Speculative Bubbles and the Cross-Sectional Variation in Stock Returns

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Abstract
Evidence suggests that rational, periodically collapsing speculative bubbles may be pervasive in stock markets globally, but there is no research that considers them at the individual stock level. In this study we develop and test an empirical asset pricing model that allows for speculative bubbles to affect stock returns. We show that stocks incorporating larger bubbles yield higher returns. The bubble deviation, at the stock level as opposed to the industry or market level, is a priced source of risk that is separate from the standard market risk, size and value factors. We demonstrate that much of the common variation in stock returns that can be attributable to market risk is due to the co-movement of bubbles rather than being driven fundamentals.

1. Introduction

Why do some stocks generate higher returns than others? Addressing this apparently simple question has been the focus of a vast amount of empirical research spanning almost half a century. Of the more popular approaches to modelling the returns on risky assets that have been proposed, a fruitful family of models stem from a modified version of the capital asset pricing model (CAPM) proposed by Fama and French (1993, 1996). Their augmented model argues that expected excess returns are related to 3 factors: (1) the market risk premium, (2) the difference in returns between a portfolio of small firm stocks and a portfolio of large firm stocks, and (3) the difference in returns between a portfolio of high book-to-market stocks and a portfolio of low book-to-market stocks. However, an important stylised feature of performance that the model was unable to explain is the tendency for those stocks with momentum to experience continued abnormal performance in the short-term (Jegadeesh and Titman, 1993). This observation has led authors (notably Carhart, 1997, in a study of the abnormal performance of US mutual funds) to add a fourth factor to the model – momentum, that Carhart defined as the difference in returns between a portfolio of winner stocks and a portfolio of loser stocks, based on the past 12 months of returns.

A further potential but as yet unexplored source of variation in stock returns could arise from the presence of speculative bubbles. Entirely separate from that on asset pricing, a body of work has sought to explain the observed phenomenon that equity market prices often rise substantially, deviating a long way from values that appear economically justifiable, for prolonged periods. Such price dynamics at first blush seem inconsistent with the efficient markets hypothesis, but proponents of rational speculative bubble theory suggest that this price behaviour is entirely consistent with market rationality since investors are compensated for the increased risk of a price collapse by ever increasing returns. According to Santoni's (1987) definition, speculative bubbles have the special characteristic that they are persistent, systematic and increasing deviations of prices from their fundamental values. The fundamentals measure the underlying or intrinsic worth of an asset based on the suitably discounted stream of cashflows that will accrue to the holder.

Speculative bubbles are generated when investors include expectations of the future price of a security in their information set. Under these conditions, the actual market price of the asset will be a function of the expected future price and vice versa. In the presence of speculative bubbles, positive expected bubble returns will lead to increased demand and will thus force prices to diverge from their fundamental values. If the expectation of positive excess returns remains unchanged and the investor is compensated for the increased risk of bubble collapse, then these excess or abnormal returns will be realised in an increasing fashion. Thus, even though investors observe that stocks are

overvalued, they are not willing to close out their positions because the bubble component offers at least the required rate of return. Prices increase slowly in the initial phase of the bubble, and then may grow exponentially before a collapse is eventually triggered as investors come to realise that the bubble-inflated price is unsustainable (see, for example, Kindelberger, 1989). Thus, as the riskiness of investing in the asset increases, so do the returns right until the point when the bubble bursts. Those who are holding the bubbly asset at this point will inevitably sustain heavy losses, but key to the formation of a bubble is that investors cannot predict precisely when it will collapse. All they can do, perhaps, is to estimate probabilistically the likelihood of collapse at any given point in time. If investors were able to forecast with precision when the bubble would collapse, the bubble would unravel back to its inception and could therefore never exist. It is important to note that, defined in this way, speculative bubbles can arise from entirely rational, forward-looking investor behaviour in the context of a limited information set, and could be thus considered consistent with orthodox asset pricing theory in the framework, for example, of Merton's (1973) intertemporal capital asset pricing model (ICAPM).

The literature on testing for the presence of bubbles in stock markets alone is extensive (see, for example, Anderson et al., 2010; Brooks and Katsaris, 2003, 2005a, 2005b; Dezhbakhsh and Demirguc-Kunt; 1990; Donaldson and Kamstra, 1996, to name but a few), and the number of papers documenting bubble existence is considerably greater than those finding against. Pastor and Veronesi (2006), on the other hand, argue that the growth of technology stock prices in the 1990s was not necessarily caused by a bubble. They demonstrate that firms' values grow with uncertainty about their future profitability. Even in the context of the simple Gordon growth model for dividends, when there is uncertainty about the future growth rate of dividends, the distribution of future dividends becomes right-skewed. So it is therefore rational for a technology stock to have a very high valuation relative to current earnings since the possibility that it will become hugely successful more than compensates for the likelihood that it will fail.

Do bubbles have any implications for asset pricing and the properties of stock returns, either theoretically or empirically? Several studies have tried to link, at least indirectly, the presence of asymmetries or anomalies in stock market returns, with the presence of speculative bubbles (see, for example, McQueen and Thorley, 1994 and Chen, Hong and Stein, 2001). Nevertheless, to our knowledge, there has been no research into the direct impact of bubbles on individual security returns or the cross-section of returns. Our study represents the first to empirically examine the impact that bubbles at the individual stock level may have on the market as a whole, and the extent to which bubble sizes can explain variations in stock returns.

The book-to-market equity and size factors have been rationalised as risk factors based on their ability to proxy for financial distress. Fama and French (1995) propose that firms with low price-to-book ratios may be in distress and have frequently suffered persistently low earnings. Indeed, Vassalou and Xing (2004) and Petkova (2006) show that the size and value factors both contain information related to financial distress and/or default risk, but these factors remain somewhat ad hoc and empirically motivated. A bubble factor, however, could proxy for risk more explicitly. From an intuitive perspective, it is easy to see why bubbles could be a priced risk factor. First, a corollary of the statement that an asset's price includes a (positive) speculative bubble is that it is trading at more than its underlying or intrinsic worth. In other words, relative to fundamentals, it is over-valued. Second, an asset whose price contains a speculative bubble is more likely, ceteris paribus, to experience a rapid and substantial fall in value, than an asset which does not. Thus, the fear of overpaying for assets and the fear that there will be a large and sudden price fall, tie in well with what many investors would consider as constituting risks.

Several tests for the presence of speculative bubbles in financial markets have been developed in recent years. Such investigations include approaches based on variance bounds (see, for example, Shiller, 1981), tests for a bubble premium (Hardouvelis, 1988), tests based on cointegration analysis (Diba and Grossman, 1988), and tests based on regime switching models (van Norden, 1996). The latter of these approaches is a direct test for the presence of bubbles, whereas the former three are indirect tests that examine the distributional properties of prices and of fundamental measures. The weight of evidence has fallen strongly in favour of the presence of rational speculative bubbles. However, virtually all existing work has focused on the *ex post* identification of evidence for or against bubbles at the aggregate stock market-wide level and at the industry level (see Anderson et al., 2010). This implies that there is significant scope for research at the company-specific level. Such research may help towards an understanding of how bubbles are formed, and how they are priced by investors.

Our research has several overlapping objectives. First, we will apply the direct bubble tests as described above to individual stock price data in order to examine the extent to which bubbles are pervasive across equity markets or are confined to specific parts of it (for example, to specific sectors or simply to individual companies). Several researchers have argued that speculative bubbles, or bubble-like behaviour, originate from high technology sectors, because the fundamental value of stocks within such high growth sectors is very difficult to estimate (Shiller, 2000, Hobijn and Jovanovic, 2001). Indeed, Cochrane (2003) argues that, "if there was a 'bubble,' or some behavioural overenthusiasm for stocks, it was concentrated on NASDAQ stocks, and NASDAQ tech and internet

stocks in particular." Our investigation will shed some light on the validity or otherwise of such arguments.

Second, we will develop a new approach to determining the returns on risky assets by reference to a stock's exposure to "bubble risk". That is, we propose that individual stocks will have particular bubble sizes, and that those stocks with the largest bubbles will have the highest probability of bubble collapse and (ceteris paribus) will command the highest returns. Thus, an augmented Fama-French (1996) approach will be employed in order to test this conjecture. The first step will be to estimate the bubble sizes for each stock. The second will be to form equally weighted portfolios of large bubble stocks and of small bubble stocks, and the difference in average return between the large bubble and small bubble portfolios will be calculated. This measure will then be used in a time-series regression alongside the Fama-French plus momentum factors in order to determine whether bubble size is a priced common risk factor. This approach is based on Fama and French's variant of the original Black, Jensen and Scholes (1972) methodology. The universe of stocks will be formed into decile portfolios ranked (separately) according to beta, the book-to-market ratio, earnings-to-price, cashflow-to-price, and bubble size. The stocks will also be formed into 25 double-sorted portfolios by size and by book-to-market. In both cases, each of the portfolios will be subjected to the same time-series regression so that the significances of the factors in each case can be compared.

The remainder of this paper is organised as follows. Section 2 describes the methods that we employ in the paper to construct fundamental values, to measure bubbles, and to incorporate them into asset pricing models, Section 3 describes the features of the dataset that we use while Section 4 presents and analyses the results and Section 5 concludes and offers suggestions for further research.

2. Methodology

2.1 Construction of Fundamentals

Central to any model of bubble behaviour is how the fundamental value of the asset is estimated. Several methods can be found in the literature, but methods based on the Gordon Growth Model or dividend multiples are most popular. We use the dividend multiple method adopted by van Norden and Schaller (1999). It is widely appreciated that the applicability of this method to high technology sectors such as biotechnology is questionable since many companies in these sectors do not currently pay dividends and may not plan to do so for many years, preferring to reward their investors with faster growing share prices. Damodaran (2000) nevertheless shows that the fundamentals of equity valuation apply to companies in such sectors just as validly as those

elsewhere, although the valuations are often noisy. It would be possible to base the fundamental measure on earnings. However, as Keating et al. (2003) point out, during the "TMT bubble" growth period of the late 1990s, stock valuations were in many cases large multiples of sales, let alone profits. Indeed, stock prices continued to rise despite firms in some cases never having made a profit and therefore it is not clear that earnings would be a better measure of fundamentals than dividends. 'Soft' factors, sometimes considered as intangible assets, such as the evaluation of patents or web traffic for internet companies, could also play a part in any practical valuation, although Keating et al. find that they are no better than accounting information at explaining the cross-section of returns. Consequently, we do not use any of these highly subjective measures here and given that dividends represent the actual cashflows received by investors, we employ them as the basis of our fundamental measure while noting their limitations. One implication of this is that any stocks not paying dividends are excluded from the analysis – this is approximately 20% of the sample during the early 1980s, rising to 40% towards the end of the period for which we have data.

According to the discounted dividend model, the fundamental value of a stock i is the present value of all of its future dividends:

$$p_{i,t}^{f} = \sum_{g=1}^{\infty} \frac{1}{(1+r)^{g}} E_{t}(d_{i,t+g})$$

(1)

where $p_{i,t}^{f}$ is the fundamental price of stock i at time t, d_{t} is the dividend in period t, r is the market discount rate (the required rate of return, which is assumed constant), and $E_{t}(.)$ is the expectations operator conditioning on information available at time t. If the actual price of the stock, $p_{i,t}^{a}$, contains a bubble, then it will follow the dynamic process:

$$p_{i,t}^{a} = p_{i,t}^{f} + b_{i,t} + u_{i,t}$$

(2)

where $b_{i,t}$ is the bubble component, and $u_{i,t}$ is a zero mean, random disturbance term.

Van Norden and Schaller (1999), vNS hereafter, show that, if dividends follow a geometric random walk (i.e. log dividends follow a random walk with drift a), and given a constant discount rate, then the fundamental value of a stock must be a multiple of current dividends:

$$p_{i,t}^{f} = \rho d_{i,t}$$

(3)

So the fundamental price of company *i* at time *t* is a multiple ρ of the dividends the company paid then, where $\rho = \dfrac{1+r}{e^{(a+\frac{\sigma^2}{2})}-1}$, *r* is the subjective discount rate, and σ^2 is the variance of the innovations in random walk with drift process for dividends.

Following vNS, our approximation of the value of ρ is to use a trailing average of the price-dividend ratio, in our case calculated over the previous five years. The relative bubble size relative to the actual price is then:

$$B_{i,t} = \frac{b_{i,t}}{p_{i,t}} = \frac{p_{i,t} - p_{i,t}^f}{p_{i,t}} = 1 - \frac{\rho d_{i,t}}{p_{i,t}} \tag{4}$$

In other words, our estimate of the relative bubble size for company *i* at time *t* is the proportional deviation from the fundamental price of the actual observed price. Thus, for example, if the dividend yield for the whole sample is 5% (price - dividend ratio = 20), and this company is currently priced at £1 but only paying a 2p dividend, we estimate the relative bubble size as 60%:

$$B_{i,t} = 1 - \frac{\rho d_{i,t}}{p_{i,t}} = 1 - \frac{20 \times 2}{100} = 0.6 \tag{5}$$

Some summary statistics for the bubble deviations, expressed as percentages of the actual stock prices, are presented in Table 1, averaged within each year for presentational ease. Given our method of constructing fundamentals using equation (4), the bubble deviation will effectively fall as the firm increases its dividend payment relative to its historical average. A firm with a large price to dividend ratio (i.e. one that has historically had a small dividend payout ratio and so has a large ρ but which substantially increases its dividend) could have a very negative bubble deviation. For example, suppose the dividend yield for the past five years is now 1% (price - dividend ratio = 100), and this company is currently priced at £1 but paying a 10p dividend, $B_{i,t}$, the bubble deviation, will be -900%. On the other hand, $B_{i,t}$ will approach 100% as the firm reduces its dividend payment relative to its historical value.

It should be apparent that the bubble deviations are bounded from above by 100% (if the entire stock price comprises a bubble and the fundamental value is zero) but they are not bounded from below and consequently the distribution of bubble deviations is has a very long lower tail and is skewed. As a result, the mean bubble deviation is negative in most years, and the median deviation, shown in column seven of Table 1, is arguably a better summary statistic. Even the median stock has a negative bubble deviation for over half of the sample years, perhaps in part reflecting the declining

propensity over time of firms to pay dividends. This should be inconsequential for our models below, however, since the stocks are compared cross-sectionally within each year separately and thus the rank ordering of bubble deviations is key rather than their absolute magnitudes.

The profile of the relative bubble sizes over time is as one would expect, reaching peaks in 1983, 1987, 1994 and 2005, before subsequent market crashes deflated the bubbles that had established. Stocks were at their most undervalued, when more than half traded at below their fundamental values, during 1988-1991, 1997-2003, and 2008 onwards. The years towards the end of our sample period, especially in 2009, are particularly interesting. Unlike the bubble collapses of late 1987 and 1998-2000, prices as a multiple of dividends were not at historically unprecedented levels in 2009. During the mid-2000s, dividend payments had been increasing at a considerable rate. Clearly, a small number of stocks were very over-valued, trading at up to ten times their fundamental values.

2.2: Cross-Sectional Regressions to determine whether Bubbles affect Returns

We first use the Fama-French (1992) approach based on a "time-series of cross-sections" with an additional variable representing the size of the bubble deviations using the following procedure. We run a cross-sectional regression of the form

$$R_{i,t} = \alpha + \beta_1 Beta_{i,t} + \beta_2 MV_{i,t} + \beta_3 BVP_{i,t} + \beta_4 MOM_{i,t} + \beta_5 DY_{it} + \beta_6 B_{i,t} + u_{i,t}$$

(6)

where R are the monthly returns, $Beta$ are the CAPM betas calucluated as the slope of a regression of log monthly returns over the previous five years against those of the FTSE All Share Index (provided that at least 24 months' of returns are available, MV are the market capitalisations, BVP is the book-to-price ratio, MOM is the previous year's return for the stock to measure the extent to which it has momentum, DY is the dividend yield, and B is the bubble deviation as a percentage of the actual price, as described above, and the subscripts denote firm i and month t.

2.3: Determining the Bubble Factor Loadings and the Risk Premia

In the spirit of Fama and French (1993), we construct a bubble factor where we sort stocks by their bubble deviations intro triclies based on the top 30%, the bottom 30% and the remaining 40% in the middle. The bubble factor, $BBMSM_t$ ("big bubble minus small bubble") is then formed by subtracting the returns on the portfolio of stocks with the smallest bubble deviations from the returns of the portfolio of stocks with the largest bubble deviations. We assume that the bubble-based portfolios are reformed monthly and are value-weighted.

For illustration, the stocks are also sorted into deciles (rather than triclies) based on the sizes of their bubble deviations, and their characteristics are shown in Table 2. The first two rows report the

returns on each portfolio when it is equally weighted (first row after the header) and value weighted (second row). It is clearly evident that there is a positive bubble premium in the sense that stocks within the largest bubble decile generate returns of 4.78% per annum (3.61% value weighted) whereas those in the lowest bubble decile produce negative returns (-2.87% or -2.35% value weighted); thus a zero cost strategy of buying the 10% of stocks with the largest bubbles and short selling the 10% of stocks with the smallest bubble deviations would yield a profit of 7.66% (5.96% if value-weighted). The third row of the table shows the average size of the company in each bubble-sorted decile by the percentage rank (so low numbers denote the smallest companies). This shows that, aside from the companies with the smallest bubble deviations, which tend to be smaller companies, there is little to choose between the rest, indicating very little association between firm size and bubble size. Similarly, aside from the decile of smallest bubble deviation companies having the highest betas, there is very little separation between the rest, so that bubble size and market risk are otherwise uncorrelated. The table also shows that, perhaps as one would expect, bubble companies tend to be growth stocks with low book-to-market ratios, and in particular the decile with the smallest bubbles have book-to-market ratios around 40% higher than those in the decile with the largest bubbles.

We then implement a variant of the two-step methodology of Fama and MacBeth (1973). In the first step we estimate a time-series version of the Carhart (1997) 4-factor model, but also including the bubble term as described above:

$$R_{i,t} = \alpha + \beta_M RMRF_{i,t} + \beta_S SMB_{i,t} + \beta_V HML_{i,t} + \beta_U UMD_{i,t} + \beta_B BBMSB_{it} + v_{i,t} \quad (7)$$

where R_{it} are the returns on the portfolios i in each month t. Following Fama and French (1993), we employ 25 two-way size and book-to-market-sorted portfolios. We also separately employ the 27 portfolios that are three-way sorted on size, book-to-market and momentum constructed by Gregory et al. (2009).[1] *RMRF*, *SMB*, *HML* and *UMD* are the factor mimicking portfolio returns for market excess returns, firm size, value and momentum respectively. If we examine the sizes of the correlations between *BBMSB* and other factors using the entire sample period they are -0.04 with *RMRF*, -0.12 with *SMB*, -0.10 with *HML* and 0.02 with *UMD*, thus demonstrating the separation of the bubble factor from the others and that it is picking up different features of the returns data.

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[1] These data were obtained from their web site at
http://xfi.exeter.ac.uk/researchandpublications/portfoliosandfactors/

We initially estimate model (7) using the first five years of data, from October 1980 to September 1985, and in the second step, we use the factor loadings in a cross-sectional regression for October 1985:

$$R_{i,t} = \alpha + \lambda_M \beta_M + \lambda_S \beta_S + \lambda_V \beta_V + \lambda_U \beta_U + \lambda_B \beta_B + e_{i,t} \tag{8}$$

We then roll the sample forward by one month and estimate a new set of betas for November 1980 to October 1985 for use in a cross-sectional regression using these betas and the sorted portfolios for November 1985. This process continues until the sorted portfolio data, which end in December 2010, are exhausted, which provides 303 data points.[2] We can interpret the second stage regression parameters, $\lambda_M, \lambda_S, \lambda_V, \lambda_U, \lambda_B$ as factor risk premia, and the latter parameter is clearly of most direct interest.

2.4: The Bubble CAPM

Returning to the partition of the stock price into its fundamental and bubble components, taking equation (2), dropping the i subscript for convenience and subtracting one from each time subscript yields:

$$p_{t-1}^a = p_{t-1}^f + b_{t-1} + u_{t-1}. \tag{9}$$

Now subtract (9) from (2), divide both sides by the actual price at time t-1, p_t^a, and set the error terms equal to their expected value of zero:

$$\frac{p_t^a - p_{t-1}^a}{p_{t-1}^a} = \frac{p_t^f - p_{t-1}^f}{p_{t-1}^a} + \frac{b_t - b_{t-1}}{p_{t-1}^a} \tag{10}$$

The term on the left hand side of (10) is just the usual proportional simple return, which we could denote by R, while terms on the right could be defined as the fundamental and bubble returns, both expressed as a proportion of the actual price:

$$r_t^a = r_t^f + r_t^b \tag{11}$$

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[2] For comparison, we also run this regression including only a constant and the bubble and also excluding the bubble. However, these results do not add any additional insights compared to the composite regression and so are not shown.

Let us now focus on the standard unconditional CAPM of Sharpe (1964) and Lintner (1965) that describes the relationship between the expected return on a stock, $E(R_i)$ and its level of market risk (β_i):

$$E(R_i) = R_f + \beta_e \left[E(R_m) - R_f \right] \tag{12}$$

We suppress time subscripts for simplicity and assume that the model holds period-by-period. Now suppose that bubbles exist at both the individual stock level and thus "bubble risk" is partly systematic (varies along with the size of the market-wide bubble) and partly idiosyncratic. The total return on a stock i is the sum of the fundamental return and of the bubble return. The return on the market portfolio can be defined similarly:

$$R_i = R_i^f + R_i^B \text{ and } R_m = R_m^f + R_m^B \tag{13a) \& (13b}$$

Given the separation of returns into the fundamental and non-fundamental components in equations (13), we can express the beta for stock i as

$$\beta_i = \frac{Cov(R_m, R_i)}{Var(R_m)} = \frac{Cov(R_m^f + R_m^B, R_i^f + R_i^B)}{Var(R_m^f + R_m^B)} \tag{14}$$

Denoting the numerator in (14) as A and the denominator as B, we can write

$$A = Cov(R_m^f, R_i^f) + Cov(R_m^f, R_i^B) + Cov(R_m^B, R_i^f) + Cov(R_m^B, R_i^B) \tag{15a}$$

and

$$B = Var(R_m^f) + Var(R_m^B) + 2Cov(R_m^f, R_m^B) \tag{15b}$$

Assume also that the following restrictions hold:

$$Cov(R_i^f, R_i^B) = 0, Cov(R_m^f, R_i^B) = 0, Cov(R_i^f, R_m^B) = 0, Cov(R_m^f R_i^B) = Cov(R_m^f R_m^B) = 0$$

These restrictions are quite sensible since, by definition, the bubble component for both a stock and the market portfolio should be independent of its fundamentals and of each other's' fundamentals.[3] (15a) and (15b) then simplify to

$$A = Cov(R_m^f, R_i^f) + Cov(R_m^B, R_i^B) \tag{16}$$

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[3] Note here that we explicitly focus on rational bubbles and rule out intrinsic bubbles of the type examined by Froot and Obstfeld (1991), which are a non-linear function of fundamentals.

and

$$B = Var(R_m^f) + Var(R_m^B) \tag{17}$$

Finally, we can write the beta from (14) as

$$\beta_i = \frac{Cov(R_m^f, R_i^f) + Cov(R_m^B, R_i^B)}{Var(R_m^f) + Var(R_m^B)} = \frac{Cov(R_m^f, R_i^f) + Cov(R_m^B, R_i^B)}{Var(R_m)} = \beta_i^f + \beta_i^B \tag{18}$$

where the fundamental beta, β_i^f and bubble beta, β_i^B are defined respectively as:

$$\beta_i^f = \frac{Cov(R_m^f, R_i^f)}{Var(R_m)} \tag{19a}$$

and

$$\beta_i^B = \frac{Cov(R_m^B, R_i^B)}{Var(R_m)} \tag{19b}$$

3. Data

Thomson Reuters Datastream provides the most comprehensive dataset on UK companies and it is from this database that we draw the majority of the series used in this study. However, more complete information on a company's line of business is available from the London Share Price Database (LSPD, constructed at the London Business School). Also, information on which companies went into administration is only available from the LSPD. We therefore combined data from these two sources using the SEDOL code to match companies. The initial task was to arrive at a 'clean' list of UK companies for as long as the data allow. For a full list of companies that have data available on Datastream, we used the lists FBRIT for live companies and DEADUK for dead companies. We then excluded all companies with

- Company accounts, earnings or share price not quoted in GBP (allowing for differing currencies would complicate calculations unnecessarily)

- industry code G17 not available on the London Share Price Database (a few older, smaller companies have no code recorded)

- Only one type of share quoted (multiple classes of share would also complicate calculations unnecessarily).

We matched companies by SEDOL code across the two databases. Where no match was found using the SEDOL code, we carried out a manual name search. If necessary we also investigated changes of name using the Companies House WebCHeck service. This resulted in 1016 live companies from FBRIT and 3296 from DEADUK, giving 4312 in total over the period January 1980 to March 2012.[4]

Having arrived at a clean list of companies, we sourced monthly total return index data (including dividend payments and adjusted for stock splits, etc.) for each company from Datastream. Unfortunately, where a company goes into administration, its return index (*RI*) often becomes stuck at the figure on its last traded day, and is not subsequently revised to zero. We therefore set the *RI* to zero for all companies whose death reason according to the LSPD was 7 (liquidation), 14 (quotation cancelled for reasons unknown), 16 (receiver appointed / liquidation), 20 (in administration / administrative receivership) or 21 (cancelled, assumed valueless), starting in the month after the LSPD's death date. Finally, where the base *RI* for month *m*-1 was not zero, we calculated the return for company *i* in month *m* in the standard fashion.

Book values are only available from January 1987 on Datastream, so all models using variables based on this start at that date. We took each company's book value per share and divided it by the share price to get the book value-to-price ratio. Fama and French (1992) allowed a six-month lag on the book values to be sure that all book values would have been publically available in the month of portfolio formation. We allowed a four-month lag however, since UK Listing Authority rules state that preliminary results must be released within 120 days of the company's year-end. Book value-to-price for company *i* in month *m* is then

$$BVP_{i,m} = \frac{BV_{i,m-4}}{P_{i,m}}$$ (20)

To calculate the Fama and French *SMB* (Small Minus Big) and *HML* (High Minus Low book value-to-price) factors, we allocate each company/month data item an *S* or *B* flag if was in the bottom or top 50% of stocks by market capitalisation, and a *H*, *M* or *L* flag if it was in the top 30%, middle 40% or bottom 30% of stocks by book value to price in that month respectively. We then calculate average returns for each of the six combinations of these flags. The return figures for each portfolio are the average value-weighted excess returns for the stocks that fall into that group in that month. The average returns are the simple averages of the three small or large returns, or the two low or high

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[4] Gregory et al. (2009) note that the number of firms listed on the main market of the London Stock Exchange has fallen from over 2000 in 1998 to half that number in 2008, although the number of firms on the Alternative Investment Market increased from around 300 to over 1500 during the same time period; our sample includes firms listed on both markets and thus the number of firms has been roughly stable over time.

returns. *SMB* for that month is then calculated as the value-weighted average return for all stocks in the small stock portfolio minus the value-weighted average return for all stocks in the large stock portfolio; *HML* is defined similarly. *RMRF* is the average value-weighted return for that month of the stocks in the six cells in Table 1, plus companies with negative book values, minus the one-month Treasury bill rate.

For tests involving the Carhart four-factor model, we calculated the momentum figure for company *i* in month *m* following Carhart (1997, p.61) as the eleven-month momentum lagged one month:

$$Mom_{i,m} = \frac{RI_{i,m-1} - RI_{i,m-12}}{RI_{i,m-12}}$$ (21)

To calculate *UMD* we allocated companies to low, medium or high momentum groups, using the bottom 30%, middle 40% and top 30% of company momentums for that month. *PR1YR* for each month is then the value-weighted average one-month subsequent returns for the high momentum group, minus the returns for the low momentum group.

4. Results
4.1 Results from the Models Incorporating Bubble Deviations Directly

The results from the cross-sectional regression of monthly returns on the firm-specific characteristics are presented in Table 3 for three separate sub-samples. The first panel presents the results for the whole sample from May 1980 to March 2012, and the second two panels present results for the shorter sub-samples January 1990 to March 2012 and January 2000 – March 2012. We choose the first of these sub-samples in order to remove any "burn in" effect from having insufficient prior observations to estimate the long term price-to-dividend ratio in the whole sample, and the second sub-sample allows us to focus on the post-technology bubble bursting period only. Several specifications are examined in each case. The first is a simple test of the CAPM, and shows that the coefficient on beta is never statistically significantly different from zero and sometimes has the wrong sign. The bubble term, on the other hand, is significant at the 1% level in every model. When the market capitalisation, book-value-to-price, and momentum terms are incorporated, neither the bubble term's magnitude nor its statistical significant are more than modestly affected.

The final row in each panel incorporates the firm's current dividend yield in the specification to tackle potential concerns that the bubble factor may simply be another way of picking this up; it has been widely documented in the empirical asset pricing literature that high yielding stocks generate higher returns on average (see, for example, Hodrick, 1992; Kothari and Shanken, 1997). It is clear from Table 3 that the inclusion of dividend yield does not reduce the impact of the bubble on the

cross-section of returns. A parameter estimate of 1.66 on the bubble factor in the most complete specification for the whole sample period implies that a 10% increase in the bubble deviation as a fraction of the actual price would lead to a 0.166% increase in average returns per month (approximately 2% per annum). It should be noted that the coefficients of determination presented in this table, while modest in size, are in accord with those of other cross-sectional stock return regressions in the extant literature.

4.2 Results from Estimation of Bubble Factor Loadings and Risk Premia

Table 4 presents the factor loadings on the standard four factors plus that for the factor-mimicking portfolio of the bubble, *BBMSB* in the context of the separate first-pass regressions for the 25 size- and BTM-sorted portfolios. The table presents the parameter estimates from the time-series regression for each portfolio on the left hand side and the corresponding *t*-ratios on the right hand side. So growth stocks are represented towards the left of the table, while they have increasing value characteristics as we move to the right. As expected, the small and value stock portfolios load positively on the SMB and HML factors respectively. All of the portfolios have similar loadings on the market risk premium, indicating that it is not a good discriminator between different equity styles. There are no clear patterns in the loadings on the momentum factor (UMD), except that small growth stocks tend to load positively while all others have negative relationships with the factor, thus confirming previous research noting the failure of the Fama-French approach to capture relative strength. Considering the bubble factor in the second panel, like those of the momentum factor, there are no very strong patterns, and indeed most of the parameter estimates are not statistically significant at even the 10% level. It does appear to be the case, however, that growth stocks load more heavily on the bubble factor than value stocks, although there is no size effect.

The second stage cross-sectional regression results are presented in Table 5. Here, the factor loadings (betas) from the first stage are used as regressors to explain the variation across the 25 two-way sorted or 27 three-way sorted portfolios for the whole sample and for the same sub-samples as those presented in Table 3. The most salient feature of the results is that none of the factor loadings are able to significantly explain the cross-sectional variation in returns. The momentum term is marginally significant in the two-way sort, but none of the other factor risk premia is statistically important and in some cases they have the wrong signs. Thus, we find, like Gregory et al. (2009), that none of the four conventional factors are priced in the three-way sorted portfolios, and only one is in the two-way sort. This reflects the emerging consensus (see also Fletcher, 2010) that asset pricing models of these sorts simply do not work well in the UK context. What Fama and French (1993; 1996) effectively show is that the model works well in capturing the

variation that it was designed to explain – when stocks are sorted into portfolios according to size and the book-to-market ratio, the size and book-to-market factors are almost tautologically able to explain the differences in returns between those portfolios better than any other factors.

How, then, can we reconcile the results of Table 3, where the bubble term appears to be a priced risk factor, with those of Table 5, where it does not? One key difference is that the former are based on cross-sectional regressions at the firm level whereas the latter use the sorted portfolios. Thus it may be the case that all of the interesting variation occurs within the sorted portfolios rather than across them, so that it is lost in the regressions of Table 5.

4.3 The Bubble CAPM

Table 6 presents the results from the second-stage Fama-MacBeth-style regressions that include the bubble and fundamental beta terms as defined in Section 3.3. The results are quite revealing, and show that, while the part of the beta based on fundamentals is statistically insignificant in all of the second pass cross-sectional regressions, the bubble beta coefficients are positive and highly significant in the CAPM regressions, although they become insignificant when the size, value and momentum terms are added in the final rows of each panel. Nonetheless these findings are indicative that correlated sentiment across stocks related to bubble formation is more important in driving the common variation in returns that can be attributed to market risk than co-movements caused by fundamentals.

5. Conclusions

This paper has shown how bubbles at the individual firm level and their covariances with bubbles at the level of the market as a whole may be important in driving stock returns. We find that bubbles have a role to play in explaining the cross-sectional variation in returns and are priced, but in a fashion unrelated to the standard four factors that are popularly employed in asset pricing tests, and so they are unable to explain the variation in returns across size, value, and momentum sorted portfolios.

While the results presented here are encouraging, it is clear that there remains much work to do in more formally embedding bubble growth and collapse into both the theoretical asset pricing literature, and as a possible explanation for what are currently viewed as pricing anomalies. The rewards for doing so could be quite considerable. Since the presence of speculative bubbles – and the risk that they may subsequently collapse – could be a driver of many phenomena in empirical asset pricing that are currently viewed as anomalies. For example, it is possible that out of phase bubbles at the individual stock level might be the cause of the long term reversals in prices first

documented by DeBondt and Thaler (1985). Along these lines, stock returns would be high when the bubble is growing, with the subsequent reversal being explained by the bubble's collapse and overshoot followed initially by an undervaluation relative to fundamentals.

Further research could focus on the drivers of bubbles at the stock level, focusing in particular on the timing of bubble inception and collapse, and the extent to which positive (negative) momentum may be driven by the slow growth (collapse) of speculative bubbles. It may be possible that knowledge of the existence of bubble deviations at the firm level could enable banks and securities firms to better manage their risks and investors to enhance their trading profits by screening out stocks where bubble collapse seems most likely along the lines suggested by Brooks and Katsaris (2005b) at the index level.

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Table 1: Summary Statistics for Bubble Deviations by Year

Year	Mean	Standard Deviation	Minimum	10th Percentile	25th Percentile	Median	75th Percentile	90th Percentile	Maximum
1980	-4.91%	19.93%	-232.48%	-22.88%	-11.68%	-2.98%	4.71%	12.07%	64.97%
1981	-8.38%	439.94%	-12079.94%	-26.78%	-10.30%	3.90%	17.77%	32.41%	90.25%
1982	-5.99%	246.53%	-7588.37%	-33.60%	-10.46%	6.12%	21.16%	38.00%	90.33%
1983	0.61%	151.85%	-3498.42%	-28.83%	-4.81%	12.21%	27.62%	45.51%	93.25%
1984	-7.59%	184.08%	-3649.67%	-37.52%	-8.52%	9.62%	25.50%	41.09%	89.25%
1985	-31.12%	411.01%	-11363.55%	-60.74%	-16.62%	5.11%	22.75%	39.43%	89.83%
1986	-24.62%	301.36%	-6892.49%	-61.15%	-12.25%	8.89%	26.69%	41.53%	90.26%
1987	3.53%	153.81%	-3729.28%	-29.16%	-2.18%	18.43%	34.76%	48.82%	85.90%
1988	-20.62%	111.97%	-2493.89%	-71.88%	-29.31%	-1.98%	15.21%	31.01%	85.80%
1989	-39.60%	357.22%	-11774.64%	-97.38%	-42.12%	-9.85%	8.26%	23.53%	88.03%
1990	-101.62%	552.70%	-15064.06%	-197.21%	-100.86%	-47.15%	-16.70%	2.98%	88.62%
1991	-118.37%	2156.15%	-32200.14%	-202.92%	-102.17%	-44.12%	-13.11%	7.94%	89.05%
1992	-64.87%	190.58%	-3269.05%	-159.85%	-72.10%	-24.62%	2.13%	18.81%	89.76%
1993	-7.86%	87.61%	-1394.73%	-60.59%	-16.88%	7.87%	25.97%	41.55%	89.98%
1994	2.44%	117.81%	-2953.71%	-35.57%	-3.22%	15.70%	32.82%	47.03%	92.28%
1995	-23.16%	281.18%	-7010.65%	-60.29%	-20.50%	1.70%	18.73%	33.83%	86.82%
1996	-26.51%	242.15%	-6385.09%	-70.42%	-28.16%	-2.55%	16.11%	33.43%	90.47%
1997	-46.70%	342.38%	-8280.74%	-104.45%	-47.07%	-11.64%	8.61%	26.25%	84.04%
1998	-79.32%	529.82%	-13870.10%	-149.45%	-74.91%	-25.62%	2.73%	24.84%	80.79%
1999	-68.06%	422.55%	-9338.57%	-142.75%	-68.14%	-19.68%	9.00%	31.30%	85.92%
2000	-51.11%	384.88%	-9796.09%	-118.17%	-56.11%	-14.64%	16.80%	42.05%	88.20%
2001	-64.90%	302.63%	-6373.69%	-151.37%	-62.99%	-19.95%	6.03%	25.91%	82.19%
2002	-78.55%	361.17%	-6064.59%	-165.10%	-67.65%	-20.19%	4.93%	25.08%	80.86%
2003	-73.76%	255.57%	-3553.80%	-177.21%	-74.31%	-18.60%	6.13%	26.57%	82.60%
2004	-27.43%	182.01%	-2934.79%	-93.51%	-26.07%	3.68%	22.74%	38.86%	78.93%
2005	-17.07%	169.89%	-3398.23%	-76.75%	-21.19%	6.87%	25.72%	37.41%	85.56%
2006	-15.75%	95.12%	-1282.31%	-80.03%	-26.90%	4.08%	24.71%	38.16%	82.28%
2007	-18.17%	89.05%	-1229.14%	-81.31%	-31.92%	-0.08%	21.35%	37.21%	75.17%
2008	-43.15%	244.54%	-3981.69%	-118.39%	-45.35%	-7.03%	15.72%	32.97%	81.15%
2009	-165.80%	552.01%	-10481.68%	-313.35%	-152.99%	-78.82%	-30.91%	-1.14%	67.57%
2010	-63.30%	340.74%	-6752.93%	-134.08%	-66.23%	-23.87%	3.22%	27.75%	78.96%
2011	-38.60%	229.08%	-4115.86%	-94.52%	-42.43%	-8.28%	14.55%	33.02%	81.69%
2012	-46.37%	295.07%	-5113.05%	-102.01%	-43.86%	-9.05%	13.59%	32.00%	84.66%

Notes: The bubble deviations are defined as the bubble size (difference between actual prices and their fundamental values, based on dividend payments as in Section 2.1) expressed as a proportion of the actual share price at that time. The bubble deviations are constructed using five-year windows to evaluate the long term price-to-dividend ratios, rolled forward on a monthly basis. The figures for each year are averages of all months in that year (the 2012 figure only averages over January – March).

Table 2: Returns and Other Characteristics for Portfolios formed on Bubble Size

	Low bubble								High bubble			p-value
	1	2	3	4	5	6	7	8	9	10	High minus low	(H minus L)
Equally weighted return	-2.87%	-1.16%	-0.05%	0.48%	1.04%	1.68%	2.26%	2.87%	3.52%	4.78%	7.66%	0.0000
Value weighted return	-2.35%	-1.23%	-0.58%	-0.07%	0.50%	0.92%	1.32%	1.97%	2.65%	3.61%	5.96%	0.0000
Size (MV % rank)	39.59%	43.86%	46.62%	48.89%	50.56%	52.43%	55.22%	56.79%	56.15%	49.84%	10.25%	0.0000
Book-to-market value	1.1125	0.9087	0.8968	0.8152	0.8319	0.8363	0.7868	0.7436	0.7643	0.7898	0.3228	0.0000
Beta	0.9164	0.8428	0.8148	0.7930	0.7923	0.7786	0.7792	0.7851	0.7981	0.8134	0.1030	0.0000

Notes: The table presents percentage annualised returns and other characteristics calculated on the basis of ranking stocks by the sizes of their bubble deviations and then allocating them to decile portfolios. The portfolios are updated annually.

Table 3: Time-series of Cross-section Results for CAPM and Fama-French Specifications including a Bubble Term

			Panel A: Whole Sample May 1980 – March 2012 (383 points)					
Intercept	Beta	Size	BVP	Mom	DY	Bubble	R^2	# obs
1.118 (0.162)***	-0.001 (0.273)	-	-	-	-	-	0.04	1143
1.467 (0.235)***	-	-	-	-	-	1.312 (0.241)***	0.02	907
1.458 (0.142)***	0.014 (0.275)	-	-	-	-	1.467 (0.241)***	0.06	823
0.801 (0.253)***	0.032 (0.276)	0.00003 (0.00003)	-0.354 (0.032)***	0.559 (0.165)***	-	-	0.06	779
2.461 (0.284)***	-0.030 (0.281)	0.00002 (0.0003)	-0.878 (0.061)***	-0.390 (0.211)*	-	1.868 (0.226)***	0.10	576
4.890 (0.274)***	-0.113 (0.279)	0.00002 (0.00003)	-0.635 (0.054)***	-1.143 (0.202)***	-0.415 (0.022)***	1.659 (0.215)***	0.11	576
			Panel B: Sub-sample January 1990 – March 2012 (267 points)					
Intercept	Beta	Size	BVP	Mom	DY	Bubble	R^2	# obs
0.637 (0.196)***	0.005 (0.325)	-	-	-	-	-	0.04	1143
1.144 (0.275)***	-	-	-	-	-	0.664 (0.057)***	0.02	810
1.118 (0.169)***	0.038 (0.332)	-	-	-	-	0.631 (0.056)***	0.06	751
0.616 (0.322)*	0.050 (0.325)	0.00003 (0.00002)	-0.451 (0.040)***	0.378 (0.214)*	-		0.06	943
2.654 (0.373)***	0.052 (0.340)	0.00001 (0.00002)	-1.225 (0.076)***	-0.697 (0.283)**	-	0.985 (0.074)***	0.10	661
4.982 (0.347)***	0.017 (0.338)	0.00001 (0.00002)	-0.891 (0.071)***	-1.519 (0.271)***	-0.413 (0.024)***	0.797 (0.068)***	0.11	661
			Panel C: Sub-sample January 2000 – March 2012 (147 points)					
Intercept	Beta	Size	BVP	Mom	DY	Bubble	R^2	# obs
0.577 (0.282)**	-0.221 (0.431)	-	-	-	-	-	0.04	1145
1.108 (0.389)***	-	-	-	-	-	0.750 (0.088)***	0.03	617
1.260 (0.219)***	-0.184 (0.450)	-	-	-	-	0.711 (0.087)***	0.08	589
1.304 (0.468)***	-0.217 (0.438)	0.000004 (0.00001)	-0.519 (0.061)***	-0.180 (0.332)	-	-	0.06	943
4.091 (0.549)***	-0.278 (0.478)	-0.00001 (0.00001)*	-1.499 (0.108)***	-1.613 (0.453)***	-	1.195 (0.114)***	0.11	524
5.416 (0.533)***	-0.399 (0.475)	-0.00001 (0.00001)*	-1.236 (0.107)***	-1.970 (0.443)***	-0.303 (0.035)***	1.034 (0.107)***	0.13	524

Notes: The R^2 and # obs (number of observations) are the averages over all of the cross-sectional regressions for a given specification.

Table 4: Factor Loadings from Time-series Regression for Model including Bubble Term

	Parameter estimates					t-ratios				
					BTM					
Size	Low	2	3	4	High	Low	2	3	4	High
Alpha										
Small	0.006	0.006	0.008	0.009	0.009	2.667	3.743	5.932	5.215	5.858
2	0.004	0.005	0.006	0.007	0.007	2.241	4.114	3.792	4.423	3.822
3	0.005	0.004	0.006	0.006	0.009	2.898	3.669	3.733	4.293	5.805
4	0.006	0.006	0.007	0.008	0.009	3.967	4.340	5.845	5.307	4.828
Big	0.007	0.005	0.005	0.007	0.007	5.525	3.110	3.916	4.589	5.368
Beta bubble										
Small	0.010	-0.0001	0.008	-0.103	-0.073	0.157	-0.002	0.263	-2.964	-1.718
2	0.014	-0.031	-0.047	-0.010	-0.044	0.286	-0.494	-1.135	-0.148	-0.718
3	0.060	0.026	0.006	-0.018	-0.066	1.079	0.870	0.164	-0.454	-1.244
4	0.027	0.022	-0.033	-0.019	-0.039	0.623	0.554	-0.796	-0.490	-0.986
Big	-0.040	0.007	0.008	-0.055	0.067	-0.984	0.209	0.221	-1.781	1.511
Beta SMB										
Small	1.072	1.000	0.839	0.769	0.746	17.921	17.760	12.836	7.031	9.207
2	0.916	0.741	0.650	0.723	0.714	9.467	10.235	5.818	7.477	6.679
3	0.883	0.546	0.554	0.465	0.740	16.578	0.074	5.331	4.371	10.331
4	0.537	0.389	0.301	0.405	0.360	7.712	6.953	4.925	6.022	3.798
Big	-0.216	-0.146	-0.013	-0.192	-0.364	-2.492	-2.715	-0.325	-4.115	-5.374
Beta HML										
Small	-0.360	-0.258	-0.039	0.107	0.262	-7.143	-2.920	-0.424	0.750	2.175
2	-0.584	-0.114	0.036	0.110	0.391	-5.957	0.085	0.297	0.938	3.750
3	-0.631	-0.151	0.058	0.266	0.470	-4.742	-1.688	0.469	2.080	5.735
4	-0.479	-0.106	0.132	0.210	0.358	-4.159	-1.841	2.029	2.881	2.965
Big	-0.408	-0.124	0.048	0.250	0.531	-3.663	-2.491	1.082	3.887	5.865
Beta UMD										
Small	0.086	-0.040	-0.008	-0.022	-0.053	1.867	-0.789	-0.139	-0.360	-0.904
2	0.064	-0.064	-0.081	-0.056	-0.089	0.895	-0.953	-1.130	-0.774	-2.048
3	-0.020	-0.088	-0.024	-0.137	-0.124	-0.201	-1.558	-0.319	-1.790	-2.450
4	0.057	-0.047	-0.049	-0.131	-0.192	1.171	-0.951	-0.773	-3.712	-2.651
Big	-0.012	-0.008	0.061	0.077	-0.022	-0.181	-0.156	0.969	2.674	-0.202
Beta RMRF										
Small	1.004	0.830	0.797	0.829	0.814	29.670	21.480	20.212	23.777	26.251
2	1.036	0.936	0.878	0.973	0.972	21.626	27.603	23.674	24.413	32.469
3	1.111	1.062	1.015	1.017	1.046	18.413	35.547	18.674	28.426	20.842
4	1.093	1.055	1.032	1.134	1.096	27.592	28.635	33.402	35.332	27.985
Big	0.886	0.954	1.053	1.023	0.898	19.633	28.676	25.812	24.687	21.165
Adjusted R²										
Small	0.797	0.751	0.816	0.818	0.846					
2	0.760	0.732	0.707	0.736	0.792					
3	0.787	0.785	0.758	0.766	0.785					
4	0.775	0.773	0.771	0.767	0.766					
Big	0.792	0.767	0.795	0.782	0.710					

Notes: Estimation over whole sample using OLS with heteroscedasticity and autocorrelation-consistent standard errors. The model is

$$R_{i,t} = \alpha + \beta_M RMRF_{i,t} + \beta_S SMB_{i,t} + \beta_V HML_{i,t} + \beta_U UMD_{i,t} + \beta_B BBMSB_{it} + v_{i,t}$$, where R_{it} are the

returns on the portfolios *i* in each month *t*. *RMRF*, *SMB*, *HML* and *UMD* are the factor mimicking portfolio returns for market excess returns, firm size, value and momentum respectively.

Table 5: Risk Premia for the CAPM and Fama-French Specifications including a Bubble Term to explain 3-Way and 2-Way Sorted Portfolios of Stocks

	λ_0	λ_M	λ_S	λ_V	λ_U	λ_B	R^2
Panel A: Whole Sample October 1980 – December 2010 (363 points)							
3-way sort	0.009	0.001	0.0010	0.003	0.002	-0.004	0.40
	(0.003)***	(0.003)	(0.0022)	(0.002)	(0.003)	(0.005)	
2-way sort	0.006	0.004	0.0004	0.005	0.006	-0.004	0.41
	(0.003)*	(0.004)	(0.0021)	(0.002)***	(0.004)*	(0.005)	
Panel B: Sub-sample January 1990 – December 2010 (252 points)							
3-way sort	0.006	0.002	0.0012	0.002	0.003	-0.003	0.40
	(0.003)*	(0.004)	(0.0025)	(0.003)	(0.004)	(0.006)	
2-way sort	0.002	0.007	0.0004	0.004	0.008	0.797	0.41
	(0.004)	(0.004)	(0.0024)	(0.003)	(0.004)**	(0.068)	
Panel C: Sub-sample January 2000 – December 2010 (147 points)							
3-way sort	0.009	-0.005	0.0034	0.006	-0.004	-0.003	0.38
	(0.003)***	(0.004)	(0.0042)	(0.005)	(0.006)	(0.009)	
2-way sort	0.0004	-0.399	-0.0013	0.010	0.007	-0.006	0.38
	(0.004)***	(0.475)	(0.0039)	(0.004)**	(0.006)	(0.009)	

Notes: *, ** and *** denote significance at the 10%, 5%, and 1% levels respectively. The stocks are value-weighted within the portfolios. The reported R^2 is the average over the cross-sectional regressions. We initially estimate the model using the first five years of data, from October 1980 to September 1985, and in the second step, we use the factor loadings in a cross-sectional regression for October 1985: $R_{i,t} = \alpha + \lambda_M \beta_M + \lambda_S \beta_S + \lambda_V \beta_V + \lambda_U \beta_U + \lambda_B \beta_B + e_{i,t}$. We then roll the sample forward by one month and estimate a new set of betas for November 1980 to October 1985 for use in a cross-sectional regression using these betas and the sorted portfolios for November 1985. This process continues until the sorted portfolio data, which end in December 2010, are exhausted, which provides 303 data points.

Table 6: Time-series of Cross-section Results for the Bubble Beta included in the CAPM and Fama-French Regressions

	Panel A: Whole Sample May 1980 – March 2012 (383 points)					
Intercept	Bubble Beta	Fundamental Beta	Size	BVP	Mom	R^2
0.957 (0.250)***	0.202 (0.083)***	-	-	-	-	0.01
0.689 (0.199)***	-	0.188 (0.140)	-	-	-	0.02
0.771 (0.196)	0.083 (0.037)**	0.124 (0.168)	-	-	-	0.03
0.862 (0.280)***	0.005 (0.035)	0.155 (0.189)	0.00001 (0.00001)	-0.763 (0.065)***	0.609 (0.175)***	0.07
	Panel B: Sub-sample January 1990 – March 2012 (267 points)					
Intercept	Bubble Beta	Fundamental Beta	Size	BVP	Mom	R^2
0.450 (0.296)	0.239 (0.109)**	-	-	-	-	0.01
0.128 (0.215)	-	0.240 (0.195)	-	-	-	0.03
0.266 (0.213)	0.134 (0.051)***	0.160 (0.233)	-	-	-	0.04
0.728 (0.327)**	0.002 (0.048)	0.235 (0.263)	0.00003 (0.00002)	-1.067 (0.085)***	0.437 (0.213)**	0.06
	Panel C: Sub-sample January 2000 – March 2012 (147 points)					
Intercept	Bubble Beta	Fundamental Beta	Size	BVP	Mom	R^2
-0.076 (0.416)	0.326 (0.176)**	-	-	-	-	0.01
-0.161 (0.324)	-	0.148 (0.226)	-	-	-	0.03
0.125 (0.310)	0.145 (0.072)**	0.026 (0.260)	-	-	-	0.04
1.272 (0.466)***	0.020 (0.067)	-0.008 (0.323)	0.000001 (0.00001)	-1.048 (0.108)***	-0.051 (0.305)	0.06

Notes: The R^2 figures presented are the averages over all of the cross-sectional regressions for a given specification.