Investment performance of shorted leveraged ETF pairs

Xinxin Jiang[a] and Stanley Peterburgsky[b,*]

[a]*Suffolk University, 73 Tremont Street, Boston, MA 02108*

[b]*Kean University, 1000 Morris Avenue, Union, NJ 07083*

*Corresponding author. E-mail: phinance@hotmail.com

Abstract

We analyze trading strategies involving triple-leveraged and inverse triple-leveraged ETF pairs by simulating daily returns over a 48 year period. Our results show that many such strategies significantly outperform the S&P 500 on a risk-adjusted basis. The Sharpe ratio appears to be maximized when shorting the bear triple-leveraged ETF and the bull triple-leveraged ETF in a 2:1 proportion, and simultaneously holding Treasuries long. In this case we find that the average annual Sharpe ratio is more than four times higher than for the S&P 500, and that the strategy outperforms the market in 43 of the 48 years.

JEL classification: G11, G17

Keywords: exchange traded funds, ETFs, pair trading, portfolio management

This version: 1/13/13

I. Introduction

Leveraged and inverse leveraged exchange traded funds (ETFs) have seen tremendous growth in popularity among investors since their introduction to financial markets in 2006. These instruments attempt to deliver specific multiples of the return on an underlying index, such as the S&P 500, on (usually) a daily basis. For example, the triple-leveraged ETF ProShares UltraPro S&P 500 (UPRO) seeks daily investment results corresponding to triple the daily performance of the S&P 500, while the inverse triple-leveraged ETF ProShares UltraPro Short S&P 500 (SPXU) seeks daily investment results corresponding to triple the inverse (i.e., negative) of the daily performance of the S&P 500. To accomplish these investment objectives, leveraged and inverse leveraged ETFs enter into futures and/or swap contracts tied to the underlying index.[1] In order to keep the target leverage constant, the funds adjust their exposure to the index by rebalancing their futures and/or swap holdings on a daily basis.

It is well established that, although leveraged and inverse leveraged exchange traded funds (henceforth, collectively, LEFTs) come close to achieving their target performance on a daily basis, long-run LETF returns generally do not equal the target leverage times the index returns (see, for example, Avellaneda and Zhang (2009), Cheng and Madhavan (2009), Lauricella (2009), Jarrow (2010), Charupat and Miu (2011), and Tang and Xu (2012)) because of the so-called constant leverage trap. In particular, both leveraged ETFs and inverse leveraged ETFs linked to *the same underlying index* often underperform the index, and by a large margin, over periods measured in months or years. Consider, for example, the performance of UPRO and SPXU during the one-week period 4/1/10 - 4/7/10 and the eighteen-month period 4/1/10 - 10/1/11. Figure 1 shows that in the short run UPRO achieved a return approximately equal to 3×

[1] The details of the process involved in constructing leveraged and inverse leveraged positions are described in
http://www.proshares.com/media/documents/components_of_leveraged_and_inverse_funds.pdf

that of the S&P 500, while SPXU attained a return close to -3× that of the S&P 500. However, in the long run both funds underperformed the index by more than 15%. Cheng and Madhavan (2009) illustrate similar underperformance for ProShares UltraShort Oil & Gas (DUG) and ProShares Ultra Oil & Gas (DIG), which are tied to the Dow Jones U.S. Oil & Gas Index with target leverage of -2× and 2×, respectively.

Our goal is to shed light on whether trading strategies involving LETF pairs have the potential to outperform the S&P 500. In particular, if the poor performance of UPRO and SPXU over long periods of time discussed above is typical, it may be possible to short both funds and generate profits higher than those offered by the index. It should be noted that the shorting strategy is not without risk. At times, one of the paired LETFs may significantly outperform the index (when held long) over an extended time period, resulting in poor pairs trade performance. For example, Figure 2 shows that during 11/21/2011 - 4/2/2012, a 50%-50% allocation to UPRO and SPXU would have produced a positive return, and therefore a shorting strategy with the same weights would have produced a negative return. On the other hand, the S&P enjoyed a return of about 18% – much better than the shorting strategy. Ultimately, whether a shorting strategy can be relied upon to consistently outperform the S&P is an empirical question. Since LETFs are relatively new products with short track records, we attempt to answer this question by simulating LETF returns.

Our research would be of only theoretical significance if it were difficult or impossible to borrow shares of LETFs, since shorting requires a simultaneous borrowing and selling of the security in question. As it turns out, many, if not most, LETFs can be shorted in practice. In fact, one of the authors of this paper has been shorting LETFs in his modestly sized online brokerage account for some time. Occasionally, the brokerage firm requests that the short position be partially or fully closed out, but such requests are infrequent. Anecdotal evidence suggests that the size of the account does play a role in determining the likelihood of it being targeted by the brokerage when shares need to be returned to a lender.

We focus specifically on UPRO and SPXU because they are very liquid funds, with average holding periods of less than 2 days and less than 3 days, respectively, and average bid-ask spreads of only 0.04% and 0.03%, respectively, and because the underlying index is the broad U.S. stock market. Other triple-leveraged and inverse triple-leveraged ETFs are somewhat less liquid and/or track either a subset of the U.S. stock market or other financial or real asset markets. In addition, UPRO and SPXU shares can be, and have been, sold short.

The rest of this article is organized as follows. In Section II we review prior literature. In Section III, we describe our simulation design. In Section IV, we present our main findings on the long-term performance of various shorted leveraged ETF pairs strategies. In Section V, we discuss the affect of dividends, tracking errors, transaction costs, fund fees and expenses, and taxes on the performance of these strategies. Finally, we summarize our research and offer concluding remarks in Section VI.

II. Literature review

Leveraged ETFs are a recent financial innovation, and the research in this area is still in its infancy. To our knowledge, this paper is the first to examine shorting strategies using LETFs. Other studies of LETFs have focused on modeling the return dynamics, examining the sources and characteristics of tracking errors, investigating the effect of LETF trading on underlying security trading, and discussing the suitability of LETFs for retail investors.

Wang (2009), Avellaneda and Zhang (2009), and Cheng and Madhavan (2009) model the stochastic process of LETF returns where the underlying index return process is assumed to evolve as a geometric Brownian motion. They show that if the prices of an LETF and the index are given by A_t and S_t, respectively, then, assuming $\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$, the price of the index at time T is $S_T = S_0 e^{(\mu - \sigma^2/2)T + \sigma\sqrt{T}z}$, while the price of the LETF at time T is $A_T = A_0 e^{(\lambda\mu - \lambda^2\sigma^2/2)T + \lambda\sigma\sqrt{T}z}$, where z is a standard normal random variable and λ is the target leverage. It can be shown that the relationship between the index return and the LETF return is $\frac{A_T}{A_0} = \left(\frac{S_T}{S_0}\right)^{\lambda} e^{(\lambda - \lambda^2)\sigma^2 T/2}$. The upshot is that since $(\lambda - \lambda^2)$ is negative for any leveraged or inverse leverage ETF, the scalar term $e^{(\lambda - \lambda^2)\sigma^2 T/2}$ is less than 1, and therefore the LETF return is less than the compounded index return. Hence, assuming index returns follow a geometric Browning motion, the LETF will lose money unless index performance is sufficiently strong.

Charupat and Miu (2011) examine a group of Canadian LETFs, and report that they have extremely small daily tracking errors. However, they define tracking errors in terms of net asset values (NAV) rather than LETF prices. Whether daily LETF returns are close to their targets remains unanswered. Tang and Xu (2012) find that daily tracking errors for a set of LETFs linked to the S&P 500, Dow Jones, and NASDAQ-100, are substantial. LETF managers consistently underleverage, and the effect on fund performance accumulates over time. The main cause of this underexposure appears to be the cost of adjusting exposure on a daily basis. Shum and Kang (2012) report that daily tracking errors for a set of LETFs linked to the Toronto Stock Exchange, gold, oil & gas, MSCI EAFE, and the S&P 500 are sizeable as well. Once again, underexposure to the underlying index appears to be the main source of deviation from target returns.

Cheng and Madhavan (2009) investigate the effect of LETFs on market volatility. They show that the required daily re-leveraging around the market close imparts additional systemic risk to the market. This phenomenon is expected to become more pronounced as aggregate LETF assets under management continue to increase. The debate as to whether LETFs are partly responsible for the recent increase in market volatility has been picked up by the media (Sorkin (2011), Kephart (2012)). Haryanto, et al. (2012) find that although LETF rebalancing demands do lead to higher market volatility, this effect is economically significant only when the market experiences a large positive or negative return by 3:30 PM. Bai, et al. (2012) examine whether two real estate-linked and four financials-linked LETFs influence the late-day volatility of 63 real estate sector stocks, and report greatest impacts for smaller, less actively traded, and more volatile stocks.

The growing sentiment that LETF return dynamics over periods longer than one day are not well understood by the average retail investor has led many financial advisors to dissuade their clients from investing in these funds. Additionally, in 2012 FINRA fined Wells Fargo, Citigroup, Morgan Stanley, and UBS a total of $9.1 million "for selling leveraged and inverse exchange-traded funds (ETFs) without reasonable supervision and for not having a reasonable basis for recommending the securities". Dulaney, et al. (2012) find that average holding periods for five LETFs they study range from 5.3 days for Direxion Developed Markets Bear 3X to 22.7 day for ProShares Ultra Russell 1000 Value Fund. Furthermore, a substantial number of investors hold these LETFs for periods longer than one quarter, suggesting they don't fully understand the long-term drag on returns caused by daily re-leveraging. The authors conclude that LETFs are unsuitable for the average retail investor.

III. Simulation design

UPRO and SPXU, the ETFs mentioned above, began trading on the NYSE on June 23, 2009. As of this writing, there is insufficient empirical data to conclusively determine whether a strategy involving positions in one or both of the funds that consistently outperforms the S&P 500 exists. Our solution is to simulate hypothetical fund returns during 1963-2010 and compare the simulated Sharpe ratios achieved by various trading strategies to the actual Sharpe ratios attained by the S&P 500 during the contemporaneous period. To ensure that our results and conclusions are accurate, we preserve a number of statistical properties related to the price dynamics of UPRO and SPXU in our simulation design. Although we work with price returns (that is, we ignore dividends), as we show in Section V our results are equally applicable to total returns. All price data we use in this study are from the Center for Research in Security Prices (CRSP).

To uncover the relevant statistical properties contained in the empirical data, we first run a pair of ordinary least squares (OLS) regressions of LETF returns on S&P 500 returns:

$$\begin{aligned} r_{UPRO,t} &= \alpha_{UPRO} + \beta_{UPRO} r_{M,t} + \varepsilon_{UPRO,t} \\ r_{SPXU,t} &= \alpha_{SPXU} + \beta_{SPXU} r_{M,t} + \varepsilon_{SPXU,t} \end{aligned} \tag{1}$$

where $r_{UPRO,t}$ is a set of UPRO returns since fund inception through 12/31/2010, $r_{SPXU,t}$ is a set of SPXU returns during the same period, and $r_{M,t}$ is a set of contemporaneous S&P 500 returns. The estimated parameters are:

$$\begin{aligned} \hat{\alpha}_{UPRO} &= .000208 \\ \hat{\beta}_{UPRO} &= 2.947 \\ \hat{\alpha}_{SPXU} &= -.000335 \\ \hat{\beta}_{SPXU} &= -2.952 \end{aligned} \tag{2}$$

Next, we calculate the first-order autocorrelation of UPRO and SPXU empirical tracking errors[2] (residuals) from the above regressions, as well as the cross-correlation and the standard deviations of the tracking errors:

$$\begin{aligned}
\varphi_{\varepsilon_{UPRO}} &= -0.4622 \\
\varphi_{\varepsilon_{SPXU}} &= -0.4890 \\
\rho_{\varepsilon_{UPRO},\varepsilon_{SPXU}} &= -0.7440 \\
\sigma_{\varepsilon_{UPRO}} &= 0.00227 \\
\sigma_{\varepsilon_{SPXU}} &= 0.00207
\end{aligned} \tag{3}$$

where $\varphi_{\varepsilon_{UPRO}}$ is the first-order autocorrelation of UPRO tracking errors, $\varphi_{\varepsilon_{SPXU}}$ is the first-order autocorrelation of SPXU tracking errors, $\rho_{\varepsilon_{UPRO},\varepsilon_{SPXU}}$ is the cross-correlation of UPRO and SPXU tracking errors, $\sigma_{\varepsilon_{UPRO}}$ is the standard deviation of UPRO tracking errors, and $\sigma_{\varepsilon_{SPXU}}$ is the standard deviation of SPXU tracking errors.

Regression results indicate that UPRO's empirical leverage is $\hat{\beta}_{UPRO} = 2.947$, which is statistically different from the target leverage of 3 at the 1% level. Likewise, SPXU's empirical leverage is $\hat{\beta}_{SPXU} = -2.952$, which is statistically different from the target leverage of -3 at the 1% level. We calibrate our simulation design to the empirical leverage rather than the target leverage.[3] That is, we imagine that hypothetical ETF returns are given by the equation,

$$\begin{aligned}
r_{UPRO,t} &= .000208 + 2.947 r_{M,t} + \varepsilon_{UPRO,t} \\
r_{SPXU,t} &= -.000335 - 2.952 r_{M,t} + \varepsilon_{SPXU,t}
\end{aligned} \tag{4}$$

---

[2] Empirical tracking errors are deviations from expected fund returns, given the empirical leverage. For UPRO and SPXU, the expected returns are $\hat{\alpha}_{UPRO} + \hat{\beta}_{UPRO} r_{M,t} = .000208 + 2.947 r_{M,t}$ and $\hat{\alpha}_{SPXU} + \hat{\beta}_{SPXU} r_{M,t} = -.000335 - 2.952 r_{M,t}$, respectively.

[3] We believe that results based on empirical leverage rather than on target leverage are most realistic, since other LETFs appear to miss their targets as well. For example, Lu, et al (2009) report that ProShares Ultra Dow 30 (DDM), ProShares UltraShort Dow 30 (DXD), ProShares Ultra S&P 500 (SSO), ProShares UltraShort S&P 500 (SDS), ProShares Ultra QQQ (QLD), and ProShares UltraShort QQQ (QID) are underleveraged relative to their targets by a statistically significant margin, while Shum and Kang (2012) document similar results for LETFs that are linked to the Toronto Stock Exchange, gold, oil & gas, MSCI EAFE, and S&P 500.

where the parameters are from (2). We simulate the tracking errors, $\varepsilon_{UPRO,t}$ and $\varepsilon_{SPXU,t}$, via the procedure described in the Appendix.[4]

Once we obtain 48 years of daily UPRO and SPXU returns via (4), where $r_{M,t}$'s are actual daily S&P 500 returns, we simulate a number of trading strategies. The strategies differ in both the default allocation of funds between the two ETFs and the rebalance thresholds. We examine the following combinations : $\left(w_{UPRO}=100\%, w_{SPXU}=0\%\right)$, $\left(w_{UPRO}=75\%, w_{SPXU}=25\%\right)$, $\left(w_{UPRO}=66.7\%, w_{SPXU}=33.3\%\right)$, $\left(w_{UPRO}=50\%, w_{SPXU}=50\%\right)$, $\left(w_{UPRO}=33.3\%, w_{SPXU}=66.7\%\right)$, $\left(w_{UPRO}=25\%, w_{SPXU}=75\%\right)$, and $\left(w_{UPRO}=0\%, w_{SPXU}=100\%\right)$, there the w's indicate the relative default weights of the two ETFs in the portfolio. The rebalance thresholds vary from no rebalancing to rebalancing only when the relative weights diverge from the default ratio by a substantial margin.

In addition to shorting UPRO and/or SPXU, every strategy involves a long position in the 3-month Treasury bill. The default total short position in the combination of UPRO and/or SPXU is equal to the long position in the Treasury bill. At the end of each trading day, the long balance is compared to the short balance. If the long balance is greater than 90% but less than 110% of the short balance, no inflow or outflow into the short subaccount takes place (although rebalancing between UPRO and SPXU may still be necessary). If the long balance is less than 90% of the short balance, a "margin call" requires the investor to decrease her short balance by "buying to cover" UPRO and SPXU in equal proportions up until the short balance and the long

[4] We cannot directly sample from a bivariate normal distribution because random sampling results in zero first-order autocorrelation. Our procedure, on the other hand, preserves the empirical first-order autocorrelations of UPRO and SPXU tracking errors.

balance equalize.[5] If the long balance is greater than 110% of the short balance, the strategy requires the investor to increase her short balance by shorting additional shares of UPRO and SPXU in equal proportions up until the short balance and the long balance equalize. In short, every trading strategy we examine contains a short UPRO/SPXU position and a long T-bill position. Once the return sequence for a particular strategy is obtained, standard portfolio analysis techniques can be used to assess the profitability of the strategy relative to the market.

IV. Simulation results

The main results of our simulations, including average annual returns, standard deviations, and Sharpe ratios are presented in Table 1. The first column shows the default ratio of UPRO to SPXU in the short subaccount. For example, 25:75 corresponds to trading strategies in which the UPRO balance is three times lower than the SPXU balance at the start of the simulation and after each rebalancing. The second column shows the rebalance thresholds. In the case of the 25:75 strategy, +5%/-5% rebalancing means that UPRO and SPXU holdings are rebalanced to the default ratio whenever the ratio breaches 20:80 or 30:70. Likewise, in the case of the 33.3:66.7 strategy, +20%/-20% rebalancing means that UPRO and SPXU holdings are rebalanced whenever the ratio breaches 13.3:86.7 or 53.3:46.7. The third, fourth, fifth, and sixth columns present each strategy's (arithmetic) average annual return, geometric average annual return, average annual standard deviation, and average annual Sharpe ratio, respectively. For annual returns, standard deviations, and Sharpe ratios, *t*-statistics from tests of each strategy's simulated results versus the S&P 500 appear below. The final three columns detail the average annual number of rebalances between UPRO and SPXU, inflows into the short subaccount, and

[5] In effect, the "maintenance margin" is assumed to be 90%, which is equal to FINRA's currently mandated maintenance margin for shorting triple-leveraged and inverse triple-leveraged ETFs. For additional information, see FINRA's Regulatory Notice 09-53, which is available at www.finra.org.

outflows from the short subaccount, respectively. For comparison purposes, annual S&P 500 data are presented at the bottom of the table.

Several important findings emerge from our simulations. First, as Table 1 shows, the best performing strategy in terms of the Sharpe ratio is the 33.3:66.7 strategy with +20%/-20% rebalancing. The investment yields an average annual Sharpe ratio of 1.3004 – more than four times that of the S&P 500. Interestingly, the 33.3:66.7 portfolio with +5%/-5% rebalancing produces a higher average return and a lower average standard deviation, yet a lower average Sharpe ratio.[6] Second, as the ratio of UPRO to SPXU increases, the annual return decreases, as does the geometric annual return. Third, as the ratio of UPRO to SPXU increases, the standard deviation decreases until we reach the 50:50 portfolio, after which it begins to increase. Fourth, rebalancing is key for all portfolios other than the 0:100 portfolio. Without rebalancing, the SPXU balance drops to near $0 over time, while the UPRO balance grows without limit. As a result, given a long enough investment horizon, the strategy loses money. In the case of 0:100, there is no investment in UPRO, and therefore the lack rebalancing is not a concern. Fifth, inflows and outflows are most frequent for strategies that don't entail rebalancing. Sixth, and not surprising, tighter rebalancing thresholds lead to more frequent rebalancing. In practice, trading triggered by inflows, outflows, and rebalancing has the potential to wipe out any profits because of transaction costs imposed on the investor. We discuss transaction costs in more detail in Section V.

Since the 33.3:66.7 strategy with +20%/-20% rebalancing performs better than any of the others as measured by the Sharpe ratio, we focus primarily on this strategy (which we refer to as

[6] Strategy A can have a lower average Sharpe ratio than strategy B despite a higher average return and a lower average standard deviation because the average Sharpe ratio is *not* the average difference between returns on the portfolio and the risk-free rate divided by the average portfolio standard deviation, but rather the average of the individual Sharpe ratios.

the base case) throughout the remainder of our paper. In Table 2 we present evidence that the base case portfolio doesn't merely outperform the S&P 500 over very long investment horizons, but also consistently outperforms the market during 1-year holding periods. The table contains a year-by-year breakdown of S&P 500 annual returns, standard deviations, and Sharpe ratios, as well as annual returns, standard deviations, and Sharpe ratios for the base case portfolio. It shows that the base case portfolio has outperformed the market in 43 of the 48 years between 1963 and 2010.

To assess potential losses from the base case strategy, we examine the top ten drawdown events over 1-day, 1-week, 1-month, and unbounded non-overlapping periods. The results are tabulated in Table 3. Panel A shows that the worst day for the portfolio during the 1963-2010 period would have been October 19, 1987. This coincides with the worst day for the S&P 500 during that period, which came to be known as Black Monday. That day the U.S. stock market lost 20.47% of its value, while the base case portfolio would have lost 32.24%. The second worst day for the portfolio would have been September 29, 2008, when it would have lost a relatively modest 14.43% of its value as the market dropped by 8.79%. Panel B shows the top weekly drawdowns, where a week is defined as 5 consecutive trading days. The worst week in recent history was the 5-day stretch that ended on Black Monday, during which the portfolio lost 38.52%, while the market sank 27.33%. Panel C tabulates analogous figures for monthly drawdowns. Once again, the worst period included Black Monday, with the portfolio losing 43.02% of its value as the market shed 28.89%. Finally, Panel D presents the top maximum drawdown events during the period we study. We find that between 5/19/2008 and 3/9/2009 the base case portfolio would have lost 58.71% of its value, as the market lost 52.58%. There are other periods of large losses as well. It is important to note, however, that investors can limit their losses by allocating part of their capital to the shorting strategy and part to other assets. In particular, by investing in the shorting strategy and the Treasury bill (in addition to the T-bill position already required by the shorting strategy) in carefully chosen proportions, an investor can realize the same Sharpe ratios as in Tables 1 and 2, while keeping drawdowns (and volatility) to the desired level.

V. Discussion

Dividends, tracking errors, transaction costs, fund fees and expenses, and taxes have important implications for the performance of the trading strategies we consider. We now discuss each of these in turn.

A. Dividends

Although we have been focusing on price returns, our results hold for total returns as well. In particular, any shorting strategy that yields better risk-adjusted performance than the S&P 500 in terms of price returns also produces better risk-adjusted performance than the market in terms of total returns. More formally, assuming no tracking errors,

$$\frac{r_p - r_f}{\sigma_p} > \frac{r_m - r_f}{\sigma_m} \text{ implies } \frac{(r_p + D_p) - r_f}{\sigma_p} > \frac{(r_m + D_m) - r_f}{\sigma_m}, \qquad (5)$$

where r_p is the annual return on the portfolio, r_f is the annual risk-free rate, σ_p is the annualized standard deviation of the portfolio, r_m is the annual return on the market, σ_m is the annualized standard deviation of the market, and D_p and D_m are the dividend yields on the portfolio and the market, respectively. Statement (5) follows because all dividends distributed by the component stocks are paid in multiple to the bull LETF by its counterparty, and have to be paid in multiple by the bear LETF to its counterparty, and the fact that,

$$\frac{(r_p + D_p) - r_f}{\sigma_p} = \frac{(r_p - r_f)}{\sigma_p} + \frac{D_p}{\sigma_p} > \frac{(r_m - r_f)}{\sigma_m} + \frac{D_p}{\sigma_p} = \frac{(r_m - r_f)}{\sigma_m} + \frac{\lambda D_m}{\lambda \sigma_m} = \frac{(r_m + D_m) - r_f}{\sigma_m}$$

, where λ is the total exposure of the portfolio to the market, and the inequality is by assumption.

If a trading strategy is subject to tracking errors, as in our case, then (5) may not follow because σ_p may not equal $\lambda\sigma_m$. However, since UPRO's and SPXU's tracking errors are very small compared to the difference in the base case portfolio's and the market's Sharpe ratios, our findings are valid with respect to total returns.

B. Tracking errors

Tracking errors arise from a number of sources. First, since triple-leveraged ETFs and inverse triple-leveraged ETFs pay 2× the risk-free interest rate and receive 4× the risk-free interest rate, respectively,[7] neither category of funds achieves its stated objective, even in theory, unless the interest rate is 0%. When *shorting* a triple-leveraged ETF, the investor in effect *pays* 3× the underlying index return and *receives* 2× the interest rate, while when shorting an inverse triple-leveraged ETF, the investor in effect receives 3× the underlying index return and pays 4× the interest rate. Consequently, interest transfers "help" short UPRO positions and "hurt" short SPXU positions. The effective interest rate exposure for an LETF shorting strategy is equal to $2 \times w_{UPRO} - 4 \times w_{SPXU}$, where w_{UPRO} and w_{SPXU} are the weights of UPRO and SPXU in the portfolio, respectively. For a 33.3:66.7 strategy, the effective interest rate exposure is -2×. Despite the drag on shorting strategy returns (for some portfolios) produced by interest transfers, all of our finding

[7] See http://www.proshares.com/media/documents/components_of_leveraged_and_inverse_funds.pdf for an overview of how LETFs achieve their desired exposure to the underlying index.

ought to hold unless interest rates are high. As of 12/31/2012, the yield on the 30-year Treasury bond was 2.95%, suggesting interest rates are expected to remain low for many years to come. Also, note that we have been implicitly assuming that shorting (including shorting of LETFs) does not generate cash that can be invested at the risk-free rate. If a brokerage pays interest on short balances, it would mitigate the negative interest rate exposure created by a short SPXU position.

Second, underleveraging (as well as overleveraging, which is not common) impacts tracking errors, since the resulting fund returns are different from target returns. However, underexposure decreases interest transfers discussed above, so it may bring actual returns closer in line with target returns.

Third, residuals…, while the negative autocorrelation of residuals…

Fourth, fund fees and expenses may exacerbate tracking errors because they may create a drag on LETF returns. Fees and expenses are discussed in more detail below.

C. Transaction costs

We can approximate transaction costs by using the data on trading frequencies, rebalance thresholds, inflow/outflow thresholds, and bid-ask spreads. In particular, not counting commissions and assuming rebalancing and inflows/outflows don't occur on the same days, average annual trading costs as a percentage of portfolio value can be expressed as,

$$\begin{aligned} TC &= \tfrac{s_{UPRO}}{2}\, freq_R RT + \tfrac{s_{SPXU}}{2}\, freq_R RT \\ &\quad + w_{UPRO}\tfrac{s_{UPRO}}{2}\, freq_{IO} IOT + w_{SPXU}\tfrac{s_{SPXU}}{2}\, freq_{IO} IOT + \tfrac{s_{Tbill}}{2}\, freq_{IO} IOT \end{aligned}, \qquad (6)$$

where s_{UPRO}, s_{SPXU}, and s_{Tbill}, are percentage bid-ask spreads for UPRO, SPXU, and Treasury bills, respectively, $freq_R$ and $freq_{IO}$ are the average annual number of rebalance and inflow/outflow transactions, respectively, RT and IOT are the rebalance thresholds and inflow/outflow thresholds, respectively, and w_{UPRO} and w_{SPXU} are the weights of UPRO and SPXU in the portfolio, respectively. According to IndexUniverse.com, the typical bid-ask spreads for UPRO and SPXU are 0.04% and 0.03%, respectively. Assuming the typical spread for T-bills is 0.08% (Chakravarty and Sarkar (2003)), average annual trading costs as a percentage of portfolio value (for the base case) is approximately,

$$\begin{aligned} TC_{.333,.667,.20} &= (\tfrac{0.04\%}{2})(1.21)(20\%) + (\tfrac{0.03\%}{2})(1.21)(20\%) \\ &\quad + 33.3\%(\tfrac{0.04\%}{2})(5.77 + 2.92)(10\%) + 66.7\%(\tfrac{0.03\%}{2})(5.77 + 2.92)(10\%) + (\tfrac{0.08\%}{2})(5.77 + 2.92)(10\%) \\ &= 0.058\% \end{aligned}$$

This is very low, and should not have a significant impact on the performance of the portfolio.

D. Fund fees and expenses

Unless the present value of expected future LETF fees and expenses is already reflected in market prices, these operating costs create a drag on LETF returns. Consequently, fees and expenses boost the performance of shorting strategies. For the funds we examine in this paper, market values are very close to NAVs, so these costs do not appear to be impounded in prices. All else equal, the higher the fees and expenses, the better the shorting strategy ought to perform. As of December 2012, UPRO's and SPXU's annual expense ratios are 0.95% and 0.93%, respectively, which ranks the two LETFs at the low end in terms of operating costs among all triple-leveraged and inverse triple-leveraged ETFs.[8]

[8] The average annual expense ratio for the universe of triple-leveraged and inverse triple-leveraged ETFs is 1.07%. The range is 0.93%–1.20%.

E. Taxes

Since short balances are marked to market, most tax obligations arising from favorable performance of our investment strategies can be postponed indefinitely. Assuming positive exposure to the market in the short subaccount (that is, a ratio of UPRO to SPXU of less than 1), inflows imply unrealized gains in the short subaccount and unrealized gains in the long subaccount, while outflows imply realized losses in the short subaccount and unrealized losses in the long subaccount. Neither situation requires immediate payment of taxes. Rebalancing usually leads to either unrealized gains for SPXU and realized losses for UPRO (when the market has been going up), or realized losses for SPXU and unrealized gains for UPRO (when the market has been going down), in which cases any tax obligations can also be postponed. Occasionally, rebalancing leads to unrealized gains for one of the LETFs and realized gains for the other (when the market has been going “sideways”), in which case taxes cannot be postponed. In sum, the tax implications of our shorting strategies are predominantly favorable.

VI. Conclusion

Leveraged and inverse leveraged exchange traded funds have become very popular financial products among investors since their introduction in 2006. These instruments allow investors to amplify directional bets on an underlying index. However, over extended periods, LETFs usually produce returns below that of the index times the leverage multiple. In fact, both bull and bear funds linked to an index often underperform the index over the same period. With this empirical observation in mind, we examine whether taking simultaneous short positions in triple-leveraged and inverse triple-leveraged ETFs can yield a higher Sharpe ratio than that of the S&P 500.

Since LETFs are a relatively new financial innovation, we simulate daily returns on a triple leveraged and an inverse triple leveraged fund over a period of 48 years. Our simulation design allows us to preserve a number of statistical properties of returns on a pair of actual funds, and calibrate them to the market returns during the period we investigate. To our knowledge, this is the first paper that attempts to explicitly estimate how these funds would have performed over the years. We find that many of the strategies we examine produce superior portfolio performance as measured by the Sharpe ratio. The optimal strategy appears to be one that shorts the bear LETF and the bull LETF in a 2:1 proportion, yielding an average Sharpe ratio that is more than four times higher than that of the S&P 500. This strategy also appears to be highly consistent in outperforming the market on a risk-adjusted basis.

Appendix

We simulate tracking errors, $\varepsilon_{UPRO,t}$ and $\varepsilon_{SPXU,t}$, via the following procedure: First, we generate two independent sequences of standard normal random variables, $z_{1,1}, z_{1,2}, ..., z_{1,N}$ and $z_{2,1}, z_{2,2}, ..., z_{2,N}$. Next, we define $z_{3,t} = -0.7440 z_{1,t} + \sqrt{(1-(-0.7440)^2)} z_{2,t}$. The parameter –0.7440 is from (3), and introduces cross-correlation between $z_{1,t}$ and $z_{3,t}$. Next, we let $\theta_{UPRO,t} = \frac{(z_{1,t} - \gamma_{UPRO} z_{1,t+1})}{\sqrt{1+\gamma_{UPRO}^2}}$ and $\theta_{SPXU,t} = \frac{(z_{3,t} - \gamma_{SPXU} z_{3,t+1})}{\sqrt{1+\gamma_{SPXU}^2}}$, where $\gamma_{UPRO} = \frac{(-1+\sqrt{1-4(-0.4622)^2}}{2(-0.4622)}$ and $\gamma_{SPXU} = \frac{(-1+\sqrt{1-4(-0.4890)^2}}{2(-0.4890)}$. The parameters -0.4622 and -0.4890 are again from (3), and introduce first-order autocorrelation (while keeping higher-order autocorrelations at zero) in $\theta_{UPRO,t}$ and $\theta_{SPXU,t}$. Finally, we center and scale the variables by letting $\varepsilon_{UPRO,t} = 0.00227 \frac{\theta_{UPRO,t} - \bar{\theta}_{UPRO,t}}{SD(\theta_{UPRO,t})}$ and $\varepsilon_{SPXU,t} = 0.00207 \frac{\theta_{SPXU,t} - \bar{\theta}_{SPXU,t}}{SD(\theta_{SPXU,t})}$, where $SD(\bullet)$ denotes sample standard deviation, bar denotes sample mean, and the parameters 0.00227 and 0.00207 are from (3).

References

Avellaneda, Marco and Stanley Zhang, 2009, Path-dependence of leveraged ETF returns, working paper, New York University.

Bai, Qing, Shaun A. Bond and Brian Hatch, 2012, The Impact of Leveraged and Inverse ETFs on Underlying Stock Returns, working paper, University of Cincinnati.

Chakravarty, Sugato and Asani Sarkar, 2003, Trading costs in three U.S. bond markets, *Journal of Fixed Income* 13, 39-48.

Charupat, Narat and Peter Miu, 2011, The pricing and performance of leveraged exchange-traded funds, *Journal of Banking and Finance* 35, 966-977.

Cheng, Minder and Ananth Madhavan, 2009, The dynamics of leveraged and inverse exchange-traded funds, *Journal of Investment Management* 7, 43-62.

Dulaney, Tim, Tim Husson, and Craig McCann, 2012, Leveraged, inverse, and futures-based ETFs, *PIABA Bar Journal* 19, No. 1.

Haryanto, Edgar, Arthur Rodier, Pauline Shum, and Walid Hejazi, 2012, Intraday share price volatility and leveraged ETF rebalancing, working paper, York University.

Jarrow, Robert A., 2010, Understanding the risk of leveraged ETFs, *Finance Research Letters* 7, 135–139.

Kephart, Jason, Market volatility? Don't blame ETFs, says ICI, *Investment News*, 23 February, 2012, accessed on 1/3/2012 at http://www.investmentnews.com/article/20120223/FREE/120229965.

Lauricella, Tom, ETF math lesson: Leverage can produce unexpected returns, *Wall Street Journal*, 4 January, 2009, accessed on 12/22/2012 at http://online.wsj.com/article/SB123111094917552317.html.

Leung, Tim and Marco Santoli, 2012, Leveraged ETFs: Admissible leverage and risk horizon, working paper, Columbia University.

Lu, Lei, Jun Wang and Ge Zhang, 2012, Long term performance of leveraged ETFs, *Financial Services Review* 21, 63–80.

Shum, Pauline M. and Jisok Kang, 2012, The long and short of leveraged ETFs: The financial crisis and performance attribution, working paper, York University.

Sorkin, Andrew Ross, Volatility, Thy Name is E.T.F., *New York Times*, 10 October, 2011, accessed on 1/3/2012 at http://dealbook.nytimes.com/2011/10/10/volatility-thy-name-is-e-t-f.

Tang, Hongfei and Xiaoqing Eleanor Xu, 2012, Solving the return deviation conundrum of leveraged exchange-traded funds, *Journal of Financial and Quantitative Analysis*, forthcoming.

Wang, Zhenyu, 2009, Market efficiency of leveraged ETFs, working paper, Federal Reserve Bank of New York.

Table 1. Performance of UPRO/SPXU/T-bill trading strategies

This table presents (arithmetic) average annual returns, geometric average annual returns, average annual standard deviations, and average annual Sharpe ratios for various UPRO/SPXU/T-bill trading strategies. The first column shows the default ratio of UPRO to SPXU in the short subaccount. The second column shows the rebalance thresholds for UPRO and SPXU. The third, fourth, fifth, and sixth columns present each strategy's average return, geometric return, standard deviation, and Sharpe ratio, respectively, as well as *t*-statistics from tests of each strategy's simulated results versus the S&P 500. The final three columns detail the average annual number of rebalances between UPRO and SPXU, inflows into the short subaccount, and outflows from the short subaccount, respectively. For comparison purposes, annual S&P 500 data are presented at the bottom of the table. All returns are price returns.
*, **, and *** denote statistical significance at the 10%, 5%, and 1% levels. All tests are two-tailed.

Default UPRO:SPXU ratio	Rebalance thresholds	Average annual return	Geometric average annual return	Average annual std. deviation	Average annual Sharpe ratio	Rebalances per year	Inflows per year	Outflows per year
0:100	none	41.93%	26.69%	43.73%	1.1519	0.00	23.65	16.52
		5.6402***		14.9434***	9.7492***			
25:75	+5%/-5%	23.80%	20.09%	21.86%	1.1341	9.04	9.83	1.38
		9.2202***		13.5047***	13.7577***			
	+10%/-10%	22.26%	19.15%	22.31%	1.1040	2.98	10.10	5.81
		9.4657***		11.1697***	12.6766***			
	+20%/-20%	20.89%	17.08%	22.26%	1.2371	0.94	9.56	5.63
		6.7878***		5.6861***	11.0818***			
	none	-17.72%	-25.66%	41.12%	-0.6312	0.00	17.38	19.52
		-3.2198***		11.9762***	-2.9447***			
33.3:66.7	+5%/-5%	16.78%	15.23%	14.72%	1.0524	12.98	5.77	2.73
		15.316***		0.2467	12.9498***			
	+10%/-10%	17.03%	15.47%	14.84%	1.1484	3.83	5.81	2.69
		10.1217***		0.5930	11.778***			
	+20%/-20%	15.04%	13.50%	14.80%	1.3004	1.21	5.77	2.92
		4.3212***		0.1270	8.2778***			
	none	-18.48%	-26.37%	41.68%	-0.9407	0.00	17.56	19.83
		-3.3087***		12.554***	-3.1293***			
50:50	+5%/-5%	6.77%	6.71%	2.22%	0.3713	14.92	0.77	0.00
		-0.4331		-13.9623***	0.3400			
	+10%/-10%	6.32%	6.20%	4.03%	0.1128	4.77	0.85	0.17
		-0.5948		-14.6261***	-0.6625			
	+20%/-20%	5.27%	4.85%	8.11%	-0.1131	1.35	1.52	0.98
		-0.7376		-15.6719***	-1.2697			
	none	-19.70%	-27.46%	42.52%	-0.7717	0.00	18.06	20.48
		-3.4689***		13.6166***	-3.4515***			
66.7:33.3	+5%/-5%	-3.73%	-5.00%	14.98%	-0.8232	12.60	3.65	4.52
		-2.4468**		3.5155***	-3.3751***			
	+10%/-10%	-4.02%	-5.66%	15.91%	-0.7568	3.85	4.02	4.83
		-2.3556**		6.3045***	-3.1358***			
	+20%/-20%	-4.79%	-8.33%	20.89%	-0.6461	1.08	6.56	7.60
		-1.923*		12.2699***	-2.658**			
	none	-20.38%	-28.13%	43.07%	-0.7884	0.00	18.44	20.88
		-3.5467***		14.3507***	-3.495***			
75:25	+5%/-5%	-8.14%	-10.67%	22.24%	-0.8208	9.54	6.60	7.96
		-2.803***		16.4868***	-3.4141***			
	+10%/-10%	-8.07%	-11.27%	23.44%	-0.7578	2.63	7.38	8.71
		-2.5905***		18.7365***	-3.1822***			
	+20%/-20%	-10.80%	-16.21%	28.54%	-0.7212	0.60	10.15	12.19
		-2.5214**		18.4442***	-2.9911***			
	none	-20.68%	-28.41%	43.26%	-0.7931	0.00	18.50	21.00
		-3.5875***		14.6149***	-3.5173***			
100:0	none	-21.30%	-29.08%	43.73%	-0.7995	0.00	18.73	21.29
		-3.6554***		15.2087***	-3.5476***			
S&P 500		7.84%	6.43%	14.70%	0.2898			

Table 2. Comparison of UPRO/SPXU/T-bill base case portfolio to S&P 500

This table contains a year-by-year breakdown of S&P 500 annual returns, standard deviations, and Sharpe ratios, as well as analogous data for a theoretical UPRO/SPXU/T-bill base case portfolio (where target leverage is achieved and tracking errors are zero), a hybrid UPRO/SPXU/T-bill base case portfolio (where empirical leverage is assumed but tracking errors are zero), and the simulated UPRO/SPXU/T-bill base case portfolio. All returns are price returns.

	S&P 500			Theoretical portfolio (base case)			Hybrid portfolio (base case)			Simulated portfolio (base case)		
Year	Return	SD	Sharpe	Return	SD	Sharpe	Return	SD	Sharpe	Return	SD	Sharpe
1963	18.89%	8.55%	1.83	15.79%	8.24%	1.52	14.98%	7.78%	1.51	19.07%	7.10%	2.23
1964	12.97%	5.26%	1.75	9.62%	3.58%	1.63	9.32%	3.67%	1.51	12.40%	3.49%	2.48
1965	9.06%	6.77%	0.74	10.03%	4.91%	1.21	9.57%	4.90%	1.12	11.53%	3.40%	2.19
1966	-13.09%	11.78%	-1.55	0.81%	14.05%	-0.31	0.32%	14.03%	-0.35	-3.37%	18.35%	-0.47
1967	20.09%	8.28%	1.85	17.54%	8.24%	1.55	17.97%	5.74%	2.31	27.37%	5.46%	4.14
1968	7.66%	8.61%	0.25	7.11%	8.75%	0.18	12.24%	8.65%	0.77	16.09%	4.57%	2.31
1969	-11.36%	9.98%	-1.84	-12.12%	18.04%	-1.06	2.20%	8.68%	-0.55	5.26%	8.05%	-0.21
1970	0.10%	15.19%	-0.51	-21.16%	17.70%	-1.64	-8.94%	19.60%	-0.86	-8.62%	18.40%	-0.89
1971	10.79%	10.21%	0.57	15.75%	6.65%	1.63	10.90%	12.05%	0.50	17.44%	8.17%	1.53
1972	15.63%	7.92%	1.44	12.98%	5.64%	1.55	13.47%	3.71%	2.50	14.99%	5.79%	1.86
1973	-17.37%	15.83%	-1.55	-0.62%	18.71%	-0.42	-18.91%	22.79%	-1.14	6.81%	16.31%	-0.02
1974	-29.72%	21.82%	-1.75	-33.29%	31.33%	-1.33	-24.55%	30.69%	-1.08	-38.75%	29.99%	-1.58
1975	31.55%	15.60%	1.58	30.31%	13.91%	1.69	39.51%	11.31%	2.89	46.10%	12.19%	3.22
1976	19.15%	11.11%	1.21	20.56%	10.08%	1.48	17.54%	5.84%	2.03	19.74%	8.16%	1.72
1977	-11.50%	9.08%	-1.86	-3.65%	11.84%	-0.77	2.09%	7.73%	-0.43	-1.84%	13.46%	-0.54
1978	1.06%	12.57%	-0.50	14.56%	17.49%	0.41	15.60%	12.34%	0.67	19.38%	17.20%	0.70
1979	12.31%	10.85%	0.16	27.32%	8.57%	1.95	19.30%	9.74%	0.89	29.10%	7.08%	2.61
1980	25.77%	16.46%	0.80	31.51%	19.15%	0.99	38.44%	15.10%	1.71	40.34%	16.20%	1.71
1981	-9.73%	13.46%	-1.95	10.78%	17.14%	-0.33	16.88%	11.57%	0.04	22.41%	10.32%	0.58
1982	14.76%	18.26%	0.09	4.51%	24.54%	-0.35	29.71%	24.42%	0.68	38.28%	21.02%	1.19
1983	17.27%	13.33%	0.60	27.66%	10.80%	1.70	26.32%	11.24%	1.51	21.42%	7.86%	1.54
1984	1.40%	12.75%	-0.76	19.40%	14.39%	0.58	19.21%	14.91%	0.54	22.71%	18.36%	0.63
1985	26.33%	10.16%	1.75	21.83%	6.65%	2.00	23.07%	7.26%	2.00	28.19%	7.89%	2.49
1986	14.62%	14.70%	0.54	24.15%	13.16%	1.32	18.07%	12.55%	0.90	24.99%	12.28%	1.48
1987	2.03%	32.15%	-0.14	-25.65%	47.56%	-0.68	-27.92%	48.58%	-0.71	-10.52%	41.65%	-0.41
1988	12.40%	17.09%	0.32	32.54%	13.50%	1.90	31.81%	13.31%	1.88	29.94%	13.44%	1.72
1989	27.25%	13.05%	1.40	29.34%	7.94%	2.56	28.21%	8.63%	2.22	34.98%	6.58%	3.94
1990	-6.56%	15.97%	-0.94	13.22%	17.33%	0.28	13.22%	18.23%	0.26	7.91%	22.17%	-0.02
1991	26.31%	14.30%	1.39	27.80%	12.24%	1.75	28.88%	12.12%	1.86	37.23%	12.62%	2.44
1992	4.46%	9.71%	0.06	9.79%	2.66%	2.21	8.78%	3.13%	1.55	9.97%	4.69%	1.29
1993	7.06%	8.63%	0.44	10.49%	5.09%	1.42	11.00%	5.72%	1.36	14.96%	6.72%	1.74
1994	-1.54%	9.84%	-0.59	8.32%	3.83%	1.06	8.08%	4.69%	0.81	10.78%	3.83%	1.70
1995	34.11%	7.81%	3.59	21.55%	4.52%	3.42	21.57%	3.66%	4.23	23.26%	4.39%	3.91
1996	20.26%	11.81%	1.26	17.43%	9.81%	1.22	15.54%	11.18%	0.91	24.33%	6.71%	2.82
1997	31.01%	18.23%	1.40	33.07%	17.48%	1.58	36.88%	10.30%	3.05	35.86%	15.68%	1.94
1998	26.67%	20.25%	1.06	32.39%	26.49%	1.02	26.59%	29.03%	0.73	3.35%	20.34%	-0.10
1999	19.53%	18.04%	0.81	26.65%	13.77%	1.58	26.98%	12.37%	1.78	31.68%	13.66%	1.96
2000	-10.14%	22.18%	-0.73	11.43%	27.19%	0.19	11.12%	24.06%	0.21	22.52%	19.51%	0.84
2001	-13.04%	21.37%	-0.82	-31.55%	30.03%	-1.20	-6.30%	29.06%	-0.37	-12.99%	32.17%	-0.54
2002	-23.37%	26.00%	-0.97	-21.95%	34.03%	-0.70	-17.54%	37.73%	-0.51	-15.44%	32.50%	-0.53
2003	26.38%	17.03%	1.48	28.95%	21.54%	1.29	26.69%	17.96%	1.42	32.61%	20.65%	1.52
2004	8.99%	11.07%	0.69	9.31%	6.67%	1.20	9.50%	3.90%	2.10	13.31%	3.96%	3.03
2005	3.00%	10.27%	-0.01	9.10%	11.01%	0.55	9.62%	8.54%	0.77	12.34%	6.83%	1.35
2006	13.62%	10.01%	0.88	14.78%	6.42%	1.55	14.99%	3.73%	2.73	16.07%	9.35%	1.20
2007	3.53%	15.93%	-0.10	20.52%	8.35%	1.85	11.86%	17.88%	0.38	21.74%	10.97%	1.52
2008	-38.49%	40.97%	-0.99	-36.07%	53.33%	-0.71	-37.08%	57.33%	-0.68	-31.91%	64.05%	-0.53
2009	23.45%	27.26%	0.85	9.27%	30.58%	0.30	2.24%	29.22%	0.07	0.71%	31.45%	0.02
2010	12.78%	18.05%	0.70	18.96%	15.23%	1.24	19.75%	14.73%	1.33	17.94%	25.51%	0.70

Table 3. Top 10 drawdown events for UPRO/SPXU/T-bill base case portfolio

This table documents the top 10 daily, weekly, monthly, and overall non-overlapping drawdown periods and associated returns for the UPRO/SPXU/T-bill base case portfolio and the S&P 500. “Weekly” and “monthly” are defined as over a period of 5 consecutive trading days and 21 consecutive trading days, respectively. Start date refers to the trading day immediately preceding the drawdown period; end date refers to the last trading day of the drawdown period. All returns are price returns.

Panel A: Daily drawdowns

Date	Portfolio drawdown	S&P 500 drawdown
10/19/1987	-32.24%	-20.47%
9/29/2008	-14.43%	-8.79%
8/31/1998	-13.33%	-6.80%
11/19/2008	-12.93%	-6.12%
10/15/2008	-11.80%	-9.03%
10/9/2008	-11.57%	-7.62%
10/26/1987	-11.20%	-8.28%
3/2/2009	-11.10%	-4.66%
9/17/2001	-10.52%	-4.92%
10/22/2008	-10.45%	-6.10%

Panel B: Weekly drawdowns

Start date	End date	Portfolio drawdown	S&P 500 drawdown
10/12/1987	10/19/1987	-38.52%	-27.33%
10/2/2008	10/9/2008	-26.98%	-18.34%
11/13/2008	11/20/2008	-26.43%	-17.43%
10/20/2008	10/27/2008	-22.88%	-13.85%
8/24/1998	8/31/1998	-21.09%	-12.03%
2/24/2009	3/3/2009	-20.24%	-9.93%
7/15/2002	7/22/2002	-19.04%	-10.68%
2/13/2009	2/23/2009	-18.16%	-10.10%
9/10/2001	9/21/2001	-17.86%	-11.60%
11/5/2008	11/12/2008	-16.62%	-10.55%

Panel C: Monthly drawdowns

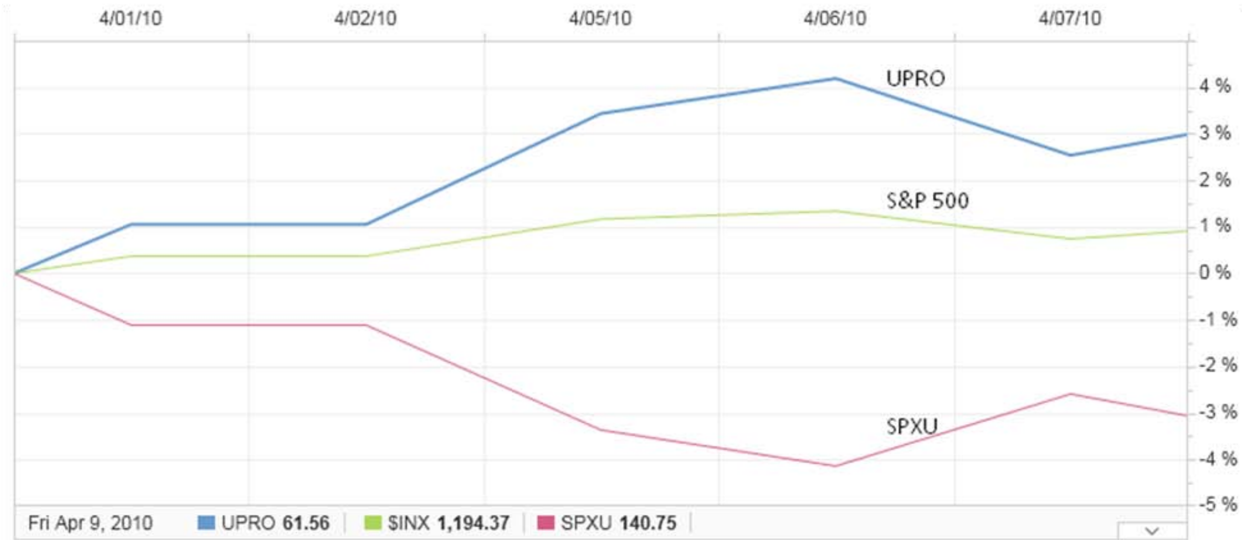
Start date	End date	Portfolio drawdown	S&P 500 drawdown
9/25/1987	10/26/1987	-43.02%	-28.89%
9/26/2008	10/27/2008	-39.58%	-30.02%
2/5/2009	3/9/2009	-32.30%	-20.02%
8/16/2001	9/21/2001	-28.41%	-18.27%
6/21/2002	7/23/2002	-28.20%	-19.35%
8/14/1974	9/13/1974	-23.38%	-15.03%
7/31/1998	8/31/1998	-23.31%	-14.58%
6/10/1974	7/10/1974	-23.28%	-14.08%
4/24/1970	5/25/1970	-23.14%	-15.13%
9/10/2002	10/9/2002	-21.67%	-14.60%

Panel D: Overall drawdowns

Start date	End date	Portfolio drawdown	S&P 500 drawdown
5/19/2008	3/9/2009	-58.71%	-52.58%
5/21/2001	10/9/2002	-49.59%	-40.83%
3/13/1974	12/6/1974	-48.27%	-34.82%

10/5/1987	10/26/1987	-43.97%	-30.61%
11/10/1969	5/26/1970	-33.39%	-29.53%
7/17/1998	8/31/1998	-27.47%	-19.34%
2/1/2001	4/4/2001	-24.87%	-19.67%
7/16/1990	10/11/1990	-24.02%	-19.92%
4/21/1966	10/7/1966	-23.40%	-20.80%
4/23/2010	7/2/2010	-18.75%	-15.99%

Figure 1. UPRO, SPXU, and S&P 500 returns during 4/1/10 - 4/7/10 (top) and 4/1/10 - 10/1/11 (bottom)



Source: http://money.msn.com

Figure 2. UPRO, SPXU, and S&P 500 returns during 11/21/2011 - 4/2/2012

Source: http://money.msn.com