Term Structure Models with Differences in Beliefs

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ABSTRACT

This paper studies both theoretically and empirically the link between macroeconomic disagreement and bond markets. Using survey data, we construct proxies of macroeconomic disagreement and find a number of novel results. First, heterogeneity in beliefs affect the price of risk so that belief dispersion regarding the real economy, inflation, and signals predict excess bond returns with $\overline{R}^2$'s between 21%- 43%. Second, disagreement explains bond volatilities with high statistical significance and $\overline{R}^2$'s $\sim$ 26% in monthly projections. Third, while around half the information contained in the cross-section of expectations is spanned by the yield curve, there remains large unspanned components important for bond pricing. Fourth, disagreement also contains significant information on trading activity: belief dispersion at 1-year horizon drives up trade at the short end of the yield curve relative to trade at the long end.

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THIS PAPER INVESTIGATES THE EMPIRICAL IMPLICATIONS OF MACROECONOMIC DISAGREEMENT for bond market dynamics. When moving from single agent to heterogeneous agent models several important properties of asset prices change. Differences in beliefs can affect the stochastic discount factor, thus equilibrium asset prices. This is important since the dynamics of macroeconomic disagreement may become a source of predictable variation in bond excess returns, drive second moments, and generate trade. A growing body of evidence indicates that heterogeneity plays an important role in a variety of settings, including equity, foreign exchange, and derivative markets. However, little is known about its affect on bond markets.

We begin with a review of the theoretical literature on economies with heterogeneous beliefs and learning, and discuss the testable implications relevant for the dynamics of bonds. In order to study these implications we build a data set that merges the historical paper archives of BlueChip surveys that contains information on the distribution of expectations of professional forecasters for a broad set of macroeconomic variables including inflation, real growth, and interest rates. This dataset is unique in that it is available at a monthly frequency, covers a long history, and it is based on a large and stable cross-section of forecasters.[1]

We address four empirical regularities that the term structure literature find difficult to reconcile with traditional homogeneous economies with no frictions. Firstly, an extensive return predictability literature has evolved from the univariate regressions of Fama and Bliss (1987) to multivariate approaches of Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009). The results suggest a substantial in-sample variability of conditional expected excess returns. The dynamics of risk compensation, however, are complex and demand a rich specification for the price of risk (see Dai and Singleton (2002)). Addressing the dynamics of risk compensation, Duffee (2002) proposes the 'essentially affine' class which allows for a flexible price of risk. Unfortunately, while this class can better match some salient features of the data they are unable to match at the same time first and second moments of yields.[2] Second, long term bond yields appear too volatile to accord with standard representative

[1] The Survey of Professional Forecasters is available only at quarterly frequency and, especially in some periods, it has a more restricted cross-section of forecasters. Previously, the commercially available BlueChip economic digital files started only in 2007.

[2] To address this issue structural models have been proposed that are capable of generating countercyclical risk premia, such as ambiguity aversion (Ulrich (2011), Gagliardini, Porchia, and Trojani (2009)), habit models (Wachter (2006) Buraschi and Jiltsov (2007)), or long run risk (Bansal and Yaron (2004)). While ambiguity models do generate rich specifications for the price of risk, the risk factors are inherently unobservable. Habit models, on the other hand, imply a tight link between past consumption and bond expected excess returns that is not fully reflected in the data. Long run risk models generate time-varying risk premia via a stochastic quantity of risk with the market price of risk held constant, a feature which is not supported by reduced form evidence.

agent models (Shiller (1979)). Furthermore, the literature has shown that the interest rate dynamics display unspanned stochastic volatility (Collin-Dufresne, Goldstein, and Jones (2009) , Li and Zhao (2006)): bond portfolios appear unable to hedge interest rate derivatives, thus suggesting a form of market incompleteness. Third, in their canonical form, affine models imply that primitive shocks underlying the economy are perfectly spanned by the yield curve. This implies that macroeconomic aggregates contain no incremental information useful for bond excess returns beyond what already contained in the cross-section of bond prices. However, this result is not supported in the data (Duffee (2011), Joslin, Priebsch, and Singleton (2011)). Finally, little is known about the link between the previous questions and the trading activity of Treasury bonds. While most of the empirical evidence focus on fitting bond yields and returns, one may argue that the dynamics of bond trading volumes are an equally important source of information to help distinguish alternative models.

Starting from a simple benchmark Vasicek general equilibrium economy we introduce multiple agents, dynamic disagreement, and learning. We derive the empirical implications arising in this economy to help understanding the features of such models and provide a comprehensive empirical study in the context of bond markets. Our empirical approach is focused on (*a*) bond risk premia; (*b*) bond volatility; (*c*) spanning properties; and (*d*) trading activity when agents have incomplete information. It is known that when agents have log-utility, bond prices can deviate from those implied by the average consensus beliefs (Xiong and Yan (2010)). The beliefs of the representative agent include an aggregation bias (Jouini, Marin, and Napp (2010)) which could make the representative agent to act as if pessimistic with respect to the consensus belief. When agents are not log-utility investors and differences in beliefs follow a dynamic process, however, trading includes also an additional intertemporal risk sharing term that makes differences in beliefs priced in equilibrium (Buraschi and Jiltsov (2006)). We derive in closed-form the term structure of bond prices for this more general case. The solution is exponential quadratic in differences in beliefs.[3] In this economy bond expected excess returns are predictable even if the benchmark homogeneous Vasicek economy has homoskedastic discount factors. Predictability is generated by time-variation in differences in beliefs. Moreover, in this economy the formation of expectations directly affects bond volatility, even if in the (fictitious) homogeneous economy volatility is constant. Finally, differences in beliefs has been used in the empirical finance literature to proxy for both disagreement and ambiguity. While it is not easy to distinguish these two approaches based on risk premia, an important element of distinction is their implications in terms of trading activity. In absence of frictions, a larger heterogeneity in beliefs induce more

[3]The first to show the importance of this class of solution in general equilibrium with stochastic differences in beliefs was Dumas, Kurshev, and Uppal (2009)

trading through risk sharing. The larger the disagreement, the greater the trading activity. Models with Knightian uncertainty and ambiguity, however, have the opposite implications: greater ambiguity induce portfolio inertia as discussed in Illeditsch (2011), de Castro and Chateauneuf (2011), and Chen, Ju, and Miao (2011).

We construct proxies of macroeconomic disagreement on both (exogeneous) economic variables (such as future real economic activity and inflation) and (endogeneous) financial variables (such as future bond prices). Indeed, in the context of a model with learning and depending on the learning mechanism, the stochastic discount factor can be an explicit function of both disagreement about fundamentals and signals. Since expected future bond yields are a function of observed signals, we use disagreement on future bond yields to reveal disagreement on signals, which are otherwise unobservable to the econometrician. Using these proxies, we obtain a number of empirical results.

First, we revisit the literature on bond return predicability and show that the cross-section of agents expectations contains economically important and statistically significant information on expected excess bond returns at a 1-year horizon. The combination of real, inflation and signal disagreement measures forecast excess bond returns with $\overline{R}^2$ equal to 42% and 21% on 2-year and 10-year bonds, respectively. We find that disagreement about the real economy is statistically significant in a number of specifications and loads positively on expected excess returns, while disagreement about inflation appears less important and is subsumed by disagreement on signals, which is always highly significant. Controlling for consensus views and realisations of fundamentals we test whether the information content in belief dispersion is subsumed by more traditional predictor variables and find the results are robust to the inclusion of a number of alternatives.

Second, we show that dispersion in interest rate forecasts is only weakly spanned by disagreement about fundamentals. This result is important since it implies the formation of *expectations* about (future) bond prices is adapted to a sigma algebra which is larger than the formation of *expectations* on economic growth and inflation themselves. This is consistent with a learning mechanism that extends the state space, generating an additional price of (signal) risk. Our empirical results support the existence of this additional source of predictable variation for expected excess returns.

Third, differences in belief provide important implications for the shape of the term structure of return volatility. Consistent with our theoretical framework, when agents are heterogeneous, a key state-variable driving individual consumption plans is the Radon-Nikodym derivative between agents beliefs. The 'level' of disagreement about future economic prospects drives the diffusion component of the Radon-Nikodym derivative; thus, generating endogeneous stochastic volatility even if fundamentals are homoskedastic. When agents have risk aversion greater than one increases in disagreement are positively correlated with changes in bond volatility. In monthly forecasts, we find that disagreement about inflation loads positively on the level of volatility with t-stats significant at the 5% level and $\overline{R}^2 \sim 29\%$ in most specifications. These results are robust to inclusion of consensus expectations and realised macro aggregates. Moving to the slope of the term structure of volatility we find that, in addition to disagreement about inflation, disagreement about signals is economically and statistically significant at the 1% level. The loading on the signal proxy is both interesting and intuitive: agents disagree on signals correlated with transitory growth rates in the run up to recessions or crises periods, which drives up volatility at the short-end. On the other hand, agents disagree about signals correlated with long run growth components at the beginning of expansions, which drives up volatility at the long-end. Again, this result is robust to inclusion of consensus expectations, macro aggregates, and lagged volatility.

Fourth, we study the spanning properties of macroeconomic disagreement. In traditional general equilibrium models of the term structure, the yield curve can be inverted to reveal the state variables that drive expected returns. Cochrane and Piazzesi (2005) uncover a tent-shape return forecasting factor from forward rates. We find that time variation in the shape of the forward curve in part represents heterogeneity in the belief structure of the economy. These belief-driven components reveal properties of the stochastic discount factor which are significant for the time variation in the price of risk, suggesting a structural interpretation for the forecasting power of the cross-section of forward rates. However, priced disagreement is only partially spanned by the yield curve in the sense that components orthogonal to the first 5 principal components of yields retains economically important information on expected returns. In a return predictability regressions including only unspanned components, disagreement about real growth and signals remain statistically significant at the 5% level while $\overline{R}^2$'s drops to between 27% and 29% on $2-5$ year bonds. Furthermore, unspanned disagreement is only weakly related to the hidden factor from Duffee (2011). Finally, we document an 'above' component constructed using only information orthogonal to both the cross-section and time-series dynamics of yields that retains important forecasting power for expected returns. This is consistent with the linear-quadratic term structure model presented in the theory section, which is non-invertible in practice, especially since the signals agents use can change over time.

Finally, we find that one-year disagreement about inflation and real growth load positively, with large statistical significance, on monthly measures of trading volume in bonds with maturity up to one year, after controlling for trade in long term bonds. Intuitively, disagreement about one-year growth rates drives up relative trade between short term versus long term bonds, consistent with an economy in which larger heterogeneity in beliefs induce agents to trade until their expected one-year marginal utilities equate and markets clear. These results are particularly interesting since they help to distinguish models with differences in beliefs from models with ambiguity, which generate opposite predictions in terms of trading activity, even if they are otherwise similar in other dimensions (expected returns).

I. Economies with Differences in Beliefs

An increasingly important part of the asset pricing literature focus on the role of heterogeneity in beliefs. In two seminal papers, Harrison and Kreps (1978) and Harris and Raviv (1993) develop a model of speculative trading based on difference of opinion in which investors receive common information but differ in the way in which they interpret information.[4] All investors in their economy agree on the nature of the information, be it positive or negative, but disagree on its importance. They show that heterogeneity in beliefs has important implications for asset prices. Similar settings have been studied by Detemple and Murthy (1994) and Zapatero (1998) in the context of a continuous time economy. Buraschi and Jiltsov (2006) allow for Bayesian learning and dynamic disagreement and show that realistic levels of heterogeneous beliefs can generate an option-implied volatility smile and help to explain the dynamics of option prices (see for a survey Basak (2005)).[5] A second stream in this literature builds on the interaction between behavioral biases and trading frictions. Scheinkman and Xiong (2003) study a model with overconfident risk-neutral agents. They show that, in this context, short-selling constraints can support rational asset price bubbles in equilibrium. Dumas, Kurshev, and Uppal (2009) use tranform analysis techniques to study the short and long run effects of sentiment risk. Additional contributions include Diether, Malloy, and Scherbina (2002), who find a negative link between differences in belief and expected stock returns which the authors interpret in favour of Miller's hypothesis. Using a different dataset, Anderson, Ghysels, and Juergens (2005) find evidence contradicting Miller's hypothesis, thus lending support to a neoclassical (i.e., risk-based) interpretation of the impact of differences in beliefs in the equity market.

Surprisingly little is known in the context of bond markets. One exception is Xiong and Yan (2010), who provide a theoretical treatment of bond risk premia in a heterogeneous

[4] Kurz (1994) motivates belief disagreement from the difficulties to distinguish different models using existing data.

[5] Equilibrium treatments of heterogeneity in beliefs include David (2008), who develops a model with counter-cyclical consumption volatility and cross-sectional consumption dispersion where agents assume different models for the underlying data generating process; (Buraschi, Trojani, and Vedolin, 2011, 2010), Bhamra and Uppal (2011); Gallmeyer and Hollifield (2008), Dumas, Kurshev, and Uppal (2009)

agent economy with log-utility investors. The authors develop a model of speculative trading in which two types of investors hold different beliefs regarding the central bank's inflation target. In the model, the inflation target is unobservable so investors form inferences based on a common signal. Although the signal is actually uninformative with respect to the inflation target, heterogeneous prior knowledge causes investors to react differently to the signal flow. Investor trading drives endogenous wealth fluctuations that amplify bond yield volatilities and generates a time varying risk premium. They provide a calibration exercise and show that a simulation of their economy can reproduce the Campbell and Shiller (1991) regression coefficients and the tent shaped linear combination of forward rates from Cochrane and Piazzesi (2005). No empirical study, however, provides empirical evidence on these questions. In what follows, we extend Xiong and Yan (2010) setting to non-myopic agents, derive closed-form solutions for the terms structure of interest rates, and investigate empirically these questions.

A. The Homogeneous Benchmark Economy

Let us consider a simple endowment economy in which agents have constant RRA preferences $u'(c_t) = e^{-\rho t} c_t^{-\gamma}$. The growth rate of endowment is a function of a vector of factors $\mathbf{g}_t$, with

$$dD_t/D_t = \beta' \mathbf{g}_t dt + \sigma_D d\mathbf{W}_t^D \tag{1}$$

$$d\mathbf{g}_t = -\kappa_g (\mathbf{g}_t - \theta) dt + \sigma_g d\mathbf{W}_t^g. \tag{2}$$

When agents have common beliefs about the data generating process, it is well known that bond prices satisfy a simple representation. This solution has been studied extensively and is known as the Vasicek (1977) model of the term structure of interest rates. Given the pricing kernel $\mathcal{M}_t^*$, with $d\mathcal{M}^*/\mathcal{M}^* = -r_t dt - \kappa' dW_t^*$, since $\mathcal{M}_t^* = u'(D_t)$ from Ito's Lemma one finds that r_t must satisfy

$$r_t = \delta + \gamma \beta' \mathbf{g}_t - \frac{1}{2} \gamma (1 + \gamma) \sigma_D^2.$$

If growth rates are constant, i.e. $\beta' \mathbf{g}_t = g_0$, so are interest rates and the term structure is flat. When g_t is stochastic, however, bond prices can be computed from the Euler equation $B_t^{(T-t)} = E_t^* \left[\frac{M_T^*}{M_t^*} \right]$, which gives rise to the simple well known affine representation $B_t^{(T-t)} = \exp \left[A_h(t,T) + G_h(t,T) \mathbf{g}_t \right]$, which implies that bond excess returns are equal to

$$rx_{t,t+dt}^{(T)} = \ -\gamma G(t,T) \sigma_D \sigma_g E \left(dW_t^D dW_t^g \right). \tag{3}$$

The dynamics of bond prices $dB_t^{(T-t)}$ and $d\mathcal{M}^*$ depend, respectively, on dW_t^g and dW_t^D. If $E\left(dW_t^D dW_t^g\right) \neq 0$ long-term bonds command a risk premium[6], which is, however, constant in this economy. A vast empirical literature have shown that the presence of factors with different spectral density can generate realistic cross-sectional shapes of the term structure. An equally vast literature, however, show that its dynamic properties are difficult to be reconciled with the data. Even when dW_t^D and dW_t^g are perfectly correlated, the simple benchmark model restricts expected excess returns to be proportional to the volatility of macroeconomic fundamentals. This tight connection makes the model able to reproduce only a small fraction of the predictable variations in expected excess returns found in the data (Duffee (2002)) as the dynamic properties of conditional volatilities depart quite substantially from those of conditional first moments. To break this link, the affine literature has investigated flexible specifications of the price of risk (as in Cheridito, Filipovic, and Kimmel (2007)). We explore a different channel of predictability which is generated by the aggregation properties of the belief dynamics of agents with different priors.

B. *Disagreement and the Term Structure*

Suppose that growth rates are unobservable and that agents agree on σ_g and θ_g but disagree on the persistence κ_g of the growth rate process. A first set of agents think that the economy is dominated by long-run risk components (very persistent shocks with a positive but very small κ_{g^1}); a second set of agents think instead that the economy is mainly exposed to temporary business-cycle shocks (real and/or monetary policy shocks with speed of mean reversion $\kappa_{g^2} > 0$). Since g_t is not observable, it may be difficult for agents to agree on the true value of κ_g and it is easy to imagine disagreement on their relative importance (see Hansen, Heaton, and Li (2008) and Pastor and Stambaugh (2000) for a discussion).[7] Let the subjective process be

$$dg_t = -\kappa_g^i (g_t - \theta_g)dt + \sigma_g d\hat{W}_t^{g,i} \qquad i = a, b. \tag{4}$$

The two subjective probability measures associated to the two posteriors are denoted as $d\mathcal{P}_t^a$ and $d\mathcal{P}_t^b$. In this context the two probability measures are absolutely continuous; the difference in beliefs between the two agents can be conveniently summarized by the Radon-

[6] A common assumption is to restrict D_t to be an affine transformation of g_t, i.e. $D_t = \exp(\beta' \mathbf{g}_t)$.

[7] Hansen, Heaton, and Li (2008) argue about the existence of significant measurement challenges in quantifying the long-run risk-return trade-off and that 'the same statistical challenges that plague econometricians presumably also plague market participants'. Pastor and Stambaugh (2000) discuss the statistical properties of predictive systems when the predictors are autocorrelated but κ is not known.

Nikodym derivative $\eta_t = \frac{d\mathcal{P}^b_t}{d\mathcal{P}^b}$, so that for any random variable X_t that is $\Im_t$-measurable,

$$E^b(X_T|\Im_t) = E^a\left(\frac{\eta_T}{\eta_t}X_T|\Im_t\right). \tag{5}$$

All agents observe the dividend process, $D(t)$, so that $\Im_t$ is common knowledge and there is no private information: agents simply agree to disagree.[8] Notice that if $\kappa^a_g > \kappa^b_g$ then agent a is optimist in states when $g_t < \theta_g$ and pessimist when $g_t > \theta_g$. Since the distribution of g_t is normal, then each agent spend the same amount of time as an optimist or pessimist.

Xiong and Yan (2010) notice that disagreement can have non trivial effects on bond prices induced by the fact that agents have ex-ante incentives to trade with each others. In turn, agents relative wealth will ex-post be affected by their trading ex-ante. The first order effect of disagreement can be immediately appreciated by noticing that, with no further assumptions:

Proposition 1 (Xiong and Yan (2010))**.** *If agents have logarithmic preferences, $u(c^i) = e^{-\rho(T-t)}\ln c^i_t$, they will trade until their wealth ratio is equal to η_T, i.e. $\eta_T = W^b_T/W^a_T$. Furthermore, in equilibrium the price of a zero-coupon bond $B^{(T-t)}_t$ with time to maturity $T-t$ is equal to the η_t-weighted average of the zero-coupon bonds prices prevailing in the (fictitious) homogeneous economies populated only by each of the two agents, $B^{(T-t),a}_t$ and $B^{(T-t),b}_t$, with*

$$B^{(T-t)}_t = \frac{1}{1+\eta_t}B^{(T-t),a}_t + \frac{\eta_t}{1+\eta_t}B^{(T-t),b}_t. \tag{6}$$

(Proof in Appendix)

One may notice that even if η_t were constant, bond prices in the heterogeneous economy would not be affine. The affine class is not robust to aggregation when agents have different probability measures. Moreover, if η_t were to be stochastic, equilibrium bond prices may differ from those prevailing in a (fictitious) economy populated by only one agent. Xiong and Yan (2010) calibrate a model with log-utility investors using an affine specification for individual agents pricing function, B^n_t, and show that a realistic parametrization can generate a rich set of cross-sectional shapes of the term structure.

While adequate for their purposes, log-preferences make agents myopic and the absence of intertemporal hedging demands can restrict the link between risk premia and the dynamics

[8] A large literature study economies where agents agree to disagree, among which Detemple and Murthy (1994), Scheinkman and Xiong (2003), Hong and Stein (2003), Anderson, Ghysels, and Juergens (2005), Buraschi and Jiltsov (2006), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), Jouini, Marin, and Napp (2010), and Banerjee and Kremer (2010).

of differences in beliefs. For instance, David (2008) show that in a heterogeneous agent economy a necessary condition for differences in belief to affect the equity risk premium is that relative risk aversion be different than 1. Given our focus, we summarize the main empirical implications when one relaxes this assumption and allows for general power utility preferences and dynamic rational learning.[9] Two main results will emerge. First, the presence of signals can increase the state-space and disagreement on these signals becomes an additional priced risk-factor above and beyond the volatility of fundamentals. This is potentially important since it can directly affect the dynamics of risk premia. Second, the solution for the term structure of bond prices can be derived in closed-form. Expected excess returns are time-varying and driven by the dynamics of the difference in beliefs η_t. We will then use these properties to guide and interpret our empirical study.

C. A General Model

The literature that accounts for incomplete information and learning, considers two types of signals. Detemple (1986), Veronesi (2000) and Buraschi and Jiltsov (2006) consider economise in which agents can improve their forecasts on $\hat{g}_t^i$ by using signals S_t^i, whose drifts are correlated with *conditional first moments* of the dividend process $E_t(dD_t)$; we will refer to these as ‘ first-order signals’. Scheinkman and Xiong (2003), Dumas, Kurshev, and Uppal (2009), and Xiong and Yan (2010) investigate, on the other hand, signals correlated with *unexpected* innovations dD_t via correlated Brownian motions; we will refer to these signals as ‘ second-order signals’ .[10] As we shall see, these two approaches give rise to solutions with similar functional forms, however, in the first economy the dynamics of signals are priced and thus expected bond returns. In the second economy, only disagreement on fundamentals is priced. To help organize the empirical analysis we follow the setup in Buraschi and Jiltsov (2006) and assume

$$dS_t^i = \left(\phi^i g_t^i + (1-\phi^i)\varepsilon_t\right) dt + \sigma_s^i dW_t^{S^i}, \tag{7}$$

$$d\varepsilon_t = dW_t^{\varepsilon}. \tag{8}$$

[9] Scheinkman and Xiong (2003), Buraschi and Jiltsov (2006), Dumas, Kurshev, and Uppal (2009), Xiong and Yan (2010), and Buraschi, Trojani, and Vedolin (2011) study economies in which a process η_t arise from investors' different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables. Kurz (1994) argues that non-stationarity of economic systems and limited data make it difficult for rational investors to identify the correct model of the economy from alternative ones.

[10] Second order signals are of the type $dS_t^i = \sqrt{1-\psi^2}\sigma_S^i dW_t^{S,i} + \psi\sigma_S^i dW_t^{g,i}$, so that the correlation between dS_t^i and dg_t is equal to ψ. They are commonly used in the ‘ overconfidence’ literature, with the parameter ψ being defined as the overconfidence when the signal is uninformative under the objective measure.

Agents are uncertain about g_t^i and compute posterior estimates (in Bayesian fashion) given initial priors and all available information $\Im_t = \sigma(D_u, S_u^1, S_u^2; 0 \geq u \geq t)$. The larger the value of ϕ^i the more weight agents place on the signals S_t^i when estimating the growth of the economy. The optimal drift forecasts can be conveniently computed by writing the economy in a state-space representation: $X_t = [\log D_t\ , S_t^1\ , S_t^2]'$ and $\mu_t = [g_t^1\ , g_t^2\ , \varepsilon]'$, with Gaussian diffusions following

$$dX_t = (A_0 + A_1\mu_t)\, dt + BdW_t^X \tag{9}$$

$$d\mu_t = (a_0 + a_1\mu_t)\, dt + bdW_t^{\mu}, \tag{10}$$

where the matrices A_0, A_1, a_0, a_1 are given in the appendix, and W_t^X and W_t^{μ} are 3 dimensional standard Brownian motions. Denote subjective posterior beliefs of the unobservables states $m_t^n := E^n(\mu_t \big| \Im_t)$ and posterior covariance matrix $\upsilon_t^n = E^n[(\mu_t - m_t^n)(\mu_t - m_t^n)' \big| \Im_t]$

Lemma 1. *(Beliefs) Under technical conditions discussed in the appendix, m_t^n and υ_t^n are $\Im_t$ measureable, unique, continuous processes solving*

$$dm_t^n = (a_0 + a_1 m_t)dt + \upsilon_t A_1' B^{-1} d\hat{W}_t^{X,n} \tag{11}$$

$$\dot{\upsilon}_t = a_1\upsilon_t + \upsilon_t a_1' + bb' - \upsilon_t A_1'(BB')^{-1} A_1 \upsilon_t \tag{12}$$

where

$$d\hat{W}_t^X = B^{-1}[dX_t - (A_0 + A_1\mu_t)\, dt]. \tag{13}$$

When $\upsilon_t \neq 0$, a rational agent will make use of observations on dS_t^i to update their prior beliefs so that m_t^n depends on the characteristics of matrix A_1, which in turn depend on the prior beliefs on both υ^i, σ_s^i, and subjective parameters. However, since agents must agree on the observables X_t, if A_0 is common it must be true that $d\hat{W}_t^{X,b} = d\hat{W}_t^{X,a} + B^{-1}A_1(m_t^a - m_t^b)dt$. Spreads in the expected unobserved states m_t drive a wedge in the perceived shocks $d\hat{W}_t^{X,n}$. This drift plays a key role in describing the difference in the probability measures of the two agents. Thus, let us define $\Psi_t \equiv B^{-1}A_1(m_t^a - m_t^b)$ as the standardized differences in belief vector. Those agents with relatively lower posterior estimates for signal drifts $m_t^{S^i,n}$ interpret any signal shock as relatively better news for productivity and will update more their posterior to higher values.

From equation 11 one can derive the diffusion process for $d\Psi_t$. One can immediately notice that when $\upsilon_t^a \neq \upsilon_t^b$ the process Ψ_t is stochastic. Moreover, when $A_1^a \neq A_1^b$ disagreement does not converge to zero asymptotically (see Appendix).[11]

[11] The intuition is nicely developed in Acemoglu, Chernozhukov, and Yildiz (2008). When agents are

D. *Individual Investor Problem*

To solve for the equilibrium SDF, consider an economy in which agents have $u'_t = c_t^{-\gamma}$, a time preference discount $\varrho_t = \exp[-\int_0^t \rho(s)ds]$, and a sequence of endowments e^i_t. When markets are complete, an equilibrium is defined by a unique stochastic discount factor $\mathcal{M}^i_t$ for each agent and a consumption plan c^i_t that solves the following intertemporal problem $\max_{\{c^i,\mathcal{M}^i\}} E^i_o \int_0^\infty \varrho_t u(c^i_t)dt$ subject to $E^i_0 \int_0^\infty \mathcal{M}^i_t \left[c^i_t - e^i_t\right] dt \leq 0$ such that markets clear, i.e. $\sum_i c^i_t = D_t$ for $\forall t$. The first order conditions imply that the optimal consumption policies are of the form $c^i_t = (\varrho_t/(\alpha_i \mathcal{M}^i_t))^{1/\gamma}$, where α_i is the Lagrange multiplier associated with the static budget constraint of agent i. It is easy to show that in equilibrium the Radon-Nikodym derivative η_t must be equal to the ratio of the stochastic discount factors of the two agents: $\eta_t = \frac{\alpha_b}{\alpha_a}\frac{u'_a(c_t)}{u'_b(c_t)} = \frac{\mathcal{M}^a_t}{\mathcal{M}^b_t}$.[12] Moreover, its diffusion satisfies (All proofs in the Appendix):

$$d\eta_t/\eta_t = -\Psi_t d\hat{W}^{X,a}_t \tag{14}$$

The stochastic evolution of the Radon-Nykodim derivative is a martingale with a diffusion coefficient which is equal to the difference in beliefs Ψ_t. An important implication follows: since the level of Ψ_t affects the η_t process, it directly affects the evolution of bond prices.[13]

E. *Bond Market Implications*

The dynamic properties of bond prices depend on the characteristics of the stochastic discount factor of the representative agent, $B^{(T-t)}_t = E^*_t(\mathcal{M}^*_T/\mathcal{M}^*_t)$, with $d\mathcal{M}^*_t/\mathcal{M}^*_t = -r_f(t)dt - \kappa^*_t d\hat{W}^{X,*}_t$. The representative investor utility function is a weighted average of each individual utilities with weight λ_t: $U^*(D(t),\lambda) := \max_{c_a(t)+c_b(t)=D(t)}\{\varrho_t u_a(c_a(t)) + \lambda_t \varrho_t u_b(c_b(t))\}$.[14] Since a necessary condition for a social optimum is that $u'_a(c_a(t)) = \lambda_t u'_b(c_b)$, from the first order condition of each individual agent, one can immediately see that this

uncertain about the signals they use to improve their forecasts, they show that observing an infinite sequence of signals does not guarantee degenerate asymptotic disagreement. This is because investors have to update beliefs about two sources of uncertainty using one sequence of signals.

[12] Consider a tradable asset with terminal payoff B_T. In equilibrium, both agents must agree on its price. Under general preferences, $u_n(c_t)$, from the Euler equation it must be true that $E^b_t\left(\frac{u'_b(c_T)}{u'_b(c_t)}B_T\right) = E^a_t\left((\frac{u'_a(c_T)}{u'_a(c_t)}B_T\right)$. Thus $E^b_t(\frac{u'_b(c_T)}{u'_b(c_t)}B_T) = E^a_t\left[\left(\frac{u'_a(c_T)/u'_a(c_t)}{u'_b(c_T)/u'_b(c_t)}\right)\frac{u'_b(c_T)}{u'_b(c_t)}B_T\right]$, which implies that $\eta_T = \frac{\alpha_b}{\alpha_a}\frac{u'_a(c_T)}{u'_b(c_T)}$.

[13] The process η_t is sometimes referred to as 'sentiment' in the behaviour finance literature.

[14] Constantinides (1982) extends Negishi (1960)'s results and proves the existence of a representative agent with heterogeneous preferences and endowments but with homogeneous beliefs. In an incomplete market setting with homogeneous agents Cuoco and He (1994) show a representative agent can be constructed from a social welfare function with stochastic weights. Basak (2000) discuss the aggregation properties in economies with heterogeneous beliefs but complete markets. He shows that a representative can be constructed from a stochastic weighted average of individuals marginal utilities.

can be achieved if the representative agent sets a stochastic weight equal to $\lambda_t = \frac{u'_a(c_a(t))}{u'_b(c_b(t))} = \frac{\alpha_a \mathcal{M}^a(t)}{\alpha_b \mathcal{M}^b(t)}$. This implies that the relative weight of the second set of agents must be proportional to the Radon-Nikodym process η_t: i.e. $\lambda_t = \frac{\alpha_a}{\alpha_b}\eta_t$. Moreover, since the Lagrange multipliers are constant, the diffusion of the Radon-Nykodym process coincides with the dynamics of the relative weight: $d\eta_t/\eta_t = d\lambda_t/\lambda_t$. This implies that ψ_t directly affects the relative marginal utility of the two set of agents in equilibrium.

It can be shown that the stochastic discount factor of the representative agent is therefore $\mathcal{M}^*_t = \alpha_a \mathcal{M}_a(t) = \lambda(t)\alpha_b \mathcal{M}_b(t)$, which is proportional to the first agent's state price density.[15] Combining the first order conditions from the individual agent's problems with the Radon-Nikodym η_t and imposing market clearing one obtains the stochastic discount factors for the representative agent:

$$\mathcal{M}^*_t = \varrho_t D_t^{-\gamma} \left(1 + \eta_t^{1/\gamma}\right)^{\gamma} \tag{15}$$

The drift of $d\mathcal{M}^*_t$ provides the risk free rate, which is equal to:

$$r_f = \rho + \gamma\beta' \underbrace{(\omega_a(\eta_t)\hat{g}^a_t + \omega_b(\eta_t)\hat{g}^b_t)}_{\text{Consensus Aggregation Bias}} - \underbrace{\frac{1}{2}\gamma(\gamma+1)\sigma_D^2}_{\text{Precautionary Savings}} + \underbrace{\frac{\gamma-1}{2\gamma}\omega_a(\eta_t)\omega_b(\eta_t)\Psi_t{}'\Psi_t}_{\text{Differences in Beliefs}}, \tag{16}$$

where $\omega_i(\eta_t) = c^i_t/D_t$ is investor's i total consumption share. This result highlights an immediate implication which is relevant for bonds markets. When $\gamma > 1$, heterogeneous expectations impact short term interest rates in two different ways: (a) by introducing an aggregation 'bias' as the representative agent expectations deviate from consensus, depending on the weights $\omega_i(\eta_t)$, and (b) a quadratic term which is explicit and increasing in the differences in beliefs Ψ_t. When $\Psi_t = 0$, the model reduces to the special case of a standard Vasicek economy in which $r_f = \rho + \gamma\beta'\hat{g}_t - \frac{1}{2}\gamma(\gamma+1)\sigma_D^2$. The cross-sectional distribution of consumption is degenerate and state prices, $\mathcal{M}^a_t = \mathcal{M}^b_t$, depend exclusively on the diffusion dD_t. Moreover, when preferences are logarithmic (i.e. $\gamma = 1$), as in Xiong and Yan (2010), the last term in equation (16) disappears and disagreement impacts the risk free rate only because of an aggregation bias due to the consumption weights $\omega_n(t)$.

The second implication is about risk premia and bond excess returns. In a Vasicek economy, the only priced shocks are $dW_D(t)$ and since dD_t/D_t is homoskedastic, the price of risk is constant. In the partial information heterogeneous economy with learning, however, the dynamics of $d\mathcal{M}^*_t$ also depend on $d\eta_t$. This creates a potential channel for Ψ_t to play a role in the predictability of bond excess returns. The intuition is simple. When agents have

[15] This follows from: $U^{*'}(D(t),\lambda(t)) = u'_a(c_a(t))\frac{\partial c_a}{\partial D} + \lambda(t)u'_b(c_b(t))\frac{\partial c_b}{\partial D} = u'_b\left(\frac{\partial c_a}{\partial D} + \frac{\partial c_b}{\partial D}\right) = \alpha_a \mathcal{M}_a(t) = \lambda(t)\alpha_b \mathcal{M}_b(t)$

different subjective beliefs, *relative* consumption is stochastic.[16] Optimistic (pessimistic) investors consume more (less) in states of high aggregate cash flows, at a lower (higher) marginal utility, because they perceive those states as more (less) likely. This also implies that the consumption volatility of the optimist is higher than the pessimist.[17] The intuition can be formalized by noticing that the stochastic discount factor of the individual agent $\mathcal{M}_t^i$ can be factorized as the product of two components, the stochastic discount factor that characterizes a Vasicek economy with no differences in beliefs $\tilde{\mathcal{M}}_t^i \equiv \varrho_t D_t^{-\gamma}$, which only depends on aggregate consumption shocks, and the stochastic consumption share $\omega_i(\eta_t)$, so that $\mathcal{M}_t^i = \tilde{\mathcal{M}}_t^i \times \omega_i(\eta_t)^{-\gamma}$. In order to finance their ex-ante individual consumption plans, pessimistic investors have to buy financial protection against low aggregate cash flow states from optimistic investors. If a negative state occurs ex-post, optimistic investors are hit twice: First, because the aggregate endowment is lower; second, because their consumption share is lower due to the protection agreement. The size of this risk transfer is proportional to the degree of disagreement among agents. In the economy with homogeneous beliefs, ω_i is constant and the discount factor is simply proportional to the marginal utility $D_t^{-\gamma}$ of aggregate consumption.[18] The most interesting aspect of this risk transfer is that it does not cancel out at the representative agent level. The prices of risk under the representatives measure are

Theorem 1.

$$\kappa_g^*(t) = \gamma\sigma_D + \omega_b(t)\psi_g(t) \qquad \kappa_s^*(t) = \omega_b(t)\psi_s(t), \tag{17}$$

which implies that bond excess returns explicitly depend on the dynamics of $\Psi_t = [\psi_g(t)\ ,\ \psi_s(t)]'$. *(Proof in Appendix).*

The instantaneous price of risk contains a compensation for both sources of disagreement. Any agent knows that, independently of his beliefs today, tomorrow other agents will likely revise their expectations in a way different than his own. The intertemporal budget constraint of this agent anticipates the fact that other agent's beliefs will affect η_t in the future. In this context, η_t is a source of risk that agents want to hedge.[19] For this reason, this agent holds less risky asset in an economy with disagreement risk than in the equivalent homogeneous economy, giving rise to a risk premium. Time variation in η_t makes the price of risk time-varying.

[16]The implied optimal consumption policies are $c_a(t) = D_t(1+\eta_t^{1/\gamma})^{-1}$ and $c_b(t) = D_t\eta_t^{1/\gamma}(1+\eta_t^{1/\gamma})^{-1}$.

[17]Notice that differences in beliefs make consumption volatility of the optimist higher. Individual consumption volatilities determined endogenously as $\sigma_{c_n} = \frac{\kappa^n}{\gamma}$.

[18]The relative consumption of the two agents c_2/c_1 is constant since now complete markets allows for perfect risk sharing.

[19]Thus, each agent's optimization includes a form of 'higher-order beliefs'.

In the special case in which signals are of second order, it is possible to show that $\kappa_s^*(t) = 0$ for $\forall t$. In this case, only disagreement on fundamentals ψ_g is priced, with $\kappa_g^*(t) \neq 0$. On the other hand, disagreement on first-order signals, in addition to disagreement on fundamentals, is priced and $\kappa_s^*(t) \neq 0$. The reason is that second order signals are simply a reason for agents to disagree on the fundamentals but they are not an independent source of pertubation of the stochastic discount factor, given ψ_g. Signals that affect the conditional first moment of the growth rate, on the other hand, enter directly as state variables in agents' belief-dependent optimal consumption plans.

Corollary (Spanning of the State Space). *Suppose that agents use $N + M$ signals for inference. Let N be the number of signals correlated with expected innovations (first-order signals) and M being the number of signals correlated with unexpected innovations of the fundamentals (second-order signals). Then, there are $1 + N$ priced state variables.*

(Proof in Appendix)

We use this implication to learn about the empirical merits of different model specifications used in the literature.

F. *The Term Structure of Bond Prices*

The price of a default-free zero coupon bond is given by $B_t^{(T-t)} = E_t^* \left(\mathcal{M}^*(T)/\mathcal{M}^*(t)\right)$.[20] One can notice that solving for the term structure of interest rates is complicated by the fact that it requires knowledge of the joint density of $D(t)$ and $\lambda(t)$, which is not available in closed-form. The solution suggested in Xiong and Yan (2010) only applies to the case of log-investors. Dumas, Kurshev, and Uppal (2009) show a method to calculate the joint Laplace transform of $D(t)$ and $\lambda(t)$ by using Radon's Theorem.[21] This can be used, in a second step, to obtain bond prices in closed-form by Fourier inversion.[22] By using this technique, one finds that the price of a zero coupon bond is the product of two terms: the first component depends on the the posterior m_D, the second component depends on the vector of differences in beliefs ψ. The following Theorem summarizes the result.[23]

Theorem 2 (The Term Structure of Bond Prices)**.** *The term structure of bond prices is*

[20]Obviously, in equilibrium, the solution can be equivalently computed with respect to any probability measure since $B(t,T) = E^a(\mathcal{M}_T^a) = E^b(\mathcal{M}_T^b) = E^*(\mathcal{M}_T^*)$.

[21]See also Buraschi, Trojani, and Vedolin (2010) for the use of this technique in a different context.

[22]The spirit of the Fourier inversion approach is similar to the one used to price derivatives in stochastic volatility models, such as Heston (1993), Duffie, Pan, and Singleton (2000), and Carr, Geman, Madan, and Yor (2003), or in interest-rate models, such as Chacko and Das (2002).

[23]One can reduce the system of ordinary differential equations for functions to a system of matrix Riccati equations, which can be linearized using Radon's Lemma. In this way, one can obtain explicit expressions for the coefficients in the exponentially quadratic solution of the Laplace transform.

equal to the product of two deterministic functions. The first is exponentially affine in the posterior growth rate of the endowment; the second is exponentially quadratic in the level of differences in beliefs:

$$B(t,T) = \varrho_{T-t} F_{m_D^a}(m_D^a, t, T; -\gamma) G(t, T, -\gamma; \psi_g; \psi_s), \tag{18}$$

$$\begin{aligned}
G(t,T,-\gamma;\psi_g;\psi_s) &\equiv \int_0^\infty \left(\frac{1+\lambda(T)^{1/\gamma}}{1+\lambda(t)^{1/\gamma}}\right)^\gamma \left[\frac{1}{2\pi}\int_{-\infty}^\infty \left(\frac{\lambda(T)}{\lambda(t)}\right)^{-i\chi} F_{\psi_g;\psi_s} d\chi\right] \frac{d\lambda(T)}{\lambda(T)}, \\
F_{m_D^a}(m_D^a,\tau,\epsilon) &= \exp(A_{m_D^a}(\epsilon,\tau) + B_{m_D^a}(\epsilon,\tau) m_D^a), \qquad (19) \\
A_{m_D^a}(\epsilon,\tau) &= \frac{1}{2}\epsilon(\epsilon-1)\sigma_D\tau - \left(\theta_g + \frac{\epsilon\gamma_D^a}{\kappa_g^a}\right)\left(e^{\kappa_g^a\tau} + \tau\right) \\
&\quad -\frac{1}{\kappa_g^a}\left(\left(\frac{\gamma_g^a}{\sigma_D}\right)^2 + \left(\frac{\phi\gamma_g^a + (1-\phi)\gamma_{g\epsilon}^a}{\sigma_S}\right)^2\right)\left(\frac{3}{2}e^{-\kappa_g^a\tau} + \kappa_g^a\tau\right) \\
B_{m_D^a}(\epsilon,\tau) &= -\frac{\epsilon(e^{-\kappa_g^a\tau} - 1)}{\kappa_g^a}, \\
F_{\psi_g,\psi_S} &= \exp\Bigg(A_\Psi(\tau) + B_\Psi(\tau)\Psi_t + \Psi_t C(\tau)\Psi_t'\Bigg), \qquad (20)
\end{aligned}$$

where $\tau = T - t$, $\Psi_t := [\psi_g(t), \psi_s(t)]'$, and A_Ψ, B_Ψ and C_Ψ are functions derived in the Appendix.

The dependence of bond prices on $m_D^a(t)$ is exponentially affine: this is due to the fact that the dividend process, conditional on an estimate for $\hat{\theta}_t$, is lognormal (Vasicek economy). Under incomplete information and learning, the term structure also explicitly depends on the difference in beliefs $\psi_g(t)$ and $\psi_s(t)$. The dependence on these factors is exponentially quadratic. It is interesting to notice the direct dependence of bond prices on $\psi_s(t)$, which occurs only in the case of first-order signals and it is absent in economies where agents use second-order signals. A number of important observations emerge which lead to our empirical questions (hypothesis).

Hypothesis 1. *In equilibrium, the date t expected excess return on period T bond is*

$$rx_{t,t+dt}^\tau = \underbrace{-\gamma B_{m_D^a}(\tau)\sigma_D\sigma_g E^*\left(d\hat{W}_t^D d\hat{W}_t^g\right)}_{Learning} + \underbrace{\omega_b(t)\psi_g(t)\sigma_{B,D}^T(t)}_{DiB\ fundamentals} + \underbrace{\omega_b(t)\psi_s(t)\sigma_{B,S}^T(t)}_{DiB\ signals} \tag{21}$$

(i) Bonds risk premia are equal to the sum of three components. The first one is a constant which depends on the extent to which dividend shocks affect growth rates. When $\gamma > 1$ and disagreement is stochastic, agents' portfolio include a hedging demand that makes expected return being a function in equilibrium of disagreement $\psi_g(t)$. Assets whose payoffs are negatively correlated with the Radon-Nikodym derivative of investors beliefs demand a disagreement premium since in bad states of the world the social planner puts more weight on more pessimistic agents. This second term is absent in Jouini, Marin, and Napp (2010).

(ii) The third term is generated by disagreement on first-order signals. Equation 21 suggests that $\psi_s(t)$ affects expected returns directly. This term is absent in economies with second order signals. Overconfidence with respect to the informativeness of signals (agents mis-judge the correlation between innovations) generates spreads in subjective measures. However, from Girsanov theorem there is no effect on the instantaneous price of risk.[24]

Hypothesis 2. *Independent of disagreement, by no-arbitrage all agents must agree on $B(t,T)$ a time t, so that $B^a(t,T) = B^b(t,T)$. However, for $t < \tau < T$ disagreement can extend to bond prices, so that $E_t^a[B(\tau,T)] \neq E_t^b[B(\tau,T)]$. Moreover, dispersion in price forecasts is due to the change of measure, $\eta_t(\psi_t^g, \psi_t^S)$, which depends on disagreement about both fundamentals and signals: $E_t^a\left[B(\tau,T)\right] = E_t^b\left[\frac{\eta_\tau}{\eta_t}B(\tau,T)\right]$.*

(i) Theorem 1 also suggests that while forward rates are uniquely determined in equilibrium at time t, expected future bond prices are agent specific and, by no arbitrage, $E_t^a[B(\tau,T)] = E_t^b[B(\tau,T)]$ only at time $\tau = t$. If ambiguity and differences in beliefs are positively correlated, it may be difficult to distinguish the implications of these two economies based on observations of bond risk premia. However, differences do exist which give rise to testable implications. In models with homogeneous expectations and ambiguity aversion, a representative agent solves a min-max problem which gives rise to a solution that depends on the extent to which the representative agent is averse to worst case events and the characteristics of its time variation. In these models expected future bond prices $E_t^i[B(\tau,T)]$ are unique $\forall i$ and $\forall \tau \geq t$, even if agents face Knightian uncertainty about the fundamentals at time t

(ii) This implication can also be used to learn about the properties of models with overconfidence based on second order beliefs. In this class of models, $E_t^a[B(\tau,T)] - E_t^b[B(\tau,T)]$ is completely explained by disagreement on fundamentals, ψ_g. We use a unique dataset on the term structure of bond yields to directly investigate the difference between disagreement on future fundamentals ψ_g versus disagreement on future bond prices.

Hypothesis 3. *Disagreement generate endogenous stochastic volatility of bond yields even if the endowment process is homoskedastic. When $\gamma > 1$, changes in disagreement directly affect both the level and slope of the term structure of volatility of (changes in) bond yields.*

[24] Dumas, Kurshev, and Uppal (2009) study also the long-run compensations of signal risk using Malliavin derivatives (impulse responses).

An important stream of the literature studies the link between conditional first and second moments of bond yields and finds that the volatility of long-term yields behave differently than of short-term yields. Even if fundamentals have constant volatility, heterogeneous agents economies give rise to endogenous stochastic bond yield volatility. This is interesting since the model generates additional empirical restrictions that do not depend on exogenous assumptions. Within the difference in beliefs literature, however, it is possible to separate (i.e., identify) different empirical implications arising from different specifications. In models with myopic investors, the level of disagreement does not have a direct effect on yield volatility since $\psi_g(t)$ does not enter the bond pricing equation as an explicit term. The effect of heterogeneity enters as a deterministic aggregation bias. In models in which disagreement on fundamentals is due to overconfident investors who rely on second order beliefs, instantaneous yield volatility is affected by date-t $\psi_g(t)$ as in (as in Dumas, Kurshev, and Uppal (2009)). If agents use first order signals, however, both date-t $\psi_g(t)$ and $\psi_s(t)$ affect directly the term structure of volatility. This result follows easily by applying Ito's lemma to equation (18) and using (14) (see the Appendix for a formal derivation).

A second implication relates to the slope of the term structure of volatility. Since disagreement on short-run (long-run) components of the economy affect mostly short (long) term yields, it also affect the slope of the term structure of volatility. In the empirical section, we use a proxy of the temporal dependence of disagreement to investigate this link.

Hypothesis 4. *Heterogeneous beliefs generate dynamics in bond prices suggestive of unspanned factors.*

An important literature investigates the extent to which the shape of the term structure spans the state variables that are responsible for the predictability of bond excess returns (Cochrane and Piazzesi (2005), Duffee (2011), Joslin, Priebsch, and Singleton (2011)). When spanning can be achieved, bond prices (or forward rates) provide sufficient information to explain the dynamics of bond excess returns. Fama and Bliss (1987) and Cochrane and Piazzesi (2005) interpret their empirical results in this context.

In our context, when $\gamma > 1$ the mapping G that links differences in beliefs to bond prices is not invertible (see equation (18)) unless one extends the state-space by redefining squared and cross terms in disagreement.[25] While theoretically possible, in practice, this is potentially challenging since agents use different signals over time whose number is potentially infinite. Thus, it may be difficult to span the state vector with a finite cross-section of bond prices.

[25] Cheng and Scaillet (2007) characterise the conditions under which one can embed linear-quadratic models in a standard affine framework. Given an augmented state-space one can estimate the parameters of the model and invert the yield curve using standard transform technics.

In the empirical section, we use disagreement data and study spanning properties in relation to the cross-section of bond prices.

Hypothesis 5. *In a heterogeneous agent economy with difference in belief agents risk share by trading state contingent claims; thus, shocks to disagreement induce an increase in trading activity.*

An additional important implication of disagreement models is that differences in beliefs are correlated with trading activity (as in Buraschi and Jiltsov (2006), Banerjee and Kremer (2010)). This implication is of particular interest since it distinguishes this class of models from single agent equilibrium term structure models, in which trading volume is indeterminate, and those with Knightian uncertainty and ambiguity.[26] It has been shown that greater ambiguity induce portfolio inertia and limited participation, as discussed in Illeditsch (2011), de Castro and Chateauneuf (2011), and Chen, Ju, and Miao (2011). We directly investigate this empirical link.

II. Data

A. Disagreement Data

We obtain measures of heterogeneity directly from surveys of market participants' expectations of future fundamentals and prices. Few sources exist with a large reliable sample period and appropriate frequencies. The Survey of Professional Forecasters (SPF), for instance, is only available at quarterly frequency. Blue Chip Economic Indicators (BCEI) does provide an extensive panel of data on expectations by agents who are working at institutions who are active in financial markets and importantly allows a simple aggregation procedure (discussed below) that mitigates problems associated with rolling forecast horizons. Unfortunately, digital copies of BCEI are only available since 2007. However, we sourced the complete original Blue Chip paper archives from Wolters Kluwer to extend the dataset back to 1.1.1990. The digitisation process required inputting around $\sim 350,000$ entries of named forecasts. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations in asset pricing. An extract from the paper archives is shown in figure 1 below.[27]

[Insert figure 1 here.]

[26] Dispersion in beliefs has been used to proxy for entropy at the individual level.

[27] We are indebted to Andrea Vedolin who was the first to indicate Blue Chip dataset to us and whose help was determinant for this project.

Each month Blue Chip carries out surveys of professional economists from leading financial institutions and service companies. Each economist is asked to forecast a set of economic fundamentals covering real, nominal, and monetary variables. The survey is conducted over the first two days of the beginning of each month. Our empirical analysis is therefore not affected by biases induced by staleness and overlapping observations between returns and responses.[28] The sample period for which we have a fully digitised dataset is 1.1.1990 - 1.12.2011. While we have forecasts for a very broad set of economic variables, in this study we focus on:

1. Real: Real GDP and Industrial Production.

2. Nominal: Consumer Price Inflation and Gdp deflator

3. Monetary: 3 Month Treasury Rate, 10 year Treasury.

Furthermore, for each variable two types of forecast are made:

1. Short-Term: an average for the remaining period of the current calendar year;

2. Long-Term: an average for the following year.

For example, in July 2003 each contributor to the survey made a forecast for the percentage change in total industrial production for the remaining 6 months of 2003 and for the percentage change to the end of 2004 (18 months ahead). The December 2003 issue contains forecasts for the remaining period of 2003 (1 month ahead) and an average for 2004 (13 months ahead). The moving forecast horizon induces a seasonal pattern in the survey which can be adjusted by first computing an implied constant maturity forecast for each individual forecaster and then adjusting any residual seasonality with an X-12 ARIMA filter (see the appendix for specific details on the procedure used). We find that combining long and short term forecasts at the individual level removes the vast majority of the observable seasonality.

[Insert figure 2 here.]

[28] An exception to the general rule was the survey for the January 1996 issue when non-essential offices of the U.S. government were shut down due to a budgetary impasse and at the same time a massive snow storm covered Washington, DC: www.nytimes.com/1996/01/04/us/battle-over-budget-effects-paralysis-brought-shutdown-begins-seep-private-sector.html. As a result, the survey was delayed a week.

The dataset has several desirable properties with respect to others used in the previous literature. First, the number of participants in the survey is stable over time. On average 51 respondents are surveyed for short term forecasts and 49 for long term forecasts with standard deviations of 1.6 and 3.3 respectively (see figure 3, the corresponding plot for long term forecasts is very similar). Only on rare occasions are survey numbers less than 40 and no business cycle patterns are visible. In the 'Survey of Professional Forecasters', on the other hand, the distribution of respondents displays significant variability: while the mean number of respondents is around 40, the standard deviation is 13 and in some years the number of contributors is as low as 9 (see figure 4).[29] While in the early 70s the number of SPF forecasters was around 60, it decreased in two major steps in the mid 1970s and mid 1980s to as low as 14 forecasters in 1990 and if one restricts the attention to forecasters who participated to at least 8 surveys, this limits the number of data point considerably. Second, while our dataset is available at a monthly frequency, SPF is available only at quarterly frequency. Third, the SPF survey has been administered by different agencies over the years, which have, over the years, changed some questions. Some of these changes crucially affected the forecasting horizon. For a detailed discussion on the issues related to SPF, see D'Amico and Orphanides (2008) and Giordani and Soderlind (2003). Other well known surveys, such as the 'University of Michigan Survey of Consumers' do not provide point estimates from individual survey respondents

[Insert figure 3 and 4 here.]

Disagreement on fundamentals. Macroeconomic disagreement is measured as the cross-sectional mean-absolute-deviation (MAD) in forecasts. We proxy for disagreement about the real economy ψ_g from the first principal component of the MAD of Industrial Production Growth and Real GDP; similarly, disagreement about inflation ψ_π is computed from the first principal component of the MAD of the gdp deflator and the Consumer Price Index. Figures 5 and 6 summarize the time series behaviour. Disagreement on the real economy has a significant business cycle component: in all previous three NBER economic recessions since 1990, ψ_g is low before the recessions and it increases to peak at the end of the recessions (this occurs in 1991, 2002, and 2009). This is interesting as large disagreement is often reported at this stage of the cycle, often related to different interpretations about initial signs of economic recovery.[30]

[29]The SPF survey has been used, among others, by Buraschi and Jiltsov (2006) and Ulrich (2010); it is available at `www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/`.

[30]In a controversial statement that attracted substantial controversy, in 1991 Normal Lamont - Chancellor of the Exchequer of the United Kingdom - labeled the initial sign of the recovery from the S&L recession

[Insert figures 5, 6, and 7 here]

Disagreement on signals. In any model, signals are intrinsically difficult to identify. However, bond yields are endogenous in equilibrium and depend on both fundamentals and the signals used to compute conditional expectations. Therefore, while prices of any tradable asset at time t are uniquely determined, disagreement on the information content of these signals contribute to a disagreement on future bond yields. We rely on this result and use disagreement on future bond yields to reveal information on the disagreement on the signals. We compute the spread between the MAD in 1-year forecasts for the 3 month Treasury rate and the MAD in 1-year forecasts about the 10 year rate (labelled ψ_s). Our proxy for signal disagreement reveals disagreement about future transitory shocks after controlling for disagreement about the long run component of the economy. As Figure 9 shows, ψ_s has a significant counter-cyclical business cycle component. It is positive and large during bad states, when agents disagree more about the short term than the steady state.[31]

Investigating how ψ_g and ψ_s are related to the principal components of the original vector of 4 macro-disagreement variables, we find that ψ_g loads strongly on the first principal component (which explains about 45% of the total variance), with high time series correlation (see figure 9). The most significant difference is the higher volatility of the first principal component, which has larger peaks during crisis periods such as during the 1998 LTCM Crisis and immediately after Lehman's default. The behaviour of ψ_s, on the other hand, has near perfect time series correlation with the third principal component (Figure 9), whose eigenvectors capture a spread between disagreement on short-term versus long-term future yields. [32]

[Insert figures 8 and 9 here] & [Insert table I here]

B. Bond Data

For Treasury bonds data, we use both the (unsmoothed) Fama-Bliss discount bonds dataset, for maturities up to five years, and the (smoothed) Treasury zero-coupon bond yields dataset of Gürkaynak, Sack, and Wright (2006) (GSW) for maturities beyond 5 years. The GSW

as 'green shoots'. Many years later, Ben Bernanke used the same words in a well-known 'CBS 60 Minutes' interview in 2009, which was counterpointed by Nouriel Roubini who argued his disagreement and labeled those signs as 'yellow weeds'. The data indeed confirm that macro-disagreement is usually pervasive in this phase of the cycle.

[31] An example of transitory shocks are signals that may reveal a change in the stance of the monetary policy in the pursuance of its dual mandate.

[32] In the interest of space we omit the eigenvector loadings. ψ^{PC_1}, however, loadings symmetrically across real and monetary disagreement, while ψ^{PC_3} loads on the spread between dispersion in short term yield versus long term yield forecasts.

data set includes daily yields for longer maturities: 1-15 years pre-1971 and 1-30 years post-1971.[33] We introduce notation along the lines of Cochrane and Piazzesi (2005) by defining the date t log price of a n-year discount bond as:

$$p_t^{(n)} = \text{log price of n-year zero coupon bond.} \tag{22}$$

The yield of a bond is defined as $y_t^{(n)} = -\frac{1}{n} p_t^{(n)}$. The date-$t$ 1-year forward rate for the year from $t+n-1$ and $t+n$ is $f_t^{(n)} = p_t^{(n)} - p_t^{(n+1)}$. The log holding period return is the realised return on an n-year maturity bond bought at date t and sold as an $(n-1)$-year maturity bond at date $t+12$:

$$r_{t,t+12}^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}. \tag{23}$$

Excess holding period returns are denoted by:

$$rx_{t,t+12}^{(n)} = r_{t,t+12}^{(n)} - y_t^{(1)}. \tag{24}$$

The realized second moments of bond returns are measured at daily frequency following Schwert (1989) and Viceira (2007) among many others. Integrated instantaneous volatility is proxied by realized volatility between month t and $t+1$ as

$$\widehat{\sigma}^2(t) = \frac{1}{n-1} \sum_{i=1}^{n} r^2(t,i). \tag{25}$$

Volatility estimates are annualised squared daily returns from the GSW dataset.

C. Volume Data

The Federal Reserve Bank of New York trades U.S. government securities with primary dealers who in turn trade on the secondary market with dealer-customers. Primary dealers report (voluntarily) weekly data on their trading activities and positions broken down counter-party or security type.[34] We compute inter-month sums of weekly trading volumes (transactions denominated in dollars) between primary dealers and secondary customers for Treasury Bills and coupon paying securities due in more than 6 years but less than or equal to 11 years. [35] All transaction data is available from `www.newyorkfed.org/markets/primarydealers.html`. The sample period is from 04.07.01 to 31.12.11.

[33] The dataset is available at: `www.federalreserve.gov/econresdata/researchdata.htm`.

[34] Primary dealers hold positions and trade in U.S. Government Securities, Federal Agency and Government Sponsored Enterprise Securities, Mortgage-backed Securities, and Corporate Securities.

[35] T-bills are issued with maturities of 28 days, 91 days, 182 days, and 364 days.

D. Macro Data

In dynamic single agent equilibrium models of the term structure, factors linked to the marginal productivity of capital account for the dynamics of interest rates. In such models, the marginal rate of substitution is tightly connected to the marginal rate of transformation and drive the term structure. Until recently, however, the search for sources of such time-variation in term premia was been carried out with limited success. An exception is Ang and Piazzesi (2003), who estimate a VAR with identifying no-arbitrage restrictions and find the combination of macro and yield curve factors improve the performance of a model including yield factors only. More recently, Ludvigson and Ng (2009) find evidence that links variations in the level of macro fundamentals, obtained from principal components of a very large set of macroeconomic variables, to the time variations of expected excess bond returns. We study the marginal contribution of disagreement after controlling for the macro-activity factors of Ludvigson and Ng (2009). Different than in their approach, however, we drop any price based information from their initial information set.[36] This allows us to interpret the factors as pure 'macro' that capture the current state of economic activity. After removing price based information from the panel we end up with 99 macro series, from which we compute the first eight principal components. A description of each series along with its data source is given in an online appendix on the authors website.

Classical understanding of risk compensation for nominal bonds also says that investors should be rewarded for the volatility of inflation and consumption growth (as opposed to the level of macro activity as in Ludvigson and Ng (2009)). We proxy for these by estimating a GARCH process for monthly log differences of CPI All Urban Consumers: Non-Durables (NSA) and Industrial Production and Capacity Utilisation: All Major Industry Groups (NSA). Finally, a well known determinant for nominal bond risk premia is time variation in real-nominal covariance which we proxy for by estimating a dynamic correlation MV-GARCH process for inflation and consumption growth.

III. Empirical Results

In the following section, we study empirically the five testable hypotheses of the heterogeneous agent model discussed above.

[36]Examples of price variables removed include: S&P dividend yield, the Federal Funds (FF) rate; 10 year T-bond; 10 year - FF term spread; Baa - FF default spread; and the dollar-Yen exchange rate. A small number of discontinued macro series were replaced with appropriate alternatives or dropped.

A. *Disagreement and Bond Risk Premia*

First, we investigate if some of the components of the time-varying stochastic discount factor that generate time-variation in risk premia are linked to the dynamics of the differences in beliefs. As summarized by HYPOTHESIS 1, when disagreement is time-varying and the risk aversion coefficient is $\gamma > 1$ differences in beliefs become an explicit source of predictability.

Our first set of regressions are based on projections of excess returns at a 12 month horizon for n-year bonds, with $n = 5$. Then we show results for a cross-section of different values of n. We run regressions of the following form:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{3} \beta_i^{(n)} \psi_{i,t}(\star) + \sum_{i=1}^{4} \gamma_i^{(n)} E_{i.t}(\star) + \sum_{i=1}^{11} \phi_i^{(n)} Macro_{i,t}(\star) + \delta \ Slope_t + \varepsilon_{t+12}^{(n)}, \tag{26}$$

where $\psi_t(\star)$ includes the set of disagreement measures as discussed above, $E_t(\star)$ is the consensus estimate of either expected inflation or expected RGDP, $Macro_t(\star)$ includes a set of controls as outlined in section D, and $Slope_t$ is the Fama-Bliss factor obtained from the forward-spot spread.

[Insert table II here.]

Table II summarizes the results. Columns (i) to (v) show results for regressions of excess returns on differences in beliefs only. We find that ψ_g explains, on its own, 11% of the variance and is statistically significant, with a t-statistics equal to 2.86. The significance of ψ_s is even stronger, with a t-statistics equal to 5.32 and an R^2 from individual regressions equal to 29%. On the other hand, we find that ψ_π is not statistically significant. When we consider a regression with all three difference in beliefs proxies, the R^2 is equal to 35% and both the ψ_g and ψ_s are statistically significant, suggesting that they capture different type of information.

In column (vi) we control for the Fama-Bliss factor. In a seminal paper, Fama and Bliss (1987) show that maturity matched forward rates contains significant information to explain the dynamics of excess bond returns. Using data spanning 1964 to 1985, they find that the forward spread can explain between 5% and 14% of the variance of bond excess returns. When we control for this factor, we find that forward spreads adds only 1% to the initial R^2: while it reduces the significance of ψ_g, the signal factor ψ_s continues to be highly significant, with a t-statistics equal 4.83.

In column (vii) and ($viii$) we control for the consensus values of inflation, gdp growth and interest rates. We want to control for these variables to check the extent to which the result is generated simply by expectations on future state of the economy, as opposed to the dispersion in expectations. In both cases, we find that consensus expectations are not statistically significant. Consensus views do not help explain bond returns. Even after controlling for these factors, ψ_s continues to have very large t-statistics.

In the last two columns we control for the first eight principal components of the Ludvigson and Ng (2009) macro-activity factors and the second moments of inflation and gdp growth. They report results with some of the highest R^2 of the current empirical literature. Updating the data set used in their paper we confirm the existence of a significant predictable component for the dynamics of excess bond return from latent factors hidden in a large panel of macro aggregates. For instance, the R^2 of a regression on ψ_π and ψ_g increases from 12% to 24%. At the same time, the R^2 of a regression on ψ_s increases from 29% to 41%. However, in both cases disagreement continues to be significant. More interestingly, the t-statistics of the slope of ψ_s increases to 5.85 after controlling for their macro factors. This is important since it highlights that the level of economic activity and the heterogeneity in beliefs capture different type of information.

To clarify the economic significance of the estimated loadings, according to the model expected excess returns are highly variable: 10-year bond returns averaged 4.98% above the risk free 1-year bond return but with a standard deviation of 2.16%. A 1-standard deviation shock to disagreement about the real economy raises expected returns on these bonds by 1.42% while a 1-standard deviation shock to disagreement about short term rates raises expected returns by 2.39%.

We then proceed to run multivariate forecasting regressions of 1-year excess returns for a cross-section of 2, 5 and 10 year maturity bonds.[37] The results are summarized in Table III. We find that the results are robust across maturities. The R^2 of regressions on 2 year bonds is even larger and equal to 42%. While the R^2 for 5 and 10-year bonds is equal to 35% and 21%, respectively. The t-statistics indicate that disagreement is statistically significant for the entire term structure. To put this in perspective, the R^2 estimated in the original article by the Fama and Bliss (1987) forward spread is 5% for the 5-year bonds and 14% for the 2-year bond. Finally, table IV repeats the predictability regressions using the disagreement principal components discussed in section A. These results confirm the existence of a priced real disagreement factor and a signal disagreement factor which are statistically and economically significant drivers of expected bond returns.

[Insert table III and IV here.]

To summarize, these results suggest that an economically sizeable proportion of time-variation in expected returns is due to changes in macroeconomic disagreement and that

[37] For the 10 year bond yields we use the GSW dataset.

this result is not subsumed by more traditional risk factors that have been studied recently in the fixed-income literature. Heterogeneity in the formation of beliefs captures a component that is linked to market clearing market and embeds a term correlated with the marginal utility of the representative. We also find that differences in beliefs on variables that proxy for signals are strongly significant. This is an interesting results that support models in which agents use first-order signals to help their inference. The following section studies the extent to which disagreement and signals and fundamentals are independent.

B. Disagreement on Fundamentals and Prices

Hypothesis 2 questions the relative importance of disagreement on fundamentals versus disagreement on future prices. This is important since disagreement on future prices cannot exist in ambiguity models even if disagreement on fundamentals serves as a proxy for Knightian uncertainty. Furthermore, if agents learn about the economy using second order signals then disagreement on fundamentals should span dispersion in future price forecasts. To address this question first consider figures 5 , 6 and 7. The dynamics of disagreement on fundamentals appear distinct from the dynamics of disagreement on prices. Forecast dispersion on short term yields typically spikes during the onset of recessions while disagreement about long term yields increases as the economy recovers. On the other hand, disagreement about inflation and real consumption growth persist throughout crisis and recessionary periods. Furthermore, considering table I we find the unconditional correlation between disagreement on signals (ψ^s) and disagreement on real growth (ψ^g) is 13%, while its correlation with disagreement about inflation (ψ^π) is -11%. We test Hypothesis 3 more formally with the following regressions:

$$\psi_t^s = -\underset{(-0.49)}{0.06}\,\psi_{\pi,t} + \underset{(1.22)}{0.04}\psi_{g,t} + \underset{(0.01)}{0.02}(\psi_{\pi,t})^2 + \underset{(1.90)}{0.27}(\psi_{g,t})^2 - \underset{(-2.44)}{1.46}\,(\psi_{\pi,t}\cdot\psi_{g,t}) + \varepsilon_t^s, \quad \overline{R}^2 = 0.14$$

$$\psi_t^s = -\underset{(1.17)}{0.02}E_t[\pi] - \underset{(-0.11)}{0.00}\,E_t[g] + \underset{(-0.65)}{0.00}\,E_t[LR] + \underset{(0.58)}{0.00}E_t[SR] + \varepsilon_t^s, \quad \overline{R}^2 = 0.07$$

where Newey-West t-statistics are reported in brackets below the point estimates, and a constant is included but not reported. The results suggest that our proxy for disagreement about signals is only weakly spanned by disagreement about the growth rate of inflation and the real economy, or consensus estimates, with $\overline{R}^2$'s of 14% and 7%, respectively. This is difficult to reconcile with single agent ambiguity models. Moreover, within the family of heterogeneous beliefs models, we learn that the cross-section of beliefs on asset prices is richer than what can be explained by disagreement on fundamentals alone, highlighting the potential importance of studying learning models with first order signals.

C. Disagreement and the Term Structure of Bond Volatility

Hypothesis 3 relates to a second set of empirical regularities that play an important role in the term structure literature: the dynamics of the conditional volatility of (changes in) bond yields. Substantial empirical evidence expose a tension between explanations of bond risk premia and their conditional second moments (Duffee (2002) and Dai and Singleton (2000)). Two set of regularities emerge from the empirical literature. First, the cross-section of the yield curve contains limited information to explain the second moment of bond yields (Collin-Dufresne and Goldstein (2002), Li and Zhao (2006)). For instance, Collin-Dufresne, Goldstein, and Jones (2009) show that while an $A_1(3)$ affine model can fit reasonably well the yield curve fails to reproduce the dynamics of yield volatilities and conclude about the importance to consider more general models. Second, the behavior of the long-end is very different from the short-end of the volatility curve. Dai and Singleton (2000) discuss the link between the properties of the mean-reversion coefficients to explain these cross-maturity patterns. Piazzesi (2005) argues that monetary factors play a key role for the specific behavior of short-end yield volatility. Kim and Singleton (2011) show that the behaviour of the volatility of long-end of the curve is very different and potentially driven by different factors. A similar observation is made in Cieslak and Povala (2011) in the context of a Wishart model. Computing the principal components for the covariance matrix of return volatilities of the vector $[y_t^{(2)} y_t^{(3)} y_t^{(4)} y_t^{(5)} y_t^{(10)}]$, we find two principal components explain virtually all realised variation: the first component (a level factor) accounts for $\sim 90\%$ and the second component (a spread factor loading positively on short term yield volatility and negatively on volatility at the long end, i.e., $\hat{\sigma}_{t,t+1}^{(2)} - \hat{\sigma}_{t,t+1}^{(10)}$) accounts for the remaining $\sim 10\%$. [38] Figures 10 and 11 plot the time series of these factors. Note, the large positive spike in PC2 in 2008 means that the term structure of volatility was downward sloping at the onset of the on-going financial crisis. Indeed, while the volatility of long dated bonds generally increases with respect to shorter bonds during expansions and declines at the onset of contractions, its high frequency dynamics are rich and difficult to explain.

[Insert figures 10 and 11 , here.]

We investigate Hypothesis 3 and study the role played by disagreement for second moments of bonds yields by running regressions for both the level, $\hat{\sigma}_{t,t+1}^{level}$, and slope of the

[38] We compute the integrated bond volatility between time t and time $t+1$ using intra-month daily data: $\hat{\sigma}_{t,t+1}^{(n)} = \frac{1}{k-1} \sum_{j=0}^{k-1} r^{(n)}(t + \frac{j}{k}, t + \frac{j+1}{k})$

term structure of volatility, $\hat{\sigma}^{slope}_{t,t+1}$:

$$\hat{\sigma}^{level}_{t,t+1} = const + \hat{\sigma}^{level}_{t-1,t} + \sum_{i=1}^{4} \beta_i \psi_{i,t}(\star) + \sum_{i=1}^{2} \gamma_i E_{i,t}(\star) + \sum_{i=1}^{3} \phi_i Macro_{i,t}(\star) + \varepsilon_{t+1}, \quad (27)$$

$$\hat{\sigma}^{slope}_{t,t+1} = const + \hat{\sigma}^{slope}_{t-1,t} + \sum_{i=1}^{4} \beta_i \psi_{i,t}(\star) + \sum_{i=1}^{2} \gamma_i E_{i,t}(\star) + \sum_{i=1}^{3} \phi_i Macro_{i,t}(\star) + \varepsilon_{t+1} \quad (28)$$

[Insert table V and VI here.]

Table V summarizes the results for the $\hat{\sigma}^{level}_{t,t+1}$. We find that differences in beliefs about inflation has strong explanatory power for the average level of yield volatility. While both ψ_g and ψ_s are insignificant, ψ_π is strongly statistically significant, with a t-statistics of 4.20 after controlling for lagged volatility $\hat{\sigma}^{level}_{t-1,t}$. The result survives even after controlling for volatility in fundamentals (inflation and gdp growth). The sign of the slope coefficient is positive, as expected for $\gamma > 1$: the larger the difference in beliefs the larger the volatility.

Table VI summarizes the results for $\hat{\sigma}^{slope}_{t,t+1}$. We find that differences in belief on inflation and signals are significant (column (ii-iv)) while ψ_g is not after controlling for lagged $\hat{\sigma}^{slope}_{t-1,t}$. The slope coefficient on ψ_π is positive while the sign of ψ_s is negative. This suggests that inflation uncertainty is mainly responsible for the average level of the term structure and the long-end of the volatility curve; the short end of the term structure is affected by signals possibly related to monetary policy, which is captured by ψ_s. The link is economically intuitive: ψ_s usually increases at the beginning of recessions and crisis periods when agents are uncertain and disagree about short-term prospects. The larger ψ_s the larger the short-end volatility, relative to the long-end. This is consistent with the general findings in Piazzesi (2005) but with the important difference that we find that it is disagreement plays a key role for understanding the volatility dynamics of long term bonds (Piazzesi mainly concentrates on short end regularities). Controlling for information in macro aggregates we find that neither the level of macro activity or the volatility of inflation and fundamentals are significant.

In general, the results are consistent with the channel discussed in heterogeneous agent literature. In addition, the fact that disagreement on signals directly affects the short-end of the curve relative to the long-end suggest that an important component of the mechanism takes place via first-order signals as opposed to an 'overconfidence' channel (i.e. second-order signals). Moreover, the results suggest that while ψ_π does not explain expected risk premia, it helps to explain average (changes in) bond yield volatility. This is interesting since it is consistent with the findings in previous literature that suggest the existence of unspanned stochastic volatility. We explore the spanning properties of disagreement in greater detail in the following section.

D. The Information 'In', 'Not In', and 'Above' the Term Structure

We now turn to study Hypothesis 4. An important stream of the literature studies the empirical properties of alternative term structure models from the perspective of their spanning characteristics. Consider, for instance, a N-factor affine term structure model admitting as a solution for bond prices $P(X_t, \tau) = \exp(a(\tau) + b(\tau)' X_t)$, where $a(\tau)$ is a scalar function and $b(\tau)$ is a N-valued function, expected excess returns to holding a T period bond are equal to $rx^{(T)}_{t,t+dt} = -b(T)'\Sigma\sqrt{S_t}\Lambda_t$, where $\Sigma\sqrt{S_t}$ is the factor loading of the affine process for the factors X_t and Λ_t is the price of risk. If the price of risk is 'completely affine', i.e. $\Lambda_t = \sqrt{S_t}\lambda_1$, then expected excess returns are proportional to factor variance, a restriction that the empirical literature finds hard to reconcile with the data. Dai and Singleton (2000) denote the admissible subfamily of completely affine models as $A_m(N)$ which are those with m state variables driving N conditional variances S_t. Specifically, elements of the state vector X_t that do not affect factor volatility (and hence bond volatility) cannot affect expected returns, thus factor variance and expected returns still go somewhat hand-in-hand. Motivated by this observation, Duffee (2002) extends the completely affine class to a set of 'essentially' affine models in which the risk factors in the economy enter the market price of risk directly and not just through their factor volatilities.[39] He suggests a specification in which $\Lambda_t = \sqrt{S_t}\lambda^0 + \sqrt{S_t^-}\lambda^X X_t$, where λ^X is an $n \times n$ matrix of constants and S^- is a diagonal matrix such that $[S_t^-]_{ii} = (\alpha_i + \beta' X_t)^{-1}$ if $\inf(\alpha_i + \beta_i' X_t) > 0$ and zero otherwise. The additional flexibility of non-zero entries in S^- translates into additional state dependent flexibility for the price of risk such that the tight link between risk compensation and factor variance is broken. A shared characteristic of the $A_m(N)$ subfamily of affine term structure models is that the cross-section of bond yields follows a Markov structure so that all current information regarding future interest rates (and thus expected returns) is summarised in the shape of the term structure today. Linear combinations of date t bond yields thus suffice to characterise date t risk factors through so-called yield curve inversion.[40] This property plays a key role in the interpretation of the results in Fama and Bliss (1987) and Cochrane and Piazzesi (2005).

While these spanning properties are a robust characteristics of the original class of this family of models, more recently an emerging literature find evidence of unspanned risk

[39] Cheridito, Filipovic, and Kimmel (2007) extend even further this class to yield models that are affine under both objective and risk-neutral probability measures without permitting arbitrage opportunities.

[40] Specifically, assume N bond yields are measured without error. Then, stacking these yields into the vector $y^N = A^N + B^N X_t$, we can solve for the risk factors through inversion as $X_t = (B^N)^{-1}\left(y^N - A^N\right)$ so long as the matrix B^N is non-singular.

factors. Ludvigson and Ng (2009) and Cooper and Priestley (2009) find that some crucial components for term structure models are unspanned by the space of yields. Moreover, Duffee (2011) and Joslin, Priebsch, and Singleton (2011) highlight the importance of studying hidden factor models, or models with unspanned macro risk, in which time variation in macro variables orthogonal to the cross-section of yields (and thus absent from date t prices) contains substantial forecasting power for future excess returns on bonds.

Hypothesis 4 relates to the discussion in this literature. Our model shows that when $\gamma > 1$ and differences in beliefs are stochastic the closed-form solution of bond prices is potentially challenging to invert even when the equivalent homogeneous economy would support an affine solution. Yields are function of a potentially large number of signals that extend the state-space. It may be difficult, therefore, to span the states vector from a finite cross-section of bond prices. Yet disagreement affects expected returns.

Thus, a natural set of questions to investigate relate to the extent to which the components of disagreement relevant for expected returns are revealed by the cross-section (as in Fama and Bliss (1987) and Cochrane and Piazzesi (2005)) versus the time-series of prices (Duffee (2011), Joslin, Priebsch, and Singleton (2011)). Proceeding in two steps, we first define the information set $G_1 \subseteq \sigma\left(PC(1-5)\right)$ and compute the unspanned component of ψ which is not explained by the cross-section of bond prices (the first five principal component, as used in Cochrane and Piazzesi (2005)): $\mathcal{UN}_\psi = \psi_t - P_j\left[\psi_t \middle| G_1\right]$.[41] Then, we proceed to test the content of unspanned, i.e. 'Not-In', disagreement as follows:

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)}\mathcal{UN}_\psi^\pi + \beta_2^{(n)}\mathcal{UN}_\psi^g + \beta_3^{(n)}\mathcal{UN}_\psi^s + \varepsilon_{t+12}^{(n)}. \tag{29}$$

Second, we define $G_2 \subseteq \left[G_1 \cup \sigma(y^{(n)})\right] \setminus G_1$ where $G_2 \sim \sigma(H_t)$ is the 'hidden' factor filtered from the time-series of prices from a 5-factor Gaussian term structure model, as in Duffee (2011).[42] Then, we estimate the component of disagreement unspanned neither by the cross-section of prices nor by information related to the hidden factor H_t. We define $\mathcal{AB}_\psi = \mathcal{UN}_\psi - P_j\left[\mathcal{UN}_\psi \middle| H_t\right]$ and test the predictive content of macroeconomic disagreement which is '*Above*' the yield curve as

$$rx_{t,t+12}^{(n)} = const + \beta_1^{(n)}\mathcal{AB}_\psi^\pi + \beta_2^{(n)}\mathcal{AB}_\psi^g + \beta_3^{(n)}\mathcal{AB}_\psi^s + \varepsilon_{t+12}^{(n)}. \tag{30}$$

Table VII reports the results of contemporaneous projections of disagreement on the first

[41]More specifically, G_1 is the sigma algebra (information set) generated by the eigenvalue decomposition of the unconditional covariance matrix of yields, or, alternatively, since there exists a linear mapping between yields and forward rates, G_1 is the space spanned by the return forecasting factor CP.

[42]We thank G. Duffee for providing the data on the hidden factor H_t.

5 principal components from an eigenvalue decomposition of the unconditional covariance matrix of yields (from the Fama and Bliss (1987) data set as in Cochrane and Piazzesi (2005)). The results show that a substantial proportion of the time-variation in disagreement about the real economy and inflation are indeed spanned by the yield curve. Specifically, ψ_g and ψ_π load significantly with $\overline{R}^2$'s of 27% and 37% respectively. The second, third and fifth principal component of bond yields are statistically significant to explain ψ_g. At the same time, the level factor is insignificant. Time variation in the *shape* of the yield curve is correlated with the heterogeneous formation of expectations. This is consistent with empirical findings in Cochrane and Piazzesi (2005) and the simulation results of Xiong and Yan (2010).

Table IX, Panel A and B, documents the impact on return predictability when one removes the component of ψ_t spanned by the yield curve. We find that more than half of the time-variation in expected returns attributable to disagreement is unspanned; moreover, the unspanned component is mostly due to signal disagreement. Expected excess return regressions on the unspanned disagreement produce R^2 all above 27%.

Is Duffee's hidden latent factor picking up the unspanned component of disagreement? To investigate this link, we run regressions of unspanned disagreement on the H_t factor (Duffee (2011)):

$$H_t = const + \sum_{i=1}^{4} \beta_i \, \mathcal{UN}_{\psi}^{i} + \varepsilon_t^i, \tag{31}$$

Table VIII summarizes the results. We find that that $\mathcal{UN}_{\psi}^{s}$ is statistically significant, with a t-statistics of 2.24. At the same time, however, the $\overline{R}^2$ does not exceed 3%. This implies that a hidden latent factor estimated from a multi-factor Gaussian specification appears unrelated to the information captured by disagreement.

Finally, we investigate equation (30). Table IX documents the predictive power of the 'above' component for expected bond returns. These factors capture information that is orthogonal to both the cross-section and the time series of bond yields (via the hidden factor). We find that $\mathcal{AB}_{\psi}$ contain substantial information for future expected bond returns, with $\overline{R}^2$ ranging from 20% to 22% for 5 year and 3 year bonds, respectively. Interestingly, however, it is the $\mathcal{AB}_{\psi}^{s}$ factor that accounts for most of this time variation, with t-statistics significant at the 1% level. We also find that this component is economically important for bond risk premia: a 1-standard deviation shock to $\mathcal{AB}_{\psi}^{s}$ increases expected excess returns on 5-year bonds by 2.38%. The results suggest that disagreement on signals is mostly 'above' the information contained in the term structure, while disagreement on the real economy and inflation is spanned.

[Insert table VII, VIII , and IX here.]

Figure 12 plots a time-series of the $\overline{R}^2$'s obtained from forecasting regressions computed with a rolling 10-year window using either CP_t or Ψ_t on the right hand side. While the degree of predictable variation due to disagreement remains fairly stable over time, CP_t is cut dramatically during periods of financial crisis. This statement is made clear by noticing the large correlation between the drop in $\overline{R}^2$ from CP_t regressions and the contemporaneous drop in the Fed Funds rate. This occurs both in the aftermath of the dot-com bubble and, again, during the recent 2007-2009 financial crisis. The apparent failure of the CP factor is interesting since its success appears to be state dependent. On March 18th, 2009 the U.S. Federal Reserve Bank announced a plan to buy almost 1.2 trillion worth of Treasury bonds to help boost lending and promote economic recovery in response to 2007 financial slowdown an imminent credit crisis. The size of the intervention was targeted to make the U.S. Fed the de-facto marginal investor in long term debt that may also have affected the spanning properties of date-t bond prices. Although this is one possible interpretation, the observed state-dependent nature of CP_t's forecasting power may be a consequence of forward-looking bias, in particular, the in-sample fit is computed using linear combinations of prices on the right to forecast prices on the left. However, it is interesting to observe that the (out-of-sample) predictive power of disagreement is not affected in these periods. This suggests that the distribution of beliefs contains unspanned economic information, which is above and beyond what contained in both consensus beliefs and the cross-section of prices. Indeed the spanning properties of prices may be reduced in period of elevated uncertainty.

E. Disagreement and Trade

In this section we examine Hypothesis 5 - the relationship between investor heterogeneity and trade by running regressions of trading volume of short term Treasury Bills, standardized by long debt volume, between primary dealers and customers. Heterogeneous beliefs models imply a positive relationship between trading volume and disagreement (see Varian (1989), Buraschi and Jiltsov (2006), and Banerjee and Kremer (2010)). This implication distinguishes this class of models from economies with ambiguity aversion. Illeditsch (2011), de Castro and Chateauneuf (2011), and Chen, Ju, and Miao (2011) show that knightian uncertainty and ambiguity generates portfolio inertia, implying that agents rebalance portfolio allocations infrequently. While ambiguity and differences in beliefs models can generate larger expected risk premia an important distinguishing feature is their implications for trading volumes.

We consider trading volume of bonds with a maturity matching the horizon of our disagreement proxies, which is one year. Since trading volume contains a significant upward trend, we standardize this series by the trading volume in long dated bonds (i.e. between 6 and 11 years).[43] Then, we run the following regressions:

$$Vol_{t,t+1} = const + \sum_{i=1}^{3} \beta_i \psi_t(\star) + \gamma Vol_{t,t-1} + \varepsilon_{t,t+1}. \tag{32}$$

Table X reports the results. Columns (i) and (ii) report baseline univariate specifications including disagreement about inflation and the real economy on the right hand side. The results show that disagreement about inflation and the real economy load positively with high statistical significance (8.88 and 3.62, respectively) with R^2's of 50% and 47%, respectively. Moving to column (iii) we run a multivariate regression that show both inflation and real disagreement are jointly significant: both factors are significant at the 1% level with an adjusted $\overline{R}^2$ of 60%. Column (iv) introduces disagreement about signals which is negative, albeit insignificant, with a relatively low R^2 of 4%. From columns (i) - (iv) we learn that while disagreement about signals is unimportant, disagreement about inflation and real consumption growth is a key determinants of trade in U.S bonds, loading positively on the trade spread between short and long ends of the yield curve. Intuitively, and as expected, larger disagreement about 1-year growth rates drives up trade at the short end. This is consistent with agents' desire to equate expected ex-ante marginal utilities. Including all disagreement measures together (column (v)) we confirm that disagreement about signals appears unimportant for the bond trading volumes. Finally, we check the robustness of the results by including lagged trading volumes. Column (vi) reports the results showing that disagreement about inflation and the real economy retain statistical significance at the 1% and 5% levels, respectively. In summary, the results on disagreement and trade suggest an economically important and statistically robust positive correlation between investor heterogeneity and trade.

[Insert table X]

IV. Learning about Ambiguity and Differences in Beliefs

The theoretical origins of disagreement and uncertainty are distinct. The last refers to unknown unknowns and studies the role of the lack of knowledge regarding the reference model on the equilibrium demand *at the individual level*. The first focuses, instead, on the pricing implications of state-contingent trading *among disagreeing agents*. Empirically, while

[43] We define $Vol_{t,t+1} = log(\frac{\$Trans_{t,t+1}(Bills)}{\$Trans_{t,t+1}(Notes)})$, the log ratio of the total dollar value of monthly transactions of Treasury Bills versus long term coupon securities.

the last relies on proxies of dispersion of *individual* priors (or empirical measures of entropy) at the level of the individual agent, the first relies on the difference in the mean forecasts of *different agents*. While these concept are different, it is reasonable to argue, however, that they are conditionally correlated. In a world of certainty, after all, agents would not disagree. For this reason differences in beliefs have been used to proxy for ambiguity. An important contribution to this literature is Ulrich (2010). He considers a single agent economy in which the investor has multiple priors about the inflation process and is ambiguity averse. The agent is assumed to observe the expected change in relative entropy between the worst-case and the approximate model for trend inflation. The observed set of multiple forecasts on trend inflation exposes the investor to inflation ambiguity. In the context of a min-max recursive multiple-prior solution, Ulrich (2010) shows that risk premia can be generated if changes in aggregate ambiguity are correlated with changes in the real value of a nominal bond. He uses the quarterly Survey of Professional Forecasters to obtain a measure of variance across individuals for inflation expectations which is then used to proxy for ambiguity at the individual level and fit the yield curve. He finds that the inflation ambiguity premium is upward sloping and peaked during the mid 1970s and early 1980s.

The empirical results regarding hypothesis 1 support both models as disagreement is positively correlated with conditional expected returns. Our empirical results on hypothesis 3 are also supportive of both models. While ambiguity in itself does not generate endogenous stochastic volatility, Ulrich (2010) show that if the entropy bound is stochastic, then yield volatilities are affected by the properties of this stochastic process, which can be interpreted as an exogenous preference shock. Ambiguity and differences in beliefs economies have different implications in terms of hypothesis 2, 4 and 5. While ambiguity cannot explain disagreement on expected future bond prices (unless one assumes heterogeneity in ambiguity), we find substantial evidence of time varying disagreement on future bond yields. Moreover, the empirical results support a positive link between differences in beliefs and bond trading volume, adding to recent empirical evidence in the equity space that show a positive link between dispersion in expectations and trade (Li and Li (2011)). Either ambiguity is less relevant for modelling bond markets or differences in beliefs may not be a good proxy for ambiguity. We prefer the latter interpretation as D'Amico and Orphanides (2008), using individual level forecasts, show that the link between differences in beliefs and ambiguity is not strong.

V. Concluding Remarks

The empirical literature highlight the difficulty that traditional homogeneous agents model have to explain four properties of bond returns. We investigate both theoretically and empirically the link between the distribution of expectations in an economy with disagreement and the dynamics of bond markets. Theoretically, we provide a framework to help understand the potential role played by disagreement. We allow for general CRRA preferences and stochastic disagreement and derive closed-form solutions for bond prices. This framework nests features of the economies of Xiong and Yan (2010) and Jouini, Marin, and Napp (2010), which can be (somewhat) interpreted as special cases for $\gamma = 1$ and for constant disagreement, respectively.

Empirically, we build a unique dataset on disagreement from the historical paper archives of BlueChip and test five hypotheses. We find that differences in belief on both fundamentals and signals is a priced risk factor, a result which is absent in representative agent models, or economies with constant beliefs/disagreement. This result is in contrast to models of differences in belief that do not explicitly consider the learning process; for example, in the very general belief and preference heterogeneity framework of Bhamra and Uppal (2011), or alternatively, when agents learn through correlated Brownian motions, as in Dumas, Kurshev, and Uppal (2009). At the same time, the dynamics of belief dispersion are tightly linked to both the level and slope of the term structure of bond volatility. Moreover, in both cases, the results suggest that disagreement about signals is as important as disagreement about fundamentals.

We also find that disagreement about signals are largely unspanned by the cross-section of bond prices. This is consistent with models in which the mapping between bond prices and state variables is not invertible, as is the case (in practice) with the linear-quadratic bond pricing solution derived in the theory section.

The findings in this paper are important since they show that information contained in the belief structure of the economy, which is not contained in consensus expectations or in macro aggregates is relevant for explaining time variation in the price of risk, second moments returns, and the dynamics of trading activity. Further, this helps to explain why single agent homogeneous economies (and their reduced form counter parts) find it difficult to fully explain some empirical properties of the term structure of interest rates.

VI. Figures

SEPTEMBER 10, 1998 ■ BLUE CHIP ECONOMIC INDICATORS ■ 3

1999 Real GDP Consensus Forecast Slips To 2.2%

SEPTEMBER 1998 Forecast For 1999	Percent Change 1999 From 1998 (Year-Over-Year)									Average For 1999			Total Units-1999		-1999-
SOURCE:	1 Real GDP (Chained) ('92 $)	2 GDP Price Index	3 Nominal GDP (Cur. $)	4 Consumer Price Index	5 Indust. Prod. (Total)	6 Dis. Pers. Income ('92 $)	7 Personal Cons. Exp ('92 $)	8 Non-Res. Fix. Inv. ('92 $)	9 Corp. Profits (Cur. $)	10 Treas. Bills 3-mo.	11 Treas. Notes 10-Year	12 Unempl. Rate (Civ.)	13 Housing Starts (Mil.)	14 Auto/Truck Sales (Mil.)	15 Net Exports ('92 $)
Bear Stearns & Co., Inc.	3.4 H	1.3	4.8	2.3	5.0 H	3.5	3.0	11.7 H	7.2	5.0	5.4	4.1 L	1.52	16.1	-314.0
First Chicago NBD Corporation	3.2	2.1	5.3	2.5	4.2	3.4	3.4	11.4	8.9 H	5.2	6.4	4.1 L	1.56	16.2	-349.1
First Union Corp	3.1	2.0	5.1	2.5	3.0	3.0	3.3	5.7	7.7	5.8 H	6.2	4.6	1.47	15.2	-234.0
Michael Evans, Kellogg School	3.1	1.6	4.7	2.5	4.0	4.0	3.9 H	9.5	4.3	5.1	5.7	4.2	1.59	16.4 H	-344.5
Eaton Corporation	3.1	1.0	4.2	2.2	2.9	2.9	2.9	8.0	5.7	5.2	5.4	4.7	1.54	14.6	-242.2
U.S. Chamber of Commerce	2.8	2.4	5.2	2.6	2.1	2.3	3.2	9.4	na	5.1	5.6	5.0	1.48	na	-310.2
National City Bank of Cleveland	2.8	2.0	4.9	2.3	3.6	3.4	3.4	5.0	4.5	5.3	5.6	4.5	1.50	15.5	-246.0
Goldman Sachs & Co.	2.8	1.3	4.2	2.4	3.6	2.9	2.7	6.8	2.9	5.5	5.9	4.1	1.48	15.3	-282.9
LaSalle National Bank	2.7	2.0	4.8	2.1	2.5	2.5	3.3	5.3	5.0	5.3	5.9	4.9	1.41	14.6	-304.0
Brown Brothers Harriman	2.7	1.8	4.5	2.4	3.1	2.9	3.3	6.9	6.1	na	na	4.7	1.49	15.2	-307.6
WEFA Group	2.6	2.7	5.4 H	2.8	3.2	3.5	2.5	9.5	3.9	5.5	6.0	4.8	1.44	14.8	-254.0
Moody's Investors Service	2.6	2.1	4.7	2.7	2.6	2.8	2.7	3.8	6.6	na	na	4.3	1.53	15.3	-252.0
Eggert Economic Enterprises, Inc.	2.6	2.0	4.6	2.0	2.7	3.3	3.5	6.5	3.5	4.9	5.3	4.7	1.44	15.2	-267.0
Bank of America*	2.5	2.2	4.7	2.5	3.0	2.8	3.3	9.9	0.9	5.0	5.6	4.4	1.39	na	-321.0
NationsBanc Montgomery Securities	2.5	2.0	4.2	2.2	2.4	2.4	2.5	4.5	3.5	4.6	5.0	4.7	1.45	14.7	-280.0
Wells Capital Management	2.5	1.6	4.2	2.5	2.6	2.7	3.0	7.1	4.1	5.0	5.7	4.6	1.53	14.8	-291.0
Cahners Economics	2.4	2.3	4.7	2.7	2.6	2.6	2.7	6.1	3.2	5.1	5.9	5.2	1.46	15.0	-225.0
Fannie Mae	2.4	2.0	4.4	2.3	2.7	2.8	3.2	8.3	2.0	5.0	5.4	4.6	1.41	na	-305.8
Georgia State University*	2.4	1.4	3.9	2.5	2.1	2.7	3.2	4.6	2.6	5.1	5.6	4.3	1.55	15.0	-334.5
Northern Trust Company	2.3	1.9	4.2	2.1	2.3	2.3	2.6	4.1	1.3	4.9	5.6	4.6	1.59	14.6	-257.4
SOM Economics, Inc.	2.3	1.3	3.7	2.0	2.6	2.7	3.2	7.8	3.3	4.9	5.4	4.2	1.51	15.2	-312.0
Chicago Capital Inc.	2.2	2.3	4.5	2.7	2.9	1.8	2.2	7.0	5.4	5.1	5.5	4.7	1.50	15.6	na
Macroeconomic Advisers, LLC**	2.2	2.1	4.3	2.4	2.5	2.7	3.1	8.0	-0.5	5.0	5.6	4.7	1.43	15.5	-301.1
Mortgage Bankers Assn. of Amer.*	2.2	2.0	4.3	2.4	2.4	2.6	3.1	7.8	-0.6	5.0	5.5	4.7	1.45	15.5	-306.0
Wayne Hummer Investments LLC*	2.2	2.0	4.2	2.4	2.4	2.4	2.3	4.5	-2.1	4.6	5.2	4.9	1.42	14.9	-287.0
Inforum - Univ. of Maryland	2.2	1.7	3.9	2.2	2.4	2.3	2.5	8.3	-2.8	4.7	5.4	4.8	1.52	15.3	-252.5
Prudential Insurance Co.	2.2	1.5	3.7	2.2	3.1	2.4	2.5	9.0	4.5	4.7	5.7	5.3 H	1.46	na	-300.0
Dun & Bradstreet	2.1	2.5	4.6	2.4	2.6	2.8	3.5	8.1	-3.0	5.6	6.1	4.4	1.33 L	15.0	-311.0
General Motors Corporation	2.1	2.3	4.5	2.4	2.8	2.6	2.6	6.0	2.0	5.2	5.9	4.9	1.45	na	-225.0
Econoclast	2.1	2.3	4.4	2.8	2.2	3.0	3.0	4.9	3.0	5.3	6.0	4.8	1.42	14.8	-265.0
Prudential Securities	2.1	1.9	4.0	2.4	2.8	2.6	3.0	5.0	5.9	5.2	na	4.8	1.48	15.1	-261.3
DuPont**	2.0	1.9	3.9	2.0	2.4	2.7	2.7	3.5	0.0	4.9	5.4	5.0	1.50	15.3	-290.0
Weyerhaeuser Company	2.0	1.6	3.6	2.3	2.3	2.0	3.5	9.0	2.0	5.1	6.2	4.5	1.59	15.0	-404.0 L
U.S. Trust Co.	2.0	1.5	3.6	2.2	2.3	3.0	2.9	4.2	-4.0	4.7	5.4	4.7	1.47	15.1	-278.0
Comerica	2.0	1.5	3.5	2.0	2.5	3.5	2.5	4.0	5.0	5.8 H	6.5 H	4.4	1.40	14.5	-190.0 H
Kellner Economic Advisers	2.0	1.0	3.0	1.2	2.1	2.5	2.6	7.0	2.0	4.9	5.2	4.7	1.45	15.0	-250.0
UCLA Business Forecasting Proj.*	1.9	3.3 H	5.2	2.4	0.3	2.3	2.2	6.9	-4.8	4.8	5.0	5.3 H	1.55	15.1	-327.8
Chrysler Corporation	1.9	2.3	4.2	2.5	2.1	2.3	2.7	6.0	4.1	4.8	5.6	5.2	1.42	na	-329.1
National Assn. of Home Builders	1.9	2.1	4.0	2.4	2.3	2.5	2.2	7.9	1.3	4.5	5.2	4.7	1.48	15.0	-302.7
PNC Bank	1.9	1.7	3.8	2.2	1.9	1.9	na	na	5.7	4.8	5.2	4.7	1.47	14.7	na
Motorola, Inc.	1.7	2.0	3.7	2.6	1.1	2.1	2.3	4.9	na	4.6	5.5	5.1	1.43	15.2	-257.6
Standard & Poors Corp.	1.7	1.8	3.6	2.5	2.5	2.6	2.6	4.5	-2.6	4.3	5.0	4.7	1.52	15.7	-307.7
Deutsche Bank Securities	1.7	1.0	2.9	1.0 L	1.0	1.9	2.2	6.0	-3.0	4.1 L	4.4 L	4.5	1.50	14.1 L	-302.0
Ford Motor Company*	1.6	2.7	4.3	2.9 H	1.3	1.6 L	2.0	7.6	na	na	na	5.2	na	na	-299.0
Turning Points (Micrometrics)	1.6	1.5	3.1	2.0	2.5	4.1 H	3.2	5.4	5.3	5.3	5.6	4.8	1.63	16.1	-391.0
Fleet Financial Group	1.5	1.5	3.1	2.3	1.0	2.0	2.8	6.7	2.0	4.2	4.9	4.9	1.44	14.9	-337.0
J P Morgan	1.4	2.0	3.3	2.4	0.9	2.2	2.5	3.1 L	5.1	4.9	5.3	4.6	1.45	14.8	-324.0
Reeder Associates (Charles)	1.1	0.9 L	2.1 L	1.4	-0.5 L	3.2	2.7	6.2	-5.7 L	4.5	6.3	4.9	1.70 H	14.5	-395.0
Chase Securities, Inc.	0.9 L	1.5	2.4	1.8	2.7	2.3	1.8 L	5.3	0.0	4.3	5.2	5.1	1.40	14.8	-300.0

Figure 1. Survey Extract, Long Term (following year) Forecasts:
Each month Blue Chip carries out surveys of professional economists from leading financial institutions and service companies. Each economist is asked to forecast a set of economic fundamentals and interest rates spanning columns 1 - 15 in the figure above. Short term forecasts are made for the remaining period of the calendar year. Long Term (next year) forecasts are made for the average quantity of interest for the following calendar year.

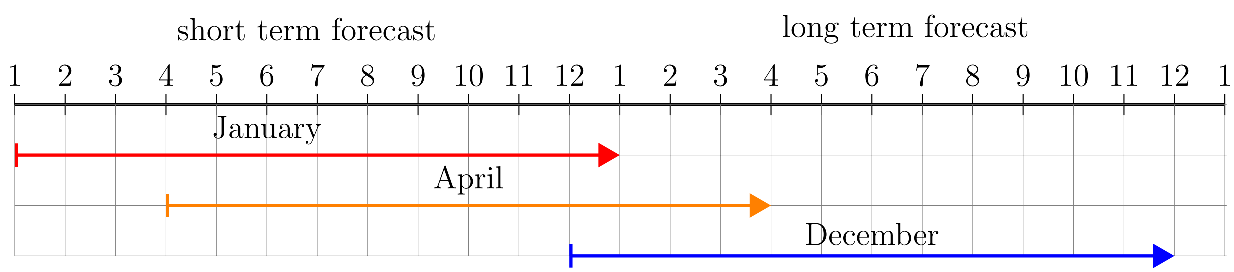
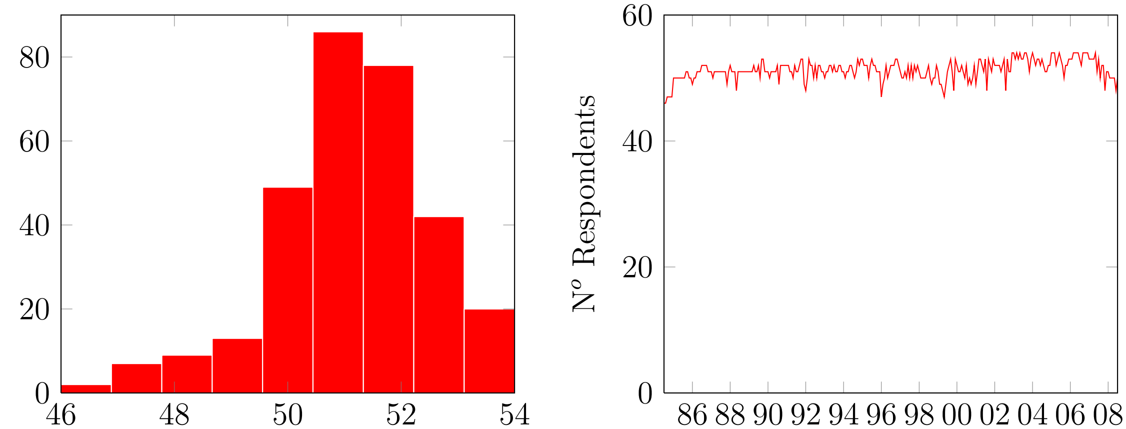


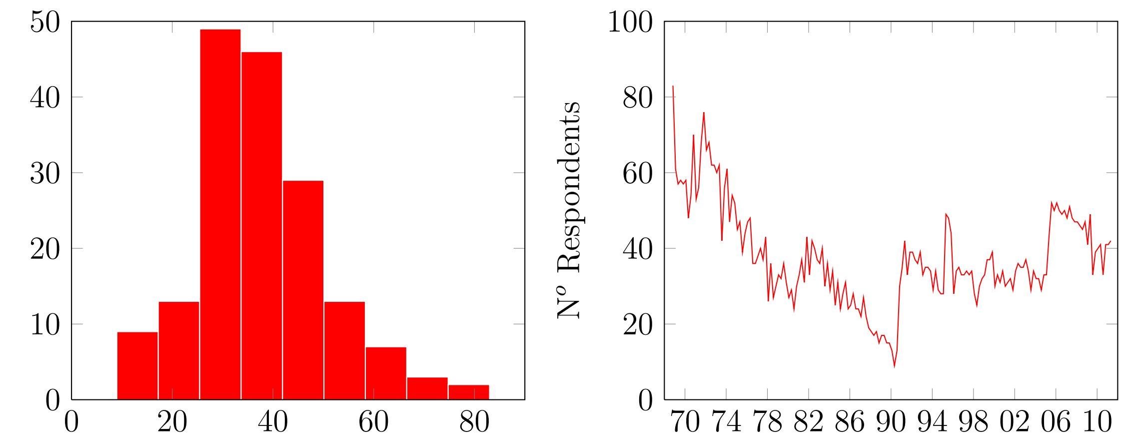
Figure 2. Constant Maturity Disagreement:
In order to construct a constant 1-year maturity disagreement measure for each forecaster, we take a weighted average of the short and long term forecasts from the BlueChip survey. Let j be the month of the year, so that $j = 1$ for January and $j = 1, 2..12$. A constant maturity disagreement is formed taking as weight $(1 - \frac{j}{12})$, for the short term disagreement (the remaining forecast for the same year), and $\frac{j}{12}$, for the long-term disagreement (the forecast for the following year).



(a) Distribution of respondent numbers

(b) Time Series of number of respondents

Figure 3. Short Term Forecast Respondent Numbers:
Panel (a): histogram displaying the distribution of the number of respondents. Panel (b): time series of number of respondents.



(a) Distribution of respondent numbers

(b) Time series of number of respondents

Figure 4. Forecast Respondent Numbers: Survey of Professional Forecasters:
Panel (a): histogram displaying the distribution of the number of respondents. Panel (b): time series of number of respondents.

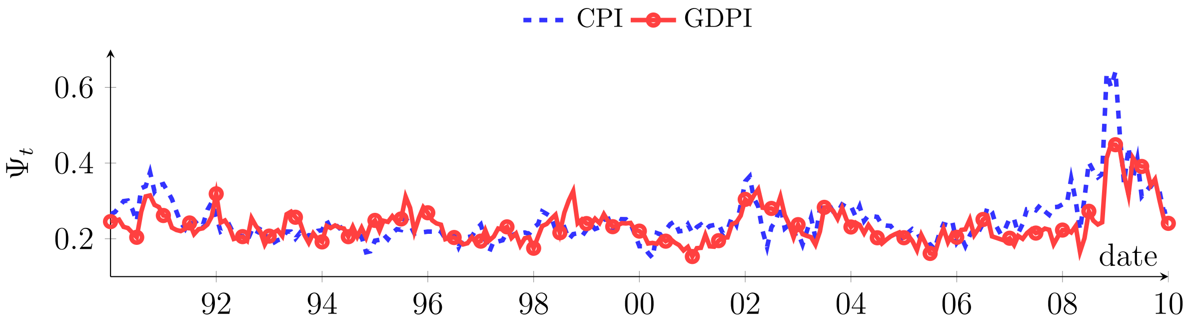


Figure 5. Disagreement on Inflation:
Time series differences in belief about the growth rate of cpi and the gdp deflator, as discussed in section II A.

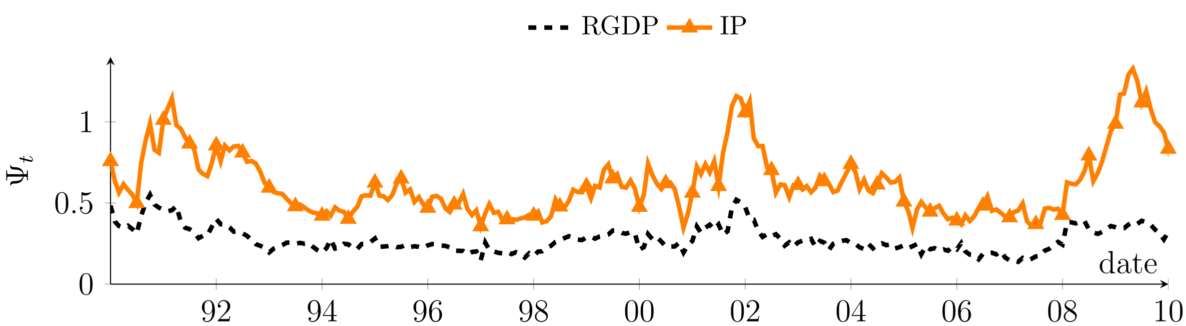


Figure 6. Disagreement on Real Growth:
Time series of differences in belief about the growth rate of industrial production and real gdp, as discussed in section II A.

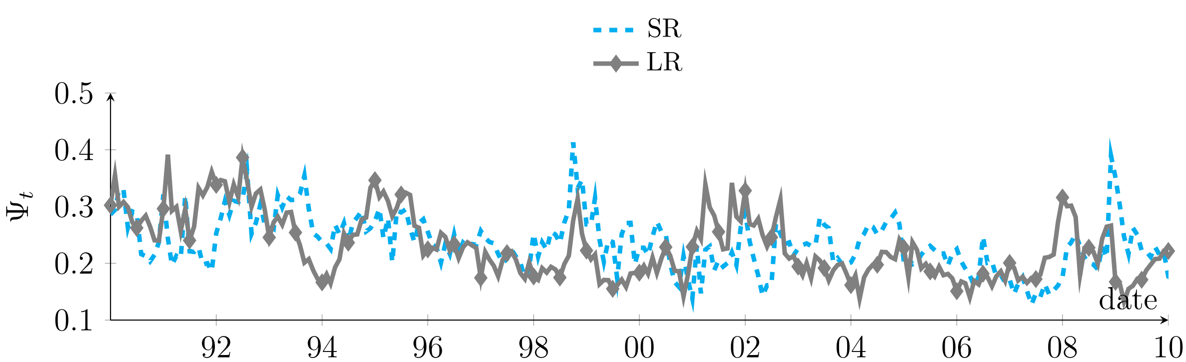


Figure 7. Disagreement on Interest Rates:
Time series of differences in belief about the level of short term and long term interest rates, as discussed in section II A.

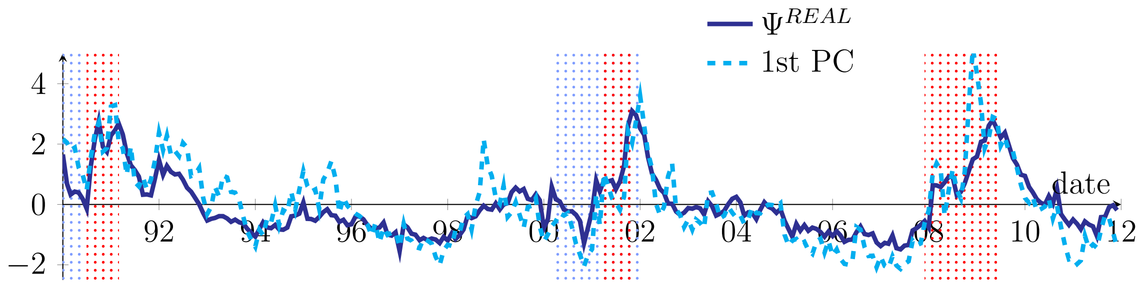


Figure 8. Real Factor versus 1st PC:
Time Series plot of the real disagreement factor, ψ_t^g against the 1st principal component of the vector $\Psi_t = [\psi_t^{REAL}, \psi_t^{INF}, \psi_t^{LR}, \psi_t^{SR}]$. Red dotted areas are NBER recession dates, while the blue dotted areas of crisis periods.

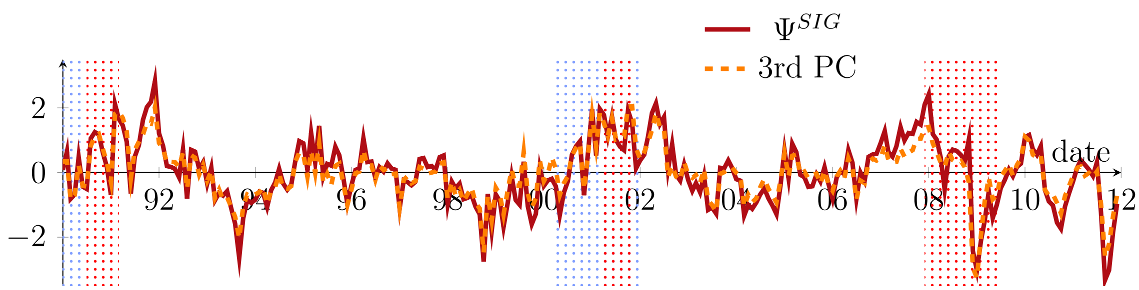


Figure 9. Signal Factor versus 3rd PC:
Time Series plot of the signal factor, ψ_t^S, against the 3rd principal component of the vector $\Psi_t = [\psi_t^{REAL}, \psi_t^{INF}, \psi_t^{LR}, \psi_t^{SR}]$. Red dotted areas are NBER recession dates, while the blue dotted areas of crisis periods.

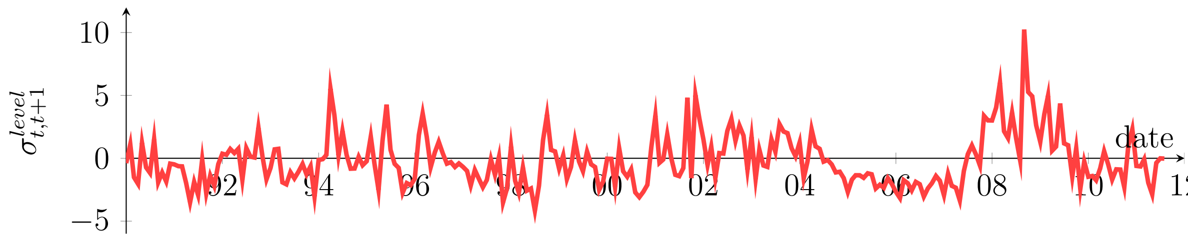


Figure 10. Volatility principal Component 1: Level
Time series of first principal component from term structure of volatility for the state vector : $[y_t^{(2)} y_t^{(3)} y_t^{(4)} y_t^{(5)} y_t^{(10)}]$.

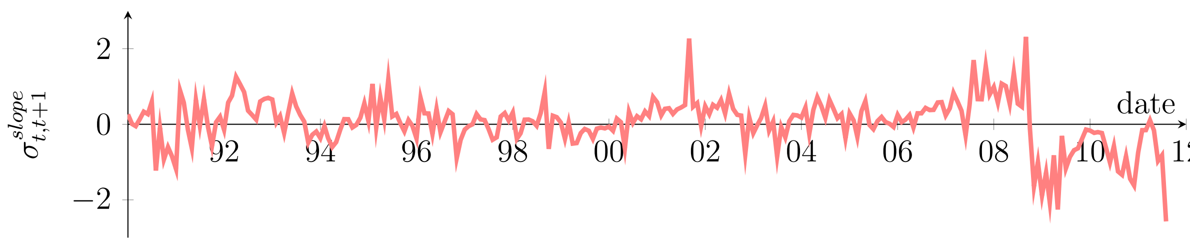


Figure 11. Volatility Principal Component 2: Slope
Time series of second principal component from the term structure of volatility for the state vector : $[y_t^{(2)} y_t^{(3)} y_t^{(4)} y_t^{(5)} y_t^{(10)}]$.

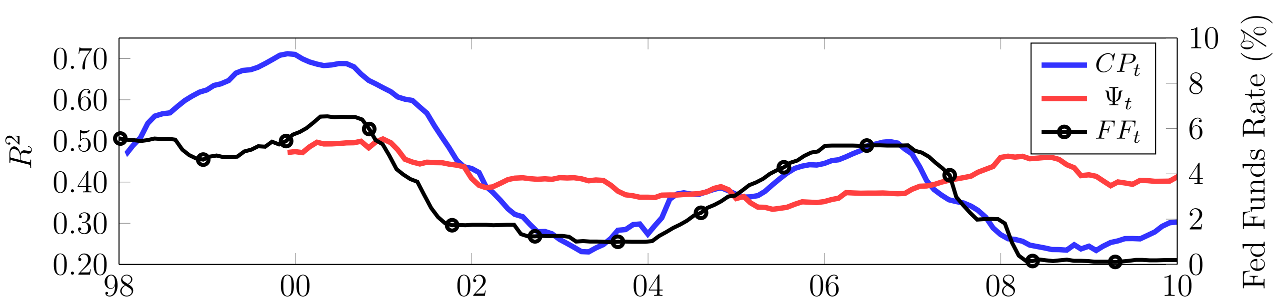


Figure 12. Rolling R^2's
Left Vertical Axis: Rolling R^2's from return predictability regressions for the forward-rate factor from Cochrane and Piazzesi (2005) and $\Psi_t = [\psi_t^{\pi}, \quad , \psi_t^{g}, \quad , \psi_t^{s}]$. R^2 statistics are computed using a 10-year rolling window of 1-year excess returns from the portfolio: $\frac{1}{4}\sum_{n=2}^{5} rx_{t,t+12}^{(n)}$. Right Vertical Axis: Effective Federal Funds rate (FF_t), H15 release.

VII. Tables

Table I. Descriptive Statistics: Disagreement

This Table reports the summary statistics for mean-absolute-deviation in economist forecasts for real, nominal, and monetary components. Sample Period: 1990.1 - 2011.12

	ψ^{RGDP}	ψ^{IP}	ψ^{CPI}	ψ^{GDPI}	ψ^{LR}	ψ^{SR}	ψ^{g}	ψ^{π}	ψ^{s}
PANEL A:									
mean	0.28	0.62	0.25	0.24	0.24	0.23	0.00	0.00	-0.06
Sdev	0.08	0.20	0.07	0.05	0.05	0.06	0.17	0.05	0.04
Skew	0.99	1.24	2.79	1.82	0.48	0.56	1.01	2.60	0.00
Kurt	3.95	4.26	15.24	7.55	3.56	2.60	3.50	12.93	3.26
AC(1)	0.90	0.93	0.85	0.83	0.70	0.85	0.94	0.87	0.71
PANEL B:									
ψ^{RGDP}	1.00								
ψ^{IP}	0.79	1.00							
ψ^{CPI}	0.47	0.54	1.00						
ψ^{GDPI}	0.44	0.60	0.70	1.00					
ψ^{SR}	0.18	0.14	0.02	0.23	1.00				
ψ^{LR}	0.42	0.28	-0.02	0.06	0.37	1.00			
ψ^{g}	0.93	0.96	0.54	0.56	0.17	0.36	1.00		
ψ^{π}	0.49	0.61	0.93	0.91	0.13	0.02	0.59	1.00	
ψ^{S}	0.17	0.09	-0.04	-0.17	-0.66	0.46	0.13	-0.11	1.00

Table II. Return Predictability Regressions:
This table reports estimates from OLS regressions of annual ($t \to t+12$) excess returns of 5-year zero-coupon bonds on disagreement factors(Ψ^i), consensus expectations ($E[\star]$), and fundamentals:

$$rx^{(5)}_{t,t+12} = const + \sum_{i=1}^{4} \beta_i \psi_t(\star) + \sum_{i=1}^{4} \gamma_i E_t(\star) + \sum_{i=1}^{11} \phi_i Macro_t(\star) + \delta Slope_t + \varepsilon_{t,t+12},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of disagreement variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

	$rx^{(5)}$									
regressor	*(i)*	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(vi)*	*(vii)*	*(viii)*	*(ix)*	*(x)*
ψ^{π}	0.48			-0.62	0.03	0.07	-0.37		-1.64	
	(0.99)			(-1.48)	(0.07)	(0.15)	(-0.95)		(-2.08)	
ψ^{g}		1.48		1.85	1.17	0.86	1.69		1.86	
		(2.86)		(3.20)	(2.53)	(1.71)	(1.97)		(2.42)	
ψ^{s}			2.38		2.22	2.27		2.41		2.35
			(5.32)		(4.78)	(4.83)		(4.91)		(5.85)
$Slope_t$						0.61				
						(1.15)				
$E(\pi)$							0.58			
							1.18			
$E(g)$							-0.01			
							(-0.02)			
$E(LR)$								1.07		
								(1.34)		
$E(SR)$								-1.01		
								(-1.26)		
$F_t^{1\to 8}$									✓	✓
σ_t^{π}									0.00	-0.17
									(0.01)	(-0.47)
σ_t^{g}									-0.26	-0.13
									(-0.70)	(-0.42)
$\rho_t^{\pi,g}$									0.45	0.29
									(1.31)	1.37)
$\overline{R}^2$	0.01	0.11	0.29	0.12	0.35	0.36	0.13	0.31	0.24	0.41
χ^2				10.30	38.51	43.60	4.89	33.79	10.23	34.21
p-value				0.01	0.00	0.00	0.09	0.00	0.01	0.00

Table III. Return Predictability Regressions:
This table reports estimates from OLS regressions of annual ($t \to t+12$) excess returns of 2, 3 4, 5, and 10-year zero-coupon bonds on disagreement factors(Ψ^i). Bond maturities 2-5 are from the Fama-Bliss dataset while the 10-year bond is taken from Gürkaynak, Sack, and Wright (2006):

$$rx_{t,t+12}^{(n)} = const^n + \sum_{i=1}^{4} \beta_i^n \psi_t(\star) + \varepsilon_{t,t+12}^n,$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of disagreement variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. Sample Period: 1990.1 - 2011.12

Maturity	$const$	ψ^π	ψ^g	ψ^s	$\overline{R}^2$	χ^2
$rx^{(2)}$	0.95	-0.07	0.46	0.71	0.42	87.57
	(6.06)	(-0.46)	(3.04)	(6.72)		0.00
$rx^{(3)}$	1.80	-0.04	0.80	1.36	0.41	70.32
	(6.31)	(-0.15)	(3.04)	(5.89)		0.00
$rx^{(4)}$	2.53	-0.09	1.08	1.88	0.39	54.44
	(6.46)	(-0.24)	(2.86)	(5.49)		0.00
$rx^{(5)}$	2.94	0.03	1.17	2.22	0.35	38.51
	(5.97)	(0.07)	(2.53)	(4.78)		0.00
$rx^{(10)}$	4.98	-0.05	1.59	3.01	0.21	15.93
	(5.35)	(-0.06)	(1.98)	(3.40)		0.00

Table IV. Return Predictability Regressions:
This table reports estimates from OLS regressions of annual ($t \to t+12$) excess returns of 2 to 5-year zero-coupon bonds on the principal components of disagreement factors.

$$rx_{t,t+12}^{(5)} = const + \sum_{i=1}^{4} \beta_i \psi_t^{PCs} + \varepsilon_{t,t+12},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of disagreement variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

Maturity	*const*	ψ^{PC^1}	ψ^{PC^2}	ψ^{PC^3}	ψ^{PC^4}	$\overline{R}^2$
rx(2)	0.90	0.28	−0.01	0.99	−0.12	0.42
	(5.64)	(2.72)	(−0.11)	(8.99)	(−0.60)	
rx(3)	1.72	0.48	−0.14	1.86	−0.23	0.40
	(5.91)	(2.65)	(−0.74)	(7.92)	(−0.62)	
rx(4)	2.42	0.61	−0.21	2.59	−0.28	0.39
	(6.13)	(2.55)	(−0.70)	(7.150	(−0.56)	
rx(5)	2.83	0.69	−0.36	2.99	−0.41	0.35
	(5.74)	(2.39)	(−0.89)	(5.98)	(−0.65)	

Table V. Bond Volatility Regressions:
This table reports estimates from forecasts of the monthly ($t \rightarrow t+1$) 1st prinicipal component of realised return volatility on disagreement factors plus controls:

$$\sigma^{level}_{t,t+1} = const + \alpha\sigma^{level}_{t-1,t} + \sum_{i=1}^{4}\beta_i\psi_t(\star) + \sum_{i=1}^{2}\gamma_i E_t(\star) + \sum_{i=1}^{3}\phi_i Macro_t(\star)$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of disagreement variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

regressor	(i)	(ii)	(iii)	(iv)	(v)	(vi)
ψ^{π}	0.29			0.25	0.01	0.31
	(4.20)			(2.45)	(0.08)	(1.96)
ψ^{g}		0.24		0.09	-0.04	0.04
		(1.64)		(0.55)	(-0.29)	(0.24)
ψ^{s}			0.06	0.07	0.11	0.10
			(0.50)	(0.59)	(1.07)	(0.83)
$E(\pi)$					0.01	
					(0.02)	
$E(g)$					-0.33	
					(-2.25)	
$E(LR)$					0.02	
					(0.07)	
$E(SR)$					-0.34	
					(-2.19)	
$F_t^{1\rightarrow 8}$						
σ_t^{π}						-0.15
						(-1.30)
σ_t^{g}						0.10
						(0.88)
$\rho_t^{\pi,g}$						0.18
						(2.05)
$\sigma^{level}_{t-1,t}$	0.47	0.49	0.53	0.47	0.42	0.46
	(8.15)	(7.12)	(8.75)	(7.38)	(6.68)	(7.03)
$\overline{R}^2$	0.29	0.29	0.28	0.29	0.30	0.38

Table VI. Bond Volatility Regressions:
This table reports estimates from forecasts of the monthly ($t \rightarrow t+1$) 2nd principal component of realised return volatility on disagreement factors plus controls:

$$\sigma^{slope}_{t,t+1} = const + \alpha\sigma^{slope}_{t-1,t} + \sum_{i=1}^{4}\beta_i\psi_t(\star) + \sum_{i=1}^{2}\gamma_i E_t(\star) + \sum_{i=1}^{3}\phi_i Macro_t(\star) + \varepsilon_{t+1},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of disagreement variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2011.12

regressor	(i)	(ii)	(iii)	(iv)	(v)	(vi)
ψ^{π}	-0.15			-0.17	-0.13	-0.04
	(-4.05)			(-5.24)	(-2.89)	(-0.71)
ψ^{g}		-0.05		0.01	0.04	0.02
		(-1.09)		(0.38)	(0.71)	(0.40)
ψ^{s}			0.13	0.13	0.13	0.15
			(3.67)	(3.91)	(3.90)	(4.23)
$E(\pi)$					0.01	
					(0.07)	
$E(g)$					0.04	
					(0.69)	
$E(LR)$					-0.03	
					(-0.29)	
$E(SR)$					0.06	
					(0.84)	
$F^{1\rightarrow 8}_t$						
σ^{π}_t						-0.04
						(-1.00)
σ^{g}_t						-0.02
						(-0.43)
$\rho^{\pi,g}_t$						-0.01
						(-0.26)
$\sigma^{slope}_{t-1,t}$	0.48	0.56	0.49	0.40	0.39	0.38
	(5.22)	(7.63)	(7.51)	(4.69)	(4.51)	(4.63)
$\overline{R}^2$	0.35	0.31	0.34	0.39	0.38	0.47

Table VII. Spanned Disagreement:
This table reports contemporaneous regressions of disagreement factors on the 5 principal components from an eigenvalue decomposition of the yield covariance matrix. The yields are 1-5 years in maturity from the Fama-Bliss data set. PC1 is as usual a level factor, PC2 is a slope factor, and PC3 is a curvature factor. PC4 and PC5 are the additional principal components shown to be economically important for bond risk premia in Cochrane and Piazzesi (2005).

$$\psi_t^i = const + \sum_{i=1}^{5} \beta_i \; PC_t^i + \varepsilon_t^i,$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2010.12

regressor	ψ^π	ψ^g	ψ^s
PC^1	-0.38	-0.14	0.20
	(-4.70)	(-1.66)	(3.06)
PC^2	0.09	0.43	0.09
	(1.42)	(6.27)	(1.19)
PC^3	-0.24	-0.20	-0.18
	(-3.16)	(-2.54)	(-2.15)
PC^4	-0.27	0.01	0.28
	(-2.83)	(0.12)	(4.12)
PC^5	0.32	0.19	-0.11
	(3.02)	(2.45)	(-1.36)
$\overline{R}^2$	0.37	0.27	0.16

Table VIII. Unspanned Disagreement and the Hidden Factor:
This table reports contemporaneous regressions of the hidden factor from Duffee (2011) on the unspanned components of disagreement.

$$Hidden_t = const + \sum_{i=1}^{4} \beta_i \ \psi^i_{\mathcal{UN}} + \varepsilon^i_t,$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2007.12

	$\psi^{\pi}_{\mathcal{UN}}$	$\psi^{g}_{\mathcal{UN}}$	$\psi^{s}_{\mathcal{UN}}$	$\overline{R}^2$
$Hidden_t$	-0.02	-0.01	0.20	0.03
	(-0.23)	(-0.14)	(2.24)	

Table IX. Return Predictability 4 : Unspanned and Above Disagreement:
This table reports estimates from OLS regressions of annual ($t \rightarrow t+12$) excess returns of 2,3,4, and 5 year zero-coupon bonds from the Fama-Bliss data set 'spanned', 'unspanned', and 'above' disagreement factors:

$$rx_{t,t+12}^{(n)} = const^{(n)} + \sum_{i=1}^{4} \beta_i^{(n)} \psi_t(\star) + \varepsilon_{t+12}^{(n)},$$

t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction. χ^2 statistics for the jointed significance of ψ_t variables are computed using 18 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. All right hand variables are standardized. A constant is included but not reported. Sample Period: 1990.1 - 2007.12

regressor	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
PANEL A: Unspanned Disagreement				
$\psi_{\mathcal{UN}}^{\pi}$	0.04	0.06	0.10	0.07
	(0.25)	(0.22)	(0.26	0.15)
$\psi_{\mathcal{UN}}^{g}$	0.33	0.56	0.66	0.67
	(2.47)	(2.65)	(2.39)	(2.0)5
$\psi_{\mathcal{UN}}^{s}$	0.60	1.20	1.69	2.11
	(5.71)	(5.42)	(5.04)	(4.69)
$\overline{R}^2$	0.28	0.29	0.28	0.27
PANEL B: Above Disagreement				
$\psi_{\mathcal{AB}}^{\pi}$	0.08	0.10	0.22	0.14
	(0.39)	(0.27)	(0.43)	(0.21)
$\psi_{\mathcal{AB}}^{g}$	0.36	0.59	0.62	0.57
	(2.05)	(1.96)	(1.55)	(1.19)
$\psi_{\mathcal{AB}}^{s}$	0.52	1.08	1.51	1.88
	(3.48)	(3.60)	(3.42)	(3.29)
$\overline{R}^2$	0.22	0.22	0.20	0.19

Table X. Trade:

This table reports estimates from forecasts of the monthly ($t \to t+1$) realised 1st principal component of return volatility on disagreement factors plus controls:

$$Vol_{t,t+1} = const + \sum_{i=1}^{3} \beta_i \psi_t(\star) + \gamma Vol_{t,t-1} + \varepsilon_{t,t+1}.,$$

where $Vol_{t,t+1}$ is the log ratio of the number of monthly transaction volumes between primary dealers and secondary customers for Treasury Bills versus coupon paying securities due in more than 6 years but less than or equal to 11 years. t-statistics, reported in ( )'s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction using 12 Newey-West lags. $\bar{R}^2$ reports the adjusted R^2. Left and right hand sides are standardized. A constant is included but not reported. Sample Period: 04.07.01 to 31.12.11

regressor	(i)	(ii)	(iii)	(iv)	(v)	(vi)
ψ^π	0.71		0.46		0.43	0.26
	(8.88)		(4.07)		(3.83)	(2.89)
ψ^g		0.68	0.39		0.40	0.18
		(3.62)	(2.69)		(2.95)	(2.10)
ψ^s				-0.22	-0.07	0.00
				(-1.14)	(-0.89)	(0.05)
$Vol_{t,t-1}$						0.51
						(8.98)
$\overline{R}^2$	0.50	0.47	0.60	0.04	0.59	0.70

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