>--</td>
<td>--</td>
<td>-0.0006</td>
<td>--</td>
<td>0.0239</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>26/02/2008</td>
<td></td>
<td>(0.0009)</td>
<td>(0.0006)</td>
<td></td>
<td>(0.0006)</td>
<td>(0.0013)</td>
<td>(0.0008)</td>
<td>(0.0014)</td>
<td>(0.0007)</td>
<td>(0.0028)</td>
</tr>
<tr>
<td>30/12/2010</td>
<td>JDM</td>
<td>--</td>
<td>--</td>
<td>0.0016</td>
<td>-0.0073</td>
<td>0.0169</td>
<td>0.0302</td>
<td>0.2974</td>
<td>--</td>
<td>67.75*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0010)</td>
<td>(0.0060)</td>
<td>(0.0032)</td>
<td>(0.0034)</td>
<td>(0.3201)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>RSJM</td>
<td>0.9852</td>
<td>0.9773</td>
<td>0.0014</td>
<td>-0.0028</td>
<td>0.0158</td>
<td>0.0210</td>
<td>0.0003</td>
<td>1.7970</td>
<td>79.89*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0071)</td>
<td>(0.0113)</td>
<td>(0.0008)</td>
<td>(0.0014)</td>
<td>(0.0007)</td>
<td>(0.0028)</td>
<td>(0.0058)</td>
<td>(0.1022)</td>
<td></td>
</tr>
</tbody>
</table>

Note: 1. (*) denotes the standard deviation estimated by the SEM algorithm.
2. JDM presents the jump diffusion model, RSJM is the regime switching jump diffusion model.
3. LRT presents the likelihood ratio test between the null hypothesis of the jump diffusion model with the alternative hypothesis of the regime switching jump diffusion model.
Table 4: The empirical studying $\delta(t_k)$ per years.

<table>
<thead>
<tr>
<th></th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>average</td>
<td>2.3168%</td>
<td>3.3999%</td>
<td>3.9141%</td>
<td>4.8831%</td>
<td>1.7008%</td>
<td>1.3407%</td>
</tr>
<tr>
<td>Interests rate</td>
<td>(3.3598*10^-6)</td>
<td>(1.2204*10^-3)</td>
<td>(5.2264*10^-6)</td>
<td>(2.4317*10^-2)</td>
<td>(2.8990*10^-3)</td>
<td>(1.5534*10^-6)</td>
</tr>
</tbody>
</table>

| Dec-06   | 0.5999%      | -2.1903%     | Dead         | Dead         | Dead         | Dead         |
|          | (7.2637*10^-7) | (8.7940*10^-5) | --           | --           | --           | --           |
| Dec-07   | 0.52626%     | -0.8044%     | -0.33427%    | Dead         | Dead         | Dead         |
|          | (2.9007*10^-7) | (6.461*10^-5) | (0.03190)    | --           | --           | --           |
| Dec-08   | Not spot     | Not spot     | Not spot     | -0.9335%     | Dead         | Dead         |
|          | --           | --           | --           | (0.0001)     | --           | --           |
| Dec-09   | Not spot     | Not spot     | Not spot     | 0.1182       | -1.5945      | Dead         |
|          | --           | --           | --           | (3.4403*10^-7) | (2.0056*10^-6) | --           |
| Dec-10   | Not spot     | Not spot     | Not spot     | 0.2021%      | -1.9734%     | -1.7148%     |
|          | --           | --           | --           | (1.9367*10^-7) | (5.07343*10^-7) | (1.6221*10^-5) |
| Dec-11   | Not spot     | Not spot     | Not spot     | 0.12889%     | -2.5380%     | -1.2981%     |
|          | --           | --           | --           | (1.5941*10^-7) | (2.41202*10^-7) | (6.0654*10^-8) |
| Dec-12   | Not spot     | Not spot     | Not spot     | -0.0214%     | -3.0687%     | -1.8320%     |
|          | --           | --           | --           | (1.3674*10^-7) | (1.9793*10^-7) | (7.9001*10^-8) |

Note: 1. ( • ) denotes the variance of estimate.
2. The average interest rates are average of 12-month Euribor (Euro Interbank Offered Rate) per months.
Table 5: The parametric estimation of the stochastic convenience yield

<table>
<thead>
<tr>
<th>Parameter of convenience yield</th>
<th>ECX Mar-11</th>
<th>ECX Jun-11</th>
<th>ECX Sep-11</th>
<th>ECX Dec-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_s$</td>
<td>0.0006</td>
<td>0.0062</td>
<td>0.0113</td>
<td>0.0144</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.0051</td>
<td>0.0622</td>
<td>0.0437</td>
<td>-0.0853</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.0005</td>
<td>0.0024</td>
<td>0.0027</td>
<td>0.0008</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.2727</td>
<td>0.2371</td>
<td>0.2593</td>
<td>0.1183</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0741</td>
<td>0.1313</td>
<td>0.2138</td>
<td>0.1134</td>
</tr>
</tbody>
</table>

Note: 1. The parameters $\kappa_s, \theta_i, \sigma_s, \rho$ are calculated through the equation (13) form 09/04/2008 to 30/12/2010.

2. RMSE refers to the relative mean square error, which is expressed in $10^{-3}$ percentages at Table 5.
Table 6: The futures option pricing errors of MRSE with the underlying ECX Dec-11 futures

<table>
<thead>
<tr>
<th>Date</th>
<th>Model</th>
<th>OTM</th>
<th>ATM</th>
<th>ITM</th>
<th>Subtotal</th>
</tr>
</thead>
<tbody>
<tr>
<td>In sample</td>
<td>BSM</td>
<td>22.4477</td>
<td>12.9187</td>
<td>4.0900</td>
<td>14.3634</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>19.1868</td>
<td>10.8241</td>
<td>3.5849</td>
<td>12.2328</td>
</tr>
<tr>
<td></td>
<td>JDM</td>
<td>3.9083</td>
<td>3.9854</td>
<td>5.9406</td>
<td>4.8755</td>
</tr>
<tr>
<td></td>
<td>RSJMc</td>
<td>5.5782</td>
<td>3.3935</td>
<td>3.7481</td>
<td>4.3004</td>
</tr>
<tr>
<td>Out sample</td>
<td>BSM</td>
<td>37.4535</td>
<td>18.2452</td>
<td>8.7145</td>
<td>24.5934</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>33.4686</td>
<td>16.4371</td>
<td>8.6214</td>
<td>22.1187</td>
</tr>
<tr>
<td></td>
<td>JDM</td>
<td>16.7969</td>
<td>10.2762</td>
<td>9.7258</td>
<td>12.7232</td>
</tr>
<tr>
<td></td>
<td>RSJMc</td>
<td>24.2441</td>
<td>10.7327</td>
<td>4.4444</td>
<td>15.5524</td>
</tr>
<tr>
<td>Total</td>
<td>BSM</td>
<td>30.8574</td>
<td>15.7239</td>
<td>6.5753</td>
<td>19.8909</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>27.2595</td>
<td>13.8355</td>
<td>6.3613</td>
<td>17.6336</td>
</tr>
<tr>
<td></td>
<td>JDM</td>
<td>12.2584</td>
<td>7.7687</td>
<td>7.9062</td>
<td>9.5004</td>
</tr>
<tr>
<td></td>
<td>RSJMc</td>
<td>17.6529</td>
<td>7.8939</td>
<td>4.0715</td>
<td>11.1843</td>
</tr>
</tbody>
</table>

Note: 1. The futures option pricing errors of MRSE is computed between the fixed models by the parametric estimating of Table 4 from 26/02/2008 to 30/12/2010 with the real option data. “In sample” denotes the real option data from 01/11/2010 to 12/30/2010. “Out sample”, the real option data is from 01/03/2011 to 28/02/2011.

2. The results are grouped under “All”(all options of the dataset), “OTM”(out-of-money, where \( S/K < 0.95 \)), “ATM”(at-the-money options, where \( 0.95 \leq S/K \leq 1.05 \)) and ITM”(in-of-money, where \( 1.05 < S/K \)).

3. The data are futures options at ECX with strike price K=12,12.5,13,13.5,14,14.5,15,15.5,16,16.5 €, Mar-11, June-11, Sep-11 and Dec-11. The total sample is 1760.
Panel A: The dynamics of the EUAs in BlueNext,

Panel B: The dynamics of the EUAs return in BlueNext

Figure 1: The dynamics of EUAs and EUAs return in BlueNext
**Panel A:** The dynamics of the EUAs at Phase II

**Panel B:** The dynamics of the EUAs return at Phase II

**Panel C:** The probabilistic dynamics of the low intensity with the EUAs return at Phase II in BlueNext

**Panel D:** The jump probabilistic dynamics with the EUAs return at Phase II in BlueNext

**Figure 2:** The dynamics of EUAs, its return, the low intensity probability and jump probability at Phase II in BlueNext
Panel A: The autocorrelation of the spot EUAs return from at Phase II in BlueNext

Panel B: The estimating autocorrelation of the spot EUAs with RSMJ model from at Phase II in BlueNext

Figure 3: The autocorrelation and the estimating autocorrelation of the spot EUAs return with RSMJ model from at Phase II in BlueNext
$\delta(t_k)$ is daily empirical convenience yield such as the equation (14) with during 2008-2010 under the average interest rate

**Figure 4: the empirical study of convenience yield**