Mergers, Capital Structure and Risk

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Abstract

We examine the terms and timing of vertical mergers when the uncertainty concerning stochastic cost of production input provides incentives to integrate. We develop a dynamic model where the acquisition is motivated by cost efficiencies and endogenously derive merger surplus. We show that during an economic downturn, merging is an alternative to bankruptcy as a solution for a downstream firm to stay in operation. The target in this model can delay the timing of a merger during economic upturn by strategically postponing its default. Our results contribute to the evidence of a U-shape pattern of merger waves. We predict industries in which pro- and counter-cyclical vertical mergers are more probable. We also provide asset pricing implications of a merger decision in different economic states.

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1. Introduction

Although the literature on mergers and acquisitions that explains why firms integrate their activities is already quite broad, there are still some aspects of the field that have received relatively little attention so far. The finance literature focuses on the timing of mergers and acquisitions, and analyses merger profitability (Lambrecht (2004), Morellec and Zhdanov (2005), Shleifer and Vishny (2003), Lambrecht and Myers (2007)). There are also papers that claim the merger decision is related to capital structure, where the post-merger leverage can increase tax benefits and therefore the firm's value (Lewellen (1971), Stapleton (1982)). Other studies discuss the role of wealth transfers, financial slack and information asymmetry (Sudarsanam (2003)). Recently Morellec and Zhdanov (2008) has taken a new approach, exploring the relationship between financial leverage and the endogenous timing of the takeover.

There is also a broad swathe of literature from the field of industrial organisation, proposing a number of reasons why firms should merge and restructure (see, e.g. Handbook of Industrial Organization). These include, for example, scale effects in the case of horizontal mergers, diversification in the case of conglomerates and cost reduction seeking in the case of vertical mergers.[1] While horizontal and conglomerate mergers have received a lot of attention from financial academics, vertical mergers remain relatively unexplored.

The economic reasons for vertical integration are best summarised in a recent OECD (2007, page 7) report: "Vertical mergers often lead to lower prices because of the elimination of double marginalisation when there is market power up and downstream pre-merger. Instead of paying a wholesale price that includes a mark up over marginal cost, the integrated firm will be able to access the input at its marginal cost. This gives it an incentive

[1] Theoretical work that addresses the reasons why firms should integrate their activities is quite extensive. The most prominent studies that discuss the boundaries of the firm include: the reduction of transaction costs due to incomplete contracting and agency costs (Coase (1937)), holdup problems and asset specificity in combination with uncertainty (Williamson (1971), 1975, 1979), property rights (Grossman and Hart (1986), Holmstrom and Roberts (1998)) and price inflexibility (Carlton (1979)).

to increase output downstream, to the benefit of consumers. Contractual incompleteness, typically attributable to difficulties firms have monitoring investment and effort to increase sales by their supplier or distributor, often means that it is more important to have margins both up and downstream to provide incentives to increase volume, rather than to have efficient contracting that provides for marginal cost pricing."

Moreover, the recent empirical findings of Garfinkel and Hankins (2011) support the hypothesis that vertical integration creates value for bidding shareholders. They found a drop in the cost measure subsequent to integration. This provides some evidence that cost reduction seeking in the case of vertical mergers is an important factor for a merger to occur.

We can therefore observe merger peaks related to extracting synergies during economic booms. However, data from the credit crunch indicates that many firms merge during recession to survive hard times. A recent empirical study by Netter, Stegemoller, and Wintoki (2011), that revises the evidence on mergers and acquisitions, suggests that "during the financial crisis of 2008 and 2009 when both equity values were depressed and debt financing was difficult to obtain, M&A activity could at worst be described as merely slowing down". An example might be the merger between Oracle and Sun Microsystems, which joined together a software company and a mission-critical computing systems company, allowing Oracle to provide integrated solutions to its clients. The merger with vertical features was announced after Sun Microsystems reported a period of losses, a decline in revenues and a drop in market capitalization and the merger achived significant efficiency gains.[2]

In this paper we present a theory of corporate transactions that focuses on vertical mergers and we show that mergers create value due to markup elimination on the upstream market. We contribute towards evidence of a U-shaped pattern of merger waves by

[2] http://ec.europa.eu/competition/mergers/cases/decisions/m55292010012120682/en.pdf

showing that firms have motives to merge not only during economic upturns but also during economic downturns. Our model emphasizes the role that capital structure plays in determining the optimal timing of vertical merger and its risk.

We develop an economic model where the endogenous merger surplus arises when the operating leverage changes. There are two firms in the model: the downstream firm and the upstream firm. The downstream firm produces a final good using the Cobb-Douglas production function that combines the technology it possesses with the intermediate input supplied by the upstream firm. Normally the upstream firm might have property rights over its product such as: "know-how", innovation or R&D. The stochastic cost of intermediate inputs affects the profits of the target and the acquirer. Two firms operating as one integrated firm can increase profits and output sold on the downstream market by reducing the markup on the upstream market.

We show that when the stochastic cost of intermediate inputs decreases, the benefits (operating synergies) associated with the merger increase, or decline with the increase of the stochastic cost of intermediate inputs. We demonstrate that when the value of a firm increases, investment into new capital becomes more attractive and the incentives for the merger become stronger.

We study how a vertical merger decision is affected by varying economic conditions in the presence of debt financing. We advance a theoretical framework where the fluctuations in the stochastic cost of the intermediate inputs determine two possible outcomes for a vertical merger to occur, during an economic upturn or downturn. Firms merge to extract synergies during an economic upturn. Merging during economic downturn reduces the risk of bankruptcy and increases debt capacity. We show that when the target firm hits its bankruptcy threshold, the bidder has incentives to merge by buying the target at liquidation value. The firm therefore has the option to merge either during economic expansion or recession. The option to merge can be exercised at a fixed sunk cost which has to be paid to cover the necessary payments associated with the merger, where the claim value is inversely related to the state variable. Therefore, these options to merge resemble put option characteristics. The typical assumption about irreversibility of a merger gives the firm incentives to postpone investment; however, by delaying exercising of the options a firm forgoes future profits.

We also provide some implications for systematic risk and dynamics of stock returns in the case of vertical mergers. We observe a decrease in the systematic risk at the time of the merger, which might be related to the significant positive synergies and risk reduction.

This article studies the vertical merger decision in a real options framework. It determines the terms and timing of mergers motivated by cost reduction. The analysis is based on a contingent claims model in the style of Lambrecht (2004), Morellec and Zhdanov (2005) and Lambrecht and Myers (2007).[3] They develop real options models to analyse the terms and timing of takeovers when firms are unlevered. Lambrecht (2004) provides a comprehensive theoretical framework of a pro-cyclical merger which is motivated by economies of scale. Morellec and Zhdanov (2005) extend the behavioural analysis of Shleifer and Vishny (2003) by constructing a two-factor model based on stock market valuations of integrating firms. Lambrecht and Myers (2007) model the disinvestment decision in declining markets and claim that takeovers impose efficient closure. Bernile, Lyandres, and Zhdanov (2011) analyse strategic incentives in the case of horizontal mergers which explain takeover activity during economic booms and recessions. They claim that a U-shaped pattern between demand and merger activity exists. We contribute to this literature by, for the first time, analysing the timing of a vertical merger decision where the surplus is derived endogenously from an economic model.

The relationship between capital structure and merger decisions is still not well understood, though there are a few recent articles. Morellec and Zhdanov (2008) present a

[3] Margrabe (1978) is an early example of modelling mergers as an exchange option with exogenous timing.

dynamic model of takeovers with two bidders, endogenous leverage and bankruptcy. Their model supports the empirical evidence that the bidder winning the contest has leverage below the industry average. Leland (2007) derives a model where only financial synergies motivate the merger decision. He claims that the magnitude of this effect depends on the firm's characteristics like default costs, firm size, taxes, and riskiness of cash flows. Hege and Hennessy (2010) present an analysis where the level of debt plays a strategic role in benefiting from larger merger share. However, there exists a trade-off between higher surplus and the resulting debt overhang which precludes efficient mergers. We contribute to this literature by illustrating how capital structure can affect an existing equilibrium between firms and how it can contribute to a U-shape pattern of merger waves.

This article also relates to the literature pioneered by Berk, Green, and Naik (1999) linking firm investment decisions to asset return dynamics. Further papers by Carlson, Fisher, and Giammarino (2004), Hackbarth and Morellec (2008) present models that explain the dynamics of risk and return by changes in a firm's characteristics such as size and book to market. In particular, the systematic risk of firms might differ as their assets and growth options have different sensitivity (beta) to market fluctuations. We contribute to this literature by analysing the implications of a vertical merger decision for the dynamics of stock returns when firms are financially levered.

The reminder of this paper is as follows. Section 2 provides the economic foundations for the theoretical framework. We develop an economic model with vertical market structure for non-integrated and integrated firms. Section 3 presents the valuation of unlevered firms. We show the properties of the solution. We determine the endogenous merger surplus which arises due to cost reduction and we derive optimal merger timing. Section 4 explains the motives and the timing of a merger when firms are levered. We illustrate the main results with a numerical example. Section 5 presents risk analysis and the asset pricing implications of the merger decision. Section 6 concludes.

2. Model Assumptions

The model assumes a vertically integrated production structure. There are two types of firms: the downstream firm, D, and the upstream firm, U. The downstream firm produces the final product using an intermediate input supplied by the upstream firm. Each firm is a monopolist in its market. This setting can be illustrated and motivated with an example of firms using innovative technologies. The upstream firm might be an R&D company that invests in the new technology. The invention of a superior know-how means that the upstream firm obtains a perpetual patent for an intermediate product and could sell it at the monopoly price to the downstream firm. The downstream firm then transforms the intermediate input into a final product and sells it at a monopoly price to the final customers. The uncertainty in this model concerns the stochastic cost of production of the intermediate input, c_t, which is a stochastic process that it is external to the economy. Therefore, when the cost of the intermediate input increases, the economy contracts.

The downstream firm has a possibility to buy the intermediate input in the market or to make the components in-house by merging with the upstream firm. We consider these two cases and specify the conditions necessary for a vertical integration in the subsequent sections.

2.1. Non-Integration: Buy option

In this subsection we consider the case when the downstream firm buys the intermediate input in the market from the upstream firm. The solutions obtained for the non-integrated case will be later compared with the case of vertically integrated firms.

2.1.1. Downstream Firm

The downstream firm produces the final output using Cobb-Douglas production function that combines technology with the intermediate input:

$$q_D = A q_U^{\alpha} \tag{1}$$

For $0 < \alpha < 1$, this equation implies that the production function exhibits diminishing returns to scale. The demand for the final product is:

$$p_D = a q_D^{-\varepsilon} \tag{2}$$

where $\varepsilon > 0$ ensures the properties of the demand function. The price elasticity of demand is constant and equals $1/\varepsilon$. Based on the standard arguments the profit of the downstream firm when it decides to buy the intermediate input is:

$$\Pi_D = p_D A q_U^{\alpha} - p_U q_U \tag{3}$$

where the parameter A is the measure of the technology level, p_D is the price of the final good on the downstream market, p_U is the price of the intermediate product on the upstream market, q_U is the quantity of the input employed in the production of the final good. The downstream firm maximizes the profit function with respect to its production input. From the first order condition it is possible to derive the demand for the intermediate input. The quantity of the input demanded by the downstream firm is therefore a function of price in the downstream market and the upstream market:

$$q_U = \left(\frac{a A^{1-\varepsilon} \alpha (1-\varepsilon)}{p_U} \right)^{\frac{1}{1-\alpha(1-\varepsilon)}} \tag{4}$$

The inverse demand function is therefore $p_U = p_D A\alpha(q_U)^{(\alpha-1)}$ and the price elasticity of demand for the upstream product is constant and equals $-1/\left[1-\alpha(1-\varepsilon)\right]$. [4] The profit function with the optimally chosen input is:

$$\Pi_D = \hat{A}^{\frac{1}{1-\hat{\alpha}}} p_U^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}}\right) \tag{5}$$

where $\hat{A} = aA^{1-\varepsilon}$, $\hat{\alpha} = \alpha(1-\varepsilon)$.

2.1.2. Upstream Firm

The upstream firm faces the following decision process. It has to decide on the price of the invented product. It is assumed here, that when the new input is created the firm has a patent of infinite duration and retains the monopoly rights over its production and sales perpetually. The instantaneous profit flow of the upstream firm is:

$$\Pi_U = p_U q_U - c_t q_U \tag{6}$$

where q_U is defined in equation (4). The state variable c_t evolves as a geometric Brownian motion with drift:

$$dc = \mu c dt + \sigma c dz \tag{7}$$

where μ and σ are constant parameters and dz is the increment of a Wiener process with a zero mean. The growth rate of the production cost is normally distributed with a mean $\mu - \frac{\sigma^2}{2}$ and a variance σ^2. It is assumed that $\mu < \frac{\sigma^2}{2}$.

The upstream firm maximizes its profits with respect to the input price [5]. The first order

[4] I assume that $\varepsilon < 1$. Otherwise, if $\varepsilon = 1$ it makes the quantity sold on the upstream market close to zero and price would be infinite.

[5] Quantity competition brings the same results in the case of a monopoly.

condition yields the optimal pricing strategy in the upstream market:

$$p_U = \frac{1}{\hat{\alpha}} c_t \tag{8}$$

The monopoly price is determined as the markup over the marginal cost. The quantity sold in the upstream market is:

$$q_U = \left(\frac{\hat{A}\hat{\alpha}^2}{c_t} \right)^{\frac{1}{1-\hat{\alpha}}} \tag{9}$$

The profit function of the upstream firm is then:

$$\Pi_U = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \pi_U(\hat{A}, \hat{\alpha}) \tag{10}$$

where: $\pi_U(\hat{A}, \hat{\alpha}) = \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{1+\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{2}{1-\hat{\alpha}}} \right)$

The profits of the upstream firm negatively depend on the input cost and positively on the level of technology of the firm, the measure of economies of scale in the production of final output and the inverse of the price elasticity of demand in the final good market .

2.1.3. Equilibrium Profit of Non-integrated Downstream Firm

Combining equation (5) with equation (8) we obtain the equilibrium profit of the downstream firm:

$$\Pi_D = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \pi_D(\hat{A}, \hat{\alpha}) \tag{11}$$

where: $\pi_D(\hat{A}, \hat{\alpha}) = \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{2\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1+\hat{\alpha}}{1-\hat{\alpha}}} \right)$

The profits of the downstream firm are a decreasing function of the input cost, and increase with technology level (where $\alpha < \frac{1}{1-\varepsilon}$ is always satisfied). The profits of the downstream firm increase with the economies of scale parameter.

2.2. Integrated Firm: Make Option

In this subsection we consider the case when the downstream firm mergers with the upstream firm. We solve for the profit flow steaming from the vertical merger. The high markup in the upstream market provides an incentive for the downstream firm to acquire the upstream firm. Therefore, an integrated firm has an access to the input at a lower price equal to the marginal cost. The decrease in the input price lowers the price of the final good and increases sales resulting in the higher profit. In the case of the integrated firm the profit is as follows:

$$\Pi_M = \hat{A} q_M^{\hat{\alpha}} - c_t q_M \tag{12}$$

The solution to the integrated firm optimization problem with respect to its production input gives:

$$q_M = \left(\frac{\hat{A}\hat{\alpha}}{c_t} \right)^{\frac{1}{1-\hat{\alpha}}} \tag{13}$$

The profit function of the integrated firm with the optimally chosen input is:

$$\Pi_M = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \pi_M(\hat{A}, \hat{\alpha}) \tag{14}$$

where: $\pi_M(\hat{A}, \hat{\alpha}) = \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} \right)$

The profits of the integrated firm are higher than the profits of the sum of two non-integrated firms due to elimination of the double markup pricing. This result is in line with the findings of the previous literature, for example Motta (2004). In particular, if the firm has access to the intermediate product at a lower cost then the price it charges in the final goods market also decreases. From the properties of the inverse demand function we can expect that a decrease in price results in a higher demand for the product. Therefore, a firm can increase its sales. Figure 1 depicts the level of prices and the output sold in the case of disintegrated and integrated firm. Panel A considers the price levels in the downstream market as a function of cost. Panel B presents the output level as a function of cost. This comparison shows that for the integrated firm the price decreases and the quantity increases which leads to a higher profit.

[Insert Figure 1 here]

3. Valuation, Merger Surplus and Optimal Merger Timing: Unlevered Firm

In the previous section we derived instantaneous cash flows generated for non-integrated and integrated firms. Here we derive the value of the unlevered firm when the marginal cost follows a stochastic process and we calculate the realized merger surplus.

3.1. Firms values without the merger option

The optimal profit flow in a general form can be written as $\Pi_i = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \pi_i(\hat{A}, \hat{\alpha})$. The value of the firm is a contingent claim where the payoff depends on the value of an underlying asset. We assume that investors are risk neutral and that there exists a risk-free asset that pays a constant interest rate r. We follow the procedure of the contingent claims valuation as discussed in Dixit and Pindyck (1994). The state variable in this model is inversely related to the contingent claim on this asset. Therefore, the value of the firm is negatively related to the price of a production input. Given the instantaneous profit the value of the unlevered firm for $i = U, D, M$ is:

$$V_i(c_t) = \mathbb{E}^c \left[\int_{\infty}^{0} e^{-rt} \Pi(t) dt \right] \tag{15}$$

where $\mathbb{E}^c$ is a conditional expectation operator assuming that the current cost shock takes the value of c(0) = c. Lemma 1 follows.

Lemma 1 *The stand-alone value of the unlevered firm where the profits are driven by stochastic cost* c_t *is:*

$$V_i(c_t) = \frac{c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \pi_i(\hat{A}, \hat{\alpha})}{r - \xi} \tag{16}$$

where $\xi = \frac{\hat{\alpha}}{\hat{\alpha}-1}\mu + 0.5\frac{\hat{\alpha}}{\hat{\alpha}-1}\left(\frac{\hat{\alpha}}{\hat{\alpha}-1} - 1\right)\sigma^2 < r$

The value of the firm is the discounted present value of the profit flow. It can be perceived as the Gordon growth model where ξ is the growth rate which depends on the model parameters.

3.2. Merger Surplus Motivated by Efficiency Seeking

In the case of vertically integrated production structure a merger of two firms can be motivated by the synergies arising from cost cutting due to markup elimination in the intermediate input. The gross surplus is defined as the difference between the value of the integrated firm and a sum of two separated entities. We show that the downstream firm can benefit from the markup elimination in the upstream market and that the surplus is always positive.

Proposition 1 *When the merger is motivated by cost efficiencies the gross merger surplus is defined as:*

$$\Omega = V_M(c_t) - V_D(c_t) - V_U(c_t) = \frac{\Pi_M - \Pi_D - \Pi_U}{r - \xi} \tag{17}$$

where $\Pi_M = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}}\right)$, $\Pi_D = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{2\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1+\hat{\alpha}}{1-\hat{\alpha}}}\right)$, *and* $\Pi_U = c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{1+\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{2}{1-\hat{\alpha}}}\right)$. *The merger surplus is always positive and* $\Pi_M > \Pi_D + \Pi_U$.

The proof is in Appendix A. The gross merger surplus is monotonically decreasing in the stochastic cost and increasing in the level of technology. Therefore, the payoff which depends on the stochastic cost has put option characteristics as the state variable is inversely related to the claim value. This implies that the increase in the growth rate of cost decreases the value of the surplus.

3.3. Value of Merger Option and Merger Threshold

In this subsection we show that there exists a first-best threshold which is derived as a tradeoff between the surplus that arises during the economic upturns and the fixed sunk cost, X, which has to be paid to cover all necessary payments associated with the integration. The state variable is inversely related to the claim value thus this option resembles a put option characteristics. Therefore, when the merger is motivated by cost efficiencies the net joint merger surplus is a decreasing function of the stochastic cost:

$$S(c_t) = max[V_M(c_t) - V_D(c_t) - V_U(c_t) - X, 0] \tag{18}$$

The decision to merge is irreversible. A firm has thus incentives to postpone its decision, however by delaying the merger it forgoes future profits. Thus, there exists a threshold when the firm decides to merge. Exercising the merger option is optimal when the state variable hits the merger threshold $\underline{c}$ from above. It can be shown that the threshold is a solution to the free boundary problem which is presented in Appendix B.

Lemma 2 *The first-best optimal threshold of central planner is:*

$$\underline{c} = \left[\frac{\lambda}{\lambda - \frac{\hat{\alpha}}{\hat{\alpha}-1}} \frac{X(r-\xi)}{\hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} + \hat{\alpha}^{\frac{2}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{2\hat{\alpha}}{1-\hat{\alpha}}} \right)} \right]^{\frac{\hat{\alpha}-1}{\hat{\alpha}}} \tag{19}$$

where λ is the negative root of the quadratic equation $z(z-1)\sigma/2 + z\mu = r$.

[Insert Figure 2 here]

Figure 2 presents the relation between the merger threshold and model parameters such as: cost uncertainty (σ), the interest rate (r), economies of scale (α), and inverse of demand elasticity (ε).

According to the real options theory uncertainty might have two potential effects. The first effect is associated with so called a hysteresis factor, which is here $[\lambda/\left(\lambda - \frac{\hat{\alpha}}{\hat{\alpha}-1}\right)]^{\frac{\hat{\alpha}-1}{\hat{\alpha}}}$. Volatility embedded in λ, increases it. Due to the fact that $\frac{\hat{\alpha}}{\hat{\alpha}-1}$ is negative, the value of $\lambda/(\lambda - \frac{\hat{\alpha}}{\hat{\alpha}-1})$ increases. Therefore, the final effect is negative and the standard effect that higher volatility delays the decision holds. As the claim value is inversely related the state variable thus option to merge is exercised at a lower marginal cost. The second effect is associated with the growth rate, ξ, as higher volatility accelerates the decision. In line with real options literature the first effect normally dominates the second. Therefore, we expect that higher volatility should delay the merger decision. Predictions are shown in Panel A Figure 2.

Inverse of demand elasticity (ε) has two effects on the merger threshold. First, it is a component of the price elasticity of demand on the upstream $(-1/\left[1-\alpha(1-\varepsilon)\right])$ and on the downstream market $(1/\varepsilon)$. An increase in epsilon decreases the price elasticity of demand and thus a price change might be explicitly transfered on the final good consumers with a lower decrease in profits. Second, the epsilon has an effect on the markup on the upstream market, which is $1/\alpha(1-\varepsilon)-1$. The less sensitive the customers are, the higher the markup on the upstream market. Therefore, incentives to merge increase as the synergies associated with the merger increase. Predictions are shown in Panel B Figure 2.

Returns to scale (α) have a decreasing effect on the merger surplus and the markup on the upstream market. A firm with a high parameter α is more efficient in transforming the intermediate input and has a lower markup on the upstream market. For firms with low α, we observe a significant delay in exercising a merger option. Therefore, vertical mergers are more likely in industries with higher markup. Predictions are shown in Panel C Figure 2. The following corollaries result immediately from Lemma 2.

Corollary 1 *Vertical mergers are more likely in industries where the demand of final good customers is less sensitive to a change in the price.*

Corollary 2 *Vertical mergers are more likely in industries where the returns to scale are decreasing and firms are operating on a higher markup.*

These results suggest that the more companies are constrained, in terms of economies of scale, the more likely vertical mergers are. This is opposite to already reported results for horizontal mergers. For example, Lambrecht (2004) predicts that horizontal mergers are more likely when firms have increasing returns to scale, in order to create positive merger synergies. Furthermore, the effect of the price elasticity has a different meaning. In the case of vertical mergers the less elastic demand means that firms can charge a higher markup. However, in the case of horizontal mergers the elasticity of demand is associated with the market power effect and the synergies effect.

4. Value of the Firm and Merger Decision: Levered Firms

In this section we introduce financial leverage into the model. The question we attempt to answer is how and to what extent financial leverage affects the merger threshold during different economic states.

The profits of firms are endogenously derived in Section 2. Each firm is now financed with equity and infinite maturity debt. A single firm issues a debt contract with a fixed coupon which is necessary to cover part of the firm's expenses. The equityholders default when they are not willing to inject more capital to cover operating loses to service debt. We assume that the debt is risky and thus the liquidation value is lower than the debt value.

The dynamic changes in the stochastic cost of the intermediate input can affect the existing equilibrium between firms in the market. Therefore, depending on the state of the economy the downstream firm encounters different incentives to integrate. We illustrate how alternative outcomes arise in the model in Figure 3. Path A in Figure 3 presents a possible way to trigger a pro-cyclical merger. The pro-cyclical merger occurs during the economic upturn when the motivation of the downstream firm to merge is related to extracting the synergies that arise in the upstream market. Denote as $\underline{c}_I$ a pro-cyclical merger threshold. Path B in Figure 3 presents a possible way to trigger a counter-cyclical merger. During the economic slowdown, one of firms can default. The upstream firm defaults before the downstream firm. Therefore, to stay in operation, the downstream firm can merge with the upstream firm upon its default. Denote as $\overline{c}_U$ a counter-cyclical merger threshold.

[Insert Figure 3 here]

This is a dynamic programming problem and can be solved by backward induction. First, we solve for post-merger values of contingent claims. Second, we solve for the pre-merger values of firms and derive the optimal merger timing during different economic states and sharing rule.

4.1. Pro-Cyclical Merger Regime

The synergies that are a consequence of a merger decision are dependent on the state of the economy. It was shown in Proposition 1 that synergies associated with efficiency seeking arise when the stochastic cost of intermediate input decreases. Therefore, the merger motivated by cost reduction occurs during the economic upturn and we call it a pro-cyclical merger.

The integrated entity generates the instantaneous profit of $\Pi_M^P(c_t)$ which is higher than the sum of joint profits of the downstream and the upstream firm.[6] Denote by $E_M^P(c_t)$, $B_M^P(c_t)$, and $V_M^P(c_t, b_M^P)$, the post-merger values of equity, debt and firm, respectively, when the option to merge is exercised at the pro-cyclical merger threshold.

There are a few possible answers to a question what can happen with the coupon of the new merged entity. The tax literature suggests that the firm increases its debt to benefit from the tax shield. There is some evidence that in the presence of competition the winning bidder levers up. We claim that in our model the new coupon is higher or equal to the sum of coupons of the downstream and the upstream firm. The argument here is that the debt capacity of the integrated firm increases after the successful merger consummation which might allow the integrated firm to renegotiate its contracted debt level. The new coupon of the merged company is $b_M^P \geq b_D + b_U$. Equityholders then select the bankruptcy threshold at $\overline{c}_M^P$.

Given the amount of the profit flow and the value of the contracted coupon we solve for the closed-form solutions of the contingent claims values which are presented in Lemma 3.

Lemma 3 *Given the merger occurred in PM regime the value of the equity is :*

$$E_M^P(c_t) = \frac{\Pi_M^P(c_t)}{r-\xi} - \frac{b_M^P}{r} - \left(\frac{\Pi_M^P(\overline{c}_M^P)}{r-\xi} - \frac{b_M^P}{r}\right)\left(\frac{c_t}{\overline{c}_M^P}\right)^{\vartheta} \tag{20}$$

the value of the firm's debt is :

$$B_M^P(c_t) = \frac{b_M^P}{r} + \left[\Phi_M^P - \frac{b_M^P}{r}\right]\left(\frac{c_t}{\overline{c}_M^P}\right)^{\vartheta} \tag{21}$$

the value of the integrated firm is :

$$V_M^P(c_t, b_M^P) = \frac{\Pi_M^P(c_t)}{r-\xi} + \left[\Phi_M^P - \frac{\Pi_M^P}{r-\xi}\right]\left(\frac{c_t}{\overline{c}_M^P}\right)^{\vartheta} \tag{22}$$

[6] Proof in Appendix A.

and the default threshold chosen by equityholders is:

$$\bar{c}_M^P = \left(\frac{\vartheta}{\vartheta - \frac{\hat{\alpha}}{\hat{\alpha}-1}} \times \frac{(r-\xi) b_M^P}{\pi_M^P(\hat{A}, \hat{\alpha}) r} \right)^{\frac{\hat{\alpha}-1}{\hat{\alpha}}} \tag{23}$$

Equation (20) has the standard interpretation of equity investors' claims as the rights to the perpetual cash flows generated by the firm and the value of the option to default with probability of $\left(c_t/\bar{c}_M^P\right)^{\vartheta}$. Equation (21) is the value of the firm's debt. The perpetual contract for bondholders guarantees the payment of the fixed coupon of b_M^P until default and in the case of the firm's bankruptcy the payment of liquidation value of Φ_M^P. The value of the firm in equation (22) is the sum of equityholders and debtholders values. The default threshold selected by equityholders, which is summarized in equation (23), has the standard properties known from the real options literature.

4.2. Counter-Cyclical Merger Regime

The economic slowdown is associated with the increase in the cost of intermediate input. The motivation for a merger during recession is twofold. First, and most importantly when cost increases one of firms might find herself in financial distress. The bankruptcy threshold of the upstream firm is lower than the one of the downstream firm. Therefore, when the upstream firm is upon default, the downstream firm is motivated to keep operating the existing supply chain in order to assure the service in the upstream market. Second, although it is suboptimal, the downstream firm is willing to capture some of the merger synergies. It is not optimal for the downstream firm to acquire before the upstream firm bankruptcy as the merger surplus is monotonically decreasing function of the state variable. The downstream firm equityholders are therefore willing to postpone the decision to merge during the economic downturn. However, when the default of the upstream firm occurs they have to make a decision in order to stay in operation as a going concern, as they are dependent on deliveries in the upstream market.[7]

Alternative for the downstream firm to merge at bankruptcy is to invest in setting up its own plant. However, the cost of investing in setting up the plant from scratch during the recession is greater or equal to buying the assets of the bankrupt company for the liquidation value.[8] Moreover, it is optimal for the downstream firm to wait till the bankruptcy of the upstream company as then the price it has to pay is equal to the liquidation value of the upstream firm. We assume that $\Phi_U < b_U/r$, therefore the debt is risky in the model. The merger transaction is a cash transfer to debtholders. Therefore, the new coupon of the merged entity is $b_M^C \geq b_D$ equityholders select the bankruptcy threshold at $\bar{c}_M^C$. The merger cost in this case increases to $X_C = X + \Phi_U$.

The counter-cyclical merger is therefore a consequence of default of the upstream firm. It is assumed that at the time of the upstream firm bankruptcy some of the know-how is lost, for example in a form of a human capital. The new technology level after the merger at bankruptcy is $A^C = (1 + \zeta)A$, where $\zeta < 0$. When firms integrate their activities upon bankruptcy of one of them the instantaneous profit of the integrated company is $\Pi_M^C(c_t)$ and the contractual debt coupon is b_M^C. $E_M^C(c_t)$, $B_M^C(c_t)$, and $V_M^C(c_t, b_M^C)$ are the post-merger values of equity, debt and firm, respectively.

The closed-form solutions of the contingent claims values are summarized in Lemma 4.

[7] In the case of bankruptcy the downstream firm equityholders might still postpone their decision to merge and subsidize the upstream firm in a form of paying a higher markup for the intermediate input. However, they might not be willing to pay the subsidy forever as the present value of the instantaneous additional payment might be higher than the lump sum cost of merger. Moreover, they might be willing to capture the synergies associated with the merger transaction.

[8] A possible explanation might be motivated by the Tobin's Q theory. During recession the market values are less than the replacement (investment) values, therefore the ratio does not suggest investment in a new plant.

Lemma 4 *Given the merger happened in CM regime the value of the equity is:*

$$E_M^C(c_t) = \frac{\Pi_M^C(c_t)}{r-\xi} - \frac{b_M^C}{r} - \left(\frac{\Pi_M^C(\overline{c}_M^C)}{r-\xi} - \frac{b_M^C}{r}\right)\left(\frac{c_t}{\overline{c}_M^C}\right)^{\vartheta} \tag{24}$$

the value of firm's debt is:

$$B_M^C(c_t) = \frac{b_M^C}{r} + \left[\Phi_M^C - \frac{b_M^C}{r}\right]\left(\frac{c_t}{\overline{c}_M^C}\right)^{\vartheta} \tag{25}$$

the value of the integrated firm is:

$$V_M^C(c_t, b_M^C) = \frac{\Pi_M^C(c_t)}{r-\xi} + \left[\Phi_M^C - \frac{\Pi_M^C}{r-\xi}\right]\left(\frac{c_t}{\overline{c}_M^C}\right)^{\vartheta} \tag{26}$$

the default threshold chosen by equityholders is:

$$\overline{c}_M^C = \left(\frac{\vartheta}{\vartheta - \frac{\hat{\alpha}}{\hat{\alpha}-1}} \times \frac{(r-\xi)b_M^C}{\pi_M^C(\hat{A},\hat{\alpha})r}\right)^{\frac{\hat{\alpha}-1}{\hat{\alpha}}} \tag{27}$$

The results presented in Lemma 4 have essentially the same interpretation as the ones reported in Lemma 3.

4.3. Pre-Merger Valuation of Firms

In this subsection we determine the pre-merger value of equity of both firms, upstream and downstream. Before the merger each firm generates a perpetual cash flow of profits and pays a coupon at each instant of time. Moreover, each firm's shareholders obtain capital gains, which are in a form of expected future changes in the equity value over each time interval. The decision to merge is associated with a tradeoff between the benefits and the sunk cost of executing a merger. The merger cost covers all expenses that are related to transaction costs such as underwriting, legal fees, and the present value of restructuring costs. These costs are fully covered by the increase in the merged firm equity value related to synergies from merging. Therefore, the equityholders choose the time when this decision benefits them.

It is assumed that after successful consummation the bidder can adjust its debt level to b_M^P or b_M^C depending on in which regime the merger occurred.

The value of each firm is determined as a solution to the free boundary problem. At the pro-cyclical merger threshold ($\underline{c}_I$) the value of the new entity becomes $E_M^P(\underline{c}_I)$ which is already known from Lemma 3. Furthermore, both firms share the new merged entity among each other according to a unique sharing rule. Denote δ as a share in the new integrated entity of the upstream firm and $1-\delta$ as a share that the downstream firm obtains. Therefore, the value of the upstream firm equity at the time of the endogenously chosen merger during economic upturn equals to a share in a new entity: $E_U(\underline{c}_I) = \delta(E_M^P(\underline{c}_I) - X)$ and the value of the downstream firm becomes: $E_D(\underline{c}_I) = (1-\delta)(E_M^P(\underline{c}_I) - X)$. However, if things go bad, the cost of intermediate input increases and the downstream firm has an opportunity to merge with the upstream firm at the counter-cyclical merger threshold $\overline{c}_U$, which is the default threshold of the upstream firm. The upstream firm equityholders liquidate the company then and their equity equals zero, $E_U(\overline{c}_U) = 0$. The downstream firm equityholders are left with a new integrated company which value was derived in Lemma 4 less the merger cost, which is $E_D(\overline{c}_U) = E_M^C(\overline{c}_U) - X_C$. The downstream firm at the counter-cyclical threshold does not negotiate on the sharing rule. We know that $\underline{c}_I < \overline{c}_U$ and the pro-cyclical merger occurs during good economic state when the cost of the production input is low. While the default occurs at the $\overline{c}_U$ when the cost of intermediate input is high. Denote by $\mathcal{L}(c_t)$ the present value of \$1 to be received the first time c_t reaches c_U conditional on reaching c_U before reaching c_I. Similar, denote by $\mathcal{H}(c_t)$ the present value of \$1 to be received the first time c_t reaches c_I conditional on reaching c_I before reaching c_U. We can now derive the optimal strategy of both firms regarding the timing of merger and the share in the new integrated company. Proposition 2 shows the results. (For details see Appendix E.)

Proposition 2 *The pro-cyclical merger threshold $\underline{c}_I$ and the sharing rule δ are are the solutions to the following problem:*

$$\partial E_U(c_t)/\partial c_t\Big|_{c=\underline{c}_I} = \delta(\partial E_M^C(c_t)/\partial c_t)\Big|_{c=\underline{c}_I} \tag{28}$$

$$\partial E_D(c_t)/\partial c_t\Big|_{c=\underline{c}_I} = (1-\delta)(\partial E_M^C(c_t)/\partial c_t)\Big|_{c=\underline{c}_I} \tag{29}$$

The counter-cyclical merger threshold $\overline{c}_U$ is the solution to the following problem:

$$\partial E_U(c_t)/\partial c_t\Big|_{c=\overline{c}_U} = 0 \tag{30}$$

where:

$$E_D(c_t) = \frac{\Pi_D(c_t)}{r-\xi} - \frac{b_D}{r} + \mathcal{L}(c_t)\left[E_M^C(\overline{c}_U) - \frac{\Pi_D(\overline{c}_U)}{r-\xi} + \frac{b_D}{r} - X\right] + \\ + \mathcal{H}(c_t)\left[(1-\delta)\left(E_M^P(\underline{c}_I) - X\right) - \frac{\Pi_D(\underline{c}_I)}{r-\xi} + \frac{b_D}{r}\right] \tag{31}$$

and:

$$E_U(c_t) = \frac{\Pi_U(c_t)}{r-\xi} - \frac{b_U}{r} + \mathcal{L}(c_t)\left[-\frac{\Pi_U(\overline{c}_U)}{r-\xi} + \frac{b_U}{r}\right] + \mathcal{H}(c_t)\left[\delta\left(E_M^P(\underline{c}_I) - X\right) - \frac{\Pi_U(\underline{c}_I)}{r-\xi} + \frac{b_U}{r}\right] \tag{32}$$

where ϑ and λ are respectively the positive and negative roots of the quadratic equation

$z(z-1)\frac{\sigma^2}{2} + z\mu = r$ *and the stochastic discount factors* $\mathcal{L}(c_t)$ *and* $\mathcal{H}(c_t)$ *are defined as:*

$$\mathcal{L}(c_t) = \frac{c_t^{\vartheta}\underline{c}_I^{\lambda} - c_t^{\lambda}\underline{c}_I^{\vartheta}}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \qquad and \qquad \mathcal{H}(c_t) = \frac{c_t^{\lambda}\overline{c}_U^{\vartheta} - c_t^{\vartheta}\overline{c}_U^{\lambda}}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \tag{33}$$

From equation (31) we can see that the equity value of the downstream firm decreases (given $\mathcal{H}(c_t) > 0$) with the increase in the share of the upstream firm in the integrated entity.

The solution to this problem is analytically not tractable. We derive some basic comparative statics in Appendix F. The results suggest that there exists a rule determining the share in the new entity and the merger threshold simultaneously between two firms. In particular, when the share of the upstream firm in the integrated equity increases, which might be associated with a higher bargaining power, the integration threshold decreases as the downstream firm waits for a higher merger surplus to exercise the option. This outcome is already known in the literature for unlevered firms (Lambrecht (2004), Hackbarth and Miao (2011)). We confirm that it holds when firms are levered. The novelty that we derive here is the effect of the change in the the default threshold of the upstream firm on the merger decision of the downstream firm during economic upturn. We show that the decision of the downstream firm equityholders is dependent not only on the share in the new equity that will belong to the upstream firm equityholders, but also on capital structure of the upstream firm. In particular, if there is any change in the parameters of the model that affects the upstream firm equityholders decision to postpone bankruptcy, the downstream firm equityholders would delay their decision to integrate during economic expansion. Proposition 3 follows.

Proposition 3 *The decision of the upstream firm equityholders to postpone (accelerate) their default, delays (speeds up) the decision of the downstream firm equityholders to integrate during economic upturn.*

Proposition 3 sheds light on the effect of risky debt on the merger decision. In particular, when the standard tradeoff between postponing the merger decision and lost future profits is affected by possibility of default of one firm, the decision to merge during economic upturn is accelerated. A higher probability of default of the upstream firm increases the value of the option to merge. Therefore, the timing of the pro-cyclical merger of the levered firm is accelerated in comparison to the unlevered firm merger threshold.

4.4. Numerical Analysis

In this subsection we confirm the comparative statics related to the analysis of pro- and counter-cyclical merger threshold and sharing rule by undertaking a numerical analysis.

[Insert Figure 4 here]

Figure 4 represents the numerical solution for the optimal pro-cyclical merger threshold during economic upturn (Panel A) and the sharing rule (Panel B) as a function of uncertainty. If the uncertainty over the input cost increases the equityholders postpone their decision to merge. This is consistent with the real options literature that high uncertainty delays the investment decision. Furthermore, the analysis reveals that the share of the target firm slightly decreases with increasing uncertainty.

[Insert Figure 5 here]

Figure 5 illustrates the effect of model parameters on the pro-cyclical merger threshold. Panel A shows that as expected higher merger cost (X) delays the timing of the pro-cyclical merger.

In Panel B we present the effect of a coupon of the upstream firm (b_U) on the merger decision of the downstream firm during economic upturn. High coupon of the upstream firm speeds up the pro-cyclical merger (low coupon of the upstream firm delays the pro-cyclical merger). Results are consistent with Proposition 3. In particular, the high coupon of the upstream firm is associated with a sooner default and the decrease in the upstream firm equity value. Therefore, the pro-cyclical merger threshold is now quicker as it is associated with a higher merger surplus.

The inverse of demand elasticity (ε) has a similar effect on the pro-cyclical merger threshold as in the case of the unlevered firm (Panel C). Therefore, for high values of epsilon the the price elasticity of demand is low and pro-cyclical mergers are more likely.

The effect of economies of scale (α) has similar properties as in the case of the unlevered firm (Panel D). In particular, the low alpha is associated with a higher markup on the upstream market and therefore the synergies from the vertical merger are higher speeding up the timing of the merger. The following corollaries emerge in the case of levered firms.

Corollary 3 *Pro-cyclical vertical mergers are more likely in industries where the demand of final good customers is less sensitive to a change in the price.*

Corollary 4 *Pro-cyclical vertical mergers are more likely in industries where the returns to scale are decreasing and firms are operating on a higher markup.*

[Insert Figure 6 here]

The basic properties of the counter-cyclical merger threshold are presented on Figure 6. According to the real options theory uncertainty might have two potential effects. A first effect is associated with the so-called hysteresis factor. Higher volatility delays the decision. A second effect is associated with the growth rate, ξ, as higher volatility accelerates the decision. In line with real options literature the first effect normally dominates the second. Therefore, we expect that higher volatility should delay the merger bankruptcy decision (default decision of the upstream firm). These predictions are confirmed in Panel A of Figure 6.

The increase in the inverse of demand elasticity (ε) delays the counter-cyclical merger threshold. An increase in epsilon decreases the price elasticity of demand and thus a price change might be explicitly transfered onto the final-good consumers with a lower decrease in profits, which delays the decision of the upstream firm equityholders to default. Second, the epsilon has an effect on the markup on the upstream market. The less sensitive the customers are the higher the markup on the upstream market and therefore less likely the default of the upstream firm. These predictions are presented on Panel B Figure 6.

The increase in the level of economies of scale (α) speeds up the counter-cyclical merger threshold as it lowers the markup on the upstream market. Therefore, the upstream firm is more likely to default sooner. The results are depicted on Panel C of Figure 6. The following corollaries emerge in the case of levered firms.

Corollary 5 *Counter-cyclical vertical mergers are more likely in industries where the demand of final good customers is more sensitive to a change in the price.*

Corollary 6 *Counter-cyclical vertical mergers are more likely in industries where the upstream firm operates on the low markup.*

The results are fairly consistent with the intuition. When the firms are more exposed to the shifts in the demand due to a price change and have a low cushion in terms of markup, the financial distress is more likely to occur.

5. Risk Analysis

Mergers are among the most important corporate events that have significant effect on stock returns as they change firms' systematic risk. Therefore, it is crucial to understand how the risk of the equity changes in the periods surrounding merger episodes. A possible answer is suggested by the real options literature that views the merger possibility as an option to invest. According to this literature exercising an investment option changes the risk of the firm. In particular, when a firm holds assets in place and a growth option to invest (with call option characteristics) at a fixed cost to get in exchange the instantaneous profit flow, its risk is inflated. When the firm exercises this option we expect that the risk decreases afterwards.

However, there is an important difference between the investment option and the option to merge. The second one involves two firms with different pre-merger characteristics. Their risk profile, capital structure, size, production capacity might be different.

In this section we perform the risk analysis for the integrated and disintegrated firms. We present the comparative statics and a numerical example to discuss the characteristics of an analytical solution.

We define the risk of a firm as in Carlson, Fisher, and Giammarino (2004) where they trace the risk profile of the firm by its beta. They prove that the beta of the firm can be derived in a form of the elasticity. It measures how the equity value changes with respect to the change in the state variable. The subsequent expression follows:

$$\beta_i = \frac{\partial E_i(c_t)}{\partial c_t} \frac{c_t}{E_i(c_t)} \tag{34}$$

where i is the subscript representing the equity value of the upstream, downstream or merged firm.

We derive the risk dynamics during the pre-merger episode for the downstream and upstream firm. Furthermore, we present post-merger betas for a combined equity. We can calculate also the risk dynamics for the debtholders and the firm value. However, it is not the purpose of this paper. We focus on the effect of the systematic risk on equity values. Proposition 4 summarizes.

Proposition 4 *Suppose $\gamma = \frac{\hat{\alpha}}{\hat{\alpha}-1}$, the risk of the levered pre-merger downstream and upstream equity respectively is:*

$$\beta_D = \gamma + \gamma \frac{F_D}{E_D(c_t)} + \gamma \frac{OD_D(c_t)}{E_D(c_t)} + \tilde{\mathcal{L}}(c_t) \frac{OM_D^C(c_t)}{E_D(c_t)} + \tilde{\mathcal{H}}(c_t) \frac{OM_D^P(c_t)}{E_D(c_t)} \tag{35}$$

where $F_D = \frac{b_D}{r}$ *and* $OD_D = \left(\frac{\Pi_D(\bar{c}_D)}{r-\xi} - \frac{b_D}{r}\right)\left(\frac{c_t}{\bar{c}_D}\right)^{\vartheta}$

$$\beta_U = \gamma + \gamma\frac{F_U}{E_U(c_t)} + \gamma\frac{OD_U(c_t)}{E_U(c_t)} + \tilde{\mathcal{L}}(c_t)\frac{OM_U^C(c_t)}{E_U(c_t)} + \tilde{\mathcal{H}}(c_t)\frac{OM_U^P(c_t)}{E_U(c_t)} \tag{36}$$

where $F_U = \frac{b_U}{r}$ *and* $OD_U = \left(\frac{\Pi_U(\bar{c}_U)}{r-\xi} - \frac{b_U}{r}\right)\left(\frac{c_t}{\bar{c}_U}\right)^{\vartheta}$

The risk of the levered pre-merger combined equity [9] *is:*

$$\beta_S = \gamma + \gamma\frac{F_S}{E_S(c_t)} + \gamma\frac{OD_S(c_t)}{E_S(c_t)} + \tilde{\mathcal{L}}(c_t)\frac{OM_D^C(c_t) + OM_U^C(c_t)}{E_S(c_t)} + \tilde{\mathcal{H}}(c_t)\frac{OM_D^P(c_t) + OM_U^P(c_t)}{E_S(c_t)} \tag{37}$$

where $F_S = \frac{b_D}{r} + \frac{b_U}{r}$, $OD_S = \left(\frac{\Pi_D(\bar{c}_D)}{r-\xi} - \frac{b_D}{r}\right)\left(\frac{c_t}{\bar{c}_D}\right)^{\vartheta} + \left(\frac{\Pi_U(\bar{c}_U)}{r-\xi} - \frac{b_U}{r}\right)\left(\frac{c_t}{\bar{c}_U}\right)^{\vartheta}$, $\tilde{\mathcal{L}}(c_t) = \frac{\vartheta c_t^{\vartheta}\underline{c}_I^{\lambda} - \lambda c_t^{\lambda}\underline{c}_I^{\vartheta}}{\underline{c}_I^{\lambda}\bar{c}_U^{\vartheta} - \bar{c}_U^{\lambda}\underline{c}_I^{\vartheta}}$, $\tilde{\mathcal{H}}(c_t) = \frac{\lambda c_t^{\lambda}\bar{c}_U^{\vartheta} - \vartheta c_t^{\vartheta}\bar{c}_U^{\lambda}}{\underline{c}_I^{\lambda}\bar{c}_U^{\vartheta} - \bar{c}_U^{\lambda}\underline{c}_I^{\vartheta}}$, $OM_D^P = (1-\delta)(E_M^P(\underline{c}_I) - X) - \frac{\Pi_D(\underline{c}_I)}{r-\xi} + \frac{b_D}{r}$, $OM_D^C = E_M^C(\bar{c}_U) - \frac{\Pi_D(\bar{c}_U)}{r-\xi} + \frac{b_D}{r} - X_C$, $OM_U^P = \delta(E_M^C(\underline{c}_I) - X) - \frac{\Pi_U(\underline{c}_I)}{r-\xi} + \frac{b_U}{r}$, $OM_U^C = -\frac{\Pi_U(\bar{c}_U)}{r-\xi} + \frac{b_U}{r}$

If the merger occurred during economic upturn the risk of the levered post-merger equity is:

$$\beta_M^P = \gamma + \gamma\frac{F_M^P}{E_M^P(c_t)} + (\gamma - \vartheta)\frac{OD_M^P(c_t)}{E_M^P(c_t)} \tag{38}$$

where $F_M^P = \frac{b_M^P}{r}$ *and* $OD_M^P = \left(\frac{\Pi_M^P(\bar{c}_M^P)}{r-\xi} - \frac{b_M^P}{r}\right)\left(\frac{c_t}{\bar{c}_M^P}\right)^{\vartheta}$

[9]The beta of a sum of the downstream and upstream equity value is the weighted average of beta of the downstream firm and beta of the upstream firm: $\beta_S = \frac{E_D}{E_D+E_U}\beta_D + \frac{E_U}{E_D+E_U}\beta_U = \frac{\frac{\partial E_D}{\partial c_t}c_t + \frac{\partial E_U}{\partial c_t}c_t}{E_D+E_U}$

If the merger occurred during economic downturn the risk of the levered post-merger equity is:

$$\beta_M^C = \gamma + \gamma \frac{F_M^C}{E_M^C(c_t)} + (\gamma - \vartheta) \frac{OD_M^C(c_t)}{E_M^C(c_t)} \tag{39}$$

where $F_M^C = \frac{b_M^C}{r}$ *and* $OD_M^C = \left(\frac{\Pi_M^C(\bar{c}_M^C)}{r-\xi} - \frac{b_M^C}{r}\right)\left(\frac{c_t}{\bar{c}_M^C}\right)^{\vartheta}$

During the merger episode the functional form of the equity value changes and therefore the systematic risk of the firm changes. Proposition 4 summarizes the main factors that affect the risk of a firm in a period preceding the merger and in a subsequent phase.

The first term in all expressions is the revenue beta (normalized to gamma) or the risk of the unlevered firm which consists of the risk of assets in place and the risk of fixed operating costs. [10]

The second term in all expressions shows the effect of financial leverage on the riskiness of the equity. The higher the debt coupon the higher the risk of the equity.

The third term in equation (38) and (39) reflects on the effect of the option to default. The higher the probability of default, that is $(c_t/\bar{c}_M^P)^{\vartheta}$, the higher the value of the option to default as a fraction of firm's equity. The debt is risky, therefore $\frac{\Pi_M^P(\bar{c}_M^P)}{r-\xi} < \frac{b_M^P}{r}$ and $\frac{\Pi_M^C(\bar{c}_M^C)}{r-\xi} < \frac{b_M^C}{r}$. Thus, the option to default has an opposite effect on the risk of the firm, it decreases the beta of the equity.[11]

Equations (35), (36), and (37) are the pre-merger betas for the downstream, upstream, and the sum of both, respectively. Pre-merger betas capture all of the above mentioned effects and additional terms relating to the effect of options to merge during economic downturn and upturn, correspondingly. The option to merge during economic downturn

[10] From Lemma 1 we can prove that the risk of the unlevered firm is γ, by taking the derivative of equation (16) with respect to the state variable.

[11] The value of a beta in absolute values never crosses zero as $\gamma \frac{\Pi_M^C(\bar{c}_M^C)}{r-\xi} - \vartheta \left(\frac{\Pi_M^C(\bar{c}_M^C)}{r-\xi} - \frac{b_M^C}{r}\right)\left(\frac{c_t}{\bar{c}_M^C}\right)^{\vartheta} < 0$

decreases the risk of the equity (reflected by the term $\tilde{\mathcal{L}}(c_t)$, which is positive and decreases the beta in the absolute value). The option to merge during economic upturn increases the risk of the equity (reflected by the term $\tilde{\mathcal{H}}(c_t)$, which is negative and increases the beta in the absolute value).

[Insert Figure 7 here]

We present the risk dynamics in Figure 7. The dotdashed line corresponds to the beta of the pre-merger downstream firm (the acquirer). The solid line corresponds to the beta of the post-merger combined equity value assuming that the merger happened during economic upturn. The dashed line corresponds to the beta of the post-merger combined equity value assuming that the merger happened during an economic downturn. This analysis reveals that during an economic upturn and downturn we can expect risk reduction at the time of integration.

6. Conclusion

This paper develops a real options framework for vertical mergers where the cost of intermediate input is stochastic. We derive the endogenous merger surplus which is a consequence of double markup elimination. We show that vertical mergers arise during economic upturns when the cost of intermediate input is low.

We demonstrate that when both firms are levered there is a propensity to merge vertically during booms and recessions. During the economic upturn the downstream firm, willing to maximize its profits, has incentives to merge as a consequence of overtaking the markup on the upstream market. During an economic downturn the downstream firm has incentives to integrate its activities with the upstream firm to keep the existing supply chain operational. Our model brings support to the existing evidence on the U-shape pattern of merger waves.

The model predicts that pro-cyclical mergers are more likely in industries where firms have high markup and demand is less elastic and counter-cyclical mergers should be expected in industries where firms have low markup and demand is more elastic.

We also address some of the implication of the capital structure for merger decision. In particular, an upstream firm can affect the decision of the downstream firm to delay the timing of a merger during economic upturn by strategically postponing its default.

We present the risk analysis of corporate event such as the decision to merge of two firms operating within the vertical market structure during economic upturn and downturn. We show that vertical mergers are associated with a decrease in systematic risk at the time of the merger.

Appendix A. Proof of Proposition 1

The gross merger surplus is defined as:

$$\Omega = V_M(c_t) - V_D(c_t) - V_U(c_t) = \frac{c_t^{\frac{\hat{\alpha}}{\hat{\alpha}-1}} \hat{A}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} + \hat{\alpha}^{\frac{2}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{2\hat{\alpha}}{1-\hat{\alpha}}} \right)}{r - \xi} \tag{A.1.}$$

As c_t, A, and $r - \xi$ are positive, in order to prove that the gross merger surplus is positive we have to prove that the expression $\left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} + \hat{\alpha}^{\frac{2}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{2\hat{\alpha}}{1-\hat{\alpha}}} \right)$ is positive.

Proof.

$$\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} + \hat{\alpha}^{\frac{2}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{2\hat{\alpha}}{1-\hat{\alpha}}} + \hat{\alpha}^{\frac{1+\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1+\hat{\alpha}}{1-\hat{\alpha}}} > 0$$

$$\left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} \right) - \hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} \right) - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} \left(\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} - \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} \right) > 0$$

$\hat{\alpha}^{\frac{\hat{\alpha}}{1-\hat{\alpha}}} + \hat{\alpha}^{\frac{1}{1-\hat{\alpha}}} < 1$, which is always satisfied.

■

Appendix B. Proof of Lemma 2

The value of the vertical merger option must satisfy the following condition:

$$rVMO(c_t) = \frac{d}{d\Delta} \mathbb{E}[VMO_{t+\Delta}] \Big|_{\Delta=0} \tag{B.1.}$$

If VMO_t is twice-continuously differentiable function of the state variable c_t, then by applying Ito's lemma we obtain:

$$rVMO(c_t) = VMO'(c_t) c_t \mu + VMO''(c_t) c_t^2 \frac{\sigma^2}{2} \tag{B.2.}$$

The general solution is:

$$VMO(c_t) = F_1 c_t^{\vartheta} + F_2 c_t^{\lambda} \tag{B.3.}$$

If $c_t \to \infty$ the value of the option converges to zero. Denote ϑ and λ as the positive and negative roots of the quadratic equation $z(z-1)\frac{\sigma^2}{2} + z\mu = r$. Therefore, the constant F_1 needs to be equal zero.

There exists a threshold $\underline{c}$ such that executing a vertical merger is optimal as soon as the variable c_t hits the threshold $\underline{c}$ from above. At the threshold the value of an option to merge is equal to the generated surplus:

$$VMO(\underline{c}, \underline{c}) = S(\underline{c}) \tag{B.4.}$$

The optimal threshold is the solution to the first-order condition (the the smooth-pasting condition):

$$\frac{\partial VMO(c_t, \underline{c})}{\partial \underline{c}} = 0 \tag{B.5.}$$

Lemma 2 follows.

Appendix C. Proof of Lemma 3 and 4

We assume that a firm is financed with equity and infinite maturity debt, paying a fixed coupon b_M^j, where $j \in P, C$. Assuming that r is a risk free rate and agents are risk-neutral, the firm's equity E_M^j and debt B_M^j must satisfy:

$$rE_M^j = \Pi_M^j - b_M^j + \frac{d}{d\Delta}\mathbb{E}[E_{M,t+\Delta}]\Big|_{\Delta=0} \tag{C.1.}$$

$$rB_M^j = b_M^j + \frac{d}{d\Delta}\mathbb{E}[B_{M,t+\Delta}]\Big|_{\Delta=0} \tag{C.2.}$$

If E^j_M and B^j_M are twice-continuously differentiable functions of the state variable c_t, then by applying Ito's lemma we get:

$$rE^j_M(c_t) = \Pi^j_M + E'(c_t)c_t\mu + E''(c_t)c_t^2\frac{\sigma^2}{2} \tag{C.3.}$$

$$rB^j_M(c_t) = b^j_M + B'(c_t)c_t\mu + B''(c_t)c_t^2\frac{\sigma^2}{2} \tag{C.4.}$$

The general solutions are:

$$E^j_M(c_t) = A_0 + A_1 c_t + A_2 c_t^{\vartheta} + A_3 c_t^{\lambda} \tag{C.5.}$$

$$B^j_M(c_t) = H_0 + H_1 c_t + H_2 c_t^{\vartheta} + H_3 c_t^{\lambda} \tag{C.6.}$$

The solution is found by the following boundary conditions. If the firm is closed then: $E(\bar{c}_i) = 0$. As $c_t \to 0$ the possibility of bankruptcy is not likely so E^j_M and B^j_M approach the unlimited liability values, thus:

$$lim_{c_t\to 0}E^j_M(c_t) = \frac{\Pi^j_M}{r-\xi} - \frac{b^j_M}{r} \tag{C.7.}$$

$$lim_{c_t\to 0}B^j_M(c_t) = \frac{b^j_M}{r} \tag{C.8.}$$

Lemma 3 and 4 follow.

Appendix D. Proof of Proposition 2

We assume that $c_0 \in (\underline{c}_I, \bar{c}_U)$. Over this region the instantaneous change in the value of the downstream and the upstream equity satisfies the Bellman equation of the following from:

$$rE_i = \Pi_i - b_i + \frac{d}{d\Delta}\mathbb{E}[E_i t + \Delta]\Big|_{\Delta=0} \tag{D.1.}$$

Applying Ito's lemma we can show that the right hand side of the above equation is equal:

$$rE_i(c_t) = \Pi_i(c_t) - b_i + \mu c_t E_i'(c_t) + \frac{\sigma^2}{2}c_t^2 E_i''(c_t) \tag{D.2.}$$

The general solutions for the downstream and the upstream firm respectively are given as:

$$E_D(c_t) = \frac{\Pi_D}{r-\xi} - \frac{b_D}{r} + A_{D1}c_t^{\vartheta} + A_{D2}c_t^{\lambda} \qquad for \qquad c_I < c_t < c_U \tag{D.3.}$$

and

$$E_U(c_t) = \frac{\Pi_U}{r-\xi} - \frac{b_U}{r} + A_{U1}c_t^{\vartheta} + A_{U2}c_t^{\lambda} \qquad for \qquad c_I < c_t < c_U \tag{D.4.}$$

where ϑ and λ are respectively the positive and negative roots of the quadratic equation $z(z-1)\frac{\sigma^2}{2} + z\mu = r$.

To determine the values of A_{D1}, A_{D2}, A_{U1}, A_{U2}, c_I and δ we use the boundary conditions for equity value of each firm at the thresholds $\underline{c}_I$ and $\overline{c}_U$. The constants (A's) are calculated from the value matching conditions.

The merger threshold during economic upturn and the unique sharing rule is then pinned down from the smooth-pasting condition at the threshold $\underline{c}_I$. The merger threshold during economic upturn is a solution to the smooth-pasting condition at the threshold $\overline{c}_U$

When $c = \underline{c}_I$ the equity value of the upstream firm equals a share δ of the integrated firm:

$$E_U(\underline{c}_I) = \delta(E_M^C(\underline{c}_I) - X) \tag{D.5.}$$

The value of the downstream firm equity is equal then at the threshold $c = \underline{c}_I$:

$$E_D(\underline{c}_I) = (1-\delta)(E_M^C(\underline{c}_I) - X) \tag{D.6.}$$

The value of the upstream firm at the pro-cyclical meregr threshold is greater the higher the δ and therefore the value of the downstream firm is lower.

When the state variable hits the upstream firm bankruptcy threshold $\overline{c}_U$ then the upstream firm equityholders liquidate the company. Their share in the new company is then zero and the downstream firm obtains 100% share in the integrated company ($\delta = 0$).

$$E_U(\overline{c}_U) = 0 \tag{D.7.}$$

To stay in operation the downstream firm can buy the upstream firm assets after liquidation and their value is pinned down by the following condition:

$$E_D(\overline{c}_U) = E_M^B(\overline{c}_U) - X_C \tag{D.8.}$$

From equations (E.5.), (E.6.), (E.7.), and (E.8.) we obtain:

$$A_{U1} = \frac{\overline{c}_U^{\lambda}\left[\delta\left(E_M^C(\underline{c}_I) - X\right) - \frac{\Pi_U(\underline{c}_I)}{r-\xi} + \frac{b_U}{r}\right] + \underline{c}_I^{\lambda}\left[-\frac{\Pi_U(\overline{c}_U)}{r-\xi} + \frac{b_U}{r}\right]}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \tag{D.9.}$$

$$A_{D1} = \frac{\underline{c}_I^{\lambda}\left(E_M^B(\overline{c}_U) - \frac{\Pi_D(\overline{c}_U)}{r-\xi} + \frac{b_D}{r} - X_C\right)}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} - \\ - \frac{\overline{c}_U^{\lambda}\left[\frac{\Pi_D(\underline{c}_I)}{r-\xi} - \frac{b_D}{r}\left(1-\delta\right)\left(E_M^C(\underline{c}_I) - X\right)\right]}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \tag{D.10.}$$

$$A_{U2} = \frac{\overline{c}_U^{\lambda}\left[\delta\left(E_M^B(\underline{c}_I) - X\right) - \frac{\Pi_U(\underline{c}_I)}{r-\xi} + \frac{b_U}{r}\right] - \underline{c}_I^{\vartheta}\left[-\frac{\Pi_U(\overline{c}_U)}{r-\xi} + \frac{b_U}{r}\right]}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \tag{D.11.}$$

$$A_{D2} = \frac{\underline{c}_I^{\vartheta}\left[E_M^B(\overline{c}_U) - \frac{\Pi_D(\overline{c}_U)}{r-\xi} + \frac{b_D}{r} - X_C\right] - \overline{c}_U^{\vartheta}\left[(1-\delta)\left(E_M^C(\underline{c}_I) - X\right) - \frac{\Pi_D(\overline{c}_U)}{r-\xi} + \frac{b_D}{r}\right]}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \tag{D.12.}$$

Denote by $\mathcal{L}(c_t)$ the present value of \$1 to be received the first time c_t reaches $\overline{c}_U$ conditional on reaching $\overline{c}_U$ before reaching $\underline{c}_I$. Similar, denote by $\mathcal{H}(c_t)$ the present value of \$1 to be received the first time c_t reaches $\underline{c}_I$ conditional on reaching $\underline{c}_I$ before reaching $\overline{c}_U$. Substituting constants into equations (E.3.) and (E.4.) we obtain:

$$\begin{aligned} E_D(c_t) = \frac{\Pi_D(c_t)}{r-\xi} - \frac{b_D}{r} + \mathcal{L}(c_t)\left[E_M^B(\overline{c}_U) - \frac{\Pi_D(\overline{c}_U)}{r-\xi} + \frac{b_D}{r} - X_C\right] + \\ + \mathcal{H}(c_t)\left[(1-\delta)\left(E_M^C(\underline{c}_I) - X\right) - \frac{\Pi_D(\underline{c}_I)}{r-\xi} + \frac{b_D}{r}\right] \end{aligned} \tag{D.13.}$$

and:

$$E_U(c_t) = \frac{\Pi_U(c_t)}{r-\xi} - \frac{b_U}{r} + \mathcal{L}(c_t)\left[-\frac{\Pi_U(\overline{c}_U)}{r-\xi} + \frac{b_U}{r}\right] + \mathcal{H}(c_t)\left[\delta\left(E_M^C(\underline{c}_I) - X\right) - \frac{\Pi_U(\underline{c}_I)}{r-\xi} + \frac{b_U}{r}\right] \tag{D.14.}$$

where the stochastic discount factors $\mathcal{L}(c_t)$ and $\mathcal{H}(c_t)$ are defined as:

$$\mathcal{L}(c_t) = \frac{c_t^{\vartheta}\underline{c}_I^{\lambda} - c_t^{\lambda}\underline{c}_I^{\vartheta}}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \quad \text{and} \quad \mathcal{H}(c_t) = \frac{c_t^{\lambda}\overline{c}_U^{\vartheta} - c_t^{\vartheta}\overline{c}_U^{\lambda}}{\underline{c}_I^{\lambda}\overline{c}_U^{\vartheta} - \overline{c}_U^{\lambda}\underline{c}_I^{\vartheta}} \tag{D.15.}$$

The smooth-pasting condition at the threshold chosen by the downstream firm equityholders helps to determine the merger pro-cyclical threshold and the sharing rule. Therefore the following set of conditions arise:

$$\left.\frac{\partial E_U(c_t)}{\partial c_t}\right|_{c=\underline{c}_I} = \delta \left.\frac{\partial E_M^C(c_t)}{\partial c_t}\right|_{c=\underline{c}_I} \tag{D.16.}$$

$$\left.\frac{\partial E_D(c_t)}{\partial c_t}\right|_{c=\underline{c}_I} = (1-\delta) \left.\frac{\partial E_M^C(c_t)}{\partial c_t}\right|_{c=\underline{c}_I} \tag{D.17.}$$

The smooth-pasting condition at liquidation threshold chosen by the upstream firm equityholders helps then to determine the counter-cyclical merger threshold. Therefore the following set of conditions arise:

$$\left.\frac{\partial E_U(c_t)}{\partial c_t}\right|_{c=\bar{c}_U} = 0 \tag{D.18.}$$

Equations (E.16.), (E.17.) and (E.18.) are non-linear in $\underline{c}_I$ and the results are not analytically tractable. The complex solution is available upon request. Numerical analysis helps to determine the necessary conditions to prove that the smooth-pasting condition is satisfied.

Appendix E. Proof of Proposition 3

In order to present the properties of the solution we show the qualitative comparative statics. By taking total derivative we aim to obtain a linear dependence of the pro-cyclical merger threshold on the sharing parameter δ.

We assume that δ changes by $d\delta$ and we verify how the endogenous variables for the upstream firm ($\underline{c}_I$, A_{U1}, and A_{U2}) change. We differentiate totally the value-matching condition for the upstream firm (equation E.5.) at the threshold $\underline{c}_I$ and $\bar{c}_U$.

$$\frac{\partial E_U(\underline{c}_I)}{\partial A_{U1}} dA_{U1} + \frac{\partial E_U(\underline{c}_I)}{\partial A_{U2}} dA_{U2} = d\delta \left(E_M^C(\underline{c}_I) - X \right) \tag{E.1.}$$

$$\frac{\partial E_U(\overline{c}_U)}{\partial A_{U1}} dA_{U1} + \frac{\partial E_U(\overline{c}_U)}{\partial A_{U2}} dA_{U2} = 0 \tag{E.2.}$$

We solve for the changes in the coefficients A_{U1} and A_{U2}, where $\Delta = \underline{c}_I^{\lambda} \overline{c}_U^{\theta} - \underline{c}_I^{\theta} \overline{c}_U^{\lambda}$:

$$dA_{U1} = -\frac{d\delta \left(E_M^C(\underline{c}_I) - X \right) \overline{c}_U^{\lambda}}{\Delta} \tag{E.3.}$$

$$dA_{U2} = \frac{d\delta \left(E_M^C(\underline{c}_I) - X \right) \overline{c}_U^{\theta}}{\Delta} \tag{E.4.}$$

Then we differentiate the smooth-pasting condition for the upstream firm at $\underline{c}_I$ (equation E.16.) and after substituting we obtain:

$$d\underline{c}_I \left(\frac{\partial^2 E_U(\underline{c}_I)}{\partial^2 \underline{c}_I} - \delta \frac{\partial^2 E_M^C(\underline{c}_I)}{\partial^2 \underline{c}_I} \right) = \frac{d\delta \left(E_M^C(\underline{c}_I) - X \right) \left(\theta \underline{c}_I^{\theta-1} \overline{c}_U^{\lambda} - \lambda \underline{c}_I^{\lambda-1} \overline{c}_U^{\theta} \right)}{\Delta} \tag{E.5.}$$

The expression in brackets on the left hand side is positive since the equity value is a convex function. The first expression in brackets on the right hand side is positive since at the threshold the claim value should be higher than the exercise price. The second expression in brackets on the right hand side is positive and Δ is positive given $\underline{c}_I < \overline{c}_U$, $\lambda < 0$, and $\theta > 1$. Therefore, the result suggest that the higher the share of the upstream firm in the integrated firm the quicker the upstream firm is willing to enter the merger.

For the downstream firm differentiating the smooth-pasting condition at $\underline{c}_I$ (equation E.17.) we obtain:

$$d\underline{c}_I \left(\frac{\partial^2 E_D(\underline{c}_I)}{\partial^2 \underline{c}_I} - (1-\delta) \frac{\partial^2 E_M^C(\underline{c}_I)}{\partial^2 \underline{c}_I} \right)$$

$$=$$

$$\frac{\left[d\delta \left(E_M^C(\underline{c}_I) - X \right) + d\overline{c}_U \underline{c}_I \left(\frac{\partial E_M^B(\overline{c}_U)}{\partial \overline{c}_U} - \frac{\partial E_D(\overline{c}_U)}{\partial \overline{c}_U} \right) \right] \left(\theta \underline{c}_I^{\theta-1} \overline{c}_U^{\lambda} + \lambda \underline{c}_I^{\lambda-1} \overline{c}_U^{\theta} \right)}{\Delta} \tag{E.6.}$$

The expression in brackets on the left hand side is positive since the equity value is a convex function. The last expression in brackets on the right hand side is negative and Δ is positive given $\underline{c}_I < \overline{c}_U$, $\lambda < 0$, and $\theta > 1$. The expression that stands with $(d\delta)$ is positive at the exercise threshold. Therefore, the change in the sharing parameter $(d\delta)$ is associated with a negative change in the integration threshold (delays it). Therefore, the results suggest that the higher the share of the upstream firm in the integrated firm the latter the downstream firm is willing to merge waiting for a larger merger surplus. The expression that stands with $(d\overline{c}_U)$ is positive as the slope of $E_M^B(\overline{c}_U)$ is less negative than the slope of $E_D(\overline{c}_U)$. Thus, the positive change in the default threshold of the upstream firm (the smaller the coupon) delays integration during economic upturn. Proposition 3 follows.

Appendix F. Proof of Proposition 4

We define the risk of a firm as in Carlson, Fisher, and Giammarino (2004) where they trace the risk profile of the firm by its beta. They prove that the beta of the firm can be derived in a form of the elasticity, thus:

$$\beta_i = \frac{\partial E_i(c_t)}{\partial c_t} \frac{c_t}{E_i(c_t)} \tag{F.1.}$$

The risk of the levered post-merger equity assuming the merger occurred during economic upturn, over the interval $0 < c_t < \underline{c}_I$ is:

$$\beta_M^P = \frac{\gamma \frac{\Pi_M^P(c_t)}{r-\xi} - \vartheta \left(\frac{\Pi_M^P(c_t)}{r-\xi} - \frac{b_M^P}{r} \right) \left(\frac{c_t}{\bar{c}_M^P} \right)^{\vartheta}}{E_M^P(c_t)} \tag{F.2.}$$

The risk of the levered pre-merger downstream and upstream equity respectively, over the interval $\underline{c}_I < c_t < \bar{c}_U$ is:

$$\beta_D = \frac{\gamma \frac{\Pi_D(c_t)}{r-\xi} + \tilde{\mathcal{L}}(c_t) OM_D^C + \tilde{\mathcal{H}}(c_t) OM_D^P}{E_D(c_t)} \tag{F.3.}$$

$$\beta_U = \frac{\gamma \frac{\Pi_U(c_t)}{r-\xi} + \tilde{\mathcal{L}}(c_t) OM_U^C + \tilde{\mathcal{H}}(c_t) OM_U^P}{E_U(c_t)} \tag{F.4.}$$

Suppose β_S is the pre-merger risk of the portfolio of two firms the upstream firm and the downstream firm and $E_S = E_U + E_D$, where $\beta_S = \frac{E_D}{E_S}\beta_D + \frac{E_U}{E_S}\beta_U = \frac{\frac{\partial E_D}{\partial c_t} c_t + \frac{\partial E_U}{\partial c_t} c_t}{E_S}$, $\underline{c}_I < c_t < \bar{c}_U$ is:

$$\beta_S = \frac{\gamma \left(\frac{\Pi_U(c_t)}{r-\xi} + \frac{\Pi_D(c_t)}{r-\xi} \right) + \tilde{\mathcal{L}}(c_t)(OM_U^C + OM_D^C) + \tilde{\mathcal{H}}(c_t)(OM_U^P + OM_D^P)}{E_S(c_t)} \tag{F.5.}$$

The risk of the levered post-merger equity assuming the merger occurred during economic downturn, $\bar{c}_U < c_t$ is:

$$\beta_M^C = \frac{\gamma \frac{\Pi_M^C(c_t)}{r-\xi} - \vartheta \left(\frac{\Pi_M^C(c_t)}{r-\xi} - \frac{b_M^C}{r} \right) \left(\frac{c_t}{\bar{c}_M^C} \right)^{\vartheta}}{E_M^C(c_t)} \tag{F.6.}$$

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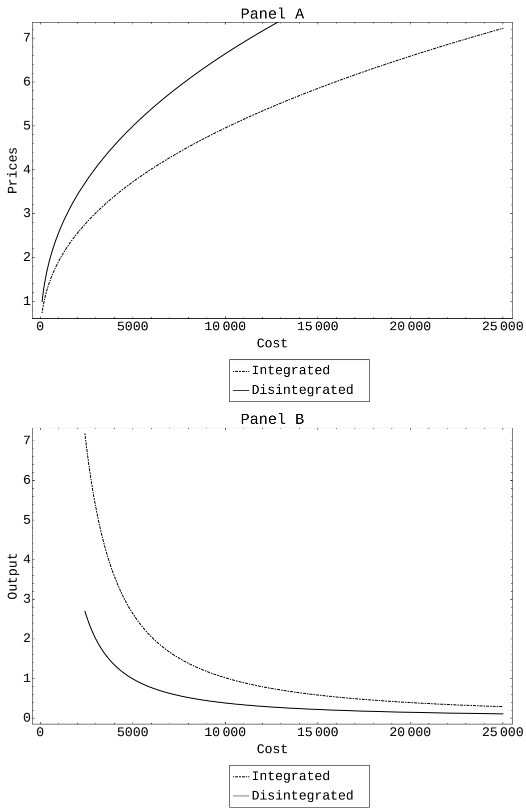

Figure 1: **Prices and output of the integrated and disintegrated firm.** For this figure, the following parameters are fixed: $A = 30$, $a = 30$, $\alpha = 0.7$ and $\varepsilon = 0.3$.

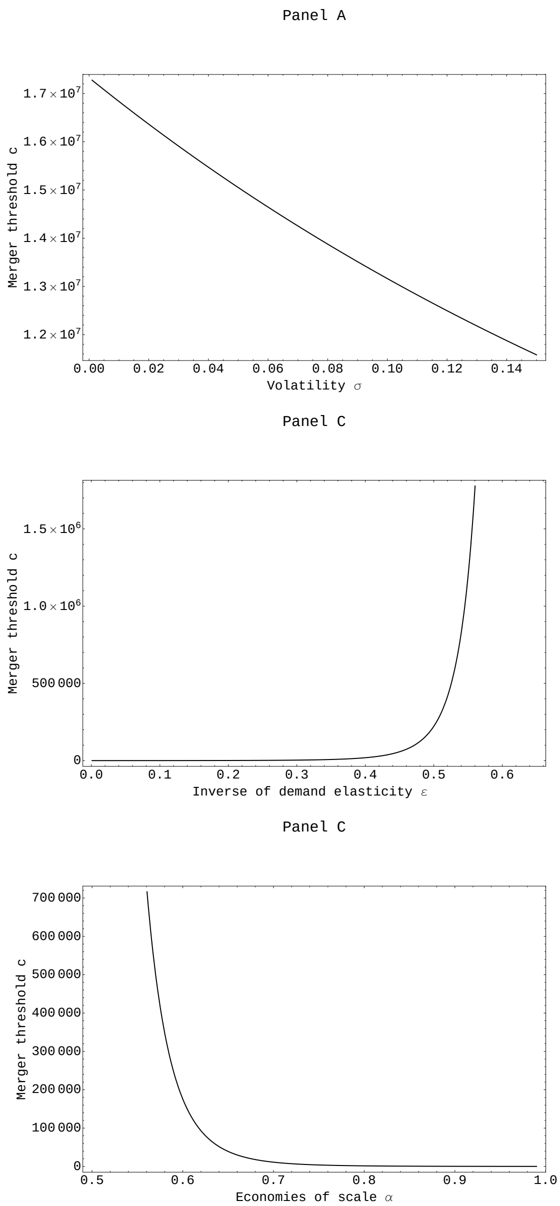

Figure 2: **Merger threshold (unlevered firm).** The figure illustrates the effect of volatility (σ), inverse of demand elasticity (ε), and economies of scale (α) on a merger threshold of the unlevered firm. The following values are set fixed: $\alpha = 0.5$, $\varepsilon = 0.6$, $\sigma = 0.2$, $A = 30$, $a = 30$, $\mu = 0.9 * \sigma^2/2$, $r = 0.06$,and $X = 50$.

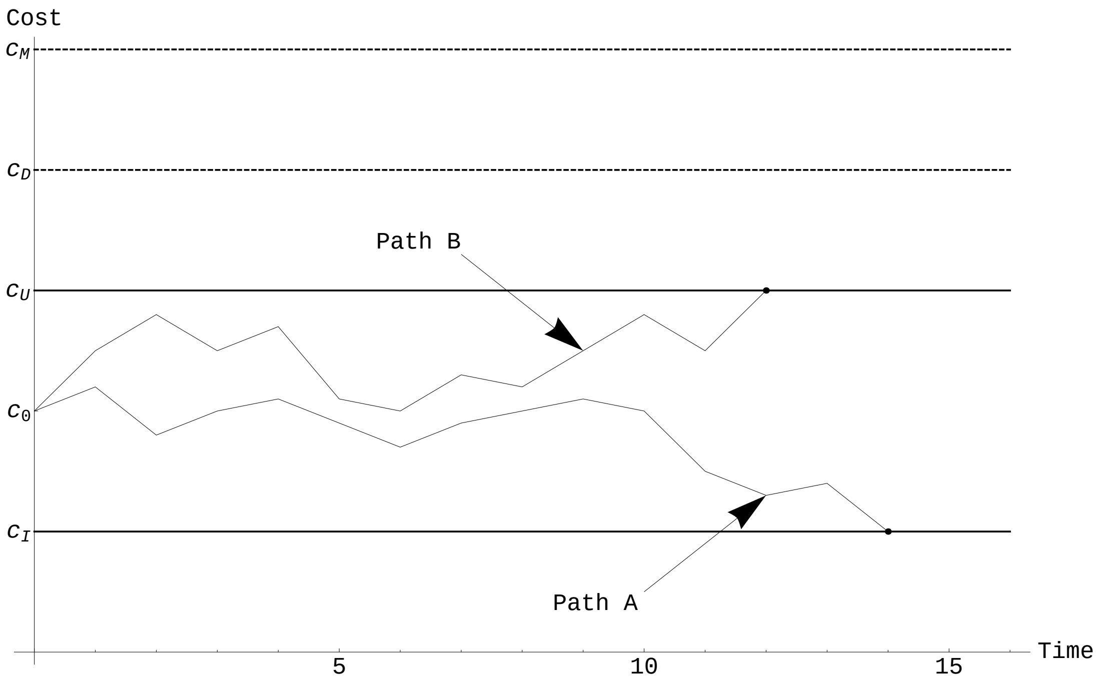

Figure 3: **Time sequence of events.** This figure plots possible paths for the stochastic process c_t. There are two possible scenarios for firms to merge. Path A represents the case in which firms integrate during the economic upturn. Path B represents the case in which firms merge during the economic downturn.

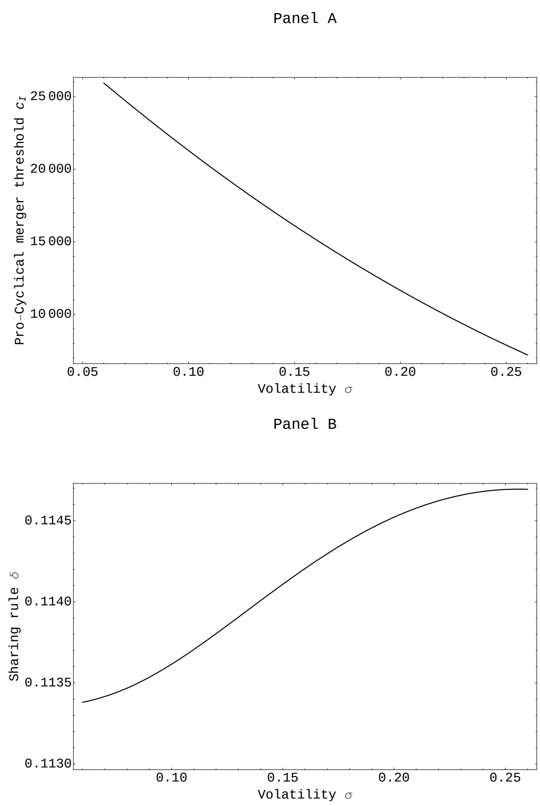

Figure 4: **Merger threshold and sharing rule.** This figure presents the solution for the pro-cyclical merger threshold $\underline{c}_I$ (Panel A) and the sharing rule δ (Panel B) as a function of volatility σ for the following parameters values: $\alpha = 0.5$, $\varepsilon = 0.6$, $A = 30$, $a = 30$, $\mu = 0.9 * \sigma^2/2$, $r = 0.06$, $b_M^P = 8.5$, $b_M^C = 7$, $b_U = 1.5$, $b_D = 7$, and $X = 50$.

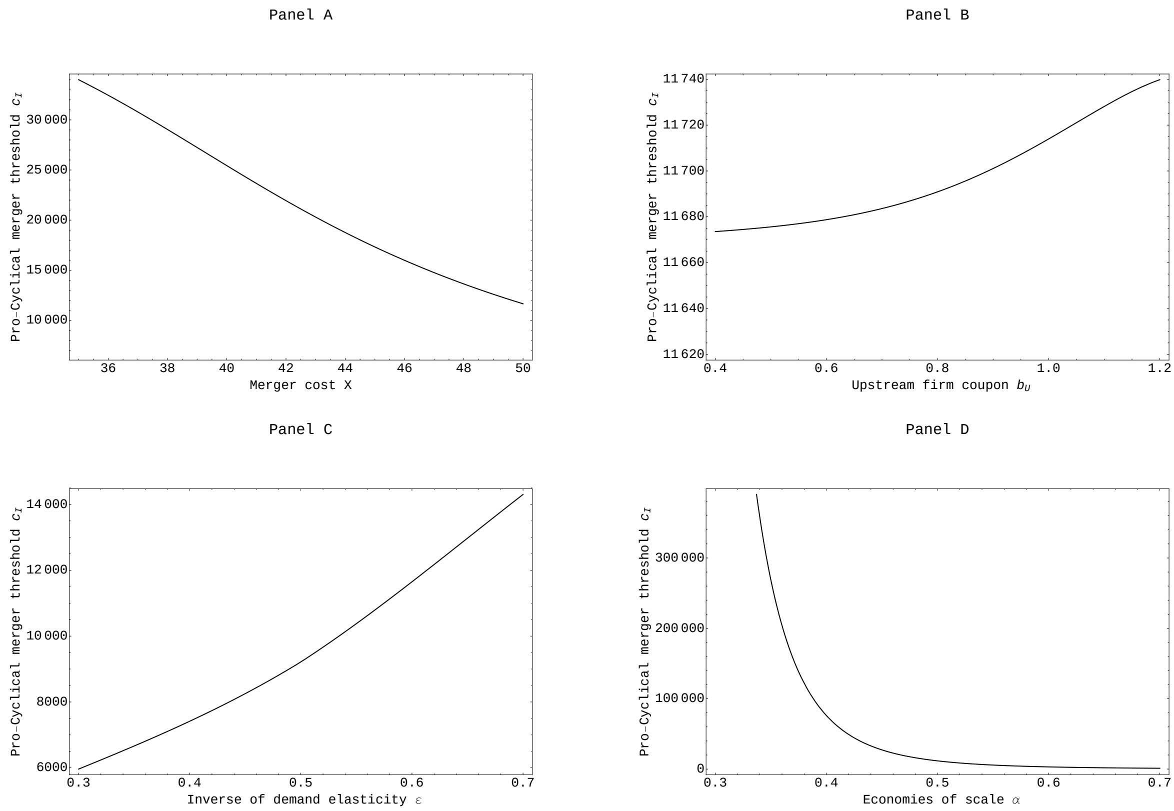

Figure 5: **Pro-cyclical merger threshold (levered firm).** The figure illustrates the effect of merger cost (X), coupon of the upstream firm (b_U), inverse of demand elasticity (ε), and economies of scale (α) on the pro-cyclical merger threshold of the levered firm. The parameters are set as in the base case.

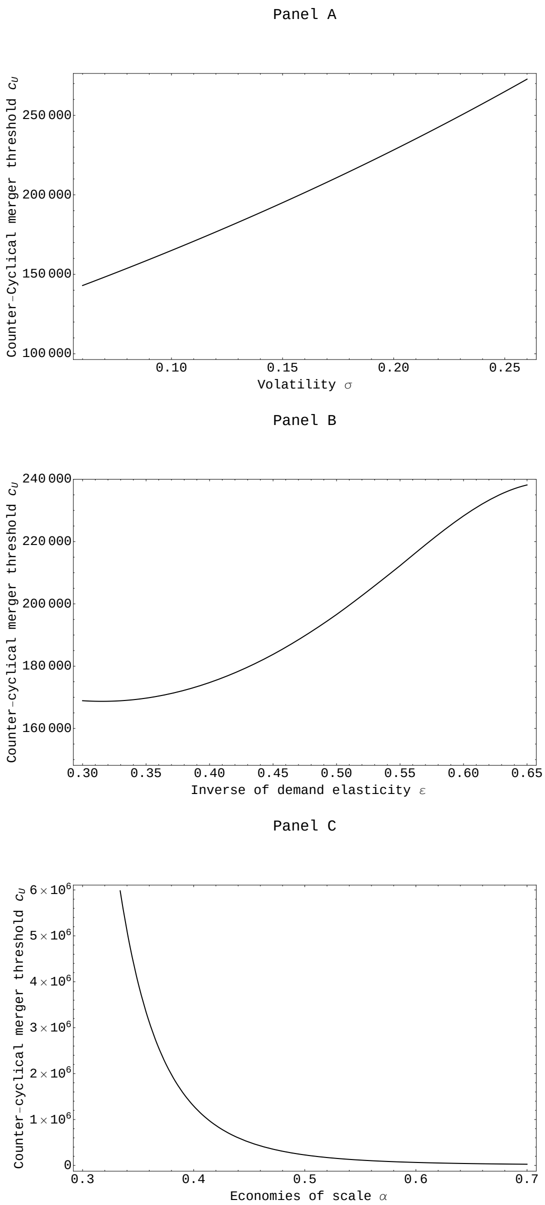

Figure 6: **Counter-cyclical merger threshold (levered firm).** The figure illustrates the effect of volatility (σ), inverse of demand elasticity (ε), and economies of scale (α) on a counter-cyclical merger threshold of the levered firm. Parameters are set as in the base case.

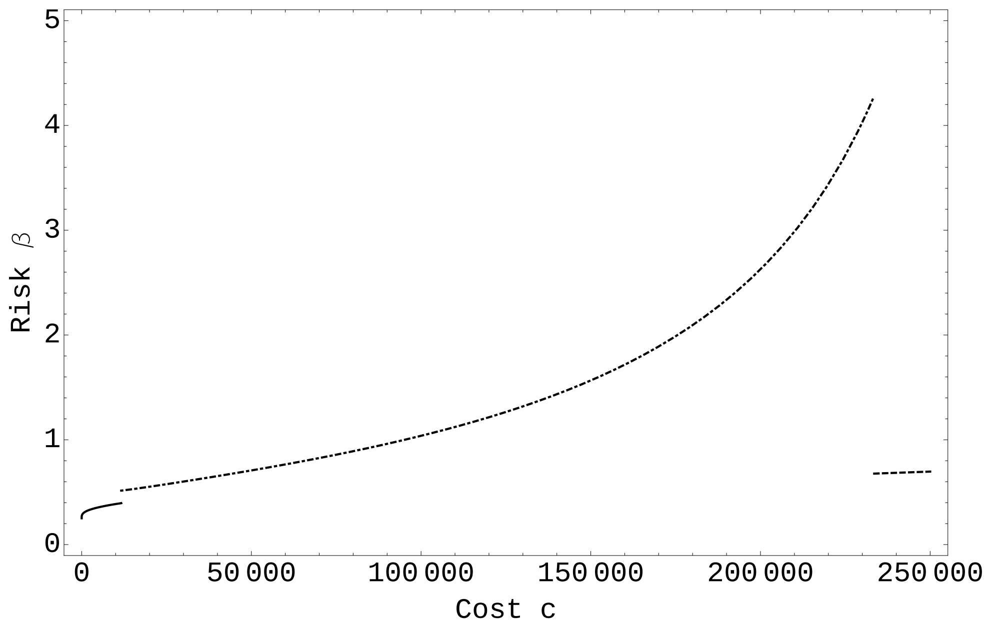

Figure 7: **Pre- and post-merger beta.** This figure presents betas as a function of cost c_t. The solid line corresponds to the beta of the post-merger combined equity value assuming that merger happened during economic upturn. The dotdashed line corresponds to the beta of the pre-merger downstream equity values. The dotted line corresponds to the beta of the post-merger combined equity value assuming that merger happened during economic downturn. The parameters are set as in the base case.