The Strategic Implementation of an Investment Process in a Funds Management Firm

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Abstract: One of several important strategic decisions that have to be made by an active funds management organization is how aggressively it implements its investment process. In this paper we model this decision on the assumption that the organization's objective is to maximise the present value of its future fee income. We then develop the model using numerical methods in order to demonstrate the impact of several endogenous and exogenous factors on this optimum active position. In particular we highlight that the most aggressive funds are likely to be embryonic funds that have the greatest potential to add value. We also establish that in the event that such funds increase their funds under management; it will be optimum for them to become more index-like. We also show how the extent to which the implementation decision for the active process is a function of a number of factors including the alpha and risk characteristics of the active component of the portfolio, the risk tolerance of the promoting organization and the relationship between performance and future fund flows. Our findings have extensive implications ranging from the choice of investment managers by investors to the functioning of capital markets

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1. Introduction

The strategic management of a funds management organisation is extremely complex and includes making decision related to the design of the investment products, the development and implementation of the investment process, the marketing of the products, and the servicing of clients. In this paper will concentrate on one important component in this long line of decision making, the aggressiveness with which a fund implements it investment process.

The starting point in our analysis is that each fund already has determined the nature of its investment product(s) along with a process designed that is consistent with its chosen investment style for managing this product. For example, the fund may have determined that will invest in large cap US equity stocks following a qualitative process based on identifying stocks with high growth potential. It has chosen an index that is representative of this style (e.g. large cap growth index) and it will both implement its process relative to the holdings in this index and also measure its performance relative to this index.

We will assume that the manager's objective for this fund is to maximise the present value of the fee income that it generates. While recognising that the achievement of this objective will be influenced by the way that the management handles all of the important areas outlined above (product design, marketing, client servicing, etc.), we will assume that the ability of a fund to attract and keep client money is largely determined by its investment outcomes. What we demonstrate in this paper is how this translates into the aggressiveness with which the fund implements its investment process. We will demonstrate that the optimal strategy will be dependent on a number of fund (and manager) characteristics and discuss how the behaviour is likely to change over the life of the fund.

In Section 2 we provide some background to the model that we develop in Section 3. We proceed in Section 4 to use a numerical example to develop some of the major implications of the model and then in Section 5 provide an example of how fund managers will behave over its life Finally, Section 6 provides us with the opportunity to reprise the paper, discuss some of its implications for investors and the market more generally and indicate areas for future research.

2. Background

From an investment perspective, the starting point in establishing any fund management organization is determining its investment products and putting in place the process, staff and infrastructure for managing the funds[1]. We assume that all of this is in place and that it defines the ability of the fund to generate excess returns. The investment choice on which we concentrate is the actual implementation of the investment process. One option is for the managers to only take small positions relative to their benchmark, thus ensuring that there is very low volatility around the benchmark return (i.e. low

[1] Other areas in which planning and implementation have to be undertaken include administration, marketing and client servicing.

tracking error). At the other extreme, the option is to deviate substantially from the benchmark portfolio which will increase the tracking error but also increase the probability of both under- and out-performance. The objective, when making the decision as to the aggressiveness with which to run the investment portfolio, is to produce a level of returns relative to the benchmark that will increase the likelihood of strong growth in funds under management and so fee income.

Cremers and Petajisto (2009), we describe the aggressiveness in the implementation of a fund's process in terms of what we call the fund's active position. Cremers and Petajisto (2009) pointed out that a fund's portfolio can be separated into two parts: a 100% position in the benchmark index and n% invested in a long/short portfolio which incorporates all of the active positions. It is this n% that we call the fund's active position and it measures the aggressiveness of the fund in implementing its investment process. In the absence of this long/short portfolio (i.e. a zero active position), we have an index fund that will realise the benchmark return with a zero tracking error (i.e. n = 0). If the amount of funds invested in the long/short portfolios represents a 100% of the total funds invested (n = 100), then there will be zero investment in the benchmark stocks. A fund's active position must lie in the range from 0% to 100% and is created by holding index stocks at a weight other than their index weight and/or by investing in stocks that lie outside of the index.

The superiority of a particular fund will depend on the ability of the manager and the quality of the process. In the short run, a number of extraneous factors are likely to swamp the importance of these attributes in terms of their impact that the active position has on realised returns. However in the longer-term, the superior funds are likely to win out with their active positions leading to better investment performance as measured by the realization of higher excess returns and/or lower tracking error.

The Setting

We commence with a new fund operated by an embryonic organisation which has little in the way of either reputation or funds under management. We assume that the fund has a style and process with the expected returns from the active implementation of the process being defined by a mean, a standard deviation and a correlation between these returns and those of the benchmark. The objective for the fund is to maximise the contribution that this fund makes to the valuation of the promoting funds management organisation.

For simplicity, we will assume a direct relationship between fee income and funds under management[2]. Numerous studies have analysed the relationship between fund investment performance and the flow of new money into the fund with the overall finding being that funds are sticky. That is, poor performing funds are given an extended period of grace they begin to lose significant funds while the returns to extremely good funds are exponential in that top performance is rewarded by a rapid influx of new funds (Siri and Tufano, 1998; Del Guercio and Tkac, 2008); Milone and Pellizzari, 2009). Both institutional and retail advisers like to see a sustained period of outperformance before they are willing to recommend a manager. The empirical evidence suggests a period of sustained good performance extending over three to five years is required to achieve growth in funds

[2] Variable costs are very low in the funds management industry and so we will ignore them for the purposes of this study.

referred to above. Equally, they are willing to tolerate relatively poor performance for a similar extended period of time before making their decision to withdraw funds.

In the light of the discussion above, the proposition put forward in this paper is that the future growth in funds under management for a particular fund is dependent on the performance achieved over a sustained period of time[3]. Once a certain level of performance is reached (and exceeded), there will be rapid growth in funds under management and so fee income. The actual magnitude of this growth in funds under management will be dependent on a number of factors but particularly the ability and willingness of the fund to absorb additional funds. There is a wide range of performance which is neither good enough to attract a lot of additional funds nor bad enough to result in a significant loss of existing funds. However, a sustained realisation of poor returns is likely to cause existing investors to withdraw a significant volume of funds and to dissuade potential investors from allocating the new funds to the manager for an extended period of time.

One issue of concern for a fund that has enjoyed significant funds growth is the impact on that continuing growth will have on its ability to sustain good performance in the future. Chen et al. (2004) indentified this as an issue for active managers and attributed it to stock illiquidity and fund manager organisation. Research undertaken by Frank Russell has found a strong negative relationship between the excess returns generated by a fund and both the level of its funds under management and the extent to which it runs concentrated portfolios with the ability of the fund to generate positive alpha. This suggests a situation where a level of funds under management will be reached beyond which more funds will diminish future performance due to both market impact and/or a limit to the number of exploitable investment opportunities. As a consequence, the ability of a fund to generate a positive outcome for investors will be compromised. This will not only limit their potential to generate the performance going forward to attract further high volumes of funds but also increase the likelihood of the fund experiencing sustained periods of underperformance which will jeopardise the existing funds that it has under management.

We started this discussion with a fund that had little in the way of either funds under management or fees. So there is little to lose in the event that the fund experiences sustained underperformance. However, if this embryonic fund does well it is likely to grow both in terms of funds under management and reputation. As a consequence the potential costs of poor performance will increase relative to the potential benefits if it is able to maintain the good performance into the future. In other words the incentives that a fund faces are likely to change over its life and this is likely to have significant implications for the way that it implements its investment strategy.

The most important consequence of this discussion is the link between performance and future fee income and that the nature of this link is likely to evolve as the circumstances of the fund change. In the next section, we develop a model that reflects the setting that we have developed above and we then proceed to use it to evaluate the implications of our model by way of several numerical examples. However before proceeding to the development of the model, we will first examine what the data tells us about the nature of the relationships in which we are interested.

[3] We realise that there are other factors such as the level of marketing and the pre-existing reputation of either the managers or the fund family that will also affect the relationship between performance and new funds flow.

Stylised Facts

We examine the CRSP data on mutual funds managing US equity portfolios over the period from 1995 to 2009 in order to glean information on their behaviour, especially in terms of the active positions that they take. As discussed previously, we like Cremers and Petajisto (2009) decompose the active portfolio into a 100% investment in the benchmark portfolio plus a long and a short portfolio of equal magnitude. The proportion that this long/short portfolio represents of the total fund (i.e. the active position) is measured as follows:

$$\text{Active position} = 0.5 \sum_{j=1}^{N} abs\,(w_{f,j} - w_{i,j})$$

where the active position is one half of the aggregate of the absolute differences between the actual fund weighting in a particular stock, $w_{f,j}$, and the index weighting for that stock, $w_{i,j}$.

We report in Table 1 the median value of a number of parameters across our sample period where we have divided the sample up on the basis of sustained performance over a three years performance (as an attempt to divide the sample into "good" funds and "bad" funds) and the level of funds under management (in order to divide the sample into large funds and small funds). We define good (bad) as being in the top (bottom) tercile by excess returns and large (small) as being in the top (bottom) tercile by funds under management.

Size	Perform-ance	Active Position	Qtr. Flows ($M)	Man. Fees	Age (yrs)	Funds $M	Excess Ret. (%pq)
Large	Good	0.804	12.27	0.75	5.7	567.6	0.45
Large	Bad	0.747	4.62	0.69	6.4	511.4	-0.57
Small	Good	0.833	0.58	0.54	4.4	13.7	0.84
Small	Bad	0.778	-0.08	0.64	4.7	13.9	--1.86

Table 1: Median values for key parameters with sample sorted by performance and funds under management

These findings support the following four propositions:

- Funds with small amounts of funds under management take larger active positions than do equivalent funds with significant funds under management
- Funds with a greater potential to add value take larger active position than do equivalent funds with a lesser potential to add value
- Small funds find it more difficult to convert a sustained period of good performance into fund under management.
- Management fees tend to increase as a fund grows and that inferior managers tend to charge higher fees, at least during the establishment period.

The other piece of analysis that we report is that we regressed active position against size (measured by the log of funds under management), fees and age where for each explanatory variable we used dummies to divide them up into good funds and bad funds. Our findings as reported in Table 2 support our previous proposition that funds become less willing to take active

positions as they grow in size. We find this particularly true for inferior funds that have by chance grown large for whom the optimum strategy would best be directed towards keeping existing clients rather than attracting new ones.

Given what we have said about funds becoming more conservative as they grow larger, one might expect this to also hold for age. This does not prove to be the case with older superior funds taking larger active positions. This highlights that age does not equate well with size and reflects that many relatively unsuccessful funds remain in operation for a long time. Finally we see a positive relationship between fees and the size of a fund's active position which is a finding that is stronger for good funds than it is for bad funds.

Variable	Coefficient	p-value
Size	-0.0063	0.0000
Size*Good	-0.0031	
Age	-0.0007	0.1705
Age*Good	0.0041	
Fees	0.0309	0.0000
Fees*Good	0.0452	
Constant	0.7747	0.0000
Adjusted R-squared	0.0910	

Table 2: The Impact of Size, Age and Fees on the Magnitude of a Fund's Active Position Differentiating between the Impact for Large and Small Funds

In terms of the focus of this paper, the most important stylised facts that we have identified are (i) superior managers will take larger active positions than inferior managers, and (ii) small funds will take larger active positions than larger funds. In Section 4 we will utilise the model developed in Section 3 to throw more light on these stylised facts

3. The Model

Within the setting established in the previous section, we develop a model that describes the aggressiveness with which a manager will implement an active investment process. Assume the benchmark return db_t/b_t is given by:

$$\frac{db_t}{b_t} = \mu dt + \sigma_b dW_t$$

with average return μ, volatility σ_b and where W_t is a standard Brownian motion. The portfolio of the active fund will diverge from the benchmark as a consequence of the active positions taken by the fund manager (say, due to holding benchmark stocks at other than their benchmark weight or investing in stocks that are outside of the benchmark). The active portion of the portfolio has a return:

$$\frac{ds_t}{s_t} = (\mu + \alpha)dt + \sigma_a dY_t$$

where α is the excess return over μ, σ_a is the volatility of the return of the active process and Y_t is a standard Brownian motion such that ρ = Cov(dW, dY).

We will refer to the investment in the benchmark b_t as the passive component and to the investment in the active portfolio s_t as the active component. We assume that the variable under control of the manager of the fund is what we have previously referred to as the active position which is measured by *w*, the proportion of the fund to be invested in the active component of the total portfolio over the time investment horizon [0,T] years. In other words, the manager can depart from a fully passive position investing a fraction *w* of the funds in s_t (while the remaining 1 − *w* remains in b_t). We also assume that *w* is decided at t = 0 and stays fixed till the end of the period, T. The funds portfolio yields a return of

$$w\frac{ds_t}{s_t} + (1-w)\frac{db_t}{b_t}$$

We can define the excess return of the fund over the benchmark as:

$$\frac{de_t}{e_t} = w\left(\frac{ds_t}{s_t} - \frac{db_t}{b_t}\right) \qquad (1)$$

We are interested in value of this excess return e_T at the end of the period. This value depends critically on the fraction *w* and on the ability of the manager to outperform the benchmark, α (and, obviously, on the other parameters of the processes s_t and b_t.)

The manager is rewarded if e_T exceeds some upper threshold *k* and is punished if e_T falls short of a lower threshold $\underline{k}$. In detail, his objective at T will be given by the random variable X_T defined as

$$X_t = e_T + gI(e_T > k) - cI(e_T < \underline{k}) = \begin{cases} e_T + g, \text{if } e_T > k \\ e_T, \quad \text{elsewhere} \\ e_T - c, \text{if } e_T < \underline{k} \end{cases}$$

where g> 0 can be interpreted as the growth in the present value of future fee income attributable to the good (above upper threshold) performance, c> 0 proxies the relative decrement of future fee income attributable to the poor (below lower threshold) performance, and $I(E)$ is a binary random variable that takes values 1 or 0 depending on the occurrence of event *E*.

Assuming that the wealth under management at $t = 0$ is u_0 and, for simplicity that fees are collected in T at a constant rate f . Then the fees raised are

$$v = fu_0(1 + r_T + X_T) = fu_0(1 + r_T) + fu_0 X_T ,$$

where r_T is the return of the benchmark portfolio. The manager does not have control over the first term as benchmark returns are out of his control and but he can optimise his fees by maximizing the second term, which depends critically on his choice of w . The same conclusion holds if we take the simplifying view the manager maximizes the present value of a perpetuity yielding v every year (at constant which will yield him the following fees:

$$fu_0(1+r_T+e_T+g) = fu_0(1+r_T+e_T)\left(1+\frac{g}{1+r_T+e_T}\right),$$

whereas a manager whose performance falls short of k only yields:

$$fu_0(1+r_T+e_T).$$

Hence, we can interpret *g* as the approximate rate of growth of wealth under management secured by the successful manager and, symmetrically, *c* is the relative decline of the wealth caused by poor performance. More generally, the manager's objective is to maximize a mean-variance function of the variable X_T, as described below.

The model incorporates the basis of the setting described in the previous section where sustained outperformance results in the fund realising a significant growth in funds under management while sustained underperformance results in a loss of existing funds under management. One may also think that the resulting change in profitability is directly applied to the compensation of the manager but we prefer to interpret the gain *g* as related to the present value of the additional future fees that the fund will obtain due to its sustained outperformance, signalled by $e_T > k$, and the cost *c* as the present value of the reduction in future fees that the fund will lose because of its sustained underperformance, signalled by $e_T < \underline{k}$. We should note that $X_T = e_T$ if $\underline{k} \leq e_T \leq k$ which means that there is a range of performance over the period which has no impact on future fee income.

We propose that the relative value of *g* as compared to *c* will be largely determined by the current size of the fund and the performance-flow curve first identified by Sirri and Tufano (1998). As we argued in the previous discussion of the setting, both prospective and existing managers are typically evaluated over periods upwards of several years and so it is reasonable to evaluate and rank a fund after some fixed and predetermined time *T*, in order to gauge the impact of performance on future fee income.

We assume that the manager is risk-averse and acts in accordance of the following mean-variance optimization problem:

$$\max_w U(w) = \max_w E[X_T] - \lambda Var[X_T], \qquad (2)$$

where λ is a risk-aversion coefficient. Intuitively, this corresponds to managers willing to trade some additional fees for a reduction in the variability of the amount that is collected, depending on their personal risk-attitude.

4. Results

This section explores the model of fund behaviour developed in the previous section. The optimisation problem cannot, to the best of our knowledge, be solved analytically. Indeed, even if there is a relatively simple expression for the mean of X_T, the variance in (2) involves both the density and the cumulative probability function of normal variables

whose distribution depends on w. Despite the fact that no close-form solution is available the model is numerically very tractable and optimal active positions under various sets of assumed values for the parameters can be computed[4].

The next subsection will describe the reference parameters. A more general numerical treatment is then provided in subsection 4.2, where we systematically vary the parameter of the reference case to explore the effects of asymmetries in the formulation of problem (6).

4.1 A reference example

The model depends on several parameters that describe the passive and active stochastic processes b_t, s_t respectively, and other structural exogenous features like g, c and the risk-aversion coefficient λ.

μ	0.05	Avg return of the benchmark
α	0.00	Avg excess return of the active process
σ_b	0.10	Volatility of the benchmark
σ_a	0.10	Volatility of the active process
ρ	0.00	Correlation of active and benchmark
T	3	Time horizon (years)
k	0.05	Upper threshold
$\underline{k}$	-0.05	Lower threshold
g	0.5	Gain if e_T exceeds k at T
c	0.05	Cost if e_T is below $\underline{k}$ at T
λ	0.3	Risk-aversion coefficient

Table 3: Values of the parameters for the reference case. The third column contains a brief description of each parameter.

The values of the parameters in our example are given in Table 3 and depict a situation where we have an active fund which might best be described as average (i.e., $\alpha = 0$) with both the benchmark and the active component of the portfolio having an expected annual return of 5% with a volatility of 10%. The two processes are uncorrelated and the performance of the fund will be evaluated after 3 years. If, at the end of the period, the fund's return exceeds that of the benchmark portfolio by 5% (k) or more, X_T is increased by $g = 0.5$. If, instead, the excess return fall short of the benchmark return by 5% ($\underline{k}$) or more, then $c = 0.05$ is subtracted to X_T. Finally, the fund maximizes a mean-variance utility function where the risk-aversion coefficient is $\lambda = 0.3$. Figure 1 shows the utility $U(w)$ as a function of the fraction invested in the active process. The manager optimally picks $w^* = 0.74$, effectively meaning that approximately three-quarters of his funds are invested in his active portfolio and one-quarter in the benchmark. This is consistent with Cremers and Petajisto (2009) who found that the average active fund position ranged

[4] The results of this section are obtained using R, available at htpp://cran.rproject.org, making use of the "integrate" and "optimizatize" functions. The code is available upon request.

between 60% and 80% while Bird et al. (2011) found that it averaged 76%.

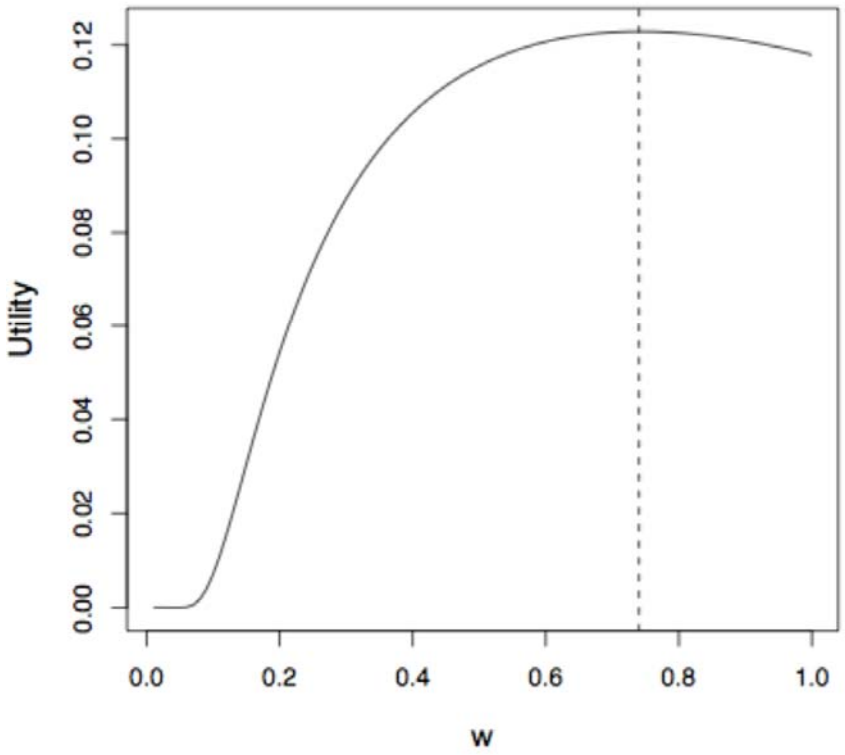

Figure 1: Utility of the manager as a function of w in the reference case. The optimal value is $w^* = 0.74$.

4.2 Robustness analysis

This subsection numerically solves the model in other cases, corresponding to different values of the parameters, and provides a thorough robustness analysis of our results. We progressively change two parameters at a time, leaving the others at their value as outlined in Table 3 (the reference case). The results as reported provide valuable insights as to how a fund will behave at various stages throughout its life cycle.

Case 1: *Varying the profit from reaching the upper threshold (g) and the loss associated with reaching the lower threshold (c).*

In Table 4 we report the optimal active position to hold for various combinations of g and c. What we see is that as g increases relative to c, the active position increases while it decreases as g decreases relative to c. It is apparent that when g ≤ c, then there is no incentive for the fund to be anything other than purely passive. This suggests that the incentive for a fund to take on an active position will decrease as *g* declines relative to *c* (and also as α declines which is covered in Case 2 and as k declines relative to k which is covered in Case 3).

w^*		Cost c				
		0.000	0.025	0.050	0.075	0.100
	0.0	0.00	0.00	0.00	0.00	0.00
	0.1	0.54	0.47	0.37	0.23	0.00
	0.2	0.65	0.60	0.55	0.49	0.43
	0.3	0.72	0.68	0.64	0.60	0.55
	0.4	0.77	0.73	0.70	0.66	0.62
Gain *g*	0.5	0.80	0.77	**0.74**	0.71	0.68
	0.6	0.83	0.80	0.77	0.75	0.72
	0.7	0.85	0.83	0.80	0.77	0.75

Table 4: Optimal active position w^* as a function of g and c. The reference case is highlighted using boldface.

Case 2: *Varying the profit associated with reaching the upper threshold (g) and the expected added value of the fund (α)*

In Table 5 we report the optimum active position for various values of g and α. The optimal active positions monotonically increase in both g and α. This demonstrates that funds with the greatest potential to add value should be the most active in implementing their investment strategy. However, it does not deny that even managers without any skill (i.e. $\alpha < 0$) should also actively manage their portfolio when the potential benefits sufficiently outweigh the potential costs (i.e. g > c). This is likely to be the case for a relatively new manager with little in the way of funds under management and so demonstrates why a "new" fund is likely to be managed quite aggressively. However as the funds under management grow, there is likely to be a decline in the performance that a manager is able to realise from their strategy and an increase in the potential cost stemming from bad performance. As a consequence, it is optimum for the fund to become less active and even "index like" when expected outperformance is reduced to 0 (i.e. $\alpha = 0$) and the downside from bad performance equates with the upside from good performance (i.e. g = c).

w^*		***Gain g***							
		0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70
	-0.03	0.00	0.00	0.00	0.25	0.33	0.38	0.42	0.45
	-0.02	0.00	0.00	0.25	0.36	0.43	0.48	0.52	0.55
	-0.01	0.00	0.00	0.38	0.48	0.54	0.59	0.63	0.66
Skill α	0	0.37	0.00	0.55	0.64	0.70	**0.74**	0.77	0.80
	0.01	0.77	0.09	0.86	0.91	0.94	0.97	0.99	1.00
	0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 5: Optimal active position w^* as a function of g and c. The reference case is highlighted using boldface.

Case 3: Varying the upper threshold (k) and the lower threshold ($\underline{k}$).

We report in Table 6 the optimum active positions for various values of k and $\underline{k}$. The numerical results point to the fact that, ceteris paribus, an increase in the upper threshold pushes the fund towards being more active. Further, for any given level for the upper threshold the aggressiveness of a fund will decrease as the lower threshold increases (i.e. the disparity between k and $\underline{h}$ increases) but that a fund's optimum active position is much more sensitive to changes in the upper threshold than it is to changes in the lower threshold. The asymmetry reflects the associated losses attributable to hitting the lower threshold (*c)* are small relative to the profits to be enjoyed from hitting the upper threshold (g). The one exception to our findings being that in instances where the upper threshold is close to zero (e.g. k = 0.01) that managers eventually move to becoming more aggressive as the lower threshold is further lowered.

w^*		***Lower threshold $\underline{k}$***							
		-0.01	-0.03	-0.05	-0.07	-0.09	-0.11	-0.13	-0.15
	0.01	0.39	0.34	0.31	0.30	0.30	0.31	0.33	0.34
	0.03	0.63	0.61	0.59	0.57	0.55	0.55	0.54	0.54
	0.05	0.77	0.76	**0.74**	0.73	0.71	0.70	0.70	0.69
Upper trh $\overline{k}$	0.07	0.88	0.86	0.85	0.84	0.83	0.82	0.81	0.80
	0.09	0.96	0.95	0.94	0.92	0.91	0.90	0.90	0.89
	0.11	1.00	1.00	1.00	0.99	0.98	0.97	0.97	0.96
	0.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 6: The values of w^* as functions of the upper and lower thresholds, k and $\underline{k}$. The reference case is boldfaced.

One of the assumptions in Table 6 is that gains from hitting the upper threshold are much greater than the costs from hitting the lower threshold (g= 0.5; c =0.05). We repeat the analysis in Table 6 but this time assuming that the gains now equal the costs (g = c = 0.05). Important other assumptions that are maintained are the manager has no skill (α = 0) and that the upper and lower thresholds are equal in magnitude. Our findings that are reported in Table 7 highlight that for a manager that has no skill there is never any incentive to pursue active management in situations where both $g \leq c$ and $\underline{k} \leq k$.

w^*		Lower threshold $\underline{k}$							
		-0.01	-0.03	-0.05	-0.07	-0.09	-0.11	-0.13	-0.15
	0.01	0.00	0.07	0.11	0.13	0.16	0.18	0.20	0.22
	0.03	0.00	0.00	0.14	0.17	0.20	0.23	0.25	0.28
	0.05	0.00	0.00	0.00	0.18	0.22	0.25	0.28	0.31
Upper trh $\overline{k}$	0.07	0.00	0.00	0.00	0.00	0.21	0.26	0.29	0.32
	0.09	0.00	0.00	0.00	0.00	0.00	0.23	0.28	0.32
	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.29
	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7: The values of w^* as functions of the upper and lower thresholds, k and $\underline{k}$, where now $c = g = 0.05$

Case 4: Varying the fund's risk aversion (λ) and the ability of the fund to generate outperformance (α)

Table 8 demonstrates the impact on a fund's risk-aversion coefficient (λ) and its potential to outperform (α) on the fund's optimum active position (w*). An important aspect of the information contained in Table 8 is that it confirms our previous empirical finding that it is the better managers who should that take the larger active positions. An inspection of the last row of Table 8 shows that the appetite of even the most capable of funds ($\alpha = 0.03$) to produce aggressive portfolios will be reduced if they have a sufficiently high aversion to taking on risk (i.e., high λ). We also learn from Table 8 that inferior funds (i.e. those with a negative α) with a large appetite to take risks (i.e. low λ) have every incentive to take relatively large active positions.

We have noted on several occasions the importance of the relative values of g and c in the determination of the optimum active position for a fund. In our base case the profits are ten-times the costs which is more consistent of an embryonic fund with little or no reputation and so with everything to gain and very little to lose. However, we are also interested in how the behaviour of a fund changes as the relative payoff changes. We have already argued in Section 2 that the ratio of g to c will decline as a fund grows and we will provide more evidence on the implications of this in Table 9 where we repeat the analysis reported in Table 8 but this time where now set the loss (c) as being equal to 0.25 (i.e., g is reduced from ten-times c to two-times c). This significant reduction in the relative benefits clearly changes the incentives to the funds with even skilful managers with a high appetite for taking risks now cutting back on the active positions that they take.

The message from Table 9 for a superior fund ($\alpha > 0$) that benefits from rapid fund growth is that they should expect to become less aggressive in the active positions as they become larger due to both a reduction in their expected α and in their relative payoff as c grows relatively to g. Of course, the inferior funds are much less likely to enjoy the growth in funds under management but the vagaries of markets cannot deny the possibility of this

happening. If by chance they do grow to being very large, then our analysis highlights that they would be well-advised to become very index-like (i.e. a closet indexer).

ω^*		Risk Aversion λ						
		0.00	0.10	0.20	0.30	0.40	0.50	0.60
	-0.03	0.55	0.48	0.43	0.38	0.34	0.00	0.00
	-0.02	0.72	0.61	0.54	0.48	0.43	0.39	0.35
	-0.01	1.00	0.81	0.68	0.59	0.53	0.48	0.43
Skill α	0.00	1.00	1.00	0.89	**0.74**	0.65	0.58	0.52
	0.01	1.00	1.00	1.00	0.97	0.81	0.70	0.63
	0.02	1.00	1.00	1.00	1.00	1.00	0.88	0.77
	0.03	1.00	1.00	1.00	1.00	1.00	1.00	0.95

Table 8: Activity w* as a function of risk-aversion (λ) and ability to outperform (α) (c = 0.05 as in the reference case). The reference case is boldfaced.

ω^*		Risk Aversion λ						
		0.00	0.10	0.20	0.30	0.40	0.50	0.60
	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	-0.01	0.77	0.52	0.37	0.00	0.00	0.00	0.00
Skill	0.00	1.00	0.84	0.58	0.44	0.34	0.24	0.11
	0.01	1.00	1.00	0.90	0.64	0.50	0.40	0.32
	0.02	1.00	1.00	1.00	0.97	0.70	0.56	0.46
	0.03	1.00	1.00	1.00	1.00	1.00	0.78	0.62

Table 9: Activity w* as a function of risk-aversion (λ) and ability to outperform (α) when c = 0.25)

Case 5: Varying the volatility σ_a of the returns of the active portfolio and the correlation ρ between the returns of the active process and the benchmark.

Table 10 displays the level of optimal active positions for various levels of σ_a and ρ. A larger volatility of "active" returns will increase the volatility of the total portfolio which suggests a reduction in the active position. A more interesting finding is that an increase in the correlation between the return on the active portfolio and that on the benchmark portfolio (ρ) also causes an increase in the optimum active position. A negative correlation would, in general, produce a larger spread between the returns on the benchmark portfolio and that from the active process resulting in an increase in the volatility of portfolio returns which curbs the incentives to take large active positions.

w^*		Active Vol σ_a						
		0.05	0.10	0.15	0.20	0.25	0.30	0.35
	-0.75	0.74	0.56	0.45	0.37	0.32	0.28	0.24
	-0.50	0.79	0.61	0.48	0.40	0.34	0.29	0.26
	-0.25	0.86	0.66	0.52	0.43	0.36	0.31	0.27
Correlation								
ρ	0.00	0.94	**0.74**	0.58	0.47	0.39	0.33	0.29
	0.25	1.00	0.86	0.66	0.52	0.43	0.36	0.31
	0.50	1.00	1.00	0.79	0.61	0.48	0.40	0.34
	0.75	1.00	1.00	1.00	0.74	0.56	0.45	0.37

Table 10: Activity w^* as a function of volatility σ_a of the active process and correlation ϱ with the benchmark. The reference case is boldfaced.

The risk attached to a funds portfolio is more commonly measured by its tracking error to the benchmark (i.e. the standard deviation of its excess returns rather than by its total risk, the standard deviation of its absolute returns). The expected tracking error for a portfolio is dependent on; (a) the aggressiveness with which a manger implements the portfolios (i.e. w), (b) the risks inherent in both the benchmark portfolio and the active portfolio and (c) the correlation between the returns on these two portfolios. We have seen throughout our discussion that there is an optimal level of aggressiveness (w^*) that i8s impacted by a number of the parameters that we have considered. In Table 11, we demonstrate how the optimum tracking error from a fund's perspective changes with variations in both the volatility of the active portfolio (σ_a) and also the correlation between the returns on the active and passive portfolio (ϱ). As is evident, this tracking error is greatest where both the correlation (ϱ) is low (i.e. most negative) and active volatility (σ_a) is low. This not only demonstrates that the managers of funds will want to pursues different tracking errors consistent with the risk characteristics of their own active portfolio but also that they will want to change the expected tracking error of their portfolio as the fund goes through its life cycle. For example, an embryonic fund will want to be very active (i.e. high w^*) and so have a high tracking error but we have seen that as that fund grows it will want to curtail its level of activity and so reduce its tracking error. This will represent quite a challenge for the fund as it will not want to signal to the market, and especially to its clients, that it has changed its behaving by becoming less active and pursuing a lower tracking error.

Tracking Error		Active Vol σ_a						
		0.05	0.10	0.15	0.20	0.25	0.30	0.35
	-0.75	0.139	0.105	0.084	0.069	0.059	0.052	0.046
	-0.50	0.137	0.105	0.083	0.069	0.058	0.050	0.044
	-0.25	0.135	0.105	0.083	0.068	0.057	0.049	0.043
Correlation								
ρ	0.00	0.133	**0.105**	0.082	0.066	0.055	0.047	0.041
	0.25	0.122	0.105	0.081	0.064	0.052	0.044	0.038
	0.50	0.100	0.100	0.079	0.061	0.048	0.040	0.034
	0.75	0.071	0.071	0.071	0.052	0.040	0.032	0.026

Table 11: The optimum tracking error as a function of volatility σ_a of the active process and correlation ρ with the benchmark. The reference case is boldfaced.

5. A Life-Cycle Example

In this section of the paper we draw upon the previous analysis to provide a specific example of the behaviour of managers at different stages in their life cycle. In particular, we consider:

- A *boutique manager* who has little in the way of funds under management. Such a manager has little to lose as a result of a sustained period of underperformance (i.e. a low c in our model) and little to prevent them being able to realise the full potential of their investment process (i.e. no erosion in the $\acute{\alpha}$ in our model)
- A *"mid-cycle" manager* who either because of skill or luck has been able to generate a sustained period of outperformance. As a consequence this manager is in the early stages of a growth spurt in funds under management resulting in the manager now having something to lose both in terms of funds being managed and a growing reputation (i.e. a rising c in our model). Also the growing funds being managed are beginning to cause a drag on performance and so the manager is already finding an erosion in the potential to add value (i.e. a reduction in $\acute{\alpha}$)[5].
- A *mature manager* who again because of skill or luck has been able to generate sufficient good performance to attract a large amount of funds. As a result the potential exist for the fund to lose a large amount of funds under management and the level of funds under management is now making it increasing more difficult for him to realise his potential to outperform (i.e. a even greater reduction in his $\acute{\alpha}$)

We further consider two managers with differing investment capabilities. We have a good manager whose investment process is expected to generate positive alphas in the long run but that this positive alpha will erode with significant growth in his funds under management. We also have a bad manager whose process is expected to generate negative alphas in the long run. However, it is far from impossible for our bad manager to generate sufficiently good investment performance to advance to the mid-cycle, if not mature, stage of the cycle. For example, a manager with an expected annualised alpha of -1.5% with a

[5] The point at which these constraints will begin to take effect will vary from market to market largely determined by the size of the market.

standard deviation of 10% has a x% probability of outperforming by 5% over three years, and a y% probability of outperforming by 5% over five years. Finally, we assume that the expected performance ($\acute{\alpha}$) of the bad manager does not erode in the event that he does significantly grow funds under management. with the growth in the value of the funds that he manages.

We use the same procedures as in Section 4 to calculate the optimal active decision (w^*) for the two managers at each of the three stages in the cycle largely using the base assumptions as set out in Table 3. The exceptions to these base assumptions are set out in Table 12 where we allow the values for $\acute{\alpha}$ and c to change through the life cycle.

	Good manager		Bad manager	
	Alpha ($\acute{\alpha}$)	Cost (c)	Alpha ($\acute{\alpha}$)	Cost (c)
Boutique	0.015	0.05	-0.015	0.05
Mid-Cycle	0.010	0.20	-0.015	0.20
Mature	0.005	0.50	-0.015	0.50

Table 12: Additional assumptions for the base case for alpha and costs.
All other values and definitions are to be found in Table 3

The optimal active positions for each of the six cases are reported in Table 13. We see that the good boutique manager should be extremely aggressive putting his entire fund into active portfolios as recommended by his process. As we have noted previously even the inferior boutique manager should take a fairly active position as they effectively have a free option given that they have nothing to lose. If the funds under management grow to a stage is reached where the manager has a significant amount of funds to lose and we are beginning to see some erosion in performance (mid-cycle), the manager has strong incentives to become less aggressive. In our example we see that the by mid-cycle the good manager has cut his active position by about a quarter while the bid manager has cut his by about a half. We assume that each of our managers keep growing and eventually a stage is reached where the potential gains from further good performance (g) are equal to the costs (c) associated with sustained underperformance. By this stage, the ability of the good manager to outperform has been seriously eroded ($\acute{\alpha}$ has fallen from 1.5% to 0.5%) while the expected underperformance of the bad manager is maintained. It is now optimal for the good manager only to maintain a very small active position while the bad manager should not push his luck any further and effectively become fully indexed. The fact that the lucky bad managers will move towards being fully indexed is not necessarily a bad thing for their clients. However, the same cannot be said for the successful good manager whose potential to add value is no longer available to its clients but, perhaps more importantly, the markets no longer benefits from the manager's ability to first identify and then correct mispricings.

	Good	Bad
Boutique	1.00	0.53
Mid-Cycle	0.73	0.29
Mature	0.08	0.00

Table 13: Optimal active positions (w*) at various stages in the life-cycle

6. Major Implications

We now draw on the previous two sections of the paper to identify some of the major implications of the model for how active managers might implement their investment process. Active fund management organizations are assumed to be risk-averse and to maximize their utility by selecting the optimal fraction of their funds to invest in the active portion of their portfolio (with the remainder being effectively invested in their benchmark portfolio). A higher active position represents a more aggressive implementation of their investment process and the discussion in Section 4 has highlighted the conditions under which this is likely to occur. Based on the literature, we initially assume in the discussion that follows that $c < g$, i.e., funds reap much larger benefits if they are able to strongly outperform than the costs incurred when they underperform. We have seen that this provides a strong incentive for even the funds that fundamentally lack skill (i.e., $\alpha \leq 0$) to take significant active positions although all funds are forced to reassess their appetite for large active positions as the potential gains (g) reduce relative to the potential costs (c).

We will start our discussion of the implications with the key stylized facts that we developed in Section 2 and then proceed to discuss some of the other major implications that can be drawn from the results reported in Section 4:

1. *New funds with little or no reputation and low funds under management are most likely to take the largest active positions* As we have shown this finding is largely the result of the fact that these funds have little to lose in the way of funds under management and/or reputation (i.e., low c), especially relative to the rewards that may well come their way (i.e., high g) if they are able to display a sustained period of impressive outperformance. We argued in Section 2 that c is likely to increase and g possibly will fall as funds under management become large. We proposed that this will provide an incentive for the fund to become more conservative in their active positions. The findings reported in Table 4 support this proposition as does a comparison between Table 8 and 9. Evans (2009) highlights that the performance of high flying young funds seems to quickly dissipate. One possible contributing factor for this is that as their funds under management grows, they now have a strong incentive to become less aggressive and which results in them being less able to replicate their initial good performance.

2. *Funds that are more likely to outperform (i.e., with higher α) have stronger incentives to be more aggressive in the implementation of their process than is the case with lesser funds (i.e., with lower α).* A fund's ability to outperform is largely reflective of its investment style, its investment process, its investment staff and its ability to coordinate all three in the implementation of its investment decisions. Not all funds are created equal on these scores but for the observer it is not an easy matter to identify the funds with the best potential as there are numerous extraneous factors that impact on fund performance, especially in the short-term.

   However, what is true is that over any time period the better funds are more likely to hit the upper threshold (k) and less likely to hit the lower benchmark ($\underline{k}$). This being the case, it is not surprising that the results that we report in Table 5 demonstrate that the optimum active position for a fund increases with its potential to outperform (i.e., funds with a higher α). We see by comparing Tables 8 and 9 that even these superior funds will reduce their optimum active positions in line with a reduction in the relative payoffs stemming from a sustained period of outperformance.

   One other issue worth contemplating here is whether it is appropriate to assume that funds that take the more active positions are the superior managers. The work of Cremers and Petajisto (2009) suggest that this might be the case as they find a positive relationship between active position and investment returns although Bird et al. (2011) find that this does not hold for all styles of managers over all periods in the market cycle. Overall, the evidence raises the possibility that managers might be a relatively good judge of the potential of their fund and so may provide signals via their investment behaviour that are useful to investors when choosing funds in which to invest.

3. *It is not only α that has implications for the optimum active positions but also the impact that the active positions have on the volatility of the portfolio's performance.* We see from Table 10 that the optimum active position decreases if the volatility σ_a of the active process increases and/or it is less correlated with the benchmark. While the first of these findings is not surprising, the second one is less intuitive and is related to our initial point that it is the impact of the active position on the risk of the whole portfolio that is important in determining the optimum level of the active position. A negative correlation increases the spread between the return on the benchmark portfolio and that on the actual portfolio (benchmark plus active position). This translates into an increase in the variance of X, which suggests a more conservative approach to implementing an active strategy.

4. *The attitude to risk of the fund management organization (and managers of individual funds) will play an important role in determining how aggressive they are in implementing the fund's investment process.* We see from Table 8 (and Table 9) that the more risk-averse organisations will be more conservative when implementing the investment process of the fund (i.e., have a lower w^*). The issue of what determines an organizations level of risk-aversion is very complex but one factor that may impact on a fund's aversion to risk is its reputation. The manager of a fund may wish to protect its reputation by becoming more conservative in the implementation of the process and

so reduce the probability of sustained underperformance[6]. This could be tested by observing the differing behaviour of new boutique fund organisations when implementing their initial products to see if it is affected by the reputation of the promoters of the funds.

One issue that we have not taken up in our discussion is the potential conflict between those that manage the fund and those that own the organisation offering the fund. Where the manager(s) does not hold a large proportion of the equity in the organisation, it is possible that his attitude to (λ) is much different to that of the organisation. For example, the organisation might run numerous funds and be more interested in the risk that a particular fund brings to its whole portfolio of funds. Thus a conflict may arise between the organisation offering a particular fund and the manager of that fund originator and the manager in terms of the aggressiveness with which the process is implemented. This conflict may become a particular problem where a fund is "required" to become less aggressive as it grows in size due to past good performance. This is likely to result in a very capable manager becoming dissatisfied due to being restrained from displaying his full potential. One way of coping with the conflicts within the organisation is by increasing the remuneration but this may not be sufficient to always solve the problem with the possible end result that the manager leaves the fund. Indeed, the manager may leave and set up his own boutique where he will have unfettered control over the implementation of the process.

5. *The way that movement in the thresholds will impact on the optimum active position.* The proposition in this paper is that sustained out (under) performance will have a positive (negative) impact on future fund flows. We have chosen to model sustained outperformance as requiring the fund to outperform by at least a specified amount (the upper threshold, k) over a particular time period. This is consistent with the findings of Sirri and Tufano (1998) that beyond some point outperformance results in a rapid flow of new funds.

   We show in Table 6 that an increase in the magnitude of the lower threshold (k) typically results in a slight decrease in the optimum active position. We might expect to find an analogous relationship between changes in the upper threshold (k) and the optimum active position with an increase in the magnitude of the upper threshold resulting in a decrease in the optimum active position. However, we find the opposite to be the case with the optimum active position (w*) actually increasing as the upper threshold (*k*) increases. On reflection, this is not such a surprising finding when one remembers that the assumed gains from outperformance (*g*) in our base case are ten-times greater than the costs (*c*) associated with underperformance. When we checked to see what happened in cases where g = c (Table 11), we found that the optimum active position no longer consistently increased as the upper threshold is increased. The optimum active position w* does initially increase where

[6] Another factor that would impact on behaviour will be the level of funds under management which will also drive a more conservative approach because it means that they organization has more at risk.

the upper threshold (k) is initially low relative to the lower threshold ($\underline{k}$) but then w* begins to decrease as k approaches $\underline{k}$ and becomes zero where $k \geq \underline{k}$.

6. *When should an active manager become passive?* The answer to this question depends on the skill of the manager (α), the relative payoffs (g cf c) and the level of the thresholds (k cf $\underline{k}$). For a manager with average skill ($\alpha = 0$), there will never be a case to be active when $g \leq c$ and $k \geq \underline{k}$. In other words, there is never the incentive for such a manager to implement an active portfolio where the expected gains in future fee income associated with outperformance are either equal or less than the expected loses in fees associated with underperformance. Of course this is not the case for a boutique manager with everything to gain and nothing to lose. The boutique manager will be relatively active when entering the market even if they lack skill. As we have seen from our example in Section 5, this will all change as a manager begins to accumulate asset under management with the incentives being for such a manager to become increasingly less active and possible eventually reach a situation where the optimum strategy would be to become totally passive

7. Summary

This paper focuses on what has been a largely neglected aspect of the operations of a funds management organization, how aggressive should it be in implementing its investment process. Our approach has been to develop a model based on the presumption that the organization's objective is to maximise the present value of its future fee income. We found that the model provides an explanation for certain behaviour that we observe and in particular:

- Those boutique funds with little reputation or funds under management are likely to be the most aggressive
- Those funds with the greatest capabilities to add value ideally would be more aggressive than inferior funds.

The suggestion being that the optimal strategy for funds whose potential is realised by way of accumulating very large funds under management would be to become "closet" indexers. We say this because the organization reaches a point where costs associated with sustained underperformance (c) increase to a level approaching the potential gains associated with sustained outperformance. Further, the ability of the fund to outperform is diluted as its funds under management reaches grow in magnitude. We use the words "closet" indexers as the fund needs to maintain the appearance of still being active because of the nature of the active mandate with its investors. Otherwise, it runs the risk of losing funds not because of poor performance but because it has dramatically strayed from its benchmark[7].

It is interesting to speculate on some of the possible implications of our findings for investors. The first insight for investors is that a willingness to take greater active positions, especially in the case of newer funds may be taken as a positive signal as to the capabilities

[7] It is interesting to speculate how a fund can become passive without giving the appearance of being passive. The answer probably lies in neutralising all of its bets on factors that are likely to cause its performance to depart from its benchmark which will necessarily mean effectively giving up on its process

of these managers. Cremers and Petajisto (2009) have identified that the more active managers produce better investment outcomes which our model suggests maybe due to the superior managers being the more aggressive[8]. This suggests that the active positions taken by a fund might provide a useful signal when trying to differentiate between the skills of managers. A number of funds of funds have been developed on the premise of constructing a portfolio of relatively new aggressively managed funds (e.g. the Russell stable of Opportunity Funds). Tracing the level of a fund's active position over time might also provide a good sign of when to divest from a fund. We have suggested that even superior funds will reduce their level of aggression in line with significant growth in its funds under management. Observing this as it occurs may provide a good signal of deterioration in the performance of the fund and so suggest looking for alternative investment opportunities.

Our findings also have implication for the functioning of markets that rely on investors identifying and exploiting investment opportunities and so correcting mispricings. Presumably the funds that realise sustained outperformance are among the better at identifying exploitable opportunities. The implications of our models are that it is likely that these funds will become less active over time resulting in the lesser funds playing a relatively larger role. This possibility combined with a trend towards more passive investing (index funds and closet indexers) and investment styles that are disruptive to markets (e.g. momentum) increases concern as to whether we are seeing a trend towards increasingly less efficient markets (Bird et al., 2011).

In closing, our paper has provided some interesting insights into an oft-neglected issue: how do funds determine the level of aggression with which they implement their investment process. In our discussion we have identified a number of areas which would well benefit from further research.

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