

Interest rate and stock return volatility indices for the Eurozone. Investors' gauges of fear during the recent financial crisis^{*}

Raquel López^a, Eliseo Navarro^b

Abstract

We suggest a methodology for the construction of a set of interest rate volatility indices for the Eurozone (EIRVIXs) based on the implied volatility quotes of caps (floors), one of the most liquid interest rate derivatives. These indices reflect the market's aggregate expectation of volatility of forward rates over both short- and long-term horizons (from one to ten years ahead).

Volatility indices in equity markets are referred to as investors' gauges of fear because they usually spike in periods of market turmoil. In this paper, we extend the empirical evidence by analyzing the effect of the recent financial crisis on short- and long-term EIRVIXs. We find that the level of short-term EIRVIXs (70%) as of April 2012 is still far from returning to the average pre-crisis value (17%) and that the crisis has also affected investors' long-term expectations of volatility. In addition, using two stock return volatility indices for the Eurozone, we find that the crisis has had a deeper impact on investors' uncertainty about the evolution of interest rates than on stock market returns.

Keywords: caps and floors, crisis, interest rates; investors' gauge of fear, volatility indices

^{*} Raquel and Eliseo acknowledge the financial support provided by Ministerio de Educación y Ciencia grant ECO2011-28134. Any remaining errors are our responsibility alone.

^a Raquel López is in the Department of Economic Analysis and Finance, University of Castilla-La Mancha (E-mail: raquel.lopez@uclm.es).

^b Eliseo Navarro is in the Department of Business Science, University of Alcalá (E-mail: eliseo.navarro@uah.es).

1. Introduction

The number of implied volatility indices has significantly increased over the last decade in the equity markets all over the world (e.g., VIX for US; VDAX, VCAC and VSTOXX for Europe; VKOSPI and India VIX for Asia; S&P/ASX 200 VIX for Australia; and New SAVI for South Africa). These indices capture the market's expectation of volatility of stock indices returns over a very short-term horizon (usually 30 calendar days) and have a number of interesting applications. They have been used for forecasting future realized volatility (see e.g., Moraux et al., 1999; Bluhm and Yu, 2001; Becker et al., 2007), to assess the underlying stock indices' market risk (Giot, 2005a), to identify profitable opportunities in the stock market (see e.g., Giot, 2005b; Banerjee et al., 2007), and to measure financial risk aversion (see e.g., Barrios et al., 2009; Beber et al., 2009; Gerlach et al., 2010; Arghyrou and Kontonikas, 2011; Oliveira et al., 2012).

In this study we suggest a methodology for the construction of a set of interest rate volatility indices for the Eurozone (EIRVIXs), which reflect the market estimate of the volatility of three- and six-month tenor forward rates over different fixed horizons – one, two, three, four, five, seven and ten years. To the best of our knowledge, there are no volatility indices calculated by exchanges or other institutions for the Eurozone fixed-income market. Thus, this paper contributes to the literature on volatility indices by covering this gap. To this end, we use data on caps (floors), one of the most liquid over-the-counter (OTC) interest rate derivative contracts (Li and Zhao, 2009). Bank for International Settlements statistics, as of December 2011, indicate that interest rate options (caps, floors, collars and corridors) were the second most-traded OTC interest rate derivatives worldwide. Moreover, the notional amount of OTC interest rate options exceeded that of exchange-traded options by nearly \$20 trillion. Sorted by currency, Euro interest rate options accounted for approximately 46% of the total amount outstanding of OTC interest rate options traded in the world.¹

Using daily data from January 2004 to April 2012, this paper analyzes how short- and long-term expectations of interest rate volatility change in response to the financial turmoil during the present crisis. Concerning short-term EIRVIXs, we observe large spikes along upward and downward slopes since the summer of 2007. As of April 2012,

¹ <http://www.bis.org/statistics/derstats.htm>

volatility levels of approximately 70% are still far from returning to the average pre-crisis value (17%). Thus, the crisis has had a deep and lasting effect on investors' short-term expectations of volatility in the fixed-income market. More interestingly, we also find that as the crisis deepened, it also eventually affected expectations of volatility five and ten years ahead: the indices initiate an upward trend in 2010.

In addition, we compare the effect of the crisis on investors' uncertainty about the future development of interest rates and stock market returns using two VSTOXX volatility sub-indices constructed from Dow Jones EURO STOXX 50 options expiring in one and two years. In the equity markets, VIX is usually called the investors' gauge of fear because it spikes during periods of market turmoil (Whaley, 2000, 2009). In this paper, we observe that VSTOXX sub-indices exhibit a lower rise than one- and two-year EIRVIXs along the crisis period and that the size of the spikes is also smaller. This finding suggests that EIRVIXs have played a greater role as investors' gauge of fear during the recent financial crisis than VSTOXX sub-indices. Based on these findings, we construct a measure of global financial risk aversion using stock return and interest rate volatility indices and we show that this indicator of risk aversion and Euro area government bond yield spreads move closely together from 2009 up to mid 2010. Finally, we prove that movements in short-term EIRVIXs and VSTOXX sub-indices are positively correlated and that changes in the interest rate volatility index might be useful for portfolio managers to improve stock return volatility forecasts.

The rest of the paper is organized as follows. The next section examines caps (floors) valuation according to the Libor Market Model (LMM), which is consistent with the market standard approach for pricing these contracts using the Black pricing formula. In Section Three, we present the methodology for the calculation of EIRVIXs. Section Four describes the database. In Section Five, we analyze the behavior and statistical properties of EIRVIXs, and compare the effect of the financial turmoil during the recent crisis on the market estimates of future volatility of interest rates and stock returns. Finally, Section Six includes the conclusions of the study.

2. Caps and floors valuation. The LMM and the Black formula

Caps and floors are portfolios of options on interest rates, caplets and floorlets. Thus, the design and valuation of caps (floors) can best be understood by first describing the options that comprise them.

Caplets (floorlets) are European-style call (put) options where the underlying asset is a forward rate agreement (FRA). An FRA is an agreement between two parties to exchange an amount of money proportional to the difference between the fixed strike rate K (set at t) and the floating interest rate (reset at time T_i) which prevails over the period $[T_i, T_i + \tau]$, $L(T_i, T_i + \tau)$, ($t \leq T_i < T_i + \tau$). The payoff of an FRA at $T_i + \tau$ is:

$$NP \cdot [L(T_i, T_i + \tau) - K] \cdot \tau, \quad (1)$$

where NP is the notional principal of the contract and τ is the tenor interval.

Caplets (floorlets) are exercised only if $L(T_i, T_i + \tau)$ is greater (smaller) than the strike K . The payoff of a caplet at $T_i + \tau$ is:

$$NP \cdot \text{Max}\{L(T_i, T_i + \tau) - K, 0\} \cdot \tau, \quad (2)$$

and the payoff of a floorlet is:

$$NP \cdot \text{Max}\{K - L(T_i, T_i + \tau), 0\} \cdot \tau. \quad (3)$$

The LMM assumes that the forward interest rate $f(t, T_i, T_i + \tau)$ follows a lognormal stochastic process (see Brigo and Mercurio, 2006 for an extensive review of LMM). Taking into account that the limiting value of the forward rate when t approaches T_i is equal to the floating interest rate $L(T_i, T_i + \tau)$, and assuming there are no arbitrage opportunities, the well-known Black (1976) pricing formulas for valuing caplets (floorlets) are derived (see e.g., Díaz et al., 2009):

$$Caplet(t, T_i, \tau, K, \sigma_{i, Black}^K) = NP \cdot [f(t, T_i, T_i + \tau) \cdot N(h_1) - K \cdot N(h_2)] \cdot P(t, T_i + \tau) \cdot \tau, \quad (4)$$

$$Floorlet(t, T_i, \tau, K, \sigma_{i, Black}^K) = NP \cdot [K \cdot N(-h_2) - f(t, T_i, T_i + \tau) \cdot N(-h_1)] \cdot P(t, T_i + \tau) \cdot \tau, \quad (5)$$

where

$$h_1 = \frac{\ln[f(t, T_i, T_i + \tau) / K] + \frac{1}{2} \cdot (\sigma_{i, Black}^K)^2 \cdot (T_i - t)}{\sigma_{i, Black}^K \cdot \sqrt{(T_i - t)}}, \quad (6)$$

$$h_2 = \frac{\ln[f(t, T_i, T_i + \tau) / K] - \frac{1}{2} \cdot (\sigma_{i, Black}^K)^2 \cdot (T_i - t)}{\sigma_{i, Black}^K \cdot \sqrt{(T_i - t)}}. \quad (7)$$

$Caplet(t, T_i, \tau, K, \sigma_{i, Black}^K)$ and $Floorlet(t, T_i, \tau, K, \sigma_{i, Black}^K)$ are the prices at t of a caplet and a floorlet, respectively, T_i is the exercise date of the option (and the maturity date of the underlying forward rate), τ is the tenor of the underlying forward rate (and $T_i + \tau$ is the maturity date of the option), $P(t, T_i + \tau)$ is the price at t of a unit-zero coupon bond with maturity at $T_i + \tau$, $N(\cdot)$ is the cumulative normal distribution, and $\sigma_{i, Black}^K$ is the so-called Black implied volatility of an option with exercise date T_i and strike K .

Black implied volatility can be understood, within the LMM, as an average of the instantaneous volatility of the log of the forward rate $f(t, T_i, T_i + \tau)$ over the period $[t, T_i]$:

$$(\sigma_{i, Black}^K)^2 = \frac{\int_t^{T_i} \sigma^2(u, T_i) du}{(T_i - t)}, \quad (8)$$

where $\sigma(t, T_i)$ is the instantaneous volatility at t of the lognormal process followed by the forward rate $f(t, T_i, T_i + \tau)$.

Caps (floors) are portfolios of caplets (floorlets) with the same strike and tenor but with consecutive maturities so that the maturity date of each caplet (floorlet) coincides with the exercise date of the following one. In the Eurozone, caps (floors) with time to expiration up to two years have a three-month tenor, whereas the tenor for caps (floors) with maturities beyond two years is six months. Thus, a two-year cap (floor) consists of a chain of seven caplets (floorlets) with exercise dates in three, six, nine, 12, 15, 18 and 21 months, whereas a three-year cap (floor) comprises five caplets (floorlets) with exercise dates in six, 12, 18, 24 and 30 months. Please note that, unlike equity options, caplets (floorlets) and caps (floors) have a constant life period.

The payoffs generated by a cap (floor) can be described as follows. On the exercise date of the first caplet (floorlet), the floating rate is observed and compared to the strike. If the floating rate is greater (smaller) than the strike, then on the second reset date the seller of the cap (floor) pays the holder the difference between the floating rate (strike) and the strike (floating rate) multiplied by the notional principal and the tenor. If the floating rate is less (more) than the strike, there is no payoff from the cap (floor). Thus, through the life of a cap (floor), payments are due at the end of each tenor interval, although the amount is known at the reset date (at the beginning of the tenor interval) when the floating interest rate is observed.²

Then, the price at time t of an n -year cap with strike K can be obtained as the sum of the values of the caplets that comprise it. That is,

$$Cap(t, T_{n-k}, K) = \sum_{i=1}^{n-k-1} caplet(t, T_i, \tau, K, \sigma_{i, Black}^K), \quad (9)$$

where k equals 4 (2) when the tenor interval is three (six) months, and $T_1, T_2, \dots, T_{n-k-1}$ are the reset dates of the cap that coincide with the exercise dates of the caplets that compose the cap and $T_{n-k} = T_{n-k-1} + \tau$, i.e., the date that the last cash flow will be due if $L(T_{n-k-1}, T_{n-k-1} + \tau) > K$. An analogous formula can be set up for the price of a floor.

However, quotations in the cap market are computed assuming that the volatility of all the caplets that compose a particular cap is the same. In fact, an n -year cap with strike K is quoted by the market through the so-called flat volatility, which is the constant value $\sigma_{n, flat}^K$ that equals the sum of the values of all the caplets that compose the cap according to the Black formula to its market price, i.e., the value $\sigma_{n, flat}^K$ such that

$$Cap(t, T_{n-k}, K) = \sum_{i=1}^{n-k-1} caplet(t, T_i, \tau, K, \sigma_{n, Black}^K). \quad (10)$$

Therefore, flat volatilities cannot be considered to be a pure measure of the future evolution of volatility of a forward rate; rather, they are a mixture of the average future

² Caps (floors) are usually defined so that the initial floating rate, even if it is greater (smaller) than the strike, does not lead to a payoff on the first reset date (Hull, 2009).

³ Actually, the market quotes flat volatilities of caps/floors. At a particular strike and for a concrete term to maturity, traders may contract the same instrument as a cap or a floor depending on their expectations.

volatilities of a set of forward rates with consecutive terms to maturity.⁴ Thus, for instance, the flat volatility of a two-year cap is a mixture of the average future volatility of three-month tenor forward rates with maturities in three, six, nine, 12, 15, 18 and 21 months.

Finally, note that according to the LMM and Equation (8), the implied volatility of caplets should be the same for all caplets with the same term to maturity, independent of the strike K . However, in practice, the implied volatility of caplets and caps (with everything else equal) varies with the strike rate, giving rise to volatility surfaces (see Jarrow et al., 2007).

3. Methodology

We develop a set of interest rate volatility indices that capture the market's expected volatility of a particular forward rate over different fixed horizons using the implied volatility quotes of caps (floors).^{5,6} However, the use of data from this market poses the problem of having to address a contract where the underlying rate is not a single forward rate but a set of forward rates with consecutive maturities. Therefore, the construction of EIRVIXs involves recovering the implied volatilities of the individual caplets that compose caps using a stripping procedure (see e.g., Hernández, 2005). This process consists of obtaining the price at time t of a caplet with a strike K and reset date T_i , $caplet(t, T_i, \tau, K, \sigma_{i, Black}^K)$, by subtracting the prices of two consecutive caps with the same strike K :

⁴ The difference between $\sigma_{i, Black}^K$ and $\sigma_{n, flat}^K$ is similar to the difference between zero-coupon rates and the yields to maturity of coupon-bearing bonds.

⁵ Stock return volatility indices are calculated using the market prices of (exchange-traded) options, rather than their respective implied volatilities, based on the concept of the fair delivery value of future realized variance suggested by Demeterfi et al. (1999). However, note that the quoted option price in the OTC market is actually implied volatility itself (i.e., implied volatility does not need to be inferred from option prices). Thus, to provide an implied volatility quote in the cap (floor) market means to give the option price, similar to how the yield to maturity of a bond is an alternative way of providing the price of the bond.

⁶ Implied volatilities of specific forward rates could be directly obtained from caplet (floorlet) quotations, however, these contracts are quite illiquid; thus, obtaining a complete enough range of caplets (floorlets) with different maturities can be complicated. Thus, the construction of EIRVIXs from caps (floors) data can give a much more accurate indication of the actual uncertainty regarding the future behavior of interest rates for a wide range of maturities, without the intrusion of the noise caused by the lack of liquidity.

$$Caplet(t, T_i, \tau, K, \sigma_{i, Black}^K) = Cap(t, T_{i+1}, K) - Cap(t, T_i, K), \quad (11)$$

where $Cap(\cdot)$ are defined as in Equation (10).

Once the price of the caplet is obtained, the Black pricing formula is used to derive the corresponding implied volatility.⁷ Note that when implementing the stripping procedure, the Black's model is used only to translate volatility quotes into option prices and vice versa. Thus, we are not making use of any of the assumptions of the Black's model. It is merely used as a tool to provide a one-to-one mapping between option prices and implied volatilities.

As stressed in the previous section, it must be noted that we obtain different implied Black volatilities for the same caplet, depending upon the strike rate K . Thus, a decision on the strikes of the caps used for the implementation of EIRVIXs needs to be made. Poon and Granger (2003) suggest using at-the-money (ATM) options because they are more liquid and less prone to measurement errors.

In the cap market, an n -year cap is said to be ATM if the strike of this instrument equals the fixed rate of a swap that has the same payment days as the cap (see e.g., Hull, 2009). However, we cannot use ATM caps in the stripping process because two consecutive caps would have different strikes to the extent that swaps with different maturities usually have different fixed rates.

Therefore, we must address the problem of determining the strike of an ATM caplet. According to the Black formula, a caplet is said to be ATM when the value of the underlying forward rate equals the strike rate. Thus, we propose using the available caps with strikes closest to the outstanding forward rate $f(t, T_i, T_i + \tau)$ defined as

$$f(t, T_i, T_i + \tau) = \left(\frac{P(t, T_i)}{P(t, T_i + \tau)} - 1 \right) \cdot \frac{1}{\tau}, \quad (12)$$

where $P(t, T_i)$ and $P(t, T_i + \tau)$ are the prices at t of unit zero-coupon bonds with maturities at T_i and $T_i + \tau$, respectively.

In particular, we will use caps with strikes immediately above and below $f(t, T_i, T_i + \tau)$, and we will refer to them as K^A and K^B , respectively, with $K^B < f(t, T_i, T_i + \tau) < K^A$.

Then, using Equation (11), we obtain the prices of caplets with strikes K^A (first out-of-the-money caplet) and K^B (first in-the-money caplet), and we derive their implied

⁷ Note that the same implied volatility is obtained when the pricing formulas for floors (floorlets) are used instead.

volatilities using the Black formula. We denote these two implied volatilities by $\sigma_{i,Black}^{K^A}$ and $\sigma_{i,Black}^{K^B}$, respectively.

Finally, we use linear interpolation to obtain EIRVIX:

$$EIRVIX(t, T_i) = \sigma_{i,Black}^{K^B} \cdot \left(\frac{K^A - f(t, T_i, T_i + \tau)}{K^A - K^B} \right) + \sigma_{i,Black}^{K^A} \cdot \left(\frac{f(t, T_i, T_i + \tau) - K^B}{K^A - K^B} \right), \quad (13)$$

where $EIRVIX(t, T_i)$ is the annualized implied volatility of a theoretical ATM caplet with a constant time to maturity, from t to T_i .

According to Carr and Lee (2003, 2009) the volatility swap rate with expiry at time T is well approximated by the ATM implied volatility maturing at the same time. A volatility swap is a contract traded OTC that pays at maturity the difference between the realized volatility of the underlying asset over the life of the contract and a fixed volatility rate (the volatility swap rate). Since the contract has zero value at the time of entry, by no arbitrage, the volatility swap rate equals the conditional risk-neutral expected value of the realized volatility over the life of the contract. Thus, $EIRVIX(t, T_i)$ approximates the conditional risk-neutral expectation of the realized volatility of the underlying forward interest rate $f(t, T_i, T_i + \tau)$ over the period $[t, T_i]$. In addition, because EIRVIX is based on the market quotes of very liquid options, it represents a consensus market view of the expected volatility of the underlying forward rate.

Using the construction method just described, we create a daily set of interest rate volatility indices for three- (six-) month tenor forward rates expiring in one and two (three, four, five, seven and ten) years. According to Duarte et al. (2005), these are the most liquid cap maturities.

According to Equation (8), each EIRVIX provides the average future volatility of a forward interest rate up to its maturity. For instance, $EIRVIX(t, 1Y)$ measures the market's assessment at any time t of the uncertainty regarding the evolution of the forward rate $f(t, t+1Y, t+1Y+3M)$ over the next year; and $EIRVIX(t, 10Y)$ would indicate the average volatility of the forward rate $f(t, t+10Y, t+10Y+6M)$ over the next ten years. Thus, unlike flat volatilities, EIRVIXs measure the volatility of specific forward rates. For instance, $\sigma_{1,flat}^K$ would be some sort of average of the future volatilities of the

forward rates $f(t, t+3M, t+6M)$, $f(t, t+6M, t+9M)$ and $f(t, t+9M, t+1Y)$ up to their respective maturities.

4. Data

For the construction of EIRVIXs we use two sets of daily data from the Eurozone fixed-income market. The first set consists of last bid-ask averages flat volatility quotes of caps (floors) for a fixed set of maturities and strikes retrieved from Bloomberg. The data supplier for these quotes is the large OTC interdealer broker ICAP. The second set consists of zero-coupon curves provided by Reuters based on the most liquid rate instruments available, a combination of deposits, liquid futures and interest rate swaps. The sample extends from January 02, 2004 to April 30, 2012.⁸

Flat volatilities correspond to caps (floors) with maturities of one to ten plus 12, 15 and 20 years and with the following range of strike rates: 0.01, 0.02, 0.0225, 0.025, 0.03, 0.04, 0.05, 0.06 and 0.07. These strikes cover the range of values of the forward rates during the sample period to ensure that there will be always a strike above and below the outstanding forward rates.⁹ Note also that the two strikes closest to the forward rate $f(t, T_i, T_i+\tau)$ can differ by only 25, 50 or 100 basis points. Thus, the assumption that the “volatility smile” is well approximated by a line that we use when the implied volatilities of near-the-money options are linearly interpolated for the construction of EIRVIX is considered reasonable due to the small range of strikes over which the interpolation is made (Fleming et al., 1995).

Note also that the stripping procedure involves using the prices of caps with maturities in one (two) years and three months, and three (four, five, seven and ten) years and six months, whereas markets only provide caps with annual terms to maturity (i.e., with an integer number of years to maturity). Therefore, interpolation and extrapolation techniques must be used to obtain flat volatilities of caps with a maturity different from those quoted.

Interpolation and extrapolation techniques are applied between caps with the same tenor interval. Thus, when the required maturity cap is below three years, we apply linear

⁸ The sample starts in January 2004 because of data availability.

⁹ The only exception occurs for the forward rate maturing in one year since values below one percent are observed since mid-2010. Thus, in this particular situation, EIRVIX is just the implied volatility of the caplet (floorlet) with a strike rate of 0.01.

interpolation/extrapolation by using the one- and two-year maturities. For the rest of maturities, we apply cubic spline interpolation based on the flat volatilities of caps with maturities of three to ten years plus 12, 15 and 20 years (see Hernández, 2005). We use linear interpolation/extrapolation only when the number of available flat volatilities is less than six. Note that these interpolation/extrapolation techniques must produce uniquely determined values of unobservable flat volatilities with any term to maturity up to ten years and six months (the maturity date of the caplet with an exercise date in ten years). See the Appendix for a detailed description of the interpolation/extrapolation procedure.

In regards to measuring the market's expectations of volatility in the equity market, we use two volatility indices distributed by STOXX Ltd, VSTOXX 12M and VSTOXX 24M. Actually, they belong to the set of sub-indices that are calculated in addition to the main index, VSTOXX, which measures volatility over the next 30 calendar days. In particular VSTOXX 12M and VSTOXX 24M are constructed based on the prices of Dow Jones EURO STOXX 50 options expiring in 12 and 24 months, respectively. Thus, they capture the market's expected volatility of the Dow Jones EURO STOXX 50 returns over the next 12 and 24 months. Sub-indices based on options with longer terms to expiration are not currently available.¹⁰

5. Empirical analysis

In this section we analyze the behavior and statistical properties of the set of EIRVIXs. Then, we analyze the role of interest rate and stock return volatility indices as investors' gauges of fear during the crisis.

5.1. Properties of EIRVIXs

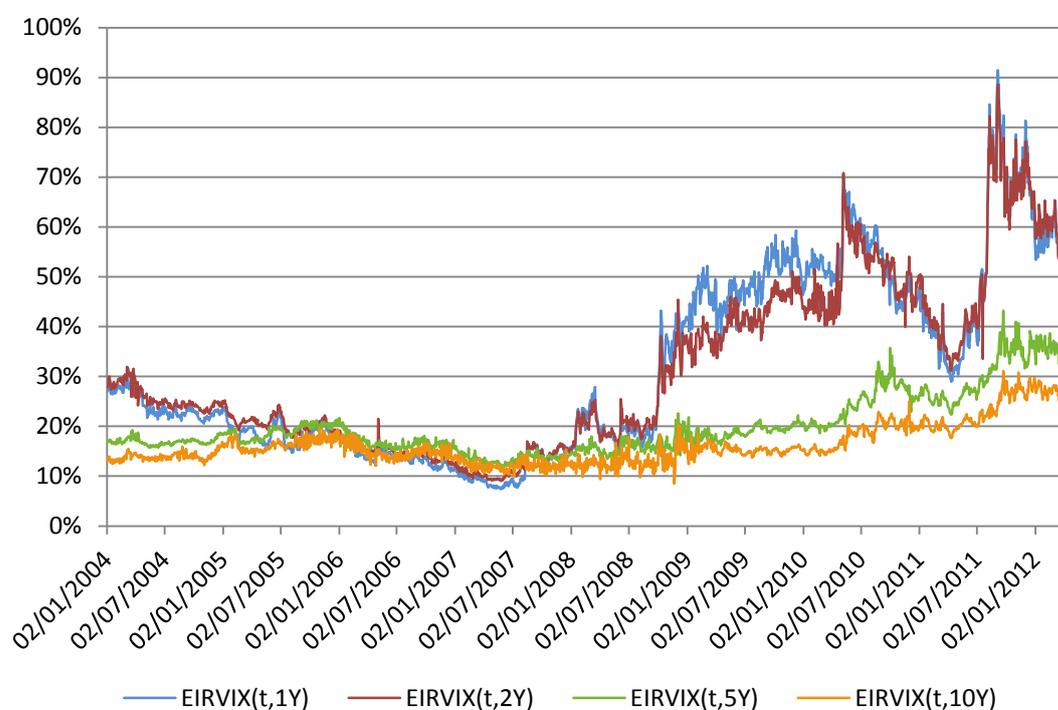
The daily evolution of EIRVIXs with times to maturity of one, two, five and ten years from January 02, 2004 to April 30, 2012 is shown in Figure 1.

We observe a decreasing pattern in EIRVIXs with the closest forecast horizons from the beginning of the sample up to approximately mid-July 2007. Then, the indices initiate an upward trend which leaves market estimates of interest rate volatility over the next one and two years at approximately 70% in May 2010. Thus, short-term EIRVIXs seem

¹⁰ Additional information on the indices can be found on <http://www.stoxx.com/index.html>

to reflect the financial turmoil since the beginning of the crisis. By May 2010, the upward trend turns a downward trend until approximately April 2011, when the level of EIRVIXs is close to 30%. Then, EIRVIXs exhibit a new outstanding rise that drives market expectations of interest rate volatility to a maximum of approximately 90% in five months. Finally, large (up and down) spikes are observed along a downward slope until April 2012, when EIRVIX's level (approximately 70%) is still far from returning to the average pre-crisis value.

Figure 1. Daily levels of EIRVIX(t,1Y), EIRVIX(t,2Y), EIRVIX(t,5Y) and EIRVIX(t,10Y) over the period from January 02, 2004 to April 30, 2012



Concerning investors' expectations of volatility over the next five and ten years, they exhibit a rise during the first half of 2010 and approximately double by the end of the sample.

Thus, two main conclusions can be drawn from the graphics. On the one hand, the fact that the indices (especially the short-term ones) spike and sharply increase during the recent financial crisis supports the interpretation of EIRVIX as a gauge of fear for fixed-income markets similar to the widely held view of VIX for equity markets (Whaley,

2000, 2009). On the other hand, the fact that long-term EIRVIXs also respond to the financial turmoil seems to suggest that investors foresee long periods of turbulence in interest rate markets. Recall that EIRVIX provides the average level of future volatility until the maturity of the underlying forward rate (see Equation (8)), and hence, only a lasting shift in the market estimates of future volatility would make long-term EIRVIXs raise.

The summary statistics of the set of interest rate volatility indices for the full sample as well as before and after the beginning of the subprime crisis are included in Table 1 (Panels A, B and C, respectively). The first subsample spans the period from January 02, 2004 to July 31, 2007 (885 observations), and the second spans the period from August 01, 2007 to April 30, 2012 (1177 observations).¹¹

The average value of all EIRVIXs increases during the crisis period. In addition, we find that the mean of all maturity EIRVIXs is quite similar before the crisis, whereas it progressively decays as the forecast horizon increases for the second subsample. Short-term EIRVIXs also show greater variability (standard deviation) than long-term ones before and during the crisis. The skewness and kurtosis measures suggest that the indices are closer to a normal distribution in the split sample than when the whole sample is considered. In any case, the Jarque-Bera test does not accept the null hypothesis of a normal distribution for any of the indices in any of the two subperiods. To investigate whether the series are stationary, the augmented Dickey-Fuller (ADF) unit root test for its most general specification (i.e., with intercept and linear trend) is performed on the logarithm of the volatility indices. The null hypothesis of a unit root is not rejected in any case.¹²

¹¹ August 2007 is usually referred to as the onset date of the subprime crisis (see e.g., Taylor and Williams, 2009; Fleming and Klagge, 2010), and a change in the values of the indices is also especially perceptible around this date.

¹² Given the likely existence of a structural break in the series during the crisis period, the modified version of the ADF test developed by Zivot and Andrews (2002) to allow for a structural break in the data is conducted. The null hypothesis is that the series follows a unit root process; the alternative hypothesis implies that the series is a trend-stationary process with a one-time break in the trend function occurring at an unknown point in time. We obtain that the null hypothesis continues not being rejected in all the cases, except for EIRVIX(t,4Y) at the 5% significance level. Moreover, we find that the structural break dates identified by the test for short-term EIRVIXs belong to July 2007.

Table 1. Summary statistics of EIRVIXs across the entire sample (Panel A) and for two subsamples: from January 02, 2004 to July 31, 2007 (Panel B) and from August 01, 2007 to April 30, 2012 (Panel C)

	EIRVIX (t,1Y)	EIRVIX (t,2Y)	EIRVIX (t,3Y)	EIRVIX (t,4Y)	EIRVIX (t,5Y)	EIRVIX (t,7Y)	EIRVIX (t,10Y)
<i>Panel A: January 02, 2004 to April 30, 2012</i>							
Observations	2062	2062	2062	2062	2062	2062	2062
Mean	0.31	0.31	0.24	0.21	0.20	0.17	0.16
Median	0.23	0.24	0.21	0.19	0.17	0.15	0.15
Maximum	0.91	0.88	0.56	0.44	0.43	0.32	0.31
Minimum	0.07	0.09	0.11	0.11	0.11	0.10	0.08
Std. Deviation	0.18	0.16	0.10	0.07	0.06	0.04	0.04
Skewness	0.65	0.78	1.14	1.18	1.38	1.42	1.34
Kurtosis	2.30	2.72	3.41	3.56	4.18	4.30	4.36
Jarque-Bera	188.18 (0.00)	219.43 (0.00)	463.57 (0.00)	507.32 (0.00)	780.69 (0.00)	842.96 (0.00)	782.66 (0.00)
ρ_1	0.99**	0.99**	0.99**	0.99**	0.99**	0.98**	0.97**
ADF	-2.31	-2.19	-2.01	-2.21	-1.81	-1.77	-2.51
<i>Panel B: January 02, 2004 to July 31, 2007</i>							
Observations	885	885	885	885	885	885	885
Mean	0.17	0.18	0.18	0.17	0.17	0.16	0.15
Median	0.16	0.19	0.19	0.18	0.18	0.16	0.15
Maximum	0.30	0.31	0.25	0.21	0.23	0.21	0.20
Minimum	0.07	0.09	0.11	0.11	0.12	0.11	0.11
Std. Deviation	0.05	0.05	0.03	0.02	0.02	0.02	0.01
Skewness	0.12	0.03	-0.72	-0.72	-0.08	-0.31	0.21
Kurtosis	2.13	2.03	2.45	2.82	2.06	2.97	2.72
Jarque-Bera	29.84 (0.00)	34.62 (0.00)	89.48 (0.00)	77.99 (0.00)	28.02 (0.00)	14.64 (0.00)	9.62 (0.00)
<i>Panel C: August 01, 2007 to April 30, 2012</i>							
Observations	1177	1177	1177	1177	1177	1177	1177
Mean	0.42	0.40	0.29	0.25	0.22	0.18	0.17
Median	0.45	0.41	0.28	0.24	0.20	0.16	0.15
Maximum	0.91	0.88	0.56	0.44	0.43	0.32	0.31
Minimum	0.09	0.11	0.12	0.12	0.11	0.10	0.08
Std. Deviation	0.17	0.16	0.11	0.08	0.07	0.05	0.05
Skewness	-0.11	0.11	0.45	0.49	0.71	0.79	0.76
Kurtosis	2.29	2.37	2.18	2.19	2.42	2.46	2.60
Jarque-Bera	27.20 (0.00)	21.72 (0.00)	73.59 (0.00)	81.01 (0.00)	116.12 (0.00)	137.05 (0.00)	121.92 (0.00)

Notes:

^a p -values of the Jarque-Bera test are inside parenthesis.

^b ρ_1 denotes the first-order autocorrelation coefficient. The significance of autocorrelations is tested with the Ljung-Box Q-statistic.

^c The ADF test is performed on the logarithm of the indices. The null hypothesis is that the series contains a unit root. The optimal lag length is determined according to the Schwarz information criterion.

^d One and two asterisks denote statistical significance at the 5% and 1% significance level, respectively.

Summary statistics for the daily log-differences of EIRVIXs are also shown in Table 2. On the one hand, the excess kurtosis found in the series is also reported by Dotsis et al.

(2007) for several equity market volatility indices in their first differences, where the non-normality may be attributed to the presence of jumps in implied volatility. On the other hand, the significant negative first-order autocorrelation supports the modeling of implied volatility indices as mean-reverting processes. All the series are stationary after differencing.

To formally investigate whether there are statistically significant differences in the distribution of the indices before and after the beginning of the crisis we apply two non-parametric tests. Panel A in Table 3 shows the results of the Wilcoxon/Mann-Whitney test for the equality of medians and the Brown-Forsythe test for the equality of variances for the series in levels. The results show evidence of significant differences in both the median and the variance between the first and second subsamples at the 1% significance level for all the indices. For the series in first log-differences (Panel B in Table 3), statistically significant differences in the medians between the first and second subsamples are unproven for all forecast horizons; however, the null hypothesis of equality of variances is rejected for EIRVIXs with time to expiration from one to five years.

Table 2. Summary statistics of first log-differences of EIRVIXs across the entire sample (Panel A) and for two subsamples: from January 02, 2004 to July 31, 2007 (Panel B) and from August 01, 2007 to April 30, 2012 (Panel C)

	EIRVIX (t,1Y)	EIRVIX (t,2Y)	EIRVIX (t,3Y)	EIRVIX (t,4Y)	EIRVIX (t,5Y)	EIRVIX (t,7Y)	EIRVIX (t,10Y)
<i>Panel A: January 02, 2004 to June 30, 2011</i>							
Observations	2061	2061	2061	2061	2061	2061	2061
Mean	0.0045	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003
Median	-0.0006	-0.0003	0.0001	0.0009	0.0002	0.0004	0.0003
Maximum	0.26	0.29	0.37	0.28	0.31	0.43	0.40
Minimum	-0.26	-0.27	-0.37	-0.27	-0.36	-0.30	-0.44
Std. Deviation	0.04	0.04	0.03	0.03	0.03	0.04	0.06
Skewness	0.27	0.40	0.10	-0.34	-0.26	0.47	-0.03
Kurtosis	6.91	10.64	16.14	15.35	14.90	14.11	8.80
Jarque-Bera	1339.76 (0.00)	5080.10 (0.00)	14831.81 (0.00)	13149.32 (0.00)	12199.07 (0.00)	10686.25 (0.00)	2890.58 (0.00)
ρ_1	-0.07**	-0.14**	-0.30**	-0.27**	-0.35**	-0.42**	-0.40**
ADF	-48.77**	-31.00**	-33.83**	-40.34**	-24.72**	-21.12**	-29.82**
<i>Panel B: January 02, 2004 to July 31, 2007</i>							
Observations	884	884	884	884	884	884	884
Mean	-0.0011	-0.0009	-0.0003	-0.0002	-0.0002	0.0000	0.0000
Median	-0.0012	-0.0007	-0.0001	0.0004	-0.0004	0.0010	-0.0006
Maximum	0.18	0.27	0.10	0.14	0.11	0.15	0.24
Minimum	-0.12	-0.22	-0.10	-0.14	-0.21	-0.14	-0.26
Std. Deviation	0.03	0.02	0.02	0.02	0.03	0.03	0.05
Skewness	0.21	0.26	0.00	0.00	-0.45	-0.02	-0.03
Kurtosis	5.38	20.90	5.14	8.48	7.39	4.96	4.76
Jarque-Bera	217.17 (0.00)	11818.61 (0.00)	169.45 (0.00)	1108.62 (0.00)	741.64 (0.00)	142.40 (0.00)	114.40 (0.00)
<i>Panel C: August 01, 2007 to April 30, 2012</i>							
Observations	1177	1177	1177	1177	1177	1177	1177
Mean	0.0016	0.0014	0.0010	0.0008	0.0008	0.0006	0.0005
Median	0.0000	0.0005	0.0005	0.0016	0.0007	-0.0003	0.0011
Maximum	0.26	0.29	0.37	0.28	0.31	0.43	0.40
Minimum	-0.26	-0.27	-0.37	-0.27	-0.36	-0.30	-0.44
Std. Deviation	0.04	0.04	0.04	0.03	0.04	0.04	0.07
Skewness	0.23	0.35	0.07	-0.40	-0.21	0.65	-0.03
Kurtosis	6.58	7.76	13.21	13.30	14.93	16.15	9.85
Jarque-Bera	640.88 (0.00)	1138.69 (0.00)	5122.49 (0.00)	5241.56 (0.00)	6991.61 (0.00)	8570.91 (0.00)	2302.72 (0.00)

Notes:

^a p -values of the Jarque-Bera test are inside parenthesis.

^b ρ_1 denotes the first-order autocorrelation coefficient. The significance of autocorrelations is tested with the Ljung-Box Q-statistic.

^c The ADF test is performed on the logarithm of the indices. The null hypothesis is that the series contains a unit root. The optimal lag length is determined according to the Schwarz information criterion.

^d One and two asterisks denote statistical significance at the 5% and 1% significance level, respectively.

Table 3. Tests of equality of medians and variances between the first and second subsamples for the indices in levels (Panel A) and in first log-differences (Panel B)

	EIRVIX (t,1Y)	EIRVIX (t,2Y)	EIRVIX (t,3Y)	EIRVIX (t,4Y)	EIRVIX (t,5Y)	EIRVIX (t,7Y)	EIRVIX (t,10Y)
<i>Panel A: Tests of equality of medians and variances for EIRVIXs in levels</i>							
Wilcoxon/Mann-Whitney test	29.77**	28.31**	23.91**	21.35**	17.78**	13.06**	11.84**
Brown-Forsythe test	615.44**	558.41**	859.66**	869.71**	629.18**	433.87**	431.19**
<i>Panel B: Tests of equality of medians and variances for EIRVIXs in first log-differences</i>							
Wilcoxon/Mann-Whitney test	1.38	1.08	0.59	1.27	0.61	0.29	0.56
Brown-Forsythe test	46.90**	122.37**	136.19**	83.82**	19.52**	1.63	0.00

Notes: The null hypothesis of the Wilcoxon/Mann-Whitney test is that the medians are equal. The null hypothesis of the Brown-Forsythe test is that the variances are equal. One and two asterisks denote rejection of the null hypothesis at the 5% and 1% significance level, respectively.

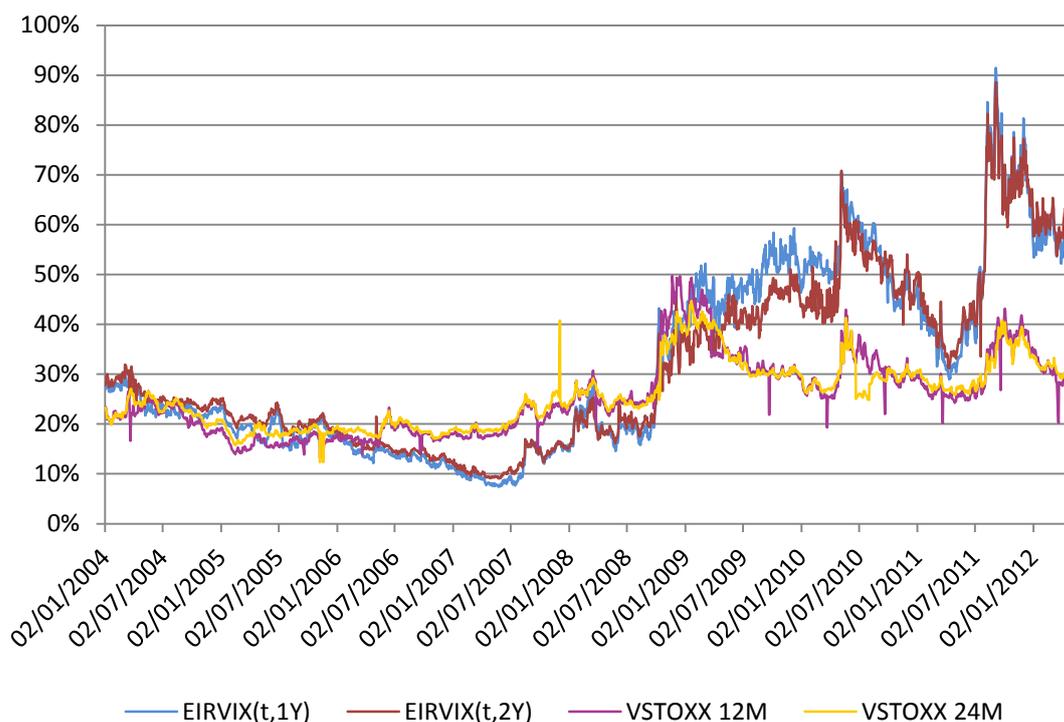
5.2. The role of interest rate and stock return volatility indices as investors' gauges of fear during the financial crisis

In this section we use EIRVIX(t,1Y) and EIRVIX(t,2Y) along with VSTOXX 12M and VSTOXX 24M to track how investors' uncertainty about the future behavior of interest rates and stock returns one and two years ahead changes in response to financial instability during the recent financial crisis.

Figure 2 plots the four mentioned indices across the sample. Similar to EIRVIXs, we can also see an increase in the VSTOXX sub-indices by the summer of 2007. However, the size of the spikes observed in the VSTOXX series along the crisis period is notably smaller than in the case of EIRVIXs. The standard deviation of VSTOXX 12M is 6% (Table 4), whereas it is 17% for EIRVIX(t,1Y). The highest volatility level (49.73%) is reached by VSTOXX 12M on November 21, 2008. Also note that for approximately one year before and after the burst of the crisis, market estimates of future volatility are higher in the equity market than in the fixed-income market. However, by March 2009, VSTOXX 12M and VSTOXX 24M decrease and from that moment until the end of the sample the stock return volatility indices remain below EIRVIXs. Moreover, by April 2012, VSTOXX sub-indices are approximately 30%, whereas the average pre-crisis level of the indices is approximately 20%. Similar findings are documented by Schwert

(2011) for the US stock market based on VIX. He shows that the recent financial crisis has had the second-largest burst of volatility after the market crash in October 1987, although it seems that stock volatility returned to more normal levels fairly quickly.

Figure 2. Daily levels of EIRVIX(t,1Y), EIRVIX(t,2Y), VSTOXX 12M and VSTOXX 24M over the period from January 02, 2004 to April 30, 2012



Two ideas may be inferred from the comparison of stock return and interest rate volatility indices. First, we find that the financial turmoil has had a deeper impact on investors' uncertainty about the future development of interest rates than on stock market returns. Put differently, EIRVIXs have played a greater role as investors' gauge of fear during the recent financial crisis than VSTOXX sub-indices. Second, it is interesting to note that both fixed-income and equity market volatility indices show a quite similar pattern over most of the sample – except from March 2009 to April 2010, when VSTOXX sub-indices start to fall while EIRVIXs keep an upward trend. Based on these findings, next we further investigate two issues.

Table 4. Summary statistics of VSTOXX 12M and VSTOXX 24M in levels and first log-differences across the entire sample (Panel A) and for two subsamples: from January 01, 2004 to July 31, 2007 (Panel B) and from August 01, 2007 to April 30, 2012 (Panel C)

	Levels		First log-differences	
	VSTOXX 12M	VSTOXX 24M	VSTOXX 12M	VSTOXX 24M
<i>Panel A: January 01, 2004 to April 30, 2012</i>				
Observations	2062	2062	2061	2061
Mean	0.25	0.25	0.0001	0.0001
Median	0.24	0.24	-0.0006	-0.0005
Maximum	0.49	0.44	0.35	0.47
Minimum	0.13	0.12	-0.44	-0.49
Std. Deviation	0.07	0.06	0.03	0.02
Skewness	0.77	0.62	-1.57	-0.49
Kurtosis	3.05	2.64	44.95	144.60
Jarque-Bera	204.47 (0.00)	143.82 (0.00)	152020.7 (0.00)	1722094.0 (0.00)
ρ_1	0.99**	0.99**	-0.11**	-0.25**
ADF	-3.29	-2.78	-23.34**	-16.24**
<i>Panel B: January 01, 2004 to July 31, 2007</i>				
Observations	885	885	884	884
Mean	0.18	0.19	0.0000	0.0000
Median	0.17	0.19	-0.0005	-0.0005
Maximum	0.26	0.27	0.29	0.39
Minimum	0.13	0.12	-0.44	-0.40
Std. Deviation	0.02	0.02	0.03	0.02
Skewness	0.79	0.91	-2.27	-0.21
Kurtosis	2.96	3.44	59.84	137.88
Jarque-Bera	93.99 (0.00)	129.49 (0.00)	119794.5 (0.00)	670163.8 (0.00)
<i>Panel C: August 01, 2007 to April 30, 2012</i>				
Observations	1177	1177	1177	1177
Mean	0.30	0.29	0.0002	0.0002
Median	0.29	0.29	-0.0006	-0.0008
Maximum	0.49	0.44	0.35	0.47
Minimum	0.15	0.21	-0.37	-0.49
Std. Deviation	0.06	0.05	0.04	0.02
Skewness	0.85	0.71	-1.30	-0.72
Kurtosis	3.28	2.80	37.76	150.12
Jarque-Bera	146.38 (0.00)	102.24 (0.00)	59616.91 (0.00)	1061587.0 (0.00)

Notes:

^a p -values of the Jarque-Bera test are inside parenthesis.

^b ρ_1 denotes the first-order autocorrelation coefficient. The significance of autocorrelations is tested with the Ljung-Box Q-statistic.

^c The ADF test is performed on the logarithm of the indices. The null hypothesis is that the series contains a unit root. The optimal lag length is determined according to the Schwarz information criterion.

^d One and two asterisks denote statistical significance at the 5% and 1% significance level, respectively.

First, we construct a measure of financial risk aversion as a linear combination of stock return and interest rate volatility indices and we analyze its relationship with the development of Euro area government bond yield spreads during the crisis. Second, we analyze whether there is a transmission of implied volatility across the Eurozone fixed-income and equity markets based on EIRVIXs and VSTOXX sub-indices. The empirical evidence on implied volatility contagion across equity markets has increased in the recent years (see e.g., Nikkinen and Sahlström, 2004; Äijö, 2008; Konstantinidi et al., 2008; Kumar, 2012; Siriopoulos and Fassas, 2012), whereas the correlation between stock return and interest rate volatility indices remains unexplored. These results have implications for portfolio management.

5.2.1. Financial risk aversion and Euro area government yield spreads

Given the investors' gauge of fear role of volatility indices, VIX and VSTOXX have been extensively used as proxies for investors' risk aversion in recent studies on Euro area government yield spreads (see e.g., Barrios et al., 2009; Beber et al., 2009; Gerlach et al., 2010; Arghyrou and Kontonikas, 2011; Oliveira et al., 2012). Particularly, Arghyrou and Kontonikas (2011) and Oliveira et al. (2012) show that the coefficients of VIX and VSTOXX, respectively, are positive and statistically significant for explaining the widening of sovereign yield spreads in the present crisis. The rationale behind these findings can be found in De Santis (2012). Higher risk aversion during the crisis has increased the demand for German bonds for safety reasons, while the yields of the bonds with higher default risk has increased. Based on the higher perception of uncertainty in the fixed-income markets over most of the crisis period, we expect that short-term EIRVIXs contains significant information about the development of Euro area government yield spreads.

The previous issue is investigated using principal component analysis. Principal component analysis allows us to decompose the behavior of a set of variables closely related into orthogonal components, the first of which captures the common factor behind the behavior of the variables. Specifically, each component is derived as a linear combination of the original data in such a way that they are in descending order of contribution for explaining the total variability of the set of variables. Thus, we use this method to analyze the relationship between the common sovereign risk factor derived from a series of Euro area government bond yield spreads and a measure of global

financial risk aversion obtained from stock return and interest rate volatility indices during the crisis period (from August 01, 2007 to April 30, 2012). See Barrios et al. (2009) for a related approach.

In particular, the common sovereign risk factor is the first principal component of the yield spreads between ten-year Euro area government bonds and the benchmark German bond. Daily data have been obtained from Reuters for the following list of countries: Belgium, Finland, France, Greece, Ireland, Italy, the Netherlands, Portugal and Spain.¹³ With respect to the financial risk aversion factor, this is the first principal component of a set of four volatility indices: two stock return volatility indices (VIX and VSTOXX) and two interest rate volatility indices (MOVE and EIRVIX(t,1Y). MOVE is computed by Merrill Lynch as a weighted average of the implied volatilities of one-month ATM options on the two-year, five-year, ten-year and 30-year US Treasury securities. The data for VIX has been collected from the website of the Chicago Board of Options Exchange (CBOE), whereas the data for MOVE has been obtained from Bloomberg.

Prior to analysis, each series is standardized to have zero mean and unit variance by subtracting the mean and dividing by the standard deviation in each case. Panel A (Panel B) in Table 5 shows the weights of each yield spread (volatility index) for each of the nine (four) principal components. The first principal component of the yield spread series explains 74% of the variation of spreads, whereas the first principal component of the volatility indices captures 54% of the total variation in the indices. The loadings on each yield spread making up the common sovereign risk factor are quite similar across countries, although the weights of Finland and the Netherlands are lower. The loadings on the four volatility indices for the risk aversion factor are all positive and show that US volatility indices contribute to the common factor to a greater extent than Euro area volatility indices. The second component, however, which explains 37% of the variability of volatility indices, only places positive weights on Euro area volatility indices. This result seems to highlight the higher and more prolonged risk aversion within the Eurozone as the financial crisis deepens in comparison with the US.

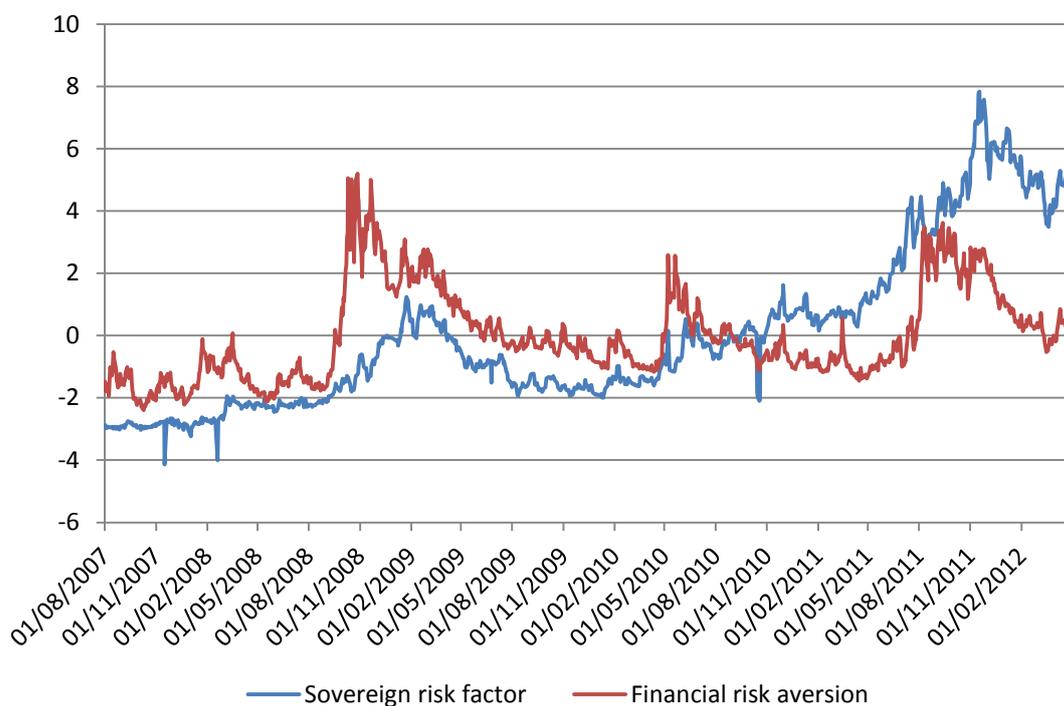
¹³ The remaining countries are not included because of data availability or because they joined the euro after August 2007. In particular, Austria is excluded from the sample because of missing data from August to the end of September 2007.

As displayed in Figure 3, global financial risk aversion and Euro area government bond yield spreads move quite close from 2009 up to mid 2010. In particular, the generalized drop in our measure of investors' risk aversion translates into lower sovereign risk. However, the behavior of the two variables notably diverges since the summer of 2011, when the sovereign risk factor sharply increases while the perceived risk in financial markets is reduced. During this period, Greece's debt crisis threatened to spread to the bigger economies of Italy and Spain. In this respect, Oliveira et al. (2012) highlight the significant roles that macroeconomic country-specific fundamentals such as the level of public debt and the current account deficit have played for explaining the rise in sovereign risk during the crisis.

Table 5. Weights of yield spreads and volatility indices for each principal component (Panels A and B, respectively). Principal components are in descending order of contribution for explaining the total variability of the set of variables. The data extends from August 01, 2007 to April 30, 2012

	Component number								
<i>Panel A: Weights of yield spreads for each principal component</i>									
	1	2	3	4	5	6	7	8	9
Belgium	0.380	0.013	-0.108	0.119	-0.264	-0.571	-0.081	-0.023	-0.652
Finland	0.205	0.633	-0.188	0.631	0.266	0.145	0.164	-0.040	0.025
France	0.367	0.094	-0.380	-0.136	-0.226	0.380	-0.654	0.247	0.085
Greece	0.362	-0.188	-0.207	-0.270	0.553	-0.025	0.340	0.537	-0.071
Ireland	0.321	-0.210	0.703	0.373	0.119	-0.085	-0.304	0.260	0.190
Italy	0.378	-0.037	-0.218	-0.048	-0.202	-0.443	0.171	-0.204	0.704
Netherlands	0.164	0.668	0.437	-0.574	-0.022	-0.062	0.026	0.006	-0.007
Portugal	0.368	-0.197	0.044	-0.142	0.430	0.203	-0.142	-0.733	-0.131
Spain	0.370	-0.153	0.171	0.048	-0.510	0.506	0.527	-0.013	-0.110
<i>Panel B: Weights of volatility indices for each principal component</i>									
	1	2	3	4					
EIRVIX (t,1Y)	0.353	0.621	0.692	0.092					
MOVE	0.555	-0.460	0.037	0.691					
VIX	0.621	-0.316	0.063	-0.713					
VSTOXX	0.424	0.548	-0.717	0.062					

Figure 3. Financial risk aversion and sovereign risk indicators from August 01, 2007 to April 30, 2012



5.2.2. Spillover effects

Volatility contagion across markets has important implications for portfolio choice and risk management because it affects option prices and hedge ratios. Moreover, if correlations across markets rise in periods of stress, then diversification benefits will be reduced. In this section, we analyze whether there is a transmission of implied volatility across the Eurozone fixed-income and equity markets based on EIRVIXs and VSTOXX sub-indices.

Table 6 shows the cross-correlations between weekly log-changes in the four indices for the entire sample (Panel A) as well as for the pre-crisis and crisis periods (Panels B and C, respectively). We find that there is a statistically significant positive correlation between changes in interest rate and stock return volatility indices over the whole sample, although this is stronger during the crisis period - the highest correlation is obtained for EIRVIX(t,1Y) and VSTOXX 12M (31%). Diebold and Yilmaz (2009) also find that volatility spillovers across stock markets become more pronounced during periods of crises. Findings have implications for option portfolio managers because they suggest a positive relationship between the change in the price of caps (floors) and the change in the price of Dow Jones EURO STOXX 50 options.

Next, we test whether there is a Granger causality relationship between changes in EIRVIX(t,1Y) and VSTOXX 12M. To this end, we estimate the following bivariate vector autoregressive (VAR) model:

$$\Delta EIRVIX(t,1Y) = a + \sum_{l=1}^k b_l \Delta EIRVIX(t-l,1Y-l) + \sum_{l=1}^k c_l \Delta VSTOXX12M_{t-l} + u_t, \quad (14)$$

$$\Delta VSTOXX12M_t = a + \sum_{l=1}^k b_l \Delta VSTOXX12M_{t-l} + \sum_{l=1}^k c_l \Delta EIRVIX(t-l,1Y-l) + u_t, \quad (15)$$

where $\Delta EIRVIX(t,1Y)$ and $\Delta VSTOXX12M_t$ stand for the first log-differences of EIRVIX(t,1Y) and VSTOXX 12M, respectively, and k denotes the number of lags used in the regression. The null hypothesis is that the c_l in Equations (14) and (15) are jointly zero. Thus, VSTOXX changes Granger-cause EIRVIX changes if lagged VSTOXX changes contain information that is not already contained in the past values of EIRVIX. Granger causality running from EIRVIX to VSTOXX can be defined in a similar way.

Table 6. Cross-correlations between weekly log-changes in EIRVIX(t,1Y), EIRVIX(t,2Y), VSTOXX 12M and VSTOXX 24M across the entire sample (Panel A) and for two subsamples: from January 02, 2004 to July 31, 2007 (Panel B) and from August 01, 2007 to April 30, 2012 (Panel C)

Cross-correlations				
<i>Panel A: January 02, 2004 to April 30, 2012</i>				
	EIRVIX(t,1Y)	EIRVIX(t,2Y)	VSTOXX 12M	VSTOXX 24M
EIRVIX(t,1Y)	1	0.85**	0.25**	0.19**
EIRVIX(t,2Y)		1	0.25**	0.15**
VSTOXX 12M			1	0.62**
VSTOXX 24M				1
<i>Panel B: January 02, 2004 to July 31, 2007</i>				
	EIRVIX(t,1Y)	EIRVIX(t,2Y)	VSTOXX 12M	VSTOXX 24M
EIRVIX(t,1Y)	1	0.83**	0.06*	0.06*
EIRVIX(t,2Y)		1	0.10**	0.07*
VSTOXX 12M			1	0.68**
VSTOXX 24M				1
<i>Panel C: August 01, 2007 to April 30, 2012</i>				
	EIRVIX(t,1Y)	EIRVIX(t,2Y)	VSTOXX 12M	VSTOXX 24M
EIRVIX(t,1Y)	1	0.86**	0.31**	0.23**
EIRVIX(t,2Y)		1	0.29**	0.18**
VSTOXX 12M			1	0.59**
VSTOXX 24M				1

Note: One and two asterisks denote statistical significance at the 5% and 1% significance level, respectively.

Table 7 reports the Chi-squared statistics and p -values from the VAR Granger Causality/Block Exogeneity Wald tests performed on a VAR(2) system. This is the lag length suggested by the Schwarz's information criterion. The absence of autocorrelation in the residual terms suggests that no longer lags for the variables are needed. Results show evidence of Granger causality running from the fixed-income to the equity market, whereas the reverse does not hold. This implies that the changes in EIRVIX($t,1Y$) might be used by portfolio managers to improve VSTOXX 12M forecasts.

Table 7. VAR Granger Causality/Block Exogeneity Wald tests between EIRVIX($t,1Y$) and VSTOXX 12M based on a VAR(2) model

Null hypothesis	Chi-sq statistic	Probability
$\Delta EIRVIX(t,1Y)$ does not Granger cause $\Delta VSTOXX$ 12M	18.09	0.00
$\Delta VSTOXX$ 12M does not Granger cause $\Delta EIRVIX(t,1Y)$	2.97	0.22

Note: $\Delta EIRVIX(t,1Y)$ and $\Delta VSTOXX12M_t$ stand for the first log-differences of EIRVIX($t,1Y$) and VSTOXX 12M, respectively.

To the best of our knowledge, this is the first study that examines whether there is a spillover of implied volatility between stock return and interest rate volatility indices within the same area. Thus, it contributes to the existing literature on implied volatility spillovers across equity markets (see e.g., Nikkinen and Sahlström, 2004; Äijö, 2008; Konstantinidi et al., 2008; Kumar, 2012; Siriopoulos and Fassas, 2012).

6. Summary and conclusions

We suggest for the first time a methodology for the construction of a set of interest rate volatility indices for the Eurozone (EIRVIXs) based on the implied volatility quotes of one of the most liquid fixed-income derivatives: caps (floors). These indices reflect the market estimate of the volatility of three- and six-month tenor forward rates over different fixed horizons – one, two, three, four, five, seven and ten years.

EIRVIXs are constructed through a two-step process. First, we apply a stripping procedure consisting of recovering the implied volatilities of the individual caplets (floorlets) that compose caps (floors), as these are the contracts that do have an underlying specific forward rate. Second, implied volatilities of near-the-money caplets

(floorlets) are linearly interpolated. Thus, each $EIRVIX(t, T_i)$ reflects the implied volatility of a theoretical ATM caplet (floorlet) with a constant time to maturity, from t to T_i . The ATM implied volatility with expiry at time T has a specific theoretical interpretation: it approximates the volatility swap rate (i.e., the conditional risk-neutral expectation of the future realized volatility of the underlying asset over the period $[t, T]$).

Volatility indices in the equity markets are referred to as investors' gauges of fear because they usually spike in periods of financial turmoil. In this paper, we extend the empirical evidence by analyzing the effect of the recent financial crisis on short- and long-term EIRVIXs. We find that the crisis has had a deep and lasting effect on investors' short-term expectations of volatility in the fixed-income market – by April 2012, volatility levels are more than three-fold the average pre-crisis value. More interestingly, we also find that as the crisis deepened, it also eventually affected expectations of volatility five- and ten-years ahead – the indices initiate an upward trend in 2010.

The first finding seems to support the interpretation of EIRVIX as investors' gauge of fear for the fixed-income market, whereas the second one might be interpreted as a signal that investors foresee long periods of turbulence in interest rate markets.

In addition, we compare the effect of the financial turmoil on investors' expectations of volatility of interest rates and stock returns over the next one and two years by using two equity market volatility indices, VSTOXX 12M and VSTOXX 24M. We observe that fixed-income and equity market volatility indices depict a quite similar pattern over most of the sample. However, VSTOXX sub-indices exhibit a lower rise than one- and two-year EIRVIXs during the crisis period and the size of the spikes is also smaller. Moreover, by April 2012, VSTOXX sub-indices are approximately 30%, whereas the average pre-crisis level of the indices is approximately 20%. This finding suggests that EIRVIXs have played a greater role as investors' gauge of fear during the recent financial crisis than VSTOXX sub-indices.

Based on these findings, we construct a measure of global financial risk aversion as a linear combination of interest rate and stock return volatility indices and we show that it moves closely together with Euro area government yield spreads from 2009 up to mid 2010. Finally, we show that changes in short-term EIRVIXs and VSTOXX sub-indices

are positively correlated, and that there is an implied volatility spillover from the fixed-income to the equity market.

Appendix

The stripping procedure involves using interpolation/extrapolation techniques to obtain the flat volatilities of caps with a maturity different from those quoted (i.e., non-annual maturity and missing annual maturity quotes) for a particular strike. One can expect to obtain a smoother fitting of the term structure of flat volatilities by using cubic spline interpolation instead of linear interpolation. However, we need to address the fact that the tenor of the underlying forward rates of the caplets that compose caps is not the same for all maturity caps (i.e., all maturity caps do not have the same structure); hence, we cannot apply cubic spline interpolation along the whole term structure.

In particular, one- and two-year caps have a shorter tenor (three months) than the rest of the caps. In this case, we extrapolate the flat volatility of the two- (one-) year cap when the required maturities are one year or two years and three months (two years). For the one year and three months maturity, we linearly interpolate between the one- and two-year maturities or extrapolate when there is one missing quote.

Then, cubic spline interpolation is applied over volatility quotes of caps with maturities of three to ten years plus 12, 15 and 20 years (i.e., the maximum number of flat volatility quotes available for a particular strike is 11). When the number of flat volatility quotes is less than six, we propose simple linear interpolation/extrapolation.

When applying cubic spline interpolation, we distinguish two possibilities. If the number of observations is greater than nine, we use two intermediate knots; otherwise, we use a single knot. Knots are positioned in such a way that the observations are uniformly distributed between knots. In particular, the position of the knots is set as follows.

Let N denote the number of available flat volatilities for a particular strike and t_1 and t_2 denote the positions of the knots. Then, we have the following:

If $N = 11$, $t_1 = 5.5$ and $t_2 = 9.5$.

If $N = 10$, t_1 is positioned at the midpoint between the third and fourth observations and t_2 at the midpoint between the seventh and eighth observations.

If $N = 9$ or $N = 8$, the unique knot is settled at the midpoint between the fourth and fifth observations.

If $N = 7$ or $N = 6$, the knot is positioned at the midpoint between the third and fourth observations.

When N is less than 6, we apply linear interpolation. For those caps with maturities out of the range of available maturities, we proceed to extrapolate. Let us denote the flat volatilities of the caps with the shortest and greatest terms to maturity by $\sigma_{Min,flat}^K$ and $\sigma_{Max,flat}^K$, respectively. Then, for caps maturing before the first available cap, we assume that flat volatilities are equal to $\sigma_{Min,flat}^K$. For caps with maturity greater than the last available cap, we assume that flat volatilities are equal to $\sigma_{Max,flat}^K$.

To give a hint of the completeness of the sample for maturity caps ranging from three to ten years plus 12, 15 and 20 years, Table 8 shows the proportion of flat volatilities available corresponding to a single day and a given strike during the whole sample. As it is considered desirable in 97% of cases, the sample is complete (eleven observations).

Table 8. Frequency of available flat volatility quotes for maturity caps ranging from three to ten years plus 12, 15 and 20 years along the sample

Number of available flat volatilities	Frequency
N = 11	0.9719
N = 10	0.0178
N = 9	0.0048
N = 8	0.0030
N = 7	0.0012
N = 6	0.0006
N = 5	0.0004
N = 4	0
N = 3	0
N = 2	0
N = 1	0

References

- Äijö, J., 2008. Implied volatility term structure linkages between VDAX, VSMI and VSTOXX volatility indices. *Global Finance Journal*. 18, 290-302. 10.1016/j.gfj.2006.11.003.
- Arghyrou, M., Kontonikas, A., 2011. The EMU sovereign-debt crisis: fundamentals, expectations and contagion. *European Commission-Economic Papers* n. 436.
- Banerjee, P. S., Doran, J. S., Peterson, D. R., 2007. Implied volatility and future portfolio returns. *Journal of Banking & Finance*. 31, 3183-3199. <http://dx.doi.org/10.1016/j.jbankfin.2006.12.007>.
- Barrios, S., Iversen, P., Lewandowska, M., Setzer, R., 2009. Determinants of intra-Euro area government bond spreads during the financial crisis. *European Economy, Economic Papers* 388.
- Beber, A., Brandt, M., Kavajecz, K., 2009. Flight-to-Quality or Flight-to-Liquidity? Evidence from the Euro-Area Bond Market. *The Review of Financial Studies*. 22, 925-957.
- Becker, R., Clements, A. E., White, S. I., 2007. Does implied volatility provide any information beyond that captured in model-based volatility forecasts? *Journal of Banking & Finance*. 31, 2535-2549. <http://dx.doi.org/10.1016/j.jbankfin.2006.11.013>.
- Black, F., 1976. The pricing of commodity contracts. *Journal of Financial Economics*. 3, 167-179. 10.1016/0304-405x(76)90024-6.
- Bluhm, H., Yu, J., 2001. Working paper, University of Auckland.
- Brigo, D., Mercurio, F., 2006. *Interest rate models : theory and practice : with smile, inflation, and credit*, 2nd ed. Springer, Berlin; New York.
- Carr, P., Lee, R., 2003. At-the-Money Implied as a Robust Approximation of the Volatility Swap Rate. Working paper, New York University.
- Carr, P., Lee, R., 2009. Volatility derivatives. *Annual Review of Financial Economics*. 1, 319-339. 10.1146/annurev.financial.050808.114304.
- De Santis, R., 2012. The Euro Area sovereign debt crisis. Safe heaven, credit rating agencies and the spread. *European Central Bank Working Paper Series* n. 1419.
- Demeterfi, K., Derman, E., Kamal, M., Zou, J., 1999. More than you ever wanted to know about volatility swaps. *Goldman Sachs Quantitative Strategies Research Notes*.
- Díaz, A., Meneu, V., Navarro, E., 2009. International evidence on alternative models of the term structure of volatilities. *Journal of Futures Markets*. 29, 653-683. 10.1002/fut.20377.
- Diebold, F. X., Yilmaz, K., 2009. Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets. *The Economic Journal*. 119, 158-171. 10.1111/j.1468-0297.2008.02208.x.
- Dotsis, G., Psychoyios, D., Skiadopoulos, G., 2007. An empirical comparison of continuous-time models of implied volatility indices. *Journal of Banking and Finance*. 31, 3584-3603. 10.1016/j.jbankfin.2007.01.011.
- Duarte, J., Longstaff, F. A., Yu, F., 2005. Risk and Return in Fixed-income Arbitrage: Nickels in front of a streamroller? *Anderson Graduate School of Management, UCLA*.
- Fleming, J., Ost diek, B., Whaley, R. E., 1995. Predicting stock market volatility: A new measure. *Journal of Futures Markets*. 15, 265-302. 10.1002/fut.3990150303.
- Fleming, M., Klagge, N., 2010. The Federal Reserve's Foreign Exchange Swap Lines. *Federal Bank of New York Current Issues in Economics and Finance*. 16, 1-7.
- Gerlach, S., Schulz, A., Wolff, G., 2010. Banking and Sovereign Risk in the Euro Area. *Centre for Economic Policy Research Discussion Paper* n. 7833.
- Giot, P., 2005a. Implied Volatility Indexes and Daily Value at Risk Models. *The Journal of Derivatives*. 12, 54-64.

- Giot, P., 2005b. Relationships Between Implied Volatility Indexes and Stock Index Returns. Are implied volatility indexes leading indicators? . *Journal of Portfolio Management*. 31, 92-100.
- Hernández, L. G., 2005. Pricing of Game Options in a market with stochastic interest rates. Thesis. School of Mathematics Georgia Institute of Technology (<http://etd.gatech.edu/theses/available/etd-03152005-044640/>).
- Hull, J., 2009. *Options, futures and other derivatives*, seventh ed. Prentice Hall, Upper Saddle River, NJ.
- Jarrow, R., Li, H., Zhao, F., 2007. Interest Rate Caps “Smile” Too! But Can the LIBOR Market Models Capture the Smile? *Journal of Finance*. 62, 345-382. 10.1111/j.1540-6261.2007.01209.x.
- Konstantinidi, E., Skiadopoulos, G., Tzagkaraki, E., 2008. Can the evolution of implied volatility be forecasted? Evidence from European and US implied volatility indices. *Journal of Banking and Finance*. 32, 2401-2411. 10.1016/j.jbankfin.2008.02.003.
- Kumar, S. S. S., 2012. A first look at the properties of India’s volatility index. *Journal of Emerging Markets*. 7, 160-176.
- Li, H., Zhao, F., 2009. Nonparametric Estimation of State-Price Densities Implicit in Interest Rate Cap Prices. *Review of Financial Studies*. 22, 4336-4376.
- Morau, F., Navatte, P., Villa, C., 1999. The Predictive Power of the French Market Volatility Index: A Multi Horizons Study. *European Finance Review*. 2, 303-320.
- Nikkinen, J., Sahlström, P., 2004. International transmission of uncertainty implicit in stock index option prices. *Global Finance Journal*. 15, 1-15. <http://dx.doi.org/10.1016/j.gfj.2003.10.007>.
- Oliveira, L., Curto, J. D., Nunes, J. P., 2012. The determinants of sovereign credit spread changes in the Euro-zone. *Journal of International Financial Markets, Institutions and Money*. 22, 278-304. <http://dx.doi.org/10.1016/j.intfin.2011.09.007>.
- Poon, S.-H., Granger, C. W. J., 2003. Forecasting Volatility in Financial Markets: A Review. *Journal of Economic Literature*. 41, 478-539.
- Schwert, G. W., 2011. Stock volatility during the recent financial crisis. *European Financial Management*. 17, 789-805.
- Siriopoulos, C., Fassas, A., 2012. An investor sentiment barometer – Greek implied volatility index (GRIV). *Global Finance Journal*. DOI: 10.1016/j.gfj.2012.03.001.
- Taylor, J., Williams, J., 2009. A Black Swan in the Money Market. *American Economic Journal: Macroeconomics*. 1, 58-83.
- Whaley, R. E., 2000. The Investor Fear Gauge. *Journal of Portfolio Management*. 26, 12-17.
- Whaley, R. E., 2009. Understanding the VIX. *Journal of Portfolio Management*. 35, 98-105.
- Zivot, E., Andrews, D. W. K., 2002. Further Evidence on the Great Crash, the Oil-Price Shock, and the Unit-Root Hypothesis. *Journal of Business and Economic Statistics*. 20, 25.