# On the distribution of European government bond returns: Empirical evidence

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#### Abstract

Due to recent turbulences in the European debt capital market, one might ask for consequences for investors. We examine which distribution assumptions hold for European government bond returns in the period from 1999 to 2011. Returns of government bonds of several European countries, inside and outside the EMU, are analysed. We fit the data to Gaussian, Student's t, skewed Student's t and stable distributions. For sovereign risk free countries we find that the Gaussian distribution is sufficient to fit the data. Since the sovereign risky government bond returns are skewed and heavy tailed, the Gaussian distribution is not flexible enough. Therefor we suggest the skewed Student's t or stable distribution to model these bonds. The results are robust to a variety of goodness of fit statistics.

**Keywords:** sovereign bond, government bond, government debt, skewed and heavy-tailed distribution, EMU

**JEL–class.:** G12, H63, C46

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### 1 Introduction

In the time prior to the European Monetary Union (EMU) yields for Euro denominated government bonds converged (see Adam et al., 2002) and prices harmonised. Government bond spreads of EMU countries that formerly issued sovereign bonds vanished after the EMU introduction. Sovereign risk (see Bernoth et al. (2004), Sgherri and Zoli (2009)) was not recognized by investors. In the present Euro crisis, this sovereign risk plays a key role and let yields of non sovereign risk and sovereign risk countries diverge again. A second effect of the EMU introduction is the significant growth of the Euro bond Market (see Pagano and Thadden, 2004). The ECB (2004) states two reasons for this development. Firstly, improvements of the budgetary balances led to low net borrowing costs. Secondly, more transparency in bond issuance resulted in a highly liquid Euro bond market. The presence of sovereign risk in European government bonds and significant growth of the European bond market force investors to improve their risk management for European government bonds.

It is well known that the first two moments are not sufficient to describe the investor's utility.<sup>1</sup> Due to limited positive returns of bonds (in contrast to equities) and a potential unlimited loss of the nominal value looking at skewness in bond returns is intuitive.<sup>2</sup> Although the origin of excess kurtosis in

<sup>&</sup>lt;sup>1</sup>Higher order moments are important for portfolio selection (see Arditti (1967), Arditti (1971) and Rubinstein (1973)). Empirical studies support that returns are not normal distributed (see Fama (1965) and Arditti (1971)).

<sup>&</sup>lt;sup>2</sup>See Gupton et al. (1997) and Basel Committee (1999, p.27).

bond returns is less intuitive, it is equally important in risk management (i.e. for VaR calculations). To support this intuition, Table 1 shows the skewness and kurtosis for sovereign risky and sovereign risk free European government bond returns. One year sovereign risk free bonds has slightly right skewed and heavy-tailed returns. The sovereign risky bonds of all maturities exhibit significant left skewness and excess kurtosis. This skewness and excess kurtosis in bond returns motivates our choice of tested distributions.

#### [insert Table 1 about here]

In our study, we empirically test the characteristics of single European government bond returns with maturities of (1,)3,5 and 10 years in the period of 1999 to 2011. Special attention is paid to skewness and excess kurtosis caused by sovereign risk. The significant rise of spreads in 2008 leads to the conclusion that we have a structural break in the data. Hence, we separately study the time of the recent Euro crisis (2008-2011). We test the assumption of normal, Student's t, skewed Student's t and stable distributed returns, to match the skewed and heavy-tailed government bond returns. Finally, we perform a variety of goodness of fit statistics to support the statistical significance of our findings.

The first string of literature, the paper is related to, is the vast literature about security return distributions. This literature is mostly concerned with equity returns. Beginning with Mandelbrot (1963) all of them have in common that they reject the Gaussian distribution assumption as a hypothesis for financial returns. Because of it's capability to exhibit heavy tails, there is empirical evidence for the Student's t distribution to give a reasonable fit for financial returns.<sup>3</sup> The skewed Student's t distribution is often used in modeling financial data, because it is able to handle skewness and excess kurtosis (see Harris and Küçükömzmen, 2001).<sup>4</sup> Since stable distributions can capture skewness and kurtosis, too, there exists a significant number of papers that suggest the usage of stable distributions for modeling equity returns. Höchstötter et al. (2005) even find that the stable distribution outperforms the skewed Student's t distribution in fitting returns of german stocks (DAX). <sup>5</sup> Rachev et al. (2000) are the only ones to study bond returns. They describe indices of US corporate bond returns with the stable distributions.

The second string of literature, driven by central bank research, is concerned with international bond markets. Cappiello et al. (2003) and Christiansen (2007) can find a nearly perfect correlation of US and European government bonds. Volatility spillover effects in the period of 1991 to 2002 are reported by Skintzi and Refenes (2006). The focus of our paper lies on the Euro government bond market. Côté and Graham (2004) find strong harmonisation of long term government bond yields caused by the EMU introduction.

 $<sup>^{3}</sup>$ See e.g. Aparicio and Estrada (2001) (European stock markets), Peiró (1994) (stock markets worldwide) or Blattberg and Gonedes (1974) (US stock market).

<sup>&</sup>lt;sup>4</sup>The skewed Student's t distribution was proposed by Hansen (1994). Further application to stock returns is given by Jondeau et al. (2007).

<sup>&</sup>lt;sup>5</sup>Further application of the stable distribution is for instance given by Rachev et al. (2005) (US stock market), Kanellopoulou and Panas (2008) (French stock market) and Höchstötter (2006) (German stock market).

Finally, in this string of literature our paper is closely related to the study of Laopodis (2008). He examines Euro and non-Euro 10 year government bond returns prior and post EMU integration in the period of 1995 to 2006. He also describes their higher order moments but does not give any application for bond return distributions. The EMU introduction brought risk into government bonds, which was formerly known only in sovereign bonds. Bernoth et al. (2004) and Sgherri and Zoli (2009) study this sovereign risk characteristics of European government bonds prior to the EMU and during the financial crisis, respectively.

The present paper contributes to the existing literature in three essential ways. To our best knowledge, we are the first to test a variety of distributions to match skewness and excess kurtosis in government bond returns. Secondly, we differ from other work in this area, by explicitly investing in single government bonds of single maturities instead of using indices. These indices cluster the duration as well as the rating, which gives a blurred view on interest rate and sovereign risk, respectively. Thirdly, we perform a variety of goodness of fit statistics in order to test the robustness of our findings. The remainder of the paper is organized as follows. In section 2 we give an overview of the government bond data under consideration. In section 3 we outline the empirical study. After motivating and defining the distributions in subsection 3.1, we present the results for the period of the European Monetary Union and for the period of the paper. The results of non Euro government bond data is provided in the appendix.

### 2 Data

To shed some light on Euro government bond characteristics, we investigate several zero coupon bonds of EMU members. The data is provided by each central bank. To explain skewness and excess kurtosis, we try to identify sovereign and interest rate risk as risk driving factors. Therefore we split the dataset in two dimensions. Firstly, the countries are clustered to either bearing sovereign risk or being sovereign risk free. Secondly, single bonds with different maturities are tested to report duration<sup>6</sup> as a measure of interest rate risk. Our dataset consists of government bonds of Germany, France, Greece, Italy, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States. We consider Germany and France to be sovereign risk free. Greece, Italy, Portugal and Spain are classified as sovereign risk bearing. This shall give a clearer view on whether skewness is exclusively caused by sovereign risk. To correct for effects that are caused by the EMU membership or the Euro currency we add Sweden, Switzerland and United Kingdom and United States, respectively.

Secondly, we explicitly study single government bonds of these countries with different maturities. Bonds with different durations shall help identifying the

<sup>&</sup>lt;sup>6</sup>Since we look at single zero coupon bonds, maturity and duration are equivalent. Therefor we do not explicitly report the durations.

contribution of sovereign risk to skewness and excess kurtosis.

Our dataset consists of government bonds with (1, )3, 5 and 10 years to maturity.<sup>7</sup> The observations are monthly starting in January 1999 through November 2011. The beginning of the period is the time where the exchange rates for EMU were fixed and exchange rate risk de facto no longer existed. This gives an overall dataset of 5364 data points.

To be able to fix the (1, )3, 5 and 10 year maturities over the period under consideration we use synthetic rather than traded bonds. These bond yields are calculated with the well known Svensson (1994) Modell which is standard for EMU countries.<sup>8</sup> The yield curve function is specified by the six parameters  $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1$  and  $\tau_2$ . The spot rate y(T) for a zero coupon bond maturing at T reads:

$$y(T) = \beta_0 + \beta_1 \frac{1 - e^{-\frac{T}{\tau_1}}}{\frac{T}{\tau_1}} + \beta_2 \left(\frac{1 - e^{-\frac{T}{\tau_1}}}{\frac{T}{\tau_1}} - e^{-\frac{T}{\tau_1}}\right) + \beta_3 \left(\frac{1 - e^{-\frac{T}{\tau_2}}}{\frac{T}{\tau_2}} - e^{-\frac{T}{\tau_2}}\right).$$

There are important central banks which use other interpolation methods for yield curve smoothing such as splines (FED). Nevertheless BIS (2005) states that the difference in estimation error are only noticeable for very short (<<1 Year) or very long (>>10 years) maturities. Since we study bonds with (1,)3,5 and 10 years to maturity the yield curve fitting method does not contribute to the estimation error significantly.

<sup>&</sup>lt;sup>7</sup>Only the government bond data of Germany, France, Spain, Switzerland, UK and US include a bond maturing after one year.

<sup>&</sup>lt;sup>8</sup>The central banks of Germany, France, Italy, Spain, Sweden and Switzerland fit the yield Curve with Svensson (1994). UK use Variable Roughness Penalty and the US use smoothing splines (see BIS, 2005). There is no information available which yield curve fitting method is used by Portugal and Greece.

### 3 Empirical Study

Having outlined the government bond data, section 3 presents the results of the empirical study. After motivating the use of more sophisticated distributions than the normal distribution, subsection 3.1 defines the distributions under consideration. Subsection 3.2 presents the results of the empirical study of the period of 1999-2011. Subsequently, the results for period of the Euro crisis are given in subsection 3.3.

#### [insert Table 2 about here]

Table 2 shows the mean, variance, skewness and kurtosis of the bond returns of Germany, France, Greece, Italy and Portugal for all analysed maturities. The mean of all bond returns is close to zero. Generally, bond returns with longer maturity show a higher variance, which is caused by a greater exposure to interest rate and credit risk. We find very little evidence for skewness of returns of sovereign risk free countries in the one year maturity. Returns with longer maturity are nearly symmetric. Secondly, we study bond returns of countries that have exposure to sovereign risk. Here, we find a strong left skewness. Similar features can be observed for the kurtosis: While the returns of Germany and France for the long maturities nearly seem Gaussian, we find excess kurtosis in the returns with one year to maturity. The return data of sovereign risky countries, particularly Greek and Portuguese, display a significant excess kurtosis. In summary, there is strong empirical evidence that motivates the use of distributions allowing for excess kurtosis and skewness, when describing European government bond returns.

#### **3.1** Distribution Assumption

Beside the normal and the Student's t distribution we consider the skewed Student's t and the stable distribution as possible candidates for describing government bond data. Normal distributions are neither able to exhibit skewness nor heavy tails. There is empirical evidence (see Table 2) that excess kurtosis is a feature of government bond returns. The Student's tdistribution is capable of modeling heavy tails. Let  $2 < \eta < \infty$  be the degree of freedom of a Student's t distribution. The normal distribution is a special case of the Student's t distribution as  $\eta$  tends to infinity.

Table 2 illustrates the need to model skewness, too. By adding the parameter  $-1 < \lambda < 1$  to the Student's t distribution, we get the skewed Student's t distribution that captures skewness and excess kurtosis. As  $\lambda$  gets closer to the left (right) bound, the distribution is left (right) skewed. The density of a skewed Student's t distribution (see Hansen, 1994) is given by

$$g(z \mid \eta, \lambda) = \begin{cases} bc \left(1 + \frac{1}{\eta - 2} \left(\frac{bz + a}{1 - \lambda}\right)^2\right)^{-(\eta + 1)/2} & z < -a/b \\ bc \left(1 + \frac{1}{\eta - 2} \left(\frac{bz + a}{1 + \lambda}\right)^2\right)^{-(\eta + 1)/2} & z \ge -a/b \end{cases}$$

with  $a = 4\lambda c(\eta - 2)/(\eta - 1)$ ,  $b^2 = 1 + 3\lambda^2 - a^2$  and  $c = \frac{\Gamma((\eta + 1)/2)}{\sqrt{\pi(\eta - 2)}\Gamma(\eta/2)}$ . The skewed Student's t distribution nests the Student's t distribution by setting

 $\lambda = 0$ . The parameter estimation is done with maximum likelihood (ML). The ( $\alpha$ -)stable distribution has four parameters: the index of stability ( $\alpha$ ), skewness- ( $\beta$ ), scale-( $\gamma$ ) and location-parameter ( $\delta$ ). It can exhibit skewness and heavy tails as well. A closed form representation does not exist in general, but it is possible to give the characteristic function. A random variable X is called stable if its characteristic function is given by (e.g., Nolan (2001))

$$E \exp(\mathrm{i}tX) = \begin{cases} \exp\left(-\gamma^{\alpha} |t|^{\alpha} \left[1 + \mathrm{i}\beta \left(\tan\frac{\pi\alpha}{2}\right) (\mathrm{sign} t) \left(|\gamma t|^{1-\alpha} - 1\right)\right] + \mathrm{i}\delta t\right), & \alpha \neq 1\\ \exp\left(-\gamma |t| \left[1 + \mathrm{i}\beta\frac{2}{\pi} (\mathrm{sign} t) \ln(\gamma |t|)\right] + \mathrm{i}\delta t\right), & \alpha = 1 \end{cases}$$

with  $0 < \alpha \leq 2, -1 \leq \beta \leq 1, \gamma > 0$  and  $\delta \in \mathbb{R}$ . As  $|\beta|$  gets bigger, the distribution becomes skewed. Lower numbers of  $\alpha$  indicate heavy tails. As  $\alpha$  tends to 2, the distribution becomes Gaussian and  $\beta$  loses its influence. Thus, in contrast to the skewed Student's *t* distribution, it is not possible to model non heavy-tailed, but skewed data appropriately. The estimation of the parameters is done by using ML techniques (see DuMouchel (1971) and Nolan (2001)).

#### 3.2 European Monetary Union

Table 3 shows the results of the parameter estimations.<sup>9</sup> We fitted the parameters of the normal, Student's t, skewed Student's t and stable distribution to the bond returns of Germany and France (Greece, Italy and Portugal) for maturities of 1,3,5 and 10 (3,5 and 10) years. For German and French

<sup>&</sup>lt;sup>9</sup>See Table 9 in the Appendix for the estimated parameters of bond returns of Spain, Sweden, Swiss, UK and USA.

government bond data we find an increasing variance for longer maturities. For the one year maturity the degrees of freedom are lower than in the Gaussian case. We calculate an  $\alpha$  of 1.757 for the German data, 1.722 for France respectively, confirming the assumption of a leptokurtic behavior from the beginning of this section. The skewness parameters  $\beta$  and  $\lambda$  show slightly right skewed returns. For the long maturities,  $\alpha$ 's close to 2 and high degrees of freedom together with  $\lambda$ 's close to zero, speak for the Gaussian hypothesis.

#### [insert Table 3 about here]

For the three countries with sovereign risk the parameters suggest a rejection of the Gaussian hypothesis. Particularly for Greece low degrees of freedom and low  $\alpha$ 's indicate heavy tails for all maturities. High negative skewness parameters confirm the presence of left skewed returns. The estimations for Italian and Portuguese return data show similar features, they are less heavytailed, though.

After calculating the parameters we can now assess the goodness of fit statistics. We apply  $\chi^2$ , Kolmogorov-Smirnov (KS), Cramer-van-Mieses (CM) and Anderson-Darling (AD) tests. Due to the partial lack of tabulation of critical values, we use simulation techniques as proposed in Borak et al. (2005) to calculate p-values and test-statistics for all distributions for the sake of consistency. The calculated p-value gives the probability that the data comes from the distribution supposed. In Table 4 the p-values of the  $\chi^2$ -test for normal, Student's t, skewed Student's t and stable distribution are given for the bond returns of Germany, France, Greece, Italy and Portugal for all analysed maturities.<sup>10</sup> For the one-year bond returns of Germany and France, distributions that exhibit skewness and excess kurtosis give a better fit than the Gaussian distribution. This confirms our hypothesis that these kinds of returns are leptokurtic. When looking at the longer maturities, we find that all distribution give similar fits in terms of goodness. Because there is neither skewness nor excess kurtosis in the data, Gaussian distributions are feasible. The only exception is Germany giving only poor results for ten-year bond returns for all distributions.

#### [insert Table 4 about here]

The goodness of fit results for government bond returns that imply sovereign risk are not as straight forward as in the risk less case. The Gaussian and Student's t distribution fail in describing the Greek and Portuguese data for every maturity. Only for the three year maturity of Italian government bond returns we find evidence for the Gaussian distribution. There is a strong fit of the stable distribution and particularly of the skewed Student's t distribution for Greek bond returns. Exclusively for the five year maturity, both distributions give only a weak fit. For Portugal both distributions, again in particular the skewed Student's t, are able to fit the characteristics of the data in all maturities in contrast to non skewed distributions. Applied

<sup>&</sup>lt;sup>10</sup>We only discuss the results of the  $\chi^2$ -test here. We get comparable results for KS, CM and AD test. Detailed results for every test inlcuding p-value and test-statistic are given in Table 11 and 12 in the Appendix.

on the Italian data, both fits are sound for three and ten years and slightly weak for the five year maturity.

#### 3.3 Euro Crisis

From an investors perspective, the euro crisis highlighted the existence of sovereign risk in euro government bond. Since market circumstances changed, sovereign risky bond returns behaved differently from then on. As a consequence, spreads of sovereign risky government bonds rose significantly and yields diverged again. This fact could lead one to conclude that there is a structural break in the time series under consideration. A graphical analysis suggests to use May of 2008 as a possible breaking point, because this is the date from which on the spreads began to diverge. We perform a Chow (1960) test for the EMU countries to verify this hypothesis and report the results in Table 5.

#### [insert Table 5 about here]

Especially for Greek and Portuguese bond returns we find indications for a structural break in all maturities, while for countries without sovereign risk there is only evidence in the short maturity.

In the following, we split time series and study the period after the structural break which corresponds to the time of the Euro crisis. Table 6 shows the estimated parameters of German, French, Greek, Italian and Portuguese government bond returns for this period. The one year to maturity bond returns of countries without sovereign risk are leptokurtic, again. Looking at the sovereign risky bonds, we find a markable negative mean, particularly for Greece. Generally, the return data is still skewed and heavy tailed. When comparing the skewness and tail parameter to the parameters of the original period, we find even greater skewness and excess kurtosis in the data.

#### [insert Table 6 about here]

Table 7 reports the p-values of the  $\chi^2$  test on normal, Student's t, skewed Student's t and stable distributions of the present data. The p-values only consider the period after the breaking point with null hypothesis that the empirical distribution equals the distributional assumption. Again, the results are similar to the ones from the original period. In general, Gaussian distributions cannot describe the features of sovereign risky bonds while stable and skewed Student's t distributions can. The skewed Student's t distribution has a slightly superior fit. The results for KS, CM and AD tests can be found in Table 13 and 14 in the appendix.

#### [insert Table 7 about here]

We summarize our findings in Table 8. Sovereign risky government bond returns need to be modeled with distributions that exhibit skewness and heavy tails. Except for the one year maturity, sovereign risk free government bonds can be described with the Gaussian distribution.

[insert Table 8 about here]

### 4 Conclusion

Prior to the EMU introduction sovereign bond spreads vanished and debt financing costs for EMU countries leveled off. However, in the recent Euro crisis sovereign risk of government bonds of EMU countries came back to investors' mind and spreads widened, significantly.

The present paper is the first step in analysing how this development should effect investors' government bond investment decisions. Firstly, we test a variety of distribution assumptions to capture the characteristics of government bond returns with and without sovereign risk for the period between 1999 and 2011. Our study shows that the returns of government bonds without sovereign risk and one year maturity is leptokurtic and can be described with distributions that exhibit skewness and excess kurtosis. For bonds with longer maturities variance increases and returns become Gaussian. For government bond returns with sovereign risk we propose the usage of distributions that can exhibit skewness and heavy tails. The skewed Student's tand the stable distribution fit the data equally well. Our findings are robust for a variety of test statistics. Secondly, we take account of the existence of a structural breaking point in May 2008. This second analyses confirms our results: Irrespective of the period under consideration there is still need to model sovereign risky bond returns with distributions that take skewness and heavy tails into account, while in most cases the Gaussian distribution satisfies non sovereign risky bond returns.

For a better understanding of how much explanatory power the sovereign

risk factor in government bonds has, we would like to extract that from government bond yields. Therefore we would like to look at sovereign spreads separately and (potentially) extract the skewness in government bond returns. This is work in progress. Since there are only a few papers dealing with government bond return distributions yet, there are some open research topics. One interesting question is which distribution suits the government returns best in a portfolio framework and in which way ko-skewness and kokortosis affect the distribution assumption.

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		$^{\mathrm{sh}}$	ort	long		
		1	3	5	10	
governign righ free	$\gamma_1$	1.06	0.23	0.06	0.13	
sovereign fisk free	$\gamma_2$	5.99	3.11	2.52	2.90	
governigen right	$\gamma_1$		-1.09	-1.66	-1.93	
sovereigh fisky	$\gamma_2$		16.15	15.21	13.26	

#### Table 1: Skewness and Kurtosis of Bond Returns

The table shows the skewness  $(\gamma_1)$  and kurtosis  $(\gamma_2)$  of European bond returns. As sovereign risk free we pool German and French government bonds. As sovereign risky we pool Greek, Portuguese and Italian government bonds. Normal distributed returns have zero skewness  $(\gamma_1 = 0)$  and a kurtosis of three  $(\gamma_2 = 3)$ .

		Mean	Var	Skew	Kurt
	1	2.61	2.21	1.16	7.45
Cormony	3	3.52	7.04	0.20	3.14
Germany	5	4.23	11.07	0.08	2.55
	10	5.07	19.41	0.26	3.01
	1	2.56	2.15	0.95	4.53
Franco	3	3.36	7.20	0.26	3.08
France	5	3.90	11.24	0.03	2.49
	10	4.27	19.20	-0.01	2.79
	3	3.45	28.78	-1.75	13.79
Greece	5	-2.86	32.52	-2.93	17.30
	10	-3.00	41.48	-1.88	11.82
	3	2.98	8.57	-0.91	7.98
Italy	5	3.17	13.04	-0.92	7.22
	10	3.03	22.23	-0.77	7.95
	3	0.40	20.93	-0.61	26.68
Portugal	5	0.30	24.90	-1.13	21.12
	10	-3.41	37.62	-3.13	20.01

Table 2: Descriptive Statistic of Government Bond Returns

The Table shows the mean, variance, skewness and kurtosis for German, French, Greek, Italian and Portuguese government bond returns. Figures of the mean and Variance are multiplied by  $10^3$ . For comparison, the normal distribution has zero skewness and a kurtosis of three.

		normal		t	skev	wed $t$	stable					
	m	$\mu$	σ	dof	dof	$\lambda$	α	$\beta$	$\gamma$	δ		
	1	2.606	2.208	11	5	0.237	1.757	0.694	1.278	2.298		
Compony	3	3.516	7.036	$\geq 10^2$	$\geq \! 10^2$	0.123	1.960	1.000	4.855	3.265		
Germany	5	4.233	11.067	$\geq 10^2$	$\geq \! 10^2$	0.059	2.000	0.765	7.800	4.233		
	10	5.070	19.413	$\geq 10^2$	$\geq 10^2$	0.115	1.952	1.000	13.355	4.243		
	1	2.563	2.147	15	7	0.305	1.722	1.000	1.277	2.097		
France	3	3.362	7.203	$\geq 10^2$	$\geq 10^2$	0.164	1.955	1.000	4.949	3.065		
	5	3.896	11.237	$\geq 10^2$	$\geq \! 10^2$	-0.045	2.000	0.332	7.920	3.896		
	10	4.265	19.203	$\geq 10^2$	$\geq 10^2$	-0.086	2.000	0.959	13.535	4.266		
	3	3.451	28.781	5	2	-0.173	1.185	-0.074	7.256	6.493		
Greece	5	-2.858	32.517	5	2	-0.503	1.263	-0.627	7.730	5.812		
	10	-2.999	41.483	5	3	-0.372	1.394	-0.560	13.572	6.527		
	3	2.984	8.567	10	5	-0.157	1.767	-0.371	4.810	3.659		
Italy	5	3.175	13.039	9	4	-0.242	1.702	-0.655	7.080	5.103		
	10	3.032	22.235	9	4	-0.279	1.632	-0.583	11.565	6.097		
	3	0.404	20.927	5	2	-0.384	1.390	-0.673	4.893	4.330		
Portugal	5	0.296	24.898	6	2	-0.403	1.471	-0.891	7.800	5.834		
i oi tugai	10	-3.406	37.619	6	3	-0.527	1.466	-0.890	13.517	6.997		

Table 3: Parameter Estimation of selected Distributions for European Bond Data Estimated parameters of the normal, Student's t, skewed Student's t and stable distribution for German and French (sovereign risk free) and Greek, Italian and Portuguese (sovereign risky) bond data. Sovereign risk free (sovereign risky) bond data is reviewed for maturities of 1, 3, 5 and 10 (3, 5 and 10) years.  $\mu$ ,  $\sigma$ ,  $\gamma$  and  $\delta$  are multiplied by 10<sup>3</sup>.

				skewed	
	m	normal	Student	Student	stable
	1	0.14	0.06	0.78	0.39
Composition	3	0.53	0.50	0.43	0.38
Germany	5	0.35	0.36	0.19	0.32
	10	0.04	0.04	0.02	0.02
	1	0.09	0.04	0.37	0.26
France	3	0.42	0.45	0.36	0.31
France	5	0.27	0.27	0.23	0.23
	10	0.13	0.15	0.15	0.12
	3	0.00	0.00	0.75	0.27
Greece	5	0.00	0.00	0.04	0.07
	10	0.00	0.00	0.83	0.56
	3	0.39	0.13	0.35	0.45
Italy	5	0.18	0.03	0.13	0.15
	10	0.03	0.01	0.53	0.24
	3	0.00	0.01	0.60	0.24
Portugal	5	0.00	0.00	0.63	0.55
-	10	0.00	0.00	0.67	0.52

Table 4: P-Values for  $\chi^2$ -Tests

P-values of  $\chi^2$  test on normal, Student's *t*, skewed Student's *t* and stable distribution for German and French (Greek, Italian and Portuguese) bond data for maturities of 1, 3, 5 and 10 (3, 5 and 10 )years with null hypothesis that the empirical distribution equals the distributional assumption.

	1	3	5	10
Germany	*10.14	2.97	2.08	1.26
France	*14.09	*4.63	2.64	1.47
Greece	-	*13.91	*33.24	*16.02
Italy	-	*5.26	*4.28	*3.49
Portugal	-	*16.40	*16.29	*33.38
Spain	*7.01	*5.02	*3.41	2.82

Table 5: Ch	low Test	for EN	IU Cou	Intries
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Chow statistic for the bond return data of Germany, France, Greece, Italy, Portugal and Spain for all maturities under consideration assuming a structural break in May 2008. A \* implies significance on a 5% level.

		nori	mal	t	skev	wed t	stable					
	m	$\mu$	σ	dof	dof	$\lambda$	α	eta	$\gamma$	δ		
Germany	1	2.199	3.068	8	3	0.285	1.517	0.580	1.428	1.513		
Franco	1	2.072	2.921	15	5	0.693	1.367	1.000	1.326	0.878		
	3	4.255	7.991	33	13	0.096	1.904	1.000	5.278	3.574		
	3	-5.941	52.057	15	4	-0.264	1.452	-0.545	26.550	4.030		
Greece	5	-22.317	55.673	11	3	-0.490	1.141	-0.824	19.392	-0.601		
	10	-24.640	70.651	16	4	-0.352	1.311	-0.782	30.988	-2.947		
	3	1.922	12.582	9	3	-0.251	1.397	-0.496	5.602	4.353		
Italy	5	2.164	18.494	8	3	-0.327	1.335	-0.716	6.863	7.291		
	10	0.711	32.276	7	3	-0.165	1.124	-0.123	10.150	4.565		
Dortugal	3	-6.360	38.110	6	2	-0.511	0.994	-0.797	8.861	5.135		
Portugal	5	-8.570	43.797	6	3	-0.510	1.099	-0.858	12.636	4.865		
	1	1.929	2.279	6	2	0.420	1.432	1.000	0.684	1.168		
ES	3	2.769	5.883	8	3	0.550	1.402	1.000	2.437	0.562		
	5	4.876	8.067	21	$\geq 10^2$	0.760	1.658	1.000	4.708	2.892		

Table 6: Parameter Estimation of selected Distributions for European Bond Data after structural Break Estimated parameters of the normal, Student's t, skewed Student's t and stable distribution for German and French (sovereign risk free) and Greek, Italian, Portuguese and Spanish (sovereign risky) bond data for the period after the structural break. The bond data is reviewed for different maturities of 1, 3, 5 and 10 years.  $\mu$ ,  $\sigma$ ,  $\gamma$  and  $\delta$  are multiplied by 10<sup>3</sup>.

				skewed	
	m	normal	Student	Student	stable
Germany	1	0.02	0.02	0.80	0.47
France	1	0.06	0.04	0.87	0.58
France	3	0.55	0.54	0.38	0.46
	3	0.42	0.39	0.61	0.41
Greece	5	0.00	0.00	0.40	0.33
	10	0.02	0.02	0.72	0.57
	3	0.25	0.23	0.35	0.18
Italy	5	0.00	0.00	0.09	0.08
	10	0.00	0.01	0.69	0.39
	3	0.01	0.01	0.42	0.38
Portugal	5	0.00	0.00	0.75	0.59
	10	0.02	0.01	0.93	0.33
	1	0.00	0.00	0.76	0.29
Spain	3	0.13	0.11	0.49	0.29
	5	0.62	0.63	0.55	0.59

Table	7:	P-	Values	for	$\chi^2$ -Tests	after	structural	Break
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P-values of  $\chi^2$  test on normal, Student's t, skewed Student's t and stable distribution for German, French, Greek, Italian, Portuguese and Spanish bond data for different maturities of 1, 3, 5 or 10 years for the period after the structural break with null hypothesis that the empirical distribution equals the distributional assumption.

	sovereign	$\operatorname{sovereign}$
	risk free	risky
1000 2011	normal	skewed $t/$
1999-2011	normai	stable
2008 2011	normal	skewed $t/$
2008-2011	normai	stable

Table 8: Concluding links between Returns, Periods and Distributions

## A Appendix

		nor	$\operatorname{rmal}$	t	skev	wed $t$		stable						
	m	$\mu$	$\sigma$	dof	dof	$\lambda$		$\alpha$	$\beta$	$\gamma$	δ			
	3	3.657	8.889	18	8	0.110	1	.874	0.130	5.713	3.522			
CH	5	4.353	12.558	$\geq \! 10^2$	$\geq \! 10^2$	0.022	2	.000	0.928	8.851	4.353			
	10	5.232	22.106	$\geq 10^2$	$\geq 10^2$	-0.053	2	.000	0.830	15.581	5.232			
	1	2.554	2.096	12	6	0.496	1	.568	1.000	1.109	1.919			
FS	3	1.864	5.730	18	8	0.221	1	.902	1.000	3.734	1.346			
ĿЭ	5	2.846	8.756	$\geq \! 10^2$	$\geq \! 10^2$	0.106	1	.954	1.000	6.012	2.478			
	10	4.087	15.077	$\geq 10^2$	$\geq 10^2$	0.191	1	.930	1.000	10.278	3.183			
	1	2.606	3.142	9	3	0.110	1	.420	0.264	1.464	2.329			
сь	3	2.982	8.782	10	4	-0.033	1	.730	-0.185	4.974	3.304			
SE	5	3.118	13.996	12	5	-0.072	1	.763	-0.192	8.292	3.510			
	10	3.040	24.180	10	5	-0.121	1	.766	-0.221	13.896	3.837			
	1	3.497	2.468	16	49	0.600	1	.791	1.000	1.508	3.077			
ШИ	3	4.364	7.076	34	14	0.043	1	.953	0.471	4.861	4.210			
υĸ	5	4.749	11.057	$\geq 10^2$	60	-0.082	2	.000	0.664	7.793	4.749			
	10	4.917	20.571	34	12	0.065	1	.958	1.000	14.221	4.163			
	1	2.770	2.459	46	90	0.592	1	.644	1.000	1.473	2.134			
TICA	3	4.098	7.240	$\geq \! 10^2$	$\geq \! 10^2$	-0.013	2	.000	0.844	5.103	4.098			
USA	5	5.023	12.716	$\geq 10^2$	$\geq 10^2$	-0.144	2	.000	0.484	8.963	5.023			
	10	6.030	23.725	15	6	-0.131	1	.789	-0.430	14.628	7.113			

Table 9: Parameters of selected Distributions for European Bond Data Estimated parameters of normal, Student's t, skewed Student's t and stable distribution for Swedish (SE), Spanish (ES), UK, USA and Swiss (CH), bond data. Maturities of 1, 3, 5 and 10 years are reviewed.  $\mu$ ,  $\sigma$ ,  $\gamma$  and  $\delta$  are multiplied by  $10^3$ .

		normal					t			skewed $t$			stable				
	m	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD
	1	<b>0.14</b> (8.87)	<b>0.00</b> (0.09)	<b>0.00</b> (0.31)	<b>0.00</b> (1.81)	<b>0.06</b> (11.47)	<b>0.01</b> (0.10)	<b>0.00</b> (0.37)	<b>0.00</b> (2.17)	<b>0.78</b> (1.05)	<b>0.66</b> (0.04)	<b>0.15</b> (0.05)	<b>0.13</b> (0.37)	<b>0.39</b> (1.91)	<b>0.37</b> (0.05)	<b>0.09</b> (0.08)	<b>0.09</b> (0.47)
	3	<b>0.53</b> (4.45)	<b>0.56</b> (0.05)	<b>0.71</b> (0.04)	<b>0.76</b> (0.25)	<b>0.50</b> (4.64)	<b>0.56</b> (0.05)	<b>0.72</b> (0.04)	<b>0.76</b> (0.25)	<b>0.43</b> (4.47)	<b>0.83</b> (0.04)	<b>0.52</b> (0.03)	<b>0.54</b> $(0.24)$	0.38 (4.56)	<b>0.58</b> (0.04)	<b>0.49</b> (0.04)	<b>0.61</b> (0.26)
Germany	5	<b>0.35</b> (6.00)	<b>0.36</b> (0.05)	<b>0.17</b> (0.08)	<b>0.23</b> (0.49)	<b>0.36</b> (6.00)	<b>0.37</b> (0.05)	<b>0.18</b> (0.08)	<b>0.25</b> (0.49)	0.19 (6.47)	<b>0.05</b> (0.06)	<b>0.03</b> (0.09)	0.05 $(0.51)$	<b>0.32</b> (6.00)	<b>0.30</b> (0.05)	<b>0.14</b> (0.09)	0.17 (0.50)
	10	<b>0.04</b> (12.34)	<b>0.02</b> (0.08)	<b>0.08</b> (0.11)	<b>0.07</b> (0.71)	0.04 (12.34)	<b>0.02</b> (0.08)	<b>0.09</b> (0.11)	<b>0.07</b> (0.71)	0.02 (13.58)	<b>0.00</b> (0.08)	<b>0.00</b> (0.15)	<b>0.00</b> (0.83)	0.02 (12.75)	<b>0.01</b> (0.08)	<b>0.02</b> (0.14)	<b>0.02</b> (0.78)
	1	<b>0.09</b> (9.87)	<b>0.00</b> (0.13)	<b>0.00</b> (0.39)	<b>0.00</b> (2.08)	<b>0.04</b> (11.38)	<b>0.00</b> (0.13)	<b>0.00</b> (0.41)	<b>0.00</b> (2.18)	<b>0.37</b> (3.66)	<b>0.00</b> (0.08)	<b>0.01</b> (0.11)	<b>0.02</b> (0.57)	<b>0.26</b> (3.01)	<b>0.01</b> (0.08)	<b>0.04</b> (0.09)	<b>0.07</b> (0.48)
	3	0.42 (5.07)	<b>0.37</b> (0.05)	<b>0.74</b> (0.03)	<b>0.77</b> (0.24)	<b>0.45</b> (5.08)	<b>0.39</b> (0.05)	<b>0.77</b> (0.03)	<b>0.78</b> (0.24)	<b>0.36</b> (5.18)	<b>0.32</b> (0.05)	<b>0.45</b> (0.04)	0.51 (0.25)	<b>0.31</b> (4.99)	<b>0.23</b> (0.06)	<b>0.50</b> (0.04)	<b>0.60</b> (0.26)
France	5	<b>0.27</b> (6.67)	<b>0.23</b> (0.06)	<b>0.18</b> (0.08)	0.24 $(0.47)$	<b>0.27</b> (6.67)	<b>0.23</b> (0.06)	<b>0.18</b> (0.08)	0.24 $(0.47)$	<b>0.23</b> (6.09)	<b>0.17</b> (0.05)	<b>0.08</b> (0.07)	0.10 $(0.43)$	<b>0.23</b> (6.69)	<b>0.19</b> (0.06)	<b>0.14</b> (0.08)	<b>0.18</b> (0.48)
	10	<b>0.13</b> (8.79)	0.15 (0.06)	<b>0.05</b> (0.12)	<b>0.07</b> (0.68)	<b>0.15</b> (8.79)	<b>0.16</b> (0.06)	<b>0.06</b> (0.12)	<b>0.08</b> (0.68)	<b>0.15</b> (7.36)	<b>0.23</b> (0.05)	<b>0.04</b> (0.08)	<b>0.05</b> $(0.52)$	<b>0.12</b> (8.86)	<b>0.13</b> (0.06)	<b>0.03</b> (0.12)	0.05 (0.68)

Table 10: Goodness-of-Fit Tests for German and French Bond Data

 $\chi^2$ , Kolmogorov-Smirnov (KS), Cramer-van-Mieses (CM) and Anderson-Darling (AD) tests on normal, Student's t, skewed Student's t and stable distribution for German and French bond data for maturities of 1, 3, 5 and 10 years. The fat values denote p-values with null hypothesis that the empirical distribution equals the distributional assumption. Numbers in brackets represent the test statistics.

			nor	mal			t				skev	ved $t$		stable				
	m	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	
	3	<b>0.00</b> (48.72)	<b>0.00</b> (0.25)	<b>0.00</b> (2.48)	<b>0.00</b> (12.79)	<b>0.00</b> (56.24)	<b>0.00</b> (0.26)	<b>0.00</b> (2.71)	<b>0.00</b> (13.56)	<b>0.75</b> (0.60)	<b>0.04</b> (0.08)	<b>0.04</b> (0.18)	<b>0.03</b> (1.54)	<b>0.27</b> (0.77)	<b>0.15</b> (0.05)	<b>0.18</b> (0.04)	<b>0.08</b> (0.35)	
Greece	5	<b>0.00</b> (89.42)	<b>0.00</b> (0.26)	<b>0.00</b> (3.24)	<b>0.00</b> (17.13)	<b>0.00</b> (101.10)	<b>0.00</b> (0.26)	<b>0.00</b> (3.46)	<b>0.00</b> (17.51)	<b>0.04</b> $(7.57)$	<b>0.10</b> (0.08)	<b>0.05</b> (0.23)	<b>0.04</b> (1.44)	<b>0.07</b> (2.90)	<b>0.05</b> (0.06)	0.15 (0.05)	0.11 $(0.35)$	
	10	<b>0.00</b> (64.74)	<b>0.00</b> (0.19)	<b>0.00</b> (1.95)	<b>0.00</b> (11.26)	<b>0.00</b> (77.87)	<b>0.00</b> (0.19)	<b>0.00</b> (2.18)	<b>0.00</b> (11.79)	<b>0.83</b> (0.54)	<b>0.52</b> (0.05)	<b>0.26</b> (0.07)	<b>0.14</b> (0.57)	<b>0.56</b> (0.39)	<b>0.66</b> (0.04)	<b>0.59</b> (0.03)	<b>0.45</b> (0.22)	
	3	<b>0.39</b> (5.35)	<b>0.06</b> (0.07)	<b>0.02</b> (0.16)	<b>0.00</b> (1.46)	0.13 (9.58)	<b>0.06</b> (0.08)	<b>0.04</b> (0.24)	<b>0.01</b> (1.96)	<b>0.35</b> (3.05)	<b>0.48</b> (0.04)	<b>0.10</b> (0.06)	<b>0.07</b> (0.44)	0.45 $(1.48)$	<b>0.67</b> (0.04)	<b>0.29</b> (0.05)	<b>0.29</b> (0.33)	
Italy	5	0.18 $(7.75)$	<b>0.01</b> (0.09)	<b>0.00</b> (0.30)	<b>0.00</b> (2.15)	<b>0.03</b> (14.00)	<b>0.02</b> (0.09)	<b>0.01</b> (0.39)	<b>0.00</b> (2.62)	0.13 (4.85)	<b>0.05</b> (0.06)	<b>0.18</b> (0.05)	<b>0.27</b> (0.31)	0.15 (3.81)	<b>0.11</b> (0.06)	<b>0.21</b> (0.06)	<b>0.35</b> (0.30)	
	10	<b>0.03</b> (12.89)	<b>0.00</b> (0.09)	<b>0.00</b> (0.35)	<b>0.00</b> (2.32)	<b>0.01</b> (18.03)	<b>0.02</b> (0.10)	<b>0.00</b> (0.44)	<b>0.00</b> (2.86)	<b>0.53</b> (1.79)	<b>0.46</b> (0.04)	<b>0.47</b> (0.03)	<b>0.46</b> (0.25)	<b>0.24</b> (1.95)	<b>0.28</b> (0.05)	<b>0.36</b> (0.04)	<b>0.33</b> (0.28)	
	3	<b>0.00</b> (19.33)	<b>0.00</b> (0.23)	<b>0.00</b> (3.36)	<b>0.00</b> (17.88)	<b>0.01</b> (30.84)	<b>0.00</b> (0.24)	<b>0.00</b> (3.63)	<b>0.00</b> (18.80)	<b>0.60</b> (1.28)	<b>0.62</b> (0.05)	<b>0.46</b> (0.06)	<b>0.35</b> (0.52)	<b>0.24</b> (1.36)	<b>0.70</b> (0.04)	<b>0.39</b> (0.03)	<b>0.29</b> (0.28)	
Portugal	5	<b>0.00</b> (30.32)	<b>0.00</b> (0.18)	<b>0.00</b> (2.03)	<b>0.00</b> (11.88)	<b>0.00</b> (42.49)	<b>0.00</b> (0.19)	<b>0.00</b> (2.29)	<b>0.00</b> (12.89)	<b>0.63</b> (1.13)	<b>0.72</b> (0.04)	<b>0.59</b> (0.04)	<b>0.49</b> (0.34)	<b>0.55</b> $(0.63)$	<b>0.71</b> (0.04)	<b>0.71</b> (0.02)	<b>0.65</b> (0.20)	
	10	<b>0.00</b> (51.40)	<b>0.00</b> (0.18)	<b>0.00</b> (1.52)	<b>0.00</b> (9.15)	<b>0.00</b> (62.68)	<b>0.00</b> (0.18)	<b>0.00</b> (1.72)	<b>0.00</b> (9.90)	<b>0.67</b> (1.22)	<b>0.66</b> (0.04)	<b>0.56</b> (0.04)	<b>0.46</b> (0.34)	<b>0.52</b> (0.70)	<b>0.33</b> (0.05)	<b>0.39</b> (0.04)	<b>0.41</b> (0.26)	

Table 11: Goodness-of-Fit Tests for Greek, Italian and Portuguese Bond Data

 $\chi^2$ , Kolmogorov-Smirnov (KS), Cramer-van-Mieses (CM) and Anderson-Darling (AD) tests on normal, Student's t, skewed Student's t and stable distribution for Greek, Italian and Portuguese bond data for different maturities of 3, 5 and 10 years. The fat values denote p-values with null hypothesis that the empirical distribution equals the distributional assumption. Numbers in brackets represent the test statistics.

			norr	nal			t				skev	ved $t$			sta	ble	
	m	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD
	1	<b>0.00</b> (21.82)	<b>0.00</b> (0.12)	<b>0.00</b> (0.55)	<b>0.00</b> (3.05)	<b>0.00</b> (24.36)	<b>0.00</b> (0.12)	<b>0.00</b> (0.65)	<b>0.00</b> (3.49)	<b>0.62</b> (1.17)	<b>0.28</b> (0.05)	<b>0.20</b> (0.05)	<b>0.15</b> (0.41)	<b>0.14</b> (1.60)	<b>0.23</b> (0.05)	<b>0.20</b> (0.04)	<b>0.20</b> (0.29)
	3	0.14 (8.66)	<b>0.02</b> (0.08)	<b>0.00</b> (0.21)	<b>0.00</b> (1.38)	<b>0.05</b> (12.29)	<b>0.05</b> (0.08)	<b>0.01</b> (0.28)	<b>0.01</b> (1.84)	<b>0.93</b> (0.39)	<b>0.72</b> (0.04)	<b>0.74</b> (0.02)	<b>0.78</b> (0.18)	<b>0.77</b> (0.35)	<b>0.86</b> (0.03)	<b>0.71</b> (0.03)	<b>0.73</b> (0.19)
Sweden	5	<b>0.36</b> (5.62)	<b>0.26</b> (0.06)	<b>0.06</b> (0.12)	<b>0.03</b> (0.86)	<b>0.21</b> (7.96)	<b>0.29</b> (0.06)	<b>0.08</b> (0.18)	<b>0.05</b> (1.20)	<b>0.97</b> (0.26)	<b>1.00</b> (0.02)	<b>0.99</b> (0.01)	<b>1.00</b> (0.09)	<b>0.82</b> (0.28)	<b>0.94</b> (0.03)	<b>0.97</b> (0.01)	<b>0.99</b> (0.09)
	10	<b>0.05</b> (11.22)	<b>0.05</b> (0.07)	<b>0.02</b> (0.16)	<b>0.00</b> (1.25)	<b>0.01</b> (15.46)	<b>0.05</b> (0.08)	<b>0.05</b> (0.23)	<b>0.02</b> (1.71)	<b>0.02</b> (9.17)	<b>0.05</b> (0.06)	<b>0.10</b> (0.06)	<b>0.09</b> (0.40)	<b>0.02</b> (8.39)	<b>0.28</b> (0.05)	<b>0.17</b> (0.06)	<b>0.19</b> (0.37)
	3	<b>0.99</b> (0.59)	<b>0.36</b> (0.05)	<b>0.18</b> (0.08)	<b>0.17</b> (0.53)	<b>0.92</b> (1.71)	<b>0.45</b> (0.05)	<b>0.19</b> (0.10)	0.15 (0.70)	0.75 $(1.52)$	<b>0.07</b> (0.06)	<b>0.13</b> (0.06)	<b>0.19</b> (0.33)	<b>0.89</b> (0.36)	<b>0.37</b> (0.05)	0.15 (0.07)	<b>0.25</b> (0.37)
Switzerland	5	<b>0.59</b> (3.91)	<b>0.94</b> (0.03)	<b>0.78</b> (0.03)	<b>0.84</b> (0.22)	<b>0.59</b> (3.91)	<b>0.95</b> (0.03)	<b>0.80</b> (0.03)	<b>0.84</b> (0.22)	<b>0.52</b> (3.74)	<b>0.80</b> (0.04)	<b>0.61</b> (0.03)	<b>0.63</b> (0.21)	<b>0.58</b> (3.92)	<b>0.93</b> (0.03)	<b>0.78</b> (0.03)	<b>0.82</b> (0.22)
	10	<b>0.79</b> (2.61)	<b>0.97</b> (0.03)	<b>0.97</b> (0.02)	<b>0.99</b> (0.12)	<b>0.81</b> (2.61)	<b>0.97</b> (0.03)	<b>0.97</b> (0.02)	<b>0.99</b> (0.12)	<b>0.67</b> (2.70)	<b>0.98</b> (0.03)	<b>0.96</b> (0.02)	<b>0.99</b> (0.11)	<b>0.76</b> (2.64)	<b>0.97</b> (0.03)	<b>0.97</b> (0.02)	<b>0.99</b> (0.12)
	1	<b>0.00</b> (21.46)	<b>0.00</b> (0.10)	<b>0.00</b> (0.48)	<b>0.00</b> (2.90)	<b>0.00</b> (22.38)	<b>0.00</b> (0.10)	<b>0.00</b> (0.52)	<b>0.00</b> (3.07)	<b>0.76</b> (1.66)	<b>0.63</b> (0.04)	<b>0.55</b> $(0.03)$	<b>0.43</b> (0.27)	<b>0.29</b> (2.20)	$\begin{array}{c} {\bf 0.43} \\ (0.05) \end{array}$	<b>0.30</b> (0.05)	<b>0.27</b> (0.32)
	3	<b>0.13</b> (8.96)	<b>0.00</b> (0.09)	<b>0.03</b> (0.14)	<b>0.03</b> (0.82)	0.11 $(9.54)$	<b>0.01</b> (0.09)	<b>0.06</b> (0.16)	<b>0.06</b> (0.96)	<b>0.49</b> (3.01)	<b>0.09</b> (0.06)	<b>0.13</b> (0.06)	<b>0.22</b> (0.32)	<b>0.29</b> (4.45)	0.02 (0.07)	<b>0.12</b> (0.08)	<b>0.21</b> (0.40)
Spain	5	<b>0.62</b> (3.64)	<b>0.77</b> (0.04)	<b>0.80</b> (0.03)	<b>0.74</b> (0.25)	<b>0.63</b> (3.76)	<b>0.80</b> (0.04)	<b>0.82</b> (0.03)	<b>0.75</b> (0.26)	0.55 (3.47)	0.16 (0.05)	<b>0.30</b> (0.04)	<b>0.30</b> (0.31)	<b>0.59</b> (2.84)	<b>0.34</b> (0.05)	<b>0.47</b> (0.05)	<b>0.52</b> (0.29)
	10	<b>0.07</b> (10.50)	<b>0.81</b> (0.04)	<b>0.56</b> (0.04)	<b>0.47</b> (0.36)	<b>0.08</b> (10.47)	<b>0.82</b> (0.04)	<b>0.60</b> (0.04)	<b>0.49</b> (0.36)	0.32 (5.27)	<b>0.12</b> (0.05)	<b>0.13</b> (0.06)	<b>0.17</b> (0.37)	<b>0.33</b> (4.36)	0.51 (0.05)	<b>0.42</b> (0.05)	<b>0.42</b> (0.32)

			norr	nal			t				skev	ved $t$			$\operatorname{stal}$	ble	
	m	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD
UK 1	1	<b>0.60</b> (3.85)	<b>0.37</b> (0.05)	<b>0.05</b> (0.12)	<b>0.01</b> (1.10)	<b>0.40</b> (5.62)	<b>0.24</b> (0.06)	<b>0.10</b> (0.14)	<b>0.02</b> (1.23)	<b>0.61</b> (3.53)	<b>0.02</b> (0.07)	<b>0.01</b> (0.14)	<b>0.01</b> (0.82)	<b>0.50</b> (1.84)	<b>0.02</b> (0.07)	<b>0.05</b> (0.09)	<b>0.02</b> (0.68)
	3	<b>0.63</b> (3.78)	<b>0.45</b> (0.05)	<b>0.50</b> (0.05)	<b>0.57</b> (0.31)	0.52 (4.41)	<b>0.49</b> (0.05)	<b>0.47</b> (0.05)	$\begin{array}{c} {\bf 0.51} \\ (0.35) \end{array}$	<b>0.36</b> (4.22)	<b>0.19</b> (0.05)	<b>0.20</b> (0.05)	<b>0.37</b> (0.27)	<b>0.45</b> (3.82)	<b>0.56</b> (0.05)	<b>0.47</b> (0.05)	<b>0.62</b> (0.26)
	5	<b>0.89</b> (1.80)	<b>0.66</b> (0.04)	<b>0.70</b> (0.04)	<b>0.77</b> (0.24)	<b>0.90</b> (1.81)	<b>0.66</b> (0.04)	<b>0.69</b> (0.04)	<b>0.78</b> (0.24)	<b>0.88</b> (1.43)	<b>0.60</b> (0.04)	<b>0.77</b> (0.02)	<b>0.83</b> (0.17)	<b>0.87</b> (1.78)	<b>0.60</b> (0.04)	<b>0.64</b> (0.04)	<b>0.74</b> (0.24)
	10	<b>0.72</b> (3.07)	<b>0.86</b> (0.04)	<b>0.73</b> (0.03)	<b>0.72</b> (0.26)	<b>0.77</b> (2.83)	<b>0.86</b> (0.04)	<b>0.67</b> (0.04)	<b>0.68</b> (0.30)	<b>0.87</b> (1.12)	<b>0.99</b> (0.03)	<b>0.99</b> (0.01)	<b>1.00</b> (0.10)	<b>0.71</b> (2.18)	<b>0.94</b> (0.03)	<b>0.93</b> (0.02)	<b>0.93</b> (0.16)
USA 1	1	<b>0.00</b> (24.91)	<b>0.00</b> (0.09)	<b>0.00</b> (0.27)	<b>0.00</b> (1.77)	<b>0.00</b> (25.43)	<b>0.01</b> (0.09)	<b>0.00</b> (0.27)	<b>0.00</b> (1.75)	<b>0.56</b> (3.66)	<b>0.81</b> (0.04)	<b>0.84</b> (0.02)	<b>0.88</b> (0.17)	0.00 (14.65)	<b>0.01</b> (0.07)	<b>0.01</b> (0.11)	<b>0.01</b> (0.72)
	3	<b>0.47</b> (4.77)	<b>0.76</b> (0.04)	<b>0.74</b> (0.03)	<b>0.65</b> (0.29)	<b>0.49</b> (4.77)	<b>0.76</b> (0.04)	<b>0.75</b> (0.03)	<b>0.64</b> (0.29)	<b>0.38</b> (4.64)	<b>0.59</b> (0.04)	<b>0.54</b> (0.03)	<b>0.38</b> (0.28)	<b>0.45</b> (4.79)	<b>0.71</b> (0.04)	<b>0.71</b> (0.03)	<b>0.57</b> (0.29)
	5	<b>0.48</b> (4.74)	<b>0.40</b> (0.05)	<b>0.20</b> (0.08)	<b>0.22</b> (0.49)	<b>0.49</b> (4.74)	$\begin{array}{c} {\bf 0.42} \\ (0.05) \end{array}$	<b>0.20</b> (0.08)	<b>0.22</b> (0.49)	0.45 (4.17)	<b>0.34</b> (0.05)	<b>0.34</b> (0.04)	<b>0.31</b> (0.30)	0.45 (4.77)	<b>0.37</b> (0.05)	<b>0.16</b> (0.08)	<b>0.18</b> (0.49)
	10	<b>0.67</b> (3.38)	<b>0.08</b> (0.07)	<b>0.05</b> (0.12)	<b>0.03</b> (0.81)	<b>0.57</b> (4.30)	<b>0.13</b> (0.07)	<b>0.07</b> (0.15)	<b>0.06</b> (1.00)	<b>0.84</b> (0.92)	<b>0.80</b> (0.03)	<b>0.66</b> (0.02)	<b>0.58</b> (0.21)	0.52 (1.37)	<b>0.19</b> (0.06)	<b>0.37</b> (0.05)	<b>0.40</b> (0.29)

Table 12: Goodness-of-Fit Tests for Swedish, Swiss, Spanish, UK and US Bond Data

 $\chi^2$ , Kolmogorov-Smirnov (KS), Cramer-van-Mieses (CM) and Anderson-Darling (AD) tests on normal, Student's t, skewed Student's t and stable distribution for Swedish, Swiss, Spanish, UK and US bond data for maturities of 1, 3, 5 and 10 years. The fat values denote p-values with null hypothesis that the empirical distribution equals the distributional assumption. Numbers in brackets represent the test statistics.

			nor	$\operatorname{mal}$			t				skev	d t		stable				
	m	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	
Germany	1	<b>0.14</b> (8.87)	<b>0.00</b> (0.09)	<b>0.00</b> (0.31)	<b>0.00</b> (1.81)	<b>0.06</b> (11.47)	<b>0.01</b> (0.10)	<b>0.00</b> (0.37)	<b>0.00</b> (2.17)	<b>0.78</b> (1.05)	<b>0.66</b> (0.04)	<b>0.15</b> (0.05)	<b>0.13</b> (0.37)	<b>0.39</b> (1.91)	<b>0.37</b> (0.05)	<b>0.09</b> (0.08)	<b>0.09</b> (0.47)	
France	1	<b>0.09</b> (9.87)	<b>0.00</b> (0.13)	<b>0.00</b> (0.39)	<b>0.00</b> (2.08)	<b>0.04</b> (11.38)	<b>0.00</b> (0.13)	<b>0.00</b> (0.41)	<b>0.00</b> (2.18)	<b>0.37</b> (3.66)	<b>0.00</b> (0.08)	<b>0.01</b> (0.11)	<b>0.02</b> (0.57)	<b>0.26</b> (3.01)	<b>0.01</b> (0.08)	<b>0.04</b> (0.09)	<b>0.07</b> (0.48)	
	3	0.42 (5.07)	<b>0.37</b> (0.05)	<b>0.74</b> (0.03)	<b>0.77</b> (0.24)	0.45 (5.08)	<b>0.39</b> (0.05)	<b>0.77</b> (0.03)	<b>0.78</b> (0.24)	<b>0.36</b> $(5.18)$	<b>0.32</b> (0.05)	<b>0.45</b> (0.04)	0.51 (0.25)	0.31 (4.99)	<b>0.23</b> (0.06)	<b>0.50</b> (0.04)	<b>0.60</b> (0.26)	

Table 13: Goodness-of-Fit Tests for German and French Bond Data after structural Break  $\chi^2$ , Kolmogorov-Smirnov (KS), Cramer-van-Mieses (CM) and Anderson-Darling (AD) tests on normal, Student's t, skewed Student's t and stable distribution for German (1 year maturity) and French (1 and 3 year maturity) bond data for the period after the structural break. The fat values denote p-values with null hypothesis that the empirical distribution equals the distributional assumption. Numbers in brackets represent the test statistics.

			nor	mal			t				$_{\rm skev}$	ved $t$		stable				
	m	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	$\chi^2$	KS	CM	AD	
Greece	3	<b>0.00</b> (48.72)	<b>0.00</b> (0.25)	<b>0.00</b> (2.48)	<b>0.00</b> (12.79)	<b>0.00</b> (56.24)	<b>0.00</b> (0.26)	<b>0.00</b> (2.71)	<b>0.00</b> (13.56)	<b>0.75</b> (0.60)	<b>0.04</b> (0.08)	<b>0.04</b> (0.18)	<b>0.03</b> (1.54)	<b>0.27</b> (0.77)	<b>0.15</b> (0.05)	<b>0.18</b> (0.04)	<b>0.08</b> (0.35)	
	5	<b>0.00</b> (89.42)	<b>0.00</b> (0.26)	<b>0.00</b> (3.24)	<b>0.00</b> (17.13)	<b>0.00</b> (101.10)	<b>0.00</b> (0.26)	<b>0.00</b> (3.46)	<b>0.00</b> (17.51)	0.04 (7.57)	<b>0.10</b> (0.08)	<b>0.05</b> (0.23)	<b>0.04</b> (1.44)	<b>0.07</b> (2.90)	<b>0.05</b> (0.06)	0.15 (0.05)	<b>0.11</b> $(0.35)$	
	10	<b>0.00</b> (64.74)	<b>0.00</b> (0.19)	<b>0.00</b> (1.95)	<b>0.00</b> (11.26)	<b>0.00</b> (77.87)	<b>0.00</b> (0.19)	<b>0.00</b> (2.18)	<b>0.00</b> (11.79)	<b>0.83</b> (0.54)	0.52 (0.05)	<b>0.26</b> (0.07)	<b>0.14</b> (0.57)	<b>0.56</b> (0.39)	<b>0.66</b> (0.04)	<b>0.59</b> (0.03)	<b>0.45</b> (0.22)	
Italy	3	<b>0.39</b> (5.35)	<b>0.06</b> (0.07)	<b>0.02</b> (0.16)	<b>0.00</b> (1.46)	0.13 (9.58)	<b>0.06</b> (0.08)	<b>0.04</b> (0.24)	<b>0.01</b> (1.96)	<b>0.35</b> (3.05)	<b>0.48</b> (0.04)	<b>0.10</b> (0.06)	<b>0.07</b> (0.44)	0.45 $(1.48)$	<b>0.67</b> (0.04)	<b>0.29</b> (0.05)	<b>0.29</b> (0.33)	
	5	0.18 (7.75)	<b>0.01</b> (0.09)	<b>0.00</b> (0.30)	<b>0.00</b> (2.15)	<b>0.03</b> (14.00)	<b>0.02</b> (0.09)	0.01 (0.39)	<b>0.00</b> (2.62)	0.13 (4.85)	0.05 (0.06)	0.18 (0.05)	<b>0.27</b> (0.31)	0.15 (3.81)	<b>0.11</b> (0.06)	<b>0.21</b> (0.06)	<b>0.35</b> (0.30)	
	10	<b>0.03</b> (12.89)	<b>0.00</b> (0.09)	<b>0.00</b> (0.35)	<b>0.00</b> (2.32)	<b>0.01</b> (18.03)	<b>0.02</b> (0.10)	<b>0.00</b> (0.44)	<b>0.00</b> (2.86)	<b>0.53</b> (1.79)	<b>0.46</b> (0.04)	<b>0.47</b> (0.03)	<b>0.46</b> (0.25)	<b>0.24</b> (1.95)	<b>0.28</b> (0.05)	<b>0.36</b> (0.04)	<b>0.33</b> (0.28)	
	3	<b>0.00</b> (19.33)	<b>0.00</b> (0.23)	<b>0.00</b> (3.36)	<b>0.00</b> (17.88)	<b>0.01</b> (30.84)	<b>0.00</b> (0.24)	<b>0.00</b> (3.63)	<b>0.00</b> (18.80)	<b>0.60</b> (1.28)	<b>0.62</b> (0.05)	<b>0.46</b> (0.06)	<b>0.35</b> (0.52)	0.24 (1.36)	<b>0.70</b> (0.04)	<b>0.39</b> (0.03)	<b>0.29</b> (0.28)	
Portugal	5	<b>0.00</b> (30.32)	<b>0.00</b> (0.18)	<b>0.00</b> (2.03)	<b>0.00</b> (11.88)	<b>0.00</b> (42.49)	<b>0.00</b> (0.19)	<b>0.00</b> (2.29)	<b>0.00</b> (12.89)	<b>0.63</b> (1.13)	<b>0.72</b> (0.04)	<b>0.59</b> (0.04)	<b>0.49</b> (0.34)	<b>0.55</b> $(0.63)$	<b>0.71</b> (0.04)	<b>0.71</b> (0.02)	<b>0.65</b> (0.20)	
	10	<b>0.00</b> (51.40)	<b>0.00</b> (0.18)	<b>0.00</b> (1.52)	<b>0.00</b> (9.15)	<b>0.00</b> (62.68)	<b>0.00</b> (0.18)	<b>0.00</b> (1.72)	<b>0.00</b> (9.90)	<b>0.67</b> (1.22)	<b>0.66</b> (0.04)	<b>0.56</b> (0.04)	<b>0.46</b> (0.34)	<b>0.52</b> (0.70)	<b>0.33</b> (0.05)	<b>0.39</b> (0.04)	<b>0.41</b> (0.26)	
	1	<b>0.00</b> (21.46)	<b>0.00</b> (0.10)	<b>0.00</b> (0.48)	<b>0.00</b> (2.90)	<b>0.00</b> (22.38)	<b>0.00</b> (0.10)	<b>0.00</b> (0.52)	<b>0.00</b> (3.07)	<b>0.76</b> (1.66)	<b>0.63</b> (0.04)	<b>0.55</b> (0.03)	<b>0.43</b> (0.27)	<b>0.29</b> (2.20)	<b>0.43</b> (0.05)	0.30 (0.05)	<b>0.27</b> (0.32)	
Spain	3	<b>0.13</b> (8.96)	<b>0.00</b> (0.09)	<b>0.03</b> (0.14)	<b>0.03</b> (0.82)	0.11 (9.54)	<b>0.01</b> (0.09)	<b>0.06</b> (0.16)	<b>0.06</b> (0.96)	<b>0.49</b> (3.01)	<b>0.09</b> (0.06)	<b>0.13</b> (0.06)	<b>0.22</b> (0.32)	<b>0.29</b> (4.45)	<b>0.02</b> (0.07)	<b>0.12</b> (0.08)	<b>0.21</b> (0.40)	
	5	<b>0.62</b> (3.64)	<b>0.77</b> (0.04)	<b>0.80</b> (0.03)	<b>0.74</b> (0.25)	<b>0.63</b> (3.76)	<b>0.80</b> (0.04)	<b>0.82</b> (0.03)	<b>0.75</b> (0.26)	0.55 $(3.47)$	<b>0.16</b> (0.05)	<b>0.30</b> (0.04)	<b>0.30</b> (0.31)	<b>0.59</b> (2.84)	<b>0.34</b> (0.05)	<b>0.47</b> (0.05)	<b>0.52</b> (0.29)	

Table 14: Goodness-of-Fit Tests for Greek, Italian, Portuguese and Spanish Bond Data after structural Break

 $\chi^2$ , Kolmogorov-Smirnov (KS), Cramer-van-Mieses (CM) and Anderson-Darling (AD) tests on normal, Student's t, skewed Student's t and stable distribution for Greek, Italian, Portuguese and Spanish bond data for different maturities of 1, 3, 5 or 10 years for the period after the structural break. The fat values denote p-values with null hypothesis that the empirical distribution equals the distributional assumption. Numbers in brackets represent the test statistics.