Conditional Currency Hedging

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ABSTRACT

This research proposes Conditional Currency Hedging based on FX risk factors as a way to improve the risk/ return trade-off of given stock, bond or commodity portfolios. In our employed sample, a conditional currency hedging framework based on Volatility, Carry Trade and Dollar Risks results in lower variance as well as higher average return of a global equity portfolio than achieved by either no, full or unconditional mean-variance hedging. This suggests that conditioning the hedging policy upon the state of risk factors could significantly enhance the performance for investors globally. Further analysis with bond and commodity performance data will follow.

JEL Classification: F31, G11, G15

Keywords: FX risk factors, Currency Hedging, Mean-variance analysis

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I. Introduction to the topic

In their long-range study on *Global Currency Hedging*, Cambpell et al. (2010) find that over- and under-hedging in certain currencies can lead to risk reductions of global equity portfolios, relative both to no and to full hedging. Using a mean-variance framework, they form optimal (zero net-investment) FX portfolios for given stock or bond portfolios. Thereby they rely on equity (bond) market betas of currencies as a measure of risk.

This paper sets out to extend the analysis by conditioning the analysis on the state of underlying FX risk factors. The question is whether the optimal exposure to different currencies is a function of (current and lagged) underlying FX risks. If so, we would expect a superior risk/ return trade-off of conditional FX hedging relative to no, full as well as unconditional hedging.

A. Risk factors and Pricing in the FX market

The FX risk pricing factors to be employed for conditioning the mean-variance analysis above are derived from a recent literature stream, which tries to account for the *Carry Trade and Momentum Puzzles* in a similar way as Asset Pricing theories in the Equity markets do. The proposed FX pricing factors focus on different potential explanations. One can distinguish conventional risk factors, FX returns-based risk factors as well as alternative (non-risk-based) explanations to account for the profitability of carry and momentum strategies (e.g. Burnside, 2011 & Burnside et al., 2011).

The traditional risk factors employed usually come from the equities realm where literature has long used them in models to price payoffs in that market. These models encompass the traditional CAPM (market excess return as sole factor), the three-factor model of Fama and French (1993), which additionally uses the value and size premia as factors, or more fundamental models employing industrial production or consumption growth data as factors.

More recently, factors have been derived from currency returns themselves, partially due to the acknowledgment that FX is a different investment class with different market participants, and should thus be priced by different risk factors than equities (e.g. Burnside, 2012). One set of pricing factors are derived from currencies sorted by their forward discount. For instance Lustig et al. (2011) form two risk factors: the level factor *DOL* (Dollar Factor) captures the average excess return of all foreign currency against the dollar; the slope factor *HML* (Carry Trade Risk Factor) is the return differential between the highest and lowest discount portfolios and captures carry trade risk. The authors can relate the latter to global equity market volatility. Menkhoff et al. (2012a) on the other hand form a direct FX Volatility risk factor *(VOL)* and are able to account for the excess returns of the carry trade therewith.

Mancini et al. (2013) have proposed a Liquidity risk factor, and have documented that high- (low-) yielding currencies offer exposure to (protection from) this risk. In their high-frequency sample from 2007-2009 the tradable liquidity risk factor *IML* (illiquid minus liquid) strongly predicts carry trade performance. Other proposed risk factors include *Currency Skewness* (e.g. Rafferty, 2011) as well as Correlation Risk *(COR* (e.g. Mueller et al., 2012). The motivation behind the

Skewness factor is the idea that investment currencies of carry trades often crash in tandem as liquidity dries up. Mueller et al. (2012) provide evidence that high (low) interest rate currencies have high (low) correlation risk exposure, that is they fare relatively worse (better) during times of increased exchange rate correlation. Finally, Burnside (2012) argues that the profitability of carry trade and momentum could be due to the possibility of rare disasters or *Peso Problems*. When these rare disasters do not occur in sample, the model cannot be calibrated properly and thus produces unaccounted-for excess returns.

Another argument states that risk factors alone cannot explain the whole outperformance of carry trade and momentum (Barroso et al., 2013). Barroso et al. (2013) for instance explain the outperformance with non-profit-maximizing market participants (such as Central Banks), a *Scarcity of Profit-seeking Capita*l and abundance of capital pursuing goals unrelated to profitability. Other potential factors cited in the literature are *Microstructure-based explanations* –such as price pressure (e.g. Burnside, Eichenbaum & Rebelo, 2011) or adverse selection (Burnside, Eichenbaum & Rebelo, 2009)–, Behavioral explanations (e.g. Burnside, Han, Hirshleifer & Wang, 2011) or *Limits to arbitrage* (e.g. Menkhoff et al., 2012b).

B. Conditional approaches in Asset Pricing

Apart from incorporating novel risk factors, recent international finance literature has also often employed conditional approaches and models.

Christiansen, Ranaldo and Söderlind (2011) for instance capture time-varying systematic risk of carry trade strategies in the form of a logistic smooth transition regression model. They apply regime-changing constants and coefficients to risk factors, based on FX volatility and liquidity regime variables. Thereby, they find a strong negative effect of volatility on carry trade performance by the direct effect of the changing constant as well as the increased correlation with traditional risk factors in high-volatility periods. This pattern is also reflected in individual currencies with investment (funding) currencies exhibiting increased positive (negative) correlation with stock returns and increased negative (positive) correlation with bond returns in times of higher volatility; this in-line with relatively constant safe haven versus investment characteristics of currencies. The conditional (regime-dependent) risk exposure thereby drastically increases the cross-sectional model fit of the asset pricing model.

Lettau, Maggiori and Weber (2014) have recently proposed a downside risk capital asset pricing model (DR-CAPM). Apart from the unconditional CAPM market beta, they additionally posit a (conditional) *downside beta*, defined by an exogenous threshold for the market return. The central finding thereby is that the currency carry trade, as well as other cross-sectional strategies, is more highly correlated with aggregate market returns conditional on low aggregate returns than it is conditional on high aggregate market returns (p. 10). The DR-CAPM can jointly explain the cross-sectional fit across different asset classes, such as currencies, equities, equity index options, sovereign bonds and commodities. Like Christiansen et al. (2011), they also employ a form of *contemporaneous* conditionality.

C. Proposed Research

C.1. Research Gap

How is the *conditional* correlation between equities, bonds and commodities with FX impacted by FX risk factors? While Campbell et al. (2010) condition upon interest differentials and find no significant difference between their baseline unconditional and conditional models, we can perform a similar analysis with the recently proposed FX risk factors as conditioning information: These encompass Carry Trade, Dollar and Momentum Performance, as well as Volatility, Liquidity and Correlation Risks. This results in a more fundamental analysis in the sense of *identifying* the nature of the underlying risk impacting the equity-FX correlation, rather than taking the equity market-beta of currencies as both risk measure as well as input to the mean-variance analysis. If the underlying premise of time-varying correlation according to the FX risk state is true, this implies lower-than-anticipated diversification benefits through static (unconditional) FX hedging as well as an outperformance of conditional relative to unconditional FX hedging.

C.2. Research Question

How do FX risk factors impact the dependence between FX and equities, bonds and commodities markets, and thus the mean-variance hedging rationale of investors in these asset classes?

C.3. Research Goal

The main research goal is to find out *if and in what way* the optimal hedging rationale for global equity, bond and commodities investors is impacted by changing FX risks. The underlying premise is that the *optimal* (variance-minimizing) FX portfolio weights are not static, but a function of the current risk state of financial markets. Thus, the basic hypothesis to be tested is whether risk factors have an impact on the mean-variance FX hedging strategy of international equity, bond and commodity portfolios through time.

The proposed paper leads to several contributions: For one, it applies the findings of recent research with regards to FX pricing factors in the context of portfolio risk management. This leads to a more fundamental understanding of the channel of various risks on the co-movement and relationship between different asset classes under different conditions. For two, concrete recommendations will be derived in terms of optimal (risk-dependent) hedging for investor in different asset classes.

II. Research Design

The proposed research is an empirical panel study on optimal hedging of international investment portfolios with changing FX positions depending on risk factors. As far as possible, the research design follows Campbell et al. (2010)'s analysis in order to ensure comparability of results as well as consistency in analysis.

A. *Risk/ pricing factors*

Given that the focus of analysis is not upon a particular risk factor, but rather on the effect of broader FX risks on the co-dependency between currencies with equities, bonds and commodities, the paper deliberately considers a variety of factors. Thereby we start with the two most established FX-based risk factors: The Dollar and Carry Trade Risk factors, as defined in Lustig et al. (2011) (or in Verdelhan, 2013). As mentioned in section 1.2 the Dollar Factor effectively measures the average excess return on all foreign currency portfolios against the dollar, while the carry trade risk factor is similar to a zero-cost strategy long in the highest forward-discount currencies and short in the lowest forward-discount currencies. As Verdelhan (2013) finds, Carry and Dollar factors reflect two distinct kinds of global shocks.

We then add Momentum and FX-Volatility risk factors, as defined in Menkhoff et al. (2012b). As Menkhoff et al. (2012b) as well as Burnside, Eichenbaum and Rebelo (2011) find, the momentum strategy has different risk-return characteristics than the carry trade and likely thus capture different fundamental risk factors. The FX volatility factor might be strongly related to the carry factor defined above, given that Lustig et al. (2011) have found the latter to be related to Equity volatility. Lastly, we add Illiquidity (Mancini et al., 2013) as well as Correlation (Mueller et al., 2014) risk factors.

In staying close to the Asset Pricing literature, we measure all risk factors on a global, instead of on a currency-specific level. This has the advantage that we measure the sensitivity of currencies with respect to global shocks, rather than idiosyncratic phenomena. On the other hand, we thereby imply (much as in a CAPM framework) relatively constant sensitivity of individual currencies to these risk factors over time.

More specifically, we employ the following measurements for the conditioning variables:

1. Dollar factor (DOL): Equally-weighted excess return of all (foreign) currencies against the dollar (Lustig et al., 2011)
2. Carry Trade factor (HML): Excess return of highest forward discount minus lowest forward discount currency portfolios (in USD) (Lustig et al., 2011)
3. Volatility (VOL) pricing factor: Average absolute log return of all available currencies in a given time interval (Menkhoff et al., 2012a)
4. Momentum factor (MOM): Excess return of best-performing minus lowest-performing (over a past holding period) currency portfolios (Menkhoff et al., 2012b)
5. Illiquidity factor (IML): Measure based on average price impact and return reversal, trading cost and price dispersion of all available currencies in a given time interval (Mancini et al., 2013)
6. Correlation factor (COR): Average of realized correlations of all available currencies in a given time interval (Mueller et al., 2012)

B. Baseline analysis

The empirical analysis is based on the estimation of risk-minimizing currency positions for exogenously given stock, bond and commodity portfolios. Risk is thereby defined as the standard deviation of the portfolio. Campbell et al. (2010) show that unconditional mean-variance analysis amounts to regressing the hedged portfolio excess returns in domestic currency on a constant and the vector of currency excess returns, and switching the sign of the slopes.

To incorporate conditionality, they interact the regressors (the currency excess returns) with the conditional information –in their case the deviation of interest rate differentials from their time-series mean. In other words, they "consider a conditional model for risk management currency demand that depends linearly on interested differentials (p.111)". We can proceed similarly in our case and regress hedged excess portfolio returns on a constant, the vector of currency excess returns as well as additionally the vectors of currency excess returns interacted with the six risk factors. Following Verdelhan (2013), we break down the carry factor into an unconditional one (HML) as well as a conditional one (HML multiplied by the interest differential of the currencies). This leads to a *conditionality in conditionality.*

B.1. Formal conditional mean-variance estimation procedure

Formally, this can be written as follows. The log portfolio excess return over the domestic interest rate is approximately equal to:

$$r_{p,t+1}^{h} - i_{1,t} = \mathbf{1}^{'}\boldsymbol{\omega}_t\left(\boldsymbol{r_{t+1}} - \boldsymbol{i_t}\right) + \left(\boldsymbol{\Psi_{\mathbf{u}}}^{'} + \boldsymbol{\Psi}^{'}_{\mathbf{c},\mathbf{t}}\right)\left(\boldsymbol{\Delta} s_{t+1} + \mathbf{i_t} - \mathbf{i_t^d}\right) + \frac{1}{2}\sum_{t}^{h}$$

$r_{p,t+1}^{h} - i_{1,t}$ is the log portfolio excess return over the domestic interest rate, $\boldsymbol{\omega}_t$ is the (n+1 x n+1) diagonal matrix of portfolio weights, $\boldsymbol{r_{t+1}}$ is the vector of log nominal asset returns in local currencies, $\boldsymbol{\Psi_{\mathbf{u}}}$ ($\boldsymbol{\Psi}_{\mathbf{c},\mathbf{t}}$) is the vector of net unconditional (conditional) currency exposures, $\boldsymbol{\Delta}\mathbf{s_{t+1}}$ is the vector of the changes in log spot exchange rates, $\mathbf{i_t}$ is the vector of log short-term nominal interest rates, $\mathbf{i_t^d}$ is the log domestic interest rate vector, and $\mathbf{1}$ is a vector of ones. All the vectors have dimension (n+1 x 1). $\frac{1}{2}\sum_{t}^{h}$ is a Jensen's inequality adjustment term.

The equation above provides an intuitive decomposition of the portfolio excess return. The first term on the right of the equation sign is the excess return on a fully hedged portfolio that has no exposure to currency risk. The second term represents the currency excess return of the investor. This is arrived at through the (unconditional plus conditional) FX portfolio weights. Note that the unconditional currency weights are static through time, while the conditional currency weights vary with the risk factors (thus the subscript t). The third term is a Jensens inequality correction, as given in the Internet Appendix of Campbell et al. (2010).

We seek to minimize the variance of this log portfolio excess return by choosing the optimal unconditional and conditional currency weights:

$$\underset{\boldsymbol{\Psi_u}, \boldsymbol{\Psi_{c,t}}}{arg\min}(\sigma^2_{p,t+1})$$

The mean-variance currency weights are formed by regressing the hedged excess returns on a constant, the vector of currency excess returns (unconditional currency weights), as well as the vector of currency returns interacted with the risk factors (conditional currency weights), and switching the sign of the slopes. That is, we regress as below (similarly to equation (4) on page 111 in Campbell et al. (2010)):

$$\begin{aligned}
&\mathbf{1}'\boldsymbol{\omega}_t\left(\boldsymbol{r_{t+1}} - \boldsymbol{i_t}\right) = \gamma_0 - \boldsymbol{\Psi}_u'\left(\boldsymbol{\Delta}s_{t+1} + i_t - i_t^d\right) \\
&- \boldsymbol{\Psi}_{DOL}'\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) DOL_{t(t+1)} \\
&- \boldsymbol{\Psi}_{HML}'\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) HML_{t(t+1)} \\
&- \boldsymbol{\Psi}_{HML}'\left[\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) * \left(i_t - i_t^d\right)\right] HML_{t(t+1)} \\
&- \boldsymbol{\Psi}_{VOL}'\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) VOL_{t(t+1)} \\
&- \boldsymbol{\Psi}_{MOM}'\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) MOM_{t(t+1)} \\
&- \boldsymbol{\Psi}_{IML}'\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) IML_{t(t+1)} \\
&- \boldsymbol{\Psi}_{COR}'\left(\boldsymbol{\Delta}s_{t+1} + \boldsymbol{i_t} - \boldsymbol{i_t^d}\right) COR_{t(t+1)} \\
&+ \varepsilon_t
\end{aligned}$$

Excess returns are as discussed above. γ_0 is the regression intercept. $DOL_{t(t+1)}, HML_{t(t+1)}, VOL_{t(t+1)}, MOM_{t(t+1)}, IML_{t(t+1)}, COR_{t(t+1)}$ are the FX risk factors at time $t(t+1)$ (scalars). $\boldsymbol{\Psi_u}$ is the (n+1 x 1) vector of estimated unconditional currency demands, $\boldsymbol{\Psi}_{DOL}, \boldsymbol{\Psi}_{HML}$, etc are the (n+1 x 1) vectors of estimated holding conditional sensitivities for each currency to the respective FX risk factor. * is the element-by-element product operator. ε_t is the residual.

The optimal net currency weights relative to full currency hedging then result from below equation:

$$\begin{aligned}
&\boldsymbol{\Psi_{RM,t}} = \boldsymbol{\Psi_u} + \boldsymbol{\Psi_{c,t}} \\
&\boldsymbol{\Psi_{c,t}} = \boldsymbol{\Psi}_{DOL} DOL_{t(t+1)} + \boldsymbol{\Psi}_{HML} HML_{t(t+1)} + \left[\boldsymbol{\Psi}_{HML} * \left(i_t - i_t^d\right)\right] HML_{t(t+1)} \\
&+ \boldsymbol{\Psi}_{VOL} VOL_{t(t+1)} + \boldsymbol{\Psi}_{MOM} MOM_{t(t+1)} + \boldsymbol{\Psi}_{COR} COR_{t(t+1)}
\end{aligned}$$

$\boldsymbol{\Psi_{RM,t}}$ is the resulting (n x 1) vector of estimated optimal foreign currency holdings at time t resulting from the regression. It is the sum of the unconditional currency holding $\boldsymbol{\Psi_u}$ and the conditional currency holding at time t $\boldsymbol{\Psi_{c,t}}$. The conditional currency holding at time t is itself the sum of the vectors of currency sensitivities multiplied by their respective risk factors.

As in Campbell et al. (2010), we are primarily interested in estimating the vector of the slopes $\mathbf{\Psi}_{DOL}, \mathbf{\Psi}_{HML}, etc.$ and testing whether they are different from zero. If they are zero (null hypothesis), we reover the purely unconditional risk management demands. If they are different from zero, the conditional risk factors do impact the optimal hedging rationale vis-à-vis unconditional hedging demands.

We can then analyze the standard deviation, mean returns, the Sharpe ratio and other risk/ return characteristics of the unhedged, hedged, unconditional mean-variance as well as conditional mean-variance portfolios. The working hypothesis is for the conditional mean-variance portfolio to have the lowest standard deviation of all portfolios. If it is statistically different (lower) than the unconditional standard deviation, it would be an indication that optimal risk management demands are impacted by conditioning on risk factors at a given time.

C. Data

C.1. Variables and sources

The data collection effort is similar as in Campbell et al. (2010). The empirical analysis is based on stock return data from Morgan Stanley Capital International (MSCI), whereby total return country indices in local currencies are used. Data on spot exchange rates, short-term interest rates, and long-term bond yields are mainly from the International Financial Statistics database (IFS) of the International Monetary Fund (IMF). Whenever such data is not available, we resort to OECD data[1]. Excess returns of all bilateral exchange rates (e.g./ CAD-EUR) are implied by the relative excess returns of the involved currencies to the USD.

The main data sample is as in Campbell et al. (2010), encompassing the Eurozone, Australia, Canada, Japan, Switzerland, the United Kingdom and the United States. These are the 7 countries where data on all of above go back longest (July 1975). The Eurozone is proxied by Germany (in terms of exchange rate, stock market as well as interest rate data) before introduction of the Euro in January 1999. A broader sample with FX, interest rates and stock data on more countries, but in a smaller timeframe (given availability) may be added to that subsequently to see whether the findings can be generalized to other countries, including Emerging Economies. Additionally to Campbell et al. (2010), we will also add various commodity index return data to analyze the hedging regime for Commodity investors.

To construct our conditional risk factors, we obtain FX data of the same set of 48 currencies as in Menkhoff et al. (2012). We directly get the DOL, HML and VOL risk factor data from the authors Menkhoff et al. (2012), underlying their article on *Currency Momentum Strategies*. The data thereby ranges from March 1976 to January 2010. After that, we construct the risk factors by the same recipe as Lustig et al. (2011) (DOL, HML) and Menkhoff et al. (2012) (VOL, MOM), respectively. We will double-check that our calculated risk factor values are historically the same as the ones we obtain directly. The IML and COR risk factors will be constructed as described by

[1]For Switzerland, in the time span from 1976:3 to 1979:12, we use OECD data given unavailable IMF data.

Mancini et al. (2013) and Mueller et al. (2012), respectively, based on our sample of 48 currencies.

Data are monthly. The sample period starts in March 1976, the earliest date for which we have data for all variables (including Risk Factors), and ends in October 2014.

C.2. Data transformation

For the baseline analysis, data are transformed as in Campbell et al. (2010) (see pages 91-95). This we do to ensure comparability of our results with our enlarged sample with the findings of Campbell et al. (2010). First we take logs of returns, as this allows for the additive portfolio return decomposition outlined in II.B before. We then form 3-month-overlapping (log) returns for stock, bond and FX excess returns. We correct for the inherent autocorrelation by forming Newey-West (HAC) standard errors with automatic lag detection.

Additionally, as a robustness test, we will form 1- and 3- month non-overlapping returns, and see how our results are affected. This will presumably have the benefit of removing or reducing the inherent autocorrelation, and possibly lead to more exact coefficient estimates (optimal currency weights).

III. Preliminary analysis and results

Our preliminary analysis is based on stock and FX excess returns as well as risk factor data (DOL, HML, VOL) from March 1976 to January 2010. This time span is employed, as the data set on risk factors obtained from Menkhoff et al. (2010) ends in January 2010. The data are transformed as discussed above.

A. Summary statistics

To get an intuition of the underlying data, we first run some descriptive statistics. It is instructive to compare these to the values in Campbell et al. (2010), whose sample ends in 2005, and see in what way they are different in our enlarged dataset including the years of the financial crisis. This is worthwile, as it impacts subsequent unconditional and conditional analysis results.

Interest rates are generally lower in all 7 countries compared to Campbell et al. (2010). This is not surprising as the added data from 2006 until 2010 falls in a timespan of extremely low interest rates, as monetary authorities worldwide grappled with the aftermath of the Financial Crisis and subsequent Recession. However, especially Euroland has much lower rates than in the sample of our baseline paper this is due to our different definition of Euroland, featuring only Germany as the Eurozone ante-1999. Campbell et al. (2010), on the other hand, construct Euroland as a stock market value-weighted index of Germany, France, Italy and the Netherlands. This different definition of Euroland before 1999 also impacts the other variables (lower excess stock returns, higher log exchange rate changes).

Excess stock returns are consistently lower than in Campbell et al. (2010)'s sample, given that 2006-2010 featured heavy stock market losses. In terms of log exchange rate changes, the data look rather similar to Campbells sample, with somewhat higher values for Euroland (as explained) and Switzerland, as the Swiss Franc appreciated strongly during the Crisis period. Excess currency returns are quite similar.

B. *Unconditional FX hedging*

In this section we proceed along the unconditional analysis in Campbell et al. (2010) to see whether and to what extent the findings can be replicated in our enlarged dataset. This forms the base model on top of which the conditional risk factors will then be introduced in the next section.

Formally, as discussed in II.B before, the unconditional analysis regresses an exogenously given stock portfolio (hedged excess returns in local currencies) on an intercept and a vector of currency excess returns (against the domestic currency of the investor) in order to find the unconditional optimal currency demands relative to full hedging currency demands. This is shown below:

$$\mathbf{1}'\boldsymbol{\omega_t}\left(\boldsymbol{r_{t+1}}-\boldsymbol{i_t}\right)=\gamma_0-\boldsymbol{\Psi}_{\mathbf{u}}'\left(\boldsymbol{\Delta}\mathbf{s_{t+1}}+\mathbf{i_t}-\mathbf{i_t^d}\right)$$

In the next three sections, we will thereby vary $\boldsymbol{\omega_t}$ —the diagonal matrix containing the exogenous equity market weights—as well as $\boldsymbol{\Psi}_{\mathbf{u}}$ —the vector containing the optimal unconditional currency weights. First we allow for only a single entry in the diagonal of $\boldsymbol{\omega_t}$ (1 domestic stock market) and estimate only single coefficients in $\boldsymbol{\Psi}_{\mathbf{u}}$ (1 currency exposure). Then we analyze the situation with only a single entry in $\boldsymbol{\omega_t}$ (1 stock market), but with multiple coefficients in $\boldsymbol{\Psi}_{\mathbf{u}}$ (multiple currencies). Lastly, we fill up the diagonal terms of $\boldsymbol{\omega_t}$ with equal weights (equally-weighted global stock portfolio) and choose the optimal weights in $\boldsymbol{\Psi}_{\mathbf{u}}$ (multiple currencies).

Table I Summary Statistics (1976:3 - 2010:1)

	Euroland	Australia	Canada	Japan	Switzerland	United Kingdom	United States
Log interest rates							
Average	4.62	8.16	6.78	2.36	2.99	7.55	5.44
Standard Deviation	1.04	1.78	1.89	1.07	1.15	1.70	1.51
Excess log stock returns in local currency							
Adjusted Average	5.51	5.94	5.26	4.24	7.64	5.91	5.89
Standard Deviation	20.54	18.91	18.15	19.79	17.85	16.89	15.73
Change in log exchange rate							
Adjusted Average	2.47	-0.28	0.03	4.26	3.36	0.01	.
Standard Deviation	11.51	11.77	6.50	12.09	12.61	11.41	.
Excess log currency returns							
Adjusted Average	1.67	2.46	1.38	1.22	0.94	2.14	.
Standard Deviation	11.66	11.89	6.55	12.39	12.87	11.60	.

Notes: Data are annualized. Stock market returns are from the Morgan Stanley Capital International database. All other variables are from IMF's IFS database, if available, or otherwise OECD data. Data are monthly, from 1976:3 to 2010:1. Interest rates are log 3-month government bill rates. Excess log stock excess returns are total local currency returns in excess of the local log nominal interest rate. The currency excess return is the return to a U.S. investor of borrowing in dollars to hold foreign currency. The Adjusted Average is formed by taking the mean of each variable and adding one-half its variance, in percentage points per annum. This gives an estimate of the mean simple excess return.

B.1. Single Country Stock Portfolios with Single Currency Exposure

The empirical analysis first examines the case of an investor who is fully invested in a specific country stock market and is considering whether exposure to other currencies could help reduce the volatility of his quarterly portfolio return.

Table II below shows the case of an investor holding an exogenous stock portfolio from a single country, and can use *just one foreign currency* to manage risk. The cells of Table II indicate the unconditionally optimal amount of U.S. dollars invested long (if positive) or short (if negative) in the respective currency (in columns), given a certain base country stock index (and currency) (in rows). We report Newey-West heteroskedasticity and autocorrelation consistent standard errors in parentheses below each optimal currency exposure, and indicate with one, two, or three stars coefficients for which we reject the null of zero at 10%, 5%, and 1% significance level, respectively.

In other words, we consider an investor who is deciding how much to hedge of the currency exposure implicit in an investment in a specific stock market, in isolation of other investments the investor might hold. For instance, the first non-empty entry in the first column corresponds to the Australian stock market and the euro, and has a value of 0.45 (significant at 1%). This means that a risk-minimizing Euroland investor with an exogenous Australian stock market portfolio and with access to The Australian dollar and the euro should buy a portfolio of euro-denominated bills worth 1.45 euros per euro invested in the Australian stock market, financing this long position by borrowing in Australian dollars. That is, he should overhedge the Australian dollar exposure incurred through his stock market holdings, and a hold a net 45% exposure to the EUR/AUD exchange rate.

Table II Optimal Currency Exposure for Single-Country Stock Portfolios: Single Currency Case

Stock Market	Euroland	Australia	Canada	Japan	Switzerland	United Kingdom	United States
Euroland		-0.47***	-0.55***	0.07	0.65*	-0.45***	-0.31
		(0.16)	(0.19)	(0.22)	(0.36)	(0.17)	(0.24)
Australia	0.45***		0.25	0.31**	0.41***	0.19	0.43***
	(0.15)		(0.17)	(0.13)	(0.14)	(0.18)	(0.14)
Canada	0.43***	-0.22		0.32*	0.42***	0.11	1.18***
	(0.12)	(0.20)		(0.17)	(0.12)	(0.16)	(0.29)
Japan	0.04	-0.31	-0.28		0.15	-0.13	-0.09
	(0.28)	(0.20)	(0.21)		(0.25)	(0.26)	(0.18)
Switzerland	-0.59**	-0.44***	-0.49***	-0.14		-0.44***	-0.39***
	(0.25)	(0.11)	(0.13)	(0.15)		(0.14)	(0.14)
United Kingdom	0.35**	-0.29**	-0.29*	0.08	0.36***		-0.04
	(0.15)	(0.14)	(0.17)	(0.14)	(0.12)		(0.21)
United States	0.05	-0.36**	-1.01***	0.06	0.13	-0.15	
	(0.17)	(0.18)	(0.26)	(0.14)	(0.12)	(0.24)	

Notes: As in Table IIIA of Campbell et al. (2010), this table shows an investor with a position in one of the seven considered stock markets, who chooses a single foreign currency position to minimize the variance of his portfolio. Rows indicate the base country stock index, while columns indicate the currencies used to manage risk. The positions shown are relative to full currency hedging, with a zero net-investment position taken in all currencies combined. The reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. Cells are obtained by regressing the hedged excess return to the row country stock market onto the excess return on the column country currency. We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure. Coefficients are marked with one, two, or three asterisks if the null of zero is rejected at 10%, 5%, and 1% signficance level, respectively.

Looking at the results, we can see that investors in the Australian and Canadian stock markets tend to profit from overhedging the exposure relative to most currencies, especially against (historically) lower-yielding reserve currencies, such as the U.S. dollar or the Swiss franc, but also the Euro (which is proxied by the German mark from 1976 to 1999). On the other hand, investors in the Euro, Swiss or U.S. stock markets oftentimes benefit from underhedging their foreign currency positions, especially against the Australian and Canadian dollars. These optimal currency exposures are driven strongly by the correlation structure of the respective currencies excess returns against the country stock markets. The Australian and Canadian dollars are historically higher-yielding currencies that often move in the same direction as stock markets. The Swiss franc and the German mark, on the other hand, have historically been lower-yielding reserve currencies with the desirable property to be negatively correlated to stock markets, particularly in stress periods.

So far these findings are in line with Campbell et al. (2010)s results. However, there are also differences in our findings, due mostly to the inclusion of the crisis period 2006-2010 in the sample. In this time span, the Japanese yen behaved as a safe-haven currency, moving against equity markets in times of stress, while the euro was a risky currency moving in-line with equity markets. This impacts our results with regards to investors in the Japan and Euroland stock markets: While Euro, Swiss and British investors in the Japanese stock market should have overhedged their yen exposure in Campbell et al. (2010)s sample, this is no longer the case. Generally, the yen should be about fully hedged by investors domiciled in the other six currencies. Also, investments into the Euroland stock market should generally be less strongly underhedged than in Campbells sample, where all but Swiss investors should have underhedged their euro exposure.

B.2. Single Country Stock Portfolios with Multiple Currencies Exposure

Table III depicts the situation for investors in the respective row stock markets that now have not one, but all of the discussed currencies at their disposal to optimally hedge their 3-month stock returns. Note that the investor basically overlays an optimal zero-net investment strategy on top of his inherent currency exposure through the stock market investment. Thus, the currency exposures in each row must add up to zero, with the diagonal term (stock market base currency) being the opposite of the sum of the other cells in the same row.

Some stylized facts emerge: Most strikingly, the Canadian dollar seems to be positively correlated to all country-stock markets and thus a short position in the Canadian dollar is optimal for investors in all seven stock markets. Less strong are the short positions in the Australian dollar and the British pound. These currency short positions are then invested in positive positions in the U.S. dollar and/ or the Swiss franc. Positions in the Japanese yen are generally near zero (fully hedged).

Compared to Campbell et al. (2010), in our analysis the optimal euro exposure is lower, usually only slightly (statistically insignificantly) above zero. On the other hand, the optimal yen exposure is only slightly negative, much closer to zero, than in Campbell et al. (2010)'s sample until 2005. This is most likely due to the aforementioned depreciation (appreciation) of the euro (yen) in the Financial crisis.

Table III Optimal Currency Exposure for Single-Country Stock Portfolios: Multiple Currency Case

Stock Market	Euroland	Australia	Canada	Japan	Switzerland	United Kingdom	United States
Euroland	0.12	-0.21	-0.60**	0.12	0.42	-0.18	0.33
	(0.32)	(0.15)	(0.26)	(0.17)	(0.27)	(0.21)	(0.32)
Australia	0.39	-0.27*	-0.66***	-0.05	0.22	-0.37**	0.74***
	(0.24)	(0.16)	(0.19)	(0.14)	(0.24)	(0.18)	(0.24)
Canada	0.19	-0.22	-0.97***	-0.10	0.39*	-0.35**	1.06***
	(0.22)	(0.14)	(0.27)	(0.14)	(0.22)	(0.16)	(0.26)
Japan	0.12	-0.30	-0.72**	-0.10	0.38	-0.12	0.73**
	(0.27)	(0.22)	(0.32)	(0.19)	(0.28)	(0.20)	(0.35)
Switzerland	0.10	-0.21*	-0.42**	0.07	0.43*	-0.11	0.14
	(0.28)	(0.11)	(0.21)	(0.15)	(0.23)	(0.20)	(0.26)
United Kingdom	0.25	-0.22*	-0.63***	-0.10	0.31	-0.08	0.47**
	(0.31)	(0.13)	(0.19)	(0.13)	(0.29)	(0.20)	(0.23)
United States	0.00	-0.09	-0.98***	-0.10	0.39*	-0.14	0.91***
	(0.23)	(0.11)	(0.22)	(0.13)	(0.20)	(0.17)	(0.26)

Notes: As in Table IIIB of Campbell et al. (2010), this table shows an investor with a position in one of the seven considered stock markets, who chooses multiple foreign currency positions to minimize the variance of his portfolio. Rows indicate the base country stock index, while columns indicate the currencies used to manage risk. The positions shown are relative to full currency hedging, with a zero net-investment position taken in all currencies combined. The reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. Cells (except diagonal terms) are obtained by regressing the excess return to the row country stock market onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms are obtained by computing the opposite of the sum of the other terms in the same row and the corresponding standard deviation. We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure. Coefficients are marked with one, two, or three asterisks if the null of zero is rejected at 10%, 5%, and 1% signficance level, respectively.

B.3. Global Portfolio with Mutiple Currencies Exposure

Lastly, we analyze the unconditionally optimal hedging rationale of an investor in a global equity portfolio with all seven above currencies at his disposal. We form an equally-weighted portfolio of all seven stock markets above, which we regress on the 6 bilateral FX excess returns against the USD. As Campbell et al. (2010) show (in the Internet Appendix), the choice of base currency does in this case not impact the analysis the optimal FX hedging rationale is independent of the base currency of the investor.

As can be seen in Table IV below, investors should have positive and economically significant positions in the U.S. dollar and the Swiss franc. For every U.S. dollar invested in the (fully hedged) global equity portfolio, the investor should optimally hold exposure amounting to 1 U.S. dollar in the U.S. dollar and the Swiss franc (0.63 + 0.36). This is financed through short positions in the Canadian dollar, and to a lesser extent the Australian dollar. The optimal positions in the euro, the Japanese yen and the British pound are insignificantly different from full currency hedging; there should be no added exposure to these currencies.

The analysis of global equity portfolios thus reaffirms the roles of the U.S. dollar and the Swiss franc as *reserve currencies* with an unconditionally beneficial (negative) correlation to global equity markets. It also confirms the reverse for the Canadian dollar: As the Canadian dollar is strongly positively correlated with global equity markets, a mean-variance investor in global equities should hold a negative exposure to it.

Campbell et al. (2010)'s findings can mostly be replicated in our new dataset, especially in terms of the positive positions in the USD and the CHF as well as the negative position in the CAD. However, we generally find even more pronounced results in terms of the positive and negative positions in these currencies, again suggesting the effect of the added sample years 2006-2010 in which the currencies move against each other more strongly than in average years. Contrary to Campbell et al. (2010), we do not find a significant positive position in the EUR, and do find a statistically significant negative position in the AUD.

Table IV Optimal Currency Exposure for an Equally Weighted Global Equity Portfolio: Multiple Currency Case

Stock Market	Euroland	Australia	Canada	Japan	Switzerland	United Kingdom	United States
Global	0.17	-0.22*	-0.71***	-0.04	0.36**	-0.19	0.63**
Portfolio	(0.20)	(0.11)	(0.19)	(0.12)	(0.18)	(0.15)	(0.25)

Notes: As in Table IV of Campbell et al. (2010), this table shows an investor holding a portfolio of stocks from all seven considered countries, with equal weights. He chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country (In this case, we chose USD as the base currency). The columns indicate the currencies used to manage risk. Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio. We run monthly monthly regressions of overlapping 3-month returns. Standard errors are corrected for autocorrelation due to overlapping intervals using the Newey-West procedure. We mark with one, two, or three asterisks coefficients for which we reject the null of zero at a 10%, 5%, and 1% significance level, respectively.

C. Introducing the FX risk factors: DOL, HML and VOL

As mentioned before, the main contribution of this paper is to analyze whether the optimal mean-variance hedging rationale of Equity, Bond and Commodities investors is impacted by certain risk factors. That is, we are looking whether conditioning upon risk factors/ state variables can improve upon the unconditional mean-variance hedging discussed above. To this end, this chapter introduces three risk factors that are discussed in current literature to explain the cross-section of currency returns (see sections I.A and II.A). We look at whether and how conditioning upon these risk factors impacts the mean-variance optimal currency exposure of investors. We interact the risk factors with currency excess returns, and see whether we can better account for global equity returns. Finally, we compare how the conditional-optimal hedging regime fares relative to unconditional-optimal and full hedging in terms of the risk of the overall investor portfolios.

We use three risk factors as conditioning state variables in our analysis: The Dollar *(DOL)*, High-Minus-Low *(HML)* and Volatility *(VOL)* risk factors. As discussed, the DOL and HML are based on Lustig et al. (2011)'s paper on Common Risk Factors in Currency Markets. DOL is defined as the average excess return of all foreign currency against the dollar, while HML is the difference in returns between the highest and lowest discount portfolios, and is a proxy for carry trade risk. Lustig et al. (2011) can relate the latter to global equity market volatility. Our third risk factor is based on global FX Volatility itself, using the proxy of Menkhoff et al. (2012), as below:

$$VOL_t = \frac{1}{T_t} \sum_{t \in T_t} \left[\sum_{k \in K_\tau}^{n} \left(\frac{\left| r_\tau^k \right|}{K_\tau} \right) \right]$$

VOL_t is the global FX volatility proxy in month t, $\left| r_\tau^k \right|$ is the absolute daily log return of currency k on day τ, K_τ denotes the number of available currencies on day τ, and T_t denotes the total number of trading days in month t.

The VOL factor thus averages absolute returns across all currencies available on any given day and averages daily values up to the monthly frequency (bilateral exchange rates against the USD).

We directly obtain the above three risk factors as calculated by Menkhoff et al. (2012) and underlying their article Carry Trades and Global Foreign Exchange Volatility. These are given at end-of-month intervals. We then adjust them to 3-month overlapping log-variables to be consistent in the analysis.

Below table V shows the summary statistics of the three risk factors (in monthly, unadjusted form). The DOL and HML can be interpreted as monthly portfolio returns in percentages: DOL as all currencies against the USD, HML as a monthly-rebalanced zero net-investment high-minus low forward discount portfolio. They can thus take positive and negative values. VOL is inherently an absolute measure and can only take on positive values. DOL and HML as well as DOL and VOL are only weakly correlated. Somewhat more pronounced is the negative correlation between HML and VOL, indicating that increasing FX volatility is negatively related to carry trade returns, which makes intuitive sense.

As can be seen in figure A.1 in the Appendix, the negative correlation between the HML and DOL returns and VOL is particularly pronounced in negative (risk) return states, when volatility increases as high-yielding currencies depreciate strongly.

Table V Summary Statistics of DOL, HML & VOL risk factors (monthly, unadjusted, 1976:3 - 2010:1)

	DOL	HML	VOL
Standard Deviations	2.28	2.58	0.15
Correlation Matrix			
DOL	1	-0.018	-0.088
HML		1	-0.188
VOL			1

Notes: This table shows the summary statistics of the DOL, HML and VOL risk factors based on Lustig et al. (2011)'s definition of DOL and HML and Menkhoff et al. (2012)'s definition of VOL. Data are monthly (end-of-month), retrieved from Menkhoff et al. (2012). DOL is the average return of all currencies against the U.S. dollar. HML is the return to a "carry trade portfolio", consisting of a long position in the highest forward discount currencies and a short position in the lowest (negative) forward discount currencies (against the USD). VOL is a measure of volatility in the foreign exchange market.

D. Conditional Currency Hedging of a Global Equity Portfolio

Having introduced the risk factors, we now look at whether they add conditioning information to a mean-variance investor in the global equity portfolio, who has the seven currencies at his disposal. In other words, we analyze whether optimal currency exposure in the seven currencies is not constant, but varying with the changing risk factors.

The baseline of comparison is the unconditional analysis from the last section (USD-based investor in global equities). We will compare these results with the results of the conditional models with each risk factor individually, as well as jointly. The conditional models, additionally to regressing on simple currency excess returns, regress on currency excess returns interacted with the risk factors. We analyze whether any of these terms have additional predictive power (significant coefficients). This would imply that the given risk factor adds conditional information to the given currency excess return, that is that the optimal currency exposure is not constant, but conditional on the varying risk state.

Table VI below shows the results of the analysis, and juxtaposes the conditional results (Panels B to E) to the unconditional results discussed in table IV earlier (Panel A)[2]. We standardize[3] the risk interaction terms, so that the sensitivity of the given optimal currency exposure to the given risk factor (a coefficient) can be read in terms of standard deviations of the interaction term.

[2] The table A.1 in the Appendix additionally shows the Conditional models with only significant coefficients included. These were arrived at by first including only the significant coefficients of the models in Table VI, and then checking significance of coefficients upon inclusion one-by-one.

[3] $\frac{x-\mu}{\sigma}$

This should ease interpretation. Other than in the unconditional case, the conditional analysis is dependent on the home currency of the investor[4]. We analyze in terms of a USD-domiciled investor who can hedge with all six bilateral exchange rates against the USD. Thereby, we cannot, as in the unconditional case, calculate the home currency position (USD) as a static residual position (the sum of the opposite of the regression coefficients), as it varies with the risk factors.

Lastly, we note that the table VI helps us understand the *sensitivity* of optimal exposure to the various currencies, as risk factors change. It does not, however, give us a good intuition about what that means in terms of actual net currency exposure on average or in different states. This intuition will helped by Table VII further down in this section.

Formally, we now add to the prior analysis (unconditional mean variance investor with an equally-weighted portfolio and multiple currencies) the three conditional interaction terms in below formula:

$$\begin{aligned}
&\mathbf{1}'\boldsymbol{\omega}_t\left(\boldsymbol{r_{t+1}}-\boldsymbol{i_t}\right)=\gamma_0-\boldsymbol{\Psi}_u'\left(\boldsymbol{\Delta}s_{t+1}+\boldsymbol{i_t}-\boldsymbol{i_t^d}\right)\\
&-\boldsymbol{\Psi}_{DOL}'\left(\boldsymbol{\Delta}s_{t+1}+\boldsymbol{i_t}-\boldsymbol{i_t^d}\right)DOL_{t+1}\\
&-\boldsymbol{\Psi}_{HML}'\left(\boldsymbol{\Delta}s_{t+1}+\boldsymbol{i_t}-\boldsymbol{i_t^d}\right)HML_{t+1}\\
&-\boldsymbol{\Psi}_{VOL}'\left(\boldsymbol{\Delta}s_{t+1}+\boldsymbol{i_t}-\boldsymbol{i_t^d}\right)VOL_{t+1}
\end{aligned}$$

As we already discussed, $\boldsymbol{\omega}_t$ is the (n+1 x n+1) diagonal matrix of stock market weights and $\boldsymbol{\Psi}_u$ is the (n+1 x 1) vector of unconditional currency weights. $\boldsymbol{\Psi}_{DOL}, \boldsymbol{\Psi}_{HML}$ and $\boldsymbol{\Psi}_{VOL}$ are the (n+1 x 1) vectors of optimal currency weight sensitivity to the global (scalar) DOL_t, HML_t and VOL_t risk factors.

[4] This has yet to be proven or shown

	Panel A: Unconditional Model	Panel B: Conditional Model (VOL)	Panel C: Conditional Model (HML)	Panel D: Conditional Model (DOL)	Panel E: Conditional Model (all)
	Estimate (Std. Error)	Estimate (Std. Error)	Estimate (St. Error)	Estimate (St. Error)	Estimate (St. Error)
EUR_USD	0.17 (0.20)	1.05** (0.49)	0.28 (0.25)	0.25 (0.18)	0.76 (0.58)
AUD_USD	-0.22* (0.11)	-0.26 (0.31)	-0.23** (0.09)	-0.17 (0.11)	-0.29 (0.21)
CAD_USD	-0.71*** (0.19)	0.22 (0.47)	-0.81*** (0.19)	-0.76*** (0.16)	0.37 (0.49)
JPY_USD	-0.04 (0.12)	-0.02 (0.21)	0.05 (0.14)	-0.11 (0.09)	0.08 (0.21)
CHF_USD	0.36** (0.18)	-0.03 (0.47)	0.14 (0.22)	0.31 (0.18)	-0.30 (0.37)
GBP_USD	-0.19 (0.15)	-0.04 (0.30)	-0.04 (0.17)	-0.15 (0.17)	0.57 (0.42)
USD	0.63** (0.25)	NA	NA	NA	NA
EUR * VOL		-5.63* (3.28)			-3.18 (4.04)
AUD * VOL		0.80 (2.25)			0.83 (1.29)
CAD * VOL		-3.62* (2.16)			-4.45** (1.98)
JPY * VOL		-0.66 (1.53)			-0.87 (1.26)
CHF * VOL		2.62 (3.48)			3.09 (2.38)
GBP * VOL		-0.17 (2.05)			-3.37 (2.16)
EUR * HML			0.01 (1.28)		0.62 (1.12)
AUD * HML			0.91 (0.82)		1.18 (0.82)
CAD * HML			0.48 (0.86)		-1.25 (1.02)
JPY * HML			-1.51** (0.71)		-0.73 (0.53)
CHF * HML			1.77 (1.18)		1.41 (1.10)
GBP * HML			-2.06* (1.12)		-3.57** (1.43)
EUR * DOL				0.01 (1.55)	-1.60 (1.80)
AUD * DOL				-1.03 (1.11)	-1.03 (1.02)
CAD * DOL				1.64* (0.90)	2.02** (0.98)
JPY * DOL				0.51 (0.70)	-0.30 (0.61)
CHF * DOL				1.29 (1.34)	2.25 (1.52)
GBP * DOL				0.01 (1.31)	0.33 (1.31)
R-squared (Adj.)	0.25 (0.24)	0.31 (0.29)	0.31 (0.29)	0.30 (0.28)	0.40 (0.36)

Notes: This table compares the unconditional and conditional currency hedging models for a mean variance investor into the equally-weighted global equity portfolio. Panel A shows the optimal currency weights of the unconditional model (same as Table IV before). Panels B-E show the conditional currency hedging models. These consist of the non-interacted currency weights on top, as well as the risk interaction terms (sensitivity to particular FX risk factor) below. These risk interaction terms are standardized, and can thus be read in terms of standard deviation. Note that the effective currency holding in conditional models varies with the values of the risk factors, and cannot be read off this table.

Table VI Optimal Currency Exposure for Equally-Weighted Global Equity Portfolio (Multiple Currencies): Unconditional Case and With Conditional Risk Factors

D.1. Conditional vs Unconditional Models

First thing to note is that indeed the (adjusted) R-squared of all conditional models are higher than in the unconditional case. This can be seen as a first potential indication that all the conditional risk factors add to the explanatory power of the model, and thus potentially to the FX hedging strategy of the equity investor[5].

Secondly, we note that by including the risk factors and interaction terms, the unconditional coefficients of many currency excess returns (such as CAD/USD or CHF/USD) vanish. This is strong indication that many of the negative and positive coefficients in the unconditional analysis (Panel A) are due to correlation between various currency and equity returns that are ultimately driven by the now included risk factors.

Thirdly we note the high (Newey-West) standard errors in the conditional models. The Newey-West standard errors are quite high relative to the conditional coefficient estimates. On top of that, particularly the introduction of the VOL risk factor additionally increases the standard errors on the unconditional coefficient estimates. These could be arising due to a variety of potential reasons: For one, we estimate a high number of cofficients, which could be multicollinear to some degree. For two, the DOL, HML and VOL time series might display inherent autocorrelation (they are arguably not random, but come in cycles of risk). For three, the correlation structure might be different in different states of the world or in different times. For four, there might in fact be a reasonable amount of randomness or noise in the coefficients. The standard errors in any case should make us wary of the danger of overfitting and thus beget additional robustness tests.

D.2. Individual risk factor-based models

We turn the attention first to the conditional models incorporating a single-risk factor, depicted in Panels B to D.

The VOL risk factor seems to drive a lot of the correlation structure (positive, negative) between currency excess returns and global equity returns. When including VOL as a risk factor, all of the significant (FX excess return) coefficients in the unconditional model vanish. On the other hand, we obtain a significant positive coefficient on the unconditional EUR/USD excess return, while obtaining negative coefficients on the EUR/USD * VOL as well as CAD/USD * VOL interaction terms. This is strong indication that a lot of the detected positive and negative unconditional hedging demands in currencies are ultimately due to currencies different sensitivity to the volatility risk factor. It also implies that by conditioning on Volatility alone, investors could potentially hedge better and simpler than unconditionally. The VOL conditional model reaches a higher R-squared than the unconditional model, and only relies on two currency positions in the EUR/USD and CAD/USD, as these react particularly negatively to volatility innovations (5.6 and 3.6 factors of standard deviation).

[5]The R-squared are obtained by the underlying regression model (regressing global equity returns on unconditional and conditional factors) *before* switching the sign of the unconditional and conditional slopes to arrive at mean-variance FX coefficients.

The HML model, other than the VOL model, leaves a lot of the unconditional partial correlation structure of the currency excess returns with equity returns intact, so that the coefficients on AUD/USD and CAD/USD stay significantly negative, even slightly amplified. It also has significantly negative JPY/USD * HML and GBP/USD * HML interaction terms, meaning that the higher the (absolute) products of these two currencies with HML excess returns, the more negative should be the position in these two currencies. Interestingly, these are two currencies whose unconditional exposure does not impact performance. Only in interaction with Carry Trade risk (HML) does the mean variance investor add exposure to these currencies.

The DOL conditional model, as the HML model, leaves some of the unconditional correlation structure of the currency excess returns with equity returns intact. The CAD/USD and the CHF/USD coefficients remain close to their unconditional values, although the CHF/USD coefficient narrowly misses significance at the 10% level. Additionally, there is a strongly positive CAD/USD * DOL term. The DOL model is quite simple. It relies mainly on one bilateral exchange rate: the CAD/USD. The higher the DOL (meaning that the Dollar depreciates relative to broad currency basket), the more positive should be the position in the CAD as a high-yielding currency strongly positively correlated with equities. Vice versa, as DOL tanks (in risk states) and the USD appreciates, the conditional mean variance investor should take a strongly negative position in the CAD/USD.

The HML and DOL risk interaction terms are somewhat less intuitive than the VOL one, as they are the product of two variables which can both be negative or positive (FX excess returns and risk factors).

D.3. Joint conditional model with all risk factors

What happens when we form a conditional model with all three risk factors combined? Are the risk factors complementary, or are the risk factors essentially proxies for the same underlying risk?

The (adjusted) R-squared of the joint model is significantly higher than of the single-risk factor models. This is a first indication that the three risk factors are proxies for different underlying sources of risk, and all add different and useful conditioning information.

Also, all of the risk factors have at least one significant interaction term with excess returns. Fourthly, of the interaction terms, the CAD/USD * DOL has positive (negatively correlated with equities) and the JPY/USD * HML, the GBP/USD * HML and CAD/USD * VOL negative (positively correlated with equities) hedging properties, meaning that the conditional mean variance investor into the global equity portfolio should take positive (negative) exposure to these terms. The positive CAD/USD * DOL is at first surprising, as the CAD/USD has a strongly negative coefficient in the unconditional model. However, interpretation is not easy, especially as the interaction terms are factors of two terms. It might arise primarily due to negative correlation with equities in states where both CAD/USD and DOL are negative or where both are positive. The negative JPY/USD * HML and GBP/USD * HML terms indicate that the higher the absolute JPY/USD, GBP/USD and HML returns become, the lower the exposure of the conditional mean-variance investor in the JPY and GBP currencies. Particularly large is the negative coefficient of the CAD/USD * VOL term, indicating that the higher the FX market volatility the more negative the position should be in the CAD/USD of the conditional investor. This makes intuitive sense, as the CAD/USD (as a risky currency against the USD) is moving down with equities in times of increased stress and volatility.

D.4. Economic significance of the risk factors on optimal currency hedging

While we have previously seen (in Table VI) that the optimal currency exposure in the 7 currencies have differing sensitivities to the risk factors (through the coefficients of the interaction terms), the table is not intuitive to interpret, as it does not show us the net optimal currency exposure in different risk states.

To get a better intuition of what the estimated models mean for effective currency exposure of a conditional mean-variance investor in global equities, we analyze the FX holdings of the estimated conditional models (Panels B to E in Table VII below) at the 5%, 50% and 95% percentiles of the risk factors and juxtapose these to the unconditional optimal currency demands of the unconditional model (Panel A in Table VII). Table A.2 in the Appendix additionally shows the 90% confidence intervals of the currency positions. This way we get a better feeling of the sensitivity of the optimal currency exposure in a currency to the changing risk factors, as well as how far it deviates from its unconditional value. The DOL and HML risk factors are thereby sorted from low to high, and the VOL risk factor in opposite direction. Low negative values of DOL and HML are indicative of risk states (stress), such as during unwinding of carry trades in the markets; the same goes for high values of VOL.

First thing to note is that some currency exposures generally seem more sensitive than others to the various risk factors. The CAD/USD exposure reacts particularly sensitively to the changing risk factors, varying drastically between different risk regimes. The different risk factors also each seem to be affect different currency pairs: while Volatility has a strong impact on the optimal EUR/USD, CAD/USD and USD exposures, the HML risk factor particularly influences the JPY/USD and CHF/USD exposures, for instance.

As discussed above, the EUR/USD exchange rate exposure is particularly affected by volatility: Optimal exposure decreases with increasing volatility, being significantly positive in calm periods, and slightly negative in high volatility periods. It is not as much affected by the HML and DOL risk factors. Relative to other currencies, the AUD/USD exposure does not vary very strongly, and is mostly at a negative value close to zero. The CAD/USD exchange rate on the other hand varies most strongly of all bilateral exchange rates with the risk factors. In bad states, it is very negative across all risk factors, and stays negative even in good states (however, much less negative; strong increase in net exposure). It is extremely strongly related to the DOL and VOL factors. The JPY/USD exchange rate is most strongly related to the HML risk factor. In times of low HML, optimal exposure is positive and in times of low HML, it is negative. This pattern might be expected of a safe, low-yielding currency.

Table VII Effective Currency Exposure depending on risk states of DOL, HML and VOL

	EUR_USD	AUD_USD	CAD_USD	JPY_USD	CHF_USD	GBP_USD	USD
Panel A: Unconditional model							
	0.17	-0.22*	-0.71*	-0.04	0.36*	-0.20	0.63*
Panel B: VOL Risk Factor							
5% (percentile)	-0.23	-0.10	-1.05*	-0.16	0.53	-0.08	1.09*
50% (percentile)	0.24	-0.16	-0.58*	-0.11	0.32	-0.06	0.34*
95% (percentile)	0.59*	-0.20	-0.23	-0.07	0.17	-0.05	-0.21
Panel C: HML Risk Factor							
5% (percentile)	0.28	-0.35*	-0.93*	0.31	-0.17	0.31	0.54
50% (percentile)	0.29	-0.18	-0.77*	-0.06	0.28	-0.19	0.63*
95% (percentile)	0.39	-0.05	-0.64*	-0.34	0.61	-0.57	0.69*
Pane D: DOL Risk Factor							
5% (percentile)	0.25	0.00	-1.25*	0.02	0.03	-0.15	1.11*
50% (percentile)	0.25	-0.18	-0.73*	-0.11	0.32	-0.15	0.60*
95% (percentile)	0.25	-0.34*	-0.29	-0.22	0.57*	-0.15	0.17
Panel E: All Risk Factors							
5% (percentile)	0.27	-0.11	-1.51*	0.08	-0.38	0.43*	1.21*
50% (percentile)	0.33*	-0.14	-0.71*	-0.10	0.25	-0.11	0.47*
95% (percentile)	0.35	-0.17	-0.06	-0.25	0.77*	-0.52*	-0.12

Notes: This table shows the effective currency exposure based on the different percentile values of the risk factors (5%, 50% and 95% percentile values). The DOL and HML risk factors are sorted in ascending order, from lowest to highest returns (very low negative values can be indicative of stress in the FX market). VOL is sorted in descending order, from highest to lowest (logic is in opposite direction, with high values indicative of stress). These are point estimates, relying on all estimated (point estimates) coefficient values, in the models of Panel B to Panel E in Table VI. Asterisks denote currency positions whose 90% confidence interval do not cross the border of zero (see Table A.2 in the Appendix, which shows the confidence intervals of currency positions of above models). This is indicative of higher certainty as regarding to whether the currency should be overhedged or underhedged in a given risk state. Unsurprisingly, it is the currencies that have significant unconditional and conditional coefficients whose confidence interval does not cross the 0-border.

Somewhat puzzling is the CHF/USD holding pattern in our HML model. Thereby, the optimal CHF holding is negative for low HML returns (in times of stress). This is counter-intuitive, given the desirable risk property of negative correlation of CHF to equity returns, the status of the CHF as a safe haven currency, as well as the unconditionally strong demand for CHF exposure. It is also the opposite of its co-movement with the VOL risk factor, where the CHF exposure intuitively increases monotonically with higher volatility. The findings thus have to be interpreted with care, and the effect of the positive HML * CHF/USD coefficient (Table VI) leading to that result should be further investigated. HML can take both positive and negative values, and thus the HML * CHF/USD interaction term is harder to interpret than the interaction term with absolute VOL.

The USD exposure seems to be particularly strongly affected by all risk factors although to a somewhat lesser extent the HML factor. In the cases of the VOL and DOL factors it seems to be a residual term with negative values (providing liquidity as a borrowing currency) in good times, and with positive exposure in negative times, as optimal exposure to investment currencies, such as the

CAD, is negative. The interpretation of the DOL factor here has to be handled with care, however: it basically is just the opposite of the performance of the DOL against all other currencies. That the USD exposure should be positively related to USD performance is clear, and does not allow the interpretation as a risk factor.

E. Conditional vs Unconditional FX hedging: Portfolio Performance

Having discussed the risk factor-based conditional models of a global equity investor, we now turn to comparing the performance of these models to complete and unconditional hedging. Campbell et al. (2010) in their study find a statistically significantly decreased standard deviation of unconditional hedging relative to full hedging. On the other hand, their conditional model based on differences to time-series averages of the forward discounts/ interest rates between countries does not decrease standard deviation significantly.

We undertake a similar performance comparison with different conditional information –our FX risk factors –, and in an enlarged dataset –adding the 2006-2010 crisis and post-crisis years. Additionally to Standard deviation, we also consider the Mean returns and the Sharpe ratio of returns under the different hedging regimes. This gives us additional insights as to what the costs are (in terms of foregone mean return) for these potential reductions in standard deviation. The Sharpe ratio as the ratio between average performance and volatility of that performance is an arguably better-suited performance measure for the different hedging regimes. With a higher Sharpe ratio-Portfolio, one can replicate any level of standard deviation of a lower Sharpe ratio-Portfolio with a higher return by simply choosing the appropriate leverage ratio.

E.1. Standard deviation, Mean and Sharpe ratios of Complete, Unconditional and Conditional hedging

As discussed in section the portfolio log excess return can be decomposed into the sum of the fully hedged asset excess returns, the currency excess returns, as well as a Jensens inequality term. Here, we approximate the portfolio log excess returns as the sum of the equity and FX excess returns, not yet incorporating the Jensens inequality term . We annualize both standard deviation and mean returns. In case of the mean return, we add one-half of the variance of returns to form the adjusted average (to approximate simple returns). The Sharpe ratio is then the ratio of annualized mean return to annualized standard devation.

$$r^h_{p,t+1} - i_{1,t} = \mathbf{1}'\boldsymbol{\omega}_t \left(\boldsymbol{r_{t+1}} - \boldsymbol{i_t}\right) + \left(\mathbf{\Psi_u}' + \mathbf{\Psi}'_{\mathbf{c},\mathbf{t}}\right)\left(\mathbf{\Delta s_{t+1}} + \mathbf{i_t} - \mathbf{i^d_t}\right) + \frac{1}{2}\sum_{t}^{h}$$

Table VIII below shows us these three measures for the equally-weighted global equity portfolio under full hedging, unconditional hedging, and conditional hedging. We perform a horserace of the unconditional and conditional models with all currencies and interaction terms, as well as with only the significant coefficients.

Table VIII Performance of Hedged Global Equity Portfolios under Full Hedging, Unconditional Hedging and Conditional Hedging regimes

	Complete Hedge	Uncond Hedge (all)	Uncond Hedge (sign.)	DOL (all)	DOL (sign.)	HML (all)	HML (sign.)	VOL (all)	VOL (sign.)	All RF (all)	All RF (sign.)
St. dev.	14.38	12.39	12.52	12.04	12.48	11.95	13.27	11.88	12.16	11.17	11.81
Mean	5.08	3.79	3.92	6.23	5.40	3.86	4.60	4.95	5.49	5.41	5.81
Sharpe Ratio	0.35	0.31	0.31	0.52	0.43	0.32	0.35	0.42	0.45	0.48	0.49

Notes: This table shows the different performance measures (Standard deviation, Mean as well as Sharpe Ratio) of the equally-weighted global equity portfolio under different hedging regimes: Full Hedging, Unconditional as well as Conditional Hedging. The unconditional and conditional models are the ones from Table VI (Panels A-E) with all currency excess returns and interaction terms included, as well as additionally with only the significant coefficients included (see Table A.1 in the Appendix).

As in Campbell et al. (2010)'s findings, the unconditional hedge does reduce the standard deviation of the global equity portfolio relative to full hedging. While complete hedging would lead to an annualized standard deviation of around 15%, the unconditional hedging strategy leads to a reduction of around 2%. This finding is robust to taking only the statistically significant coefficients on the unconditional model. These are the AUD/USD with a negative exposure (-), the CAD/USD with a negative position (-) and the CHF/USD with a positive exposure (+). However, unconditional hedging does not only decrease the standard deviation, but also the average returns by nearly a third (roughly 1.5%). Overall, thus, the Sharpe ratio of the optimal unconditional hedging is actually significantly lower (0.27) than in the complete hedging scenario (0.34).

What does Conditional Hedging do to performance? The standard deviations of all models are lower than in the Complete Hedging and also the Unconditional Hedging regime. The average returns of the strategy are all higher than in the Unconditional Hedging regime. Compared to the Complete Hedge, the average returns are sometimes lower, sometimes higher. This leads to higher Sharpe ratios across all Conditional models compared to the Unconditional and Complete Hedging cases (with the exception of the HML model, which has a lower Sharpe ratio than the Complete Hedge). The short answer thus is that Conditional Hedging improves the risk/ return trade-off by both reducing volatility as well as often increasing expected returns.

We now analyze the discussed models in more detail. The VOL model leads to the lowest standard deviation of all single risk factor-conditional models with 12.40%. The mean return with around 4.6% is slightly lower than under complete hedging, still leading to an improved Sharpe ratio relative to unconditional and complete hedging. The result is robust to including only the significant currency and interaction term coefficients (EUR(+), AUD(-), EUR*VOL(-), CAD*VOL(-)), which actually increases the Sharpe ratio slightly.

The HML model leads to a similar standard deviation of around 12.5%, but also the lowest mean return of all conditional models with 3.83%. The standard deviation is thus lower than under complete hedging, but the mean return is also lower, leading to a Sharpe ratio of 0.31. This Sharpe ratio is better than the unconditional one, but lower than the complete hedging one. The result is robust to only including significant coefficients (AUD(-), CAD(-), EUR(+), JPY*HML(-),

GBP*HML(-), AUD*HML(+), CHF*HML(+)).

The DOL model with all coefficients included again achieves a standard deviation of around 12.5%, while at the same having the highest mean return of all hedging models by far: 6.65%. This results in a much higher Sharpe ratio of 0.53. When only including the significant coefficients (CAD(-), CHF(+), AUD(-), CAD*DOL(+)), the model has a slightly higher standard deviation, and quite strongly decreased mean returns of around 5%. However, even then its Sharpe ratio is still the highest of the single factor conditional models.

Lastly, we analyze the conditional model with all three risk factors combined. This model achieves the lowest standard deviation of all models with 11.6%, around 1.5 percentage points lower than the unconditional model. It also has a mean return of around 5.6%, which is nearly 2% higher than the unconditional model return of 3.6%. Consequently, the Sharpe ratio is close to 0.5, clearly outperforming the unconditional Sharpe ratio of slightly less than 0.3. The findings are robust to including only the significant coefficients (EUR(+), CAD*VOL(-), EUR*VOL(-), CAD*DOL(+), GBP*HML(-)).

E.2. Graphing the performance

Another way to look at the performance of the different hedging regimes is to plot and inspect them. Below figures A.1 and 2 show the excess returns of the equally-weighted portfolio under complete, unconditional and conditional hedging.

As can be seen, both the unconditional and conditional hedging curves show somewhat lower extreme negative returns during crisis periods, especially towards the latter half of the sample (mid 1990s to now). For instance, during the Asian financial crisis of 1998, the unconditionally and conditionally hedged returns do not go down to below -20%, as in the full hedging regime.

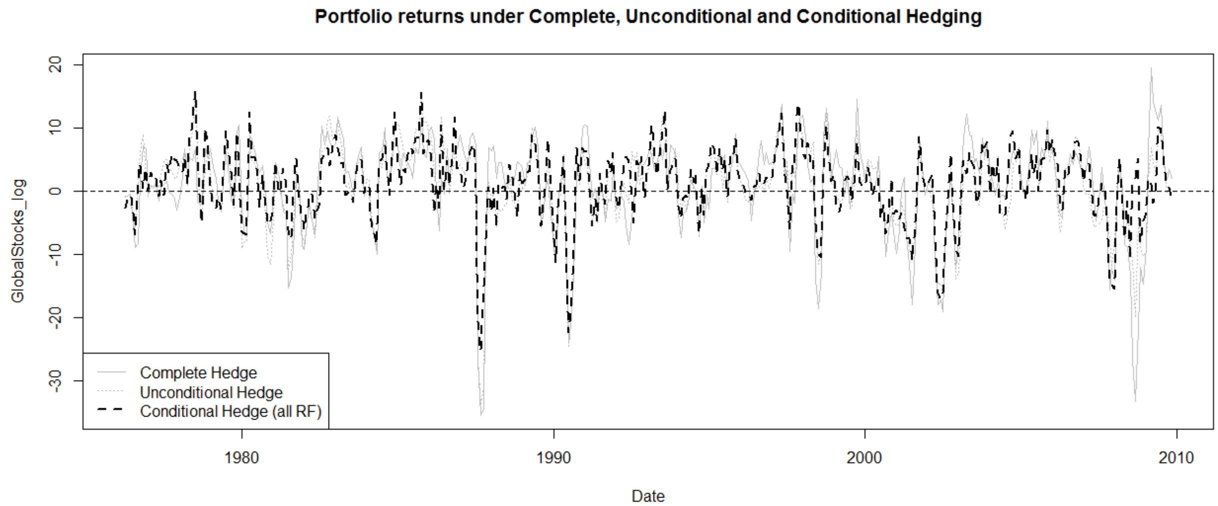
Another way to state this is that the unconditional correlation properties of currencies work to the advantage of the holder of the global equity portfolio in crisis periods: while stock markets tank, the combined currency exposures of the unconditional investor goes up and somewhat ameliorates the drawdown. The short position in risk currencies (such as the CAD) and the long position in safe haven currencies (such as the CHF) gains value in crisis periods.

Conditionally hedged returns perform visibly better than unconditionally hedged returns in two ways: For one, they perform better in crisis periods. Especially pronounced is the effect in September 2008, when the equally-weighted portfolio returns tank by nearly 30%, the unconditionally hedged returns go down by nearly 20%, while the conditionally hedged returns are even slightly positive due to the extreme counteracting effect of the conditional currency exposure. For two, the conditionally hedged returns are generally a bit higher during normal non-crisis periods, leading to a better risk-return trade-off across different risk states.

E.3. Interpretation

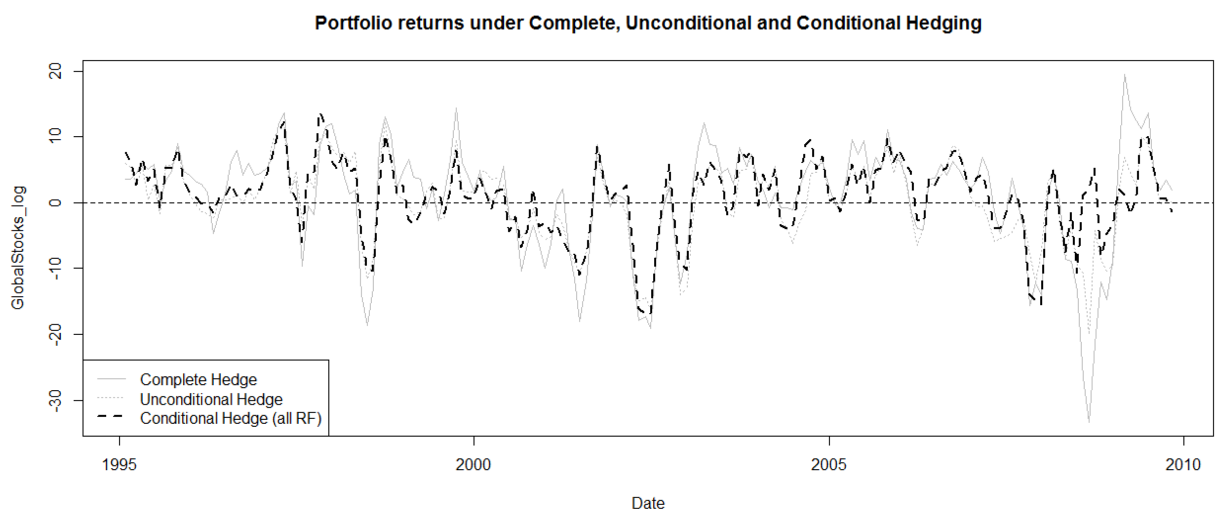
In summary, the conditional hedge outperforms its complete and unconditional counterparts in terms of the risk/ return trade-off. While the unconditional model decreases the volatility of the

Figure 1. Portfolio performance of different hedging regimes (1976:3 - 2010:1)



Notes: This figure shows the 3-month forward looking returns of the global equity portfolio under the three discussed hedging regimes at each month from 1976:3 to 2010:1. Completely hedged returns are in solid grey, unconditionally hedged returns in dashed grey, and conditionally hedged returns (incorporating all risk factors) in dashed black. The conditionally hedged returns are visibly less volatile, and perform better in crisis periods.

Figure 2. Portfolio performance of different hedging regimes (1995:1 - 2010:1)



Notes: This figure shows the 3-month forward looking returns of the global equity portfolio under the three discussed hedging regimes at each month from 1995:1 to 2010:1. Completely hedged returns are in solid grey, unconditionally hedged returns in dashed grey, and conditionally hedged returns (incorporating all risk factors) in dashed black. The conditionally hedged returns are visibly less volatile, and perform better in crisis periods.

global equity portfolio, it does so at the cost of also decreasing average returns. The resulting Sharpe ratio is actually lower than in the complete hedging case. This puts a question mark on the findings of Campbell et al. (2010) that equity fund investors should unconditionally hedge their currency exposure. Simply levering up the portfolio less, or holding more cash (proportionately inverse to the volatility of complete to unconditional hedging), might in fact lead to better performance.

The conditional model, on the other hand, decreases the volatility of the global equity portfolio even further than unconditional hedging, while at the same time not eating away at expected returns relative to full hedging. This leads to an improved Sharpe ratio of the conditional hedging regime, clearly outperforming both full as well as unconditional hedging.

IV. Findings and Further Analysis planned

A. Findings

As has been shown in recent literature, the average outperformance of higher-yielding currencies against lower-yielding currencies (Carry Trade) is driven by various risk factors (e.g. Lustig et al., 2011; Mancini et al., 2013; Menkhoff et al., 2012a & b; Mueller et al., 2012; Verdelhan, 2013). The average outperformance of these currencies is thus compensation for higher systematic risk in the sense of covariance with these risk factors.

Based on these findings, our analysis asks a related, but different question: What are the effects of these risk factors on the currency hedging rationale of international equity, bond and commodity investors? What happens to correlation between equities, bonds and commodities on the one hand and currencies on the other hand when risk factors change, and what impact does that have on a mean-variance investor?

Based on our preliminary analysis of a global equity portfolio and 7 major currencies, we find that the positive (negative) co-dependence of high-yielding (low-yielding) currencies with equities is indeed driven by different risk factors: In our sample these are the Volatility (VOL), Carry Trade (HML) and Dollar (DOL) risk factors. As the FX risk factors change, so do the expected returns across the equities and currencies realms, and so does the correlation between the two markets. This again impacts the optimal currency exposure of a mean-variance equity investor.

Thus, our analysis strongly suggests that the co-dependence of different financial markets (in this case, FX and equities) is driven by risk factors. This is significant in academic terms. It is also meaningful for asset and risk management practice. This is particularly the case if we can identify the valuable conditioning information in advance of the changed correlations and expected returns; more on this in the next section. In any case, it helps us better understand the dynamics between various markets under different risk conditions, and should thus help us to tail-hedge for instance.

The findings are also significant in that they pinpoint which currency exposures are particularly sensitive to what kind of risk. For instance, the optimal EUR/USD position is strongly impacted by Volatility risk, while the CAD/USD is generally very sensitive to all kinds of risk. This finding might be very useful for targeted hedging of particular risks: If an asset manager is particularly concerned about increasing volatility, he should drastically decrease his net EUR/USD position (or even go short) for instance. This is even more significant, as the conditional models are quite sparse, and beat the unconditional hedging performance with at most two or three currency weights. This is tantamount to ease of implementation.

A.1. Qualification of Findings

Two major qualifications have to be made regarding the findings of our preliminary analysis. For one, there are very large (Newey-West) standard errors of the coefficients with regards to unconditional and conditional currency exposure (Table VI). The size of the standard errors makes inference for some currency-risk factor combinations difficult; namely for those, for which the 90% confidence interval passes the 0-threshold in terms of effective currency exposure (see Table A.2 in the Appendix) for both the unconditional and conditional models. As a robustness test, we have thus tested how performance is affected when we only include statistically significant coefficients in both the unconditional and the conditional hedging models (Table VIII shows the performance of the models in Table A.1 in the Appendix –which generally holds up well.

The standard errors have multiple reasons to come about: For one, there is large autocorrelation in the overlapping 3-month returns we have used for our analysis so far by design. We have used these to ensure comparability with Campbell et al. (2010) who do this for their analysis. As an easy fix, we will look at how the analysis changes when we use non-overlapping returns (see next section).

But another, in a sense more fundamental reason for the large standard errors could be due to currencies changing their inherent risk characteristics in terms of unconditional and conditional correlation properties with the equities markets over time. As a case in point, the yen (euro) was an overhedged (underhedged) currency in Campbell et al. (2010)'s sample, indicating negative (positive) correlation properties for global equity investors from 1975-2005. In the sample period added from 2006-2010, the picture reversed: then yen as a safe-haven currency gained strongly against both the euro and the equities markets in the crisis. As much as risk characteristics of currencies change through time, the coefficients of our analysis will change too. This implies potentially fruitful research into country-specific risk factors (such as fiscal or monetary data of a country) added to or interacted with our global FX-market risk factors.

The second major qualification is how far the conditional models can actually be implemented by investors. Our conditional hedging models indicate what the optimal currency weights would be in different risk states. However, the conditionality built-in into these models is contemporaneous to the equity and FX excess returns. It is thus an open question to what extent investors can make use of the conditional risk factors. In terms of downside risk management, it is valuable in itself to know how the systematic currency risk characteristics change as the risk factors change. This is a question of risk preferences. However, to what extent one can actually approximate the returns of the optimal conditional models throughout the risk states (as in Table VIII) is a question that should be further examined. It strongly depends on the persistence as well as the ability to forecast the risk factors through time. It is a stylized fact that both Volatility as well as the performance of the Carry Trade comes in cycles, whereby periods of relatively stable Volatility / Carry Trade Performance drastically change to very different levels.

B. *Further Analysis Planned*

Further to our above analysis, our findings should be both corroborated as well as added to.

In terms of validating and corroborating our above results, we will firstly look at non-overlapping returns at various different frequencies (as opposed to the current 3-month overlapping returns, as in Campbell et al., 2010), and will see whether and in what way our results are affected. Newey-West standard errors might decrease, as autocorrelation decreases, improving the inference. We will use a different definition of the euro, not relying on the German mark before 1999, but value-weighting it with different European currencies ante-1999 (similarly to Campbell et al., 2010). We will transform the risk factors to log-likelihoods in order to simplify interpretation of the risk factors themselves as well as the interaction terms with currency excess returns. We will enlarge the sample to include the most recent 4 years 2011-2014. We will perform further robustness tests, such as looking at subsamples, and performing pseudo-out of sample forecasting to see how valuable a model calibrated in one time period really is in the next.

Importantly, we also want to extend our analysis. Further to equities, we will additionally look at how bond and commodity investors and their hedging rationales are affected by changing risk factors. We will introduce further risk factors –the Momentum, Illiquidity and Correlation factors discussed in sections I.A and II.A. We will analyze other performance measures, such as downstate standard deviations and maximum drawdowns, and will particularly focus on the risk properties of the different hedging regimes in crisis periods. Potential considerations, rather for future research, are the introduction of country-specific risk factors to account for the changing risk properties of currencies through time and of transaction costs to analyze the net effects of portfolio re-balancing following the changing risk factors.

Importantly, we will focus on the *tradability* of the discussed conditional models. In the current models, the investor at time t forms currency weights based on (equity, currency, etc.) excess returns as well as risk factors in $t + 1$. The insights would be even more enticing, if they could be traded in real-time, based on data available at time t when the portfolio weights are chosen. To this end, we will condition upon *contemporaneous and lagged* risk factors at time t, rather than $t + 1$.

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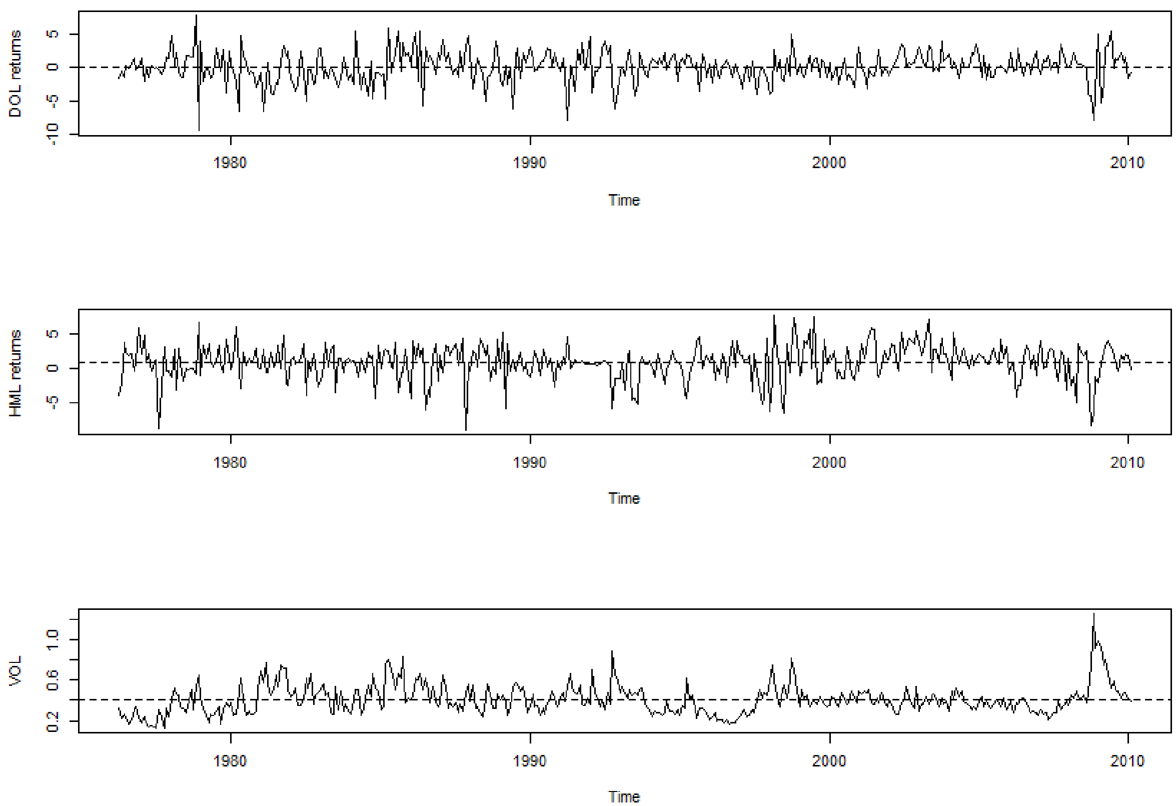
Appendix A. Further Graphs and Tables

Figure A.1. VOL, HML & DOL risk factors through time (1976:3 - 2010:1)



Notes: This figure shows the raw end-of-month VOL, HML & DOL risk factors through time. These are obtained from Menkhoff et al.,2012. Strong upticks in VOL are usually accompanied by strongly negative DOL and HML returns.

	Panel A: Unconditional Model	Panel B: Conditional Model (VOL)	Panel C: Conditional Model (HML)	Panel D: Conditional Model (DOL)	Panel E: Conditional Model (all)
	Estimate (Std. Error)	Estimate (Std. Error)	Estimate (St. Error)	Estimate (St. Error)	Estimate (St. Error)
EUR_USD		0.99*** (0.20)	0.43*** (0.14)		0.95** (0.61)
AUD_USD	-0.24*** (0.07)	-0.17* (0.10)	-0.24*** (0.09)	-0.20* (0.11)	
CAD_USD	-0.73*** (0.12)		-0.81*** (0.18)	-0.74*** (0.17)	
JPY_USD				-0.11 (0.09)	
CHF_USD	0.38*** (0.05)			0.38*** (0.13)	
GBP_USD				-0.15 (0.17)	
USD	0.59** (0.25)	NA	NA	NA	NA
EUR * VOL		-3.53** (1.48)			-3.13* (1.67)
AUD * VOL					
CAD * VOL		-2.92*** (0.48)			-3.96*** (0.52)
JPY * VOL					
CHF * VOL					
GBP * VOL					
EUR * HML					
AUD * HML			1.40** (0.55)		
CAD * HML					
JPY * HML			-1.56*** (0.54)		
CHF * HML			1.87*** (0.60)		
GBP * HML			-2.01** (0.83)		-2.47** (1.02)
EUR * DOL					
AUD * DOL					
CAD * DOL				1.30*** (0.46)	1.84** (0.80)
JPY * DOL					
CHF * DOL					
GBP * DOL					
R-squared (Adj.)	0.24 (0.24)	0.30 (0.29)	0.30 (0.29)	0.27 (0.26)	0.34 (0.33)

Notes: This table compares the statistically significant optimal currency weights in unconditional and conditional hedging models. Panel A shows the (significant) optimal currency weights of the unconditional model. Panels B-E show the (significant) coefficients of conditional currency hedging models. These consist of the non-interacted currency weights on top, as well as the risk interaction terms (sensitivity to particular FX risk factor) below. These risk interaction terms are standardized, and can thus be read in terms of standard deviation. Note that the effective currency holding in conditional models depends on the risk factors, and cannot be read off this table.

Table A.1 Unconditional and Conditional Currency Hedging models with significant coefficients

	EUR_USD	AUD_USD	CAD_USD	JPY_USD	CHF_USD	GBP_USD	USD
Panel A: Unconditional model							
	0.17	-0.22*	-0.71*	-0.04	0.36*	-0.20	0.63*
Confidence Interval	(-0.15, 0.50)	(-0.40, -0.03)	(-1.03, -0.39)	(-0.23, 0.15)	(0.07, 0.65)	(-0.44, 0.05)	(-0.23, 1.03)
Panel B: VOL Risk Factor							
5% (percentile)	-0.23	-0.10	-1.05*	-0.16	0.53	-0.08	1.09*
Confidence Interval	(-0.79, 0.33)	(-0.41, 0.20)	(-1.63, -0.47)	(-0.44, 0.12)	(-0.05, 1.11)	(-0.45, 0.30)	(0.68, 1.50)
50% (percentile)	0.24	-0.16	-0.58*	-0.11	0.32	-0.06	0.34*
Confidence Interval	(-0.31, 0.80)	(-0.47, 0.14)	(-1.16, -0.01)	(-0.39, 0.17)	(-0.25, 0.90)	(-0.43, 0.30)	(0.03, 0.65)
95% (percentile)	0.59*	-0.20	-0.23	-0.07	0.17	-0.06	-0.21
Confidence Interval	(0.04, 1.15)	(-0.51, 0.10)	(-0.81, 0.35)	(-0.35, 0.21)	(-0.40, 0.75)	(-0.42, 0.31)	(-0.64, 0.22)
Panel C: HML Risk Factor							
5% (percentile)	0.28	-0.35*	-0.93*	0.31	-0.17	0.31	0.54
Confidence Interval	(-0.44, 1.00)	(-0.61, -0.08)	(-1.50, -0.35)	(-0.12, 0.73)	(-0.85 , 0.52)	(-0.27, 0.88)	(-0.04, 1.11)
50% (percentile)	0.29	-0.18	-0.76*	-0.06	0.28	-0.19	0.63*
Confidence Interval	(-0.44, 1.00)	(-0.44, 0.09)	(-1.34, -0.19)	(-0.49, 0.36)	(-0.40, 0.96)	(-0.77, 0.39)	(0.38, 0.93)
95% (percentile)	0.29	-0.05	-0.64*	-0.34	0.62	-0.57	0.69*
Confidence Interval	(-0.43, 1.01)	(-0.31, 0.21)	(-1.21, -0.07)	(-0.76, 0.09)	(-0.07, 1.30)	(-1.14, 0.01)	(0.34, 1.14)
Pane D: DOL Risk Factor							
5% (percentile)	0.25	0.00	-1.25*	0.02	0.03	-0.15	1.11*
Confidence Interval	(-0.27, 0.77)	(-0.30, 0.31)	(-1.71, -0.79)	(-0.25, 0.28)	(-0.39, 0.45)	(-0.51, 0.20)	(0.65, 1.56)
50% (percentile)	0.25	-0.18	-0.73*	-0.11	0.32	-0.15	0.60*
Confidence Interval	(-0.27, 0.77)	(-0.48, 0.12)	(-1.20, -0.27)	(-0.38, 0.15)	(-0.10, 0.74)	(-0.51, 0.21)	(0.26, 0.94)
95% (percentile)	0.25	-0.34	-0.29	-0.22	0.57*	-0.15	0.17
Confidence Interval	(-0.27, 0.78)	(-0.64, 0.03)	(-0.76, 0.17)	(-0.49, 0.04)	(0.15, 0.99)	(-0.50, 0.21)	(-0.48, 0.83)
Panel E: All Risk Factors							
5% (percentile)	0.27	-0.11	-1.51*	0.08	-0.38	0.43*	1.21*
Confidence Interval	(-0.35, 0.90)	(-0.45, 0.23)	(-2.20, -0.81)	(-0.25, 0.41)	(-0.95, 0.19)	(0.08, 0.79)	(0.53, 1.88)
50% (percentile)	0.33*	-0.14	-0.71*	-0.10	0.25	-0.11	0.47*
Confidence Interval	(0.02, 0.65)	(-0.28, 0.01)	(-0.93, -0.49)	(-0.24, 0.04)	(-0.08, 0.57)	(-0.31, 0.10)	(0.25, 0.70)
95% (percentile)	0.35	-0.17	-0.06	-0.25	0.77*	-0.52	-0.12
Confidence Interval	(-0.26, 0.95)	(-0.50, 0.16)	(-0.63, 0.51)	(-0.50, 0.00)	(0.08, 1.46)	(-0.83, -0.20)	(-0.65, 0.41)

Notes: This table shows the effective currency exposure based on the different percentile values of the risk factors (5%, 50% and 95% percentile values). The DOL and HML risk factors are sorted in ascending order, from lowest to highest returns (very low negative values can be indicative of "stress" in the FX market). VOL is sorted in descending order, from highest to lowest (logic is precisely in opposite direction, with high values indicative of "stress"). Shown are point estimates, as well as 90% confidence intervals, relying on all estimated coefficient values and Newey-West Standard errors, in the models of Table 4.

Table A.2 Effective Currency Exposure depending on risk states of DOL, HML and VOL