Insider trading, stochastic liquidity, and equilibrium prices

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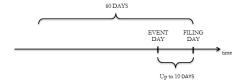
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DO MEASURES OF STOCK LIQUIDITY REVEAL THE PRESENCE OF INFORMED TRADERS?

- Measures of trading liquidity should be informative about the presence of adverse selection (Glosten and Milgrom, 1985; Kyle, 1985; Easley and O'Hara, 1987)
- For example, Kyle (1985) proposes seminal model of insider trading:
 - ullet Insider knows terminal value of the firm that will be revealed to all at T.
 - Market maker absorbs total order flow (informed + noise) at price set to break even.
 - ⇒ Insider trades proportionally to undervaluation and inversely to time and price impact.
 - ⇒ In equilibrium, price responds to order flow linearly.
 - \Rightarrow Price impact (Kyle's λ) should be higher for stocks with more severe adverse selection
 - ⇒ Price volatility is constant and independent of noise trading volatility.
- Several empirical measures of adverse selection proposed in the literature. (e.g., Glosten, 1987; Glosten and Harris, 1988; Hasbrouck, 1991)
- Question: how well do these measures perform at picking up the presence of informed trading?

EMPIRICAL MOTIVATION

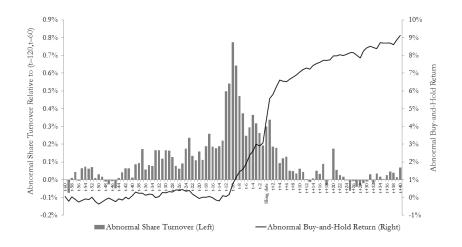
In recent paper 'Do prices reveal the presence of informed trading?,' we collect data on informed trades from Schedule 13D filings — Rule 13d-1(a) of the 1934 Securities Exchange Act that requires the filer to "... describe any transactions in the class of securities reported on that were effected during past 60 days..."



Find that:

- Trades executed by Schedule 13D filers are informed:
 - Announcement returns
 - Profits of Schedule 13D filers
- Measures of adverse selection are lower on days with informed trading

Buy-and-Hold Abnormal Return



Two month excess return is around 9%

DO INFORMED TRADES MOVE STOCK PRICES?

	days with informed trading (1)	days with no informed trading (2)	difference (3)	t-stat (4)
excess return	0.0064	-0.0004	0.0068***	9.94
turnover	0.0191	0.0077	0.0115***	21.67

IS ADVERSE SELECTION HIGHER WHEN INFORMED TRADE?

	(t-60,t-1)	(t-420,t-361)	diff				
Adverse Selection Measures							
$\lambda * 10^6$	19.0011	22.3285	-3.3274*** [-3.36]				
pimpact	0.00659	0.00664	-0.00005 [-0.21]				
cumir	0.0015	0.0017	-0.0002** [-2.16]				
trade — related	0.0691	0.0686	0.0005 [0.24]				
illiquidity	0.4611	0.5025	-0.0413*** [-4.12]				
pin	0.4385	0.4943	-0.0559*** [-13.1]				
Other Liquidity N	∕leasures		. ,				
rspread	0.0095	0.0109	-0.0014*** [-4.69]				
espread	0.0162	0.0175	-0.0012*** [-2.99]				
baspread	0.0219	0.0239	-0.0020*** [-4.85]				

IS ADVERSE SELECTION HIGHER WHEN INFORMED TRADE?

	days with informed trading (1)	days with no informed trading (2)	difference (3)			
Adverse Selectio	n Measures					
$\lambda*10^6$	14.3311	20.1644	-5.8334***			
pimpact	0.0060	0.0064	[-8.38] -0.0004** [-2.18]			
cumir	0.0013	0.0015	-0.0002**			
trade — related	0.0654	0.0673	[-2.06] -0.0019 [-0.99]			
Other Liquidity Measures						
rspread	0.0081	0.0089	-0.0008***			
espread	0.0145	0.0155	[-3.43] -0.001*** [-3.25]			

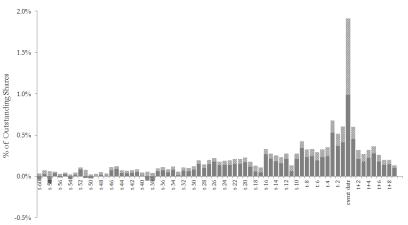
Summary of Empirical Paper

- Schedule 13D filers have valuable information when they purchase shares of targeted companies
- Thus, the information asymmetry is high when Schedule 13D filers purchase shares
- We find that excess return and turnover are higher when insiders trade, which seems to indicate that they have price impact
- However, we find that measures of information asymmetry and liquidity indicate that stocks are more liquid when informed trades take place
- This evidence seems at odds with our intuition and common usage in empirical literature.
 - Biais, Glosten, and Spatt (2005): "As the informational motivation of trades becomes relatively more important, price impact goes up. [page 232]"

THE MECHANISM

- Why do traditional microstructure measures of informed trading fail to capture Schedule 13D trading activity?
 - Activists trade on days with high liquidity ("select when to trade")
 - Activists' trades generate endogenous liquidity ('latent liquidity', or Cornell and Sirri's (1992) 'falsely informed traders').
 - Activists use limit orders ("select how to trade")
- Find evidence for selection (when to trade):
 - \bullet Aggregate S&P 500 volume (+) and return (-) forecasts trading by insiders.
 - Abnormally high volume when they trade.
- Find evidence for use of limit orders:
 - Subset of uniquely matched trades in TAQ show that activist trades often classified as sells by Lee-Ready algorithm (especially during pre-event date).

ABNORMAL SHARE TURNOVER - REVISITED



- 38 Average Percentage of Outstanding Shares Purchased by Schedule 13-D Filers
- ■Unexplained Abnormal Volume as Percentage of Outstanding Shares

THEORETICAL CONTRIBUTION

- We extend Kyle's (insider trading) model:
 - general noise trading volatility process.
 - possibly random horizon with general intensity.
- Main results:
 - Equilibrium price may exhibit endogenous 'excess' stochastic volatility.
 - Price impact (Kyle's lambda) is stochastic: lower (higher) when noise trading volatility increases (decreases) and path-dependent.
 - Market depth is a martingale when horizon is fixed: price impact is a submartingale expected to increase over time to reflect liquidity timing option.
 - When the horizon is random, market depth is a sub-martingale expected to grow at arrival intensity.
 - Informed trade more aggressively when noise trading volatility is higher and price impact is lower.
 - More information makes its way into prices when noise trading volatility is higher: volume and volatility are positively related.
 - Time series averaged lambda is a poor measure of total execution costs paid by uninformed investors.

Related Literature

- Kyle (1985), Back (1992)
- Admati-Pfleiderer (1988)
- Foster-Viswanathan (1990), (1993)
- Back-Pedersen (1998)
- Hong-Rady (2002)
- Madhavan, Richardson and Roomans (1998)
- Clark (1973), Tauchen and Pitts (1983), Gallant, Rossi and Tauchen (1992), Lamoureux and Lastrapes (1990)

Insider

- We follow Back (1992) and develop a continuous time version of Kyle (1985)
- Risk-neutral insider's maximization problem:

$$\max_{\theta_t} \mathbb{E}\left[\int_0^T (v - P_t) \theta_t dt \, | \mathcal{F}_t^Y, v \right] \tag{1}$$

- \bullet As in Kyle, we assume there is an insider trading in the stock with perfect knowledge of the terminal value υ
- It is optimal for the insider to follow absolutely continuous trading strategy (Back, 1992).
- Can extend to T unpredictable stopping time with intensity ρ_t .

Market Maker

- The market maker is also risk-neutral, but does not observe the terminal value. Instead, he has a prior that the value v is normally distributed $N(\mu_0, \Sigma_0)$
- The market maker only observes the total order flow (prior to T):

$$dY_t = \underbrace{\theta_t dt}_{informed} + \underbrace{\sigma_t dZ_t}_{uninformed}$$

$$\underbrace{\sigmarder \ flow}_{order \ flow} + \underbrace{\sigma_t dZ_t}_{uninformed}$$

$$(2)$$

• where σ_t is the stochastic volatility of the uninformed order flow:

$$\frac{d\sigma_t}{\sigma_t} = m(t, \sigma^t)dt + \nu(t, \sigma^t)dM_t$$

and M_t is orthogonal (possibly discontinuous) martingale.

Since the market maker is risk-neutral, equilibrium imposes that

$$P_t = \mathbf{1}_{\{t < T\}} \mathbf{E} \left[v \,|\, \mathcal{F}_t^{\mathsf{Y}} \right] + v \mathbf{1}_{\{t \ge T\}} \tag{3}$$

• We assume that the market maker and the informed investor observe σ_t .

Preview of Results

- Can we simply 'paste' together Kyle economies with different noise-trading volatilities?
- No! The insider will optimally choose to trade less in the lower liquidity states, because he anticipates the future opportunity to trade more when liquidity is higher.
- Of course, the market makers foresee this and adjust prices accordingly...
- Equilibrium ...?

SOLVING FOR EQUILIBRIUM

• First, we conjecture a trading rule for the insider:

$$\theta_t = \beta_t (\upsilon - P_t)$$

 Second, derive the dynamics of the stock price consistent with the market maker's filtering rule, conditional on a conjectured trading rule of the insider

$$dP_t = \lambda_t dY_t$$

- Then we solve the insider's optimal portfolio choice problem, given the assumed dynamics of the equilibrium price
- Finally, find fixed point, such that conjectured rule by the market maker is consistent with the insider's optimal choice
- Difficulty: λ_t, β_t are stochastic processes!

Equilibrium with fixed horizon T

Price impact is stochastic:

$$\lambda_t = \sqrt{\frac{\Sigma_t}{G_t}} \tag{4}$$

ullet where Σ_t is remaining amount of private information

$$\Sigma_t = \mathrm{E}\left[(\upsilon - P_t)^2 \,|\, \mathcal{F}_t^Y \right] = \Sigma_0 e^{-\int_0^t \frac{\sigma_s^2}{G_s} ds} \tag{5}$$

and G_t solves recursive equation

$$\sqrt{G_t} = E\left[\int_t^T \frac{\sigma_s^2}{2\sqrt{G_s}} ds \,|\, \mathcal{F}_t^M\right] \tag{6}$$

Optimal strategy of insider is:

$$\theta_t = \frac{1}{\lambda_t} \frac{\sigma_t^2}{G_t} (v - P_t) \tag{7}$$

Equilibrium stock price process:

$$dP_t = \underbrace{\frac{\sigma_t^2}{G_t}}(v - P_t)dt + \sqrt{\frac{\Sigma_t}{G_t}}\sigma_t dZ_t$$
 (8)

Properties of the G_t solution

- \bullet G_t is the crucial quantity of expected noise to characterize equilibrium:
 - If $\underline{\sigma} \leq \sigma_t \leq \overline{\sigma}$ then there exists a bounded solution such that:

$$\underline{\sigma}^{2}\left(T-t\right)\leq G_{t}\leq \overline{\sigma}^{2}\left(T-t\right)$$

If there exists a bounded solution then it is unique and:

$$G_t = \mathrm{E}_t \left[\int_t^T \sigma_s^2 ds - \int_t^T \Sigma_s d \left[\frac{1}{\lambda} \right]_s \right]$$

- Thus $\textit{G}_t = \mathrm{E}_t \left[\int_t^T \sigma_s^2 ds \right]$ iff λ_t is deterministic, else <.
- For several special cases we can construct an explicit solution and characterize equilibrium further:
 - σ_t general martingale.
 - σ_t deterministic growth rate (m_t) .
 - σ_t continuous time Markov Chain.

Properties of equilibrium with fixed horizon

- $\lim_{t\to T} P_t = v$ a.s. 'bridge' property of price in insider's filtration.
- Market depth $(1/\lambda_t)$ is martingale.
- Price impact (λ_t) is a submartingale (increases on average due to liquidity timing option).
- $d\Sigma_t = -dP_t^2$ (stock price variance is high when information gets into prices faster, which occurs when noise trader volatility is high).
- Total profits of the insider are equal to $\sqrt{\Sigma_0 G_0}$.
- Realized execution costs of uninformed can be computed pathwise as

$$\int_0^T (P_{t+dt} - P_t) \sigma_t dz_t = \int_0^T \lambda_t \sigma_t^2 dt$$

 Unconditionally, expected aggregate execution costs of uninformed equal insider's profits (but not path by path).

Equilibrium with Random Horizon T

- ullet With random horizon T with intensity ho_t (adapted to \mathcal{F}_t^M) the main changes are:
- G_t solves recursive equation:

$$\sqrt{G_t} = E\left[\int_t^\infty \frac{e^{-\int_t^s \rho_u du} \sigma_s^2}{2\sqrt{G_s}} ds \,|\, \mathcal{F}_t^M\right] \tag{9}$$

- If σ_t and ρ_t are uniformly bounded then a bounded solution exists.
- If a bounded solution exists it is unique.
- Market depth is a super martingale:

$$\mathrm{E}_t[d\frac{1}{\lambda_t}] = \frac{\rho_t}{\lambda_t} dt$$

- Expected change in price impact balances liquidity timing option with random termination risk.
- Σ_t is a strictly decreasing process with $\lim_{t \to \infty} \Sigma_t = 0$.
- Prices are never fully revealing: there is an announcement jump at $T < \infty$.

General martingale dynamics (fixed T)

Suppose uninformed order flow volatility is unpredictable (a martingale):

$$\frac{d\sigma_t}{\sigma_t} = \nu(t, \sigma^t) dM_t, \tag{10}$$

- Then can solve $G(t) = \sigma_t^2(T t) = \int_t^T \mathrm{E}[\sigma_s]^2 ds \leq \mathrm{E}[\int_t^T \sigma_s^2 ds]$,
- Price impact is: $\lambda_t = \frac{\sigma_v}{\sigma_t}$ where $\sigma_v^2 = \frac{\Sigma_0}{T}$ is the annualized prior variance.
- The trading strategy of the insiders is $\theta_t = \frac{\sigma_t}{\sigma_v(T-t)}(v-P_t)$
- Equilibrium price dynamics are identical to the original Kyle (1985) model:

$$dP_t = \frac{(v - P_t)}{T - t} dt + \sigma_v dZ_t. \tag{11}$$

IMPLICATIONS OF MARTINGALE DYNAMICS

This example shows we can extend Kyle's equilibrium by simply 'plugging-in' stochastic noise trading volatility:

- Market depth varies linearly with noise trading volatility,
- Insider's strategy is more aggressive when noise trading volatility increases,
- Both effects offset perfectly so as to leave prices unchanged (relative to Kyle):
 - Prices display constant volatility.
 - Private information gets into prices linearly and independently of the rate of noise trading volatility (as in Kyle).
- ⇒ In this model empirical measures of price impact will be time varying (and increasing over time on average), but do not reflect any variation in asymmetric information of trades.

Constant Expected growth rate

 We assume that uninformed order flow volatility follows a geometric Brownian Motion:

$$\frac{d\sigma_t}{\sigma_t} = mdt + \nu dW_t, \tag{12}$$

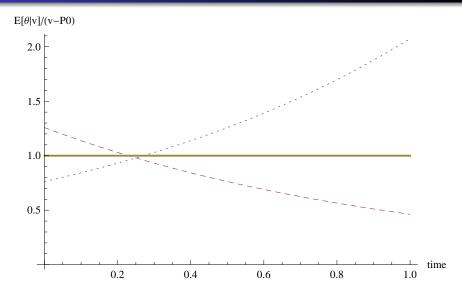
- We can solve for $G(t) = \sigma_t^2 B_t$ where $B_t = \frac{e^{2m(T-t)}-1}{2m}$,
- Then price impact is: $\lambda_t = rac{e^{mt}}{\sigma_t} \sqrt{rac{\Sigma_0}{B_0}}$
- The trading strategy of the insider is: $\theta_t = \frac{\sigma_t}{e^{mt}B_t}\sqrt{\frac{B_0}{\Sigma_0}}(v-P_t)$
- Equilibrium price dynamics:

$$dP_t = \frac{(v - P_t)}{B_t}dt + e^{mt}\sqrt{\frac{\Sigma_0}{B_0}}dZ_t. \tag{13}$$

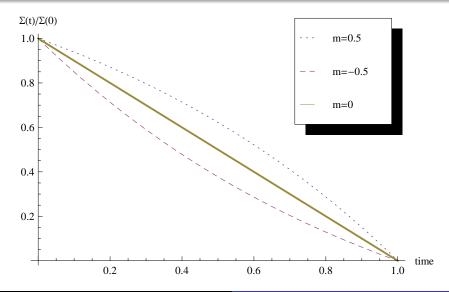
IMPLICATIONS OF CONSTANT GROWTH RATE

- As soon as there is predictability in noise trader volatility, equilibrium prices change (relative to Kyle):
 - Price volatility increases (decreases) deterministically with time if noise trading volatility is expected to increase (decrease).
 - Private information gets into prices slower (faster) if noise trading volatility is expected to increase (decrease).
- Interesting separation result obtains:
 - Strategy of insider and price impact measure only depends on current level of noise trader volatility.
 - Equilibrium is independent of uncertainty about future noise trading volatility level (ν) .
 - As a result, equilibrium price volatility is deterministic
 - Private information gets into prices at a deterministic rate, despite measures of price impact (and the strategy of the insider) being stochastic!

IMPLICATIONS OF CONSTANT GROWTH RATE



Information revelation



Constant *m* and Random Horizon with constant of

- if ho > m are constant then $G(t) = rac{\sigma_t^2}{2(
 ho m)}$
- Information flows into prices at a constant rate: $\Sigma_t = \Sigma_0 e^{-2(\rho-m)t}$
- Market depth is given by: $\frac{1}{\lambda_t}=e^{
 ho t}\sigma_t e^{-mt} \frac{1}{\sqrt{2(
 ho-m)\Sigma_0}}$
- The trading strategy of the insider is: $\theta_t = \frac{2(\rho-m)}{\lambda_t}(v-P_t)$
- Stock price dynamics are given by an Ornstein-Uhlenbeck process:

$$dP_{t} = \underbrace{2(\rho - m)}_{\kappa > 0} (v - P_{t}) dt + e^{-(\rho - m)t} \sqrt{\Sigma_{0} 2(\rho - m)} dZ_{t} + (v - P_{t}) d\mathbf{1}_{\{T \leq t\}}. \quad (14)$$

GENERAL DETERMINISTIC GROWTH RATE

• Suppose that noise trading volatility has deterministic drift m_t :

$$\frac{d\sigma_t}{\sigma_t} = m_t dt + \nu(t, \sigma^t) dW_t \tag{15}$$

- Then: $G(t) = \sigma_t^2 \int_t^T e^{\int_t^u 2m_s ds} du = \int_t^T \mathrm{E}[\sigma_s]^2 ds \leq \mathrm{E}[\int_t^T \sigma_s^2 ds],$
- Private information enters prices at a deterministic rate (Σ_t is deterministic).
- Equilibrium price volatility is deterministic: $\sigma_P(t) = \mathrm{E}[\sigma_t] \sqrt{\frac{\Sigma_0}{G_0}}$
- Implications:
 - ullet With deterministic m_t , shocks in noise trading volatility do not affect price volatility.
 - With $\nu_t \neq 0$ price impact is stochastic decreasing with noise trading volatility.
 - \Rightarrow need $m_t \neq 0$ and stochastic to get stochastic price volatility and meaningful correlation between price volatility, volume, and price impact.

MEAN REVERSION

 We assume that uninformed order flow log-volatility follows an Ornstein-Uhlenbeck process:

$$\frac{d\sigma_t}{\sigma_t} = -\kappa \log \sigma_t dt + \nu dW_t. \tag{16}$$

• Series expansion solution for $G(t) = \sigma_t^2 A(T - t, x_t, \kappa)^2 < \mathbb{E}[\int_t^T \sigma_s^2 ds]$ where

$$A(\tau, x, \kappa) = \sqrt{T - t} \left(1 + \sum_{i=1}^{n} (-k\tau)^{i} \left(\sum_{j=0}^{i} x^{j} \sum_{k=0}^{i-j} c_{ijk} t^{k} \right) + O(\kappa^{n+1}) \right), \tag{17}$$

where the c_{ijk} are positive constants that depend only on ν^2 .

- Price impact is stochastic and given by: $\lambda_t = \frac{\sqrt{\Sigma_t}}{\sigma_t A(T-t, \mathsf{x}_t, \kappa)}$.
- The trading strategy of the insider is: $\theta_t = \frac{\sigma_t}{\sqrt{\Sigma_t}A(T-t,x_t,\kappa)}(v-P_t)$.
- private information enters prices at a stochastic rate: $\frac{d\Sigma_t}{\Sigma_t} = -\frac{1}{A(T-t,x_t,\kappa)^2}dt$.
- Stock price dynamics follow a three factor (P, x, Σ) Markov process with stochastic volatility given by:

$$dP_t = \frac{(v - P_t)}{A(T - t, x_t, \kappa)^2} dt + \frac{\sqrt{\Sigma_t}}{A(T - t, x_t, \kappa)} dZ_t.$$
 (18)

MEAN-REVERSION

• The first term in the series expansion of the $A(\tau, x, \kappa)$ function is instructive:

$$A(\tau, x, \kappa) = \sqrt{\tau} \left(1 - \frac{\kappa}{2} \tau \left(\frac{\nu^2 \tau}{6} + x\right)\right) + O(\kappa^2). \tag{19}$$

- With mean-reversion ($\kappa \neq 0$) uncertainty about future noise trading volatility (ν) does affect the trading strategy of the insider, and equilibrium prices.
- When x=0 (where vol is expected to stay constant), the higher the mean-reversion strength κ the lower the A function. This implies that mean-reversion tends to lower the profit of the insider for a given expected path of noise trading volatility.
- If $\kappa > 0$ then A is decreasing in (log) noise-trading volatility (x_t) and in uncertainty about future noise trading volatility ν . This implies that stock price volatility is stochastic and positively correlated with noise-trading volatility.
- Equilibrium price follows a three-factor Bridge process with stochastic volatility.
- Private information gets incorporated into prices faster the higher the level of noise trading volatility, as the insider trades more aggressively in these states.
- Market depth also improves, but less than proportionally to volatility.

A TWO-STATE CONTINUOUS MARKOV CHAIN: FIXED T

• Assume uninformed order flow volatility can take on two values σ^{s_t} in state $s_t = H, L$ with $\sigma^L < \sigma^H$:

$$ds_{t} = (H - s_{t})dN_{L}(t) - (s_{t} - L)dN_{H}(t)$$
(20)

where $N_i(t)$ is a standard Poisson counting process with intensity η_i (i = H, L).

• The solution is $G(t, \sigma_t) = \mathbf{1}_{\left\{\sigma_t = \sigma^H\right\}} G^H(T-t) + \mathbf{1}_{\left\{\sigma_t = \sigma^L\right\}} G^L(T-t)$, where the deterministic functions G^H, G^L satisfy the system of ODE (with boundary conditions $G^H(0) = G^L(0) = 0$):

$$G_{\tau}^{L}(\tau) = (\sigma^{L})^{2} + 2\eta_{L}(\sqrt{G^{H}(\tau)G^{L}(\tau)} - G^{L}(\tau))$$
 (21)

$$G_{\tau}^{H}(\tau) = (\sigma^{H})^{2} + 2\eta_{H}(\sqrt{G^{H}(\tau)G^{L}(\tau)} - G^{H}(\tau))$$
 (22)

- We compute execution costs of uninformed numerically in this case.
- Show that uninformed execution costs can be higher when noise trading volatility is higher (and Kyle lambda is actually lower).

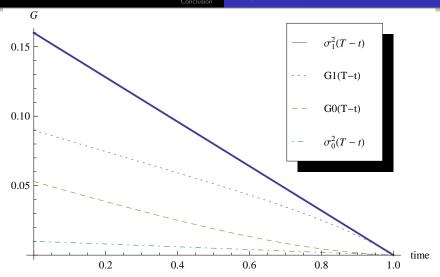


FIGURE: G function in high and low state

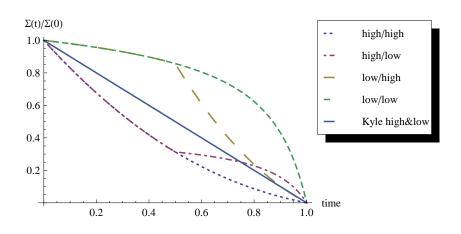


FIGURE: Four Private information paths

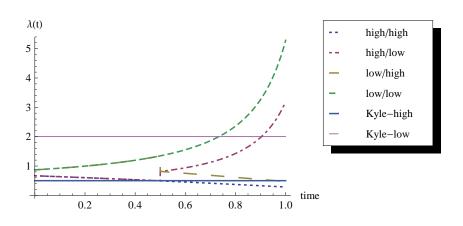


FIGURE: Four paths of price impact λ_t

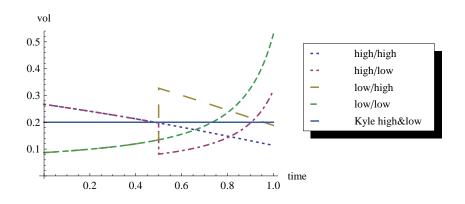


FIGURE: Four paths of Stock price volatility

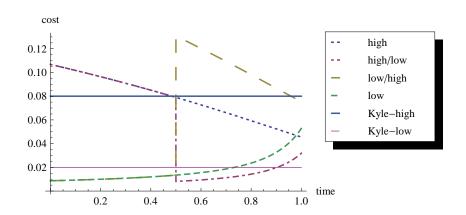


FIGURE: Four paths of uninformed traders execution costs

Noise trading volatility paths:	нідн/нідн (1)	LOW/LOW (2)	ніgн/low (3)	LOW/HIGH (4)		
Panel A: Aggregate execution costs						
TOTAL	0.078	0.017	0.054	0.057		
Path Dependent	0.047/0.031	0.005/0.012	0.047/0.007	0.005/0.052		
Panel B: 'Number Total Path Dependent	0.16	0.01	$0.085 \\ 0.08/0.005$	$0.085 \\ 0.005/0.08$		
Panel C: Normalized aggregate execution costs						
Total	0.487	1.740	0.636	0.671		
Path Dependent	0.587/0.387	1/2.4	0.587/1.4	1/0.65		
Panel D: Averag TOTAL PATH DEPENDENT	0.487	1.740	1.023 $0.584/1.462$	0.853 1.06/0.646		
Panel E: Average stock price volatility						
Total	0.195	0.174	0.190	0.182		
Path Dependent	0.234/0.156	0.106/0.242	0.234 / 0.146	0.106 / 0.258		

Main Take-aways

- Average price-impact is not informative about execution costs paid by uninformed.
- Normalizing by 'abnormal' trading volume is crucial.
- Even so, average execution costs to uninformed are path-dependent.
- Stock volatility and price-impact are negatively related in changes, but not necessarily in levels (≠ inventory trading cost model).
- Stock volatility and volume are positively related in changes, but not in levels.
- Price-impact is not sufficient statistic for rate of arrival of private information.

A TWO-STATE CONTINUOUS MARKOV CHAIN: RANDOM HORIZON

- Suppose that intensity and volatility are $\rho(s_t)$ and $\sigma(s_t)$ where s_t follows two state Markov Chain $(s_t = H, L)$. Assume $\rho(H) > \rho(L)$ and $\sigma(H) > \sigma(L)$.
- Then $G(s_t) = \frac{\sigma(s_t)^2}{\kappa(s_t)}$ with G(H) > G(L) and $\kappa(H) > \kappa(L)$.
- Stock price dynamics are given by modulated jump diffusion:

$$dP_t = \kappa(s_t)(v-P_t) dt + e^{-\frac{1}{2} \int_0^t \kappa(s_u) du} \sqrt{\Sigma_0 \kappa(s_t)} dZ_t + (v-P_t) d\mathbf{1}_{\{T \leq t\}}.$$

- Private information flows into prices: $rac{\Sigma_t}{\Sigma_0} = e^{-\int_0^t \kappa(s_u) du}$
- Market depth is given by $\frac{1}{\lambda_t} = \frac{1}{\sqrt{\kappa(s_t)\Sigma_0}} \sigma(s_t) e^{\frac{1}{2} \int_0^t \kappa(s_u) du}$
- The trading strategy of the insider is: $\theta_t = \frac{\kappa(s_t)}{\lambda_t}(v-P_t)$

Conclusion

- Recent empirical paper finds that standard measures of adverse selection and stock liquidity fail to reveal the presence of informed traders
- Propose extension of Kyle (1985) to allow for stochastic noise trading volatility (and random horizon):
 - Insider conditions his trading on 'liquidity' state.
 - Price impact measures are stochastic and path-dependent (not necessarily higher when more private information flows into prices).
 - Total execution costs can be higher when measured average price impact is lower.
 - Predicts complex relation between trading cost, volume, and stock price volatility.
 - Generates stochastic 'excess' price volatility driven by non-fundamental shocks.
- Future work:
 - Better measure of liquidity/adverse selection?
 - Model of activist insider trading with endogenous terminal value. Why the 5% rule?
 - Risk-Aversion, Residual Risk and Announcement returns.
 - Absence of common knowledge about informed presence.