Investment Horizons and Asset Prices under Asymmetric Information *

Elias Albagli

(Preliminary, comment welcome)

October 30, 2012

Abstract

I study a fully dynamic rational expectations economy with asymmetric information, where agents have finite investment horizons; T. Horizons affects asset prices through two key mechanisms: as T increases, 1) the age-adjusted risk aversion of the average investor falls, and 2) the risk transfer from forced liquidators into voluntary buyers drops. There are typically two equilibria: a stable equilibrium in which higher T lowers price volatility, and an unstable one with the opposite properties. Moreover, equilibria that fail to exist for low T can be recovered for high enough lifespan. Along the stable equilibrium, increasing T lowers price volatility and alleviates the uncertainty of uninformed investors. The low risk environment induces aggressive trading by the informed, which impound their knowledge into prices. Expected returns and return volatility are similar to an economy with full-information about fundamentals, even if the informed are relatively few. For short horizons, cautious trading disaggregates information from prices and the economy approaches one with no private information. Consistent with evidence of increased fund liquidations during episodes of financial distress, the results suggest that heightened volatility and uncertainty can be explained by the shortening of investor horizons in a rational economy with asymmetric information.

JEL codes: E23, E32, G12, G14, G23.

Keywords: Investment horizons, asymmetric information, asset prices.

* University of Southern California, Marshall FBE. F: 213-7406418. albagli@marshall.usc.edu.

1 Introduction

It has long been recognized that short investment horizons can be a destabilizing force in asset markets. Vast research in the limits of arbitrage underscores how sophisticated traders investing other people's money have good reasons to be concerned about short term performance, perhaps even more so than about long-run returns. While this view has come a long way in the past 20 years, much less understood is the role played by short investment horizons during financial crises. In particular, what role does short-term speculation have in jointly explaining the following stylized facts observed during episodes of market turbulence: i) a tightening of funding constraints of financial intermediaries in the form of capital outflows, and the consequent increase in fund liquidations; ii) sharp declines in asset prices and increases in volatility and expected returns; and iii) spikes in market- and survey-based measures of economic uncertainty? This paper contributes to filling this gap by providing a model where investors trade an infinitely lived asset, but have (arbitrary) finite investment horizons, T. The model then studies how variations in horizons affect asset returns in equilibrium. In other words, the exercise performed here captures fact i) above –increased fund outflows during crises– by reducing the effective investing horizons of traders, and studies how it relates to facts ii) and iii).

The model is based on the dynamic rational expectations equilibrium analysis pioneered by Wang (1994). The economy is made up of a unit measure of competitive investors who trade an infinitely-lived asset to maximize utility of lifetime consumption under CARA preferences. There are two types of investors: those who observe private information about the persistent component of the dividend process (informed investors), and those who must infer it from dividends and prices (uninformed investors). Investors also differ in their age. At any point in time, there are T generations of investors coexisting. $T-1$ groups of equal mass (aged $1, 2, ...T-1$) are still active in the market and can take voluntary positions, while the oldest generation (aged T) is exiting the economy and must unwind its positions at prevailing prices. The net supply of the asset is random and causes prices to fluctuate for reasons orthogonal to fundamentals. This prevents prices from fully revealing the information observed by informed investors.

In general terms, the central finding of the paper is that investment horizons matter a great deal for asset pricing and market efficiency. The generalized OLG economy developed here highlights two key and novel mechanisms that account for this result. The first relates to the "pricing" of risk, which I label the *age-adjusted risk aversion* effect. As investors live for more periods, they are willing to absorb the liquidations of the dying generation at lower expected returns, since they can smooth their consumption over a longer lifespan and are less exposed to temporary price deviations caused by the random asset supply. The second mechanism relates to the "quantity" of risk that active investors must bear in equilibrium, which I label the *risk transfer* effect. As investors live longer, the relative size of the oldest generation (forced liquidators) shrinks in relation to the mass of active investors (voluntary traders). Since both mechanisms work in the same direction, longer investment horizons lower the the risk premium, and mitigate price volatility arising from random supply innovations. Moreover, since price volatility is dominated by innovations in fundamentals for long horizons, prices are more informative for investors who observe only public information. The market is then not only more stable, but also more efficient in the informational sense as investment horizons increase.

More specifically, the paper makes three contributions. The first is the characterization of existence, multiplicity, and stability properties of linear equilibria in generalized OLG models. By building a parsimonious environment able to encompass arbitrary investing horizons and heterogeneous information structures, the model nests a wide variety of economies whose equilibrium properties have not been addressed in prior work. In terms of existence, the results show that for any given set of parameters describing preferences and stochastic processes, economies which fail to exhibit equilibria for low investment horizons will always have equilibria for large enough T. The reverse interpretation of this result is that market equilibrium can break down as horizons shorten. Regarding multiplicity, a finite investment horizon economy generically exhibits two equilibria (whenever equilibria exists), a result consistent with the findings of Spiegel (1998) and Watanabe (2008) for 2-period OLG economies.[1] These include a stable, low volatility equilibrium where innovation in the asset supply have small price impact, and an unstable equilibrium where they cause high volatility in returns. In this respect, the contribution of the present paper is describing the evolution of pricing moments along these equilibria as a function of the investment horizon. Along the stable, low volatility equilibrium, increases in T lower the price impact of asset supply innovations, decreasing price volatility. As $T \to \infty$, the low volatility equilibrium converges smoothly to the case studied by Wang (1994). Along the unstable equilibrium however, longer investment horizons leads to unbounded increases in price volatility. In the limit, this second equilibrium vanishes as $T \to \infty$. Intuitively, as investors live for more periods, both the increased willingness to take risks and the larger proportion of voluntary trades with respect to forced liquidations makes the high volatility equilibrium increasingly "difficult" to sustain.

As a second contribution, the paper shows that the asset pricing implications of different investment horizons rely crucially on the two afore-mentioned mechanisms, both of which are –to the best of my knowledge– new to the literature. To understand the first mechanism, the *age-adjusted risk aversion* effect, consider a hypothetical economy where all investors are infinitely lived. In this economy, the marginal propensity to consume out of wealth is the ratio between the net and gross interest rate, r/R. This coefficient is precisely how agents price the uncertainty regarding wealth fluctuations –the "age-adjusted" risk aversion parameter of all agents corresponds to $\gamma \cdot r/R$, where γ is the CARA parameter.[2] In the other extreme case in which agents live only 2 periods, the marginal propensity to consume wealth is one, and the effective risk aversion equals γ. In the present model, the pricing of risk depends on the age of the investor. Importantly, as horizons increase, the average age-adjusted risk aversion declines. The economics behind the second mechanism –the *risk transfer* effect– are as follows. Consider once again an economy with infinitely-lived agents. Because investors always trade voluntarily, there is no forced transfer of risk between generations, and all agents bear the aggregate risk proportionally. In the other extreme when agents live for two periods, the dying generation (in mass 1/2) must unload all its positions into a single younger generation (also in mass 1/2). In other words, the whole aggregate risk of the economy must exchange hands every period! The generalized OLG economy studied here essentially spans the whole intermediate region of investment horizons left out by the extreme cases, and shows how increases in T lead to a reduction in the relative transfer of risk from the dying to all other generations.

[1] Two equilibria arise in the single asset case, as studied here. In the N-risky asset case, there exists 2^N equilibria.

[2] This is the economy considered by Wang (1994).

The third contribution of the paper is the characterization of asset price informativeness and uncertainty as a function of investors' horizons. I study the behavior of asset prices along the stable equilibrium for three generic economies: a full-information benchmark where all investors observe the persistent component of dividends; a no-information economy in which all investors observe only the history of dividends and prices; and the asymmetric-information economy where only a relatively small mass of investors has access to persistent payoff information. A comparison between these economies reveals the following results: a) For long horizons, the intermediate economy behaves similarly to the full information benchmark. The low risk environment implied by large T induces active trading of informed investors in response to private information, which then gets impounded into prices. Uninformed investors extract precise information from prices, which reduces the level of uncertainty of the average agent. In this economy, price movements are largely driven by fundamental volatility, and expected returns and return volatility closely mimic the full-information case. b) For short investment horizons, the asymmetric information economy approaches the no-information benchmark. The high risk implied by small T leads informed investors to trade more cautiously, disaggregating information from prices and increasing uncertainty about fundamental asset values for the uninformed. In this economy, price movements are largely driven by supply innovations, and expected returns and return volatility line up closely with the no-information case.

The model presented here is most closely related to the literature studying trading in OLG environments. De Long et al. (1990), as well as Spiegel (1998), study economies with 2-period lived investors. Spiegel (1998) is closest to the present paper as in his model all investors are rational, and the random component of returns comes from (rather small) random innovations in the asset supply. Watanabe (2008) extends Spiegel's model to introduce asymmetric information about forthcoming dividends. In all these models however, investor horizons are fixed. The discussion on how the economy can transition between episodes of high and low price volatility therefore remains, by construction, relegated to an equilibrium switching argument *only*. He and Wang (1995), and Cvitanić et al. (2006), study finite horizon economies with incomplete information. Since agents derive utility only from terminal wealth, the age-adjusted risk aversion coincides with the CARA parameter in both papers. Moreover, in these papers all investors grow old simultaneously, so there is no risk transfer from dying to active generations. The two main forces at work in the present paper are therefore quite different.

Other related papers study the impact of short-term investors in the context of 3-period models. In Froot et al. (1992), investors might choose to study information unrelated to fundamentals to the extent it can predict short-term price movements. Kondor (2012) studies an economy with short-term traders, focusing on how public disclosures can simultaneously increase divergence of (rational) beliefs about returns while lowering the conditional uncertainty about fundamentals. Cespa and Vives (2012) focus on how persistent noise trading can generate two equilibria even in a finite horizon economy. Also, in an earlier version of this paper, I study the impact of increased fund liquidations during downturns in effectively lowering investors' horizon, and its implications for price informativeness and expected returns (Albagli, 2009). It is of course difficult to compare the results obtained in a fully dynamic model from those derived from finite horizon environments. The key difference with these papers remains in that the present analysis allows for varying investment horizons –indeed, such variation constitutes the baseline of the results discussed– whereas 3-period models have a rigid lifespan structure.[3]

Finally, the work by Chien et al. (2012) is also related. They build an economy where some investors are intermittent portfolio re-balancers. When equity prices drop following bad shocks, this forces a relatively small mass of sophisticated investors to bear aggregate risk disproportionally, inducing further price declines and an increase in expected returns. The economy they consider has symmetric information, infinitely lived agents and CRRA preferences, so the forces at work are quite different. Nevertheless, a varying mass of investors who must absorb aggregate risk is a common theme in both papers, and so the findings presented here are complementary to their work.

The rest of the paper is organized as follows. Section 2 lays out the model and describes the equilibrium concept. Section 3 presents the characterization of existence, multiplicity, and stability, for economies with symmetric information. Section 4 then focuses on how investment horizons affect the stable equilibrium of the asymmetric information economy, discussing its implications for expected returns, price volatility and uncertainty. Section 5 concludes. All proofs are contained in the appendix.

2 A Generalized OLG model with Asymmetric Information

2.1 Basic Setup

2.1.1 Securities and Payoffs

Time is discrete and runs to infinity. There is a risk-free asset in perfectly elastic supply yielding a gross return of $R = 1+r$, and one risky asset paying an infinite stream of dividends $\{D_\tau\}_{\tau=1}^{\infty}$. Dividends follows a mean-reverting process with unconditional mean $\bar{F}$ and persistence ρ_F (with $0 \leq \rho_F \leq 1$):

$$\begin{aligned} D_t &= F_t + \varepsilon_t^D, \text{ with} \quad (1) \\ F_t &= (1-\rho_F)\bar{F} + \rho_F F_{t-1} + \varepsilon_t^F. \quad (2) \end{aligned}$$

F_t is the persistent payoff component. Due to the disturbances ε_t^D and ε_t^F however, the value of F_t is not revealed by the observation of dividends.

The risky asset supply is given by θ_t, a mean-reverting stochastic process described by

$$\theta_t = (1-\rho_\theta)\bar{\theta} + \rho_\theta \theta_{t-1} + \varepsilon_t^\theta, \quad (3)$$

where $\bar{\theta} \geq 0$ is its unconditional mean, ρ_θ denotes its persistence (with $0 \leq \rho_\theta \leq 1$), and ε_t^θ is a white noise disturbance. The error vector $\epsilon_t \equiv [\varepsilon_t^D \;\; \varepsilon_t^F \;\; \varepsilon_t^\theta]'$ is serially uncorrelated, and follow a joint normal distribution with mean zero, and variance-covariance matrix $\Sigma = \text{diag}(\sigma_D^2, \sigma_F^2, \sigma_\theta^2)$.

[3] In Albagli (2009), changes in investing horizons are partly captured through comparative statics in the mass of investors that are forced to liquidate due to households' early withdrawals.

2.1.2 Investors

The mass of investors in the economy is normalized to unity. A fraction μ of these, labeled uninformed investors, have access to publicly available information only. Letting $\underline{h}_t \equiv \{h_{t-s}\}_{s=0}^{+\infty}$ denote the complete history of a variable h up to time t, public information corresponds to the history of dividends and prices of the risky asset represented by the filtration $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$. The complement share of investors (in mass $1-\mu$) are informed: in addition to public information Ω_t^U, they observe the contemporaneous realization of the persistent dividend component, F_t.

The population in the economy follows a generalized overlapping generation structure with a stationary age distribution. That is, at time t, a mass $1/T$ of investors aged T is dying, which is replaced by an equal mass of new-born investors who live for T periods. Hence, at any point in time the economy has T different generations of investors coexisting, aged $j = 1, 2, ...T$ years. The mix between informed and uninformed investors is assumed to be the same in each generation, so that the economy displays a constant age/information distribution. Investors maximize utility of lifetime consumption: $\sum_{s=1}^{T} \beta^s U(C_{t+s})$, where period-utility is given by negative exponential preferences $U(C) = -e^{-\gamma \cdot C}$ with equal CARA coefficient across generations/investor types. All investors are born with exogenous wealth w_0.

2.1.3 Asset Markets

Investors can take long or short positions in the risky asset at any time during their active trading years $j = \{1, 2...T-1\}$, for which they can borrow (or save) unlimited amounts in the risk-free asset. The dying generation aged T, however, must liquidate accumulated positions as it may not leave the economy with a net debt to any other investor. Denoting $X_{j,t}^U$ and $X_{j,t}^I$ the demand of uninformed and informed investors aged j in period t, the aggregate demand for the asset is given by

$$AD : X_t \equiv \frac{1}{T}(\mu \cdot \sum_{j=1}^{T-1} X_{j,t}^U + (1-\mu) \cdot \sum_{j=1}^{T-1} X_{j,t}^I). \tag{4}$$

Investors are price-takers and submit price-contingent demand orders (generalized limit orders) to a "Walrasian auctioneer", who then sets a price P_t for the risky asset such that all orders are satisfied. Defining the dollar net excess return of investment in the risky asset as $Q_{t+1} \equiv D_{t+1} + P_{t+1} - RP_t$, the wealth of investor aged j consuming $C_{j,t}^i$ and demanding $X_{j,t}^i$ (for $i = \{U, I\}$) evolves according to

$$W_{j+1,t+1}^i = (W_{j,t}^i - C_{j,t}^i)R + X_{j,t}^i Q_{t+1}. \tag{5}$$

2.2 Equilibrium Characterization

2.2.1 Recursive Representation

The key object to solve is the risky asset price, P_t. I conjecture (and later confirm) that the setup described above leads to the following linear price equation:

$$P_t = p_0 + \hat{p}_F F_t^U + p_F F_t + p_\theta \theta_t, \tag{6}$$

where $F_t^U \equiv \mathbb{E}[F_t|\Omega_t^U]$ is defined as the uninformed investors' forecast of the persistent dividend component F_t, conditional on publicly available information. To characterize the equilibrium, it is useful to begin by writing the evolution of the main state variables in a recursive form. Let $\Psi_{t+1} \equiv [1 \;\; F_{t+1} \;\; \theta_{t+1}]'$. Given equations (1), (2), and (3), the evolution of Ψ_{t+1} can be written as

$$\Psi_{t+1} = A_\psi \Psi_t + B_\psi \epsilon_{t+1}^U, \tag{7}$$

where A_ψ and B_ψ are matrices of proper order, and the vector $\epsilon_{t+1}^U \equiv [\varepsilon_{t+1}^D \;\; \varepsilon_{t+1}^F \;\; \varepsilon_{t+1}^\theta \;\; \tilde{F}_t^U]'$ is the expanded error vector faced by the uninformed investors, who in addition to the exogenous shocks face uncertainty coming from their own forecast errors about F_t; $\tilde{F}_t^U \equiv \hat{F}_t^U - F_t$ (see the Appendix). Most of the equations of interest, including the evolution of beliefs, optimal demands, and prices, can be expressed in terms of this recursive representation.

The solution approach builds on the standard techniques used in CARA-normal REE setups.[4] These models apply a guess-verify procedure which consists of 3 steps. First, conjecture that prices are linear in the underlying shocks. Based on this conjecture, update beliefs of investors (posterior means and variance) about future dividends and prices. Second, derive the optimal demands of investors. Third, impose market clearing and solve for the conjectured price coefficients in terms of the underlying parameters.

More formally, for any filtration Ω, let $H(x|\Omega) : R \to [0, 1]$ denote the conditional posterior cdf of a random variable x. Let (j, i) denote the age/information type of each investor in the economy, with $j = \{1, 2, ...T\}$, and $i = \{U, I\}$, and let the filtration $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$ and $\Omega_t^I = \{\underline{D}_t, \underline{P}_t, F_t\}$ represent the information available at time t to uninformed and informed investors, respectively.[5] The equilibrium concept is as follows:

A competitive rational expectations equilibrium is: 1. A price function given by (6), 2. A risky asset demand $X_{j,t}^i = x(P_t, \Omega_t^i, j)$ by investor (j, i), 3. Posterior beliefs $H(\Psi_t|\Omega_t^U)$ and $H(\Psi_t|\Omega_t^I)$ for uninformed and informed investors, respectively, such that $\forall\ (j, i)$: (i) Asset demands are optimal given prices and posterior beliefs; (ii) The asset markets clear at all times; and (iii) Posterior beliefs satisfy Bayes law.

2.2.2 Traders' Problem

For an investor aged j in period t, with information given by the filtration Ω_t^i, the problem is given by

$$\max_{x,c} \quad \mathbb{E}[-\sum_{s=0}^{T-j} \beta^s e^{-\gamma C_{j+s,t+s}} \mid \Omega_t^i], \;\; \text{s.t.} \;\; W_{j+1,t+1}^i = (W_{j,t}^i - C_{j,t})R + X_{j,t}^i Q_{t+1}, \;\; W_{1,t}^i = w_0. \tag{8}$$

This optimization remains analytically tractable as long as the evolution of future wealth, conditional on information, is normally distributed. The value function then takes a known form in terms of its dependence on the first and second conditional moments of investors' beliefs about the state variables driving future returns. With this tractable value function representation, asset demands and consumption/savings policies can be determined in closed form (see the discussion in Wang (1994) for more details).

[4] See Vives (2008) for a textbook discussion.

[5] Whether we allow informed investors to observe the complete history $\underline{F}_t$, or just the current value F_t, is irrelevant since $\{\theta_t, F_t\}$ are sufficient statistics for predicting future returns.

We now check whether future excess returns Q_{t+1} are indeed conditionally normally distributed. For informed investors, this is immediate. Because the informed also observe the public information available to the uninformed, they know the value of the current forecast F_t^U. Since they also observe F_t privately, the price reveals the current realization of the aggregate supply, θ_t. It is then straightforward to show that Q_{t+1} is conditionally gaussian for the informed investors.

For the uninformed, beliefs must be characterized with dynamic filtering methods. Note that from the price equation in (6), uninformed investors can back out a noisy signal about F_t, after subtracting the constant as well as the contribution of their own forecasts to the price. Let's label this signal the *informational content of price*, given by $p_t \equiv F_t + \lambda \cdot \theta_t$, with $\lambda \equiv p_\theta / p_F$. Together with the dividend in equation (1), these signals constitute the vector of public information about the state vector Ψ_t. Using the recursive characterization, the signal vector can be written as

$$S_t \equiv [D_t \;\; p_t]' = A_s \Psi_t + B_s \epsilon_t^U. \tag{9}$$

The next theorem describes the evolution of uninformed investors beliefs, showing that forecast errors follow a normal distribution. Specifically, let $\mathbb{O} \equiv \mathbb{E}[(\Psi_t - \mathbb{E}[\Psi_t \mid \Omega_t^U])(\Psi_t - \mathbb{E}[\Psi_t \mid \Omega_t^U])' \mid \Omega_t^U]$ denote the variance of the state vector, conditional on public information. Then,

Theorem 1 (Beliefs with public information): *The distribution of the state vector Ψ_t, conditional on the filtration $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$, is normal with mean $\mathbb{E}[\Psi_t \mid \Omega_t^U]$ and variance $\mathbb{O}$, where*

$$\mathbb{E}[\Psi_t \mid \Omega_t^U] = A_\psi \mathbb{E}[\Psi_{t-1} \mid \Omega_{t-1}^U] + K(S_t - \mathbb{E}[S_t \mid \Omega_{t-1}^U]), \tag{10}$$

and the conditional variance and projection matrix K jointly solve

$$\mathbb{O} = (I_3 - KA_s)(A_\psi \mathbb{O} A_\psi' + B_\psi \Delta B_\psi'), \tag{11}$$

$$K = (A_\psi \mathbb{O} A_\psi' + B_\psi \Delta B_\psi') A_s' (A_s (A_\psi \mathbb{O} A_\psi' + B_\psi \Delta B_\psi') A_s' + B_s \Delta B_s')^{-1}, \tag{12}$$

$$\Delta = \mathrm{diag}(\sigma_D^2, \sigma_F^2, \sigma_\phi^2, \mathbb{O}(2,2)).$$

Once we have checked that uninformed investors' beliefs follow a conditional gaussian distribution, we can state the results characterizing the value functions and the optimal consumption and investment policies chosen by different types of investors.

Theorem 2 (consumption/investment policies): *Let $W_{j,t}^I$ and $W_{j,t}^U$ denote wealth of a j-aged informed and uninformed investor, respectively. Let $M_t \equiv [1 \;\; F_t \;\; \theta_t \;\; \tilde{F}_t^U]'$ and $M_t^U \equiv [1 \;\; \hat{F}_t^U \;\; \hat{\theta}_t^U]'$ denote the current projection of informed and uninformed investors about the expanded state vector, respectively. Then,* *1. The value function and optimal rules of the informed investor correspond to*

$$J^I(W^I_{j,t}; M_t; j; t) = -\beta^t e^{-\alpha_j W^I_{j,t} - V^I_j(M_t)}, \tag{13}$$

$$X^I_{j,t} = (\frac{A_Q}{\alpha_{j+1}\Gamma^I_{j+1}} - \frac{h^I_{j+1}}{\alpha_{j+1}\Gamma^I_{j+1}}) \cdot M_t, \tag{14}$$

$$C^I_{j,t} = c^I_{j+1} + (\frac{\alpha_{j+1}R}{\alpha_{j+1}R + \gamma})W^I_{j,t} + \frac{M'_t m^I_{j+1} M_t}{2(\alpha_{j+1}R + \gamma)}. \tag{15}$$

2. The value function and optimal rules of the uninformed investor correspond to

$$J^U(W^U_{j,t}; M^U_t; j; t) = -\beta^t e^{-\alpha_j W^U_{j,t} - V^U_j(M^U_t)}, \tag{16}$$

$$X^U_{j,t} = (\frac{A^U_Q}{\alpha_{j+1}\Gamma^U_{j+1}} - \frac{h^U_{j+1}}{\alpha_{j+1}\Gamma^U_{j+1}}) \cdot M^U_t, \tag{17}$$

$$C^U_{j,t} = c^U_{j+1} + (\frac{\alpha_{j+1}R}{\alpha_{j+1}R + \gamma})W^U_{j,t} + \frac{M^{U\,\prime}_t m^U_{j+1} M^U_t}{2(\alpha_{j+1}R + \gamma)}. \tag{18}$$

Future returns, Q_{t+1}, depend on the contemporaneous state variables F_t and θ_t, but also on the uninformed investors' projection about these variables. This can be conveniently reduced to a dependence on the expanded state vector $M_t \equiv [1 \;\; F_t \;\; \theta_t \;\; \tilde{F}^U_t]'$, which includes uninformed investors' forecast error about current state variables.[6] While this vector is perfectly observed by the informed investors, it is observed with noise by the uninformed investors. Conditional on their information however, their forecast error is a zero-mean, normally distributed random variable. Hence, for both investor types future returns are linear in these projections (M_t for the informed, M^U_t for the uninformed), plus additional white noise error with gaussian distribution. The problem then remains tractable and value functions and optimal policies have the closed-form expression stated above.

Optimal portfolios take the form found in other dynamic CARA-gaussian models. Take the case for informed investors. The term $A_Q/(\alpha_{j+1}\Gamma^I_{j+1})$ is a mean-variance efficient portfolio capturing the tradeoff between expected returns (numerator) and risk (denominator), where α_{j+1} is the age-dependent risk aversion coefficient, and Γ^I_{j+1} is the renormalized covariance matrix of returns. In simple terms, this ratio reflects the response in investors demand coming from an increase in expected returns. The second term is a hedging component arising from the fact that innovations in contemporaneous returns affect expected returns further into the future. More precisely, the error innovation ϵ_{t+1} not only affects returns Q_{t+1}, but also the value function at $t+1$, giving rise to an additional source of risk (see Wang (1994) for more details). What makes this particular problem different from other dynamic REE environments is of course the dependence of these components on the age of the investor.

The price equilibria conjectured in (6) is the solution of a fixed-point problem. Starting from an initial price vector conjecture P, the equilibrium conditions deliver a new price vector $P' = F(P)$, where the functional $F(\cdot)$ is implicitly defined by the agents optimization problem together with the market clearing requirement. An equilibrium is then a fixed point $P^* = F(P^*)$. The solution method, barring

[6] This is because the forecast error of the uninformed about F_t is perfectly colinear with her forecast error about θ_t.

some special cases commented below, relies on numerical procedures. Beginning with a known terminal value function for the dying generation, one can iteratively compute the value functions at earlier ages for each investor to find the optimal consumption and investment rules for all the different ages actively interacting in the asset market. Equilibrium prices can then be solved by imposing the market clearing condition:

$$\frac{1}{T}(\mu \cdot \sum_{j=1}^{T-1} X_{j,t}^{U} + (1-\mu) \cdot \sum_{j=1}^{T-1} X_{j,t}^{I}) = \theta_t. \tag{19}$$

In the next two sections I analyze the properties of the equilibrium and discuss the implications of varying investment horizons for particular classes of economies.

3 Symmetric Information Economies

This section characterizes the equilibrium and discusses the implications of varying the investment horizon T, for economies with symmetric information. I study both the cases where the mass of uninformed agents is $\mu = 1$ (no-information economy) and $\mu = 0$ (full information economy). Apart from being more tractable and allowing for the derivation of some analytical results, these economies convey much of the intuition about the mechanisms that are triggered from variations in investment horizons, and so provide a natural starting point for the analysis.

3.1 Existence and multiplicity of equilibria

Table 1 introduces the baseline parameters that I will use throughout (unless otherwise stated). Although the purpose of the paper is not to provide a detailed calibration exercise for fitting asset pricing data, some features of the parameter choice are relevant to discuss. I have chosen the variance of the persistent dividend process as a normalization (equal to one) and made it relatively persistent ($\rho_F = 0.95$). In comparison, the temporary dividend component is relatively volatile ($\sigma_D = 3$).[7] The average net supply of the risky asset, $\bar{\theta}$, is normalized to one, in accordance to the measure of agents in the economy. Its standard deviation σ_θ is set to 10% of its unconditional value.

Table 1: Baseline parameters

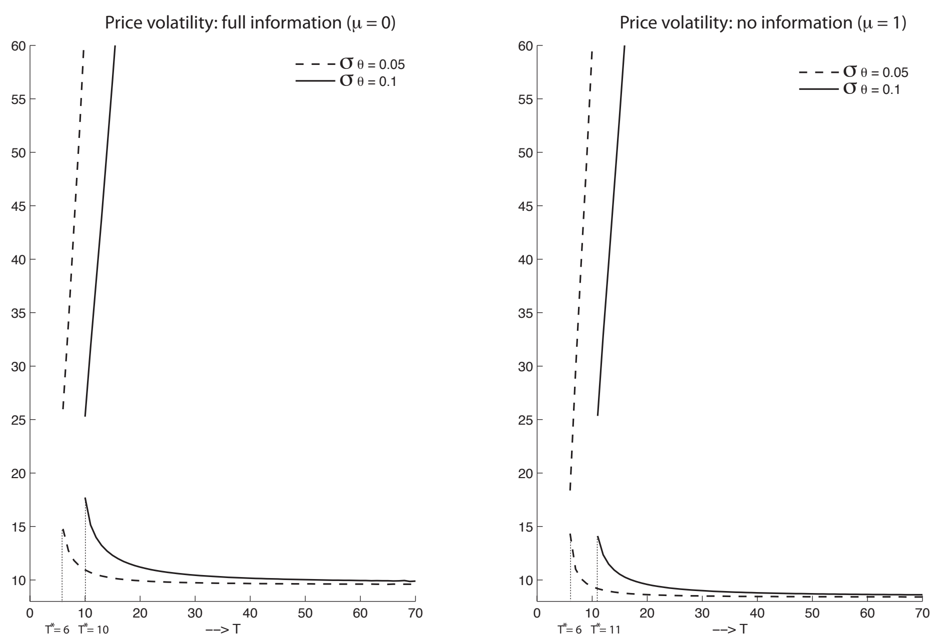
σ_D	σ_F	σ_θ	ρ_F	ρ_θ	γ	r	β	F	θ
3	1	0.1	0.95	0.6	1	0.05	1/(1.05)	20	1

Figure 1 plots the relation between the volatility of prices P_{t+1}, given an information set which conditions on public information $\{\underline{D}_t, \underline{P}_t\}$, and the investment horizon, T. The right panel considers the

[7]Although the transitory dividend component volatility does not enter the price equation (6) directly, it makes inference of the persistent dividend component F_t more difficult for the uninformed investors.

no-information economy ($\mu = 1$), while the left panel plots the full-information economy ($\mu = 0$). The first thing to note from the figure is that an equilibrium does not exist for all investment horizons. Under the baseline parameters (solid lines), the full-information economy exhibits equilibria starting from the critical horizon of $T^* = 10$ onwards, which coincides with the critical T^* for the no-information economy. The fact that overlapping generations models for the stock market can fail to exhibit equilibria is discussed by Spiegel (1998), Watanabe (2008), and Banerjee (2011). As the figure suggest, for the traditional OLG setup in which $T = 2$, conditions for existence are indeed quite stringent and one must adjust some of the key parameters related to risk. Spiegel (1998), for instance, assumes a very small volatility of the random supply; σ_θ. The dotted lines show equilibria for $\sigma_\theta = 0.05$, for which both economies reach the existence region at lower values of T^*.

Figure 1: Existence and multiplicity



The second apparent feature of the equilibria is multiplicity. For each investment horizon equal or larger than the minimum required for existence, there are generally 2 equilibria.[8] Across these equilibria, only the price coefficient associated with innovations in the persistent dividend component ($\hat{p}_F$ for $\mu = 1$, and p_F for $\mu = 0$) remains the same. The rest of the parameters vary reflecting two self-fulfilling equilibria. In one equilibrium, the price impact of the random supply is relatively small (p_θ is small, in absolute value). If investors trading in the current period believe this equilibrium will prevail in the future, they require a modest compensation for taking on the other side of random supply innovations, which then have minor effects on prices and returns. As a result of this low risk environment, the average price of the security is high, reflected by a large coefficient p_0. But the situation might well be the converse. If investors expect

[8] Although one could adjust parameters such that for the critical horizon T^* the two equilibria coincide.

future supply innovations to have large effect on future prices, they will become reluctant to absorb the current (stochastic) supply of the stock, whose price will then respond to innovations in θ with a large, negative coefficient. The high risk faced by investors is compensated through a large average premium (a low, or even negative value of p_0 in the price equation).

Although similar existence and multiplicity results have been discussed in 2-period OLG models, little is known about how *variations* in investor horizons can affect the properties of the equilibria in a generalized T-horizon economy. Figure 1 shows that, along the low volatility equilibrium, increasing the lifespan of investors reduces price volatility. As mentioned earlier, this is due to two main mechanisms at work: the *age-adjusted risk aversion* effect, and the *risk transfer* effect, which I will explain in more detail momentarily. The high volatility equilibria, in contrast, exhibits the exact opposite features. As investors live for longer horizons, price volatility increases along this equilibrium. Intuitively, as investors live longer, it takes an increasingly volatile security to induce investors to demand the high levels of compensation that are consistent with such volatility in equilibrium

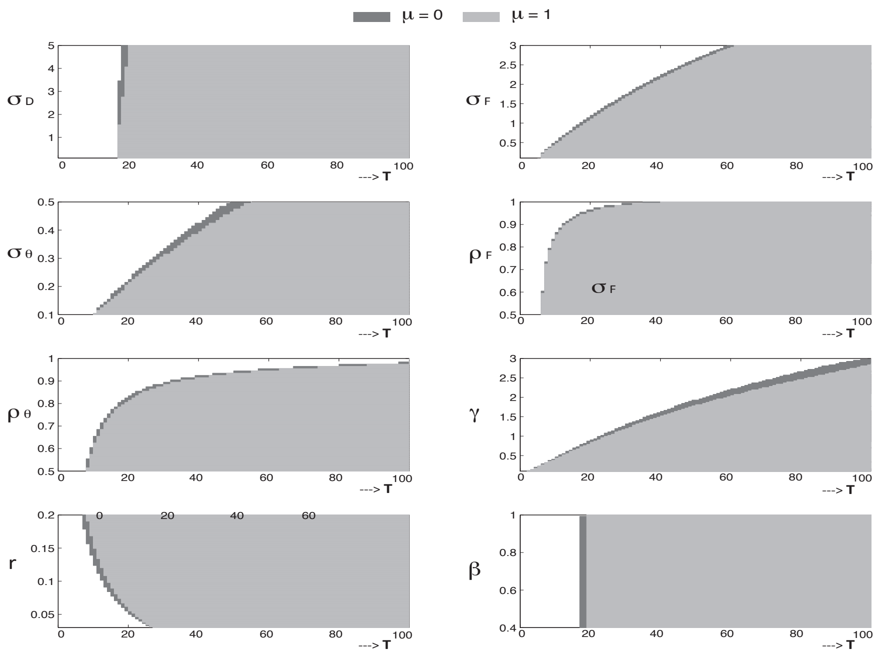
There are additional features of the high volatility equilibrium that make it a less suitable candidate for equilibrium selection. First, as noted by Spiegel (1998) and made apparent in Figure 1, price the volatility along the low volatility equilibrium is decreasing in the variability of the supply shock (σ_θ), while it is increasing along the high-volatility equilibrium (compare the solid vs. dashed lines in the figure). Second, the extreme levels of asset price volatility for longer lifespans make it increasingly hard to find positive average prices. Third, while the low-volatility equilibrium converges smoothly to the infinite horizon economy, the high volatility equilibrium vanishes as $T \to \infty$. Actually, it is possible to show this result analytically for the symmetric information economies under particular parameter values (see the Appendix for the proof).

Proposition 1 (infinite horizon limit): Let $\mu = 0$, or $\mu = 1$. As $T \to \infty$,
a. An equilibrium always exists.
b. The equilibrium is unique.

The result in proposition 1 is of theoretical interest on its own. To my knowledge, a formal proof of the existence and uniqueness of equilibria in dynamic REE settings when $T \to \infty$ has not been stated in prior work. Wang (1994), for instance, states that an equilibrium price equation similar to expression (6) can be solved for numerically. Whether this is the case for all possible parameters, or whether the solution is unique, is left an open question. The economics behind this result –the survival of the low volatility equilibrium only, in the limit– seem intuitive. When investors are infinitely lived, they can always accommodate a highly volatile asset price by voluntarily buying when it is underpriced, and selling when it is overpriced. Hence, the attractiveness of the highly volatile asset cannot remain a feature of the equilibrium, as competitive agents will bid the price up when it is underpriced, and down when it is overpriced. In equilibrium, only a moderate level of price volatility is sustainable. This volatility corresponds to the limit of the stable equilibrium in the finite horizon economy.

Figure 2 gives a more general picture of the equilibrium existence regions. Starting from the benchmark

Figure 2: Existence regions



parameters, each panel shows the critical investment horizon T^* that sustains equilibria, for both the full- and no-information economy. Parameters that increase the volatility of the dividend process (higher σ_D, σ_F, or an increase in the persistence ρ_F) increase the fundamental risk of the security, and require increasing critical horizons T^* for existence. Similarly, increases in non-fundamental risk related to the random supply shock (higher σ_θ, or an increase in the persistence ρ_θ) also shift up the minimal required horizon. Naturally, risk aversion γ also increases T^*, while the converse is true for the risk-free rate, since prices respond less to fundamental innovations when investors discount future flows at a higher rate. Finally, although β matters for the consumption path chosen by investors, it has no effect in their trading decisions (as trading choices are wealth-independent), and hence no impact on prices or the critical horizon T^*. Moreover, for all parameter values, the full-information information economy reaches an equilibrium at a (weakly) lower value of T^* than the no-information economy, since in the former investors have less uncertainty about the fundamental dividend process.

3.2 Stability

Another crucial feature that differs across the high and low volatility equilibria is stability. I will define an equilibrium to be stable if, starting from a small perturbation of the price coefficient associated with the supply shock, the resulting price coefficient after one iteration of the agents' optimization problem leads to a new price vector with a smaller deviation from the initial equilibrium. That is, from a small perturbation $p_\theta^{'} \neq p_\theta^*$, an equilibrium is *stable* if $|p_\theta^{''} - p_\theta^*| < |p_\theta^{'} - p_\theta^*|$, and *unstable* if $|p_\theta^{''} - p_\theta^*| > |p_\theta^{'} - p_\theta^*|$, where $p_\theta^{''}$ is the element of the price vector $P^{''}$ associated with supply innovations, and $P^{''} = F(P^{'})$.

For the general case, stability can only be inferred from numerical simulations. As discussed in more detail in the appendix, the low-volatility equilibrium is (numerically) stable according to the above definition of stability, while the high-volatility equilibrium is unstable. A special case which lends easily to an analytical derivation is the symmetric information economy in the standard OLG model with $T = 2$.

Proposition 2 (stability): Let $\mu = 0$, or $\mu = 1$. For $T = 2$,
a. The low-volatility equilibrium is stable.
b. The high-volatility equilibrium is unstable.

The result in Proposition 2 sheds light onto the fragile nature of the high-volatility equilibrium. Indeed, if the initial price conjecture contains even a slight deviation from the equilibrium value, the "tatonnement" process set in motion by the iteration $P^{''} = F(P^{'})$ will set the economy further and further away from the initial equilibrium. In the case where the perturbation is positive (i.e., $p^{'}_{\theta} > p^{*}_{\theta}$), the economy will converge to the stable, low volatility equilibrium. When negative, the economy diverges.

Given that stability is yet another appealing equilibrium characteristic that adds to the list of desirable properties discussed above, I will focus the attention on the low-volatility equilibrium in the remainder of paper.

3.3 Variations in investment horizons: two key mechanisms

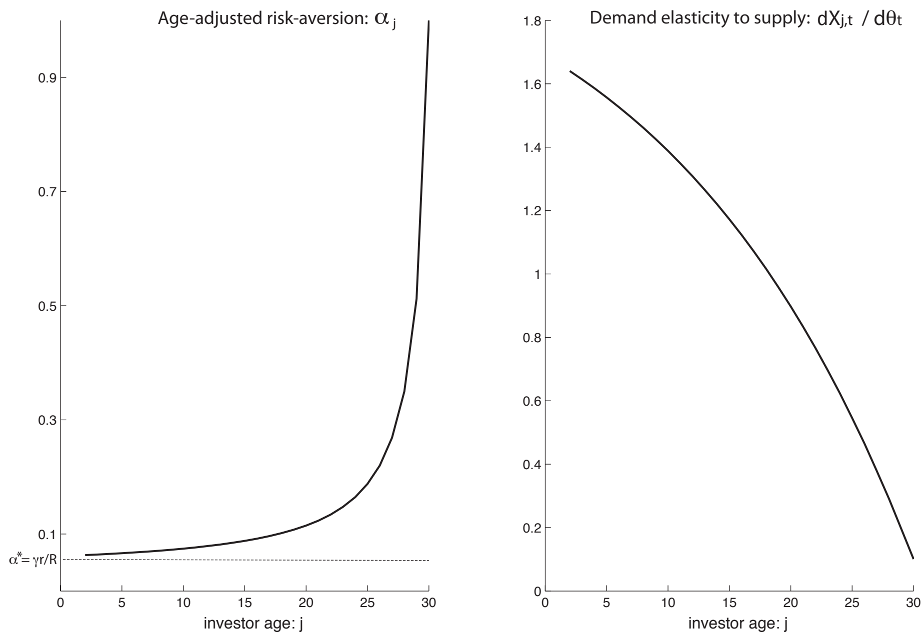
I now analyze in more detail the two main mechanisms driving the relation between price volatility and investment horizons, in the case of economies with symmetric information.

3.3.1 Age-adjusted risk aversion effect

The first mechanism is related to the changes in the *pricing* of risk induced by changes in T, which I will refer to as the *age-adjusted risk aversion* effect. A key driver of the action in dynamic trading environment is not the concern of investors about future dividends, but about future *price changes*. In this sense, a 2-period OLG economy represents a rather extreme case in which investors buying the asset face a single future trading round where terminal wealth is completely determined. When investors live for more periods however, they are less affected by price fluctuations at a particular point in time since they are not forced to unwind their portfolio at adverse prices –unless they have reached the terminal date T. In the words of De long et al. (1990), this is akin to receiving "dividend insurance": investors who purchase assets with high fundamental value diminish their total risk exposure to non-fundamental shocks, the more so the longer they live and consume dividends.

To understand this mechanism more formally, consider the optimal portfolio decisions of the informed investors in the full-information economy by setting $\mu = 0$ (an equivalent argument holds for other information structures). The left panel of Figure 3 plots the age-adjusted risk aversion coefficient, α_{j+1}, for an investment horizon $T = 30$. Inspection from the numerical results reveal this is the key source of variation in the denominator in the demand expression (14). As shown in the appendix, α_{j+1} can be

Figure 3: Age-adjusted risk aversion effect



solved recursively through the equation:

$$\alpha_j = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma} \tag{20}$$

The economics behind this expression can be understood as follows. Take the case of an investor aged $j = 30$. This investor is now retiring and hence her value function is just determined by the utility of terminal wealth. In other words, the value of $\alpha_{30} = \gamma$. When this investor was aged $j = 29$ one period ago, this is the value of α that applies for her portfolio decisions, just as in a static CARA investment problem. At the other extreme, for a newly born investor aged $j = 1$, the current risk of her transaction Γ^I_{j+1} is not accounted one-for-one in terms of its utility (value-function) impact, since she still has many periods to recover from fluctuations in returns. This does not mean trading is risk-less: changes in wealth resulting form the risky portfolio choice will affect her wealth, current consumption and value function. But as the consumption policy expression (15) shows, current consumption choices are only affected by changes in current wealth through

$$\frac{\partial C^I_{j,t}}{\partial W^I_{j,t}} = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma} = \alpha_j,$$

which is *precisely* the age-adjusted risk aversion coefficient used to price risk in the demand equation (14).

As an illustrative reference for this mechanism, consider the extreme case of a hypothetical economy in which investors live for infinite periods (as analyzed by Wang (1994)). In this case, the value of α can be found as the stationary solution to (20), corresponding to $\alpha^* = \gamma r/R$. As shown in Figure 3, the age-adjusted risk-aversion of a newly born investor in the finite horizon economy converges towards the infinitely-lived investor (age-adjusted) risk aversion coefficient, α^*. For an investor with an intermediate age, say $j = 20$, the situation is somewhat in between the static benchmark and the infinite horizon economy. As investors get older, their willingness to take risk is diminished. Indeed, this differential response of investors demands as age increases is plotted in the right panel of Figure 3, which shows the elasticity of demands to innovations in supply. As investors age, they become more cautious in their positions and absorb a decreasing proportion of the aggregate supply. If we now compare two economies in which investors live for different periods, the longer lived economy will have, at any point in time, traders which on average have more trading rounds left in them. As Figure 3 makes clear, the willingness to take on risks by the average investor will be larger in such economy.

To my knowledge, this mechanism has not been studied formally in overlapping generations models of the financial market. A related discussion appears in De long et al. (1990), who state that increasing the age of the 2-period lived sophisticated investors in their model should lead to more risk-taking, and hence a diminished price impact from the positions taken by the "noise traders". This exercise is however not formally carried out in that paper, nor in the successive models of trading in OLG environments, which always consider 2-period lived agents.[9]

3.3.2 Risk transfer effect

The second mechanism is related to changes in the *amount* of risk that must be absorbed in equilibrium by all active generations, induced by changes in T. I label this mechanism the *risk transfer* effect. In an economy in which agents live for T periods, at each point in time there are $T-1$ generations of voluntary investors, compared to a single generation of retirees. In consequence, the relative risk transfer between these generations is the ratio $1/(T-1)$. This mechanism can be illustrated by a convenient decomposition of the market-clearing condition in (19). Continuing with the full-information economy, this gives

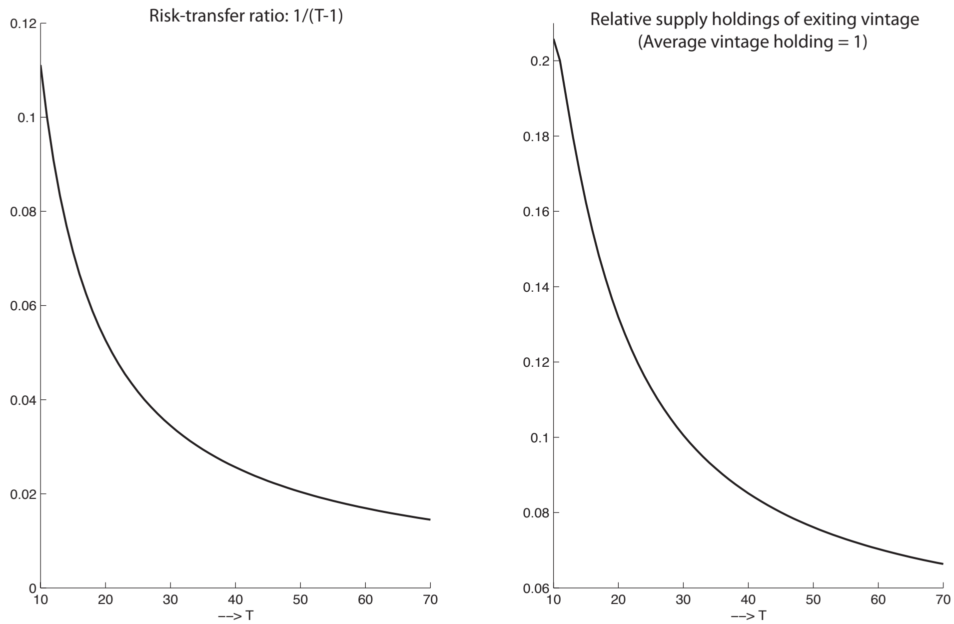
$$\begin{aligned} \frac{1}{T}(X^I_{1,t} + X^I_{2,t} + \cdots + X^I_{T-1,t}) &= \underbrace{X^I_{1,t-1} + X^I_{2,t-1} + \cdots + X^I_{T-1,t-1}}_{\theta_{t-1}} + \underbrace{(1-\rho_\theta)(\bar{\theta} - \theta_{t-1}) + \varepsilon^\theta_t}_{\Delta\theta_t} \quad (21) \\ \Rightarrow \qquad \frac{1}{T}(X^I_{1,t} + \Delta X^I_{2,t} + \cdots + \Delta X^I_{T-1,t}) &= X_{T-1,t-1} + \Delta\theta_t \end{aligned}$$

The demand in the left-hand side of (21) is composed of all current active investors. Of these, investors aged $2, 3 \cdots, T-1$ were also present in the previous period, and hence their *net demands* correspond to $\Delta X^I_{2,t} = X^I_{2,t} - X^I_{1,t-1}$ for the investor currently aged 2, $\Delta X^I_{3,t}$ for the investor aged 3, and so on. For all these investors, the change in net positions is voluntary. Only for the investor aged $T-1$ in the previous period the net demand is exogenously set at $-X^I_{T-1,t-1}$. In equilibrium, the negative of this amount, plus the supply innovation $(1-\rho_\theta)(\bar{\theta} - \theta_{t-1}) + \varepsilon^\theta_t$, must be absorbed by changes in the net positions of

[9]Closer in this respect are the dynamic, finite horizon models developed by He and Wang (1995), and Cvitanić et al. (2006). As time elapses, agents learn more about the fundamental value of the asset –which is a single terminal payoff. However, since in both models agents derive utility only from consumption of terminal wealth, they price changes in current wealth one-to-one (i.e., the marginal propensity to consume wealth changes is 1). Therefore, the age-adjusted risk aversion coefficient coincides with the CARA parameter.

all active investors. Hence, a relative risk transfer from a mass of $1/T$ retirees to a mass of $(T-1)/T$ voluntary investors: a risk transfer ratio of $1/(T-1)$. For illustrative purposes, the left panel of Figure 4 plots the risk-transfer ratio as a function of the investment horizon.

Figure 4: Risk transfer effect

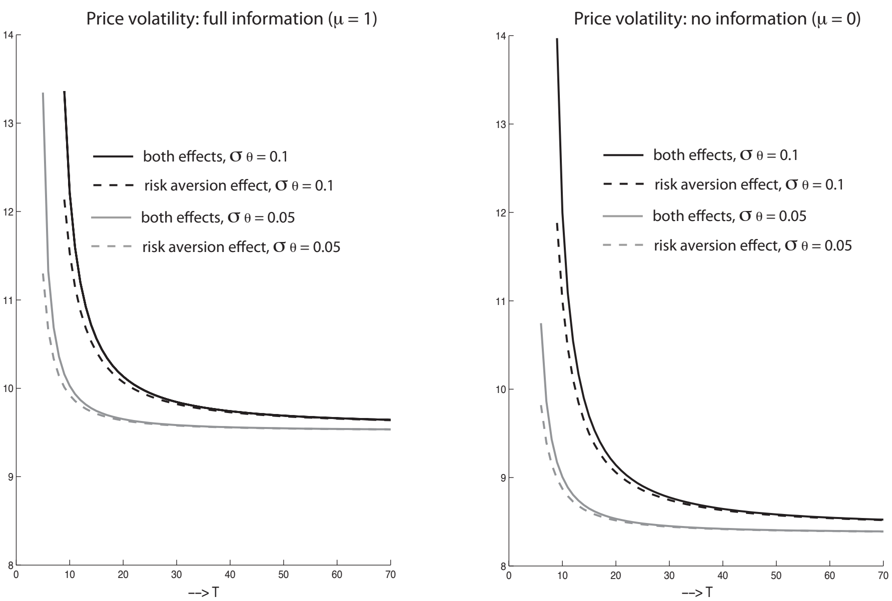


As the right panel of Figure 3 showed earlier, however, the dying vintage of investors who exit the economy hold less than their "fair share" of the supply –which amounts to $1/T$. As investors age, the increase in the age-adjusted risk aversion leads to a progressive reduction in asset holdings, compared with the positions of their younger counterparts. This is made explicit by the right panel of Figure 4, which plots the (equilibrium) holdings of the risky asset supply of the exiting vintage, relative to the average holding. When $T = 70$, the exiting generation constitutes a 1.4% of the active traders population (i.e., 1/69), but holds only about 7% of share of risky assets held by the average investor. It follows that when this vintage of investors exit the economy, they will increase the supply of the asset that must be held by the other investors by only $1.4\% \cdot 7\% \approx 0.1\%$. This gradual reduction in the risky positions of a given vintage suggests that the transfer of risk from the dying generation to all others is a smooth process. The fact however remains that, at any point in time, all vintages who hold less than the average share of the supply are transferring risk to those who hold more, an effect induced by the anticipation of each vintage that they will in fact die in a finite number of periods. Hence, the risk transfer ratio $1/(T-1)$ remains as the relevant statistic to account for the importance of this effect.

These two mechanisms shed light on why the results of OLG models with $T = 2$ are so extreme and specific. On the one hand, current participants in the market must purchase an asset which they must completely unwind in a single future trading round (the *age-adjusted risk aversion* effect). But moreover, when they do liquidate in the next period, their entire positions must be purchased by a generation with equal mass as them (the *risk transfer* effect). Simply put, the whole aggregate risk of the economy must forcefully exchange hands every period!

To gauge the relative importance of these two mechanisms, I now perform a decomposition of the change in price volatility through the following exercise. I fix the mass of active (voluntary) investors in the market to one (which corresponds to the infinite horizon economy). I then vary the investment horizon T, letting only the average demands of investors change as a result of the *age-adjusted risk aversion* effect– hence, shutting off the *risk transfer* mechanism. Figure 5 shows the volatility of prices (conditional on

Figure 5: Relative contributions of the effects



public information) that emerges in these economies. As before, I include both the baseline parameters and a case with lower volatility of the random supply, $\sigma_\theta = 0.05$. As the figure makes clear, both effects are important in delivering the change in price volatility due to changing investment horizons. For the baseline parameters (black lines), the contribution of the *risk transfer* effect is about 33% of the total price volatility increase produced by shrinking T from 70 to 10, in the case of the full-information economy. For the no-information economy, the contribution of the *risk transfer effect* reaches 39% over the same range. For economies that admit equilibria at lower investment horizons –which is the case when $\sigma_\theta = 0.05$ (gray lines)– the importance of the *risk transfer* effect grows larger, since changes in the relative risk transfer ratio, $1/(T-1)$, become more significant as T approaches the origin. For the full- and no-information economy, the *risk transfer* effect account for 66% and 42% of total price volatility changes, respectively.

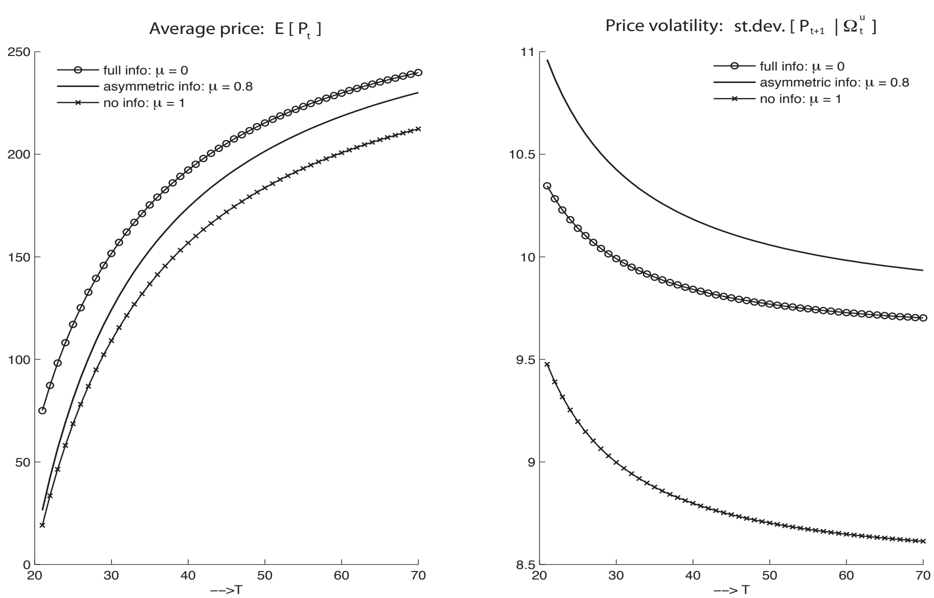
4 Asymmetric Information Economies

This section analyzes the effects of changes in investment horizons for the more general asymmetric information economy. The endogenous price signal $p_t \equiv F_t + \lambda \cdot \theta_t$ derived in section 2 will be relatively more informative about the persistent dividend component when the ratio $\lambda \equiv p_\theta / p_F$ is small (in absolute magnitude). But from the discussion in section 3, we learned that as horizons shrink, volatility increases due to the heightened price response to supply innovations. It follows that in the asymmetric information economy, the informational role of markets will be diminished as the lifespan of investors is reduced. This section describes how pricing moments and market efficiency in the asymmetric information economy behave as a function of T, contrasting them with the symmetric information benchmarks of section 3.

4.1 Asset pricing moments

The left hand side of Figure 6 plots the unconditional average of prices for three different economies: the full information economy (circled line), the no-information benchmark (crossed line), and the asymmetric information case when 20% of investors are informed (plain line). The ordering of mean prices across these economies follow a one-to-one ordering with the amount of fundamental information available to agents: prices are the highest for the full-information economy, intermediate for the asymmetric information economy, and the lowest for the no-information case.

Figure 6: Average prices and volatility



Interestingly, the right panel of the figure reveals that price volatility (the standard deviation of future prices, conditional on current public information) might not follow the same ordering. In particular, the no-information economy displays the least amount of price volatility, whereas the asymmetric-information economy is the most volatile. The reason behind this result is intuitive. For the no-information economy, prices convey less information about fundamentals and hence vary less to reflect changes in the underlying persistent dividend component. As the investment horizon shrinks, volatility spikes but mostly reflecting an increased role of supply shocks. In the full-information economy, volatility is larger for all investment horizons due to the high impact that fundamentals have on prices when they can be observed without noise. Once again, as horizons grow short, the increased importance of supply shocks raise price volatility.

The asymmetric information economy, on the other hand, inherits the behavior of the two benchmark economies to a different degree depending on the investment horizon under consideration. For relatively long investment horizons, volatility is higher than in the no-information economy since prices respond more to changes in fundamentals. For the parameters chosen, it is also above the volatility displayed by the full-information economy, as prices react more to innovations in supply when fundamental uncertainty is larger. Moreover, as T decreases, the volatility of prices in the asymmetric information case increases with a sensitivity similar to that of the no-information economy, reflecting the increased uncertainty of uninformed investors and the increased reaction of prices to non-fundamental innovations.

Figure 7: Risk premium and return volatility

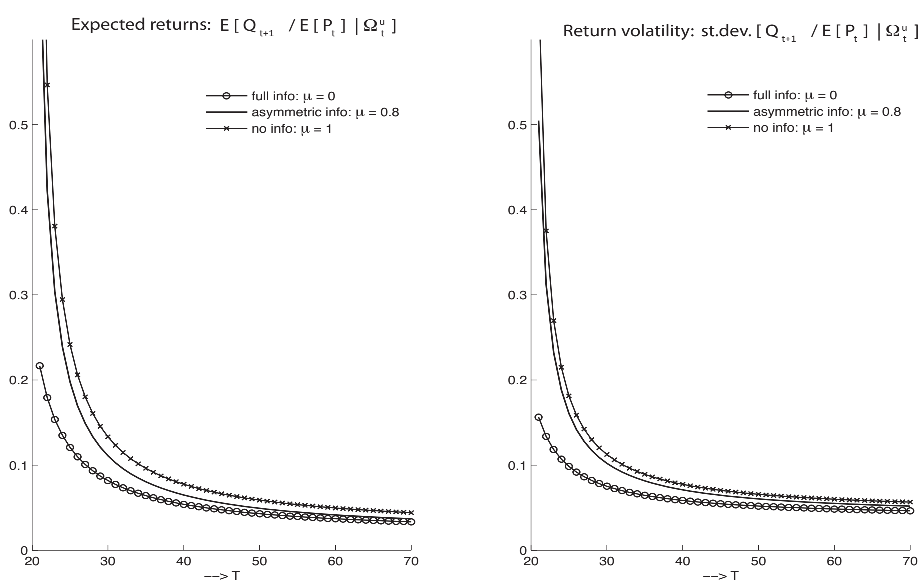


Figure 7 plots the expected return (risk premium) and the volatility of returns across the different economies.[10] The following patterns emerge in this figure which are independent of the parameters chosen. First, both expected returns and return volatility are the lowest for the full information economy. Indeed, although prices are relatively volatile in this economy, prices are also the highest, so the volatility of returns is diminished. Regarding the risk premium, it is intuitive that since investors have the most information about fundamentals in this case, their willingness to participate in the market is the highest, which is reflected by lower required returns. Second, the risk premium and return volatility is the highest for the

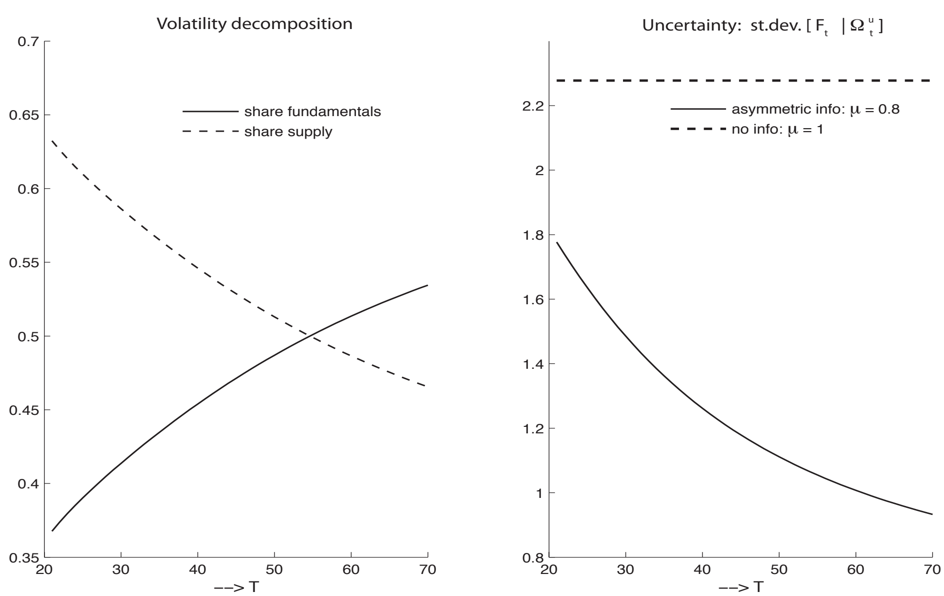
[10]Since prices can be negative or arbitrarily close to zero in linear price models, I define the expected return as the ratio between the expected dollar return conditional on public information, $\mathbb{E}[Q_{t+1}|\Omega_t^u]$, and the unconditional mean price, $\mathbb{E}[P_t]$.

no-information economy, since agents face the largest amount of uncertainty regarding the fundamental value of the asset. Third, the asymmetric information economy lies strictly in between the symmetric information benchmarks. I will now explain in more detail the economic forces behind this last key result.

4.2 Market efficiency

Figure 8 plots the decomposition of price volatility (left panel) and the resulting uncertainty of the uninformed investors (right panel), as a function of investment horizons. The share of price fluctuations due to fundamentals is calculated as the fraction of the price variance explained by innovations in F_t, while the non-fundamental share includes the fraction of price variance coming from both supply shocks, and the forecast errors incurred by the uninformed investors (both shares sum to one). Clearly, fundamental volatility dominates for relatively large values of the investment horizon, while the converse is true for short lifespans. In consequence, the informational content of prices is diminished as investment horizons shorten, and the uncertainty of the uninformed investors increases. As the right panel of the figure shows, the uncertainty of the uninformed (the standard deviation of F_t, conditional on the history of public information) nearly doubles as horizons fall from $T = 70$ to $T = 20$.

Figure 8: Variance decomposition and uncertainty



Indeed, this figure provides the central intuition for the behavior of investors' required returns in the asymmetric information economy plotted in Figure 7. In particular, for long investment horizons, the risk premium under asymmetric information comes very close to the returns required in the full-information economy, whereas the no-information economy stands out with a larger compensation for risk. However, as horizons shorten, the asymmetric information economy approaches the no-information economy, both demanding significantly higher compensation for purchasing the security than the full-information economy.

What explains the apparent switching of the asymmetric information economy between these different benchmark regimes? Precisely the endogenous nature of information in rational expectations environments. In a competitive risk-averse framework, the central force determining how aggressively informed investors react to their private knowledge is risk. For long investment horizons, both the *age-adjusted risk aversion* and the *risk transfer* effect lead to investor demands which are relatively elastic to expected returns. In consequence, the market is deep and supply innovations have modest price impacts. In response to this low risk environment, competitive CARA-investors who posses superior knowledge will trade on this information aggressively, and information that is originally received privately by, say, 20% of the trader population will mostly find its way into the price. Conversely, for short investment horizons, the aforementioned effects incite cautious trading and demands become relatively inelastic to private information about future returns. In consequence, information that is private to a group of investors *remains private* to such group. This explains why asset prices in the asymmetric information economy approach the behavior exhibited by the full-information economy for long horizons, since in this case the economy resembles one which a large group of investors are, in effect, pretty well informed in equilibrium. On the contrary, shrinking investment horizons set in motion a process of information *disaggregation* from prices, and the knowledge of the population becomes in effect much closer to the no-information benchmark –with asset pricing moments following suit.

The above fact can also be explained in terms of the excess sensitivity of the risk premium to changes in T in the asymmetric information economy, relative to the symmetric knowledge benchmarks. This excess sensitivity is explained by the interdependence between the uninformed investors' level of uncertainty and the risk faced by the informed. To understand this link, let's describe in more detail the consequences of a reduction in the demand elasticity of informed investors to private information. If these agents respond less to information, the equilibrium price reveals less information about the fundamental F_t, and uncertainty about fundamentals is higher for the uninformed. In response, the willingness to make the market is reduced for these agents, which require a larger compensation for bearing a larger amount of risk. In consequence, supply innovations are met with wider price fluctuations. The increased volatility of prices, in turn, increases the risk for the informed, inducing them to react even less strongly to private information. This last observations closes the interdependence, or "spiral", between the risk faced by the informed investors, and uncertainty of the uninformed.

However, one should note that there are limits to the strength of this spiral effect. Indeed, the uncertainty of the uninformed in the asymmetric information economy is capped above by the knowledge of investors in the no-information benchmark, which do learn about the persistent component F_t form the observation of dividends. More interestingly, when horizons shrink and the uncertainty of the fundamental process grows larger, at the same time the price is becoming relatively *more* informative about the noisy supply. Since this second source of price variation takes the center stage for relatively short horizons (Figure 8, left panel), knowing relatively more about this second component becomes increasingly important. In effect, the feedback loop between price volatility and investor uncertainty grows weaker due to this counteracting force.[11]

I conclude this section with some remarks on quantitative issues. First and foremost, CARA-normal models are obviously not designed to capture realistic variations in asset pricing moments as the more traditional theoretical asset pricing literature. This paper is no exception, and the central results should be interpreted as an illustration of the qualitative mechanisms that are triggered by changes in effective investment horizons. That said, the proposed mechanism of varying horizons does seem to play an important role, at least within the confines of the model.

One could, of course, be suspicious about how big changes in investment horizons need to be for pricing moments to show interesting variations. Is a reduction from 70 to 20 periods (say, years) a reasonable proxy for what happens at the business cycle frequency? In this respect, note that the *inverse* of the investment horizon corresponds exactly to the fraction of investors which are liquidating forcefully at any given time (the mass of the dying generation). In this light, an increase in liquidations from 1.42% (i.e., 1/70) to 5% (i.e., 1/20) does not seem exaggerated if one considers the empirical findings of fund liquidations during periods of financial distress. For instance, Ben-David et al. (2012) find that hedge funds reduced their exposure in equity markets in almost 30% during the 2008:Q3-Q4 contraction, which corresponded roughly to 1% of all outstanding equities. Importantly, their results indicate that most of this selling was actually *forced* by investors withdrawing financing. Carhart et al. (2002) study attrition rates in the mutual fund industry, and find that while 3.6% of funds disappear yearly on average over their sample, the standard deviation is quite high, at 2.4%. Chen et al. (2008) measure distress selling of troubled –but still alive– mutual funds, and report average distress-driven sales between 0.6-1% of mutual funds holdings at quarterly frequency (when using the asset-weighted measure of outflows). Moreover, this fraction spikes considerably (nearly doubles) during the mayor episode of financial turbulence covered in their sample: the demise of LTCM in 1998. Taken together, variations in forced liquidations between 1% and 5% per year seem to be in the ball park of the magnitudes reported in these studies for variation at the business cycle frequency.

The second observation relates more generally to the quantitative implications of endogenous information aggregation in explaining asset pricing moments. As Figure 7 shows, although the asymmetric information economy reacts much more to changes in investment horizons than the full-information economy, one could argue that the no-information economy does a reasonable job in delivering comparable variations in risk premium and return volatility. What is then the value added of a (more complex) model that highlights asymmetric information?

The answer is that symmetric information benchmarks have no implications for market efficiency. For the no-information economy, prices contain no additional information about the persistent dividend process –over and above from what can be learned from dividends. In the full-information economy, on the other hand, the price system is irrelevant as a source of information since agents already know the value of fundamentals. It is in this respect that the analysis of asymmetric information in financial markets becomes crucial. On one hand, it stresses that while return volatility will be high when horizons shorten, it is the non-fundamental part of volatility that takes the center stage during these episodes,

[11] For a more detailed description of this counteracting mechanism, see the discussion in Avdis (2011).

and that this matters for the allocative role of financial markets only to the extent that prices convey valuable information to investors. On the other, it gives a plausible explanation for why most measures of economic uncertainty often used in empirical work tend to spike during contractions and episodes of financial distress.

5 Conclusions

This paper analyzed the asset pricing implications of finite investment horizons in financial markets. The main message is twofold. First, horizons matter for both the pricing of risk of the average investor (the *age-adjusted risk aversion* effect) and the amount of risk that must be held in equilibrium by the active investors in a generalized OLG economy (the *risk transfer* effect). Both mechanism have the potential of delivering interesting variations in expected returns and return volatilities when we interpret periods of fund withdrawals as an effective shortening of investors' horizons.

Second, investment horizons matter for market efficiency. While long horizons incite informed investors to release their private information into prices through the trading process, the high risk environment triggered by short investment horizons can significantly reduce asset price informativeness. In particular, even if the fraction of informed investors is relatively low, the degree of information contained in prices can show ample variations across different horizon regimes. Economies where effective investment horizons are long behave similarly to a hypothetical case in which all investors observe economic fundamentals in real time, while they align much closer with a no-information benchmark when horizons are short. This findings suggests information *disaggregation* from the price process can be an important mechanism for understanding variations in economic uncertainty more specifically, and market efficiency more generally.

There are several avenues in which one can extend and complement the current analysis. First, although the focus of the paper is studying the asset pricing effects of varying investment horizons, the analysis is limited to the comparison between steady states of different economies, rather than actual time variation in horizons within the same economy. While a very interesting question to study, this type of analysis is difficult to handle within rational expectations models as equilibrium prices lose their linear form. This makes the standard solution methods inapplicable and the analysis becomes intractable.

A related extension which *can*, in principle, be analyzed under the current framework is the effects of deterministic changes in population demographics. The analysis suggests that a generation of "baby boomers" entering the economy could lead to higher asset prices and reduced uncertainty early in their life, but to declining prices, heightened volatility and poor informational efficiency as the generation approaches old age. Although the equilibrium in this economy would exhibit time-dependent price coefficients, the analysis remains tractable as long as these demographic changes are perfectly anticipated.

Finally, one could also study the incentives to acquire information for different investment horizons. While investors have more incentives to become informed when prices are less reliable, these are also the times when the lifespan over which they plan to use such information is shorter. This leads to non-trivial predictions about the effects of investment horizons on endogenous information acquisition. I leave these important questions for future research.

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6 Appendix

Proof of Theorems 1 and 2:

To characterize beliefs, optimal consumption and investment rules, and equilibrium prices, one must first find the matrices A_ψ and B_ψ in expression (7). From equations (1), (2), and (3), the evolution of Ψ_{t+1} is given by (7) as long as

$$A_\psi = \begin{bmatrix} 1 & 0 & 0 \\ F_0 & \rho_F & 0 \\ \theta_0 & 0 & \rho_\theta \end{bmatrix}, \text{ and } \quad B_\psi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

where $F_0 \equiv (1-\rho_F)\bar{F}$, and $\theta_0 \equiv (1-\rho_\theta)\bar{\theta}$. I will prove Theorem's 1 and 2 through the following 4 steps. 1) For given coefficients in the price equation, solve the Bayesian filtering problem of the uninformed, and find the autoregressive process of their forecast errors. 2) Find the recursive representation of the conditional state vectors M_t and M_t^U, and the conditional distributions of future excess returns, Q_{t+1}. 3) Solve optimal demands, and impose market clearing to find equilibrium prices. The resulting price function is then found by equating the market clearing price to the initial price conjecture.

Step 1: Together with the coefficients in the price equation (6), the recursive representation in (7) leads directly to the results in Theorem 1, which is just the Bayesian updating of beliefs described by the Kalman filter. A derivation of the Kalman filter can be found in most advanced statistics textbooks.[12] Writing $\mathbb{E}[\Psi_t \mid \Omega_t^U] \equiv \Psi_t^U$ and $\Psi_t^U - \Psi_t \equiv \tilde{\Psi}_t^U$ for notational convenience, the evolution of the uninformed investors' forecast error vector can be found from manipulation of (10):

$$\begin{aligned} \tilde{\Psi}_{t+1}^U &= A_U \cdot \tilde{\Psi}_t^U + B_U \cdot \epsilon_{t+1}^U, \text{ with} \\ A_U &\equiv (I_3 - KA_s)A_\Psi, \text{ and } B_U \equiv (K(A_sB_\Psi + B_s) - B_\Psi). \end{aligned}$$

But notice that the observation of the price implies forecast errors about F_t and θ_t are perfectly colinear for the uninformed; i.e, $F_t + \lambda \cdot \theta_t = F_t^U + \lambda \cdot \theta_t^U$, or $\tilde{\theta}_t^U = -\lambda^{-1} \cdot \tilde{F}_t^U$. This allows to rewrite the vector $\tilde{\Psi}_{t+1}^U$ as $\tilde{\Psi}_{t+1}^U = [0 \;\; \tilde{F}_{t+1}^U \;\; \tilde{\theta}_{t+1}^U]' = A_U \cdot [0 \;\; \tilde{F}_t^U \;\; \tilde{\theta}_t^U]' + B_U \cdot \epsilon_{t+1}^U$, or

$$\begin{aligned} \tilde{F}_{t+1}^U &= \rho_U \cdot \tilde{F}_t^U + b_U \cdot \epsilon_{t+1}^U, \\ \text{with} \quad & \rho_U \equiv A_U(2,2) - \lambda^{-1}A_U(2,3), \text{ and } b_U \equiv B_U(2,:). \end{aligned} \tag{22}$$

[12] See Ljungqvist and Sargent (2000) for applications in macroeconomics.

Step 2: Using (22), the evolution of the conditional state vectors M_t and M_t^U can now be found:

$$\begin{aligned} M_{t+1} &= A_M \cdot M_t + B_M \cdot \epsilon_{t+1}, & (23)\\ M_{t+1}^U &= A_M^U \cdot M_t^U + B_M^U \cdot \epsilon_{t+1}^U, & (24)\\ \text{with} \quad A_M &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ (1-\rho_F)\bar{F} & \rho_F & 0 & 0 \\ (1-\rho_\theta)\bar{\theta} & 0 & \rho_\theta & 0 \\ 0 & 0 & 0 & \rho_U \end{bmatrix}, \quad B_M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ B_U(2,1) & B_U(2,2) & B_U(2,3) \end{bmatrix}, \\ A_M^U &= A_\Psi, \ \ B_M^U = K(A_s A_\Psi l_0 v_0 + A_s B_\Psi + B_s), \ \ l_0 \equiv [0 \ \ -1 \ \ -\lambda^{-1}]', \ \text{and } v_0 \equiv [0 \ 0 \ 0 \ 1]. \end{aligned}$$

Expressions (23) and (24) can now be used to express the conditional moments of future returns for informed and uninformed investors. Note that the forecast F_t^U by in the price equation (6) can replaced to express prices as a function of current state variables and the forecast error of the uninformed,

$$P_t = p_0 + p_1 \cdot F_t + p_2 \cdot \theta_t + p_3 \cdot \tilde{F}_t^U \equiv P \cdot M_t, \qquad (25)$$

where $p_1 = \hat{p}_F + p_F$, $p_2 = p_\theta$, $p_3 = \hat{p}_F$, and $P \equiv [p_0 \ \ p_1 \ \ p_2 \ \ p_3]$. Moreover, writing the dividend in (1) as $D_{t+1} = A_D \cdot \Psi_{t+1} + B_D \cdot \epsilon_{t+1}$, with $A_D \equiv [0 \ 1 \ 0]$, $B_D \equiv [1 \ 0 \ 0]$, the future return Q_{t+1} can now be written as:

$$\begin{aligned} Q_{t+1} &= A_Q \cdot M_t + B_Q \cdot \epsilon_{t+1}, & (26)\\ \text{with} \quad & A_Q \equiv A_D A_M + P \cdot (A_M - I_4 R), \text{ and } \ B_Q \equiv (A_D + P) \cdot B_M + B_D. \end{aligned}$$

For uninformed investors, the corresponding expression in terms of their conditional expectations and forecast errors is:

$$Q_{t+1} = A_Q^U \cdot M_t^U + B_Q^U \cdot \epsilon_{t+1}^U, \qquad (27)$$

$$\text{with} \ \ A_Q^U \equiv A_Q m_0, \ \ B_Q^U \equiv A_Q m_1 v_0 + B_Q m_0', \ \ m_0 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}', \text{and } m_1 \equiv [0 \ \ -1 \ \ \lambda^{-1} \ \ 1]'$$

Expression (26) and (27) confirm that future returns indeed follow a conditional gaussian distribution for both informed and uninformed investors.

Step 3: Optimal investment and consumption policies can now be found by applying a known result on value functions in gaussian-exponential environments (see Vives, 2008). In particular, the conjectured value function at $t+1$ for an informed investor aged j at time t takes the form

$$J^I(W_{j+1,t+1}^I; M_{t+1}; j+1; t+1) = -\beta^{t+1} \cdot \exp\{-\alpha_{j+1} W_{j+1,t}^I - \frac{1}{2} M_{t+1}' V_{j+1}^I M_{t+1}\}.$$

The value function at t then takes the form

$$J^I(W_{j,t}^I; M_t; j; t) = \max_{\{X,C\}} - \exp\{-\gamma C\} - \beta \delta_{j+1}^I \exp\{-\alpha_{j+1} R(W_{j,t}^I + C) - \alpha_{j+1} X A_Q M_t \tag{28}$$
$$-\frac{1}{2} M_t' v_{j+1}^{I,aa} M_t + \frac{1}{2}(\alpha_{j+1} X B_Q + M_t' v_{j+1}^{I,ab} M_t)(\Xi_{j+1}^I)^{-1}(\alpha_{j+1} X B_Q + M_t' v_{j+1}^{I,ab} M_t)')\}, \text{ with}$$
$$v_{j+1}^{I,aa} \equiv A_M' V_{j+1}^I A_M; \;\; v_{j+1}^{I,bb} \equiv B_M' V_{j+1}^I B_M; \;\; v_{j+1}^{I,ab} \equiv A_M' V_{j+1}^I B_M; \tag{29}$$
$$\Xi_{j+1}^I \equiv \Sigma^{-1} + v_{j+1}^{I,bb}, \text{ and } \delta_{j+1}^I \equiv |\Sigma \cdot \Xi_{j+1}^I|^{-1/2}.$$

The optimal consumption/investment policies can now be found through differentiation. Letting $\Gamma_{j+1}^I \equiv B_Q(\Xi_{j+1}^I)^{-1} B_Q'$, we get

$$X_{j,t}^I = \frac{A_Q - h_{j+1}^I}{\alpha_{j+1}\Gamma_{j+1}^I} \cdot M_t, \tag{30}$$

$$C_{j,t}^I = c_{j+1}^I + \left(\frac{\alpha_{j+1} R}{\alpha_{j+1} R + \gamma}\right) \cdot W_{j,t}^I + \frac{M_t' m_{j+1}^I M_t}{2(\alpha_{j+1} R + \gamma)}, \text{ where} \tag{31}$$

$$c_{j+1}^I \equiv \log(\gamma/(\beta \delta_{j+1}^I \alpha_{j+1} R))/(\alpha_{j+1} R + \gamma); \;\; h_{j+1}^I \equiv B_Q(\Xi_{j+1}^I)^{-1} v_{j+1}^{I,ab\,\prime}, \text{ and} \tag{32}$$
$$m_{j+1}^I \equiv g_{j+1}'(\Gamma_{j+1}^I)^{-1} g_{j+1} + v_{j+1}^{I,aa} - v_{j+1}^{I,ab}(\Xi_{j+1}^I)^{-1} v_{j+1}^{I,ab\,\prime}.$$

The matrices V_{j+1}^I and scalars α_j, for $j = 1, 2 \ldots T$, can be solved recursively by setting $\alpha_T = \gamma$, $V_T^I = 0_4$, and using the recursive equations found in the f.o.c.s:

$$\alpha_j = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma}, \text{ and } V_j^I = m_{t+1}^I \cdot \left(\frac{\alpha_j}{\alpha_{j+1} R}\right) + 2 \cdot i_{1,1} \cdot \left(\gamma c_{j+1}^I + \log \frac{\alpha_j}{\gamma}\right). \tag{33}$$

where $i_{1,1}$ is a 4x4 matrix with all element equal to zero except for the first, which equals one. For the uninformed investors the procedure is similar, but one must replace M_t, A_Q, B_Q, and Σ, for M_t^U, A_Q^U, B_Q^U, and Δ, respectively. This allows solving investment and consumption policies using the appropriate superscript U instead of I.

We can now impose the market clearing condition (19). Note that although the conditional state vector has different dimensions for the informed and uninformed investors, the uninformed forecasts can be replaced by the actual values of the state variables plus the forecast noise. This leads to the following equation that must be satisfied by the price:

$$P = Y \cdot Z^{-1}, \quad \text{where} \tag{34}$$

$$Z \equiv [(A_M - I_4 R) \cdot ((m_0 \bar{w}_0^U - m_1 v_0 \bar{w}_1^U) \cdot (m_0' + x_0) + I_4 \bar{w}_0^I) - B_M((m_0' \bar{w}_1^U \cdot (m_0' + x_0) + I_4 \bar{w}_1^I)],$$

$$
\begin{aligned}
Y &\equiv [A_\theta - (A_D A_M m_0 \bar{w}_0^U - (A_D A_M m_1 v_0 + (A_D B_M + B_D) m_0^{'}) \bar{w}_1^U) \cdot (m_0^{'} + x_0) \\
&\quad - A_D A_M \bar{w}_0^I + (A_D B_M + B_D) \bar{w}_1^I], \;\; \bar{w}_0^U \equiv \frac{1}{T} \sum_{s=2}^{T} (\alpha_s \Gamma_s^U)^{-1}, \;\; \bar{w}_0^I \equiv \frac{1}{T} \sum_{s=2}^{T} (\alpha_s \Gamma_s^I)^{-1}, \\
\bar{w}_1^U &\equiv \frac{1}{T} \sum_{s=2}^{T} (\Xi_s^U)^{-1} v_s^{U,ab\,'} (\alpha_s \Gamma_s^U)^{-1}, \; \bar{w}_1^I \equiv \frac{1}{T} \sum_{s=2}^{T} (\Xi_s^I)^{-1} v_s^{I,ab\,'} (\alpha_s \Gamma_s^I)^{-1}, \\
A_\theta &\equiv [\bar{\theta} \;\; 0 \;\; 1 \;\; 0], \;\; \text{and } x_0 \equiv [0 \;\; 1 \;\; -\lambda^{-1}]^{'} \cdot v_0.
\end{aligned}
$$

Equation (34) is the fixed point problem that determines the equilibrium price vector (whenever an equilibrium exists). To see this, notice that both the (1x4) matrix Y and the (4x4) matrix Z depend on an initial conjecture of the price vector P_0 through their dependence on A_M, B_M, A_Q, B_Q, A_M^U, B_M^U, A_Q^U, and B_Q^U, all function of P_0. The fixed point in (34) that defines the solution can then be written as $P^* = Y(P^*)Z(P^*)^{-1} = F(P^*)$, where the functional $F(\cdot)$ is implicitly defined by the f.o.c's in the dynamic optimization of investors together with the market clearing requirement.

Proof of Proposition 1:

This proof consists of three steps. 1) Derive the price equation that arises from the market clearing condition. This leads to a quadratic equation for the random supply coefficient p_θ in the price equation (6). I will show here that both roots of the equation depend on a particular coefficient of the value function matrix V (the stationary value function matrix in the infinite horizon case). This coefficient is the ninth element of the matrix, associated with the utility (value function) impact of random supply innovations, which I label V_θ. 2) Derive a second equation which describes the element V_θ as a function of the price coefficient p_θ. An equilibrium is then a pair $\{p_\theta, V_\theta\}$ satisfying both equations. 3) show that a) an intersection between these functions always exists (part a) of the proposition), and b) it is unique (part b) of the proposition). Since the problem of investors in the no-information economy can always be represented as an equivalent full-information economy with scaled up noise, and the proof is valid for all possible parameter values, I will prove here the proposition for the full-information economy only.

Step 1: In the infinite horizon case with symmetric (full) information, there exists only one type of investor whose asset demand (equation (30)) can be restated (dropping the age and information subscripts) as:

$$
X_t = \frac{A_Q - B_Q \Xi^{-1} v^{ab\,'}}{\alpha \Gamma} \Psi_t,
$$

where $A_Q = [-p_0 r + p_F F_0 + p_\theta \theta_0 + F_0 \quad p_F(\rho_F - R) + \rho_F \quad p_\theta(\rho_\theta - R)]$ and $B_Q = [1 \quad (1 + p_F) \quad p_\theta]$ are the row vectors associated with the loadings of future returns on the vector of contemporary state variables, and future disturbances, respectively. It is straightforward to show that for this economy,

$$\Xi = \begin{bmatrix} \sigma_D^{-2} & 0 & 0 \\ 0 & \sigma_F^{-2} & 0 \\ 0 & 0 & \sigma_\theta^{-2} + V_\theta, \end{bmatrix}, \quad \Gamma = \sigma_D^2 + (1+p_F)^2\sigma_F^2 + p_\theta^2 \frac{1}{\sigma_\theta^{-2} + V_\theta}, \quad \text{and} \quad \alpha = \frac{\gamma r}{R}.$$

Moreover, from equation (29), it can be shown that the second term in the demand's numerator corresponds to $B_Q \Xi^{-1} v^{ab\,\prime} = [p_\theta \frac{V_{1,3} + V_\theta \theta_0}{\sigma_\theta^{-2} + V_\theta} \quad 0 \quad p_\theta \frac{\rho_\theta V_\theta}{\sigma_\theta^{-2} + V_\theta}]$, where $V_{1,3}$ is the 3rd (and 7th) term of the (symmetric) matrix V. The market clearing condition then reads

$$\frac{[-p_0 r + p_F F_0 + p_\theta(\theta_0 \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + V_\theta} - \frac{V_{1,3}}{\sigma_\theta^{-2} + V_\theta}) + F_0 \quad p_F(\rho_F - R) + \rho_F \quad p_\theta(\rho_\theta \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + V_\theta} - R)]}{\gamma(r/R)(\sigma_D^2 + (1+p_F)^2\sigma_F^2 + p_\theta^2 \frac{1}{\sigma_\theta^{-2} + V_\theta})} = [0 \quad 0 \quad 1],$$

which gives rise to three equations determining the price coefficients p_0, p_F and p_θ:

$$0 = -p_0 r + p_F F_0 + p_\theta(\theta_0 \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + V_\theta} - \frac{V_{1,3}}{\sigma_\theta^{-2} + V_\theta}) + F_0 \tag{35}$$

$$0 = p_F(\rho_F - R) + \rho_F \tag{36}$$

$$p_\theta(\rho_\theta \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + V_\theta} - R) = p_\theta^2 \frac{\gamma(r/R)}{\sigma_\theta^{-2} + V_\theta} + \gamma(r/R)(\sigma_D^2 + (\frac{R}{R - \rho_F})^2 \sigma_F^2). \tag{37}$$

Expression (37) is the first equation we will use to find the equilibrium pair $\{p_\theta, V_\theta\}$. Defining $k \equiv (2\gamma/R)(r/R)^2(\sigma_D^2 + (\frac{R}{R-\rho_F})^2\sigma_F^2)^{1/2}$, and $\sigma^{-2} \equiv \sigma_\theta^{-2}\frac{R-\rho_\theta}{R}$, we can rewrite the two roots of (37) as

$$p_{\theta,1}^{+}(V_\theta) = -\frac{R^2(\sigma^{-2} + V_\theta)}{2\gamma r} + \frac{R^2(\sigma^{-2} + V_\theta)}{2\gamma r}\sqrt{1 - k^2 \frac{\sigma_\theta^{-2} + V_\theta}{(\sigma^{-2} + V_\theta)^2}}, \tag{38}$$

$$p_{\theta,1}^{-}(V_\theta) = -\frac{R^2(\sigma^{-2} + V_\theta)}{2\gamma r} - \frac{R^2(\sigma^{-2} + V_\theta)}{2\gamma r}\sqrt{1 - k^2 \frac{\sigma_\theta^{-2} + V_\theta}{(\sigma^{-2} + V_\theta)^2}}. \tag{39}$$

Step 2: to find the dependence of the value function term V_θ on the model parameters and the price coefficient p_θ, we adapt equation (33) to the infinite horizon case to write

$$V = \frac{m}{R} + 2 \cdot i_{1,1} \cdot (\gamma c + \log \frac{\alpha}{\gamma}),$$

but since $i_{1,1}$ is zero for all terms besides the first, the expression for the ninth term in V reduces to

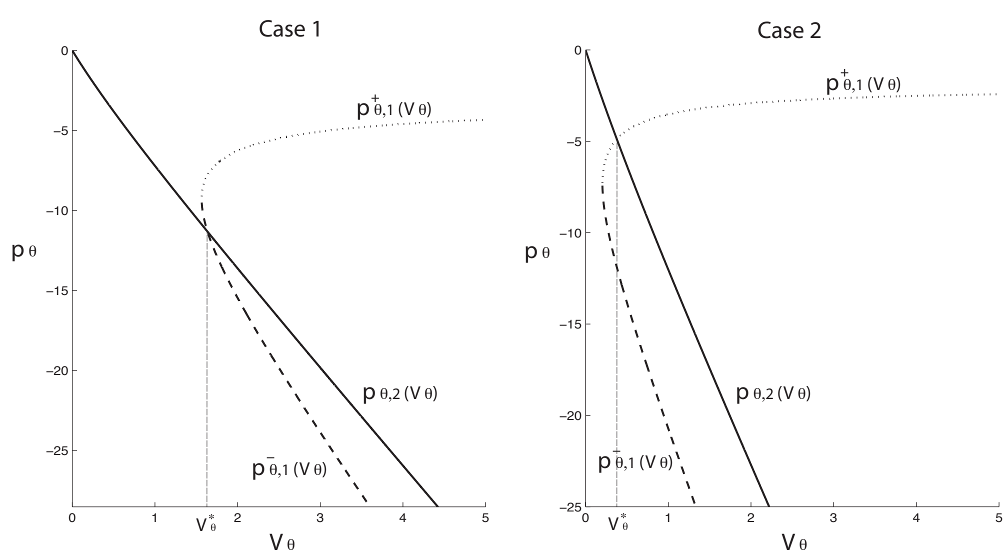
$$V_\theta = \frac{m(3,3)}{R} = p_\theta(\rho_\theta \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} - V_\theta} - R)\frac{r\gamma}{R^2} + V_\theta \rho_\theta^2 \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} - V_\theta}, \quad \text{or}$$

$$p_{\theta,2}(V_\theta) = -V_\theta \frac{R}{\gamma r}(1 + \frac{\rho_\theta(1-\rho_\theta)\sigma_\theta^{-2}/R}{\sigma^{-2} + V_\theta}) \tag{40}$$

Step 3: I now study equations (37) and (40). Figure 9 provides the loci of these equations in the $\{V_\theta, p_\theta\}$ space. To find existence, it suffices to show that over the range of values of V_θ for which (37) has a real solution, it intersects either of the two roots of equation (40) at least once. Uniqueness then amounts to showing that this intersection is a singleton. I will prove existence and uniqueness by establishing the following facts:

i) $\partial p^+_{\theta,1}(\cdot)/\partial V_\theta > 0$;
ii) $\partial p^-_{\theta,1}(\cdot)/\partial V_\theta < 0$, $\partial^2 p^-_{\theta,1}(\cdot)/\partial V_\theta^2 > 0$, and $\lim_{V_\theta \to \infty} \partial p^-_{\theta,1}(\cdot)/\partial V_\theta = \frac{-R^2}{\gamma r}$,
iii) $\partial p_{\theta,2}(\cdot)/\partial V_\theta < 0$, $\partial^2 p_{\theta,2}(\cdot)/\partial V_\theta^2 > 0$, and $\lim_{V_\theta \to \infty} \partial p_{\theta,2}(\cdot)/\partial V_\theta = \frac{-R}{\gamma r}$, and
iv) At V_θ^*, $|\partial p^-_{\theta,1}(V_\theta^*)/\partial V_\theta| > |\partial p_{\theta,2}(V_\theta^*)/\partial V_\theta|$, where V_θ^* satisfies $p^-_{\theta,1}(V_\theta^*) = p_{\theta,2}(V_\theta^*)$, whenever an intersection between $p_{\theta,2}(\cdot)$ and the negative root $p^-_{\theta,1}(\cdot)$ exists.

Figure 9: Infinite horizon economy with symmetric information



Facts i)-iii) imply that an intersection between equations (37) and (40) always exist. This is because equation $p_{\theta,2}(\cdot)$ in (40), which begins in the origin and has a strictly negative slope that converges to a constant, will either intersect the negative root $p^-_{\theta,1}(\cdot)$ in (39) (case 1), whose strictly negative slope converges to a constant of larger absolute magnitude than the limit of $p_{\theta,2}(\cdot)$, or it will intersect the positive root $p^+_{\theta,1}(\cdot)$ in (38) (case 2), which has strictly positive slope.

Fact iv) provides the uniqueness result. To see this, notice that in case 1, the condition that $|\partial p^-_{\theta,1}(V^*_\theta)/\partial V_\theta| > |\partial p_{\theta,2}(V^*_\theta)/\partial V_\theta|$ implies that the two lines can only intersect once. If this was not true, then at least in one of these intersections condition iv) would be violated. Regarding case 2, whenever equation $p_{\theta,2}(\cdot)$ intersects the positive root $p^+_{\theta,1}(\cdot)$, condition iv) implies it cannot also intersect the negative root $p^-_{\theta,1}(\cdot)$, even once. This is because whenever $p^+_{\theta,1}(\cdot)$ and $p_{\theta,2}(\cdot)$ intersect, the first intersection with $p^-_{\theta,1}(\cdot)$ would be from above, implying $|\partial p^-_{\theta,1}(V^*_\theta)/\partial V_\theta| < |\partial p_{\theta,2}(V^*_\theta)/\partial V_\theta|$, or a violation of condition iv). Hence, condition iv) establishes the global uniqueness of the solution. I now prove each of these claims.

To establish fact i), derive equation (38) w.r.t. V_θ, which yields the result immediately. For fact ii), simply derive the negative root $p^-_{\theta,1}(\cdot)$ in (39) twice, which gives

$$
\begin{aligned}
\partial p^-_{\theta,1}(\cdot)/\partial V_\theta &= -\frac{R^2}{2\gamma r}[1 + \frac{2(\sigma^{-2}+V_\theta)-k^2}{2((\sigma^{-2}+V_\theta)^2 - k^2(\sigma_\theta^{-2}+V_\theta))^{1/2}}] < 0, \\
\partial^2 p^-_{\theta,1}(\cdot)/\partial V_\theta^2 &= \frac{R^2}{2\gamma r}[\frac{k^2\sigma_\theta^{-2}\rho_\theta/R + k^4/4}{((\sigma^{-2}+V_\theta)^2 - k^2(\sigma_\theta^{-2}+V_\theta))^{3/2}}] > 0, \quad \text{and} \\
\lim_{V_\theta\to\infty} \partial p^-_{\theta,1}(\cdot)/\partial V_\theta &= \frac{-R^2}{\gamma r}.
\end{aligned}
$$

Similarly, for fact iii) we derive equation $p_{\theta,2}(\cdot)$ in (40) twice to find

$$
\begin{aligned}
\partial p_{\theta,2}(\cdot)/\partial V_\theta &= -\frac{R}{\gamma r}[1 + \frac{\sigma^{-2}\sigma_\theta^{-2}\rho_\theta(1-\rho_\theta)/R}{(\sigma^{-2}+V_\theta)^2}] < 0, \\
\partial^2 p_{\theta,2}(\cdot)/\partial V_\theta^2 &= \frac{R}{\gamma r}[\frac{2\sigma^{-2}\sigma_\theta^{-2}\rho_\theta(1-\rho_\theta)/R}{(\sigma^{-2}+V_\theta)^3}] > 0, \quad \text{and} \\
\lim_{V_\theta\to\infty} \partial p_{\theta,2}(\cdot)/\partial V_\theta &= \frac{-R}{\gamma r}.
\end{aligned}
$$

Finally, to establish fact iv), let's define the following objects:

$$
a(V_\theta) \equiv \frac{\sigma_\theta^{-2}\rho_\theta(1-\rho_\theta)/R}{(\sigma^{-2}+V_\theta)^2}, \quad b(V_\theta) \equiv \frac{V_\theta}{\sigma^{-2}+V_\theta}, \quad c(V_\theta) \equiv 1 - k^2\frac{\sigma_\theta^{-2}+V_\theta}{(\sigma^{-2}+V_\theta)^2}, \quad \text{and} \quad d(V_\theta) \equiv \frac{R(1-b(V_\theta))}{R-\rho_\theta b(V_\theta)}.
$$

Intersection of equations $p^-_{\theta,1}(\cdot)$ and $p_{\theta,2}(\cdot)$ at $V_\theta = V^*_\theta$ implies we can write

$$
-\frac{R^2}{2\gamma r}(\sigma^{-2}+V^*_\theta)(1+c(V^*_\theta)^{1/2}) = -\frac{R}{\gamma r}V^*_\theta(1+a(V^*_\theta)), \quad \text{or} \tag{41}
$$

$$
c(V^*_\theta) = \frac{4}{R}b(V^*_\theta)(1+a(V^*_\theta))(\frac{b(V^*_\theta)(1+a(V^*_\theta))}{R} - 1) + 1. \tag{42}
$$

We now compare the slopes between equations $p^-_{\theta,1}(\cdot)$ and $p_{\theta,2}(\cdot)$ at $V_\theta = V^*_\theta$. Manipulation of $\partial p^-_{\theta,1}(V^*_\theta)/\partial V_\theta$ using the equality condition in (42) allows to write (omitting the dependence on V_θ)

$$\partial p^-_{\theta,1}(\cdot)/\partial V_\theta|_{V_\theta=V^*_\theta} = -\frac{1}{\gamma r}\frac{[b^2(1+a)^2 + b(1+a)\frac{R-b(1+a)}{R}\rho_\theta]}{\frac{2}{R}b(1+a)-1}, \tag{43}$$

while the slope of equation $p_{\theta,2}(\cdot)$ takes the form

$$\partial p_{\theta,2}(\cdot)/\partial V_\theta|_{V_\theta=V^*_\theta} = -\frac{R}{\gamma r}(1+a(1-b)). \tag{44}$$

Proving condition iv) then amounts to establishing

$$\frac{[b^2(1+a)^2 + b(1+a)\frac{R-b(1+a)}{R}\rho_\theta h]}{\frac{2}{R}b(1+a)-1} > 1+a(1-b). \tag{45}$$

but notice that $a(\cdot)$ can be expressed as $\rho_\theta(1-b(\cdot))\frac{1-\rho_\theta}{R-\rho_\theta}$. Using this last transformation in (43) and (44) gives after some (tedious) manipulation, the result in (45). This completes the proof.

Proof of Proposition 2:

Since the problem of investors in the no-information economy can always be represented as an equivalent full-information economy with scaled up noise, I will prove the proposition for the full-information economy only. Let $P^* \equiv [p^*_0 \quad p^*_F \quad p^*_\theta]$ be an equilibrium price vector, and $P' \equiv [p^*_0 \quad p^*_F \quad p'_\theta]$ represent a perturbation from the equilibrium. When $T = 2$, the value function matrix V is the zero matrix, and $\alpha = 1$. Let $m \equiv 2\frac{\gamma}{R-\rho_\theta}(\sigma^2_D + (\frac{R}{R-\rho_F})^2\sigma^2_F)^{1/2}$. The solution for the price coefficient associated with the supply shock, p_θ, is then given by the following roots

$$p^{*,+}_\theta = -\frac{\sigma^{-2}_\theta(R-\rho_\theta)}{2\gamma}(1-\sqrt{1-m^2\sigma^2_\theta}), \tag{46}$$

$$p^{*,-}_\theta = -\frac{\sigma^{-2}_\theta(R-\rho_\theta)}{2\gamma}(1+\sqrt{1-m^2\sigma^2_\theta}). \tag{47}$$

Let's now analyze the effects of perturbing the equilibrium by setting $p'_\theta \neq p^*_\theta$. From the agents optimization and market clearing condition, we get a new implied price coefficient p''_θ given by

$$p''_\theta = -\frac{\gamma(\sigma^2_D + (\frac{R}{R-\rho_F})^2\sigma^2_F)}{R-\rho_\theta} - \frac{(p'_\theta)^2}{R-\rho_\theta},$$

from which we can subtract the initial perturbation p'_θ to write

$$p''_\theta - p'_\theta = -(p'_\theta - p^*_\theta)(1 + \frac{\gamma\sigma^2_\theta(p'_\theta + p^*_\theta)}{R-\rho_\theta}).$$

If we now evaluate this expression at $p'_\theta \to p^*_\theta$ along the low volatility equilibrium in the positive root (equation (46)), we find

$$p''_\theta - p'_\theta|_{p'_\theta \to p^{*,+}_\theta} = -(p'_\theta - p^*_\theta)\sqrt{1 - m^2\sigma^2_\theta},$$

and so $|p''_\theta - p^*_\theta| < |p'_\theta - p^*_\theta|$ and the equilibrium is stable. Along the high-volatility equilibrium in the negative root (equation (47)), we get

$$p''_\theta - p'_\theta|_{p'_\theta \to p^{*,-}_\theta} = (p'_\theta - p^*_\theta)\sqrt{1 - m^2\sigma^2_\theta},$$

and so $|p''_\theta - p^*_\theta| > |p'_\theta - p^*_\theta|$ and the equilibrium is unstable. This completes the proof.

Numerical solution method:

I now describe the numerical methodology used to find the equilibrium coefficients in the price function of equation (6). The problem consists in finding the (possibly multiple) price coefficients that satisfy the fixed point representation in expression (34). As initial price coefficients, I use a matrix whose rows correspond to different starting values for the price vector, where the only element changing across rows is the coefficient associated with the random supply, p_θ. Due to its convergence properties, the low volatility equilibrium is straightforward to find as initial guesses using relatively low values of p_θ (in absolute magnitude) quickly converge to the the same fixed point price vector.

The complications arise when trying to find the price coefficients in the unstable price vector. To find it I postulate successively decreasing (more negative) supply coefficients as an initial guesses. If the iteration brings the price coefficient to the low-volatility equilibrium, this means that the conjectured supply coefficient is still not negative enough. Beyond some threshold, the postulated value of p_θ is too negative and the iteration diverges. This implies that the second equilibrium value of p_θ must lie in between the last initial guess that produced a convergence, and the first guess that produced the divergence. I then zoom into this region, creating a new matrix of price vectors that span this narrower range of price coefficients.

I then repeat the process, every time defining a new range of price coefficients between the last converging and first diverging row vector of the initial matrix. After a few iterations, the range of values where the second equilibrium p_θ lies can be made arbitrarily narrow, providing an arbitrarily close approximation to the true equilibrium value.